

THE UNIVERSITY OF MICHIGAN  
COLLEGE OF LITERATURE, SCIENCE, AND THE ARTS  
Department of Mathematics

Final Report

CONTINUOUS TRANSFORMATIONS AND INTEGRAL MANIFOLDS

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## Final Report

## I. RESEARCH PAPERS

During the entire period of research Sept. 1, 1965-Oct. 31, 1968 the following 17 Progress Reports have been issued and distributed. A star denotes that the paper has appeared.

- \*1. Garth W. Warner, Quasi additive set functions and nonlinear integration over a variety, May 1966. A Ph.D. thesis at The University of Michigan. Only the introduction was issued as a report and distributed. The main part of the thesis has appeared as: (a) The Burkil-Cesari integral, Duke Mathematical Journal, 35, 1968, 61-78; (b) The generalized Weierstrass-type integral, Annali Scuola Normale Sup. Pisa (2) 22, 1968, 164-192.
- \*2. L. Cesari, Existence theorems for multidimensional problems of optimal control, March 1966. An invited lecture delivered at the "International Symposium on Differential Equations and Dynamical Systems," Mayaguez Puerto Rico, December 1965. Differential Equations and Dynamic Systems, Academic Press 1967, 115-132.
- \*3. L. Cesari, Smoothness properties of periodic solutions in the large of nonlinear hyperbolic differential systems, May 1966. Funkcialaj Ekvacioj, 9, 1966, 325-338.
4. L. Cesari, A remark on uniform stability, May 1966. The main part of this report will appear in the third edition of Cesari's book "Asymptotic Behavior and Stability Problems in Ordinary Differential Equations, Springer Verlag, 1st ed. 1959, 2nd ed. 1963.
- \*5. L. Cesari, Existence theorems for Lagrange and Pontryagin problems of the calculus of variations and optimal control. More dimensional extensions in Sobolev spaces, June 1966. Lectures delivered at the International Summer Center, Bressanone, Italy, June 1966. C.I.M.E. Edizioni Cremonese, 87-175.
- \*6. L. Cesari, Existence theorems for multidimensional Lagrange problems, January 1967. Journal of Optimization Theory and Applications, 1, 1967, 87-112.
- \*7. L. Cesari, Multidimensional Lagrange and Pontryagin problems. An invited lecture delivered at the Conference on the Mathematical Theory of Control, Los Angeles, January 1967. Mathematical Theory of Control, Academic Press 1967, 272-284.
- \*8. L. Cesari, Sobolev spaces and multidimensional Lagrange problems of optimization. Annali Scuola Normale Sup. Pisa, 22, 1968, 193-227.

- \*9. L. Cesari, Multidimensional Lagrange problems of optimization in a fixed domain and an application to a problem of magnetohydrodynamics. Archive Rat. Mech. Anal. 29, 1968, 81-104.
- 10. L. Cesari, Existence theorems for optimal controls of the Mayer type, August 1967. In process of publication in SIAM Journ. Control.
- 11. L. Cesari, Remarks on the implicit function theorem for orientor fields, January 1968.
- \*12. L. Cesari, Results and problems in the theory of distributed parameter systems, March 1968. An invited lecture. Joint Automatic Control Conference, Ann Arbor, June 1968, 1121-1122.
- 13. L. Cesari, Existence theorems for Lagrange problems in Sobolev spaces, April 1968. An invited lecture at the Symposium on Nonlinear Functional Analysis, April 16-18, 1968, Chicago. In process of publication.
- 14. L. Cesari, Optimization with partial differential equations in canonic form and conjugate problems, July 1968. In process of publication in Archive Rat. Mech. Anal.
- 15. L. Cesari, A nonlinear problem in potential theory, August 1968. In process of publication in Michigan Math. Journ.
- 16. L. Cesari, Functional analysis and differential equations, September 1968. An invited lecture at the Conference on Qualitative Theory of Nonlinear Differential and Integral Equations, Madison, Wisconsin, August 1968. In process of publication.
- 17. L. Cesari, J. La Palm, and T. Nishiura, Remarks on some existence theorems for optimal control. In process of publication in Journal of Optimization Theory and Applications.

## II: SUMMARY OF THE RESEARCH ACCOMPLISHED IN THE FRAME OF THE PROJECT

In previous papers Cesari (Trans. Am. Math. Soc. 124, 1966, 369-412, 413-429) had proved a number of existence theorems for lumped parameter control problems of optimization with bounded or unbounded control space. In particular, Cesari had extended to such problems the Nagumo-Tonelli existence theorem for free problems.

Papers 5, 10, and 17 continue the same line of work for lumped parameter systems. In particular, in 10 the author shows that the consistent use of suitable variants, proposed by Cesari, of Kuratowski's upper semicontinuity condition for variable sets leads to a number of new existence theorems, and moreover to a reinterpretation of parallel results of E. J. McShane and T. Nishiura. The same variants of Kuratowski's upper semicontinuity condition have been later studied by Pavol Brunovski (SIAM J. Control, 6, 1968, 174-185) and by A. LaSota and C. Olech (Bull. Acad. Polon. Sci. 14, 1966, 615-621). In 17 the authors prove further existence theorem for optimal solutions by a refinement of the same conditions.

Papers 2, 3, 6, 7, 8, 9, 12, 13 concern existence theorems for distributed parameter control problems of optimization, where the solutions are manifolds of suitable dimension. In 2 the author studies distributed parameter systems with compact control space, and proves a Filippov-type existence theorem and other existence theorems by a new argument and consistent use of Sobolev spaces technique. In 6 the author proves existence theorems in situations where the control space is unbounded but  $L_p$ -compactness arguments with  $p > 1$  can be used. In 8 the author proves existence theorems, in particular a Nagumo-Tonelli-type theorem, in situations where the control space is unbounded and only  $L_1$ -integrability can be assured. This is an extremely difficult case. In 8 the author proves also existence theorems with both unbounded domain and unbounded control space. In 9 the author first reformulates the results above for distributed parameter systems with partial differential equations written in the Dieudonné-Rashevski form. This a rather general form, which has been proposed, independently and in completely different settings, by J. Dieudonné (e.g. Foundations of Modern Analysis, Academic Press 1960) and by P. K. Rashevski (Geometric Theory of Partial Differential Equations, Moscow 1948). The author then applies the entire technique in 9 to a problem of magnetohydrodynamics (cold plasma) proposed by K. A. Lurie (1963-64) with both unbounded domain and unbounded control space, and the equations written in the Dieudonné-Rashevski form.

In paper 14 the author gives a theoretical form of the maximum principle for distributed parameter systems with partial differential equations written in the Dieudonné-Rashevski form. The results seem to have a generality far above that obtained so far in this difficult field. The students B. A.

Suryanarayana and David Cowles are applying these ideas in a number of particular situations in their thesis.

Garth Warner's report No. 1 and J. Breckenridge's thesis concern surface area and continuous parametric manifolds. In 1 the author develops a very general approach to integration. A number of previous integration theories are included in his study, which also applies to theoretical questions of calculus of variations and control. In Breckenridge's thesis the same technique is developed with applications to measure theoretical analysis in surface area and theory of currents.

Reports 3, 15, and 16 concern a method based on functional analysis for existence and error bounds of solutions of boundary value problems in ordinary and partial differential equations. The method was initiated by Cesari in 1939. Report 16 is a survey of papers by different authors in the last ten years within the frame of Cesari's method. In 15 the author gives an existence theorem for solutions of the problem  $\Delta u = f(x,y,u)$  in a circle of the  $xy$ -plane and  $u = 0$  on the boundary. Preliminary work on Bessel functions which was needed in this research is contained in C. D. Stocking's thesis. In addition C. D. Stocking gives error bounds for the first Galerkin approximation to the problem above. The error, within certain limitations, is remarkably small.

### III. Ph. D. THESES PREPARED IN THE FRAME OF THE RESEARCH PROJECT

Garth Warner was supported by the present grant in summer 1966 while preparing his thesis for publication.

James La Palm completed November 1967 his Ph.D. thesis: Existence theorems for problems of optimal control and the calculus of variations with exceptional sets.

Abraham Naparstek completed April 1968 his Ph. D. thesis: Periodic solutions of certain weakly nonlinear hyperbolic partial differential equations.

John Breckenridge is nearing completion of his Ph. D. thesis on measure theoretical problems of Lebesgue area theory. We expect to discuss it during this academic year.

B. A. Suryanarayana is nearing completion of his Ph. D. thesis on optimal control problems with hyperbolic partial differential equations. We expect to discuss it during this academic year.

Charles D. Stocking is nearing completion of his Ph. D. thesis on nonlinear boundary value problems in a circle and related questions on Bessel functions. We expect to discuss it during this academic year.

Three other students Tom Angell, Richard Baum, and David Cowles are preparing their Ph.D. theses on theoretical aspects of optimal control theory and the calculus of variations.

#### IV. LECTURES

Since August 1965 Cesari was invited to give lectures at the following institutions and conferences:

International Symposium on Differential Equations and Dynamical Systems, Mayaguez, Puerto Rico, Dec. 27-30, 1965. Sponsored by US-AFOSR and Brown University, Providence, R. I.

International Summer Center, Bressanone, Italy. An eight lecture summer course, June 10-18, 1966.

Conference on the Mathematical Theory of Control, Los Angeles, California, Jan. 30-Feb. 1, 1967. Sponsored by US-AFOSR and the University of Southern California, Los Angeles.

California Institute of Technology, Pasadena, California, Feb. 2, 1967.

University of California, Berkeley, Feb. 3; San Diego (La Jolla, Feb. 7; Los Angeles, Feb. 8, 1967.

Institute Henri Poincaré, Sorbonne, Paris, March 15-April 30, 1967. Lectures in the Lions-Schwartz seminar.

Technische Universitaet, Berlin (West), May 5, 1967.

Euratom, Ispra, Computing Center, May 8, 9, 1967.

Universities of Pavia, May 10, and Firenze, May 15, 1967.

Academy of Science, Department of Mathematics, Prague, May 21-26, 1967.

Tulane University, New Orleans, March 29, 1968.

Symposium on Nonlinear Functional Analysis, Chicago, April 16-18, 1968.

Northwestern University, Evanston, May 1, 1968.

Joint Automatic Control Conference, Ann Arbor, June 26-28, 1968.

Conference on Qualitative Theory of Nonlinear Differential and Integral Equations, Madison, Wisconsin, August 19-23, 1968.



V. PROGRAM OF LECTURES AT THE UNIVERSITY OF MICHIGAN

In the frame of the research work of the present project the following scientists have been invited to speak at The University of Michigan:

Hubert Halkin, University of California, San Diego (La Jolla).

A. Avez, University of Paris, Paris.

Minoru Urabe, Kyusyu University, Fukuoka, Japan.

Emilio O. Roxin, University of Buenos Aires (now at the Rhode Island State University, Kingston, R. I.).

Stephen A. Williams, California Institute of Technology (now at UCLA).

Hans Lewy, University of California, Berkeley.

Felix Browder, University of Chicago.

Czeslaw Olech, University of Krakow and Brown University.

Pavol Brunovski, University of Bratislava and University of Minnesota.

Peter D. Lax, New York University.

James Serrin, University of Minnesota.

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