CONTROL OF A LEADSCREW DRIVEN FLEXIBLE ROBOT ARM

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ABSTRACT

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High performance requirements in robotics have led to the consideration of structural flexibility in robot arms. Here the dynamics of a spherical coordinate robot arm, whose last link is very flexible, is studied for the purpose of control. The assumed modes method is employed to represent the deflection of any point on the flexible link, and the Lagrangian approach is used to obtain the unconstrained equations of motion. The robot arm, considered in this work, has two joints driven by a lead screw transmission mechanisms. The kinematic constraints associated with the nonbackdrivable lead screws are also considered.

An integral plus state feedback controller is derived based on a linearized version of the rigid body model of the robot arm. The controller is then implemented on the rigid and flexible model. The rationale is to simulate the controllers currently used in industrial robots and to assess the interrelationships
between the robot arm structural flexibility and the controller design. The simulation results illustrate the differences between leadscrew driven and unconstrained axes of the robot; they indicate the trade-off between speed and accuracy; and show potential instability mechanisms due to the interaction between the controller and the robot structural flexibility.

The integral plus state feedback controller, derived for the rigid body model of the robot arm, is extended to include the flexible motion. The objective is to introduce additional damping into the flexible motion. This is done by using additional sensors to measure the compliant link vibrations and feed them back to the controller. The effect of control and observation spillover is examined, and found not to present a serious practical problem. The simulation results show that additional damping in the flexible motion can be achieved by including the flexible motion in the control action. Experimental evaluation showed excellent agreement with the results of the digital simulations.
To my parents
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CHAPTER 1

INTRODUCTION

This chapter provides an overview of the research in robot arm modeling, dynamics, and control. The motivation for high precision and high speed robot operation is discussed and some of the problems are identified. The current state-of-the art in robot arm dynamic modeling and control is reviewed, including studies of robot arm flexibility. Finally, the objectives of this study are set forth, and the organization of the dissertation is summarized.

1.1. Motivation

Until the 1980’s, the use of robots was primarily limited to industrial and research environments which are hazardous or unpleasant for human workers. In recent years, the evolution of the mechanical arm from teleoperator and crane to present day industrial and space robots has increased their implementation in almost every industrial area. However, the poor endpoint positional accuracy of current robots have restricted their applications to tasks that are error tolerant. Many tasks, such as assembly of high precision parts require positional tolerances on the order of 0.0254mm (0.001”). Existing manipulators can perform tasks
with a repeatable position on the order of 0.254mm (0.01”).

The operation of high-precision robots is severely limited by the dynamic deflection which persists for a period of time after a move is completed. The settling time required for this residual vibration delays subsequent operations, thus conflicting with the demand for increased productivity. These conflicting requirements have rendered the robotic assembly task to be a challenging research problem.

The performance problems in robotics are related to the transmission mechanisms (i.e. backlash, friction and compliance), the structural deformation (both static and dynamic), and the massive design of the robot arm.

1.2. Previous Work in the Dynamic Modeling of Robots

The automation of assembly tasks will be greatly enhanced if robots can operate at higher speeds with greater positioning accuracy. These goals cannot be achieved with the existing massive robot designs, which make them slow and heavy. Many robot arms are made to be massive for increased rigidity. To improve their structural performance, payloads and operating speeds are kept fairly low. In other words, the rigid body assumption is justified by conceeding low performance.

For higher operating speeds, mechanisms should be made light-weight to reduce the driving torque requirements and enable the robot to respond faster. However, lighter members are likely to elastically deform, thus making it a necessity to take into consideration the dynamic effects of the distributed link
flexibility. This is because high speed operation leads to high inertial forces which in turn cause vibration and deteriorate accuracy. This difficulty may be eased by fabricating the moving members of manipulators in fiber reinforced composite materials which can result in high structural stiffness and strength with low mass [1]; however the basic problem remains.

In an attempt to improve the overall performance of manipulators, some researchers have treated the aforementioned problems separately. A direct drive arm is used by Asada [2] to solve the backlash and friction problems. The compliance problem in the drive mechanism is discussed by Kuntze and Jacobus [3]. Zalucky and Hardt [4] used a micromanipulator to correct for the link static deflection.

The first step in improving the performance of robots is to obtain a reasonably accurate dynamic model. The general practice has been based on rigid body analysis where all the robot links are assumed to be rigid [5-9]. This is understandable since even the rigid body model is complex, nonstationary and highly nonlinear. However, when the robot is operated at high speed, the vibration induced by the large inertial and external forces can no longer be neglected and the rigid body assumption becomes invalid.

Until recently, the inclusion of the link's flexibility in the dynamic model of the robot arm is done by solving for the rigid body motion and the flexible motion separately [10]. Generally the flexible motion is small, and its effect on the rigid body motion is neglected. Therefore, only the effect of the rigid body motion on the flexible motion is considered. This is done by solving the rigid
body equations for inertial forces which in turn are introduced as excitation sources to the elastic problem. This approach is adequate for modeling fairly rigid structures. However, an accurate dynamic model for a very flexible structure would require all the coupling terms between the flexible and the rigid body motions to be retained. This is done by using coupled reference position and elastic deformation models. Some researchers have used finite element techniques to describe the elastic deformation [11-16]. Others have used global methods such as the assumed modes method, Rayleigh-Ritz method, etc. [17-18]. These approximate techniques can be used to yield a set of equations which represent the combined rigid and flexible motion.

1.3. Overview of Robot Controller Design

Robots are inherently very complex structures. Their rigid body dynamic models exhibit a nonlinear and nonstationary behavior. In today's robots, the desired end-effector position is converted by real-time kinematic computation into the equivalent angles that each joint must assume [19]. The joints are then simultaneously driven to their assigned angles by separate conventional servo loop controllers. However, the implementation of conventional linear control techniques have led to poor performance because of both the inherent geometric nonlinearities of these systems, and the dependence of the system dynamics on the characteristic of the manipulated objects. Therefore, a sophisticated controller design is needed to ensure the robot desired performance.
In the control of rigid robots, adaptive, nonlinear feedback, and optimal control techniques have been investigated. Adaptive control theory [20-25] has been proposed as a promising solution to the nonlinearity and nonstationarity problems. The main task of the adaptive controller is to adjust the feedback gains of the manipulator so that its closed loop performance characteristics closely match the desired ones. However, the large required computation time has restricted their applications to simulation studies.

Nonlinear feedback control, or the "computed torque" method [26], has led to better performance over conventional control techniques in computer simulations. The controller is based on an idealized model of the manipulator. This is a severe drawback because when the "idealized" control law is used with an actual plant, the validity of the scheme is subject to doubt and the robustness of the controller becomes of great importance.

A time-optimal control strategy is studied in [27] to enable faster movement of the robot arm. Kahn and Roth [28] showed that this technique results in a two-point boundary value problem. As a consequence, the computation must be repeated for each new set of initial and final conditions used in the numerical solution. In addition, the numerical algorithm yields an optimal solution that is a function of time and does not account for any unexpected disturbances which may act on the system. These drawbacks have rendered the implementation of the time-optimal control technique difficult, if not impossible, for real time application in feedback control of robots. However, this technique has turned out to be a powerful tool for off-line trajectory planning.
In an attempt to relax these difficulties, a suboptimal control strategy is often used in digital simulations. In this technique, linearized version of the dynamic model of the manipulator is obtained for which an analytical optimal control solution can be found. Many of these sophisticated control techniques suffer from excessive computation time requirements which make them unattractive for real time applications using current microprocessor technology [29].

Besides the difficult issues encountered in rigid robots, a new problem is created when robot compliance is included in the dynamic model. It emanates from the distributed nature of the link mass and elasticity. An infinite number of degrees of freedom are required to specify the position of every point on the elastic link. However, due to physical limitations, only a finite number of sensors and actuators can be mounted on the flexible body. Thus resulting in the problem of controlling a large dimensional system with a smaller dimensional controller [30-31].

Meckl and Seering [32-33] proposed two types of forcing functions to achieve fast response with negligible residual vibrations. The first type allows vibration to occur during the move but would stop the motion in such a way as to eliminate any residual vibration. This consists of a bang-bang action with multiple switching points. The second type avoids the excitation of the resonant modes of the structure as it moves. This is done by using a forcing function with no frequency components matching the system natural frequencies. This work is still confined to research laboratories; it can be used off-line to generate the trajectory of the robot arm that satisfies a criterion considering both the response time and
the residual vibration.

By and large, the research in the closed loop control of flexible manipulators can be divided into two categories. The first uses additional sensors to measure the flexible motion (i.e. assumes all state variables to be available) \([34-37]\). This enables the inclusion of the flexible motion in the control action, thus achieving better positional accuracy with the existing joint motors. The second category employs a micromanipulator along with additional sensors to compensate for static and dynamic structural deflections \([4], [38-39]\). This concept gives the control system designer more capabilities to improve the robot arm performance at additional hardware cost.

1.4. Dissertation Overview

The purpose of this work is to improve the end-effector positional accuracy by measuring and feeding back its transverse vibration. The spherical coordinate robot arm considered in this study is schematically illustrated in Fig. 1-1.

Two types of controllers are implemented. The first is a conventional linear controller based on the rigid body model. The rationale is to simulate the controllers currently used in industrial robots and to assess the interrelationships between the robot arm flexibility and the controller design. The second control system uses the measured flexible motion to introduce additional damping in the flexible structure.
Figure 1-1. A schematic of the physical system.
In the next chapter, the dynamic modeling of the robot arm including the transverse elastic displacements of the last link is presented in detail. The derivation of the constraint equations imposed by the leadscrews are derived in Chapter 3.

Chapter 4 includes the design of the rigid body controller and the results of the investigation of the interrelationships between the structural flexibility and the controller design.

In Chapter 5, the rigid and flexible motion controller is designed. The separate and combined effects of control and observation spillover are shown.

Chapter 6 presents the experimental apparatus along with the corresponding results. A comparison and discussion of the theoretical and experimental results is also made.

Finally, Chapter 7 summarizes the work done, presents the main conclusions, and states some prospective research topics.
CHAPTER 2

DYNAMIC MODELING OF THE ROBOT ARM

Typically, the dynamic modeling of a flexible robot arm considers the flexible motion to be small and neglects its effect on the rigid body motion. This technique gives an accurate model for fairly rigid structures. However, in the case of very flexible manipulators, all the coupling terms between the rigid and the flexible motions need to be retained. This is done by using coupled reference position and elastic deformation models.

In this chapter, the dynamics of a spherical coordinate robot arm, whose last link is flexible, is studied for the purpose of control. The assumed modes method is employed to represent the deflection of any point on the flexible link. Lagrange's method represents a very systematic approach to obtain the equations of motion of such a complicated system in the simplest manner possible.

2.1. Modeling

The physical system considered in this work is a spherical coordinate robot, which has two revolute joints and one prismatic joint (see Fig. 1-1). It consists of an arm connected to a gear driven rotating base. The arm is made of two beams
such that the second beam can move axially into the first. The entire arm is free
to rotate around the horizontal axis passing though the pivot point 0 and parallel
to the unit vector $\sim K$ (see Fig. 2-1). The $r$ and $\theta$ coordinates have lead screw
transmission mechanisms, whereas power is transmitted to the $\phi$ coordinate
through a gear train. All axes are driven by dc servo motors.

The second beam consists of a thin, lightweight and very flexible aluminum
rod. The rationale is to exaggerate the flexibility problem which leads to a better
understanding of the interaction between the structural flexibility and the con-
troller design. This will also narrow the gap between the natural frequencies of
the elastic modes and the servo loop frequencies, thus reducing the numerical
computation problem associated with stiff systems and the problem of sampling
rapidly enough to characterize the high frequencies of the flexible arm.

The first and second beams of the robot arm have length $L_1$ and $L_2$ respectively as shown in Fig. 2-1. A reference inertial frame $(I, J, K)$ fixed at point 0, is
shown along with a non-inertial, body fixed, rotating reference frame $(i, j, k)$. The equations are derived in terms of the latter to keep the mass moment of
inertia constant throughout the rigid body motion of the robot.

The robot arm can rotate with angular velocity $\dot{\phi}$ around the vertical axis
passing through $J$ and $\dot{\theta}$ around the horizontal axis along $k$. The second beam
has one additional degree of freedom $r$ which allows it to move axially into the
first beam. The payload at the end-of-arm is modeled by a concentrated mass
$m_r$ at the end of the second beam.
Figure 2-1. Arm geometry and coordinates.
The general modeling problem for articulated flexible manipulators has been studied by others [11-16]. The constraint equations for a slot joint which allows for a combined translational and rotational motions is discussed in [40]. In this study, the prismatic joint constraint is handled by considering the portion of the second beam protruding from the first beam to have a flexible cantilever beam like behavior, whereas the first beam and the part of the second beam located inside the first beam are considered to undergo rigid body motion only. Referring to Fig. 1-1, this modeling assumption is justified by the structural properties of the first beam which is stiff in flexure. The protruding portion of the second beam is relatively flexible. It is more rigid in compression than in flexure. Therefore, the longitudinal vibrations of the flexible portion of the second beam are neglected and only transverse vibrations are considered in addition to the rigid body motion. Since the elastic deformation is small, the variations of the mass moment of inertia due to the flexible motion are assumed to be negligible.

2.2. Equations of Motion

Lagrange’s equation [41-42] is implemented to obtain the dynamic model of this complicated system. This method considers the system as a whole rather than its individual components, thus eliminating the need to calculate interacting forces. It formulates the problem in terms of two scalar functions, namely, work and kinetic energy.

The position vector of the mass center for the rigid portion of the robot arm becomes important in the derivation of its kinetic energy expression. For the
first beam, one has,

\[ R'_{1} = \left( \frac{L_{1}}{2} - a \right) \hat{i} \tag{2-1} \]

where the star superscript indicates a value related to the center of mass. \( L_{1} \) is the total length of the first beam. \( a \) is the length of the portion of the first beam lying to the left of the pivot point 0 (see Fig. 2-1). The position vector of the mass center of the part of the second beam located inside the first beam is

\[ R'_{2} = z'' \hat{i} \tag{2-2} \]

where \( z'' = L_{1} + \frac{r}{2} - \frac{L_{2}}{2} \), \( L_{1} \) is the length of the portion of the first beam lying to the right of the pivot point 0, \( L_{2} \) is the total length of the second beam, and \( r \) is the length of the part of the second beam protruding from the first beam. The position vector of any point on the protruding part of the second beam is

\[ R_{g} = y' \hat{i} + V \hat{j} + W \hat{k} \tag{2-3} \]

where \( y' = L_{1} + y \) and \( y' = i = j \). The assumed modes method [41] is used to obtain the expressions for the transverse vibrations \( V \) and \( W \). They are written as a linear combination of admissible functions, \( \Phi_{i}(y) \), of spatial coordinates, multiplied by time dependent generalized coordinates \( \varphi_{i}(t) \).

\[ V = \sum_{i=1}^{n} \Phi_{i}(y) \varphi_{1i}(t) \tag{2-4} \]

\[ W = \sum_{i=1}^{n} \Phi_{i}(y) \varphi_{2i}(t) \]

An approximation to the dynamics of infinite dimensional systems is often based on the fact that low frequencies alone are adequate to describe the flexible
behavior of the system. Therefore, \( n \) is selected to be 2, and the admissible functions are chosen to be the first two eigenfunctions of a clamped-free beam. Referring to Fig. 1-1, these boundary conditions are chosen because the second beam slides inside the first beam and is supported by a teflon sleeve which only permits axial motion. The clamped-free beam eigenfunctions \( \Phi_1 \) and \( \Phi_2 \) have the following general form which is described in [43], [44].

\[
\Phi_i = \cos \left( \frac{\epsilon_i y}{r} \right) - \cos \left( \frac{\epsilon_i y}{r} \right) - \alpha_i \left[ \sin \left( \frac{\epsilon_i y}{r} \right) - \sin \left( \frac{\epsilon_i y}{r} \right) \right]
\]  

(2-5)

where the values of \( \epsilon_i \) and \( \alpha_i \) for each mode are given in [43]. Note that an additional approximation has been introduced in that the solution is not separable in space and time when \( r(t) \) is not constant. However, this forms a useful approximation as long as \( r(t) \) satisfies the Euler-Bernoulli beam assumptions. That is \( r \) must always be much greater than the beam cross sectional dimensions, the deflection terms \( V \) and \( W \) don't exceed one tenth of \( r \), and rotary inertia and shear deformation effects are ignored. Substituting (2-4) and (2-5) into (2-1) to (2-3) gives the complete form for the position vectors. These results are then used to obtain the velocity terms from:

\[
\dot{\mathbf{r}_i} = \frac{d\mathbf{r}_i}{dt} + \mathbf{\Omega} \times \mathbf{r}_i
\]

(2-6)

where \( \mathbf{\Omega} \) is the rotation vector of the \( (\mathbf{i}, \mathbf{j}, \mathbf{k}) \) basis and \( \mathbf{r}_i \) is the position vector of any particle along link \( i \).
The velocity terms are then used to develop the kinetic energy expressions for each component of the robot-arm including the payload, \( m_p \). The latter is considered to be a point mass, and does not include rotation around its own mass center. The total kinetic energy will be

\[
T_i = \sum_{i=1}^{\text{n}} T_i + T_p = \sum_{i=1}^{\text{n}} \frac{1}{2} \int (\dot{R}_i \cdot \dot{R}_i) dm_i + \frac{1}{2} m_p (\dot{R}_p \cdot \dot{R}_p) \tag{2-7}
\]

where \( n \) is the number of links in the robot arm, \( T_i \), is the kinetic energy of link \( i \) and \( T_p \) is the kinetic energy of the payload.

The total potential energy consists of the strain energy of the second beam [45] and the potential energy associated with the rigid body motion. The strain energy includes terms due to flexural rigidity.

\[
\frac{1}{2} \int_0^y EI(y) \left[ \frac{\partial^2 V(y,t)}{\partial y^2} \right]^2 dy + \frac{1}{2} \int_0^y EI(y) \left[ \frac{\partial^2 W(y,t)}{\partial y^2} \right]^2 dy \tag{2-8}
\]

and terms due to the axial force:

\[
\frac{1}{2} \int_0^y T(y,t) \left[ \frac{\partial V(y,t)}{\partial y} \right]^2 dy + \frac{1}{2} \int_0^y T(y,t) \left[ \frac{\partial W(y,t)}{\partial y} \right]^2 dy \tag{2-9}
\]

where \( T(y,t) \) is the axial force in the second beam which arises due to gravity and inertial forces. The latter consist of centripetal acceleration and flexible motion effects. However, the variation of the axial force due to the flexible motion is neglected and one obtains,
\[
T(y,t) = \rho A_d (\ddot{\phi} \cos^2 \theta + \dot{\phi}^2)(L_1 r + \frac{r^2}{2}) \left[ 1 - \frac{L_1 y + \frac{y^2}{2}}{L_1 r + \frac{r^2}{2}} \right] \]
\[
+ m_p (L_1 + r) \left[ \ddot{\phi} \cos^2 \theta + \dot{\phi}^2 \right] + \left[ m_p + \rho A_d (r - y) \right] g \sin \theta
\]

A compressive \( T(y,t) \) would tend to decrease the transverse stiffness of the beam while a tensile \( T(y,t) \) would have the opposite effect. The total potential energy will be:

\[
V_t = \left[ m_1 \left( \frac{L_1}{2} - d \right) + m_{d_1} + m_{H1} (L_1 + L_2) + m_2 \left( L_1 + r - \frac{L_2}{2} \right) \right]
\]
\[
+ m_p (L_1 + r) g \sin \theta + \rho A_2 g \cos \theta \int_0^r V(y) dy + m_p V |_{L_1 + r} g \cos \theta
\]
\[
+ \frac{1}{2} \int_0^r E I(y) \left[ \frac{\partial^2 V(y,t)}{\partial y^2} \right]^2 dy + \frac{1}{2} \int_0^r E I(y) \left[ \frac{\partial^2 W(y,t)}{\partial y^2} \right]^2 dy
\]
\[
+ \frac{1}{2} \int_0^r T(y,t) \left[ \frac{\partial V(y,t)}{\partial y} \right]^2 dy + \frac{1}{2} \int_0^r T(y,t) \left[ \frac{\partial W(y,t)}{\partial y} \right]^2 dy
\]

where \( m_1 \) and \( d_1 \) are respectively the mass and the distance from the \((i,j,k)\) origin to the mass center of the leadscrew that drives the second beam and the \( m_{H1} \) is its housing's mass.

The coefficients in the kinetic and potential energy expressions, given in equations (2-7) and (2-11), involve the integration of complicated functions with respect to the spatial coordinate \( y \). Some of the terms encountered in the kinetic
energy expression have the following form,

\[
\begin{align*}
\int_0^r \Phi_i(y) dy & \quad \int_0^r y \Phi_i(y) dy \\
\int_0^r \Phi_i(y) \cdot \Phi_j(y) dy & \quad \int_0^r \Phi_i(y) \Phi_j(y) dy \\
\int_0^r \Phi_i(y) dy & \quad \int_0^r y \Phi_i(y) dy
\end{align*}
\]

where \(i = 1,2\) and \(j = 1,2\). The integration terms appearing in the potential energy expression can be written in the following general form,

\[
\int_0^r \left( \frac{d^2 \Phi_i(y)}{dy^2} \right) \cdot \left( \frac{d^2 \Phi_j(y)}{dy^2} \right) dy \quad \text{for} \quad \{i = 1,2\} \quad \{j = 1,2\}
\]

where \(\Phi_i(y)\) is the eigenfunction previously defined in equation (2-5). These complex functions are integrated numerically using Gaussian quadrature of high order (40 points) [46].

The virtual work principle is implemented to obtain an expression for the generalized forces. If the forces exerted on the robot arm are divided into conservative forces, which are directly related to the potential energy, \(V_i\), and nonconservative forces which are not, then the virtual work expression could be written as, [41]

\[
\delta W = \delta W_c + \delta W_{nc}
\]

where the subscripts \(c\) and \(nc\) denote terms due to the conservative and nonconservative forces respectively. \(\delta W_c\) is related to the potential energy \(V_i\) as follows:
\[ \delta W_c = \delta V_l = - \sum_{i=1}^{n} \frac{\partial V_l}{\partial x_i} \delta x_i \]  

(2-13)

where \( x_i \) is the \( i^{th} \) generalized coordinate that can be either one of the rigid body degrees of freedom \( r, \theta \) and \( \phi \) or one of the flexible motion coordinates \( q_{11}, q_{12}, q_{21} \) and \( q_{22} \). The free body diagram of the robot arm, which shows the nonconservative forces, is depicted in Fig. 2-2. The expression for \( \delta W_{nc} \) can be easily obtained.

\[
\delta W_{nc} = \sum_{i=1}^{n} Q_{nc} \delta x_i = R \cdot \delta R_i \bigg|_{r=0} + F_c \cdot \delta P_2 \bigg|_{L_1} + T_3 \cdot \delta \phi
\]

\[ = F_c \delta r - F_{ax} \delta \theta + T_3 \delta \phi \]  

(2-14)

where \( Q_{nc} \) is the \( i^{th} \) nonconservative generalized force. \( \delta \chi \) is the virtual rotation vector of \( \Omega \). \( R \) is the reaction force at the pivot point. \( F_c, F_{ax} \) and \( T_3 \) are the

---

**Figure 2-2.** Free body diagram of the robot arm.
joint driving force and torques. Now that the kinetic energy, potential energy and the virtual work expressions are derived, the equations of motion can be readily obtained by implementing the following form of Lagrange's equation

\[
\frac{d}{dt} \left( \frac{\partial T_i}{\partial \dot{x}_i} \right) - \frac{\partial T_i}{\partial x_i} + \frac{\partial V_i}{\partial x_i} = Q_{nc} \tag{2-15}
\]

The resulting equations of motion are seven highly nonlinear, coupled, second order ordinary differential equations. The reader is referred to Appendix A for the listing of the unconstrained equations.

2.3. Summary

A brief description of the physical system is given along with the definition of the coordinate axes and the generalized coordinates. The assumed modes method is implemented to model the flexible motion of the last link. Expressions for the virtual work, kinetic and potential energies are derived. Lagrange's equation is employed to obtain the unconstrained equations of motion.

Some of these equations can be greatly simplified once the role played by the leadscrews and the effect of the rigid body motion on the flexible motion and vice-versa, are understood. These leadscrew constraints are considered in the next chapter.
CHAPTER 3

LEADSCREW DYNAMIC MODELING AND CONSTRAINTS

The robot arm considered in this work has two joints driven by lead screw transmission mechanisms which are used to transmit power by converting angular motion to linear motion. In this chapter, the kinematic constraints associated with a lead screw are introduced to investigate the behavior of a lead screw driven flexible robot arm in the presence of coulomb friction and the self locking condition (i.e., the nonbackdrivability of the lead screw).

3.1. Overview of the Static Model of a Lead Screw

A detailed derivation of the static model of a lead screw can be found in [47]. A brief summary of the modeling procedure is included to provide the required background for the next section.

A free body diagram of an unrolled single thread of a lead screw is shown in Fig. 3-1. \( F_a \) is the resultant of all the forces exerted on the lead screw in the axial direction. \( N \) is the normal force and \( P \) is the force required to raise or lower the axial load, \( F_a \), on the thread surface. The friction force, \( f_f \), is considered to be of the simplest form of dry coulomb friction,
Figure 3-1. Free body diagram of an unrolled thread.

\[ f_\parallel = -\mu \mid N \mid \text{sgn}(\dot{z}) \]  

(3-1)

where \( \mu \) is the coefficient of friction and \( \dot{z} \) represents the velocity along the rough surface of the lead screw thread. The system is in equilibrium under the action of these forces. Therefore, the minimum force, \( P \), required to raise or lower the axial load, \( F_x \), can be obtained from the summation of forces in the vertical and horizontal directions. The expression for \( P \) to raise the load has the following form,

\[
P = \frac{F_x \left[ \frac{l}{\pi d_m} + \mu_s \right]}{1 - \left( \frac{\mu_s l}{\pi d_m} \right)}
\]

(3-2a)

where \( \mu_s \) is the static coefficient of friction. To lower the load, \( P \) becomes
\[ P = \frac{F_{es} \left[ \mu_s - \left( \frac{l}{\pi d_m} \right) \right]}{1 + \left( \frac{\mu_s l}{\pi d_m} \right)} \]  

(3-2b)

where \( l \) is the lead of the lead screw and \( d_m \) is its mean diameter. The expression given in (3-2) represent the force required to overcome static friction and the geometric effect. This force, \( P \), can be directly related to the torque, \( T \), applied on the lead screw by,

\[ T = \frac{P d_m}{2} \]  

(3-3)

In the case where \( \mu_s < \left( \frac{l}{\pi d_m} \right) \) in (3-2b), the force required to lower the axial load becomes negative. This causes the screw to spin without external effort. However, most lead screws are nonbackdrivable in the absence of the applied force, \( P \). This results in satisfaction of the following inequality,

\[ \mu_s > \tan \psi_1 = \frac{l}{\pi d_m} \]  

(3-4)

where \( \psi_1 \) is the thread helix angle. The inequality in (3-4) is often referred to as the self-locking condition.

3.2. Generalization of the Static Model

The static model of the lead screw derived in [47] is useful for determining the torque required to overcome static friction and geometric effects for the load configuration given in Fig. 3-1. This model needs to be modified if different configurations are considered.
Figure 3-2. Transition of the leadscrew housing motion from lower to upper thread depending on the sign of the normal force, \( N \).

In this work, the leadscrew is assumed to satisfy the self-locking condition. The friction force has the same form as in (3-1) and the axial force, \( F_{ax} \), can be either tensile or compressive. As a consequence, the normal force, \( N \), can now be positive or negative. This sign change in \( N \) can be physically interpreted as having the leadscrew housing sliding on the upper thread's surface if it was originally on the lower surface or vice versa. This is illustrated in Fig. 3-2. The equations of motion for both the upper and lower thread surfaces can be combined in the following general form,

\[
-F_{ax} \sin \psi_{1} - \mu |N| \text{sign}(z) + P \cos \psi_{1} = m_{f} \ddot{z} \tag{3-5}
\]

where \( m_{f} = 0 \) is a fictitious mass, and \( z \) is the coordinate along the thread sur-
face. It can be directly related to the rigid body degree of freedom driven by the corresponding lead screw through simple geometric relations. The expression for the normal force, $N$, is obtained from the summation of forces in the vertical direction,

$$N = F_{ax} \cos \psi_1 + P \sin \psi_1$$  \hspace{1cm} (3-6)

In this work, the axial load, $F_{ax}$, consists of inertial and gravitational terms. It can be written as

$$F_{ax} = F_{ax}(x, \dot{x}, \ddot{x}, g)$$  \hspace{1cm} (3-7)

where $x^T = [r, \theta, \phi, q_{11}, q_{12}, q_{21}, q_{22}]$ is the generalized coordinate vector and $g$ is the gravitational acceleration. Substituting (3-6) and (3-7) into (3-5), the general form of the equation of motion becomes

$$\ddot{x}_i = f_i[x, \dot{x}, \ddot{x}, g, \text{abs}(x, \dot{x}, \ddot{x}, g)]$$  \hspace{1cm} (3-8)

where $\text{abs}(x, \dot{x}, \ddot{x}, g)$ function represents the absolute value of the normal force $N$ in (3-5). The term $\ddot{x}_i$ can be either $\ddot{r}$ or $\ddot{\theta}$ since only two of the rigid body degrees of freedom $r$ and $\theta$ are driven by lead screw transmission mechanisms. The $f_i$'s are very complex, highly nonlinear functions which do not have closed form solutions. Instead, they are solved numerically on a digital computer. However, most numerical algorithms require that the equations be written in either of the following forms,

$$\ddot{x} = f(x, z, t)$$  \hspace{1cm} (3-9)

or reduced to the first order form
\[ z = f(z, t) \]  

(3-10)

where now \( z^r = [r, \theta, \phi, q_{11}, q_{12}, q_{21}, q_{22}, \dot{r}, \dot{\theta}, \dot{\phi}, \dot{q}_{11}, \dot{q}_{12}, \dot{q}_{21}, \dot{q}_{22}] \). However, (3-8) cannot be written in either of these two forms due to the presence of the acceleration vector \( \ddot{z} \) on the right hand side. This difficulty would not exist had it not been for the absolute value term of the normal force, \( N \), in equation (3-5). To overcome this problem, separate equations are written to describe the motion on the upper and lower thread surfaces of the leadscrew. By writing the force balance equations from the appropriate free body diagram in Fig. 3-2, the lower thread equation of motion can be obtained,

\[
m_f \ddot{z} = -F_{as} \left[ \sin \psi_1 + \mu \cos \psi_1 \text{sgn}(\dot{z}) \right] + P \left[ \cos \psi_1 - \mu \sin \psi_1 \text{sgn}(\dot{z}) \right]
\]

(3-11)

Similar procedures are followed to derive the equation of motion on the upper thread of the leadscrew,

\[
m_f \ddot{z} = F_{as} \left[ \sin \psi_1 - \mu \cos \psi_1 \text{sgn}(\dot{z}) \right] + P \left[ \cos \psi_1 + \mu \sin \psi_1 \text{sgn}(\dot{z}) \right]
\]

(3-12)

This results in having two equations of motion for each degree of freedom driven by a leadscrew mechanism.

### 3.3. Coulomb Friction Effect

The effect of friction can be clearly identified at the beginning of the motion or when the response of the degree of freedom driven by a leadscrew is oscillatory. If the leadscrew is to rotate as soon as the input torque is applied, then the latter must be large enough to overcome the combined effects of static friction and geometry. Otherwise, the leadscrew would remain at a standstill, thus
causing a delay in the system response.

In the case of an oscillatory response, the time at which the maximum overshoot occurs corresponds to the position where the system comes to a complete halt (i.e. velocity is zero). This results in a sudden jump in the required driving torque caused by the sharp increase in the magnitude of the friction force due to the transition from a dynamic to a static coefficient of friction. This may reshape the entire response if the leadscrew is driven by a controller. The latter, being unaware of this sudden increase in the required driving torque, would supply an input torque that is not large enough to overcome the resistive torque of the static friction. As a result, the system would remain at rest waiting for the control effort to build up. Under these conditions, the system is said to fall in the "stopping region." This is illustrated in Fig. 3-3. For some special cases, analytical expressions for the upper and lower bound of that region are developed in [48].

3.4. Self-Locking Condition Effect

In this work, a self-locking condition is assumed; that is, once the leadscrew is subjected to an axial load, it will not rotate unless the applied input torque is large enough to overcome a certain resistive torque, $T_R$. In the case of compressive axial force, the latter would have the following form,

$$T_R = \frac{F_{ss} d_m}{2} \left( \mu_s + \tan \psi_1 \right) \left( 1 - \mu_s \tan \psi_1 \right)$$  (3-13a)

for raising the load, and
\[ T_R = \frac{F_\alpha d_N}{2} \frac{(\mu_s - \tan \psi_1)}{(1 + \mu_s \tan \psi_1)} \]  

for lowering the load. The self-locking condition plays an important role in a multi degree of freedom system where the axial force exerted on the leadscrew is partially due to coupling terms between the degree of freedom driven by the leadscrew and the rest of the system degrees of freedom. In the absence of an input torque, the leadscrew which is nonbackdrivable, will not rotate regardless of the magnitude of the axial force. This renders the degree of freedom driven by the leadscrew transmission mechanism completely decoupled from the rest of the system degrees of freedom. However, this nice property is lost with the application of an input torque. Since the latter is directly related to the axial force which in turn depends on the coupling terms.

To recapitulate, the self-locking assumption of the leadscrew transmission mechanism leads to the following,

1. The degree of freedom driven by the leadscrew is completely decoupled from the rest of the system degrees of freedom in the absence of the input torque.
2. Once the input torque is applied, the coupling terms between the system degrees of freedom are retained through the axial force.

3.5. Constrained Equations of Motion

The gear train that drives the \( \phi \) motion or the base joint is assumed to be ideal (i.e. no backlash or friction). Therefore, the physical system does not impose any constraint on \( \phi \). In this work, only the leadscrew constraints are con-
Figure 3-3. Effect of coulomb friction on an oscillatory response

considered. As a result, the equations of motion for $r$ and $\theta$, derived in Chapter 2 and listed in Appendix A, need to be modified. Newton's method is employed to compute the axial forces exerted on the leadscrews that drive $r$ and $\theta$. The latter, which consist of inertial and gravitational terms, have the general form given in equation (3-7). The constrained $r$ and $\theta$ equations of motion are obtained by substituting $F_n$ by its value in equations (3-11) and (3-12). The interested reader is
referred to Appendix B for the listing of the constrained equations.

In an attempt to check the consistency of these equations, a dynamic model based on the rigid body assumption of the robot arm is derived in appendix C. It shows that the rigid body model can be obtained as a special case from the general equations of motion presented in appendices A and B. This provides a partial confirmation of the validity of the modeling procedure.

3.6. Summary

A brief description of the static model of a leadscrew has been presented along with some modifications to allow for the handling of the dynamic case. Effects of coulomb friction and the self-locking condition on the behavior of a leadscrew driven rigid body degree of freedom are discussed. Finally, the constrained equations of motion for $r$ and $\theta$ are derived.

Now that the derivation of the dynamic model of the flexible robot arm is complete, the interrelationships between the structural flexibility and the controller design will be investigated in the next chapter.
CHAPTER 4

RIGID BODY CONTROLLER

This chapter presents the derivation of an integral plus state feedback controller based on a linearized version of the rigid body model of the robot arm. The controller is then implemented on the rigid and flexible model derived in Chapters 2 and 3. The rationale is to simulate the controllers currently used in industrial robots and to assess the interrelationships between the robot arm structural flexibility and the controller design.

4.1. Design Of The Integral Plus State Feedback Controller

In most existing industrial robots, the nonlinear terms in the equations of motion are not considered in the design of the control system. This design practice is valid as long as the robot arm is restricted to slow motion. However, when manipulators are operated at high speed, the effect of geometric nonlinearities and the dependence of system dynamics on the characteristic of the manipulated objects become significant, thus leading to a degradation of the overall robot arm performance.
A conventional linear controller is implemented in this work to simulate the controllers currently used in industrial robots. It is designed based on a linearized version of the rigid body equations of motion derived in Appendix C. Throughout this work, the payload is kept constant and the rigid body degrees of freedom are restricted to the vicinity of the equilibrium state. This is done to reduce the effects of the variations in the manipulated objects and the nonlinear terms on the linear controller. Thus leading to better assessment of the interrelationships between the robot arm structural flexibility and the controller design.

The first step in designing the controller is to express the rigid body equations of motion in terms of state variables which are defined as follows:

\[
\begin{align*}
y_1 &= r \\
y_2 &= \theta \\
y_3 &= \dot{r} \\
y_4 &= \dot{\theta} \\
y_5 &= \phi \\
y_6 &= \dot{\phi}
\end{align*}
\]  

(4-1)

The general form of the nonlinear state equations, based on the rigid body equations of motion given in Appendix C, can be written as,

\[
\dot{\tilde{y}} = f(\tilde{y}, \tilde{u})
\]

(4-2)

where \(\tilde{y}\) is the state vector and \(\tilde{u}\) is the control vector. The function \(f\) is continuously differentiable in the \(y_i\)'s. It is expanded in a Taylor series about the equilibrium state \(\tilde{y}^e = [r_e, \theta_e, \phi_e, 0, 0, 0]\) to give,

\[
\begin{align*}
f_i(\tilde{y}, \tilde{u}) &= f(\tilde{y}_e, \tilde{u}_e) + \sum_{j=1}^{n} \left( \frac{\partial f_i}{\partial y_j} \right)_{\tilde{y}_e} (y_j - y_{je}) + \\
&\quad \sum_{l=1}^{n} \left( \frac{\partial f_i}{\partial u_l} \right)_{\tilde{u}_e} (u_l - u_{le}) + \text{[ higher order terms]}
\end{align*}
\]

(4-3)

Define \(\delta u = u_i - u_{le}\) and \(\delta y = y_i - y_{je}\). Substitute (4-3) into (4-2) and retain the
first order terms only, to obtain,

\[
\delta y = A \delta y + B \delta u
\]  

(4-4)

Equation (4-4) represents a first order linear approximation of (4-2). The matrices \(A\) and \(B\) can be obtained from,

\[
A = \begin{bmatrix}
\frac{\partial f_1}{\partial y_1} & \cdots & \frac{\partial f_1}{\partial y_6} \\
\frac{\partial f_2}{\partial y_1} & \cdots & \frac{\partial f_2}{\partial y_6} \\
\frac{\partial f_3}{\partial y_1} & \cdots & \frac{\partial f_3}{\partial y_6}
\end{bmatrix}, \quad \text{and } B = \begin{bmatrix}
\frac{\partial f_1}{\partial u_1} & \cdots & \frac{\partial f_1}{\partial u_6} \\
\frac{\partial f_2}{\partial u_1} & \cdots & \frac{\partial f_2}{\partial u_6} \\
\frac{\partial f_3}{\partial u_1} & \cdots & \frac{\partial f_3}{\partial u_6}
\end{bmatrix}
\]  

(4-5)

Note that each rigid body degree of freedom driven by a lead screw has two equations representing its motion on the lower and upper thread. This results in two sets of linear equations. The \(A\) and \(B\) matrices are listed in Appendix D.

Next the design of a linear controller for the rigid body axes \(r, \theta, \phi\) is considered. In modeling such a controller, the dynamics associated with joint sensors and actuators are neglected. It is assumed that \(r, \theta, \phi\) and their time derivatives are available, and that the control torques \(T_1, T_2, T_3\), and \(T_4\) can be applied to drive each axis. A simple linear controller is used. It is a multiple input - multiple output integral plus state feedback controller whose control vector, \(u\), can be written as,

\[
u = -K^S \delta y + K^I \int (R - C_d) dt
\]  

(4-6)

where \(K^S\) is the \(3 \times 6\) state feedback gain matrix, \(K^I\) is the \(3 \times 3\) integral gain matrix and \(R^T = [R_1, R_2, R_3, 0, 0, 0]\) is the desired reference input. The term \(C_d\)
represents the system output where the $C$ matrix has the following form

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

By defining three additional state variables as,

$$\begin{align*}
y_7 &= \int (y_1 - R_1) dt \\
y_8 &= \int (y_2 - R_2) dt \\
y_9 &= \int (y_3 - R_3) dt
\end{align*}$$

(4-7)

the control vector, $u$, becomes

$$u = -KZ$$

(4-8)

where $Z = [y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9]$. The $3 \times 9$ gain matrix $K$ now has the following form:

$$K = \begin{bmatrix} K^S \\ K^I \end{bmatrix}$$

(4-9)

The gains are chosen based on the linear model described in (4-4) and (4-5).

The selected gain matrix is of the following form,

$$K = \begin{bmatrix} k_{11}^S & k_{12}^S & 0 & k_{14}^S & 0 & 0 & k_{11}^I & 0 & 0 \\ k_{21}^S & k_{22}^S & 0 & 0 & k_{25}^S & 0 & 0 & k_{22}^I & 0 \\ 0 & 0 & k_{33}^S & 0 & 0 & k_{35}^S & 0 & 0 & k_{33}^I \end{bmatrix}$$

This controller decouples the linearized system, thus allowing us to carry out a design procedure for each axis independently to achieve a damping ratio of $\xi = 1$ and desired servo-loop frequencies of $\omega_r$, $\omega_\theta$, and $\omega_\phi$ for the $r$, $\theta$, and $\phi$ axes respectively [49]. The formulations needed to evaluate the gain matrices of the lower and upper thread motions are given in Appendix D. This simple controller is then
applied to the robot arm as modeled by both the rigid body and the combined rigid body and flexible equations of motion. This is illustrated in the block diagram in Fig. 4-1. The computer code written for the digital simulation is listed in Appendix F. The simulation results are presented in the next section.

4.2. Results and Discussion

The dynamic equations of the flexible robot arm along with the equations obtained from the controller lead to a set of seventeen complex, coupled, highly nonlinear stiff equations. The difficulty in handling stiff problems is that most conventional numerical algorithms for solving ordinary differential equations require a time increment equal in measure to the minimum time constant of the system while the problem range which is the difference between initial and final
value of time is equal in measure to the maximum system time constant. As a result, the problem cannot be run to completion in a reasonable number of steps. Gear's method [50], used here, is well suited to handle stiff systems since it automatically changes the step size depending upon the region where the solution is relatively active.

The purpose of the simulation studies is to investigate the inter-relationships between the robot structural flexibility and the controller design. These include the effect of constraints due to transmission mechanisms, forced excitation due to inertial forces, and potential instability mechanisms such as resonance. The standard set of physical system parameters used in the computer program are listed in Table 1.

The results obtained for the rigid and flexible motion are shown in Fig. 4-2 to 4-13. The first three plots show the critically damped behavior of the rigid body degrees of freedom $r$, $\theta$, and $\phi$. This illustrates the good performance of the linear controller in the vicinity of the equilibrium point. The plot in Fig. 4-5 represents the motion of the flexible coordinate $q_{11}(t)$. In the transient response, one notes the following:

1. The excitation of the structural vibration is due to the effect of the rigid body motion. This excitation decreases with time due to the damping introduced by the rigid body controller, and to the diminishing effect of the inertial forces associated with the rigid body motion as it approaches the steady state.
### TABLE 1

<table>
<thead>
<tr>
<th>Standard Set of Physical System Parameters</th>
<th>VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of the first beam (m_1)</td>
<td>0.698 Kg</td>
</tr>
<tr>
<td>Mass of the second beam (m_2)</td>
<td>0.0429 Kg</td>
</tr>
<tr>
<td>Mass of the Payload (m_p)</td>
<td>0.05 Kg</td>
</tr>
<tr>
<td>Cross sectional area of the second beam (A_2)</td>
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</tr>
<tr>
<td>Length of the first beam (L_1)</td>
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</tr>
<tr>
<td>Length of the second beam (L_2)</td>
<td>0.5 m</td>
</tr>
<tr>
<td>Gravitational acceleration (g)</td>
<td>9.81 (m/sec^2)</td>
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<tr>
<td>Aluminum density (\rho)</td>
<td>2707 (Kg/m^3)</td>
</tr>
<tr>
<td>Flexural rigidity (EI)</td>
<td>5.67 (Pa)</td>
</tr>
<tr>
<td>Reference position for (r)</td>
<td>0.4 m</td>
</tr>
<tr>
<td>Reference position for (\theta)</td>
<td>0 rad</td>
</tr>
<tr>
<td>Reference position for (\phi)</td>
<td>0 rad</td>
</tr>
<tr>
<td>Desired reference position for (r)</td>
<td>0.5 m</td>
</tr>
<tr>
<td>Desired reference position for (\theta)</td>
<td>0.5 rad</td>
</tr>
<tr>
<td>Desired reference position for (\phi)</td>
<td>0.5 rad</td>
</tr>
<tr>
<td>Servo natural frequency for (r) (\omega_{nr})</td>
<td>4 (rad/sec)</td>
</tr>
<tr>
<td>Servo natural frequency for (\theta) (\omega_{n\theta})</td>
<td>4 (rad/sec)</td>
</tr>
<tr>
<td>Servo natural frequency for (\phi) (\omega_{n\phi})</td>
<td>8 (rad/sec)</td>
</tr>
</tbody>
</table>
(2) The natural frequency of the flexible mode decreases with time due to the increase of the span of the flexible beam and to the vanishing effect of the rigid body inertial forces applied to the robot arm as modeled by both the rigid body and the combined rigid body and flexible equations of motion.

(3) The steady state response is a small amplitude sustained oscillation around a negative value. The latter represents the static deflection due to gravity while the sustained oscillation shows the decoupled response of the elastic motion from the rigid body motion at steady state due to the effect of the self locking condition of the leadscrew.

The motion of \( q_{12}(t) \) is illustrated in Fig. 4-6. It has one additional feature. The magnitude of the vibratory motion is on the order of \( 10^{-3} m \) while the one for \( q_{11}(t) \) is on the order of \( 10^{-4} m \). This is due to the large amount of energy required to excite the higher modes. Coupling with the first mode is also evident.

Figures 4-7 and 4-8, which illustrate the flexible motion represented by \( q_{21}(t) \) and \( q_{22}(t) \), indicate a very important aspect of this particular robot design. The transient responses show the large excitation induced by the rigid body motion, and then die out with time. These interesting results can be interpreted as follows:

(1) Gravity doesn’t affect the flexible motion in the \( q_{21}(t) \) and \( q_{22}(t) \) directions. Therefore, their motions are expected to either die out to zero with time or oscillate around zero (i.e. no static deflection).
Figure 4-2. r displacement [meter] obtained from the rigid body controller in the base run.
Figure 4-3. \( \theta \) response obtained from the rigid body controller in the base run.
Figure 4-4. $\phi$ response obtained from the rigid body controller in the base run.
Figure 4-5. Flexible motion coordinate $q_{11}(t)$ in response to the rigid body controller in the base run.
Figure 4-6. Flexible motion coordinate $q_{12}(t)$ in response to the rigid body controller in the base run.
(2) Since there is no constraint on the rigid body motion in the \( \phi \) direction, the flexible motion in the \( \varphi_2(t) \) and \( \varphi_2(t) \) directions act as a disturbance source on the rigid body motion in the \( \phi \) direction. These disturbances are compensated for by the rigid body controller.

Figures 4-9 to 4-11 represent the control torques \( T_1, T_2, \) and \( T_3 \). The oscillations in the control signals for the base and second joints show the effect of the flexible motion on the rigid body motion. The control torques \( T_1 \) and \( T_2 \) for the \( r \) and \( \theta \) axes respectively are set to zero once the steady state is reached. This is due to the nonbackdrivable characteristic of the leadscrew transmission mechanism which satisfies the self locking conditions. The total deflection of the end effector in the vertical and the horizontal directions (i.e. \( V \) and \( W \) in equation (2-4)) are shown in Figs. 4-12 and 4-13 and found to be dominated by the first mode and nearly identical in shape to Figs. 4-5 and 4-7.

Additional runs are made with modifications to the standard set of parameters to study the behavior of the system in the following areas:

(1) \textit{Instability mechanisms}: The flexural rigidity, \( EI \), is reduced to 0.44 Pa. This can be achieved by constructing the flexible beam of polymers. As a result, the natural frequencies of the flexible beam are lowered to values that are close to the servo loop frequencies. The main results are shown in Fig. 4-14 to 4-18. The important features of this run are:

(a) In Figs. 4-14 to 4-16, the \( \phi \) response has small overshoot. The control torques for the base and second joints are very oscillatory. This illus-
Figure 4-7. Flexible motion coordinate $q_{21}(t)$ in response to the rigid body controller in the base run.
Figure 4-8. Flexible motion coordinate $q_{22}(t)$ in response to the rigid body controller in the base run.
Figure 4-9. Control signal for the prismatic joint obtained from the rigid body controller in the base run.
Figure 4-10. Control signal for the second joint obtained from the rigid body controller in the base run.
Figure 4-11. Control signal for the base joint obtained from the rigid body controller in the base run.
Figure 4-12. Total vertical deflection, \( v \), obtained from the rigid body controller in the base run.
Figure 4-13. Total horizontal deflection, w, obtained from the rigid body controller in the base run.
trates the effect of the flexible motion on the rigid body motion.

(b) The flexible motion frequency is reduced and the magnitude of the oscillation in both the vertical and the horizontal directions increased. (see Fig. 4-17 and 4-18).

(c) Due to the small value of $EI$ and to the proximity of the servo-loop frequency $\omega_s$ to the fundamental beam natural frequency, the amplitude of these oscillations grow with time. This is clearly shown in Fig. 4-18. Such proximity is not likely to occur in practice, but would lead to very poor performance when it does.

Note that the integration is interrupted at 1.24 seconds since this case could not be run to completion with a reasonable computation time due to the divergence of the solution.

(2) Speed accuracy trade off: The servo loop natural frequencies for $r$, $\theta$, and $\phi$, are raised from $\omega_r = 4$, $\omega_\theta = 4$ and $\omega_\phi = 8$ rad/sec in the base run, to $\omega_r = 8$, $\omega_\theta = 8$, and $\omega_\phi = 16$ rad/sec. Comparing the numerical results obtained with those of the base run, an increase on the order of 2 is observed in the amplitudes of the flexible motion coordinates. This is illustrated in Fig. 4-19 and 4-20 where the response of the first elastic mode in the horizontal and vertical directions are given. Unlike the base run, the responses of $\theta$ and $\phi$ (see Fig. 4-21 and 4-22) show the effects of the flexible motion on the rigid body motion. This is also reflected in Fig. 4-23 and 4-24 where the control torques for the base and second joints become more oscillatory then
Figure 4-14. 

Rotation [radian] vs. time [second].

Response obtained from the rigid body controller in the instability run.
Figure 4-15. Control signal for the second joint obtained from the rigid body controller in the instability run.
Control torque
$T_3(t) \text{ [N-M]}$

Figure 4-16. Control signal for the base joint obtained from the rigid body controller in the Instability run.
Figure 4-17. Total vertical deflection, $v$, obtained from the rigid body controller in the instability run.
Figure 4-18. Total horizontal deflection, $w$, obtained from the rigid body controller in the instability run.
Figure 4-19. Flexible motion coordinate $q_{11}(t)$ in response to the rigid body controller in the speed-accuracy trade-off.
Figure 4-20. Flexible motion coordinate $q_2(t)$ in response to the rigid body controller in the speed-accuracy trade-off.
Figure 4-21. \( \theta \) response obtained from the rigid body controller in the speed-accuracy trade-off.
Figure 4-22. The response obtained from the rigid body controller in the speed-accuracy trade-off.
Figure 4-23. Control signal for the second joint obtained from the rigid body controller in the speed-accuracy trade-off.
Control torque
$I_3(t)$ [N-M]

Figure 4-24. Control signal for the base joint obtained from the rigid body controller in the speed-accuracy trade-off.
Figure 4-25. \( \theta \) response obtained from the rigid body controller with the payload increased to 78.5 g.
their counterparts in the base run.

(3) **Effect of leadscrew on the controller design:** The payload mass is increased from 50g in the base run to 78.5g and the gains of the rigid body controller are retuned to compensate for this new payload mass. The latter increases the flexible motion inertial terms, causing large fluctuations in the magnitude of the normal force, $N$, and even in some cases changing its sign. Recall from the leadscrew constraint derivation that changes in the sign of $N$ result in switching of the motion from the lower to the upper thread or vice versa. This will, depending on the type of motion, either increase or decrease sharply the required torque to rotate the leadscrew which may be under tension or compression. Thus causing a deterioration in the controller performance in the vicinity of the transition period. This is clearly shown in Fig. 4.25 where the notch in the $\theta$ response is caused during the transition of the motion from one surface to another.

### 4.3. Summary and Conclusions

An integral plus state variable feedback controller, based on the linearized version of the rigid body model of the robot arm, is derived. This simple controller design is considered to be representative of the robot controllers currently in use. The dynamic model and the controller design are used as the basis for the simulation studies. The latter demonstrates the potential mechanisms by which the robot structural dynamics and controller design can interact.
The effects of constraints, due to the leadscrew transmission mechanisms, give the robot arm a cantilever beam-like behavior in the \( r \) and \( \theta \) directions which leads to some sustained vibratory motion of \( q_{11}(t) \) and \( q_{12}(t) \) at steady state. The absence of the constraint in the \( \phi \) direction allows the strain energy to be dissipated by the rigid body controller. The simulation results demonstrate the following additional features: (i) There is a potential for instability if the servo-loop frequencies approach the natural frequency of the robot arm, and (ii) High speed operation deteriorates the accuracy of the system, and results in a trade-off between speed and accuracy.

These results are very useful for designing and evaluating potential approaches to the flexible robot arm control problem. The latter will be discussed and treated in the next chapter.
CHAPTER 5

DESIGN AND IMPLEMENTATION OF THE RIGID
AND FLEXIBLE MOTION CONTROLLER

The interrelationships between the structural flexibility and the controller design, obtained in Chapter 4, provide guidance in developing the multiple-input multiple-output controller for the rigid and flexible motion of the robot arm. Towards achieving this goal, a simple case, involving the controller design of a three lumped mass system, is treated. The latter simulates the control problem of a compliant beam whose flexible motion is approximated by two elastic modes. The rationale is to start with the control of a relatively simple system as a preliminary step towards the general control problem of the flexible robot arm.

5.1. Description of the Three Lumped Mass System

In an attempt to gain insight into the control problem of the flexible robot arm, a simple system consisting of three lumped masses connected by massless springs is considered (see Fig. 5-1). This system can be viewed as a lumped model of an articulated compliant beam. The rationale is to scale down the complexity of the dynamic model of the robot arm, to get better insight into the controller
Figure 5-1. A schematic of the three lumped mass model.
performance, while working with a simple linear model, and to reduce the cost of computer runs.

The first mass reflects the rotational rigid body degree of freedom of the joint whereas the second and third masses represent the first and second elastic modes of transverse vibration. No structural damping is considered in the discrete model and the springs represent the structural stiffness of the beam. The three lumped masses are supported in the vertical direction to include the effect of gravity. The control force is applied to the first mass only.

Denoting the displacements of the first, second and third masses by \( z_1, z_2 \) and \( z_3 \) respectively. The equations of motion can be written as

\[
\begin{align*}
    m_1 \ddot{z}_1 &= k_1(z_2 - z_1) + m_1 g + F_1 \\
    m_2 \ddot{z}_2 &= -k_1(z_2 - z_1) + k_2(z_3 - z_2) + m_2 g \\
    m_3 \ddot{z}_3 &= -k_2(z_3 - z_2) + m_3 g
\end{align*}
\]

where \( k_1 \) and \( k_2 \) are the structural stiffness terms and \( g \) is the gravitational acceleration.

5.2. Controller Design for the Lumped Model

Two types of controllers are used for the three lumped mass model. The first is based solely on the motion of the first mass. This gives a good indication of the effect of the rigid body controller on the flexible system which is represented by the second and third masses. The second controller is designed based on the motions of the first and second masses, thus, including the flexible motion into the control action. The third mass, representing the second elastic mode, is
intentionally not considered in the controller design to study the effect of control spillover.

In both controller designs, an integral plus state feedback controller is implemented. The gains of the rigid body controller are computed based on the equation of motion for the first mass,

\[ m_1 \ddot{x}_1 = F_1 + m_1 g \quad (5-2) \]

This equation is identical to (5-1a) with the flexibility term \( k_1 (x_2 - x_1) \) set to zero. The control force, \( F_1 \), has the following form,

\[ F_1 = - \begin{bmatrix} k_1^* & k_2^* \\ k_2^* & k_1^* \end{bmatrix} \begin{bmatrix} x_1 \\ \dot{x}_1 \end{bmatrix} - k_1^* \int (x_1 - R_1) dt \quad (5-3) \]

By substituting (5-3) into (5-2), the gains are computed to achieve a damping ratio of \( \xi = 0.53 \) and a desired natural frequency for the first mass. The controller is then applied to the three lumped mass model.

The rigid and flexible motion controller is designed based on the linear equations of motion of the first and second masses,

\[ m_1 \ddot{x}_1 = F_1 + m_1 g + k_1 (x_2 - x_1) \quad (5-4a) \]
\[ m_2 \ddot{x}_2 = m_2 g - k_1 (x_2 - x_1) \quad (5-4b) \]

These equations are similar to (5-1a) and (5-1b) with the coupling term \( k_2 (x_3 - x_2) \) set to zero. Define the state variables to be,

\[ y_1 = x_1 \quad y_2 = x_2 \quad y_3 = \dot{x}_1 \quad y_4 = \dot{x}_2 \quad y_5 = \int (x_1 - R_1) dt \]

the equations of motion become,
\[
\begin{bmatrix}
\dot{y}_1 \\
\dot{y}_2 \\
\dot{y}_3 \\
\dot{y}_4 \\
\dot{y}_5 \\
\dot{y}_6
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
-\frac{k_1}{m_1} & \frac{k_1}{m_1} & 0 & 0 & 0 & 0 \\
\frac{k_1}{m_2} & -\frac{k_1}{m_2} & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
y_1 \\
y_2 \\
y_3 \\
y_4 \\
y_5 \\
y_6
\end{bmatrix} + 
\begin{bmatrix}
0 \\
0 \\
\frac{1}{m_1} \\
0 \\
0 \\
-R_1
\end{bmatrix}
\begin{bmatrix}
F_1 + g \\
g
\end{bmatrix}
\] (5-5)

In matrix notation, this can be written as,

\[
\dot{\tilde{y}} = \tilde{A}\tilde{y} + \tilde{B}F_1 + \tilde{W}
\] (5-6)

where the vector, \(\tilde{W}\), shows the effect of gravity. Note that with this notation, the integral action becomes embedded in the state equations. Thus, the integral plus state variable feedback controller can simply be written as,

\[
F_1 = -\tilde{K}\tilde{y}
\] (5-7)

where the gain matrix \(\tilde{K} = [k_1 \ k_2 \ k_3 \ k_4 \ k_5]\). Substitute (5-7) into (5-6) to get,

\[
\dot{\tilde{y}} = \tilde{A}\tilde{y} + \tilde{B}F_1 + \tilde{W} = (\tilde{A} - \tilde{B}\tilde{K})\tilde{y} + \tilde{W}
\] (5-8)

The eigenvalues of \((\tilde{A} - \tilde{B}\tilde{K})\) matrix can be arbitrarily assigned by the state feedback \(F_1 = -\tilde{K}\tilde{y}\), as long as the system given by (5-6) is controllable. First, the state equations are converted to the controllable canonical form. This is done by transforming the state vector \(\tilde{y}\) to a new state vector \(\tilde{z}\) using a constant transformation matrix \(T\) so that

\[
\tilde{y} = T\tilde{z}
\] (5-9)

The differentiation of this equation yields,
\[ \dot{y} = T \dot{Z} \]  

(5-10)

Substituting these values into (5-6) to get

\[ \dot{Z} = T^{-1}ATZ + T^{-1}BF_1 + T^{-1}W \]  

\[ = A'Z + B'F_1 + T^{-1}W \]  

(5-11)

where \( A' = T^{-1}AT \) and \( B' = T^{-1}B \). Let \( t_i \) be the \( i^{th} \) column of \( T \), i.e.

\[ T = \begin{bmatrix} t_1, t_2, \ldots, t_n \end{bmatrix} \]  

(5-12a)

Use the formulation given in [51], [52], [53], [54] to compute the \( t_i \)'s

\[ t_n = B \]
\[ At_n = t_{n-1} - a_1 t_n \]
\[ \vdots \]
\[ \vdots \]
\[ At_2 = t_1 - a_{n-1} t_n \]
\[ At_1 = -a_n t_n \]  

(5-12b)

where the \( a_i \)'s are the coefficients of the characteristic polynomial of \( A \). This choice of \( T \) leads to the following general form of the matrices \( A' \) and \( B' \),

\[ A' = T^{-1}AT = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -a_5 & -a_4 & -a_3 & -a_2 & -a_1 \end{bmatrix} \]

and \( B' = T^{-1}B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \)  

(5-13)
In the new state equations, the control signal \( F_1 \) becomes

\[
F_1 = -K'Z = -K'T^{-1}y = -Ky
\]  
(5-14)

where \( K' = [k_1', k_2', k_3', k_4', k_5'] \). Use (5-14) in (5-11),

\[
\dot{Z} = (A' - B'K')Z + T^{-1}W
\]  
(5-15)

the closed loop matrix can be written as,

\[
A' - B'K' = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
(-a_6 - k_1') & (-a_4 - k_2') & (-a_3 - k_3') & (-a_2 - k_4') & (-a_1 - k_5')
\end{bmatrix}
\]  
(5-16)

The gains are computed by matching the entries of \( A' - B'K' \) with those of the desired matrix \( A'_d \). The latter has the following form,

\[
A'_d = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
-a_{45} & -a_{44} & -a_{43} & -a_{42} & -a_{41}
\end{bmatrix}
\]  
(5-17)

where \( a_i \)'s are the \( i^{th} \) coefficient of the desired characteristic polynomial. The desired eigenvalues are assigned according to settling time and percent overshoot considerations.

The gain matrix, \( K' \), of the new state vector is directly related to the gain matrix, \( K \), of the original state vector through equation (5-14). This leads to,

\[
K = K'T^{-1}
\]  
(5-18)

The merits of the two controllers, derived in this section, are compared by applying them on the system of three lumped masses.
5.3. Results and Discussion of the System with Three Lumped Masses

The gains for the rigid body controller are computed to achieve a damping ratio $\xi = 0.53$ and a natural frequency, $\omega_n = 15 \text{ rad/sec}$. This controller is applied to the three lumped masses and the results are illustrated in Fig. 5-2 to 5-4. The response of the first mass, $z_1$, is very oscillatory due to the effect of coupling with the second and third masses. Note that the three lumped mass model (see Fig. 5-1) represents a conservative system. That is, once the system is excited, it should oscillate indefinitely. However, the decay in the $z_2$ and $z_3$ responses, in Fig. 5-3 and 5-4, is caused by the damping induced by the rigid body controller.

For best evaluation of the rigid and flexible motion controller, a reduced model, consisting of the first and second mass, is initially used. The exclusion of the third mass eliminates the effect of control spillover. Figures 5-5 and 5-6 represent the responses of the first and second mass respectively. The overshoot is due to the assignment of the desired damping ratio $\xi = 0.53$. The oscillation seen in Fig. 5-2 and 5-3 is completely eliminated due to the additional damping introduced by the rigid and flexible motion controller.

Finally, to show the effect of control spillover, the rigid and flexible motion controller is applied on the three lumped masses. This is illustrated in Fig. 5-7 to 5-9. The responses of $z_1$ and $z_2$, which represent the displacements of the first and second mass, become more oscillatory then in Fig. 5-5 and 5-6. This deterioration in the response is expected due to the effect of control spillover. However, the performance of this controller, even in the presence of control
Figure 5-2. First mass response obtained from the rigid body controller.
Figure 5-3. Second mass response obtained from the rigid body controller.
Figure 5-4. Third mass response obtained from the rigid body controller.
Figure 5-5. First mass response obtained from the rigid and flexible motion controller in the reduced order model case.
Figure 5-6. Second mass response obtained from the rigid and flexible motion controller in the reduced order model case.
Figure 5-7. First mass response obtained from the rigid and flexible motion controller in the control spillover case.
$X_2$ displacement
[meter]

Figure 5-8. Second mass response obtained from the rigid and flexible motion controller in the control spillover case.
Figure 5-9. Third mass response obtained from the rigid and flexible motion controller in the control spillover case.
spillover, is still superior to the rigid body controller (see Fig. 5-2 to 5-4). The oscillatory motion is approximately reduced to half.

At steady state, the deviation of the second and third mass displacements from the desired value is caused by the static deflection of the springs.

5.4. Linearization of the Rigid and Flexible Equations of Motion

The first step in designing the rigid and flexible motion controller for the robot, is to obtain a linearized version of the equations describing the combined rigid and flexible motions. The latter have the following general form,

$$M(z)\dddot{z} + F(z, \dot{z}) = F'(T)$$  

(5-19)

where \(z' = [r, \theta, \phi, q_{11}, q_{21}, q_{22}]\) and \(T' = [T_1, T_2, T_3]\). The inertia matrix \(M(z)\) is not diagonal. Therefore, the linearization procedure, outlined in section 4-1, cannot be applied directly. An intermediate step, involving the algebraic computation of the inverse inertia matrix, is required to transform (5-19) to the form given in equation (4-2). To avoid this difficulty, another procedure, which is also based on Taylor series expansion, is implemented.

Suppose \(z_n(t), \dot{z}_n(t), \ddot{z}_n(t)\) and \(\dddot{z}_n(t)\) satisfies the second order differential equation given in (5-19) for \(t \geq 0\). Define

$$z(t) = z_n(t) + \delta z(t)$$

(5-20)

$$T(t) = T_n(t) + \delta T(t)$$

Substitute (5-20) into (5-19), to get

$$M(z_n + \delta z)[\dddot{z}_n + \dddot{\delta z}] + F(z_n + \delta z, \dot{z}_n + \delta \dot{z}) = F'(T_n + \delta T)$$  

(5-21)
Expand by Taylor series the nonlinear terms around \(z_\ast\) and \(T_\ast\). This yields,

\[
\begin{align*}
\left[ M(\bar{z}_\ast) + \frac{\partial M(z_\ast)}{\partial \bar{z}} \delta \bar{z} \right] \dot{\bar{z}} + \left[ F(\bar{z}_\ast, \dot{\bar{z}}_\ast) + \frac{\partial F(z_\ast, \dot{z}_\ast)}{\partial z} \delta z \right] + F'(T_\ast) \delta T + \text{higher order terms}
\end{align*}
\]

or

(5-22)

\[
\frac{\partial F(\bar{z}_\ast, \dot{z}_\ast)}{\partial \bar{z}} \delta \bar{z} = F'(T_\ast) \delta T + \text{higher order terms}
\]

Rearranging the terms,

\[
\left\{ M(\bar{z}_\ast) \ddot{\bar{z}} + F(\bar{z}_\ast, \dot{\bar{z}}_\ast) - F'(T_\ast) \right\} + M(\bar{z}_\ast) \dddot{\bar{z}} + \frac{\partial M(z_\ast)}{\partial \bar{z}} \ddot{\bar{z}} + \frac{\partial M(z_\ast)}{\partial z} \dot{\bar{z}} + \frac{\partial F(z_\ast, \dot{z}_\ast)}{\partial \bar{z}} \ddot{\bar{z}} + \frac{\partial F(z_\ast, \dot{z}_\ast)}{\partial z} \dot{\bar{z}} + \text{higher order terms}
\]

(5-23)

Assuming that the solution, \(z_\ast\), is at equilibrium (i.e. \(\dot{z}_\ast = \ddot{z}_\ast = 0\)) and retaining only first order terms, equation (5-23) becomes

\[
M(\bar{z}_\ast) \ddot{\bar{z}} + \frac{\partial F(z_\ast, 0)}{\partial \bar{z}} \delta \bar{z} + \frac{\partial F(z_\ast, 0)}{\partial z} \delta z = \frac{\partial F'(T_\ast)}{\partial \bar{z}} \delta \bar{T}
\]

(5-24)

which can be rearranged to

\[
\ddot{\bar{z}} = -M^{-1}(z_\ast) \frac{\partial F(z_\ast, 0)}{\partial \bar{z}} \delta \bar{z} - M^{-1}(z_\ast) \frac{\partial F(z_\ast, 0)}{\partial z} \delta z + M^{-1}(z_\ast) \frac{\partial F'(T_\ast)}{\partial \bar{z}} \delta \bar{T}
\]

(5-25)

where \(M^{-1}(z_\ast)\) is the inverse of the inertia matrix \(M(z)\) evaluated at \(z_\ast\). In matrix form, equation (5-25) can be written as,

\[
\begin{bmatrix}
\ddot{\bar{z}} \\
\dot{\bar{z}}
\end{bmatrix} = 
\begin{bmatrix}
O_{7 \times 7} & I_{7 \times 1} \\
-M^{-1}(z_\ast) \frac{\partial F(z_\ast, 0)}{\partial \bar{z}} & -M^{-1}(z_\ast) \frac{\partial F(z_\ast, 0)}{\partial z}
\end{bmatrix}
\begin{bmatrix}
\delta \bar{z} \\
\delta \bar{T}
\end{bmatrix} + 
\begin{bmatrix}
O_{7 \times 3} \\
-M^{-1}(z_\ast) \frac{\partial F'(T_\ast)}{\partial \bar{z}}
\end{bmatrix} \delta \bar{T}
\]

(5-26)

The linearized form of the equations of motion for the rigid and flexible motion can be readily obtained by evaluating equation (5-26).
5.5. Derivation of the Rigid and Flexible Motion Controller

This section, is a very important one, in that it contains a detailed discussion of how the flexible motion is treated in the design of the rigid and flexible motion controller.

Any compliant link would undergo static and dynamic deflections. There are two approaches with which the static deflection problem can be handled. In the first approach, the static deflection is computed theoretically and then taken into consideration in specifying the desired end effector final position to the controller. In the second approach, additional sensors and actuators are used. The static deflection is measured and corrected for by the additional actuator referred to as a "straightness servo" in [4]. In this work, no attempt is made to compensate for the static deflection, as this can be treated as a separate problem.

The control objective is to introduce additional damping into the flexible motion. This is done by using additional sensors to measure the compliant link dynamic vibrations and feed them back to the controller. Theoretically, any flexible system has an infinite number of elastic modes. Due to physical limitations, a limited number of sensors and actuators can be applied, thus restricting the controller design to the few critical modes. Note that the outputs of the sensors would contain information about the unmodeled as well as the modeled modes. This is referred to as observation spillover. Similarly, the control action would affect both the modeled and unmodeled modes leading to the control spillover phenomenon. Based on the work done by Balas in [30], it is expected that the higher unmodeled modes can cause detrimental effect on the system response. It
can even lead to instability. To examine the effect of observation and control spillover, only the first elastic mode in the vertical and horizontal directions are considered in the controller design. The second mode is considered to be a representative of the higher unmodeled modes.

The rigid and flexible motion controller is designed based on the linearized equations of motion given in (5-26). An integral plus state feedback controller is implemented. The integral action is applied on the joint angles to insure that they reach their desired values with zero steady state error. Define the state vector, \( \tilde{y} \), and the control vector, \( \tilde{u} \), to be,

\[
\tilde{y} = \begin{bmatrix} \frac{\delta x}{\delta t} \\ \frac{\delta \tilde{y}}{\delta t} \end{bmatrix} \quad \tilde{u} = \delta T
\]  

(5-27)

Then equation (5-26) can be written in the following general form,

\[
\tilde{y} = Ay + Bu
\]  

(5-28)

This represents a linear, time invariant, multi-input multi-output system. Three additional state variables are introduced to facilitate the implementation of the integral plus state feedback controller,

\[
y_{11} = \int (r - R_1) dt \quad y_{12} = \int (\theta - R_2) dt \quad y_{13} = \int (\phi - R_3) dt
\]

This embeds the integral action into the state equations. Thus allowing the control action to be expressed as,

\[
\tilde{u} = -K_F \tilde{y}
\]  

(5-29)

where \( K_F \) is the feedback gain matrix of the rigid and flexible motion controller.
Substitute (5-29) into (5-28), to get

$$\dot{\tilde{y}} = (A - BK^\varepsilon)\tilde{y}$$ \hspace{1cm} (5-30)

The eigenvalues of the closed loop matrix, $(A - BK^\varepsilon)$, can be arbitrarily assigned by the controller, defined in (5-29), as long as the system given by (5-28) is controllable [55]. Note that for a given set of eigenvalues, the solution for the gain matrix is not unique.

The eigenvalues determine the stability and the speed of the response where the closed loop eigenvectors reshape the transient response. The non-uniqueness of the state feedback gain matrix, $K^\varepsilon$, in pole assignment provides the designer the freedom to partially select the closed loop eigenvectors [56], [57].

Standard procedures employ a transformation matrix to convert the linearized system to controllable canonical form for arbitrary eigenvalues assignment. Note that the set of eigenvectors are assigned by the particular choice of the transformation matrix. One such systematic method [53] is used to compute the gain matrix $K^\varepsilon$.

First, a state feedback is introduced to make the system controllable by a single component of the $u$ vector. This converts the multi-input system to single input one. Second, the method established for single-input systems, which was used earlier in the three lumped mass case, is employed to assign the desired closed loop eigenvalues. Note that the gain matrix is not unique for a given set of eigenvalues. The methodology, described here, does not make any attempt to use the freedom given by the nonuniqueness of the state feedback gain to
partially assign the closed loop eigenvectors. The latter, which determines the shape of the transient response, ends up being arbitrarily assigned; thus, leading, in most cases, to poor transient response.

This method is employed on the system given by (5-28). Although, the steady state response followed exactly the reference input, the transient response was very oscillatory. Most of the control effort was carried out by a single component of the control vector, \( u \), thus rendering this particular control design to be completely unacceptable from a practical point of view.

Being aware of these difficulties, the control problem is now approached from a physical point of view. No transformation matrix is used. Therefore, all the state variables have physical interpretations. The main task of the controller is to move the robot arm from one position to another with the least amount of vibration possible. From the results of the rigid body controller (see Fig. 4-2 to 4-13), the rigid body degrees of freedom \( r, \theta \) and \( \phi \) exhibit a critically damped behavior with zero steady state error. However, the transverse deflection in both the vertical and the horizontal directions are oscillatory. In light of this, the rigid and flexible motion controller is obtained by expanding the rigid body controller with the emphasis on introducing additional damping into the flexible system. To do this, the terms in equation (5-26) which represents the open loop case, have to be physically interpreted. \( M^{-1}(z_0) \frac{\partial F(z_0,0)}{\partial z} \) consists of all the stiffness terms whereas \( M^{-1}(z_0) \frac{\partial F(z_0,0)}{\partial z} \) contains all the damping terms. By evaluating equation (5-28) and including the integral action one obtains,
\[
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & | & 1 & 0 & 0 & 0 & 0 & | & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & | & 0 & 1 & 0 & 0 & 0 & | & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & | & 0 & 0 & 1 & 0 & 0 & | & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & | & 0 & 0 & 0 & 1 & 0 & | & 0 & 0 & 0 \\
\begin{bmatrix}
A_{q1} & A_{q2} & 0 & A_{q4} & 0 & | & 0 & 0 & 0 & 0 & 0 & | & 0 & 0 & 0 \\
A_{q1} & A_{q2} & 0 & A_{q4} & 0 & | & 0 & 0 & 0 & 0 & 0 & | & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & A_{q4} & | & 0 & 0 & 0 & 0 & 0 & | & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & A_{q4} & | & 0 & 0 & 0 & 0 & 0 & | & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & A_{10,5} & | & 0 & 0 & 0 & 0 & 0 & | & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & | & 0 & 0 & 0 & 0 & 0 & | & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & | & 0 & 0 & 0 & 0 & 0 & | & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & | & 0 & 0 & 0 & 0 & 0 & | & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
- & - & - \\
B_{q1} & B_{q2} & 0 \\
B_{q1} & B_{q2} & 0 \\
0 & 0 & B_{q3} \\
B_{q1} & B_{q2} & 0 \\
0 & 0 & B_{10,3} \\
- & - & - \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
u_1 \\
u_2 \\
u_3
\end{bmatrix}
\]

where the \( A_{ij} \) and \( B_{ij} \) terms represent the open loop terms. This yields

\[
M^{-1}(z_0) \frac{\partial F(z_0, 0)}{\partial z} = 0.
\]

That is no structural damping is considered. This is consistent with the robot arm dynamic modeling assumption. Applying the rigid
body controller,

\[ u = -K' \dot{y} \]

where \( K' \) represents the feedback gain matrix of the rigid body controller.

\[
K' = \begin{bmatrix}
  k'_{11} & k'_{12} & 0 & 0 & 0 & k'_{16} & 0 & 0 & 0 & k'_{1,11} & 0 & 0 \\
  k'_{21} & k'_{22} & 0 & 0 & 0 & k'_{27} & 0 & 0 & 0 & k'_{2,12} & 0 \\
  0 & 0 & k'_{35} & 0 & 0 & 0 & k'_{38} & 0 & 0 & 0 & k'_{3,15}
\end{bmatrix}
\]  

(5-32)

the closed loop matrix \((A - BK')\) becomes,

\[
(A - BK') = \begin{bmatrix}
  0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
  A_{a1} - \alpha_1 & A_{a2} - \alpha_1 & 0 & A_{a4} & 0 & -\alpha_1 - \alpha_1 & 0 & 0 & 0 & -\alpha_1 - \alpha_1 & 0 \\
  A_{a7} - \alpha_1 & A_{a72} - \alpha_1 & 0 & A_{a74} & 0 & -\alpha_1 - \alpha_1 & 0 & 0 & 0 & -\alpha_1 - \alpha_1 & 0 \\
  0 & 0 & -\alpha_1 & 0 & A_{a8} & 0 & 0 & -\alpha_1 & 0 & 0 & -\alpha_1 \\
  A_{a1} - \alpha_1 & A_{a2} - \alpha_1 & 0 & A_{a4} & 0 & -\alpha_1 - \alpha_1 & 0 & 0 & 0 & -\alpha_1 - \alpha_1 & 0 \\
  0 & 0 & -\alpha_1 & 0 & A_{a10,5} & 0 & 0 & -\alpha_1 & 0 & 0 & -\alpha_1 \\
  1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]  

(5-33)

where \( \alpha_1 \) terms represent the entries altered or created by the rigid body controller. The portion of the closed loop matrix that corresponds to \( M^{-1}(z_*) \frac{\partial F(z_*, 0)}{\partial z} \)
has nonzero terms. The latter represent the damping introduced by the rigid body controller in both the rigid and flexible motions. This is consistent with the decaying oscillation obtained in the transverse deflection for both the vertical and horizontal directions (see Fig. 4-5 to 4-8).

Since the main emphasis of the rigid and flexible motion controller is to introduce additional damping into the flexible system, then the gain matrix of the rigid body controller is modified to give

\[
K' = \begin{bmatrix}
 k_{11}' & k_{12}' & 0 & 0 & 0 & k_{16}' & 0 & k_{19}' & 0 & k_{1,11}' & 0 & 0 \\
 k_{21}' & k_{22}' & 0 & 0 & 0 & k_{27}' & 0 & k_{29}' & 0 & k_{2,12}' & 0 & 0 \\
 0 & 0 & k_{53}' & 0 & 0 & 0 & k_{58}' & 0 & k_{5,10}' & 0 & 0 & k_{5,15}'
\end{bmatrix}
\]  

(5-34)

The corresponding closed loop matrix can be written as follows,

\[
(A - BK') = \begin{bmatrix}
 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
 \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
 A_{61}' - \alpha_1 & A_{62}' - \alpha_1 & A_{64}' & 0 & A_{66}' & -\alpha_1 & -\alpha_1 & 0 & -\alpha_2 & 0 & -\alpha_1 & -\alpha_1 & 0 \\
 A_{71}' - \alpha & A_{72}' - \alpha & A_{74}' & 0 & A_{76}' & -\alpha_1 & -\alpha_1 & 0 & -\alpha_2 & 0 & -\alpha_1 & -\alpha_1 & 0 \\
 0 & 0 & -\alpha_1 & 0 & A_{86}' & 0 & 0 & -\alpha_1 & 0 & -\alpha_2 & 0 & 0 & -\alpha_1 \\
 A_{91}' - \alpha_1 & A_{92}' - \alpha_1 & A_{94}' & 0 & A_{96}' & -\alpha_1 & -\alpha_1 & 0 & -\alpha_2 & 0 & -\alpha_1 & -\alpha_1 & 0 \\
 0 & 0 & -\alpha_1 & 0 & A_{10,6}' & 0 & 0 & -\alpha_1 & 0 & -\alpha_2 & 0 & 0 & -\alpha_1 \\
 \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]  

(5-35)
where \( A_i \)'s terms are obtained from the linearized version of the rigid and flexible motion equations. \( a_i \) terms represent the entries altered or created by the rigid body controller and \( a_2 \) terms represent the entries created to induce more damping into the flexible system. Note that the selection of the gains \( k_{f_1}, k_{f_2}, \) and \( k_{f_{10}} \) should be done very carefully, since they simultaneously introduce new terms in the rigid and flexible motion equations. Thus, the larger these gains are, the more damping of the transverse deflections can be achieved and the greater is the effect of the flexible motion on the rigid body motion. This causes the latter to become oscillatory. Therefore a compromise should be made in introducing as much damping as possible while keeping the effect of the flexible motion on the rigid body motion to a minimum.

5.6. Results and Discussion of the Rigid and Flexible Motion Controller

The rigid and flexible motion controller is first applied on a reduced order model where only one elastic mode is considered to represent the flexible motion in each the horizontal and the vertical directions. The rationale is to eliminate the effect of observation and control spillover. The results are illustrated in Fig. 5-10 to 5-17. The first three plots show the expected critically damped response of the rigid body degrees of freedom \( r, \theta \) and \( \phi \). They are identical to their counterparts in the base run of the rigid body controller (see Fig.4-2 to 4-4). The first mode in the vertical and the horizontal directions are presented in Fig. 5-13 and 5-14. The oscillations are due to the inertial forces emanating from the fast and
Figure 5-10. The response obtained from the rigid and flexible motion controller in the reduced order model case.
Figure 5-11. The response obtained from the rigid and flexible motion controller in the reduced order model case.
Figure 5-12. The response obtained from the rigid and flexible motion controller in the reduced order model case.
Figure 5-13. Flexible motion coordinate $q_{11}(t)$ in response to the rigid and flexible motion controller in the reduced order model case.
Figure 5-14. Flexible motion coordinate $q_{21}(t)$ in response to the rigid and flexible motion controller in the reduced order model case.
Figure 5-15. Control signal for the prismatic joint obtained from the rigid and flexible motion controller in the reduced order model case.
Control torque \( T_2(t) \) (N·m)

Figure 5-16. Control signal for the second joint obtained from the rigid and flexible motion controller in the reduced order model case.
Figure 5-17. Control signal for the base joint obtained from the rigid and flexible motion controller in the reduced order model case.
sudden rigid body motions. However, these vibrations have rapidly died out, thus reflecting the additional amount of damping introduced by including the flexible motion into the control action. Finally, Fig. 5-15 to 5-17 show the applied control torques $T_1, T_2$ and $T_3$. The oscillations in the control signals show the effect of the flexible motion on the rigid body motion.

Additional runs are made to study the following effects:

(1) *Control spillover*: The rigid and flexible motion controller is now applied on the full rigid and flexible motion model. (i.e. two elastic modes are used to represent the transverse vibration in both the vertical and horizontal directions). This is done to assess the effect of control spillover on the second uncontrolled elastic mode. The main results are shown in Fig. 5-18 to 5-23. The first four plots represent the first and second elastic modes in the vertical and horizontal directions respectively. Even though the responses corresponding to the first mode become more oscillatory then the previous case, the overall performance of the flexible motion coordinates show a reduction of approximately 50% in magnitude when compared with their counterparts in the base run of the rigid body controller. The large oscillation in the control torques $T_2$ and $T_3$, shown in Fig. 5-22 and 5-23, reflects the greater effect of the flexible motion on the rigid body motion.

(2) *Control and observation spillover*: This run simulates an actual situation where both control and observation spillover can be present. In the experimental set up, accelerometers mounted at the end effector, are implemented to measure the tip acceleration. The latter is then integrated twice to yield
Figure 5-18. Flexible motion coordinate $q_{11}(t)$ in response to the rigid and flexible motion controller in the control spillover case.
Figure 5-19. Flexible motion coordinate $q_{12}(t)$ in response to the rigid and flexible motion controller in the control spillover case.
Figure 5-20. Flexible motion coordinate $q_{21}(t)$ in response to the rigid and flexible motion controller in the control spillover case.
Figure 5-21. Flexible motion coordinate $q_{22}(t)$ in response to the rigid and flexible motion controller in the control spillover case.
Figure 5-22. Control signal for the second joint obtained from the rigid and flexible motion controller in the control spillover case.
Figure 5-23. Control signal for the base joint obtained from the rigid and flexible motion controller in the control spillover case.
the transverse deflections \( V \) and \( W \) and their time derivatives \( \dot{V} \) and \( \dot{W} \). Assuming that the flexible motion is entirely due to the first mode, then \( V \) and \( W \) can be written as

\[
V = \Phi_1 q_{11}(t) \quad W = \Phi_1 q_{21}(t)
\]  
(5-36)

The flexible motion coordinates \( q_{11}(t) \) and \( q_{21}(t) \) become,

\[
q_{11}(t) = \frac{V}{\Phi_1} \quad \text{and} \quad q_{21}(t) = \frac{W}{\Phi_1}
\]  
(5-37)

where \( \Phi_1 \) is the eigenfunction of a clamped free beam, previously defined in equation (2-5), evaluated at the end effector. In digital simulation, the observation spillover is introduced as follows,

\[
q_{11}(t) = \frac{\Phi_1(r) q_{11}(t) + \Phi_2(r) q_{22}(t)}{\Phi_1(r)} = \frac{V(r)}{\Phi_1(r)}
\]  
(5-38)

\[
q_{21}(t) = \frac{\Phi_1(r) q_{21}(t) + \Phi_2(r) q_{22}(t)}{\Phi_1(r)} = \frac{W(r)}{\Phi_1(r)}
\]  
(5-39)

Thus the \( q_{11}(t) \) and \( q_{21}(t) \) measurements, that are fed back to the controller, are no longer representatives of the first mode only. Instead, they contain informations about both the first and second elastic modes. Figures 5-24 to 5-27 present the main results of this run. In Fig. 5-24 and 5-26, the responses of the first mode in the vertical and horizontal directions are almost identical to those in the control spillover case. However, the uncontrolled second mode exhibits more oscillatory behavior in the horizontal direction while it becomes unstable in the vertical direction. This instability is obtained due to the combined effect of control and observation spillover on the second mode.
Figure 5-24. Flexible motion coordinate $q_{11}(t)$ in response to the rigid and flexible motion controller in the control and observation spillover.
Figure 5-25. Flexible motion coordinate $q_{12}(t)$ in response to the rigid and flexible motion controller in the control and observation spillover.
Figure 5-26. Flexible motion coordinate $q_{21}(t)$ in response to the rigid and flexible motion controller in the control and observation spillover.
Figure 5-27. Flexible motion coordinate $q_{22}(t)$ in response to the rigid and flexible motion controller in the control and observation spillover.
Balas discusses the stability issue under these conditions in [30].

3. Control and Observation spillover in the presence of structural damping. Any physical system possesses a certain amount of structural damping. The latter is introduced into the dynamic model of the flexible robot arm by using Rayleigh's dissipation function, $F$, which can be written in the following general form,

$$ F = \frac{1}{2} \sum_{r=1}^{n} \sum_{s=1}^{s} c_{rs} \dot{q}_r \dot{q}_s $$  \hspace{1cm} (5-40)

where $c_{rs}$ is the damping coefficient. Considering only diagonal terms (i.e. $c_{rs} = 0 \text{ for } r \neq s$) equation (5-40) becomes,

$$ F = \frac{1}{2} \left( c_1 \dot{q}_{11}^2 + c_2 \dot{q}_{12}^2 + c_3 \dot{q}_{21}^2 + c_4 \dot{q}_{22}^2 \right) $$  \hspace{1cm} (5-41)

The values of $c_{rs}$'s are determined experimentally and found to be 0.103 kg rad/sec which corresponds to a damping ratio, $\xi = 0.0145$. Viscous damping forces can be derived from Rayleigh's dissipation function, $F$, using the following formulation,

$$ Q_i = -\frac{\partial F}{\partial \dot{z}_i} $$  \hspace{1cm} (5-42)

where $z_i$ is the $i^{th}$ generalized coordinate. Dividing the nonconservative forces $Q_{nc}$ in equation (2-15) into those dissipative type and those of impressed upon the system by external forces, Lagrange's equation become

$$ \frac{d}{dt} \left( \frac{\partial T_i}{\partial \dot{z}_i} \right) - \frac{\partial T_i}{\partial z_i} + \frac{\partial V_i}{\partial z_i} + \frac{\partial F}{\partial z_i} = Q_{ncd} $$  \hspace{1cm} (5-43)
where $Q_{intD}$ denotes generalized forces due only to external forces exerted on the system. The rigid and flexible motion controller is then applied to the modified robot arm model described in equation (5-43). The results are shown in Fig. 5-28 to 5-31. These plots are identical to their counterparts in the undamped model with control and observation spillover case except for the response of the second mode in the vertical deflection, $q_{1d}(t)$, which becomes stable.

5.7. Summary

This chapter provides the design of the rigid and flexible motion controller for the robot arm. First the control of a three lumped mass system, which represents a discrete model for a single compliant link, is developed. The rationale is to start with the control of relatively simple system and progress gradually towards the general nonlinear problem.

Two types of controllers are employed in the lumped model study. The first, simulating the rigid body controller, is based entirely on the motion of the first mass. In the design of the second controller, the dynamics of both the first and second masses are considered. The third mass is deliberately left out to study the effect of control spillover. The main results obtained, can be stated as follows,

(1) The rigid body controller provides some damping of the flexible motion.
Figure 5-28. Flexible motion coordinate $q_{11}(t)$ in response to the rigid and flexible motion controller in the control and observation spillover with structural damping included.
Figure 5-29. Flexible motion coordinate $q_{12}(t)$ in response to the rigid and flexible motion controller in the control and observation spillover with structural damping included.
Figure 5-30. Flexible motion coordinate $q_{21}(t)$ in response to the rigid and flexible motion controller in the control and observation spillover with structural damping included.
Figure 5-31. Flexible motion coordinate $q_{22}(t)$ in response to the rigid and flexible motion controller in the control and observation spillover with structural damping included.
(2) In the absence of observation and control spillover, the rigid and flexible motion controller completely eliminates the vibratory motion.

(3) In the presence of control spillover, the rigid and flexible motion controller reduces the oscillation observed in the rigid body controller case by almost half.

The rigid and flexible motion controller is designed based on a linearized version of the general equations of motion. Only the first mode of the transverse deflection in both the vertical and horizontal directions are considered in the control action. Higher modes are left out to study the effect of control and observation spillover. The controller objective is to add more damping to the flexible motion while maintaining a good rigid body motion performance. The following conclusions can be drawn:

(1) In the ideal case where there is no observation or control spillover, the flexible motion recovers rapidly from the sudden excitation induced by the rigid body motion inertial forces.

(2) In the presence of control spillover, the overall performance of the flexible motion shows a reduction of approximately 50% in magnitude when compared with their counterparts in the base run of the rigid body controller.

(3) In the presence of observation and control spillover, identical results are obtained for the flexible motion as in the control spillover case except for the response of the uncontrolled second mode in the verti-
cal direction which becomes unstable.

(4) In the presence of structural damping with observation and control spillover; the response of the uncontrolled second elastic mode in the vertical direction becomes stable.

A comment is in order here. One should note that even in the presence of observation and control spillover, the rigid and flexible motion controller provides an additional damping capable of reducing by approximately 50% the magnitude of the flexible motion oscillation detected in the rigid body controller.

There is a need for experimental validation of the dynamic model of the robot arm, as well as evaluation of the rigid body controller versus rigid and flexible motion controller. This will be the subject of the next chapter.
CHAPTER 6

EXPERIMENTAL RESULTS AND COMPARISON

TO SIMULATION RESULTS

The flexible robot arm control problem has been investigated using analytical methods and digital simulation. There is a need for experimental evaluation of both the dynamic model of the robot arm and the controller design. This chapter experimentally compares the rigid and flexible motion controller with the rigid body controller to evaluate the merit of measuring and feeding back the flexible motion.

6.1. Experimental Set Up

The three degree of freedom spherical coordinate laboratory robot, described in section 2.1, is used for the experimental work. It is interfaced to an IBM/PC microcomputer. A Tecmar Lab Tender interface board which has built-in 8 bit analog to digital and digital to analog converters is used. The schematic of the experimental set up is given in Fig. 6-1. Three DC servo motors, driven by power amplifiers, are used as actuators. The control torques are transmitted to the links by leadscrews for the $r$ and $\theta$ axes and a gear train for the $\phi$ axis. One
Figure 6-1. A schematic of the experimental set up.
tachometer generator and one optical encoder [58], [59] is mounted on each joint to measure the position and velocity of its corresponding rigid body degree of freedom. The optical encoder signals are read by the computer through digital counters. Each joint has a different size of counter associated with it according to their range of operation. A 20 bit counter is used to record the number of counts generated by the optical encoder monitoring the $\phi$ rotation whereas 12 bit and 20 bit counters are used to record the $\theta$ and $r$ motions respectively. The contents of any counter can be strobed into a tri-state buffer. The latter can be accessed by the computer through differential drivers. Differential receivers are also used to reduce the noise on the control lines originating with the microcomputer.

The part of the second beam protruding from the first beam, which is considered to be flexible in the digital simulation, is made of a thin aluminum rod. The payload is kept constant. It consists of a mounting stud and two Kistler piezotron accelerometers (model 8606A100). The latter are used to measure the end effector transverse vibration in both the vertical and the horizontal directions. The stability conditions for such systems where the sensors are not colocated with the actuators are discussed in [60]. The acceleration signals are passed through a Kistler piezotron coupler (model 5120). The latter has a built in Kistler low pass filter (model 5318A22) with a break frequency set to 60 HZ. This attenuates the higher mode signals, thus reducing the effect of observation spillover. The dc component of the coupler's output which represents the end effector acceleration due to the rigid body motion needs to be filtered out. This
is done by passing the acceleration signals through two cascaded high pass Krohn-Hite filters (model 3322) with a break frequency set to 3 HZ. Finally, the total deflection of the end effector and its time rate are obtained by integrating twice the accelerometer signals using analog double integrators [61], [62]. These integrators are used to devote most of the sampling period to the computation of the control action. The schematic drawing of the circuit board is given in Appendix E. The flexible motion signals are digitized by the 8 bit analog to digital converter whose resolutions are 0.2176 mm/count for the total deflection, \( V \), and 16.314 mm/sec/count for \( \dot{V} \).

The prismatic joint poses a very challenging problem. As the second beam slides inside or outside the first beam, the length of the flexible part varies. Thus, causing the beam natural frequencies to change significantly. This problem is not considered in this study. Therefore, the length of the part of the second beam protruding from the first beam, \( r \), is kept constant. Due to hardware difficulties, the base joint governing the \( \phi \) rotation was not operational, thus restricting the experimental work to the second joint only. That is only the rigid body coordinate \( \theta \) and the flexible motion in the vertical direction, \( V \), can be controlled. The horizontal deflection, \( W \), would also be affected by the controller due to its coupling with \( V \). The \( \theta \) rotation is measured by an optical encoder with 825 counts per revolution (Optisyn model 77-4-003-625AA). The latter is enhanced by connecting the encoder shaft to a gear train with gear ratio of 8:1 to yield a resolution of 31.25 counts per degree of \( \theta \).
In designing the second beam, the following constraints are taken into consideration:

1. The capability of the small dc servo motor mounted on the second joint.
2. The computation time required for the control algorithm.
3. The speed of the microcomputer used.

To satisfy the first constraint, the second beam should be lightweight. The control algorithm or the speed of the microcomputer impose a very stringent constraint on the design of the second beam. In the experimental work, the flexible motion is considered to be dominated by the first elastic mode. In order to avoid aliasing [63], the sampling theorem must be satisfied. That is the sampling period must be at most half the harmonic motion period. Thus, the fundamental frequency, which is fixed once a certain design for the second beam is adopted, would set the maximum sampling period. However, for high fundamental frequency, the period, which is inversely proportional to the natural frequency, becomes very small. Depending on the speed of the microcomputer or the control algorithm used, the upper bound of the sampling period imposed by the design of the second beam, may or may not be large enough to perform the control loop. Therefore, in this work, the second beam is designed to have a low fundamental frequency. This would give ample time to try different control algorithms. For the experimental setup employed here, the length of the second beam is chosen to be 0.5m and its diameter is 0.635cm so that its fundamental frequency would
be approximately 6 Hz. The control algorithm used is an integral plus state feedback controller whose control loop requires a small amount of computation. The sampling period used is 0.024 sec. and is well below the upper bound imposed by the sampling theorem.

6.2. Experimental Evaluation of Model Parameters

In contrast to the digital simulation, the dynamics of the actuators and sensors are considered in the experimental work. The robot arm is treated as a pure inertia loading on the motor shaft. The armature controlled DC motor used in the experimental set up is very common in servo control systems. The derivation of its dynamic model is well documented in the literature [49]. The corresponding transfer function can be written as follows,

\[
\frac{\dot{\theta}}{u} = \frac{K}{\tau \theta + 1} \tag{6-1}
\]

where \(u\) is the input voltage to the motor, \(\tau\) is the time constant, and \(K\) is the overall gain of the motor, the power amplifier and the tachometer generator. The parameters \(K\) and \(\tau\) are determined experimentally from open loop runs. For a constant step input \(u(t) = A\) volt and initial condition \(\dot{\theta}(t = 0) = 0\), the response of the system described by equation (6-1) can be given by

\[
\theta(t) = KA(1 - e^{-t/\tau}) \tag{6-2}
\]

At steady state, \(\theta_{ss}\) is equal to \(KA\). The gain, \(K\), can now be easily determined since \(\theta_{ss}\) corresponds to the steady state value of the open loop response of the system to a step input. The time constant, \(\tau\), corresponds to the time at which
the open loop response reaches $KA(1 - e^{-t})$. Several open loop runs are made, where different step inputs are given to the system and the sensors values are stored in the computer. The average values for $K$ and $\tau$ are found to be 2.284 degree/sec/volt and 0.15 sec respectively.

6.3. Design and Performance of the Rigid Body Controller

The model described by equation (6-1) is first expressed in terms of state variables which are defined as follows:

$$
\begin{align*}
y_1 &= \theta \\
y_2 &= \dot{\theta} \\
y_3 &= \int (\theta - R_2) dt
\end{align*}
$$

(6-3)

where $R_2$ is the desired position of $\theta$. The state variable $y_3$ is introduced to imbed the error signal into the state equations. The latter would have the following form,

$$
\begin{bmatrix}
\dot{y}_1 \\
\dot{y}_2 \\
\dot{y}_3
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 \\
0 & -\frac{1}{\tau} & 0 \\
1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
y_1 \\
y_2 \\
y_3
\end{bmatrix} +
\begin{bmatrix}
0 \\
\frac{K}{\tau} \\
0
\end{bmatrix} u +
\begin{bmatrix}
0 \\
0 \\
-R_2
\end{bmatrix}
$$

(6-4)

In matrix form, equation (6-4) can be expressed as

$$
\dot{y} = Ay + Bu + R
$$

(6-5)

Using the procedure outlined in [64], the state equations are converted to difference equations which can be written as,
\[ y(t + 1) = Py(t) + Qu(t) \]

where \( T \) is the sampling period. \( P \) and \( Q \) are obtained from

\[
P = e^{AT} = I + AT + \frac{AT^2}{2!} + \frac{AT^3}{3!} + \ldots + \frac{A^iT^{i+1}}{(i+1)!} + \ldots
\]

\[
Q = \int_0^T e^{A\xi}d\xi B = \left\{ IT + \frac{AT^2}{2!} + \frac{AT^3}{3!} + \ldots + \frac{A^iT^{i+1}}{(i+1)!} + \ldots \right\}B
\]

Two different values of \( i = 100 \) and \( i = 300 \), are considered in the series. No significant differences in the results are observed, thus, for \( i = 100 \) the series has converged.

Next the design of the rigid body controller is considered. An integral plus state feedback controller is used. The control signal \( u(t) \) becomes,

\[
u(t) = -\begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix} = -K^Ty(t)
\]

where \( K^T \) is the transpose of the controller gain matrix. The closed loop difference equations are obtained by substituting (6-8) into (6-8)

\[ y(t + 1) = (P - QK^T)y(t) + R \]

The gains are selected to yield a damping ratio \( \xi = 1 \) and desired servo loop frequency \( \omega \). The controller software used in the experimental studies is listed in Appendix G.

The rigid body controller is applied on the robot arm to rotate it from its initial position \( \theta(0) = 0 \), which corresponds to the horizontal position, to a certain
desired position, $R_2$. Several runs are made. It is found that for $R_2$ greater than 20 degrees the effect of nonlinearity becomes significant and the response is oscillatory. Typical results of the rigid body controller are shown in Fig. 6-2 to 6-4 for $R_2 = -20$ degrees. The first plot shows the $\theta$ response. It has a small overshoot which persists for a while due to the effect of friction. Figures 6-3 and 6-4 represent the control voltage applied to the motor and the total vertical deflection of the end effector. The maximum deflection observed is approximately $\pm 3.5$mm. The control signal shows some saturation. This is desirable since it serves the purpose of driving the robot arm at its highest speed. The pulse like pattern observed in Fig. 6-3 and 6-4 illustrate the poor resolution obtained from the 8 bit analog to digital and digital to analog converters.

An additional run is performed to study the effect of friction. This is done by driving the robot arm beyond its linear range and the response becomes oscillatory with a large amount of overshoot. This is illustrated in Fig. 6-5. At the peak of the oscillation, $\dot{\theta}$ is zero. This results in a sudden increase in the required driving torque due to the change from dynamic to static coefficient of friction. The control signal, illustrated in Fig. 6-6, is unable to respond quickly to the sharp variation in the resistive torque would cause the robot arm to come to a complete halt waiting for the control effort to build up. This is consistent with what is observed in the simulation result in Fig. 3-3.
Figure 6-2. $\theta$ response obtained from the rigid body controller in the experimental work.
Figure 6-3. Control signal for the second joint obtained from the rigid body controller in the experimental work.
Figure 6-4. Total vertical deflection in response to the rigid body controller in the experimental work.
Figure 6-5. Response showing the effect of friction in the experimental work.
Figure 6-6. Control signal for the second joint showing the effect of friction in the experimental work.
6.4. Design and Results of the Rigid and Flexible Motion Controller

In the experimental work, the observation and control spillover are always present. The control action affects all elastic modes whereas the accelerometer signals contain information from all modes. A low pass filter with a break frequency set to 60 HZ attenuates the effect of higher modes. However, it does not eliminate the observation spillover problem entirely. Fortunately, the physical system exhibits some light structural damping. As seen in Chapter 5, this provides a margin of stability for the uncontrolled modes.

The objective of the rigid and flexible motion controller is to introduce more damping into the flexible motion. This is done by following the same controller design approach used in Chapter 5. That is expanding the rigid body controller to include the feedback signal of the derivative of the flexible motion coordinate, $\dot{q}_{11}(t)$. The control signal becomes

$$
\begin{equation}
\mathbf{u}(t_k) = -[k_1, k_2, k_3, k_4] \begin{bmatrix} \nu_1(t_k) \\ \nu_2(t_k) \\ \nu_3(t_k) \\ \nu_4(t_k) \end{bmatrix} = -K^T \dot{y}
\end{equation}
$$

(6-10)

where the $\mathbf{y}(t_k)$ in the augmented state vector $\mathbf{y}$ represents $\dot{q}_{11}(t)$. The latter is obtained by assuming that the flexible motion is dominated by the first elastic mode. Therefore, from the assumed modes method, the total vertical deflection, $V$, can now be written as

$$
V = \Phi_v(y)q_{11}(t)
$$

(6-11)

its time derivative $\dot{V}$, is
\[ \dot{V} = \Phi_1(\nu)\dot{q}_{11}(t) + \Phi_1(\nu)\ddot{q}_{11}(t) \]  

(6-12)

In the experimental study, the length of the flexible beam is kept constant. This yields

\[ \dot{\Phi}_1(\nu) = 0 \]  

(6-13)

Thus leading to

\[ \dot{V} = \Phi_1(\nu)\ddot{q}_{11}(t) \]  

(6-14)

Since the accelerometers are located at the end effector, then

\[ \ddot{q}_{11}(t) = \frac{\dot{V}}{\Phi_1(\nu)} \]  

(6-15)

where \( \Phi_1(\nu) \) is the value of the assumed mode shape evaluated at the end effector. \( \dot{V} \) is the time rate of the total deflection. It is obtained by integrating the accelerometer signal once using the hardware integrator. Equation (6-15) provides a means to compute the newly introduced state variable \( \nu_4 = \ddot{q}_{11}(t) \) required for the rigid and flexible motion controller. Only the gain \( k_4 \) needs to be selected since the rest of the gains \( k_1, k_2 \) and \( k_3 \) retain the same values that they hold in the rigid body controller. The gain \( k_4 \) is selected by trial and error. A large value for \( k_4 \) would increase the effect of the flexible motion on the rigid body motion. This would have an adverse effect on the response of the rigid body coordinate \( \theta \). Therefore a compromise is reached where the gain \( k_4 \) is selected to be large enough to damp out the flexible motion within an acceptable tolerance while keeping the effect of the flexible motion on the rigid body motion to minimum. The rigid and flexible motion controller is applied to the robot arm and the
Figure 6-7. \( \theta \) response obtained from the rigid and flexible motion controller in the experimental work.
Figure 6-8. Control signal for the second joint obtained from the rigid and flexible motion controller in the experimental work.
Figure 6-9. Total vertical deflection in response to the rigid and flexible motion controller in the experimental work.
results are shown in Fig. 6-7 to 6-9. The first plot represents the overdamped $\theta$ response. The applied control voltage, shown in Fig. 6-8, has two important features:

(1) The control signal fluctuates sharply during the period where $\theta$ approaches its desired value. In response to these oscillations, the DC motor introduces a series of pulses into the system. This results in reduction of the amplitude of the vertical deflection. Thus leading to additional damping in the flexible motion.

(2) A small position error exists after $t = 3$ sec. The control effort is unable to overcome the resistive torque, induced by the friction and geometry effects of the leadscrew, and brings the $\theta$ response to a standstill. The integral action, which tries to correct for the position error, starts building up the control action. This phenomenon is clearly observed in the control signal plot (Fig. 6-8).

Finally, Fig. 6-9 shows a reduction in the maximum deflection of the end effector of approximately 75% as compared to the rigid body controller results shown in Fig. 6-4.

6.5. Conclusions

In experiments, the rigid and flexible motion controller reduced the transverse deflection in the vertical direction by approximately 75% over the rigid body controller even in the presence of observation and control spillover.
The effect of friction, observed in Figs. 6-5 and 6-6, is very consistent with the results obtained in the digital simulation.

Note that in the digital simulation, an approximate reduction of 50% in the vertical deflection is obtained versus an approximate 75% reduction in the experimental work. This may in part be due to the fact that in the experimental work only the second joint is used while in the digital simulation all three joints are moved simultaneously.

6.6. Summary

This chapter provides a description of the experimental set up. A set of difference equations, which represents a simple linear model of the servo positioner, are obtained. Several open-loop runs are performed to measure the open loop characteristics of the motor, the power amplifier and the tachometer generator. Both a rigid body controller and rigid and flexible motion controller are implemented. A comparison between their performances is made. Also the effect of friction is experimentally demonstrated.
CHAPTER 7

SUMMARY AND CONCLUSIONS

This chapter provides a brief summary of the dissertation content. The main conclusions are outlined and some prospective research topics, from unsolved issues encountered in the present research, are stated.

7.1. Summary

An overview of the research done in robot arm modeling, dynamics, and control is presented. In this work, the dynamics of a spherical coordinate robot arm, whose last link is very flexible, is studied for the purpose of control. The assumed modes method is employed to represent the deflection of any point on the flexible link. All the coupling terms between the rigid and the flexible motions are retained. This is done by using coupled reference position and elastic deformation models. The kinetic and potential energy expressions are developed. Finally, Lagrange's method is implemented to obtain the unconstrained equations of motion.

The robot arm considered in this work has two joints driven by leadscrew transmission mechanisms. The kinematic constraints associated with a leadscrew
are introduced to investigate the behavior of a leadscrew driven flexible robot arm in the presence of coulomb friction and the self locking condition (i.e. the leadscrew is nonbackdrivable).

An integral plus state feedback controller is derived based on a linearized version of the rigid body model of the robot arm. The controller is then implemented on the rigid and flexible model. The rationale is to simulate the controllers currently used in industrial robots and to assess the interrelationships between the robot arm structural flexibility and the controller design.

In an attempt to gain insight into the control problem of the flexible robot arm, a simple case, involving the controller design of a three lumped mass system is considered. The latter simulates the control problem of a compliant beam whose flexible motion is approximated by two elastic modes. This allows the study of the controller performance while working with a simple linear model.

The objective in designing the rigid and flexible motion controller is to introduce additional damping into the flexible motion. The integral plus state feedback controller, derived for the rigid body model of the robot arm, is extended to include the flexible motion. This is done by using additional sensors to measure the compliant link dynamic vibrations and feed them back to the controller. The effect of control and observation spillover is examined by considering one elastic mode in both the vertical and horizontal directions in the controller design. The second mode in each direction is considered to be representative of the higher unmodeled modes.
Finally, an experimental evaluation is performed on both the rigid and flexible motion controller and the rigid body controller to assess the merit of measuring and feeding back the flexible motion.

7.2. Major Contributions

The major contributions of this work are:

(1) The kinematic constraints associated with a leadscrew are developed and the behavior of a leadscrew driven flexible robot arm in the presence of coulomb friction and the self locking condition is investigated.

(2) The investigation of the interrelationships between the robot arm structural flexibility and the controller design has led to the following conclusions:

   (a) Drive characteristics influence coupling between flexible and rigid body motions.

   (b) Rigid body controllers can provide some damping into the flexible motion, and the reduction of arm vibration depends on drive characteristics.

   (c) Interactions between structural design and controller design should be considered.

(3) The inclusion of the flexible motion in the control action results in additional damping into the flexible motion and the important issue of observation and control spillover is found to present no serious practical problem.
(4) The dynamic model of the leadscrew driven flexible robot arm and controller design are evaluated experimentally and found to be in good agreement with the digital simulation results.

7.3. Prospective Research Topics

The following issues, which were encountered in this work, need to be addressed in future research:

1 - The problem of developing a general procedure to obtain a dynamic model of a prismatic joint in a flexible robot arm should be addressed.

2 - Using additional sensors to measure the flexible motion, the merits of introducing additional damping into the flexible motion by different types of controllers need to be examined.

3 - The issue of using additional sensors and actuators should be studied.
APPENDIX A

THE UNCONSTRAINED EQUATIONS OF MOTION

The resulting seven nonlinear, coupled, second order ordinary differential equations of the unconstrained motion are listed in this appendix. The equation of motion for $r$ is,

$$\ddot{r} = \ddot{\bar{r}} = 0 \quad \text{for} \quad F_z = 0$$

and

$$-\rho A_2 \ddot{r}^2 + \rho A_2 (L_2 - r) \ddot{r} + \rho A_2 \left[ \dddot{r} + r^2 + (\dot{r} \dot{\cos} \theta + \ddot{r} \cos \theta - \dot{r} \ddot{\theta} \sin \theta) [.78q_{21} + .43q_{22}] \right. \right.$$

$$+ \dddot{\theta} [.78q_{11} + .43q_{12} - \ddot{\theta} [.78q_{11} + .43q_{12}]$$

$$+ 0.61(\dot{r} \dot{q}_{11} + 2r \dot{q}_{11} \dot{q}_{11}) + 0.67(\dot{r} \dot{q}_{11} \dot{q}_{12} + \ddot{r} \dot{q}_{11} \ddot{q}_{12} + \dot{r} \dot{q}_{11} \ddot{q}_{12}) + 0.5(\dot{r} \dot{q}_{11} \dot{q}_{12} + \ddot{r} \dot{q}_{11} \ddot{q}_{12})$$

$$+ 0.5q_{11} \dot{q}_{11} \dot{q}_{12} + \dot{r} \dot{q}_{12} \ddot{q}_{12} + \ddot{r} \dot{q}_{12} \ddot{q}_{12} + \dot{r} \dot{q}_{12} \ddot{q}_{12} + \ddot{r} L_1 [.78q_{11} + .43q_{12}] + \dot{\theta} L_1 [.78q_{11} + .43q_{12}]$$

$$+ (\dddot{r} + \ddot{r}) [.35q_{11} - .25q_{12}] + \dot{\theta} r [.35q_{11} - .25q_{12}]$$

$$- (\dot{r} \dot{r} \sin \theta + \dot{\theta} \dot{q}_{11} \sin \theta)(.5q_{11} \dot{q}_{21} + .65q_{11} \dot{q}_{22} - .65q_{12} \dot{q}_{21} + .5q_{12} \dot{q}_{22})$$

$$- \dot{\theta} \dot{r} \cos \theta [.5q_{11} \dot{q}_{21} + q_{11} \dot{q}_{21} + .65q_{11} \dot{q}_{22} + q_{11} \dot{q}_{22}] - .65(\dot{q}_{12} \dot{q}_{21} + q_{12} \dot{q}_{21})$$

$$+ .5q_{11} \dot{q}_{22} + q_{12} \dot{q}_{22}) + 0.61(\dot{r} \dot{q}_{21} + 2r \dot{q}_{21} \dot{q}_{21}) + 0.67(\dot{r} \dot{q}_{21} \dot{q}_{22} + \ddot{r} \dot{q}_{21} \ddot{q}_{22} + \dot{r} \dot{q}_{21} \ddot{q}_{22})$$

$$+ 0.18(\dot{r} \dot{q}_{22} + 2r \dot{q}_{22} \dot{q}_{22}) + (\dot{\theta} \dot{r} \sin \theta + \dot{\theta} \dot{q}_{11} \sin \theta)(.5q_{21} \dot{q}_{11} + .65q_{21} \dot{q}_{12})$$

$$- \dot{\theta} \dot{r} \cos \theta (.5q_{21} \dot{q}_{11} + q_{21} \dot{q}_{11}) + 0.65(\dot{q}_{21} \dot{q}_{12} + q_{21} \dot{q}_{12})$$

$$- .65q_{22} \dot{q}_{11} + .5q_{22} \dot{q}_{12} + \dot{\theta} \dot{r} \sin \theta (.5q_{21} \dot{q}_{11} + q_{21} \dot{q}_{11}) + 0.65(\dot{q}_{21} \dot{q}_{12} + q_{21} \dot{q}_{12})$$

$$- .65(\dot{q}_{22} \dot{q}_{11} + q_{22} \dot{q}_{11}) + 0.5(\dot{q}_{22} \dot{q}_{12} + q_{22} \dot{q}_{12}) - L_1 (\dot{\theta} \cos \theta - \dot{\theta} \dot{\sin} \theta) [.78q_{21} + .43q_{22}]$$

$$- L_1 \dot{\theta} \cos \theta (.78q_{21} + .43q_{22}) - (\dot{\theta} \cos \theta - \dot{\theta} \dot{\sin} \theta) [.35q_{21} - .25q_{22}]$$
\[ \dot{\phi} \cos\theta \left[ 0.35(\dot{r}_{21} + r_{21}^2) - 0.25(\dot{r}_{22} + r_{22}^2) \right] + 0.5(\dot{r}_{21} \dot{q}_{21} + r_{21} \ddot{q}_{21}) + 0.65(\dot{r}_{21} \dot{q}_{22} + r_{21} \ddot{q}_{22}) - 0.65(\dot{r}_{22} \dot{q}_{21} + r_{22} \ddot{q}_{21}) + 0.5(\dot{r}_{22} \dot{q}_{22} + r_{22} \ddot{q}_{22}) + m_r \left( \dddot{r} + 2(\dot{\phi} \cos\theta - \dot{\phi} \sin\theta)(q_{21} - q_{22}) \right) \\
+ 2(\dot{\phi} \cos\theta)(q_{21} - q_{22}) - 2(\dot{q}_{11} - q_{12}) - 2\dot{\phi}(q_{11} - q_{12}) + \frac{\rho A_2}{2} \left[ \ddot{r}^2 + \dot{\theta}^2 \left( L_1 - \frac{L_2}{2} + \frac{r}{2} \right) \right] \\
+ \phi \left[ L_1 - \frac{L_2}{2} + \frac{r}{2} \right]^2 \cos^2 \theta - \frac{\rho A_2}{2} (L_2 - r) \left[ \ddot{\theta} \left( L_1 - \frac{L_2}{2} + \frac{r}{2} \right) \right] \\
+ \phi \left[ L_1 - \frac{L_2}{2} + \frac{r}{2} \right] \cos^2 \theta + \frac{\rho A_2 (L_2 - r)^2}{8} (\phi^2 \cos^2 \theta + \ddot{\theta}^2) \\
- \frac{\rho A_2}{2} \left( \dddot{r}^2 + (\ddot{q}_{21}^2 + q_{21}^2) + \ddot{q}_{21}^2 \right) + \ddot{q}_{21}^2 \left( 0.78q_{11} + 0.43q_{22} \right) + 2r \dot{\phi} \cos\theta \left( 0.78q_{21} + 0.43q_{22} \right) \\
- 2\dot{r} \dot{\phi}(q_{11} - q_{12}) - 2\dot{\phi} \dot{q}_{21} + 2\dot{q}_{11} + 2\dot{q}_{12} + 2\ddot{L}_1 \left[ 0.78q_{11} + 0.43q_{12} \right] + 2\dot{\theta} \left( 0.35 \dot{r}_{21} - 0.25 \dot{r}_{22} + 1.14q_{11} + 0.182q_{12} \right) \\
- 2\dot{\phi} \sin\theta \left( 0.5 \dot{q}_{11} \dot{q}_{21} + 0.65 \dot{q}_{12} \dot{q}_{21} - 0.65 \dot{q}_{11} \dot{q}_{22} + 0.5 \dot{r}_{12} \dot{q}_{22} + \dot{q}_{11} \dot{q}_{21} + \dot{q}_{12} \dot{q}_{22} \right) \\
+ \ddot{\theta} \left( L_1^2 + 2L_1 r + r^2 \right) - 2\ddot{L}_1 \dot{q}_{21} + 2(\dot{\theta}(q_{21} - q_{22}) + \dot{q}_{21} \dot{q}_{21} + 1.3 \dot{r}_{21} \dot{q}_{22} - 1.3 \dot{r}_{22} \dot{q}_{21} + \ddot{r}_{22} \dot{q}_{21} + \ddot{q}_{21} + \ddot{q}_{22} + \dot{q}_{21} \dot{q}_{22} + 2\dot{\theta} \sin\theta \left( \dot{q}_{11} + q_{12} \right) \\
+ \ddot{\theta} \left( 0.5 \dot{q}_{11} \dot{q}_{21} + 0.65 \dot{q}_{12} \dot{q}_{21} - 0.65 \dot{q}_{11} \dot{q}_{22} + 0.5 \dot{r}_{12} \dot{q}_{22} + \dot{q}_{11} \dot{q}_{21} + \dot{q}_{12} \dot{q}_{22} \right) \\
- 2L_1 \dot{\phi} \cos\theta \left( 0.78q_{21} + 0.43q_{22} \right) - 2\dot{\phi} \cos\theta \left( 0.35 \dot{r}_{21} - 0.25 \dot{r}_{22} + 1.14q_{11} + 0.182q_{12} \right) \\
+ \ddot{\phi} \sin^2 \theta \left( 0.78q_{11} + 0.43q_{12} \right) - 4\dot{\phi} \sin\theta \left( 0.57q_{21} + 0.091q_{22} \right) \\
+ \ddot{\phi} \sin^2 \theta \left( \dot{q}_{11}^2 + \dot{q}_{12}^2 - 2\dot{\phi} \sin\theta \left( L_1 \dot{q}_{21} + 0.43q_{12} \right) - 4\dot{\phi} \sin\theta \left( 0.57q_{11} + 0.091q_{12} \right) \\
+ (L_1^2 + 2L_1 r + r^2) \phi^2 \cos^2 \theta - \frac{m_r}{2} \left( 4\ddot{\phi} \left( \dot{q}_{11} - q_{12} \right) + 2\ddot{\theta} \left( L_1 + r \right) - 4\dot{\phi} \sin\theta \left( q_{21} - q_{22} \right) \\
- 4\phi \cos\theta \left( \dot{q}_{21} - \dot{q}_{22} \right) - 4\dot{\phi} \sin \theta \cos \theta \left( q_{11} - q_{12} \right) + 2(L_1 + r) \phi^2 \cos^2 \theta \right) + m_H \dddot{r} \\
- (\phi^2 \cos^2 \theta + \ddot{\theta}) m_H (L_1 + r - L_2) = F_i - (m_2 + m_r + 0.2) \rho A_2 \cos\theta \left( 0.78q_{11} + 0.43q_{12} \right) \\
+ \frac{E_1}{r^4} \left[ 18.5(q_{21}^2 + q_{22}^2) + 728.28(q_{11}^2 + q_{12}^2) \right] \text{ for } F_i \neq 0 \]
The equation of motion for \( \theta \) is,
\[
\ddot{\theta} = \dot{\theta} = 0 \quad \text{for} \quad F_{rr} = 0
\]
and
\[
\left[ \frac{m_1 L_1^2}{12} + m_1 \left( \frac{L_1}{2} - d \right)^2 + c + m_H (L_1 + r - L_2)^2 \right] \left( \ddot{\theta} + \dot{\phi}^2 \sin \theta \cos \theta \right)
+ \rho A_2 (L_2 - r) \dot{\phi} \left( L_1 - \frac{L_2}{2} + \frac{r}{2} \right) \dot{r} + \rho A_2 (L_2 - r) \dot{\phi} \left( L_1 - \frac{L_2}{2} + \frac{r}{2} \right)^2
- \rho A_2 \left( L_1 - \frac{L_2}{2} + \frac{r}{2} \right)^2 \frac{\rho A_2 (L_2 - r) \dot{\phi} \dot{r}}{4} + \rho A_2 (L_2 - r) \ddot{\phi}
+ \frac{\rho A_2}{2} \left( 2(\ddot{\theta} + \dot{\phi} \dot{r})(q_{11}^2 + q_{12}^2) + 2 \dot{\theta}(2 q_{11} \dot{q}_{11} + 2 q_{12} \dot{q}_{12}) - 2(\ddot{r} + \dot{r}^2)(.78 q_{11} + .43 q_{12})
- 2 \dot{r}(.78 q_{11} + .43 q_{12}) - 2 \phi (\dot{r} \cos \theta + \dot{\phi} \cos \theta - \dot{\phi} \theta \sin \theta)(q_{11} \dot{q}_{21} + q_{12} \dot{q}_{22})
- 2 \phi \dot{r} \cos \theta(q_{11} \dot{q}_{21} + q_{12} \dot{q}_{22} + q_{12} \dot{q}_{22} + q_{12} \dot{q}_{22}) + 2 L_1 [.78 \dot{r} q_{11} + .78 \dot{r} q_{11} + .43 \dot{r} q_{12} + .43 \dot{r} q_{12}
+ .78 \dot{r} q_{11} + .78 \dot{r} q_{11} + .43 \dot{r} q_{12} + .43 \dot{r} q_{12}] + 2 (\ddot{r} + \dot{r}^2)(.35 q_{11} + .25 q_{12}) + 2 \dot{r}(.35 q_{11} + .25 q_{12})
+ 4 \dot{r}(.57 q_{11} + .09 q_{12}) + 2 \dot{r}^2(.57 \dot{q}_{11} + .09 \dot{q}_{12})
+ 2 \dot{\theta} \left( L_1 \ddot{r} + L_1 \dot{r}^2 + \frac{q_{12}^2}{3} \right) + 2 \dot{\theta}(L_1 \ddot{\phi} + 2 L_1 \dot{\phi} \dot{r} + \dot{r}^2)
- 2 L_1 (\dot{\phi} \sin \theta + \dot{\phi} \cos \theta)(.78 \dot{r} q_{21} + .43 \dot{r} q_{22}) + 2 L_1 \dot{\phi} \sin \theta [.78 (\dot{r} q_{21} + \dot{r} q_{21}) + .43 (\dot{r} q_{22} + \dot{r} q_{22})]
- 2 (\dot{\phi} \cos \theta)(\dot{r} \sin \theta + 2 \dot{r} \dot{\phi} \cos \theta)(.57 \dot{q}_{21} + .09 \dot{q}_{22}) - 2 \phi \dot{r} \sin \theta(.57 \dot{q}_{21} + .09 \dot{q}_{22})
+ \frac{m_p}{2} \left[ 8 \theta (q_{11}^2 + q_{12}^2 - 2 q_{11} q_{12}) + 8 \theta (2 q_{11} \dot{q}_{11} + 2 q_{12} \dot{q}_{12} - 2 \dot{q}_{11} \dot{q}_{12} - 2 \dot{q}_{11} \dot{q}_{12})
- 4 \dot{r}(q_{11} - \dot{q}_{12}) - 4 \dot{r}(q_{11} - \dot{q}_{12}) - 8 (\dot{\phi} \cos \theta - \dot{\phi} \theta \sin \theta)(q_{11} \dot{q}_{21} - q_{11} \dot{q}_{21} - q_{12} \dot{q}_{21} + q_{12} \dot{q}_{21})
- 8 \dot{\phi} \cos \theta(q_{11} \dot{q}_{21} + q_{12} \dot{q}_{21} + q_{12} \dot{q}_{22} - q_{12} \dot{q}_{21} - q_{12} \dot{q}_{21} + q_{12} \dot{q}_{22})
+ 4 \dot{r}(q_{11} - \dot{q}_{12}) + 4 \dot{r}(L_1 + r)(q_{21} - \dot{q}_{21}) + 2 \dot{\theta} (L_1 + r)^2 + 2 \dot{\theta} (L_1 + r)^2
- 4 (\ddot{\theta} \sin \theta + (L_1 + r) \dot{\phi} \sin \theta + (L_1 + r) \dot{\phi} \cos \theta)(q_{21} - q_{22}) - 4 (L_1 + r) \dot{\phi} \sin \theta(q_{21} - q_{22})
+ \rho A_2 (L_2 - r) \ddot{\phi} \left( L_1 - \frac{L_2}{2} + \frac{r}{2} \right)^2 \cos \theta \sin \theta + \frac{\rho A_2 (L_2 - r)^3}{12} \dot{\phi} \cos \theta \sin \theta
\[-\frac{\rho A^2}{2} \left\{ -2\ddot{r}\cos\sin\theta(q_{21}^2 + \dot{r}^2) - 2\dot{r}\dot{\phi}\cos\sin(0.78q_{21} + 0.43q_{32}) + 2\dot{r}\dot{\sin}\theta(q_{11}q_{21} + q_{12}q_{22}) \\
- 2\dot{\phi}\cos\sin(0.78q_{21} + 0.43q_{32}) - 2\dot{\phi}\cos(0.78q_{21} + 0.43q_{32}) + 2\ddot{r}\sin\theta\cos(0.78q_{21} + q_{22}) \\
+ 2\dot{r}\cos\sin(0.78q_{21} + 0.43q_{32}) - 2\dot{r}\cos(0.78q_{21} + 0.43q_{32}) + 2\dot{\phi}\sin\theta(0.78q_{21} + q_{22}) \\
+ 2\dot{\phi}\sin(0.78q_{21} + 0.43q_{32}) + 2\dot{\phi}\cos(0.78q_{21} + 0.43q_{32}) + 2\dot{r}\cos\sin(0.78q_{21} - 0.25q_{32}) \\
+ 2\dot{r}\cos(0.78q_{21} + 0.43q_{32}) + 2\dot{\phi}\sin\theta(0.78q_{21} + q_{12}) - 2\dot{\phi}\sin\theta(0.78q_{11} + q_{12}) \\
+ 0.43q_{12}) - 2\dot{\phi}\cos^2\theta - \sin^2\theta(0.78q_{11} + 0.43q_{12}) - 2\cos\sin\theta\left(\frac{L_1^2r + L_1^2r^2 + \frac{r^2}{3}}{3}\right) \right\} \\
- \frac{m_r}{2} \left\{ -8\dot{\phi}\cos\sin\theta(q_{21}^2 + q_{22}^2 - 2q_{21}q_{22}) - 4\dot{r}\sin\theta(q_{21} - q_{22}) \\
+ 8\dot{\phi}\cos\sin(0.78q_{21} - 0.43q_{32} + q_{12}q_{21} + q_{12}q_{22}) - 8\dot{\phi}\cos(0.78q_{21} + 0.43q_{32} + q_{12}q_{21} + q_{12}q_{22}) \\
- 4(L_1 + r)\cos\sin\theta(q_{21} - q_{22}) + 8\dot{\phi}\sin\theta(0.78q_{21} + q_{22}^2 - 2q_{21}q_{22}) + 8\dot{\phi}\cos(0.78q_{21} - 0.43q_{32} + q_{12}q_{21} + q_{12}q_{22}) \\
+ 8\dot{\phi}\sin\theta(0.78q_{11} + q_{12}^2 - 2q_{11}q_{12}) - 4\dot{\phi}(L_1 + r)(\cos^2\theta - \sin^2\theta)(0.78q_{11} + 0.43q_{12}) \\
- 2(L_1 + r)^2\dot{\phi}\cos\sin\theta \right\} = -a_{e}F_{e} \left\{ m_1 \left[ L_1 \frac{L_1}{2} - a \right] + m_1 d_1 + m_H(L_1 + r - L_2) \\
+ m_2 \left[ L_1 + r - \frac{L_2}{2} \right] + m_2(L_1 + r) \right\} g\cos\theta - \rho A_2 g\sin\theta \left[ 0.78q_{11} + 0.43r_{12} \right] \\
- m_r g\sin\theta \left[ 2q_{11} - 2q_{12} \right] \quad \text{for} \quad F_{e} \neq 0 \]
The equation of motion for $\phi$ is,

$$\left[ m_1 \left( \frac{L_1}{2} - 2\phi \right)^2 + \frac{m_1 L_1^2}{12} + c + m_{H_1}(L_1 + r - L_2)^2 \right] \left( \dot{\phi} \cos^2 \theta - 2\dot{\phi} \dot{\theta} \cos \theta \sin \theta \right)$$

$$- \rho A_2 \dot{\phi} \cos^2 \theta \left[ L_1 - \frac{L_2}{2} + \frac{r}{2} \right] + \rho A_2 \dot{\phi} \cos^2 \theta (L_2 - r) \left[ L_1 - \frac{L_2}{2} + \frac{r}{2} \right]^2$$

$$+ \rho A_2 \dot{\phi} \cos \theta (L_2 - r) \left[ L_1 - \frac{L_2}{2} + \frac{r}{2} \right] - 2\rho A_2 \dot{\phi} \sin \theta \cos \theta (L_2 - r) \left[ L_1 - \frac{L_2}{2} + \frac{r}{2} \right]^2$$

$$- \frac{\rho A_2}{4} \dot{\phi} \cos \theta (L_2 - r)^2 + \frac{\rho A_2}{12} \phi \cos \theta (L_2 - r)^3$$

$$- \frac{\rho A_2}{6} \phi \sin \theta \cos \theta (L_2 - r)^3 + 2\rho \dot{\phi} \cos \theta m_{H_1}(L_1 + r - L_2)$$

$$+ \rho A_2 \left[ \dot{\phi} \cos \theta + \dot{\phi} \cos \theta - 2\dot{\phi} \dot{\theta} \cos \theta \sin \theta \right] \left( q_{21}^2 + q_{22}^2 \right) + \dot{\phi} \cos \theta (2q_{21} \dot{q}_{21} + 2q_{22} \dot{q}_{22})$$

$$+ \left[ \dot{r} \cos \theta + \dot{\theta} \cos \theta - \dot{r} \sin \theta \right] (0.78q_{21} + 0.43q_{22}) + \dot{r} \cos \theta (0.78q_{21} + 0.43q_{22})$$

$$- \left[ \dot{\theta} \cos \theta + \dot{\theta} \cos \theta - \dot{r} \sin \theta \right] (q_{11q_{21}} + q_{12q_{22}}) - \dot{r} \cos \theta (q_{11q_{21}} + q_{11q_{21}} + q_{12q_{22}} + q_{12q_{22}})$$

$$- \left[ \dot{r}^2 \sin \theta + \dot{r} \sin \theta + \dot{r} \cos \theta \right] (0.5q_{11q_{21}} + 0.65q_{11q_{21}} - 0.65q_{11q_{21}} + 0.5q_{12q_{22}})$$

$$- \dot{r} \sin \theta (0.5q_{11q_{21}} + q_{11q_{21}}) + 0.65(q_{11q_{22}} + q_{11q_{22}}) - 0.65(q_{11q_{21}} + q_{12q_{21}})$$

$$+ 0.5(q_{12q_{22}} + q_{12q_{22}}) - (\dot{r} \sin \theta + \dot{r} \cos \theta) \left[ \dot{q}_{11q_{21}} + \dot{q}_{11q_{21}} + \dot{q}_{12q_{22}} \right] - \sin \theta \left[ \dot{q}_{11q_{21}} + \dot{q}_{11q_{21}} + \dot{q}_{12q_{22}} \right]$$

$$+ \dot{q}_{12q_{22}} + q_{12q_{22}} - L_1 \left[ \dot{r} \sin \theta + \dot{r} \sin \theta + \dot{r} \cos \theta \right] (0.78q_{21} + 0.43q_{22})$$

$$- \dot{r} \sin \theta (0.78q_{21} + 0.43q_{22}) - (\dot{r}^2 \sin \theta + 2\dot{r} \dot{\sin \theta} + \dot{r} \dot{\cos \theta} \cos \theta) \left[ 0.5q_{21} + 0.09q_{22} \right]$$

$$- \dot{r}^2 \sin \theta (0.5q_{21} + 0.09q_{22}) + \left[ \dot{\phi} \sin \theta + \dot{\phi} \sin \theta + 2\dot{\phi} \dot{\sin \theta} \cos \theta \right] (q_{21} + q_{22})$$

$$+ \dot{\phi} \sin \theta (2q_{21} \dot{q}_{21} + 2q_{22} \dot{q}_{22}) + \left[ \dot{r}^2 \sin \theta + \dot{r} \sin \theta + \dot{r} \cos \theta \right] (0.5q_{21} \dot{q}_{11} + 0.65q_{21} \dot{q}_{12})$$

$$- 0.65q_{22} \phi + 0.5q_{22} \phi + \dot{r} \sin \theta \left[ 0.5q_{21} \phi + q_{21} \phi + 0.65q_{21} \phi \right]$$

$$- 0.65(q_{22} \phi + q_{22} \phi) + 0.5(q_{22} \phi + q_{22} \phi) + \dot{r} \sin \theta \left[ q_{21} \phi + \dot{q}_{21} \phi + q_{22} \phi + \dot{q}_{22} \phi \right]$$

$$+ \left[ \dot{r} \sin \theta + \dot{r} \cos \theta \right] \left[ \dot{q}_{21} \phi + \dot{q}_{22} \phi \right] + L_2 \dot{\sin \theta} (0.78q_{21} + 0.43q_{22} + 0.78q_{21} + 0.43q_{22})$$

$$- L_1 \dot{\cos \theta} \left[ 0.78 \phi + \dot{r} \phi \right] + 0.43(r \phi + \dot{r} \phi) + 0.78(r \phi + \dot{r} \phi) + 0.43(r \phi + \dot{r} \phi)$$

$$- \left[ \dot{r}^2 \cos \theta + \dot{r} \sin \theta \right] (0.35q_{21} - 0.25q_{22}) - \dot{r} \cos \theta \phi (0.35q_{21} - 0.25q_{22})$$
\[
- \left[ 2r \cos \theta - r^2 \sin \theta \right] (0.57 \dot{q}_{21} + 0.09 \dot{q}_{22}) - r^2 \cos \theta (0.57 \dot{q}_{21} + 0.09 \dot{q}_{22}) \\
+ (\dot{\theta} \sin \theta + \dot{\phi} \sin \theta + 2 \dot{r} \dot{\theta} \cos \theta)(q_{11} \dot{q}_{11} + q_{12} \dot{q}_{12}) + 2 \dot{\phi} \sin \theta (q_{11} \dot{q}_{11} + q_{12} \dot{q}_{12}) \\
- 2L_1 \left[ \dot{\phi} \sin \theta \cos \theta + \dot{\phi} \cos \theta - \dot{\phi} \sin \theta \cos \theta \right] (0.78q_{11} + 0.43q_{12}) \\
- 2 \dot{r} L_1 \sin \theta \cos \theta (0.78q_{11} + 0.43q_{12}) - 2 \left[ \dot{\phi} \cos \theta - 2 \dot{r} \sin \theta \cos \theta + 2 \ddot{r} \dot{\phi} \sin \theta \cos \theta \right] \\
+ 2 \dot{\phi} \cos \theta (0.57q_{11} + 0.09q_{12}) - 2 \dot{\phi} \sin \theta \cos \theta (0.57q_{11} + 0.09q_{12}) \\
+ (L \dot{r} \ddot{r} + 2L_1 \dddot{r} + r^2 \dot{r}) \dot{\phi} \cos \theta + \left[ L \dddot{r} + L \dot{r}^2 + \frac{r}{3} \right] (\dot{\phi} \cos \theta - 2 \dot{r} \sin \theta \cos \theta) \\
+ m_r \left[ 4 \dot{\phi} \cos \theta - 2 \dot{r} \dot{\phi} \sin \theta \cos \theta \right] (q_{21} \dot{q}_{21} + q_{22} \dot{q}_{22} - 2q_{21}q_{22}) \\
+ 4 \dot{\phi} \cos \theta (2q_{21} \dot{q}_{21} + 2q_{22} \dot{q}_{22} - 2q_{21}q_{22}) + 2 \left[ \dot{r} \cos \theta - \dot{\phi} \sin \theta \right] (q_{21} - q_{22}) \\
+ 2 \dot{r} \cos \theta (q_{21} - q_{22}) - 4 \left( \dot{\phi} \cos \theta - \dot{\phi} \sin \theta \right) (q_{11} \dot{q}_{21} + q_{12} \dot{q}_{22} + q_{12} \dot{q}_{22}) \\
+ 4 \dddot{q}_{11} + q_{11} \dddot{q}_{21} + q_{12} \dddot{q}_{22} - q_{12} \dddot{q}_{22} - q_{11} \dddot{q}_{22} - q_{12} \dddot{q}_{22} + q_{12} \dddot{q}_{22} + q_{12} \dddot{q}_{22} \\
- 4 \dddot{q}_{11} + q_{11} \dddot{q}_{21} + q_{11} \dddot{q}_{22} - q_{12} \dddot{q}_{22} + q_{12} \dddot{q}_{22} + q_{12} \dddot{q}_{22} \\
+ 4 \dddot{q}_{11} + q_{11} \dddot{q}_{21} + q_{11} \dddot{q}_{22} - q_{12} \dddot{q}_{22} + q_{12} \dddot{q}_{22} + q_{12} \dddot{q}_{22} \\
+ 4 \dddot{q}_{11} + q_{11} \dddot{q}_{21} + q_{11} \dddot{q}_{22} - q_{12} \dddot{q}_{22} + q_{12} \dddot{q}_{22} + q_{12} \dddot{q}_{22} \\
+ 4 \dddot{q}_{11} + q_{11} \dddot{q}_{21} + q_{11} \dddot{q}_{22} - q_{12} \dddot{q}_{22} + q_{12} \dddot{q}_{22} + q_{12} \dddot{q}_{22} \\
- 2 \left[ \dot{\phi} \sin \theta (L_1 + r) + \dot{\phi} \sin \theta + \dot{\phi} \cos \theta (L_1 + r) \right] (q_{21} - q_{22}) - 2 \dot{\phi} \sin \theta (L_1 + r) (q_{21} - q_{22}) \\
+ 4 (\dot{\phi} \sin \theta + \dot{r} \dot{\phi} \cos \theta) \left[ q_{21} \dot{q}_{21} + q_{22} \dot{q}_{22} - 2q_{21}q_{22} \right] + 8 \dot{\phi} \sin \theta \left( q_{21} \dot{q}_{21} + q_{22} \dot{q}_{22} - 2q_{21}q_{22} + q_{21}q_{22} \right) \\
+ 4 \dot{\phi} \sin \theta \left( q_{21} \dot{q}_{21} + q_{22} \dot{q}_{22} - 2q_{21}q_{22} + q_{21}q_{22} \right) + 4 \dot{\phi} \sin \theta \left( q_{21} \dot{q}_{21} + q_{22} \dot{q}_{22} - 2q_{21}q_{22} + q_{21}q_{22} \right) \\
- 2 \dddot{q}_{21} + q_{21} \dddot{q}_{11} - q_{22} \dddot{q}_{11} - q_{22} \dddot{q}_{11} + q_{22} \dddot{q}_{12} + q_{22} \dddot{q}_{12} + q_{22} \dddot{q}_{12} - 2 \dddot{q}_{21} - q_{21} \dddot{q}_{11} \\
+ 8 \dot{\phi} \sin \theta (q_{11} \dot{q}_{11} + q_{12} \dot{q}_{12} + q_{12} \dot{q}_{12} - q_{11} \dot{q}_{12} - q_{11} \dot{q}_{12} - q_{11} \dot{q}_{12}) - 4 \left[ \dot{\phi} \sin \theta \cos \theta (L_1 + r) + \dot{\phi} \cos \theta \cos \theta (L_1 + r) \right] \\
- \dot{\phi} \sin \theta \cos \theta (L_1 + r) + \dot{\phi} \sin \theta \cos \theta (L_1 + r) (q_{11} - q_{12}) + \dot{\phi} \sin \theta \cos \theta (L_1 + r) (q_{11} - q_{12}) \\
+ \dot{\phi} \left( L_1 + r \right) \cos \theta + 2 \left( L_1 + r \right) \dot{r} \cos \theta - 2 \dot{r} \dot{\phi} \sin \theta (L_1 + r) \right] \dot{\phi} \sin \theta \cos \theta \right) + \dot{\phi} \left( m_{H_2} g^2 + L_1 \right) = T_1 $
The equation of motion for \( q_{11}(t) \) is,

\[
\frac{\rho A_2}{2} \left\{ \dot{r}^2 + \ddot{r} \right\} (q_{11} - 1.3q_{12}) + \dot{r} \left( \dot{q}_{11} - 1.3\dot{q}_{12} \right) + 2\dot{r}\ddot{q}_{11} + 2r\dddot{q}_{11} + 1.56L_1(\ddot{\theta} r + \dot{\theta} r) \\
+ 1.14(\ddot{\theta} r^2 + 2\dot{\theta}rr) - 2(\dot{\phi} rsin\theta q_{21} + \dot{\phi} \dot{r}rcos\theta q_{21} + \phi rsin\theta q_{22} + \phi rsin\theta q_{22}) - 2\dot{\theta}^2 r q_{11} \\
+ 1.56rr\dot{\theta} + 2\dot{\phi} \dot{r}rcos\theta q_{21} - 1.52\dot{r}^2 q_{11} - 1.06\dot{\theta}^2 q_{12} - rr\dot{q}_{11} - 1.3rr\dot{q}_{12} - 1.56\dot{\theta} r L_1 - 0.7\dot{\theta}rr \\
+ 2.6\dot{\phi} rsin\theta q_{22} - 2\phi rsin\theta q_{21} - 2\phi rsin\theta q_{11} + 1.56\phi^2 L_1 rsin\theta cos\theta + 1.14\phi^2 r^2 sin\theta cos\theta \right\} \\
+ \frac{m_f}{2} \left\{ 8(q_{11} - q_{12}) + 4\dot{\theta} (L_1 + r) + 4\dot{r} - 8(\phi \sin\theta + \phi \cos\theta) \right\} q_{21} - q_{22} \\
- 8\phi \sin\theta (q_{21} - q_{22}) - 8\phi^2 (q_{11} - q_{12}) + 4\dot{r}^2 + 8\phi \dot{\theta} \cos\theta (q_{21} - q_{22}) - 8(\dot{q}_{21} - \dot{q}_{22}) \phi \sin\theta \\
- 8\phi^2 \sin^2\theta (q_{11} - q_{12}) + 4\phi^2 \sin\theta \cos\theta (L_1 + r) \right\} = -0.68m_f^2 g \cos\theta - 2m_p g \cos\theta - \frac{12.362}{r^3} EI q_{11} \\
- \rho A_2 (\phi^2 \cos^2\theta + \dot{\phi}^2) \left\{ L_1 + \frac{r}{2} \right\} \left\{ \frac{4.65}{r} q_{11} - \frac{7.38}{r} q_{12} \right\} \\
+ \rho A_2 (\phi^2 \cos^2\theta + \dot{\phi}^2) \left\{ (3.08L_1 + 1.13r) q_{11} - (6.96L_1 + 3r) q_{12} \right\} \\
- \left[ m_f (L_1 + r)(\phi^2 \cos^2\theta + \dot{\phi}^2) + (m_p + \rho A_2 r^2) g \sin\theta \right] \left\{ \frac{4.65}{r} q_{11} - \frac{7.38}{r} q_{12} \right\} \\
+ \rho A_2 g \sin\theta \left[ 3.08q_{11} - 6.96q_{12} \right]
\]
The equation of motion for \( q_{12}(t) \) is,

\[
\begin{align*}
\rho A_2 \left( r^2 + r^2 \right) (0.65q_{11} + 0.5q_{12}) + \rho A_2 \left( r^2 \right) (0.65q_{11} + 0.5q_{12}) &+ \ddot{q}_{12} + \ddot{r}_{12} + 0.43L_1 (\dot{\theta} r + \dot{\theta} r) \\
+ 0.09(2\dot{r}r + \dot{\theta} r^2) - (\dot{\theta} \sin \theta q_{22} + \dot{\theta} \cos \theta q_{22} + \dot{\theta} r \sin \theta q_{22} + \dot{\theta} r \cos \theta q_{22}) &- \dot{\theta} r q_{12} + 0.43r \dot{r} \dot{\theta} \\
+ r \dot{\theta} \cos \theta q_{22} - 0.53r^2 q_{11} &- 4.34r^2 q_{12} + 0.65rr \dot{q}_{11} - 0.5rr \dot{q}_{12} - 0.43r \dot{\theta} L_1 + 0.25rr \dot{\theta} \\
- 1.3(\dot{r} \sin \theta q_{22} - r^2 \sin \theta q_{12} + 0.43r \dot{\theta}^2 \sin \theta \cos \theta L_1 + 0.09r \dot{\theta}^2 \sin \theta \cos \theta) &
\end{align*}
\]

\[
+ m_r \left[ 4(q_{12} - q_{11}) - 2(\dot{\theta} (L_1 + r) + \dot{\theta} r) - 4(\dot{\theta} \sin \theta + \dot{\theta} \cos \theta)(q_{22} - q_{21}) - 4 \sin \theta (q_{22} - q_{21}) \\
- 4 \dot{\theta}^2 (q_{12} - q_{11}) - 2\dot{r} \dot{\theta} + 4 \dot{\theta} \cos \theta(q_{22} - q_{21}) - 4 \sin \theta (q_{22} - q_{21}) - 4 \dot{\theta}^2 \sin \theta (q_{12} - q_{11}) \\
- 2 \dot{\theta}^2 \sin \theta \cos \theta (L_1 + r) \right] = -1.43m_2 \frac{g \cos \theta}{r^3} + 2m_r \frac{g \cos \theta}{r^3}
\]

\[
- \rho A_2 \left( \dot{\theta}^2 \cos \theta + \dot{\theta} r \right) \left[ L_1 r + \frac{r^2}{2} \right] \left[ -\frac{7.38}{r} q_{11} + \frac{32.42}{r} q_{12} \right] \\
+ \rho A_2 \left( \dot{\theta}^2 \cos \theta + \dot{\theta} r \right) \left[ \left( -6.96L_1 + 3r \right) q_{11} + (23.77L_1 + 9.73r) q_{12} \right] \\
- \left[ m_r (L_1 + r) (\dot{\theta}^2 \cos \theta + \dot{\theta} r) + (m_r + \rho A_2 r) g \sin \theta \right] \left[ -\frac{7.38}{r} q_{11} + \frac{32.42}{r} q_{12} \right] \\
+ \rho A_2 g \sin \theta \left[ -6.96q_{11} + 23.77q_{12} \right]
\]
The equation of motion for $q_{21}(t)$ is,

$$\begin{align*}
\rho A_2 \left(0.5(r^2q_{21} + \ddot{r}q_{21} + \dot{r}q_{21}) - 0.65(r^2q_{22} + \ddot{r}q_{22} + \dot{r}q_{22}) + \dot{r}q_{21} + \ddot{r}q_{21}
+ \phi \sin\theta q_{11} + \phi \cos\theta q_{11} + \phi \sin\theta q_{11} + \phi \cos\theta q_{11}
- 0.78 \left[r^2\phi L_1 \cos\theta + r^2 \phi L_1 \sin\theta - r^2 \phi \cos\theta - r^2 \phi \sin\theta\right]
- \phi \cos^2\theta q_{21} - 0.78 \phi \cos\theta + r^2 \phi \cos\theta q_{11} + \phi \sin\theta(0.5r^2q_{11} - 0.65\dot{r}q_{11})
+ 0.78 \phi \sin\theta L_1 \phi \sin\theta + 0.57 r^2 \phi \sin\theta - r^2 \phi \sin^2\theta q_{21} - 0.76 r^2q_{21} - 0.53 r^2q_{22} - 0.53 r^2q_{22} - 0.65 \dot{r}q_{22}
- \phi \sin\theta(0.5r^2q_{11} + 0.65\dot{r}q_{11}) + 0.78 \phi \sin\theta q_{11} + 0.35 r^2 \phi \cos\theta + m_r \left\{4(q_{21} - q_{22})
+ 4(\dot{q}_{21} - \dot{q}_{22})\right\}
+ 4(\dot{q}_{21} - \dot{q}_{22})q_{21} + 4(\dot{q}_{21} - \dot{q}_{22})q_{22} - 2 \left[r^2 \phi \cos\theta + (L_1 + r) \phi \cos\theta\right]
- (L_1 + r) \phi \cos\theta - 4 \phi \cos^2 \theta q_{21} - 4 \phi \cos^2 \theta q_{22} - 2 \phi \cos\theta + 4 \phi \cos\theta q_{11} - 4 \phi \sin\theta q_{11} + 4 \phi \sin\theta q_{11} - 4 \phi \sin\theta q_{12}
+ 2(L_1 + r) \phi \sin\theta - 4 \phi \sin^2 \theta q_{21} - 4 \phi \sin^2 \theta q_{22}\right) = \frac{-12.362}{r^3} EI q_{21} - \rho A_4 (\phi^2 \cos^2 \theta + \phi^2) \left[L_1 r + \frac{r^2}{2}\right]
\times \left[4 \frac{r}{r} q_{21} - 7.38 \frac{r}{r} q_{22} \right] + \rho A_2 (\phi^2 \cos^2 \theta + \phi^2) \left[(3.08 L_1 + 1.13 r) q_{21} - (6.96 L_1 + 3 r) q_{22}\right]
- \left[m_r (L_1 + r) (\phi^2 \cos^2 \theta + \phi^2) + (m_r + \rho A_2 r) \phi \sin\theta\right] \left[4 \frac{r}{r} q_{21} - 7.38 \frac{r}{r} q_{22}\right]
\end{align*}$$
The equation of motion for \( q_{22}(t) \) is,

\[
\rho A_2 \left[ 0.65(r^2 q_{21} + \ddot{r} q_{21} + r \dddot{q}_{21}) + 0.5(r \dddot{q}_{22} + r^2 \dot{q}_{22} + r \dddot{q}_{22}) \\
+ r \dddot{q}_{22} + \dot{r} q_{22} + \dot{r} \dot{q}_{22} + \dot{\phi} r \sin \theta q_{12} + \dot{\phi} \dot{\phi} \cos \theta q_{12} + \dot{\phi} r \sin \theta q_{12} + \dot{\phi} r \sin \theta \dot{q}_{12} \right] \\
- 0.43 L_1 (r \dddot{\phi} \cos \theta + r \ddot{\phi} \cos \theta - \dot{r} \dot{\phi} \sin \theta) - 0.09 (2r \ddot{r} \ddot{\phi} \cos \theta + r^2 \dddot{\phi} \cos \theta - r^2 \ddot{\phi} \sin \theta) - r \dddot{\phi} \cos \theta q_{22} \\
- 0.43 r \dddot{\phi} \cos \theta + r \ddot{\phi} \cos \theta q_{12} + 0.43 r \ddot{\phi} L_1 \dot{\phi} \sin \theta + 0.09 r^2 \ddot{\phi} \sin \theta - r \dddot{\phi} \sin \theta q_{22} - 0.53 r^2 q_{21} \\
= 4.34 r^2 q_{22} + 0.65 r \dddot{q}_{22} + \dot{r} q_{22} + \dot{\phi} \sin \theta (1.3 r \dddot{q}_{11} + \dot{r} \dddot{q}_{11}) + 0.43 r L_1 \dot{\phi} \cos \theta - 0.25 r \dddot{\phi} \cos \theta \\
+ m_r \left[ 4(q_{22} - \ddot{q}_{22}) + 4(\dot{\phi} \sin \theta + \dot{\phi} \cos \theta) (q_{12} - \ddot{q}_{11}) + 4 \dot{\phi} \sin \theta (q_{12} - \ddot{q}_{11}) \right] \\
+ 2 \left[ r \ddot{\phi} \cos \theta + (L_1 + r) \ddot{\phi} \cos \theta - (L_1 + r) \dot{\phi} \sin \theta \right] - 4 \ddot{\phi} \cos \theta q_{22} + q_{21} \\
+ 2 \ddot{r} \ddot{\phi} \cos \theta + 4 \dot{\phi} \cos \theta (q_{12} - \ddot{q}_{11}) + 4 \dot{\phi} \sin \theta (q_{12} - \ddot{q}_{11}) - 2 \dot{\phi} (L_1 + r) \dot{\phi} \sin \theta - 4 \ddot{\phi} \sin \theta (q_{22} - q_{21}) \right] \\
= -\frac{485.519}{r^8} E I q_{22} - \rho A_2 (r^2 \cos^2 \theta + \dot{\theta}^2) \left[ L_1 r + \frac{r^2}{2} \right] \left\{ -\frac{7.38}{r} q_{21} + \frac{32.42}{r} q_{22} \right\} \\
+ \rho A_2 (r^2 \cos^2 \theta + \dot{\theta}^2) \left[ - (6.96 L_1 + 3 r) q_{21} + (23.77 L_1 + 9.73 r) q_{22} \right] \\
- \left[ m_r (L_1 + r) (r^2 \cos^2 \theta + \dot{\theta}^2) + (m_r + \rho A_2 r) g \sin \theta \right] \left\{ -\frac{7.38}{r} q_{21} + \frac{32.42}{r} q_{22} \right\} \\
+ \rho A_2 g \sin \theta \left\{ -6.96 q_{21} + 23.77 q_{22} \right\}
\]
APPENDIX B

THE CONSTRAINED EQUATIONS OF MOTION

FOR $\theta$ AND $r$

Only the listing of the affected equations is shown here (i.e. $r$ and $\theta$ equations). The constrained equation of motion for $r$ on the lower thread is

\[
\left\{ (m_2 + m_r + m_H) \dot{\theta} \sin \theta - \left[ m_2 \left( L_1 + r - \frac{L_2}{2} \right) + m_r (L_1 + r) \right] + m_r (L_1 + r) \right. \\
+ m_H \left( L_1 + r - L_2 \right) \left( \dot{\theta}^2 + \dot{\phi}^2 \cos^2 \theta \right) + 2 \dot{\phi} \cos \theta \left[ (0.78 \dot{q}_{21} + 0.43 \dot{q}_{22}) m_2^\prime \right. \\
- 2 \ddot{\phi} \left[ m_2^\prime (0.78 \dot{q}_{11} + 0.43 \dot{q}_{12}) + 2 m_r (\dot{q}_{11} - \dot{q}_{12}) \right] + \ddot{\phi} \cos \theta \left[ m_2^\prime (0.78 \dot{q}_{21} + 0.43 \dot{q}_{22}) \\
+ 2 m_r (\dot{q}_{21} - \dot{q}_{22}) \right] + (\dot{\phi}^2 \sin \theta \cos \theta - \ddot{\theta}) \left[ m_2^\prime (0.78 \dot{q}_{11} + 0.43 \dot{q}_{12}) \\
+ 2 m_r (q_{11} - q_{12}) \right] \right) \left[ \tan \psi_1 + \mu \text{sgn}(r) \right] + \left\{ \frac{4 \pi}{I_d \mu} J_{ij} (1 - \mu \tan \psi_1) \\
+ (m_2 + m_r + m_H) (\mu + \tan \psi_1) \right\} r \right. \\
+ \left. \left\{ 2 \rho A_2 \dot{\phi} \cos \theta (0.78 \dot{q}_{21} + 0.43 \dot{q}_{22}) \\
- 2 \ddot{\phi} A_2 (0.78 \dot{q}_{11} + 0.43 \dot{q}_{12}) \right\} (\tan \psi_1 + \mu) = \frac{2 T_1}{d_m} (1 - \mu \tan \psi_1 \text{sgn}(r)) \right. \\
\]

where $m_2^\prime$ is the mass of the part of the second beam protruding from the first beam and $\mu$ is the dynamic coefficient of friction.
The constrained equation of motion for $r$ on the upper thread is,

$$
\left\{ (m_2 + m_p + m_{H_1})g \sin \theta - \left[ m_2 \left( L_1 + r - \frac{L_2}{2} \right) + m_p \left( L_1 + r - \frac{L_2}{2} \right) \right] + m_p \left( L_1 + r - \frac{L_2}{2} \right) \right\} \dot{\phi} \cos \theta + \dot{\theta}^2 + 2 \dot{\phi} \cos \theta \left[ (0.78 \dot{q}_{21} + 0.43 \dot{q}_{22}) m_2^2 + 2 m_p \left( \dot{q}_{21} - \dot{q}_{22} \right) \right] - 2 \ddot{\theta} \left[ m_2^2 \left( 0.78 \dot{q}_{11} + 0.43 \dot{q}_{12} \right) + 2 m_p \left( \ddot{q}_{11} - \ddot{q}_{12} \right) \right] + \dot{\phi} \cos \theta \left[ m_2^2 \left( 0.78 \dot{q}_{21} + 0.43 \dot{q}_{22} \right) + 2 m_p \left( \dot{q}_{21} - \dot{q}_{22} \right) \right] + \ddot{m}_p \left( 0.78 \dot{q}_{11} + 0.43 \dot{q}_{12} \right)
$$

$$
+ 2 m_p \left( \dot{q}_{11} - \dot{q}_{12} \right) \right\} (\tan \psi_1 - \mu \text{sgn}(r)) + \left\{ \frac{4 \pi}{l d_a} J_i (1 - \mu \tan \psi_1) + (m_2 + m_p + m_{H_1}) (\mu + \tan \psi_1) \right\} \ddot{r} + \left\{ 2 \rho A \frac{r \dot{\phi} \cos \theta (0.78 \dot{q}_{21} + 0.43 \dot{q}_{22})}{d_a} - 2 \dot{r} \rho A \frac{(0.78 \dot{q}_{11} + 0.43 \dot{q}_{12})}{d_a} \right\} (\mu + \tan \psi_1) = \frac{2 T_i}{d_a} (1 + \mu \tan \psi_1 \text{sgn}(r)).
$$

An intermediate result for the constrained equation of motion for $\theta$ is presented here to define the quantity $R_{st}$,

$$
R_{st} = m_{H_2} \dot{\theta} - \left[ \frac{a}{2} m_1 L_1^2 + m_1 \frac{L_1^2}{12} + m_1 d_1 + m_2 \left( L_1 + r - \frac{L_2}{2} \right) \right] + m_{H_2} \left( L_1 + r - \frac{L_2}{2} \right)
$$

$$
+ m_2 \left( L_1 + r \right) \dot{\phi} \cos \theta - \ddot{\theta}^2 \sin \theta \cos \theta \left[ \frac{m_1 L_1^2}{12} + c + m_2 \left( L_1 + r - \frac{L_2}{2} \right)^2 \right]
$$

$$
+ \frac{m_2 L_2^2}{12} + \frac{m_1 \left( L_1^2 - a \right)}{2} + m_{H_2} \left( L_1 + r - L_2 \right)^2 + m_p \left( L_1 + r \right)^2 \right] - \frac{\rho A}{a} \left[ 0.78 \dot{r} L_1 \dot{q}_{11} + 1.56 \dot{r} L_1 \dot{q}_{12} + 0.43 \ddot{r} L_1 q_{12} + 0.86 \ddot{r} L_1 \dot{q}_{12} + 0.78 r L_2 q_{11}
$$

$$
+ 0.43 r L_2 \dot{q}_{11} - 0.43 \left( r \dot{q}_{11} + \ddot{r} q_{11} + \ddot{r} q_{11} \right) - 0.68 \left( r \dot{q}_{12} + \ddot{r} q_{12} + \ddot{r} q_{12} \right)
$$

$$
+ 0.57 \left( 2 \ddot{r} q_{11} + r \dot{q}_{11} \right) + 0.09 \left( 2 \ddot{r} q_{12} + r \dot{q}_{12} \right) - 2 \frac{m_p}{a} \left( \left( r + L_1 \right) \left( \ddot{q}_{11} - \ddot{q}_{12} \right) - \ddot{r} \left( \dot{q}_{11} - \dot{q}_{12} \right) \right)
$$

$$
+ \frac{2 \dot{\phi} A \sin \theta}{a} \left[ 0.78 r L_1 + 0.57 r^2 \right] \dot{q}_{21} + (0.43 r L_1 + 0.09 r^2) \dot{q}_{22} + (0.78 r L_1 + 0.35 r) \dot{q}_{21}
$$

$$
+ (0.43 L_1 \dot{r} - 0.25 r r \dot{r}) \dot{q}_{22} \right] + \frac{4 \dot{m}_p \sin \theta}{a} \left( L_1 + r \right) \left( \dot{q}_{21} - \dot{q}_{22} \right).
$$
\[\begin{align*}
&+ \frac{\rho A_2 \sin \theta}{a} \left[ (0.78rL_1 + 0.57r^2)q_{21} + (0.43rL_1 + 0.09r^2)q_{22} \right] \\
&+ \frac{2m_f}{a} \ddot{\phi} \sin \theta \left[ (q_{21} - q_{22})(L_1 + r) \right] + \frac{\rho A_2}{a} \phi^2 \sin^2 \theta \left[ (0.78rL_1 + 0.57r^2)q_{11} \right] \\
&+ (0.43rL_1 + 0.09r^2)q_{12} \right] + \frac{2m_f}{a} \phi^2 \sin^2 \theta (q_{11} - q_{12})(L_1 + r) \\
&- \frac{\rho A_2}{a} \phi^2 \cos^2 \theta \left[ (0.78rL_1 + 0.57r^2)q_{11} + (0.43rL_1 + 0.09r^2)q_{12} \right] \\
&- \frac{2m_f}{a} \phi \cos^2 \theta (L_1 + r)(q_{11} - q_{12}) + \frac{2\phi \rho A_2 \cos \theta}{a} \left[ r(q_{11}\ddot{q}_{21} + q_{12}\ddot{q}_{22}) + r(0.5q_{11}q_{21} + 0.5q_{12}q_{22} \\
&+ 0.65q_{12}q_{21} - 0.65q_{11}q_{22}) \right] + \frac{8m_f}{a} \phi \cos \theta \left[ q_{11}\ddot{q}_{21} - q_{11}\ddot{q}_{22} - q_{12}\ddot{q}_{21} + q_{12}\ddot{q}_{22} \right] \\
&+ \frac{\phi \rho A_2 \cos \theta}{a} r(q_{11}\ddot{q}_{21} + q_{12}\ddot{q}_{22}) + \frac{4m_f}{a} \phi \cos \theta \left[ q_{11}\ddot{q}_{21} - q_{12}\ddot{q}_{21} - q_{11}\ddot{q}_{22} + q_{12}\ddot{q}_{22} \right] \\
&+ \frac{m_f}{a} \phi^2 \sin \theta \cos \theta (q_{11}^2 + q_{12}^2) + \frac{4m_f}{a} \phi \sin \theta \cos \theta \left[ q_{11}^2 + q_{12}^2 - 2q_{11}q_{12} \right]
\end{align*}\]

where \(m_f^L\) and \(m_f^R\) are the masses of the parts of the first beam lying to the left and to the right of the pivot point \(0\) respectively.

The constrained equation of motion for \(\theta\) on the upper thread is,

\[
\begin{align*}
R_a(\tan \psi_1 + \mu \text{sgn} \dot{\theta}) &+ \left\{ \frac{2\ddot{\phi}}{a} \left[ m_{H_1}(L_1 + r - L_2) + m_2 \left( L_1 + r - \frac{L_2}{2} \right) \right] \\
&+ m_f (L_1 + r) \right\} - \frac{8m_f}{a} \left[ q_{11}\ddot{q}_{11} - q_{11}\ddot{q}_{12} + q_{12}\ddot{q}_{12} - q_{12}\ddot{q}_{11} \right] - \frac{2\rho A_2 \dot{\theta}}{a} \left[ 0.5\ddot{r}(q_{11}^2 + q_{12}^2) \right] \\
&+ r(q_{11}\ddot{q}_{11} + q_{12}\ddot{q}_{12}) \right\} (\tan \psi_1 + \mu) + \frac{2T_2}{d_m} \left( 1 - \mu \text{sgn}(\dot{\theta}) \right) (\tan \psi_1) \\
&= \left\{ \frac{4\pi a}{d_m} J_{12} (1 - \mu \tan \psi_1) \right\} + \frac{m_{H_2} a}{\sin \psi_1 \cos \psi_1} + \left[ \frac{1}{a} \left( m_1 \left( L_1^2 - a \right) + \frac{m_2 L_1^2}{12} \right) + c \\
&+ m_2 \left( L_1 + r - \frac{L_2}{2} \right) \right] + \frac{2m_2 L_2}{12} + m_{H_1}(L_1 + r - L_2)^2 + m_f (L_1 + r)^2 \\
&+ \frac{m_f}{a} (q_{11}^2 + q_{12}^2) + \frac{4m_f}{a} (q_{11}^2 + q_{12}^2 - 2q_{11}q_{12}) \right\} \ddot{\theta}
\end{align*}\]

where \(J_{12}\) is the mass moment of inertia of the second joint leadscrew around its axis.
The constrained equation of motion for $\theta$ on the lower thread is,

\[
R_m(tan\psi_1 - \mu sgn(\dot{\theta})) + \left\{ \frac{-2r\dot{\theta}}{a} \left[ m_H^2 (L_1 + r - L_2) + m_2 \left( L_1 + r - \frac{L_2}{2} \right) + m_r (L_1 + r) \right] \right. \\
- \frac{8m_r}{a} \left[ q_{11} \ddot{q}_{11} - q_{11} \ddot{q}_{12} + q_{12} \ddot{q}_{12} - q_{12} \ddot{q}_{11} \right] - \frac{2Ap}{a} \left[ 0.5 \dot{r}(q_{11}^2 + q_{12}^2) \\
+ r(q_{11} \ddot{q}_{11} + q_{12} \ddot{q}_{12}) \right] (\mu + tan\psi_1) + \frac{2T^2}{d_m} (1 + \mu tan\psi_1 sgn(\dot{\theta})) \\
= \left\{ \frac{m_H^2}{\sin\psi_1 \cos\psi_1} + \left[ \frac{1}{a} \left( \frac{m_1 L_1^2}{12} + c + m_2 (L_1 + r - \frac{L_2}{2}) \right)^2 + m_1 \left( L_1^2 - a^2 \right) \right. \\
+ \frac{m_2 L_2^2}{12} + \dot{m}_H(L_1 + r - L_2)^2 + m_r (L_1 + r)^2 \right) + \frac{m_r^2}{a} (q_{11}^2 + q_{12}^2) \\
+ \frac{4m_r}{a} (q_{11}^2 + q_{12}^2 - 2q_{11}q_{12}) (\mu + tan\psi_1) + \frac{4\pi a}{Id_m} J_m (1 - \mu tan\psi_1) \right\} \ddot{\theta}.
\]
APPENDIX C

THE RIGID BODY EQUATIONS OF MOTION

OF THE ROBOT ARM

In an attempt to check the work done for the general case, the equations of
the rigid body motion of the robot arm are derived. The system is now greatly
simplified since it has only three degrees of freedom (i.e. r, θ and φ). The robot
arm is treated as an assembly of rigid bodies whose mass centers become impor-
tant in the derivation of their kinetic energies (see Fig. 2-1). For the first beam,
one has,

\[ R_1' = \left( \frac{L_1}{2} - a \right) i \]

where the star superscript indicates a value related to the center of mass. \( L_1 \)
is the total length of the first beam and \( a \) is the length of the portion of the first
beam lying to the left of the pivot point 0. The position vectors of the mass
center of the second beam and the payload are,

\[ R_2' = \left( L_1 + r - \frac{L_2}{2} \right) i \]  \hspace{1cm} (C-2)

\[ R_p = (L_1 + r) i \]  \hspace{1cm} (C-3)

where \( L_1 \) is the length of the portion of the first beam lying to the right of the
pivot point 0. \( r \) is the length of the part of the second beam protruding from the first beam and \( L_2 \) is the total length of the second beam. The position vectors of the mass center of the leadscrew and its housing that produce the translational motion of the second beam are,

\[
R'_{i,i} = d_{i,i}
\]

\[
R_{H,1} = (L_1 + r - L_2)_i
\]

where \( d_{i,i} \) is the distance from the origin of \((i, j, k)\) to the mass center of the leadscrew. The leadscrew housing is assumed to be a point mass located at the beginning of the second beam.

**KINETIC ENERGY**

The velocity terms can be derived from,

\[
\dot{R}'_{i,i} = \frac{dR'}{dt} + \Omega \times R',
\]

\[
\Omega = \dot{\phi}(\sin \theta i + \cos \theta j) + \dot{\theta} k
\]

where \( \Omega \) is the rotation vector of the \((i, j, k)\) basis and \( R' \) is the position vector of the mass center for the \( i^{th} \) body. The velocity terms are then used to develop the kinetic energy for each part of the robot arm including the payload, \( m_p \). The latter is considered to be a point mass, and does not include rotation around its own mass center. Since the first beam and the leadscrew that drives the second beam undergo fixed point rotation, then their kinetic energies can be defined by:
where $\mathbf{H}_1$ and $\mathbf{H}_2$ are the angular moments of momentum of the first beam and the lead screw respectively around point $O$. The kinetic energy of the second beam has rotational and translational parts,

\[ T_2 = \frac{1}{2} \mathbf{H}_2 \cdot \dot{\mathbf{H}}_2 + \frac{1}{2} \mathbf{\Omega} \cdot \mathbf{H}'_2 \]

where $\mathbf{H}'_2$ is the angular moment of momentum around the mass center of the second beam. Since the payload and the lead screw housing are considered to be point masses and do not undergo rotation about their own mass centers, then their kinetic energies would simply be written as,

\[ T_3 = \frac{1}{2} m_p \mathbf{\dot{R}}_p \cdot \mathbf{\dot{R}}_p + \frac{1}{2} m_{H_1} \mathbf{\dot{R}}_{H_1} \cdot \mathbf{\dot{R}}_{H_1} \]

where $m_{H_1}$ and $m_p$ are the masses of the lead screw housing and the payload respectively. The kinetic energy of the lead screw and its housing that are used to rotate the first beam around the pivot point $O$ can be obtained from

\[ T_4 = \frac{1}{2} \mathbf{\Omega}_1 \cdot \mathbf{H}'_{1_2} + \frac{1}{2} \mathbf{\Omega}_1 \cdot \mathbf{H}'_{H_2} \]

where $\mathbf{H}'_{1_2}, \mathbf{H}'_{H_2}$ are the angular moments of momentum of the lead screw and its housing respectively around point $O$. The expression for $\mathbf{\Omega}_1$ can be written as

\[ \mathbf{\Omega}_1 = \dot{\phi} \mathbf{J} \]

The total kinetic energy would then be
\[ T_i = \sum_{i=1}^{4} T_i \]  

(C-12)

**POTENTIAL ENERGY**

The datum line is chosen to be aligned with the \( \mathbf{j} \) unit vector. It is horizontal and passes through the point \( O \). The potential energy associated with the rigid body motion arises from the gravitational acceleration, \( g \), and is:

\[
V_t = \left\{ m_1 \left( \frac{L_1^2}{2} - a \right) + m_i d_i + m_H (L_1 + r - L_2) \right. \\
+ \left. m_2 \left( L_1 + r - \frac{L_2}{2} \right) + m_r (L_1 + r) \right\} g \sin \theta
\]  

(C-13)

**VIRTUAL WORK PRINCIPLE**

The virtual work principle is employed to obtain the generalized forces. If the forces exerted on the robot arm can be divided into conservative forces, which are directly related to the potential energy expression \( V_t \), and nonconservative forces which are not, then the virtual work term can be written as \([42]\):

\[
\delta W = \delta W_c + \delta W_{nc}
\]  

(C-14)

where the subscripts \( c \) and \( nc \) denote terms due to conservative and nonconservative forces respectively. \( \delta W_c \) is related to \( V_t \) as follows:

\[
\delta W_c = -\delta V_t = -\sum_{i=1}^{n} \frac{\partial V_t}{\partial z_i} \delta z_i
\]  

(C-15)

where \( z_i \) is the \( i^{th} \) generalized coordinate which, in this case, can be any of the rigid body degrees of freedom \( r, \theta \) and \( \phi \). \( \delta W_{nc} \) can be written in the following
\[ \delta W_{nc} = \sum_{i=1}^{n} Q_{inc} \delta x_i \]  

(C-16)

where \( Q_{inc} \) represent nonconservative generalized forces. \( \delta W_{nc} \) expression can be easily derived from the free body diagram depicted in Fig. 2-2.

\[
\delta W_{nc} = R \cdot \delta R' + F_i \cdot \delta R + T_{\phi} \cdot \delta \phi + P_{ext} \cdot \delta \phi \]

(C-17)

where \( \delta \phi \) is the virtual rotation vector of \( \phi \). Comparison of (C-16) and (C-17) yields,

\[
Q_{inc} = F_i \\
Q_{inc} = -aF_{ext} \\
Q_{\phi nc} = T_{\phi}
\]

(C-18)

LAGRANGE'S EQUATIONS

Now that the expressions for the nonconservative generalized forces, the total kinetic and potential energies are derived; the equations of motion can be obtained using the following form of Lagrange's equation,

\[
\frac{d}{dt} \left( \frac{\partial T_i}{\partial \dot{x}_i} \right) - \frac{\partial T_i}{\partial x_i} + \frac{\partial V_i}{\partial x_i} = Q_{inc}
\]

(C-19)

Substitute (C-12), (C-13) and (C-18) into (C-19) to get the following:

\[
(m_{H_1} + m_2 + m_r) \ddot{r} - (\dot{\phi}^2 \cos^2 \phi + \ddot{\phi}) \left[ m_{H_1}(L_1 + r - L_2) + m_2 \left( L_1 + r - \frac{L_2}{2} \right) + m_r (L_1 + r) \right] = F_i - g \sin \phi (m_2 + m_r + m_{H_1})
\]

(C-20)
The unconstrained equation of motion for $\theta$ is,

\[
\begin{aligned}
\left( \frac{m_1 L_1^2}{12} + m_1 \left( \frac{L_1}{2} - a \right)^2 \right) + c + m_{H_1}(L_1 + r - L_2)^2 + m_2 \left( L_1 + r - \frac{L_2}{2} \right)^2 + \frac{m_2 L_2^2}{12} \\
+ m_r (L_1 + r)^2 \frac{\ddot{\theta}}{g} + 2r \dot{\theta} \left( m_{H_1}(L_1 + r - L_2) + m_2 \left( L_1 + r - \frac{L_2}{2} \right) + m_r (L_1 + r) \right) \\
+ \phi^2 \sin \theta \cos \theta \left( \frac{m_1 L_1^2}{12} + m_1 \left( \frac{L_1}{2} - a \right)^2 \right) + c + m_{H_1}(L_1 + r - L_2)^2 \\
+ m_2 \left( L_1 + r - \frac{L_2}{2} \right)^2 + \frac{m_2 L_2^2}{12} + m_r (L_1 + r)^2 \right) = -aF_{\text{ext}} + g \cos \theta \left( m_1 \left( \frac{L_1}{2} - a \right) \\
+ m_r (L_1 + r - L_2) + m_2 \left( L_1 + r - \frac{L_2}{2} \right) + m_r (L_1 + r) \right)
\end{aligned}
\]

(C-21)

where $c$ is the mass moment of inertia of the prismatic joint leadscrew with respect to the $j$ and $k$ directions. Finally the unconstrained equation of motion for $\phi$ can be written as follows,

\[
\begin{aligned}
\ddot{\phi} \left[ \frac{m_1 L_1^2}{12} + m_1 \left( \frac{L_1}{2} - a \right)^2 \right] + \phi + m_{H_1}(L_1 + r - L_2)^2 + m_2 \left( L_1 + r - \frac{L_2}{2} \right)^2 \\
+ \frac{m_2 L_2^2}{12} + m_r (L_1 + r)^2 \cos^2 \theta + (m_{H_2} a^2 + L_2) \right) - 2 \dot{\phi} \cos \theta \sin \theta \left( \frac{m_1 L_1^2}{12} \\
+ m_1 \left( \frac{L_1}{2} - a \right)^2 \right) + m_{H_1}(L_1 + r - L_2)^2 + c + m_2 \left( L_1 + r - \frac{L_2}{2} \right)^2 \\
+ \frac{m_2 L_2^2}{12} + m_r (L_1 + r)^2 \right) + 2 \dot{\phi} \cos^2 \theta \left( m_{H_1}(L_1 + r - L_2) \\
+ m_2 \left( L_1 + r - \frac{L_2}{2} \right) + m_r (L_1 + r) \right) = T_3
\end{aligned}
\]

(C-22)
CONSTRAINED EQUATIONS OF MOTION

The only constraints, that are considered in this study, are the ones imposed by the leadscrew transmission mechanisms. Therefore, only the \( r \) and \( \theta \) equations will be affected. The constrained equations of motion can be obtained by applying the formulation derived in Chapter 3 on (C-20) and (C-21). The modified \( r \) equation would have the following form for upper thread motion

\[
\left\{- (m_2 + m_r + m_{H_1}) \sin \theta + (\dot{\phi}^2 \cos^2 \theta + \dot{\phi}^2) \right\} \left[ m_r (L_1 + r) + m_2 \left( L_1 + r - \frac{L_2}{2} \right) \right] \\
+ m_{H_1} (L_1 + r - L_2) \right\} (\tan \psi_1 - \mu \text{sgn}(\dot{r})) + \frac{2T_1}{d_m} (1 + \mu \tan \psi_1 \text{sgn}(\dot{r})) \\
= \left\{ (m_2 + m_r + m_{H_1}) (\tan \psi_1 + \mu) + \frac{4\pi}{ld_m} J_i (1 - \mu \tan \psi_1) \right\} \ddot{r}
\]

(C-23)

and for lower thread motion

\[
\left\{- (m_2 + m_r + m_{H_1}) \sin \theta + (\dot{\phi}^2 \cos^2 \theta + \dot{\phi}^2) \right\} \left[ m_r (L_1 + r) + m_2 \left( L_1 + r - \frac{L_2}{2} \right) \right] \\
+ m_{H_1} (L_1 + r - L_2) \right\} (\tan \psi + \mu \text{sgn}(\dot{r})) + \frac{2T_1}{d_m} (1 - \mu \tan \psi_1 \text{sgn}(\dot{r})) \\
= \left\{ (m_2 + m_r + m_{H_1}) (\tan \psi_1 + \mu) + \frac{4\pi}{ld_m} J_i (1 - \mu \tan \psi_1) \right\} \ddot{r}
\]

(C-24)

where \( \psi_1 \) is the helical angle of the thread, \( \mu \) is the dynamic coefficient of friction, \( T_1 \) is the applied control torque, \( d_m \) and \( l \) are the mean diameter and the lead of the leadscrew respectively. \( J_i \) is the mass moment of inertia of the prismatic joint leadscrew around its axis.
The constrained equation of motion for $\vartheta$ on the lower thread is:

$$\left\{ \begin{align*}
&\dot{\vartheta} = -\frac{g \cos \theta}{a} \left( -\frac{a}{2} m_1^2 + m_1^2 \frac{L_1}{2} + m_1 d_1 + m_{H_1}(L_1 + r - L_2) + m_2 \left( L_1 + r - \frac{L_2}{2} \right) \right) \\
&+ m_r (L_1 + r) \left( -\frac{g \sin \theta \cos \theta}{a} \left[ c + m_1 \frac{L_1^2}{12} + m_1 \left( \frac{L_1}{2} - c \right)^2 + m_{H_1}(L_1 + r - L_2)^2 \right] \\
&+ \frac{m_2 L_2^2}{12} + m_2 \left( L_1 + r - \frac{L_2}{2} \right) + m_r (L_1 + r)^2 \right) \left( \tan \psi_1 - \mu \text{sgn}(\dot{\vartheta}) \right) \\
&- \frac{2 r \dot{\vartheta}}{a} \left[ m_{H_1}(L_1 + r - L_2) + m_2 \left( L_1 + r - \frac{L_2}{2} \right) + m_r (L_1 + r) \right] \left( \tan \psi_1 + \mu \right) \\
&+ \frac{2 T_2}{d_m} (1 + \mu \tan \psi_1 \text{sgn}(\dot{\vartheta})) = \left\{ -\frac{m_{H_1} a}{\sin \psi_1 \cos \psi_1} + \frac{1}{a} \left[ c + \frac{m_1 L_1^2}{12} + m_1 \left( \frac{L_1}{2} - c \right)^2 \right] \\
&+ m_{H_1}(L_1 + r - L_2)^2 + \frac{m_2 L_2^2}{12} + m_2 \left( L_1 + r - \frac{L_2}{2} \right)^2 + m_r (L_1 + r)^2 \right) \left( \tan \psi_1 + \mu \right) \\
&+ \frac{4 \pi a}{d_m} J_{\psi_1} (1 - \mu \tan \psi_1) \right\} \ddot{\vartheta}
\end{align*} \right.$$  

(C-25)

The constrained equation of motion for $\vartheta$ on the upper thread is:

$$\left\{ \begin{align*}
&\dot{\vartheta} = -\frac{g \cos \theta}{a} \left( -\frac{a}{2} m_1^2 + m_1^2 \frac{L_1}{2} + m_1 d_1 + m_{H_1}(L_1 + r - L_2) + m_2 \left( L_1 + r - \frac{L_2}{2} \right) \right) \\
&+ m_r (L_1 + r) \left( -\frac{g \sin \theta \cos \theta}{a} \left[ c + m_1 \frac{L_1^2}{12} + m_1 \left( \frac{L_1}{2} - c \right)^2 \right] \\
&+ \frac{m_2 L_2^2}{12} + m_2 \left( L_1 + r - \frac{L_2}{2} \right)^2 + m_{H_1}(L_1 + r - L_2)^2 + m_r (L_1 + r)^2 \right) \left( \tan \psi_1 + \mu \text{sgn}(\dot{\vartheta}) \right) \\
&- \frac{2 r \dot{\vartheta}}{a} \left[ m_{H_1}(L_1 + r - L_2) + m_2 \left( L_1 + r - \frac{L_2}{2} \right) + m_r (L_1 + r) \right] \left( \tan \psi_1 + \mu \right) \\
&+ \frac{2 T_2}{d_m} (1 - \mu \tan \psi_1 \text{sgn}(\dot{\vartheta})) = \left\{ -\frac{m_{H_1} a}{\sin \psi_1 \cos \psi_1} + \frac{1}{a} \left[ c + \frac{m_1 L_1^2}{12} + \frac{m_2 L_2^2}{12} \right] \\
&\right.  
\end{align*} \right.$$  

\[ \text{(C-26)} \]
+ m_1 \left( \frac{L_1}{2} - a \right)^2 + \frac{4\pi a}{ld_\alpha} J_{\alpha} (1 - \mu \tan \psi') \delta

(C-26)

where \( m_f \) and \( m_r \) are the masses of the parts of the first beam lying to the left and to the right of the pivot point 0 respectively. \( J_z \) is the mass moment of inertia of the second joint lead screw around its axis.
APPENDIX D

RIGID BODY CONTROLLER GAIN MATRICES

Evaluation of equation (4-5) leads to the following general form for the $A$ and $B$ matrices,

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & A_{42} & 0 & 0 & 0 \\ A_{51} & A_{52} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & B_{41} & 0 \\ 0 & B_{51} & 0 \\ 0 & B_{52} & 0 \end{bmatrix}$$

Upper thread motion for $r$ and $\theta$ yields the following entries for $A$ and $B$ matrices,

$$A_{42} = \frac{- (m_2 + m_r + m_H) \cos \theta \left( \tan \psi_1 - \mu \text{sgn}(r) \right)}{(m_2 + m_r + m_H) \left( \tan \psi_1 + \mu \right) + \frac{4\pi}{l_{\text{d}}}} J_i (1 - \mu \tan \psi_1)$$

$$A_{51} = \left\{ \begin{array}{l} - \frac{g \cos \theta}{a} \left( m_{H_1} + m_2 + m_r \right) \left( \tan \psi_1 + \mu \text{sgn}(\dot{\theta}) \right) \alpha_{2t} - \frac{2}{a} \left[ m_{H_1} (L_1 + r_t - L_2) ight. \\
+ m_2 \left( L_1 + r_t - \frac{L_2}{2} \right) + m_r (L_1 + r_t) \right] \left[ m_{H_2} - \frac{g \cos \theta}{a} \left[ \frac{m_1^2 L_1}{2} - \frac{a}{2} \right] m_1 + m_1^2 \right] \\
+ m_{H_1} (L_1 + r_t - L_2) + m_2 \left( L_1 + r_t - \frac{L_2}{2} \right) + m_r (L_1 + r_t) \right\} \right\}$$

$$\times \left( \tan \psi_1 + \mu \right) \left( \tan \psi_1 + \mu \text{sgn}(\dot{\theta}) \right) / \alpha_{2t}^2.$$
\[ A_{\theta 2} = \left\{ \frac{g \sin \theta}{a} \left[ \frac{m_2 L_1}{2} + m_1 d_1 + m_H (L_1 + r_e - L_2) + m_2 \left( L_1 + r_e - \frac{L_2}{2} \right) \right] - \frac{a}{2} m_T + m_p (L_1 + r_e) \right\} \left[ \tan \psi_1 + \mu \text{sgn}(\hat{r}) \right] / \alpha_{2\theta} \]

\[ B_{41} = \frac{2(1 + \mu \tan \psi_1 \text{sgn}(\hat{r}))}{d_m \left( m_2 + m_p + m_H \right) (\tan \psi_1 + \mu) + \frac{4\pi}{id_m} J_i(1 - \mu \tan \psi_1)} \]

\[ B_{\theta 2} = \frac{2(1 - \mu \tan \psi_1 \text{sgn}(\hat{r}))}{\alpha_{2\theta} d_m} \]

where

\[ \alpha_{2\theta} = \frac{m_H a}{\sin \psi_1 \cos \psi_1} + \frac{1}{a} \left[ c + \frac{m_1 L_1^2}{12} + m_H (L_1 + r_e - L_2)^2 + \frac{m_2 L_2^2}{12} + m_1 \left( \frac{L_1}{2} - a \right)^2 \right. \]

\[ + \left. m_2 \left( L_1 + r_e - \frac{L_2}{2} \right)^2 + m_p (L_1 + r_e)^2 \right\} (\tan \psi_1 + \mu) + \frac{4\pi a}{id_m} J_i(1 - \mu \tan \psi_1) \]

Any variable with a subscript \( \epsilon \) should be evaluated at the equilibrium point.

Lower thread motion for \( r \) and \( \theta \) gives the following entries for \( A \) and \( B \) matrices.

\[ A_{\theta 2} = \frac{-(m_2 + m_p + m_H \mu \cos \psi_1 (\tan \psi_1 + \mu \text{sgn}(\hat{r}))}{(m_2 + m_p + m_H) (\tan \psi_1 + \mu) + \frac{4\pi}{id_m} J_i(1 - \mu \tan \psi_1)} \]

\[ A_{\theta 1} = \left\{ \frac{\mu \cos \psi_1}{a} \right\} \left[ m_H^2 + m_2 + m_p \right] (\tan \psi_1 - \mu \text{sgn}(\hat{r})) \alpha_{2\theta} - \frac{2}{a} \left[ m_H (L_1 + r_e - L_2) + m_2 \left( L_1 + r_e - \frac{L_2}{2} \right) + m_p (L_1 + r_e) \right] \left[ m_H^2 - \frac{\mu \cos \psi_1}{a} \left[ \frac{m_1^2 L_1}{2} - \frac{a m_T^2}{2} + m_1 d_1 \right. \right. \]

\[ \left. + m_H (L_1 + r_e - L_2) + m_2 \left( L_1 + r_e - \frac{L_2}{2} \right) + m_p (L_1 + r_e) \right] \left( \tan \psi_1 + \mu \right) (\tan \psi_1 - \mu \text{sgn}(\hat{r}))} / \alpha_{2\theta}^2 \]
\[ A_{\theta 2} = \left\{ \frac{g \sin \theta_e}{a} \left[ \frac{m_1^2 L_1}{2} + m_1 d_{11} + m_{H1}(L_1 + r_e - L_2) + m_2 \left( L_1 + r_e - \frac{L_2}{2} \right) \right] - \frac{a}{2} m_1^r + m_r (L_1 + r_e) \right\} (\tan \psi_1 - \mu \text{sgn}(\dot{\theta})) / a_{2r} \]

\[ B_{41} = \frac{2(1 - \mu \tan \psi_1 \text{sgn}(\dot{r}))}{d_m \left[ (m_2 + m_r + m_{H1})(\tan \psi_1 + \mu) + \frac{4\pi}{l_d} J_1(1 - \mu \tan \psi_1) \right]} \]

\[ B_{62} = \frac{2(1 + \mu \tan \psi_1 \text{sgn}(\dot{\theta}))}{a_{2r} d_m} \]

Since \( \phi \) is driven by a gear train, then one value for \( B_{63} \) is obtained,

\[ B_{63} = \frac{1}{a_{3r}} \]

where

\[ a_{3r} = \left[ \frac{m_1 L_1^2}{12} + m_1 \left( \frac{L_1}{2} - r \right)^2 + c + m_{H1}(L_1 + r_e - L_2)^2 + m_2 \left( L_1 + r_e - \frac{L_2}{2} \right)^2 \right. \]

\[ + \frac{m_2 L_2^2}{12} + m_r (L_1 + r_e)^2 \left. \cos^2 \theta_e + m_{H2} a_2^2 + J_{1z} \right] \]

The gain matrix of the rigid body controller is,

\[ K = \begin{bmatrix} k_{11}^s & k_{12}^s & 0 & 0 & k_{14}^s & 0 & 0 & 0 \\ k_{21}^s & k_{22}^s & 0 & 0 & k_{24}^s & 0 & 0 & k_{22}^l \\ 0 & 0 & k_{33}^s & 0 & 0 & k_{55}^s & 0 & 0 \end{bmatrix} \]
where

\[
K_{i1}^1 = \frac{W_{i1}^2 + 2P_{d1}\xi_{d1} W_{d1}}{B_{d1}}
\]

\[
K_{i2}^2 = \frac{W_{i2}^2 + 2P_{d2}\xi_{d2} W_{d2} + A_{d2}}{B_{d2}}
\]

\[
K_{i3}^3 = \frac{W_{i3}^2 + 2P_{d3}\xi_{d3} W_{d3}}{B_{d3}}
\]

\[
K_{i1}^2 = \frac{A_{d2}}{B_{d1}}
\]

\[
K_{i2}^3 = \frac{A_{d1}}{B_{d2}}
\]

\[
K_{i4}^4 = \frac{2\xi_{d1} W_{d1} + P_{d1}}{B_{d4}}
\]

\[
K_{i5}^5 = \frac{2\xi_{d2} W_{d2} + P_{d2}}{B_{d5}}
\]

\[
K_{i6}^6 = \frac{2\xi_{d3} W_{d3} + P_{d3}}{B_{d6}}
\]

\[
K_{i1}^1 = \frac{P_{d1} W_{d1}^2}{B_{d1}}
\]

\[
K_{i2}^2 = \frac{P_{d2} W_{d2}^2}{B_{d2}}
\]

\[
K_{i3}^3 = \frac{P_{d3} W_{d3}^2}{B_{d3}}
\]

where \(\xi_{d}\) and \(W_{d}\) are the desired damping ratio and servo loop frequency. \(P_{d}\) is the pole assigned for the integral controller.
APPENDIX E

SCHEMATIC OF THE DOUBLE INTEGRATOR CIRCUIT
APPENDIX F

LISTING OF THE COMPUTER CODE

FOR THE DIGITAL SIMULATION
C********** MAIN PROGRAM IDENTIFICATION *********

C
C
C THIS PROGRAM IS WRITTEN TO SIMULATE THE RIGID AND FLEXIBLE
C MOTION OF A ROBOT ARM. SEVEN DEGREES OF FREEDOM ARE USED IN
C THIS STUDY. THE FIRST THREE DEGREES OF FREEDOM, DENOTED Y(1)
C THROUGH Y(3), ARE USED TO SIMULATE THE RIGID BODY MOTION,
C WHILE THE LAST FOUR DEGREES OF FREEDOM, Y(4) THROUGH Y(7), ARE
C USED TO SIMULATE THE FLEXIBLE MOTION WHICH IS TRUNCATED AFTER
C ITS SECOND MODE. THE SEVEN HIGHLY NONLINEAR SECOND ORDER
C ORDINARY DIFFERENTIAL EQUATIONS OF MOTION ALONG WITH THE STATE
C FEEDBACK CONTROLLER EQUATIONS ARE TRANSFORMED TO SEVENTEEN FIRST
C ORDER ORDINARY DIFFERENTIAL EQUATIONS. THE LATTERS ARE THEN
C SOLVED SIMULTANEOUSLY USING GEAR'S METHOD.

C******************************************************************************

***** FUNCTION IDENTIFICATION *****

C
C THIS FUNCTION EVALUATES THE SGN FUNCTION
C OF THE ABSOLUTE VALUE OF ITS ARGUMENT.

C******************************************************************************

FUNCTION ASGN(X)
REAL*8 X,ASGN,EPS1
ASGN=0.000
EPS1=0.0000000000000001
IF(DABS(X).GT.EPS1) GO TO 100
ASGN=0.000
GO TO 101
100 ASGN=1.000
101 RETURN
END

C******************************************************************************

C
C THIS FUNCTION EVALUATES THE SGN FUNCTION OF
C ITS ARGUMENT.

C******************************************************************************

FUNCTION SGN(X)
REAL*8 X,SGN,EPS
SGN=0.000
EPS=0.0000000000000001
IF(X.GT.EPS) GO TO 151
IF(DABS(X).LE.EPS)GO TO 153
SGN=-1.000
GO TO 152
153 SGN=0.000
GO TO 152
151  SON=1.000
152  RETURN
END

******************************************************************************

********** MAIN PROGRAM BODY **********

REAL*8 TOUT, EPS, DM, H, Y(17), T0, KS11N, KS14N, IK11N
REAL*8 WNR, WNTETA, WNFETA, POLE1, POLE2, POLE3, ZETA1
REAL*8 ZETA2, ZETA3, MU, MH1, M1, M2, MP, L1, L2, KSY1
REAL*8 C1, C2, C3, J1L, L1, R1, R2, R3, KS12P, KS21P, KS21N, TT3, TT4
REAL*8 A51P, A2E, TH, M1L, M1R, LT, IL2, CONST2, CONST3
REAL*8 RDOMAX, TDOMAX, RDPER, TDPER, ER1PER, ER2PER, TA1, TA2, A52P
REAL*8 ATT1, ATT2, TT1, MUS, R11, A42N, B1N, A51N, A52N, A2, ROW, EI
REAL*8 TW2, B2N, A42P, CONST, TW3, S
REAL*8 A, AA1, AA2, AA3, AA4, INERTI, ML, DL, TLIMIT
REAL*8 ATN1, ATN2, ABN1, ABN2, PHEE1, PHEE2, V, W

COMMON /GEAR9/ NSTEP, NFE, NJE
  1 IK22P, KS33, KS36, IK33, KS12P, KS21P
COMMON /GAINKN/ KS11N, KS14N, IK11N, KS22N, KS25N,
  1 IK22N, KS12N, KS21N
COMMON /CONTL/ C1, C2, C3
COMMON /DATA/ M1, M2, MP, MH1, L1, L2, MU, G, KSY1, M1L, M1R, LT, IL2
  1 R1, R2, R3, LO, J1L, L, DM, TT3, TT4
COMMON /DATA1/ ATT1, ATT2, MUS, CONST2, CONST3
  1 ML, DL, INERTI, TT1, TA1, TA2
COMMON /SINGUL/ S, A2, ROW, EI
COMMON /DT/ RDOMAX, TDOMAX, RDPER, TDPER, ER1PER, ER2PER, CONST

REAL*4 TI(2010), Y12(2010), X(2010), AT1, YA(2010), Z(2010), ATTT
REAL*4 D1(2010), D2(2010)

DATA YES, NO/3YES, 2HND/
WRITE(7,10)
10  FORMAT(' 1',.3X,'TIME',.9X,'r',.14X,'THETA',.11X,'PHY',.11X,
1  'Q1(T)',.10X,'Q12(T)',.9X,'Q21(T)')

READ(8,71)R1, R2, R3, Y(1), Y(2)
READ(8,71)Y(3), Y(4), Y(5), Y(6), Y(7)
71  FORMAT(5F10.5)
72  FORMAT(6F10.5)
READ(8,72)WNR, WNTETA, WNFETA, POLE1, POLE2, POLE3
**DATA**

TT4=0.00D
TT3=0.00D
TT1=0.00D
TA1=0.00D
TA2=0.00D
TH=0.00D
N=1700
I=000
T0=0.00D
H=0.000001D0

**INITIAL CONDITION OF THE STATE VECTOR.**

\[ \text{CONST} = y(1) + (R1 - y(1))/2.00D \]
\[ \text{CONST}2 = y(2) + (R2 - y(2))/2.00D \]
\[ \text{CONST}3 = y(3) + (R3 - y(3))/2.00D \]
TLIMIT = 0.01D0
Y(8) = 0.00D
Y(9) = 0.00D
Y(10) = 0.00D
Y(11) = 0.00D
Y(12) = 0.00D
Y(13) = 0.00D
Y(14) = 0.00D
Y(15) = 0.00D
Y(16) = 0.00D
Y(17) = 0.00D

**DESIRMED SYSTEM CHARACTERISTICS.**

TOUT = 0.0001D0
EPS = 0.000001D0
MF = 22
INDEX = 1
MIR = 0.454D0
ML = 0.24363D0
M1 = ML + MIR
IL2 = 0.001074D0

**PHYSICAL SYSTEM PARAMETERS.**

ATN1 = 1.8751041D0
ATN2 = 4.6940911D0
ABN1 = 0.7340955D0
ABN2 = 1.01846644D0
MH1 = 0.071D0
MP = 0.050D
L1 = 0.233D0
KS1 = 0.06997D0
L0 = 0.125D0
JL = 0.00000044641D0
DM = 0.00721D0
L = 0.0015875D0
INERTI = 0.0006940D0
ML = 0.06870D0
DL = 0.0870D0
LT = L1 + L0

*** ERROR CRITERION ***

R11 = L1 + R1
ATT1 = R1 - Y(1)
ATT2 = R2 - Y(2)
ER1PER = 0.0000127DO
ER2PER = ER1PER / R11

PHEE1 = DCOSH(ATN1) - DCOS(ATN1) - ABN1*(DSINH(ATN1) - DSIN(ATN1))
PHEE2 = DCOSH(ATN2) - DCOS(ATN2) - ABN2*(DSINH(ATN2) - DSIN(ATN2))
A = M2 + MP + MH1
PIN = INERTI + MH1 * ((L1 + CONST - L2)**2 + (M1*(LT**2))/12.000 +
1 * (M2*(L2**2))/12.000 + M1*(((L1/2.000) - LD)**2) + M2*+
1 * (L1 + CONST - (L2/2.000))**2) + MP*(((L1 + CONST)**2) +
1 * M1*R1)/2.000 + M2*(L1 + CONST - (L2/2.000)) +
1 * MH1*(L1 + CONST - L2) - (L0**2)*2.000)
AA1 = MH1*LO)*((DSIN(KSY1)*DCOS(KSY1))
AA2 = (PIN*(DTAN(KSY1) + MU))/LO
Tw2 = 12.56600*DL)/L*DM
Tw3 = M2*(((L1 + CONST - (L2/2.000))) + MP*-(L1 + CONST)
1 + MH1*(L1 + CONST - L2)

A2E = AA1 + Tw2*LO*(1.000 - (MU*DTAN(KSY1))) + AA2
AA3 = A1*(DTAN(KSY1) + MU) + Tw2*(1.000 - MU*DTAN(KSY1))
AA4 = MH1 - (PIN*DCOS(CONST2))/LO

B3 = 1.000*(PIN*(((DCOS(CONST2)**2) + (MH1*LO**2 + 1L2))
IK33 = POLE3*((WNFETA**2))/B3
KS33 = (WNFETA**2)*2.000*POLE3*ZETA3*WNFETA)/B3
KS36 = (2.000*ZETA3*WNFETA*POLE3)/B3

A2P = -(G*A*DCOS(CONST2)*DTAN(KSY1) + MU))/AA3
BIP = (2.000*(1.000 - (MU*DTAN(KSY1))))/(AA3*DM)
A5P = (((G*DCOS(CONST2))/LO)*A1*(DTAN(KSY1) + MU)*A2E - (((1
2.000*LO)*((DTAN(KSY1) + MU)**2) + Tw3*AA4/G))/A2E**2)
A5P = (PIN*G*DSIN(CONST2)*DTAN(KSY1) + MU)/(LO*A2E)
B2P = (2.000*(1.000 - MU*DTAN(KSY1)))/(DM*A2E)

IK11P = (POLE1*((WNR)**2))/B1P
IK22P = (POLE2*((WNTE**2)))/B2P

KS11P = ((WNR)**2)*2.000*POLE1*ZETA1*WNR)/B1P
KS22P = ((WNTE**2)*2.000*POLE2*ZETA2*WNTE + A5P))/B2P

KS12P = A42P/B1P
KS21P = A51P/B2P

KS14P = (2.000*ZETA1*WNR*POLE1)/B1P
KS25P = (2.000*ZETA2*WNTE*POLE2)/B2P
A42N*-(G*AINDCOS(CONST2)*(DTAN(KSY1)-MU))/AA3
B1N*-(2.000*(1.000*(MU*DTAN(KSY1)))/(AA3*DM)
A51N*-(G*AINDCOS(CONST2))/LG*1*(DTAN(KSY1)-MU)*A2E*)(
1.000/LG)*(((DTAN(KSY1)**2)-(MU**2))*(TW3*AA4*G))/(A2E**2)
A52N**-(PIIN*G*OSIN(CONST2)*(DTAN(KSY1)-MU))/LG*A2E)
B2N*-(2.000*(1.000*UM*DTAN(KSY1)))/(DM*A2E)

IK11N*(POLE1*(WNR**2))/B1N
IK22N*(POLE2*(WNTETA**2))/B2N

IK11N*(WNR**2)*2.000*POLE1*ZETA1*WNR)/B1N
IK22N*(WNTETA**2)*2.000*POLE2*ZETA2*WNTETA+A52N)/B2N

KS12N=A42N/B1N
KS21N=A51N/B2N

KS14N*(2.000*ZETA1*WNR+POLE1)/B1N
KS25N*(2.000*ZETA2*WNTETA+POLE2)/B2N

15 IF(TH.LT.0.500)GO TO 343
INDEX*2

*** CALL GEAR SUBROUTINE TO SOLVE THE EQUATIONS OF MOTION. ***

343 CALL DGEAR (N,TO,H,Y,TOUT,EP3,MF,INDEX)

THE CONDITION FOR THE VARIABLE S IS INCLUDED TO POINT OUT THE SINGULARITY OF THE INERTIA MATRIX WHENEVER IT OCCURS AND THEN TO HALT THE EXECUTION OF THE PROGRAM.

IF(S.EQ.0.000) GO TO 101
GO TO 102

101 WRITE(7,103) S
103 FORMAT(1X,'S=',F6.4)
GO TO 34

102 WRITE(7,11) TOUT,Y(1),Y(2),Y(3),Y(4),Y(5),Y(6)
11 FORMAT(1X,F7.4,2X,5(E14.8,2X),E14.8,6X)

*** STORE THE NUMERICAL RESULTS FOR PLOTTING. ***

I=I+1
TH=TH+1.000
Y1(I)=Y(1)
Y2(I)=Y(2)
Y3(I)=Y(3)
Y4(I)=Y(4)
Y5(I)=Y(5)
Y6(I)=Y(6)
Y7(I)=Y(7)
Y8(I)=Y(8)
Y9(I)=Y(9)
Y10(I)=Y(10)
Y11(I)=Y(11)
Y12(I)=Y(12)
Y13(I)=Y(13)
Y14(I)=Y(14)
Y15(I)=C1
Y16(I)=C2
Y17(I)=C3
V=PHEE1*Y(4)+PHEE2*Y(5)
W=PHEE1*Y(6)+PHEE2*Y(7)
D(I)=V
D2(I)=V
X(I)=(L1+Y(I))*DCOS(Y(2))*DCOS(Y(3))-V*DSIN(Y(2))*DCOS(Y(3))
1 +W*DSIN(Y(3))
YA(I)=(L1+Y(I))*DSIN(Y(2))*V*DCOS(Y(2))
Z(I)=(L1+Y(I))*DCOS(Y(2))*DSIN(Y(3))-V*DSIN(Y(2))*DSIN(Y(3))
1 -W*DCOS(Y(3))

Z(I) SHOULD BE CONSIDERED NEGATIVE, HOWEVER THE COORDINATE
SYSTEM ADOPTED BY THE PLOTTING ROUTINE IS THE LEFT HANDED
SYSTEM (i.e. X IS HORIZONTAL AND POINTING TO THE RIGHT, Y
IS VERTICAL AND POINTING UPWARD. Z IS PERPENDICULAR TO
THE PLANE OF THE SCREEN AND POINTING TOWARDS THE SCREEN.)
FOR THIS REASON Z(I) IS CONSIDERED POSITIVE IN ORDER TO
MAKE IT AGREE WITH THE PHYSICAL SYSTEM.

T1(I)=TOUT
WRITE(6,208) TOUT
208 FORMAT(2X,'TOUT=',F7.4)
IF (INDEX.EQ.0) GO TO 13
WRITE(7,12) INDEX
12 FORMAT(/,17X,'ERROR RETURN WITH INDEX=','I7)
GO TO 34
13 TOUT=TOUT+.0037DD
IF(TOUT.GT.TLIMT)GO TO 206
GO TO 207
206 TLIMIT=TLIMIT+.01DD
WRITE(6,203)
203 FORMAT(/,2X,'WOULD YOU LIKE TO HALT THE EXECUTION? (Y/N)'
READ(5,204) ATT
204 FORMAT(A4)
IF(ATT.EQ.,YES)GO TO 205
207 IF(TOUT.LE.6.000) GO TO 15

*** OUTPUT THE REST OF THE NUMERICAL RESULTS. ***

205 WRITE(7,446)
446 FORMAT(/,7X,'TIME',10X,'Q22(T)',13X,'r DOT',12X,
1 'THETA DOT',11X,'PHY DOT',11X,'Q11(T) DOT',9X,
1 'Q12(T) DOT')

DO 444 ILL=1,1
WRITE(7,445) T(I),Y(I),Y(I),Y(I),Y(I),Y(I),Y(I)
1 ,Y(I)
445 FORMAT(5X,F7.4,5X,6(E14.8,5X))
444 CONTINUE

WRITE(7,556)
556 FORMAT(/,7X,'TIME',8X,'Q21(T) DOT',9X,'Q22(T) DOT',8X,
1 'r CONTROLLER',5X,'THETA CONTROLLER',4X,'PHY CONTROLLER')
DO 555 IT = 1, 1
   WRITE(7,592) IT(I), Y13(I), Y14(I), Y15(I), Y16(I), Y17(I)
   592 FORMAT(5X,F7.4,5X,5(E14.8,5X))
555 CONTINUE

C
   1 /, 2X, 'B2N=', E14.8, 3X, 'B3=', E14.8, 3X, 'K11P=', E14.8,
   1 /, 2X, 'K22P=', E14.8, 3X, 'K33=', E14.8, 3X, 'KS11P=', E14.8,
   1 /, 2X, 'KS22P=', E14.8, 3X, 'KS14N=', E14.8, 3X, 'KS25N=', E14.8,
   1 /, 2X, 'KS21P=', E14.8, 3X, 'KS12N=', E14.8, 3X, 'KS21N=', E14.8,
   1 /, 2X, 'KS33=', E14.8)

C
   WRITE(7,93) RDMAX, TDMAX, RDPER, TDPER
93 FORMAT(2X, 'RDMAX=', F20.10, 4X, 'TDMAX=', F20.10, /,
   1 2X, 'RDPER=', F20.10, 4X, 'TDPER=', F20.10)
   C
   WRITE(5,587)
587 FORMAT(///, 15X, 'POSITION OF THE END EFFECTOR.'..//)

C
   WRITE(5,588)
588 FORMAT(///, 7X, 'TIME', 10X, 'X COORD.', 10X, 'Y COORD.', 10X,
   1 'Z COORD.', ///)

C
   DO 589 IK = 1, 1
   WRITE(7,590) IK, X(IK), YA(IK), Z(IK)
590 FORMAT(5X,F7.4,6X,3(E14.8,4X))
589 CONTINUE

C
   WRITE(7,600)
600 FORMAT(//, 3X, 'TIME', 6X, 'V DEFLECTION', 7X, 'W DEFLECTION')
   DO 601 IKL = 1, 1
   WRITE(7,602) IKL, D1(IKL), D2(IKL)
602 FORMAT(2X,F7.4,3X,2(E14.8,5X))
601 CONTINUE

C
   *** MAKE THE 2-D PLOTS. ***
   CALL PLTF(I, Y1, Y2, Y3, Y4, Y5, Y6, Y7, Y8, Y9, Y10, Y11,
   1 Y12, Y13, Y14, Y15, Y16, Y17, D1, D2, TI)
   C
   *** MAKE THE 3-D PLOT. ***
   CALL WRTF6(I, I1, X, YA, Z)

C
   WRITE(6,111)
111 IF(YES .EQ. 0) GO TO 34
   CALL DIMP(I, X, YA, Z)

CT
   WRITE(7,16) NSTEP, NFE, NUE
16 FORMAT(///, 'PROBLEM COMPLETED IN', I10, 2X,
   1 'STEPS', ///, 21X, I10, 2X, 'F EVALUATIONS', ///)
SUBROUTINE DIFFUN (N,TIME,Y,YDOT)

*********** PROGRAM IDENTIFICATION ***********

THIS SUBROUTINE DEFINES THE DIFFERENTIAL EQUATIONS OF MOTION

******************************************************************************

*********** VARIABLE IDENTIFICATION ***********

M1=MASS OF THE FIRST BEAM
M2=MASS OF THE SECOND BEAM
MP=MASS OF THE PAYLOAD
MH1=MASS OF THE HOUSING OF THE LEADScrew USED
TO MOVE THE SECOND BEAM IN THE r DIRECTION.
MH2=MASS OF THE HOUSING OF THE LEADScrew USED
TO DRIVE THE ROBOT ARM IN THE TETA DIRECTION.
C1=TRANSLATIONAL CONTROLLER FORCE
C2=ROTATIONAL CONTROLLER TORQUE AROUND K VECTOR
C3=ROTATIONAL CONTROLLER TORQUE AROUND THE VERTICAL AXIS
L1=LENGTH OF THE FIRST BEAM
L2=LENGTH OF THE SECOND BEAM
ZETA1=DAMPING RATIO IN THE r DIRECTION
ZETA2=DAMPING RATIO IN THE TETA DIRECTION
ZETA3=DAMPING RATIO IN THE FETA DIRECTION
WNR=NATURAL FREQUENCY FOR r
WNteta=NATURAL FREQUENCY FOR TETA
WNfeta=NATURAL FREQUENCY FOR FETA
R1=REFERENCE INPUT IN THE r DIRECTION
R2=REFERENCE INPUT IN THE TETA DIRECTION
R3=REFERENCE INPUT IN THE FETA DIRECTION
IK11=GAIN FOR THE INTEGRAL CONTROLLER
KS11=GAIN FOR THE STATE FEEDBACK CONTROLLER.
Y(1)=DISPLACEMENT IN THE r DIRECTION
Y(2)=DISPLACEMENT IN THE TETA DIRECTION
Y(3)=DISPLACEMENT IN THE FETA DIRECTION
Y(4)=Q11(t) DISPLACEMENT.
Y(5)=Q12(t) DISPLACEMENT.
Y(6)=Q21(t) DISPLACEMENT.
Y(7)=Q22(t) DISPLACEMENT.
Y(15) TO Y(17) ARE THE INTEGRAL OF THE ERROR BETWEEN
THE DESIRED AND THE COMPUTED POSITIONS AND VELOCITIES.
A2=CROSS SECTIONAL AREA OF THE SECOND BEAM
ROW=DENSITY OF THE UNIFORM SECOND BEAM

******************************************************************************

*********** STORAGE BLOCK ***********
REAL*8 M1, M2, MP, L1, L2, R1, R2, R3, Y(N), YDOT(N), A4, B4
REAL*8 KY1, MH1, C1, C2, C3, ACC3, BCC, MUS, K11N, K14N
REAL*8 ACC7, JL1, L, KS25N, DM, KS33, KS36, IK22N, KS12N, KS1N
REAL*8 IK11P, IK22P, IK33, ACC9, M1L, M1R, LT, IL2, ATT1, ATT2
REAL*8 ERR1, ER2, ERR2, ERR3, ERR2P, ERR2PER, RDMAX, TDMAX, RDPER, TDPER, SGN, ASGN
REAL*8 T11, T13, T14, ABC1, ACC12, ACC13, ACC14, CONST2, CONST3
REAL*8 ERR1, ERR2, KS12P, KS21P, ML, DL, INERTI

REAL*8 TIME, CENTAC, DIST, PALL(7), S, PART30, PART31, PART32
INTEGER IV(7), IU, IN, IM, IY, IJH, JUH, K, BC, DC, PART33, PART34
REAL*8 PBY(7), A2, ROW, A13, A14, A15, ROW7, ROW8, PART35, PART36
REAL*8 ROW1, ROW2, ROW3, ROW4, ROW5, ROW6, PART37, PART38, PART39
REAL*8 W2P, AL1, AL2, AL10, AL11, E1, ROW10, ROW9, PART40, PART41
REAL*8 PART50, PART51, B(7, 7), T(7, 7), F(7, 7), D(7), YVEXIT(7)
REAL*8 PTB(7, 7), PTF(7), PTD(7), PTBY(7), A(7, 7), PART60, PART61
REAL*8 PART1, PART2, PART3, PART4, PART5, PART62, PART63, PART64
REAL*8 PART6, PART7, PART8, PART9, PART65, TMN1, TMN2

COMMON /SINGUL/ S, A2, ROW, EI
 1 IK22P, KS33, KS36, IK33, KS12P, KS21P
COMMON /GAINKN/ KS11N, KS14N, IK11N, KS22N, KS25N,
 1 IK22N, KS12N, KS1N
COMMON /CONTLY/ C1, C2, C3
COMMON /DATA/ M1, M2, MP, MH1, L1, L2, MU, GM, KY1, M1L, M1R, LT, IL2
 1 R1, R2, R3, LO, JL1, L, DM, TT3, TT4
COMMON /DATA1/ ATT1, ATT2, MUS, CONST2, CONST3
 1 ML, DL, INERTI, TT1, TA1, TA2
COMMON /DT/ RDMAX, TDMAX, RDPER, TDPER, ERR1, ERR2, ERR2P, ERR2PER, CONST

TT1 IS INTRODUCED TO DETECT THE FIRST TIME THE INTEGRATION IS CARRIED OUT.

IF (TT1, GT, 0.5D0) GO TO 200
YDOT(B) = 0.000
YDOT(9) = 0.000
YDOT(10) = 0.000
YDOT(11) = 0.000
YDOT(12) = 0.000
YDOT(13) = 0.000
YDOT(14) = 0.000
C1 = 0.000
C2 = 0.000
C3 = 0.000

200 M = 7
A13 = 0.76D0
A14 = 0.53D0
A15 = 0.34D0
AL1 = 0.78D0
AL2 = 0.43D0
AL10 = 12.362D0
AL11 = 485.519D0
INITIALIZE OF THE T MATRIX.

DO 222 I=1,7
   DO 222 J=1,7
   T(I,J)=0.000
222 CONTINUE

C  **********************************************************************
C  ********** SORT BLOCK **********
C  **********************************************************************

C CENTAC=(Y(10)**2)+((DCOS(Y(2))**2)+(Y(9)**2)
DIST=(L1*(Y(1))+(Y(1)**2)/2.000)
BC=L1*(Y(1)-L2/2.000)
RDW2=L1-(L2/2.000)*(Y(1)/2.000)
ROW3=L2-Y(1)
ROW4=L1-Y(1)
ROW5=(L1**2)*Y(1)+L1*(Y(1)**2)+(Y(1)**3)/3.000
ROW6=ROW1/2.000
ROW7=MP*ROW4
ROW8=((ROW1*Y(1))+MP)*G*DSIN(Y(2))
ROW9=ROW1*G*DSIN(Y(2))
ROW10=ROW1*Y(10)**2)*DSIN(Y(2))*DCOS(Y(2))
W2=ROW1*Y(1)

A4=M2+MP+MH1
ACC9=MH1/(DSIN(KSY1))*(DCOS(KSY1))
ACCT=(12.5660*UL1)/(L*DM)
B4=M2+BC+MP*ROW4
BC*(M1*RM1)+(L1+Y(1)-L2)
ACC3=((M1*RM1)/2.000)+MP*(L1+Y(1))+(M1*Y(1)-(L2/2.000))
ML=ML*DL+Y(1)-(M1*L1)-(L2/2.000)
DC=(M1*(L1+Y(1)))*12.000)+(M2*(L2**2))/12.000+M2*(L1+Y(1))
1-(L2/2.000))**2)*MP*((L1+Y(1)**2)+INERTI+MH1*(L1+Y(1))
1-L2)**2)+Y1*(L1/2.000)-L0**2)

C C CHECK WHETHER THE r MOTION IS ON THE UPPER OR LOWER THREAD.
C C PART30=(2.000+C1)/DM-ACC7*YDOT(8))*DSIN(KSY1)*(A4*(G*
1 DSIN(Y(2))+YDOT(8))-BCC(((Y(10)**2)+(DCOS(Y(2))**2))+
1 (Y(9)**2)+2.000*
1 Y(10)+DCOS(Y(2))*(0.7800+Y(13)+0.4300*Y(14))*M2P+2.000*MP*(
1 Y(13)-Y(14))/2.000+ROW1+Y(8)*Y(10)+DCOS(Y(2))*(0.7800*Y(6)+
1 0.4300*Y(7)-2.000*Y(9)+M2P*(0.7800+Y(11)+0.4300*Y(12)))+
1 2.000*MP*(Y(11)-Y(12)))*DCOS(KSY1)
PART31=(-2.000*Y(8)+Y(9)*ROW1*(0.7800+Y(4)+0.4300*Y(5)))+
1 YDOT(10)+DCOS(Y(2))*(M2P*(0.7800+Y(6)+0.4300*Y(7)))+2.000*MP
1 *Y(6)+Y(7)))-(YDOT(9)-Y(10)**2)+DSIN(Y(2)))+DCOS(Y(2)))*
1 (M2P*(0.7800+Y(4)+0.4300*Y(5))+2.000*MP*(Y(4)-Y(5)))
1+DCOS(KSY1)
N1=PART30+PART31
IF(N1.GE.0.000)GO TO 535
GO TO 600
C  *** LOWER THREAD LOGIC FOR 0 MOTION. ***
C
535 IF(TTI.GT.0.5D0)GO TO 542
536 IF(AT1)536,537,538
537 C1=K51N*(Y(1)-CONST)-K51N*Y(8)+IK1N*Y(15)-K512N*Y(2) GO TO 542
538 C1=K51P*(Y(1)-CONST)-K51P*Y(8)+IK1P*Y(15)-K512P*Y(2) GO TO 547
542 IF(TT3.GT.0.5D0)GO TO 571
543 IF(SGN(Y(8)))539,540,541
544 C1=K51N*(Y(1)-CONST)-K51N*Y(8)+IK1N*Y(15)-K512N*Y(2) GO TO 546
545 C1=K51P*(Y(1)-CONST)-K51P*Y(8)+IK1P*Y(15)-K512P*Y(2) GO TO 546
546 IF(TT1.LT.0.5D0)GO TO 552
547 AT1=R1-Y(1)
548 IF(AT1)549,550,551
549 C1=K51N*(Y(1)-CONST)-K51N*Y(8)+IK1N*Y(15)-K512N*Y(2) GO TO 552
550 C1=0.0D0 GO TO 547
551 C1=K51P*(Y(1)-CONST)-K51P*Y(8)+IK1P*Y(15)-K512P*Y(2)
552 IF(C1)543,547,545
C
C  *** COMPUTATION OF THE MINIMUM ***
C  *** TORQUES FOR 0 ***
C
543 ABC1=1.0D0*MUS*(DTAN(KSY1))
544 ACC14=DTAN(KSY1)+MUS
545 TMIN1=-(A4**2+DSIN(Y(2))+GCS*((Y(10)**2)+((DCOS(Y(2))**2)+
546 1.0D0+Y(10)+DCOS(Y(2))))*(0.78D0+Y(13)+0.4D0)+
547 0.5D0+Y(13)-Y(14))+2.0D0+Y(9)*M2P*(0.78D0+
548 0.4D0)+Y(11)-Y(12))+Y(10)**2)+
549 DSIN(Y(2))*DCOS(Y(2))-YDPT(9)*M2P*(0.78D0+Y(4)+0.4D0+Y(5))
550 +2.0D0+M2P*(Y(4)-Y(5))-YDPT(10)*DCOS(Y(2))*M2P*(0.78D0+Y(6)+
551 0.4D0)+Y(7))+2.0D0+M2P*(Y(6)-Y(7)))*(DM/2.0D0)*(ACC1/ABC1).
552 IF(C1.LT.TMIN1)GO TO 546
553 GO TO 547
545 ABC1=1.0D0-MUS*(DTAN(KSY1))
546 ACC14=DTAN(KSY1)+MUS
547 TMIN1=-(A4**2+DSIN(Y(2))+GCS*((Y(10)**2)+((DCOS(Y(2))**2)+
548 1.0D0+Y(10)+DCOS(Y(2))))*(0.78D0+Y(13)+0.4D0)+
549 0.5D0+Y(13)-Y(14))+2.0D0+Y(9)*M2P*(0.78D0+
550 0.4D0)+Y(11)-Y(12))+Y(10)**2)+
551 DSIN(Y(2))*DCOS(Y(2))-YDPT(9)*M2P*(0.78D0+Y(4)+0.4D0+Y(5))
552 +2.0D0+M2P*(Y(4)-Y(5))-YDPT(10)*DCOS(Y(2))*M2P*(0.78D0+Y(6)+
553 0.4D0)+Y(7))+2.0D0+M2P*(Y(6)-Y(7)))*(DM/2.0D0)*(ACC1/ABC1).
554 IF(C1.GT.TMIN1)GO TO 546
555 GO TO 547
546 IF(TA1.LT.0.5D0)GO TO 31
547 IF(RDMAX.GE.DABS(Y(8)))GO TO 30
548 RDMAX=DABS(Y(8))
30 TA1=TA1+1.0D0
31 RPDPER=RDMAX+1.0D0
32 ERR1=DABS(R1-Y(1))
33 IF((DABS(Y(8)).LT.RPDPER).OR.(ERR1.GT.ERRPER))GO TO 34
547 Y(1)=R1
548 C1=0.0D0
549 Y(15)=0.0D0
TT3=1.000
GO TO 547

C

34 PART38=DTAN(KSY1)+MU*SGN(Y(8))
PART39=1.000-(MU*DTAN(KSY1))*SGN(Y(8))
A(1,1)=A4*(MU*DTAN(KSY1))+ACC7*(1.000-MU*DTAN(KSY1))
A(1,2)=-(M2P*(0.78DO*Y(4)+0.43DO*Y(5))+2.000*MP*(Y(4)-Y(5)))+
1 PART38
A(1,3)=DCOS(Y(2))*(M2P*(0.78DO*Y(6)+0.43DO*Y(7))+2.000*MP*+
1 (Y(6)-Y(7)))*PART38
A(1,4)=0.000
A(1,5)=0.000
A(1,6)=0.000
A(1,7)=0.000
B(1,1)=(-2.000*RDW1*Y(10)*DCOS(Y(2))*0.78DO*Y(6)+0.43DO*+
1 Y(7))+2.000*Y(9)*RDW1*(0.78DO*Y(4)*0.43DO*Y(5))*(MU+
1 DTAN(KSY1))
B(1,2)=-(BCC*Y(9)+2.000*(M2P*(0.78DO*Y(11)+0.43DO*Y(12))-*+
1 2.000*MP*(Y(11)-Y(12)))+PART38
B(1,3)=-(BCC*Y(10)*((DCOS(Y(2)))*2)-2.000*DCOS(Y(2)))(+
1 0.78DO*Y(13)+0.43DO*Y(14)+M2P+2.000*MP*(Y(13)-Y(14)))-Y(10)
B(1,4)=DSIN(Y(2))*(M2P*(0.78DO*Y(4)+0.43DO*Y(5))+2.000*+
1 MP*(Y(4)-Y(5)))+PART38
B(1,5)=0.000
B(1,6)=0.000
B(1,7)=0.000
F(1)=(0.200*C1*PART39)/DM
D(1)=A4+G*DSIN(Y(2))*PART38
GO TO 548

547 Y(8)=0.000
A(1,1)=1.000
A(1,2)=0.000
A(1,3)=0.000
A(1,4)=0.000
A(1,5)=0.000
A(1,6)=0.000
A(1,7)=0.000
B(1,1)=0.000
B(1,2)=0.000
B(1,3)=0.000
B(1,4)=0.000
B(1,5)=0.000
B(1,6)=0.000
B(1,7)=0.000
F(1)=0.000
D(1)=0.000
GO TO 548

C

C

LOGIC FOR THE \( r \) MOTION ON THE UPPER THREAD.

C

600 IF(TT1.GT.0.500)GO TO 601
IF(ATT1)602,603,604
602 C1=KS1NP*(Y(1)-CONST)-KS14P*Y(8)+IK1NP*Y(15)-KS12P*Y(2)
GO TO 601
603 C1=C0 000
GO TO 605
604 C1=KS1NP*(Y(1)-CONST)-KS14N*Y(8)+IK1N*Y(15)-KS12N*Y(2)
601 IF(TT3.GT.0.500)GO TO 606
IF(SGN(Y(8)))607,608,609
607 C1=KS1P*Y(1)-CONST)-KS14P*Y(8)+IK11P*Y(15)-KS12P*Y(2)
GO TO 610
609 C1=KS1N*Y(1)-CONST)-KS14N*Y(8)+IK11N*Y(15)-KS12N*Y(2)
GO TO 610
608 IF(TT.LT.0.5DO)GO TO 611
ATT=1+Y(1)
IF(ATT)1612,613,614
612 C1=KS1P*Y(1)-CONST)-KS14P*Y(8)+IK11P*Y(15)-KS12P*Y(2)
GO TO 611
613 C1=0.O00
GO TO 605
614 C1=KS1N*Y(1)-CONST)-KS14N*Y(8)+IK11N*Y(15)-KS12N*Y(2)
611 IF(C1)6165,605,617
615 ABC1=1.000*MUS*(DTAN(KSY1))
ACC14=DTAN(KSY1)*MUS
TMIN1=-(A4*G*SIN(Y(2)))*(BCC*((Y(10)**2)))*(DCOS(Y(2))**2)
1 (Y(9)**2)) 1.000*Y(10)*DCOS(Y(2))*(0.7800*Y(13)+0.4300*
1 (Y(14)) YMP*Y(13)-Y(14)))+2.000*Y(9)*M2P*(Y(8)+0.7800*
1 (Y(11)+0.4300*Y(12)-2.000*MP*(Y(11)-Y(12)))-((Y(10)**2)
1 DSIN(Y(2))**2)*DCOS(Y(2))-YDFT(Y(9))*(M2P*(Y(8)+0.4300*Y(5))
1 +2.000*MP*(Y(4)-Y(5))-YDFT(10)*DCOS(Y(2))*(M2P*(0.7800*Y(6)+
1 0.4300*Y(7)))+2.000*MP*(Y(6)-Y(7))))*(DM/2.000)*(ACC14/ABC1)
IF(C1)611,605,617
GO TO 617
616 ABC1=1.000*MUS*(DTAN(KSY1))
ACC14=DTAN(KSY1)*MUS
TMIN1=-(A4*G*SIN(Y(2)))*(BCC*((Y(10)**2)))*(DCOS(Y(2))**2)
1 (Y(9)**2)) 1.000*Y(10)*DCOS(Y(2))*(0.7800*Y(13)+0.4300*
1 (Y(14))) M2P*2.000*MP*(Y(13)-Y(14)))+2.000*Y(9)*M2P*(0.7800*
1 (Y(11)+0.4300*Y(12)-2.000*MP*(Y(11)-Y(12)))-((Y(10)**2)
1 DSIN(Y(2))**2)*DCOS(Y(2))-YDFT(Y(9))*(M2P*(0.7800*Y(4)+0.4300*Y(5))
1 +2.000*MP*(Y(4)-Y(5))-YDFT(10)*DCOS(Y(2))*(M2P*(0.7800*Y(6)+
1 0.4300*Y(7)))+2.000*MP*(Y(6)-Y(7))))*(DM/2.000)*(ACC14/ABC1)
IF(C1)611,605,617
GO TO 610
610 IF(TA1.LT.0.5DO)GO TO 619
IF(RDMAX.GE.DABS(Y(8)))GO TO 620
619 RDMAX=DABS(Y(8))
620 TA1=TA1+1.000
RDPER=RDMAX+*0.000100
ERR1=DABS(R1-Y(1))
IF(DABS(Y(8)).GT.RDPER).DR.(ERR1.GT.ER1PER))GO TO 621
606 Y(1)=R1
C1=0.000
Y(15)=0.000
TT=1.000
GO TO 605
621 PART38=DTAN(KSY1)-MU*SGN(Y(8))
PART38=1.000*(MU*DTAN(KSY1)+SGN(Y(8))
A(1,1)=A4*(MU*DTAN(KSY1)*ACC7*(1.000-MU*DTAN(KSY1))
A(1,2)=-(M2P*0.7800*Y(4)+0.4300*Y(5))+2.000*MP*(Y(4)-Y(5))
1 PART38
A(1,3)=DCOS(Y(2))+(M2P*(0.7800*Y(6)+0.4300*Y(7)))+2.000*MP*
1 Y(6)-Y(7)))*PART38
A(1,4)=0.000
A(1,5)=0.000
A(1,6)=0.000
A(1,7)=0.000
B1(1,1)=-(2.000*RD1*Y(10)+DCOS(Y(2))*(0.7800*Y(6)+0.4300*)
\begin{align*}
1 \ Y(7)) &= 2.000^*Y(9)*RDW1*(0.7800^*Y(4)+0.4300^*Y(5)) * (MU^* \\
1 \ DTAN(KSY1)) &
1 \ B(1,2) &= (BC^*Y(9)^*2.000^*M2P*(0.7800^*Y(11)^*0.4300^*Y(12)))^* \\
1 \ B(1,3) &= (DC^*Y(10)^*(*DCOS(Y(2))^*0.2000^*DCOS(Y(2)))^* \\
1 \ O.7800^*Y(13)^*0.4300^*Y(14)^*M2P*(0.2000^*M2P*(Y(13)^*Y(14)))^*Y(10)^* \\
1 \ D(*) &= DSIN(Y(2))^*DCOS(Y(2))*(M2P*(0.7800^*Y(4)^*0.4300^*Y(5)))^*2.000^* \\
1 \ MP^*Y(4)^*Y(5))^*^*PART38 \\
1 \ B(1,4) &= 0.000 \\
1 \ B(1,5) &= 0.000 \\
1 \ B(1,6) &= 0.000 \\
1 \ B(1,7) &= 0.000 \\
1 \ F(1) &= ((2.000^*C1^*PART39)/DM \\
1 \ D(*) &= ^*A^*G^*DSIN(Y(2)^*PART38 \\
1 \ GO TO 548 \\
1 \ Y(8) &= 0.000 \\
1 \ A(1,1) &= 1.000 \\
1 \ A(1,2) &= 0.000 \\
1 \ A(1,3) &= 0.000 \\
1 \ A(1,4) &= 0.000 \\
1 \ A(1,5) &= 0.000 \\
1 \ A(1,6) &= 0.000 \\
1 \ A(1,7) &= 0.000 \\
1 \ B(1,1) &= 0.000 \\
1 \ B(1,2) &= 0.000 \\
1 \ B(1,3) &= 0.000 \\
1 \ B(1,4) &= 0.000 \\
1 \ B(1,5) &= 0.000 \\
1 \ B(1,6) &= 0.000 \\
1 \ B(1,7) &= 0.000 \\
1 \ F(1) &= 0.000 \\
1 \ D(*) &= 0.000 \\
1 \ CHECK WHETHER THE THETA MOTION IS OCCURING \\
1 \ ON THE UPPER OR LOWER THREAD. \\
1 \ PART32 = ((2.000^*C2)/DM-ACC7^*LO^*YDOT(9)^*DSIN(KSY1)^*+MH1^*G \\
1 \ -((ACCS^*G^*DCOS(Y(2)))^*LO)-((YDOT(9)^*+(Y(10)^**2)^*DSIN(Y(2)^* \\
1 \ 1 \ DCOS(Y(12)))^*LO)^*DC= ((2.000^*Y(8)^*Y(9)))^*LO)^*BC= (ROW1/LO)^* \\
1 \ O.7800^*YDOT(8)^*L1^*Y(4)^*1.5600^*L1^*Y(8)^*Y(11)^*0.4300^*YDOT(8)^* \\
1 \ L1^*Y(5)^*0.8600^*Y(8)^*Y(12)^*L1^*+0.7800^*Y(1)\*YDOT(11)^*L1^* \\
1 \ O.4300^*Y(1)^*L1^*YDOT(12)^*0.4300^*((Y(8)^**2)^*Y(4)^*Y(1)^*YDOT(8)^* \\
1 \ Y(4)^*Y(1)^*Y(8)^*Y(11)^*-0.6800^*((Y(8)^**2)^*Y(5)^*Y(1)^*YDOT(8)^* \\
1 \ Y(5)^*Y(1)^*Y(8)^*Y(12)^*)^*0.5700^*(2.000^*Y(1)^*Y(8)^*Y(11)^*Y(11)^**2)^* \\
1 \ Y^*YDOT(11)^*0.0900^*(2.000^*Y(1)^*Y(8)^*Y(12)^*Y(11)^**2)^*YDOT(12)^* \\
1 \ ))^*DCOS(KSY1)^* \\
1 \ PART33 = ((2.000^*MP)/LO)^*RDW4^*(YDOT(11)^*YDOT(12)^*YDOT(8)^* \\
1 \ ((Y(4)^*Y(5)^*YDOT(10)^*DSIN(Y(2)^*RDW1)/LO)^*((0.7800^* \\
1 \ Y(1)^*L1^*0.5700^*(Y(1)^**2)^*Y(13)^*((O.4300^*Y(1)^*L1^*0.0900^*( \\
1 \ Y(1)^**2)^*Y(14)^*0.7800^*Y(8)^*L1^*0.3500^*Y(1)^*Y(8)^*Y(6)^* \\
1 \ 0.4300^*L1^*Y(8)^*0.2500^*Y(1)^*Y(8)^*Y(7))^*DCOS(KSY1)^* \\
1 \ PART34 = ((4.000^*Y(10)^*DSIN(Y(2)^)*MP^*ROW4)/LO)^*(Y(13)^*Y(14)^* \\
1 \ ((RDW1/LO)^*YDOT(10)^*DSIN(Y(2)^)/LO)^*((0.7800^*Y(1)^*L1^*0.5700^* \\
1 \ Y(1)^**2)^*Y(6)^*0.4300^*Y(1)^*L1^*0.0900^*(Y(1)^**2)^*Y(7)^) \\
1 ^+((2.000^*MP)^*YDIT(10)^*DSIN(Y(2)^)/LO)^*ROW4^*(Y(6)^*Y(7)^)^* \\
1 \ RDW1/LO)^*(((Y(10)^*Y(10)^**2)^*DSIN(Y(2)^)**2)^*Y(9)^**2)^* \\
1 \ ((0.7800^*Y(1)^*L1^*0.5700^*(Y(1)^**2)^*Y(4)^*0.4300^*Y(1)^*L1^* \\
1 \ *0.0900^*(Y(1)^**2)^*Y(5)^)^*DCOS(KSY1) \\
1 \ PART35 = ((2.000^*MP)/LO)^*[((Y(10)^*Y(10)^**2)^*DSIN(Y(2)^)**2)^* \\
1 \ (Y(9)^**2)^*Y(4)^*Y(5)^*RDW4^*(RDW1/LO)^*(((Y(10)^**2)^*((}
1 DCOS(Y(2))*2)+Y(4)*4*(0.4300*Y(1)+1.0*0.9090*(Y(1)*2)+Y(5))
1 -((2.000*MP*ROW4)/LO)*((Y(10)*2)+DCOS(Y(2))*2)+
1 (Y(9)*2)+Y(4)*4*(0.4300*Y(10)*ROW1+DCOS(Y(2)))/LO)*1+
1 Y(1)*Y(4)*Y(10)+Y(5)+Y(14)+Y(8)*(0.5000*Y(4)+Y(6)+0.5000*Y(5))
1 *(Y(7)+0.6500*Y(5)+Y(6)+0.6500*Y(4)+Y(7)+)*DCOS(KSY1)
PART36=((8.000*MP*Y(10)*DCOS(Y(2)))/LO)*Y(4)*Y(13)-Y(4)*
1 Y(14)-Y(5)*Y(13)+Y(5)*Y(14)-((8.000*MP*Y(9))/LO)*Y(4)*
1 Y(11)-Y(4)*Y(12)+Y(5)*Y(12)-Y(5)*Y(11)-(2.000*
1 Y(9)+ROW1)/LO)*(0.5000*Y(8)+((Y(4)*2)+(Y(5)*2)))+Y(1)*
1 *(Y(4)*Y(11)+Y(5)*Y(12))*((YD0T(10)*DCOS(Y(2))/ROW1)/LO)
1 *(Y(4)*Y(6)+Y(5)*Y(7)+)*DCOS(KSY1)
PART37=((4.000*YD0T(10)*DCOS(Y(2)))/LO)*Y(4)*Y(6)-Y(5)*
1 Y(6)*Y(4)*Y(7)+Y(5)*Y(7)-(2.000/L0)*YD0T(9)=-Y(10)**2)+
1 DSIN(Y(2)))*DCOS(Y(2))*Y(4)*Y(5)**2)+Y(4)**2)-((4.000*MP/L0)
1 *(YD0T(9)-Y(11)**2)DSIN(Y(2))*DCOS(Y(2))*Y(4)**2)+
1 *(Y(5)**2)-2.000*Y(4)+Y(5)+)*DCOS(KSY1)
N2=PART32+PART33+PART34+PART35+PART36+PART37
IF(N2.GE.0.000)GO TO 700
GO TO 800
700 IF(TI1.GT.0.500)GO TO 701
IF(T1.TO2)702,703,704
702 C2=-K521*Y(Y+1)-K522P*Y(Y+2)-CONST2)-K525P*Y(Y+9)+1K22P*Y(Y+16)
GO TO 701
703 C2=0.000
GO TO 705
704 C2=-K521*Y(Y+1)-K522P*Y(Y+2)-CONST2)-K525P*Y(Y+9)+1K22P*Y(Y+16)
IF(T1.GT.0.500)GO TO 706
IF(SGN(Y(9)))/707,708,709
707 C2=-K521*Y(Y+1)-K522P*Y(Y+2)-CONST2)-K525P*Y(Y+9)+1K22P*Y(Y+16)
GO TO 710
709 C2=-K521*Y(Y+1)-K522P*Y(Y+2)-CONST2)-K525P*Y(Y+9)+1K22P*Y(Y+16)
GO TO 710
708 IF(TI1.LT.0.500)GO TO 711
ATT2=R2-Y(Y+2)
IF(ATT2)712,713,714
712 C2=-K521*Y(Y+1)-K522P*Y(Y+2)-CONST2)-K525P*Y(Y+9)+1K22P*Y(Y+16)
GO TO 711
713 C2=0.000
GO TO 705
714 C2=-K521*Y(Y+1)-K522P*Y(Y+2)-CONST2)-K525P*Y(Y+9)+1K22P*Y(Y+16)
711 IF(C2)715,705,717
715 ACC12=1.000*MUS*(DTAN(KSY1))
ACC13=DTAN(KSY1)*MUS
PART60=-(MH1)-G*(ACC13+G*DCOS(Y(2)))/LO)*-(Y(10)**2)+
1 DSIN(Y(2))*DCOS(Y(2)))/LO)-DC-(ROW1)/LO)*0.7800*YD0T(8)*
1 (Y(4)+2)+1.5600*Y(8)+Y(14)+0.4300*YD0T(8)
1 *L1*(Y(5)-0.8600*Y(8)+Y(12)+L1+0.7800*Y(1)+YD0T(11)+L1+
1 0.4300*Y(1)+L1*YD0T(12)-0.4300*Y(8)+2*Y(4)+Y(1)+YD0T(8)*
1 Y(4)+Y(1)+Y(8)+Y(14)+0.6800*Y(8)+Y(5)+YD0T(8)*
1 Y(5)+Y(1)+Y(8)+Y(12)+0.5700*Y(2000*Y(1)+Y(8)+Y(11)+Y(11)**2)+
1 YD0T(11)-0.0900*Y(2.000*Y(1)+Y(8)+Y(12)+Y(11)**2)*YD0T(12))
PART61=-(2.000*MP/LO)*(ROW1*YD0T(11)-YD0T(12)-YD0T(8)*
1 (Y(4)+Y(5))=2.000*Y(5)+YD0T(2)+ROW1)/LO)*0.7800*Y(1)+
1 Y(1)+L1*0.5700*Y(1)+YD0T(11)+Y(13)+0.4300*Y(1)+L1+0.0900*(
1 Y(1)+Y(8)+Y(1)+Y(8)+L1+0.3500*Y(1)*Y(8)+Y(6)+
1 (Y(4)+L1*Y(8)-0.2500*Y(1)+Y(8)+Y(7)))
PART62=-(4.000*Y(10)*DSIN(Y(2))/MP*ROW4)/LO)*Y(13)-Y(14)
1 *(Y(10)+YD0T(10)-DSIN(Y(2))/LO)*0.7800*Y(1)+L1+0.5700*Y(1)
1 *(Y(1)+Y(8)+Y(6)+0.4300*Y(1)+L1+0.0900*(Y(1)+Y(8))+Y(7))
1 (*((Y(4)**2)+(Y(5)**2))*(((4.000*MP*Y(10)+DSIN(Y(2)))*DCOS(Y(2))))
1 /LO)*((Y(4)-Y(5)**2))*PART40
B(2,3)=PART50-PART51
B(2,4)=0.000
B(2,5)=0.000
B(2,6)=0.000
B(2,7)=0.000
F(2)=(-2.000*PART41+C2)/DM
D(2)=(-(ACC3*G*DCOS(Y(2)))/LO)+WH1*G)*PART40
GO TO 721
705 Y(9)=0.000
A(2,1)=0.000
A(2,2)=1.000
A(2,3)=0.000
A(2,4)=0.000
A(2,5)=0.000
A(2,6)=0.000
A(2,7)=0.000
B(2,1)=0.000
B(2,2)=0.000
B(2,3)=0.000
B(2,4)=0.000
B(2,5)=0.000
B(2,6)=0.000
B(2,7)=0.000
F(2)=0.000
D(2)=0.000
GO TO 721

C
C LOGIC FOR THE THETA MOTION CARRIED ON THE UPPER THREAD.

C

800 IF(TT1.GT.0.500)GO TO 801
IF(ATT2)802,803,804
802 C2=-K521N*Y(1)-K522N*(Y(2)-CONST2)-K525N*Y(9)+IK22N*Y(16)
GO TO 801
803 C2=0.000
GO TO 805
804 C2=-K521P*Y(1)-K522P*(Y(2)-CONST2)-K525P*Y(9)+IK22P*Y(16)
801 IF(TT4.GT.0.500)GO TO 806
IF(SGN(Y(9)))807,808,809
807 C2=-K521N*Y(1)-K522N*(Y(2)-CONST2)-K525N*Y(9)+IK22N*Y(16)
GO TO 810
809 C2=-K521P*Y(1)-K522P*(Y(2)-CONST2)-K525P*Y(9)+IK22P*Y(16)
GO TO 810
808 IF(TT1.LT.0.500)GO TO 811
ATT2=R2-Y(2)
IF(ATT2)812,813,814
812 C2=-K521N*Y(1)-K522N*(Y(2)-CONST2)-K525N*Y(9)+IK22N*Y(16)
GO TO 811
813 C2=0.000
GO TO 805
814 C2=-K521P*Y(1)-K522P*(Y(2)-CONST2)-K525P*Y(9)+IK22P*Y(16)
811 IF(C2)815,805,817
815 ACC12+1.000*MUS*(DTAN(KSY1))
ACC13=DTAN(KSY1)-MUS
PART60=(-WH1*G-((ACC3*G*DCOS(Y(2)))/LO)-(((Y(10)**2)*
1 DSIN(Y(2)))*DCOS(Y(2))))/LO+(ROW1/LO)*(+0.78DO+YDOT(8))*
1 LY+Y(4)+0.56DO*L1+Y(8)+Y(11)+0.43DO*YDOT(8)
1 LY+Y(5)+0.86DO*Y(8)+Y(12)+L1+0.78DO*Y(1)*YDDT(11)*L1+
1 *Y(4)\*Y(5))*ROW4*+(ROW1/LD)*((Y(10)**2)*((1 DCOS(Y(2)**2)+2)\*{(0.78DO\*Y(1)**1+L0.57DO\* 
1\*(Y(1)**2))\*Y(4)+((0.43DO\*Y(1)**1+L+0.09DO\*Y(1)**2))\*Y(5)) 
1-((2.00DO*MP\*ROW4)/L0)*((Y(10)**2)*{(DCOS(Y(2)**2)/L0)}* 
1\*(Y(4)\*Y(5)+((Y(13)**1)+Y(5)**1+Y(14)**1)+(Y(4)\*Y(7)**1)+ 
1\*(Y(7)**1-0.65DO\*Y(5)**1+Y(6)**1-0.65DO\*Y(4)**1+Y(7)**1)) 
1+((8.00DO\*MP\*Y(10)+DCOS(Y(2)**2)/L0)*Y(4)+Y(13)-Y(4)* 
1(Y(14)-Y(5)**1+Y(13)**1+Y(5)**1+Y(14)) 
1+((YDOT(10)+DCOS(Y(2)**1)*ROW1/LD)*Y(1)**1 
1(Y(4)**1+Y(6)**1+Y(5)**1) 
1PART65=(((4.000*YDOT(10)+DCOS(Y(2)**1)*MP)/L0)*Y(4)-Y(6)-Y(5)**1 
1(Y(6)-Y(4)**1+Y(5)**1)+M2P/LD)*Y(10)**2* 
1DSIN(Y(2)**1)*DCOS(Y(2)**2)*((Y(4)**2)+(Y(5)**2))\*{(4.000*MP)/L0} 
1\*(Y(10)**2)*DSIN(Y(2)**2)+DCOS(Y(2)**2)*((Y(4)**2)+ 
1\*(Y(5)**2)+2.0000\*Y(4)**1+Y(5)))) 
1TMIN2=PART60+PART61+PART62+PART63+PART64+PART65*(DM/2.000)* 
1ACC13/ACC12 
1IF(C2 GT TMIN2)GO TO 810 
GO TO B05 
810 IF(TA2 LT 0.500)GO TO 818 
IF(TDMAX GE DABS(Y(9)))GO TO 819 
818 TDMAX=DABS(Y(9)) 
819 TA2+TA2+1.000 
TDPER=TDMAX*0.00001DO 
ERR2=DABS(R2-Y(2)) 
IF(DABS(Y(9))<.GT.TDPER).OR.(ERR2<.GT.ERR2PER))GO TO 820 
806 Y(2)=R2 
C2=0.000 
Y(16)=0.000 
T4=1.000 
GO TO B05 
820 PART40=DTAN(KSY1)**1+MUSGN(Y(9)) 
PART41=1.0000*MU*DTAN(KSY1)**1+SGN(Y(9)) 
A(2,1)=((ROW1/LD)*((0.78DO\*L1-0.43DO\*Y(1)**1)*Y(4)+((0.43DO\*L1 
1-0.68DO\*Y(1)**1)*(2.000*MP)/L0)+Y(4)**1))**PART40 
A(2,2)=ACCS*L0+ACC7*L0*(1.000-MU*DTAN(KSY1)*+(DTAN(KSY1)+MU) 
1)*/(DC/LD)+(M2P/L0)+((Y(4)**2)+(Y(5)**2))/(4.000*MP)/L0 
1*/(Y(4)**2))**2 
A(2,3)=-(((ROW1*DSIN(Y(2)**2)))/L0)*((0.78DO\*Y(1)**1+L0.57DO 
1Y(1)**2))**2)*Y(6)+((0.43DO\*Y(1)**1+L+0.09DO\*Y(1)**2))**Y(7))** 
1-2.000*MP*DSIN(Y(2)**2))\*L0*ROW4*Y(6)-Y(7))/(ROW1\*DCOS(Y(2)**2) 
1/L0+Y(1)**2+Y(4)**1+Y(5)**1+Y(7))/(4.000*MP*DCOS(Y(2)**2))\*/L0* 
1(Y(4)**1-Y(5)**1)*Y(6)**1+Y(7)**1)**PART40 
A(2,4)=(ROW1/LD)*((0.78DO\*Y(1)**1+L0.57DO**1+Y(1)**2))+(2.000* 
1MP*ROW4)/L0)**PART40 
A(2,5)=((ROW1/LD)*((0.43DO\*Y(1)**1+L0.09DO**1+Y(1)**2))**2))-(2.000* 
1*MP*ROW4)/L0)**PART40 
A(2,6)=0.000 
A(2,7)=0.000 
B(2,1)=-(ROW1/LD)*((1.56DO\*L1+O.71DO* 
1Y(1))**1+1+0.86DO\*L1-0.50DO\*Y(1)**1+Y(12)-(0.43DO\*Y(4)+ 
10.68DO\*Y(5))**1+Y(8))**PART40 
B(2,2)=-(((8.00DO*MP)/L0)*Y(4))**1*(Y(11)**1-Y(4)**1+Y(12)**1+Y(12)* 
1Y(5)**1)**1*(2.000*ROW1)/L0*(0.50DO*Y(8)**1((Y(4)**2)+ 
1Y(5)**2))**1*(Y(4)**1+Y(11)**1+Y(5)**1+Y(12)**1))-(2.000*Y(8)**1BCCO/L0) 
1*(MU*DTAN(KSY1))**1 
PART50=-(Y(10)**1)*DSIN(Y(2))**1)*DCOS(Y(2))**1)*DC*(2.000*ROW1** 
1DSIN(Y(2)**1)/L0)*((0.78DO\*Y(1)**1+L0.57DO**1+Y(1)**2))**Y(13)+ 
10.43DO**1+L1+0.09DO**1+Y(1)**2))**Y(14)**(0.78DO\*Y(8)**1+L1+0.35DO}
1 *Y(1)*Y(8)*Y(6)+(-0.43DD*Y(8)*L1+0.25DD*Y(1)*Y(8))*Y(7)**((1.4.*MP*DSIN(Y(2))*ROW4)/L0)*(Y(13)-Y(14))+((ROW1*Y(10))*
1((DSIN(Y(2))*2)/L0)*((0.78*Y(1)*L1+0.57*Y(Y(1))*2)*Y(4)
1+((0.43*Y(1)*L1+0.09DD*Y(1))*2)*Y(5))+((2.000*MP*Y(10))*
1DSIN(Y(2))*2)/L0)*ROW4*(Y(4)-Y(5)))*PART40
PART40=((ROW1*Y(10)*((DCOS(Y(2))*2)/L0)*((0.780*Y(1)*L1
1+0.57*Y(Y(1))*2)*Y(4)+0.430*Y(1)*L1+0.09DD0*Y(1))*2))
1*(2.000*MP*ROW4*Y(10)*((DCOS(Y(2))*2)/L0)*(Y(4)-Y(5))
1+((2.000*ROW*DCOS(Y(2)))/L0)*((Y(1)*(Y(4)*Y(13)+Y(5))*Y(14))
1+(0.500*Y(4)*Y(4)*Y(6)+0.500*Y(5)*Y(7)+0.6500*Y(5)*Y(6)-0.6500
1*Y(4)*Y(7)))+(8.000*MP*DCOS(Y(2))/L0)*Y(4)*Y(13)-Y(4)*Y(14)
1-(Y(5)*Y(13)+Y(5)*Y(14))+((M2*Y(10)*DSIN(Y(2)))*DCOS(Y(2)))/L0)
1+(Y(4)**2)+(Y(5)**2)+(4.000*MP*Y(10)*DSIN(Y(2)))*DCOS(Y(2))
1/L0)*((Y(4)-Y(5))*2)**PART40
B(2,3)=PART50-PART51
B(2,4)=0.000
B(2,5)=0.000
B(2,6)=0.000
B(2,7)=0.000
F(2)=((2.000*PART41*C2)/OM
D(2)=((-((ACC3*G*DCOS(Y(2)))/L0)+MH1*G)*PART40
G0 TD 721
805 Y(9)=0.000
A(2,1)=0.000
A(2,2)=1.000
A(2,3)=0.000
A(2,4)=0.000
A(2,5)=0.000
A(2,6)=0.000
A(2,7)=0.000
B(2,1)=0.000
B(2,2)=0.000
B(2,3)=0.000
B(2,4)=0.000
B(2,5)=0.000
B(2,6)=0.000
B(2,7)=0.000
F(2)=0.000
D(2)=0.000
721 C3=-K33*(Y(3)-CONST3)-K36*(Y(10)+IK33*Y(17)
A(3,1)=ROW1*(Y(1)+DCOS(Y(2)))+(0.7800*Y(6)+
1.0,43DD*Y(7)-1.3000*Y(1)*DSIN(Y(2))/((Y(4)*Y(7))-(Y(5)*Y(6))-
L1+DCOS(Y(2))=(0.7800*Y(6)+0.43DD0*Y(7))-
1+Y(1)*DSIN(Y(2))+(0.3500*Y(6)+0.2500*Y(7)))**MP*2.000*
1DCOS(Y(2)))=(Y(6)-Y(7))
A(3,2)=ROW1*(Y(1)*DSIN(Y(2))+(Y(4)+Y(6)+Y(5)+Y(7))
L1+Y(1)*DSIN(Y(2))+(0.7800*Y(6)+0.43DD0*Y(7))-
1+(Y(1)**2)+DSIN(Y(2))+(0.5700*Y(6)+0.0800*Y(7)))+MP*
L1-(4.000*DCOS(Y(2)))+((Y(4)+Y(6)-Y(4))+(Y(7)-Y(5))*Y(6)+
1+Y(5)*Y(7)+2.000*(L1+Y(1))*DSIN(Y(2))/((Y(6)-Y(7))
A(3,3)=I2+MH1+(L0)**2)+INERTI+MH1*(L1+Y(1)-L2)**2)+
1((M1-LT**2)/12.000)+(((LT/2.000)-L0)**2)*M1))
1((DCOS(Y(2))**2)+ROW1*ROW3*(ROW2*2)**(DCOS(Y(2)))**2)
1((ROW1*ROW3)**2)**(DCOS(Y(2)))**2)/12.000*ROW1*(Y(1)*
1+((DCOS(Y(2))**2)+(Y(6)+Y(7)**2)+Y(1)+(DSIN(Y(2))**2)
1+(2.000*L1*DSIN(Y(2)+DCOS(Y(2)))+Y(10)+0.7800*Y(4)+0.4300*
1Y(5)-2.000*DSIN(Y(2)))+DCOS(Y(2)+(Y(1)**2)+(0.5700*
1Y(4)+0.0800*Y(5)+ROW5*(DCOS(Y(2)))**2)+MP*(4.0000*
1+(DCOS(Y(2))**2)*((Y(6)*2+y(7)**2-2.000*Y(6)+Y(7))+}
1 4.000*((DSIN(Y(2)))**2)*(Y(6)**2+Y(7)**2-2.000*Y(6)*
Y(7)+Y(4)**2-2.000*Y(4)*Y(5)-4.000*DSIN(Y(2)))*
DCOS(Y(2))+ROW1*(Y(4)*Y(5))*DCOS(Y(2))*DCOS(Y(2))*
(A(3.4)-ROW1*(Y(1)*DSIN(Y(2)))*Y(6)-MP*4.000*DSIN(Y(2))*
(Y(6))-Y(7))
A(3.5)-ROW1*(Y(1)*DSIN(Y(2)))*Y(6)-MP*4.000*DSIN(Y(2))*Y(7)
1-Y(6))
A(3.6)-ROW1*(Y(1)*DSIN(Y(2)))*Y(4)-L1*DCOS(Y(2))*0.7800*
Y(1)-Y(2)**2+DCOS(Y(2))*0.5700+MP*4.000*DSIN(Y(2)))*Y(4)
1-Y(5))=2.000*ROW4*DCOS(Y(2)))
A(3.7)=ROW1*(Y(1)*DSIN(Y(2)))*Y(5)-L1*
DCOS(Y(2)))*0.4300*Y(1)-(Y(1)**2)*DCOS(Y(2))
1=0.0900*MP*(4.000*DSIN(Y(2)))*Y(5)-Y(4)+2.000
1=ROW4*DCOS(Y(2))}
A(4.1)=ROW1*(Y(1)/2.000)*Y(4)+1.300*Y(5)
A(4.2)=ROW6*(1.5600*L1*Y(1)+1.1400*(Y(1)**2))
1+MP=(2.000*ROW4)
A(4.3)=ROW1*DSIN(Y(2))*Y(1)+Y(6)-MP*4.000
1+DSIN(Y(2)))*Y(6)-Y(7))
A(4.4)=ROW1*(Y(1))=4.000+MP
A(4.5)=4.000*MP
A(4.6)=0.000
A(4.7)=0.000
A(5.1)=ROW1*(Y(1))*(0.6500*Y(4)+0.5000*Y(5))
A(5.2)=ROW1*(0.4300*L1*Y(1)+0.0900*(Y(1)**2)).
1-ROW4*2.000*ROW4
A(5.3)=ROW1*DSIN(Y(2))*Y(1)+Y(7)-4.000*MP*
1+DSIN(Y(2)))*Y(7)-Y(6)
A(5.4)=4.000*MP
A(5.5)=ROW1*(Y(1))=4.000*MP
A(5.6)=0.000
A(5.7)=0.000
A(6.1)=ROW1*(Y(1))*(0.500+Y(6)-0.6500*Y(7))
A(6.2)=0.000
A(6.3)=ROW1*(Y(1)*DSIN(Y(2)))*Y(4)-1.7800*Y(1)
DCOS(Y(2)))*L1*0.5700*(Y(1)**2)*DCOS(Y(2)))*MP*4.000*
1+DSIN(Y(2)))*Y(4)-Y(5)-2.000*ROW4*DCOS(Y(2)))
A(6.4)=0.000
A(6.5)=0.000
A(6.6)=ROW1*(Y(1))=4.000*MP
A(6.7)=MP*4.000
A(7.1)=ROW1*(Y(1))*(0.6500*Y(6)+0.5000*Y(7))
A(7.2)=0.000
A(7.3)=ROW1*(Y(1)*DSIN(Y(2)))*Y(5)-0.4300*L1*Y(1)
DCOS(Y(2)))*0.0900*(Y(1)**2)*DCOS(Y(2)))*MP*4.000*DSIN(Y(2))
1+Y(5)-Y(4)+2.000*ROW4*DCOS(Y(2))
A(7.4)=0.000
A(7.5)=0.000
A(7.6)=4.000*MP
A(7.7)=ROW1*(Y(1)+4.000*MP

*** COMPUTATION OF THE CORIOLIS, CENTRIPETAL, ***
*** GRAVITY AND CONTROL TERMS. ***

PART1=ROW1*Y(10)*(ROW2**2)*((DCOS(Y(2)))**2)
+ROW1*Y(10)*ROWe*((DCOS(Y(2)))**2)-((ROW1*
1+ROW2)*2)/4.000*Y(10)*((DCOS(Y(2)))**2)+ROW1*
1+Y(10)+((DCOS(Y(2)))**2)+Y(6)**2
\[
\begin{align*}
1 \cdot (y(4) \cdot 2 + y(5) \cdot 2 - 2.000 \cdot y(4) \cdot y(5) - 2.000 \cdot (\text{row}4 \cdot 2)) + \\
4.000 \cdot y(9) \cdot \text{dsin}(y(2)) + (y(4) \cdot y(6) - y(4) \cdot y(7) - \\
y(5) \cdot y(6) + y(5) \cdot y(7) - 4.000 \cdot \text{dcos}(y(2)) \cdot (2.000 \cdot y(11)) + \\
y(6) \cdot y(4) \cdot (y(13) - 2.000 \cdot y(11)) \cdot y(7) \cdot y(4) \cdot y(14) - \\
2.000 \cdot y(12) \cdot y(6) - y(5) \cdot y(13) + 2.000 \cdot y(12) \cdot y(7) + \\
y(5) \cdot y(14) - 2.000 \cdot y(9) \cdot \text{row}4 \cdot \text{dcos}(y(2)) \cdot (y(6) - y(7)) + \\
4.000 \cdot \text{dcos}(y(2)) \cdot (y(13) \cdot y(4) - y(13) \cdot y(5) - y(14) \cdot y(4)) + \\
y(14) \cdot y(5) - (y(10) \cdot (\text{dcos}(y(2)) \cdot 2) \cdot \text{row}4 \cdot (-y(10)) + \\
1 \cdot (\text{dcos}(y(2)) \cdot 2) \cdot \text{row}4 \cdot (y(4) - y(5)) \cdot 4.000)
\end{align*}
\]

\[
\begin{align*}
\text{B(3,2)} &= \text{PART6} \cdot \text{PART7} \cdot \text{PART8} \cdot \text{PART9} \cdot 2.000 \cdot y(10) \\
1 \cdot \text{dcos}(y(2)) \cdot \text{dsin}(y(2)) \cdot (\text{inerti} + \text{mh1} \cdot ((l1 \cdot y(1) - l2) \cdot 2)) \\
\text{B(3,3)} &= \text{row}1 \cdot (y(1) \cdot 2.000 \cdot y(6) \cdot y(13) \cdot y(7) \cdot y(14)) + \\
1 \cdot (\text{dsin}(y(2)) \cdot 2) \cdot 2.000 \cdot (y(4) \cdot y(11) + y(5)) + \\
y(12) \cdot 2.000 \cdot y(1) \cdot \text{dsin}(y(2)) \cdot \text{dcos}(y(2)) \cdot (0.7800) + \\
1 \cdot (y(1) \cdot y(12) + 0.4300 \cdot l1 \cdot y(12) + 0.5700 \cdot y(1) \cdot y(11) + 0.0900) + \\
y(1) \cdot y(12) \cdot \text{mp} \cdot (8.000 \cdot y(6) \cdot y(13) \cdot y(7) \cdot y(14) - y(13)) + \\
y(1) \cdot (y(6) \cdot y(14) + 0.0900 ((\text{dsin}(y(2)) \cdot 2) \cdot 2) \cdot (y(4) \cdot y(11)) + \\
y(5) \cdot y(12) \cdot y(11) \cdot y(5) \cdot y(4) \cdot y(12) - 4.000 \cdot \text{dsin}(y(2)) \cdot \\
1 \cdot \text{dcos}(y(2)) \cdot \text{row}4 \cdot (y(11) - y(12))
\end{align*}
\]

\[
\begin{align*}
\text{B(3,4)} &= 0.000 \\
\text{B(3,5)} &= 0.000 \\
\text{B(3,6)} &= 0.000 \\
\text{B(3,7)} &= 0.000 \\
\text{B(4,1)} &= \text{row}6 \cdot (y(8) \cdot (y(4) - 1.300 \cdot y(5)) \cdot y(9) \cdot y(1)) + \\
3.1400 \cdot 2.000 \cdot y(10) \cdot \text{dsin}(y(2)) \cdot y(6) - 2.000 \cdot y(8) + \\
1 \cdot (a13 \cdot y(4) + a14 \cdot y(5)) + 2.600 \cdot y(10) \cdot \text{dsin}(y(2)) + \\
y(1) \cdot y(7) + \text{mp} \cdot 4.000 \cdot y(9) + \\
\text{B(4,2)} &= \text{row}1 \cdot y(9) \cdot y(1) \cdot y(4) - 4.000 \cdot \text{mp} \cdot y(9) \cdot (y(4) - y(5)) + \\
\text{B(4,3)} &= \text{row}6 \cdot (-2.000 \cdot y(10) \cdot ((\text{dsin}(y(2)) \cdot 2) \cdot 2) \cdot y(1) + \\
1 \cdot y(4) \cdot y(10) \cdot \text{dsin}(y(2)) \cdot \text{dcos}(y(2)) \cdot y(1) \cdot (1.5600) + \\
1 \cdot l1 \cdot y(12) \cdot \text{mp} \cdot (-4.000 \cdot y(10) \cdot ((\text{dsin}(y(2)) \cdot 2) \cdot 2) + \\
\text{B(4,4)} &= \text{row}4 \cdot 2.000 \cdot y(8) + \\
\text{B(4,5)} &= \text{row}6 \cdot 2.600 \cdot y(1) \cdot y(8) + \\
\text{B(4,6)} &= \text{row}6 \cdot y(10) \cdot 4.000 \cdot \text{dsin}(y(2)) \cdot y(1) - 8.000 + \\
1 \cdot \text{mp} \cdot y(10) \cdot \text{dsin}(y(2)) + \\
\text{B(4,7)} &= 8.000 \cdot \text{mp} \cdot y(10) \cdot \text{dsin}(y(2)) + \\
\text{B(5,1)} &= \text{row}1 \cdot (y(8) \cdot (0.6500 \cdot y(4) + 0.500 \cdot y(5)) + \\
1 + 0.8600 \cdot y(9) \cdot y(1) \cdot y(10) \cdot \text{dsin}(y(2)) \cdot y(7) + \\
y(8) \cdot (a14 \cdot y(4) + a15 \cdot y(5)) - 1.3000 \cdot y(10) + \\
\text{dsin}(y(2)) \cdot y(1) \cdot y(6) \cdot \text{mp} \cdot 4.000 \cdot y(9) + \\
\text{B(5,2)} &= \text{row}1 \cdot (-y(9) \cdot y(1) \cdot y(5)) - 4.000 \cdot \text{mp} + \\
y(9) \cdot (y(5) \cdot y(4)) + \\
\text{B(5,3)} &= \text{row}1 \cdot (-y(1) \cdot y(10) \cdot ((\text{dsin}(y(2)) \cdot 2) \cdot 2) + \\
y(5) \cdot y(10) \cdot \text{dsin}(y(2)) \cdot \text{dcos}(y(2)) \cdot y(1) + \\
1 \cdot (0.4300 \cdot l1 \cdot 0.0900 \cdot y(11)) + \text{mp} \cdot (-4.000) + \\
y(10) \cdot ((\text{dsin}(y(2)) \cdot 2) \cdot 2) \cdot (y(5) \cdot y(4)) - 2.000 + \\
y(10) \cdot \text{dsin}(y(2)) \cdot \text{dcos}(y(2)) \cdot \text{row}4 + \\
\text{B(5,4)} &= \text{row}1 \cdot 1.300 \cdot y(1) \cdot y(8) + \\
\text{B(5,5)} &= \text{row}1 \cdot y(8) + \\
\text{B(5,6)} &= \text{mp} \cdot 8.000 \cdot y(10) \cdot \text{dsin}(y(2)) + \\
\text{B(5,7)} &= \text{row}1 \cdot 2.000 \cdot y(10) \cdot \text{dsin}(y(2)) \cdot y(1) + \\
1 \cdot 8.000 \cdot \text{mp} \cdot y(10) \cdot \text{dsin}(y(2)) + \\
\text{B(6,1)} &= \text{row}1 \cdot y(8) \cdot (0.500 \cdot y(6) - 0.6500 \cdot y(7)) + \\
y(10) \cdot \text{dsin}(y(2)) \cdot y(4) \cdot y(10) \cdot \text{dcos}(y(2)) + \\
1 \cdot (-1.5700 \cdot y(1)) \cdot y(10) \cdot \text{dsin}(y(2)) \cdot y(1) + \\
(-1.300 \cdot y(5)) + (-y(8) \cdot (a13 \cdot y(6) + a14 \cdot y(7))) + \\
1 \cdot \text{mp} \cdot (-4.000 \cdot y(10) \cdot \text{dcos}(y(2)))
\end{align*}
\]
B(6.3)=ROW1*(2.000*Y(1)*Y(10)*DCOS(Y(2))*Y(4))*
1.56DO+Y(1)+L1*DSIN(Y(2))*Y(10)+1.14DO*(Y(1)**2)
1*Y(10)DSIN(Y(2))+MP*(8.000*Y(10)*DCOS(Y(2))
1*(Y(4)-Y(5))+ROW4*4.000*Y(10)*DSIN(Y(2))
B(6.3)=ROW1*Y(1)*Y(10)-Y(6)*4.000*MP-Y(10)
1*(Y(6)-Y(7))
B(6.4)=2.000*ROW1*Y(10)*DSIN(Y(2))*Y(1)+8.000
1*MP-Y(10)*DSIN(Y(2))
B(6.5)=8.000*MP-Y(10)*DSIN(Y(2))
B(6.6)=ROW1*Y(8)
B(6.7)=ROW1*1.300*Y(1)*Y(8)
B(7.1)=ROW1*Y(8)*(0.6500*Y(6)+0.5000*Y(7))*Y(10)
1*DSIN(Y(2))*Y(5)*0.8600*Y(1)*Y(10)*DCOS(Y(2))
1*Y(8)*(A14*Y(6)+A15*Y(7))+1.300*Y(10)*DSIN(Y(2))
1*Y(1)*Y(4)
1*MP*4.000*Y(10)*DCOS(Y(2))
B(7.2)=ROW1*(2.000*Y(1)+Y(10)*DCOS(Y(2))*Y(5)+
0.1800*Y(1)*Y(10)+DSIN(Y(2))+0.8600*Y(1)*Y(1)
1*Y(10)*DSIN(Y(2))+MP*(8.000*Y(10)*DCOS(Y(2))
1*(Y(5)-Y(4))=4.000*ROW4*Y(10)*DSIN(Y(2))
B(7.3)=ROW1*Y(1)*Y(10)*Y(7)-4.000*MP*Y(10)*
1*(Y(7)-Y(6))
B(7.4)=MP*8.000*Y(10)*DSIN(Y(2))
B(7.5)=ROW1*2.000*Y(10)*DSIN(Y(2))*Y(1)+8.000
1*MP*Y(10)*DSIN(Y(2))
B(7.6)=ROW1*1.300*Y(1)*Y(8)
B(7.7)=ROW1*Y(8)
F(3)=C3
F(4)=0.000
F(5)=0.000
F(6)=0.000
F(7)=0.000
D(3)=0.000
D(4)=ROW1*Y(1)*G*DCOS(Y(2))*AL1=-2.000*MP*G
1*DCOS(Y(2))-E1*Y(1)*AL10)/(Y(1)**3)
1-((ROW1*CENTAC*DIST*(4.6500*Y(4)-7.3800*Y(5)))/Y(1))
1-ROW1*CENTAC*(3.0800*L1+1.1300*Y(1))*Y(4)-6.9600*L1
1+3.000*Y(1)*Y(5)-((ROW7*CENTAC+ROW8)*(4.6500*Y(4)-
1-7.3800*Y(5)))/(Y(1)+ROW8*(3.0800*Y(4)-6.9600*Y(5)))
D(5)=ROW1*Y(1)*G*DCOS(Y(2))*AL2=2.000*MP
1*G*DCOS(Y(2))-E1*Y(5)*AL11)/(Y(1)**3)
1-((ROW1*CENTAC*DIST*(5.3800*Y(4)+32.4200*Y(5)))/Y(1))
1-ROW1*CENTAC*(-6.9600*L1+3.000*Y(1))*Y(4)*23.7700*L1
1+9.7300*Y(1)*Y(5)-((ROW7*CENTAC+ROW8)*7.3800*Y(4)+
1-32.4200*Y(5))/(Y(1)+ROW9*(-6.9600*Y(4)+23.7700*Y(5)))
D(6)=ROW1*Y(1)*Y(5))/Y(1)**3-((ROW1*CENTAC*DIST*(6.6500*Y(6)-
1-7.3800*Y(7)))/(Y(1)+ROW1*CENTAC*((3.0800*L1+1.1300*Y(1))
1+Y(6)-6.9600*L1+3.000*Y(1))*Y(7))/Y(1)+ROW9*(1
3.0800*Y(6)-6.9600*Y(7))
D(7)=AL11*E1*(Y(7))/Y(1)**3-((ROW1*CENTAC*DIST*(7.3800*
1*Y(6)-32.4200*Y(7)))/(Y(1)+ROW1*CENTAC*(-6.9600*L1
1+3.000*Y(1))*Y(6)+23.7700*L1+9.7300*Y(1))*Y(7))
1-((ROW7*CENTAC+ROW8)*7.3800*Y(6)+32.4200*Y(7))
1*/Y(1))=ROW9*(-6.9600*Y(6)+23.7700*Y(7))
YVECT(1)+Y(8)
YVECT(2)+Y(9)
YVECT(3)+Y(10)
YVECT(4)+Y(11)
YVECT(5)+Y(12)
YVEC(6)=Y(13)
YVEC(7)=Y(14)

*** COMBINING THE TERMS ON THE LEFT HAND SIDE TO ONE SINGLE VECTOR. ***

DO 60 IN=1,M
   PBY(IN)=0.000
   DO 61 IM=1,M
      PBY(IN)*PBY(IN)+(-B(IN,IM))*YVEC(IM)
   61 CONTINUE
   DO 60 IY=1,M
      PALL(IY)=0.000
      PALL(IY)*PBY(IY)+F(IY)*D(IY)
60 CONTINUE

*** LU DECOMPOSITION OF THE (7,7) INERTIA MATRIX. ***

CALL DLUD(M,7,A,7,T,IV)
IF(IV(M).EQ.0) GO TO 90
   GO TO 91
90 S=0.000
   GO TO 14
91 S=-1.000

*** SOLVING FOR THE ACCELERATION TERMS. ***

CALL DBS(M,7,T,IV,PALL)

*** THE EQUATIONS OF MOTION IN FIRST ORDER FORM. ***

YDOT(1)=Y(8)
YDOT(2)=Y(9)
YDOT(3)=Y(10)
YDOT(4)=Y(11)
YDOT(5)=Y(12)
YDOT(6)=Y(13)
YDOT(7)=Y(14)
YDOT(8)=PALL(1)
YDOT(9)=PALL(2)
YDOT(10)=PALL(3)
YDOT(11)=PALL(4)
YDOT(12)=PALL(5)
YDOT(13)=PALL(6)
YDOT(14)=PALL(7)
YDOT(15)=R1-Y(1)
YDOT(16)=R2-Y(2)
YDOT(17)=R3-Y(3)
TT1=TT1+1.000

14 RETURN

END

C
C**********************************************************************
C
SUBROUTINE PEDERV (N,T,Y,PD,NO)
RETURN
END
SUBROUTINE PLTF(I,Y1,Y2,Y3,Y4,Y5,Y6,Y7,Y8,Y9,Y10,Y11,
                      Y12,Y13,Y14,Y15,Y16,Y17,DI,D2,TI)

*** SUBROUTINE IDENTIFICATION ***

THIS SUBROUTINE IS WRITTEN TO MAKE TWO DIMENSIONAL CURVES.

REAL*4 XBOX(50),YBOX(50),H1,H2,Y1(20),Y2(20)
REAL*4 Y3(20),Y4(20),Y5(20),Y6(20),Y7(20),Y8(20)
REAL*4 Y9(20),Y10(20),Y11(20),Y12(20),Y13(20)
REAL*4 Y14(20)
REAL*4 TI(20),X1(100),X2(100),X3(100),X4(100),X5(100)
REAL*4 D1(20),D2(20)
REAL*4 X8(100),X9(100),X10(100),X11(100),X12(100)
REAL*4 X13(100)
REAL*4 X14(100),X15(100),Z1,Z2,Z3,Z4,Z5,Z6,Z7,Z8
INCT=1

DATA FOR DRAWING THE BOX THAT WILL CONTAIN FOUR PLOTS.
DATA XBOX/0.0,0.5,0.5,0.0,0.0/,YBOX/0.0,0.0,0.0,1.0,1.0,0.0/
DATA H1,H2/0.12,0.1/

READ THE COORDINATES OF THE ORIGIN FOR THE FOUR PLOTS.
READ(8,45)Z1,Z2,Z3,Z4,Z5,Z6,Z7,Z8

FORMAT(8F5.3)

HEAD TITLES OF THE PLOTS.
READ(8,44)(X1(IL),IL=1,20)
READ(8,44)(X2(IL),IL=1,20)
READ(8,44)(X3(IL),IL=1,20)

SET THE LETTERING SIZE FOR THE TITLE ON A LINEAR AXIS.

CALL PLOTS
CALL PAXTL(H1)

SET THE LETTERING SIZE FOR THE NUMBERING ON A LINEAR SCALE.

CALL PAXVAL(H2)

DRAW THE CONTOUR WHICH SURROUNDS THE FOUR FIGURES.

CALL PLINE(XBOX,YBOX,5,1,0,0)

READ THE TITLES OF THE PLOTS. NOTE THAT THE LATTERS ARE
UPDATED EVERY TIME FOUR PLOTS ARE DRAWN.
C
READ(8,44)(X4(IL),IL=1,20)
READ(8,44)(X5(IL),IL=1,20)
READ(8,44)(X6(IL),IL=1,20)
READ(8,44)(X7(IL),IL=1,20)
READ(8,44)(X8(IL),IL=1,20)
READ(8,44)(X9(IL),IL=1,20)
READ(8,44)(X10(IL),IL=1,20)
READ(8,44)(X11(IL),IL=1,20)
READ(8,44)(X12(IL),IL=1,20)
IF(ICOUNT.GT.4)GO TO 371
READ(8,44)(X13(IL),IL=1,20)
READ(8,44)(X14(IL),IL=1,20)
READ(8,44)(X15(IL),IL=1,20)
371 CONTINUE
WRITE UP OF THE OVERALL TITLE OF THE RUN.
CALL PSYM(1.05,10.5,0.25,X1,0.0,40,0)
CALL PSYM(1.75,10.15,0.25,X2,0.0,40,0)
CALL PSYM(1.05,9.8,0.25,X3,0.0,40,0)

C
C
PLOTTING THE CURVES.

C
C
IF(ICOUNT.GT.1)GO TO 500
CALL PLTCUR(I, TI, Y1, X4, Z1, 22)
CALL PLTCUR(I, TI, Y2, X5, Z3, 24)
CALL PLTCUR(I, TI, Y3, X6, Z5, 26)
CALL PLTCUR(I, TI, Y4, X7, 27, 28)
GO TO 503

C
C
500 IF(ICOUNT.GT.2)GO TO 501
CALL PLTCUR(I, TI, Y5, X4, Z1, 22)
CALL PLTCUR(I, TI, Y6, X5, Z3, 24)
CALL PLTCUR(I, TI, Y7, X6, Z5, 26)
CALL PLTCUR(I, TI, Y8, X7, 27, 28)
GO TO 503

C
C
501 IF(ICOUNT.GT.3)GO TO 502
CALL PLTCUR(I, TI, Y9, X4, Z1, 22)
CALL PLTCUR(I, TI, Y10, X5, Z3, 24)
CALL PLTCUR(I, TI, Y11, X6, Z5, 26)
CALL PLTCUR(I, TI, Y12, X7, 27, 28)
GO TO 503

C
C
502 IF(ICOUNT.GT.4)GO TO 504
CALL PLTCUR(I, TI, Y13, X4, Z1, 22)
CALL PLTCUR(I, TI, Y14, X5, Z3, 24)
CALL PLTCUR(I, TI, Y15, X6, Z5, 26)
CALL PLTCUR(I, TI, Y16, X7, 27, 28)
GO TO 503

C
C
504 CALL PLTCUR(I, TI, Y17, X4, Z1, 22)
CALL PLTCUR(I, TI, Y1, X1, Z1, 2)
REAL*4 Z1, 2, TI(220), Y1(220), X1(100)

CALL PSSCALE(3.0, 0.5, TIMIN, TIF, TI, I, 1)
CALL PSSCALE(3.0, 0.5, YMIN, YIF, Y1, I, 1)
CALL PLTOFS(TIMIN, TIF, YMIN, YIF, Z1, 2)
CALL PLINE(TI, Y1, I, 1, 0.0, 1)
CALL PAXIS(Z1, 2, 'TIME: ', 1 [SECOND]', -17.3, 0,
1 0.0, TIMIN, TIF, 0.5)
RETURN
END

THIS SUBROUTINE MAKES 2 PLOTS ON THE SAME PLOTTING SPACE:

SUBROUTINE PLTDCU(I, TI, Y1, Y2, X1, Z1, 2)
REAL*4 Z1, 2, TI(220), Y1(220), Y2(220), X1(100)

CALL PSSCALE(3.0, 0.5, TIMIN, TIF, TI, I, 1)
CALL PSSCALE(3.0, 0.5, YMIN, YIF, Y1, I, 1, Y2, I, 1)
CALL PLOFS(TIMIN,TIF,Y1MIN,Y1F,Z1,Z2)
CALL PLINE(T1,Y1,I,1,0,0,1)
CALL PDSLNO(T1,Y2,I,1,0,0,1)
CALL PAXIS(Z1,Z2,"TIME, t [SECOND]",-17,3.0,1.0,0.0,TIMIN,TIF,0.5)
CALL PAXIS(Z1,Z2,X1,25,3.0,90.0,Y1MIN,Y1F,0.5)
RETURN
END
SUBROUTINE DIM3(I,X,YA,Z)

REAL*4 XMAX,YAMAX,ZMAX,XMIN,YAMIN,ZMIN
REAL*4 YES,ATT,QVMIN,QVMAX,AT
INTEGER*4 N,I

DATA PIC1,PIC2,YES/HPIC1,HPIC2,3YES/
N=1
XMAX=0.0
YAMAX=0.0
ZMAX=0.0
XMIN=0.0
YAMIN=0.0
ZMIN=0.0

SEARCH FOR THE MAXIMUM VALUE IN THE X ARRAY.
DO 30 IJ=1,N
IF(IJ.EQ.1)GO TO 31
IF(XMAX.GE.X(IJ))GO TO 30
31 XMAX=X(IJ)
30 CONTINUE

SEARCH FOR THE MAXIMUM VALUE IN THE YA ARRAY.
DO 32 IJ=1,N
IF(IJ.EQ.1)GO TO 33
IF(YAMAX.GE.YA(IJ))GO TO 32
33 YAMAX=YA(IJ)
32 CONTINUE

SEARCH FOR THE MAXIMUM VALUE IN THE Z ARRAY.
DO 34 IJ=1,N
IF(IJ.EQ.1)GO TO 35
IF(ZMAX.GE.Z(IJ))GO TO 34
35 ZMAX=Z(IJ)
34 CONTINUE

SEARCH FOR THE MINIMUM VALUES IN THE X,YA,AND Z ARRAYS.
DO 36 IJ=1,N
IF(IJ.EQ.1)GO TO 37
IF(XMIN.LE.X(IJ))GO TO 36
37 XMIN=X(IJ)
36 CONTINUE

DO 38 IJ=1,N
IF(IJ.EQ.1) GO TO 39
IF(YMIN.LE.YA(IJ)) GO TO 38
39 YMIN = YA(IJ)
38 CONTINUE
C DO 40 IJ = 1,N
IF(IJ.EQ.1) GO TO 41
IF(ZMIN.LE.Z(IJ)) GO TO 40
41 ZMIN = Z(IJ)
40 CONTINUE
C DETERMINE THE LENGTH OF THE AXES TO BE PLOTTED IN THE THREE
C DIMENSIONAL PLOTS.
C IF(XMAX.GE.YAMAX) GO TO 50
IF(YAMAX.GE.ZMAX) GO TO 52
GO TO 53
50 IF(XMAX.GE.ZMAX) GO TO 51
53 OVMAX = ZMAX
GO TO 54
51 OVMAX = XMAX
GO TO 54
52 OVMAX = YAMAX
C IF(XMIN.LE.YAMIN) GO TO 60
IF(YAMIN.LE.ZMIN) GO TO 62
GO TO 63
60 IF(XMIN.LE.ZMIN) GO TO 61
63 OVMIN = ZMIN
GO TO 64
61 OVMIN = XMIN
GO TO 64
62 OVMIN = YAMIN
64 IF(ABS(OVMAX).GE.ABS(OVMIN)) GO TO 81
AXLGH = ABS(OVMIN)
GO TO 82
81 AXLGH = ABS(OVMAX)
82 FACTOR = 0.8/AXLGH
DO 666 ILF = 1,N
X(ILF) = X(ILF)*FACTOR
YA(ILF) = YA(ILF)*FACTOR
Z(ILF) = Z(ILF)*FACTOR
666 CONTINUE
C WRITE(7,432)XMAX,YAMAX,ZMAX,XMIN,YAMIN,ZMIN
432 FORMAT(2X,'XMAX=',F15.8,10X,'YAMAX=',F15.8,10X,'ZMAX=',F15.8,10X,
1 /2X,'XMIN=',F15.8,10X,'YMIN=',F15.8,10X,'ZMIN=',F15.8)
WRITE(7,433)OVMAX,OVMIN
433 FORMAT(2X,'OVMAX=',F15.8,10X,'OVMIN=',F15.8)
C WRITE(7,77)FACTOR,AXLGH
77 FORMAT(//,5X,'FACTOR=',F15.7,7X,'AXLGH=',F15.7)
C THIS IS AN INTERACTIVE PORTION OF THE PROGRAM TO ALLOW THE
C USER TO VIEW THE THREE DIMENSIONAL PLOT FROM THE DESIRED
C ANGLE.
C WRITE(6,627)
627 FORMAT(2X,'WHAT ARE THE DESIRED ANGLES TO VIEW THE PLOT ?')
WRITE(6,676)
   FORMAT(2X,'THE ANGLE FOR THE X-AXIS IS: (DEGREES)')
READ(5,845)AX
WRITE(6,901)
901   FORMAT(2X,'THE ANGLE FOR THE Y-AXIS IS: (DEGREES)')
READ(5,902)AY
902   FORMAT(2X,'THE ANGLE FOR THE Z-AXIS IS: (DEGREES)')
READ(5,617)AZ
617   FORMAT(F15.7)
   AAX=(AX*3.141592654)/180.0
   AAY=(AY*3.141592654)/180.0
   AAZ=(AZ*3.141592654)/180.0
C      START PLOTTING THE THREE DIMENSIONAL PLOT.
C
CALL IGINIT
CALL IGBONS(PIC1)
CALL IGBONS(PIC2)
CALL IGMA(-0.8,0.,0.)
CALL IGDA(0.8,0.,0.)
CALL IGMA(0.,-0.8,0.)
CALL IGDA(0.,0.8,0.)
CALL IGMA(0.,0.,-0.8)
CALL IGDA(0.,0.,0.8)
CALL IGMA(X(1),Y(1),Z(1))
DO 45 IT=2,N
   CALL IGDA(X(IT),Y(IT),Z(IT))
45 CONTINUE
CALL IGTXT('<RSCALE>',2.0)
CALL IGMA(4.,-0.05,0.)
CALL IGTXT('<X<E>')
CALL IGMA(-0.05,-0.05,0.)
CALL IGTXT('<O<E>')
CALL IGMA(0.05,4.0)
CALL IGTXT('<Y<E>')
CALL IGMA(-0.05,0.,4)
CALL IGTXT('<Z<E>')
CALL IGTRAN(PIC2,'ROTY',AAY,'ROTX',AAX,'ROTZ',AAZ)
CALL IGENDS(PIC2)
CALL IGTXT('<RSCALE>',2.0)
CALL IGMA(-0.4,-0.85,0.)
CALL IGTXT('FIGURE 10: END EFFECTOR POSITION'
   '<CRLF>FOR THE CASE OF R=2.0 METER,
   '<CRLF>THETA=0.5 RAD. AND PHI=0.5 RAD.<E>')
CALL IGENDS(PIC1)
CALL IGRON('TERMINAL')
C
      THIS INTERACTIVE PORTION OF THE PROGRAM ALLOWS THE USER
      TO MAKE HARD COPY OF THE THREE DIMENSIONAL PLOT IF DESIRED
      AND TO SPECIFY ANOTHER SET OF ANGLES IF THE PLOT IS TO BE
      VIEWED FROM DIFFERENT ANGLE.
C
WRITE(6,48)
48   FORMAT(2X,'DOES THE PLOT LOOKS GOOD ?',/,,2X,
   'WOULD YOU LIKE TO MAKE A HARD COPY? (YES/NO)')
READ(5,49)ATT
49   FORMAT(A4)
IF(ATT.EQ.YES)GO TO 888
GO TO 889
888 CALL IGDROM('CALCOMP')
889 WRITE(6,324)
324 FORMAT(2X,'WOULD YOU LIKE TO TRY DIFFERENT ANGLES? (Y/N)')
READ(5,325)AT
325 FORMAT(A4)
IF(AT.EQ.YES) GO TO 44
RETURN
END

.75 6.05 4.75 6.05 0.75 1.525 4.75 1.525
RIGID AND FLEXIBLE MOTION CASE,
DESIRED POSITION : R=0.5 M
THETA=0.5 RAD. AND PHI=0.5 RAD.
  r DISPLACEMENT [METER]
  THETA ROTATION [RAD]
  PHI ROTATION [RAD]
  DISPLACEMENT. q (t) [M]
  DISPLACEMENT. q (t) [M]
  DISPLACEMENT. q (t) [M]
  VELOCITY, r [M/SEC]
  VELOCITY, q (t) DISPLACEMENT.
  VELOCITY, q (t) DISPLACEMENT.
  VELOCITY, q (t) DISPLACEMENT.
  VELOCITY, q (t) DISPLACEMENT.
ANGULAR VELOCITY, [RAD/SEC]
ANGULAR VELOCITY, [RAD/SEC]
VELOC. q (t) [M/SEC]
VELOCITY, q (t) [M/SEC]
ANGULAR VELOCITY, [RAD/SEC]
ANGULAR VELOCITY, [RAD/SEC]
FIGURE 1. GENERAL MOTION,
  r DISPLACEMENT.
FIGURE 2. GENERAL MOTION,
  THETA DISPLACEMENT.
FIGURE 3. GENERAL MOTION,
  PHI ROTATION.
FIGURE 4. GENERAL MOTION,
  DISPLACEMENT. q (t) DISPLACEMENT.
FIGURE 5. GENERAL MOTION,
  q (t) DISPLACEMENT.
FIGURE 6. GENERAL MOTION,
  q (t) DISPLACEMENT.
FIGURE 7. GENERAL MOTION,
  q (t) DISPLACEMENT.
FIGURE 8. GENERAL MOTION,
  r VELOCITY.
FIGURE 9. GENERAL MOTION,
  ANGULAR VELOCITY.
FIGURE 10. GENERAL MOTION,
  ANGULAR VELOCITY.
FIGURE 11. GENERAL MOTION,
  q (t) VELOCITY.
FIGURE 12. GENERAL MOTION,
  q (t) VELOCITY.
FIGURE 13. GENERAL MOTION,
  VELOCITY, q (t) [M/SEC]
FIGURE 14. GENERAL MOTION,
  VELOCITY, q (t) [M/SEC]
CONTROL TORQUE, T1 [N-M]
CONTROL TORQUE, T2 [N-M]
FIGURE 13. GENERAL MOTION, q(t) VELOCITY.
FIGURE 14. GENERAL MOTION, q(t) VELOCITY.
FIGURE 15. GENERAL MOTION, CONTROL TORQUE FOR r.
FIGURE 16. GENERAL MOTION, CONTROL TORQUE FOR T3 [N-M]
V DEFORMATION [M].
W DEFORMATION [M].
FIGURE 17. GENERAL MOTION, CONTROL TORQUE FOR .
FIGURE 18. GENERAL MOTION, VERTICAL DEFORMATION V.
FIGURE 19. GENERAL MOTION, HORIZONTAL DEFORMATION W.
APPENDIX G

LISTING OF THE COMPUTER CODE

FOR THE EXPERIMENTAL WORK
INTEGR*2 ENC2V,CJ1,CJ2,DACVAL,DACHN,N,TERMCT,SOURCF
1,ADCAVL,ADCHN

COMMON /FRQ1/FREQ,N,DT
COMMON /ENCBL/CJ1,CJ2,ENC2V
COMMON /DACBL/DACHN,DACVAL
COMMON /ADCBL/ADCHN,ADCVL
COMMON /COUNT/TERMCT,SOURCF
COMMON /STEP/K

OPEN(17,FILE="DATA.TXT",STATUS="NEW")

CALL INIT
CALL CLK(FRQ,FREQCK)
DT=1.0/FREQCK
WRITE(*,50)TERMCT,SOURCF,FREQCK,DT
50 FORMAT(2X,THE TERMINAL COUNT IS =',I6,2X,
THE SOURCE FREQUENCY IS =',I2,2X,
FREQUENCY DESIRED IS =',F15.8,2X,
' Sampling Period is =',F15.8,2X)

CALL CLOCK

WRITE(*,10)
 FORMAT(2X,ENTER JOINT 2 CHANNEL NUMBER,(REAL NUMBER))
READ(*,11)DACH
11 FORMAT(F15.8)
DACHN=IFIX(DACH)

WRITE(*,12)DACHN
32 FORMAT(2X,CHANNEL NUMBER CHOSEN FOR JOINT 2 IS ,I3,2X,
IS THIS SELECTION CORRECT ? (Y/N))
READ(*,13)AF
13 FORMAT(A1)
IF(AF.EQ."N")GO TO 14

DACVAL=0
CALL DAC

CJ1=2
CJ2=3

FREQ=FREQCK
CALL CLSP

STOP
END
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51

52

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Pass One  No Errors Detected

52 Source Lines
SUBROUTINE CLKFRQ(FREQK)

THIS ROUTINE IS TO INPUT CLOCK FREQUENCY, AND THEN
DETERMINE TERMCT AND SOURCF FOR THE LABTENDER CLOCK

INTEGER TERMCT,SOURCF
COMMON COUNT/TERMCT,SOURCF

SOURCF = 5

4 WRITE(*,7)
7 FORMAT('Enter Frequency of Clock Interrupts (Hz)/'
     'Typical value: 1000. Maximum: 5000. '/
     'Include a decimal point.')
6 READ (*,8) FREQK
8 FORMAT(F15.8)
17 IF(FREQK.LE.5000.) GOTO 9
19 WRITE(*,*) 'Clock frequency must be less than 5000. Hz.'
20 GOTO 4
21 CONTINUE
22
23 C TESTCT = REAL VALUE FOR TERMCT, NECESSARY IF TERMCT > 32767.
24 10 TESTCT = AINT((1.10**SOURCF)/FREQK)
25
26 C WRITE STATEMENTS FOR VARIOUS INTERMEDIATE VALUES ARE COMMENTED
27 C OUT IN THIS PROGRAM TO MAKE THEM INVISIBLE TO THE USER. YOU MAY WISH
28 C TO USE THE WRITES TO SEE HOW THE PROGRAM WORKS, BUT DO NOT LEAVE
29 C THEM IN YOUR FINAL PROGRAM.
30
31 WRITE(*,15) TESTCT,SOURCF
32 15 FORMAT(' TESTCT = ',F12.3,' SOURCF = ',I2)
33
34 C CHECK TO BE SURE TESTCT AND SOURCF ARE WITHIN THE DESIRED RANGE.
35 IF (TESTCT.GT.32767.) THEN
36 SOURCF = SOURCF - 1
37 GOTO 10
38 ELSEIF (TESTCT.LT.2) THEN
39 WRITE(*,17) FREQK
40 17 FORMAT(' Clock Frequency of ',F12.3,' is too high.')
41 GOTO 4
42 ELSEIF (SOURCF.LT.2) THEN
43 WRITE(*,18) FREQK
44 18 FORMAT(' Clock Frequency of ',F12.3,' Hz is too low.')
45 GOTO 4
46 ELSE
47 CONTINUE
48 TERMCT = INT(TESTCT)
49 ENDIF
50
51 WRITE(*,35) TERMCT,SOURCF
52 35 FORMAT(' TERMCT = ',I6,' SOURCF = ',I2)
> 53 C  FROUT = (1*10**SOURCF)/REAL(TERMCT)
> 54 C  FREQK = FROUT/1000.
> 55 C  ERROR = ((FROUT - FREQCK)/FREQCK)*100.
> 56 C  WRITE(*,40) FREQCK,FROUT,ERROR,FREQK
> 57 C  40 FORMAT(' COUNTER FREQUENCY OUT = ',F12.3/
> 58 C   ' ERROR = ',F12.3,' %';/
> 60 C   ' FREQ. IN KHz = ',F12.3)
> 61
> 62 RETURN
> 63 END

> Name  Type  Offset  P Class
> AINT
> FREQCK  REAL  0  INTRINSIC
> INT
> SOURCF  INTEGER*2  2  /COUNT /
> TERMCT  INTEGER*2  0  /COUNT /
> TESTCT  REAL  192

> Name  Type  Size  Class
> CLKFRQ
> COUNT  4  SUBROUTINE

> Pass One  No Errors Detected
> 63 Source Lines
SUBROUTINE CLSLIP

*** THIS SUBROUTINE PERFORMS THE CLOSED LOOP RUN. ***
*** WHICH CONSISTS OF THE RIGID BODY CONTROLLER. ***
*** THE LATTER IS A POINT TO POINT CONTROLLER. ***

INTEGER N,K,CJ1,CJ2,ENC2V,DACVAL,DACHN,ICL,M(3000),J,
1 IFREQ,K1,K2,K3,ADCVAL,ADCHN,NF

DIMENSION P(420),V(420),E(420),EI(420),VI(840),RM1(420),T(1000)
DIMENSION PV(840)

COMMON /FRO1/FREQ,N,DT
COMMON /ENCBL/CJ1,CJ2,ENC2V
COMMON /DACBL/DACHN,DACVAL
COMMON /ADCBL/ADCHN,ADCVAL
COMMON /STEP/K

IFREQ=IFX(FREQ)

WRITE(*,10)
FORMAT(20X,*** CLOSED LOOP RUN. ****,/20X,
1 **** RIGID BODY CONTROLLER ONLY. ****,/2X,
1 *** CHANNEL NUMBER CHOSEN FOR q dot IS 0 ****,/,
1 2X,**** CHANNEL NUMBER CHOSEN FOR q IS 1 ****,/2X,
1 ENTER CLOSED LOOP REFERENCE VALUE IN DEGREES; /2X,
1 TYPICAL VALUE IS BETWEEN 10 AND -20.0 DEGREES.)
READ(*,11)CL

D=1.0
ICL=IFX(CL*31.25)
AK1=-57.37/(D*31.25)
AK2=-8.75/(D*31.25)
AK3=-51.52/(D*31.25)

WRITE(*,9)
FORMAT(2X,ENTER DURATION OF THE RUN. (SECONDS)/,
12X,***** MAXIMUM 10 SECONDS. *****)
READ(*,8)DN
N=IFX(FREQ*DN)
NF=N+IFREQ5

WRITE(*,14)CL,ICL,D,N,DT,AK1,AK2,AK3
FORMAT(2X,CLOSED LOOP REFERENCE VALUE=,*F15.8,DEGREES;*,
12X,CLOSED LOOP REFERENCE VALUE=,*I7,COUNTS;*,2X,
1 SCALING FACTOR=,*F10.5,3X,N=,*I7,2X,DT=,*F15.8;*,30X,
1 CONTROLLER GAINS ARE: *,2X,K1=,*F15.5,3X,K2=,*F15.5,
1 K3=,*F15.5;*,2X,ARE THEY CORRECT? (Y/N))
READ(*,15)AT1
FORMAT(A1)
IF(AT1.NE."Y")GO TO 16
INITIALIZATION OF THE POSITION VECTOR, THE VELOCITY VECTOR AND
THE ERROR VECTOR.
K=1
CALL ENC
ADCHN=0
CALL ADC
VI(K)=ADCVAL
ADCHN=1
CALL ADC
PV(K)=ADCVAL
P(K)=ENC2V
POS=P(K)
POST=P(K)
ERROR=0.0
E(K)=0.0
V(K)=0.0
E(K)=ENC2V-ICL
M(K)=0
K=2
CALL ENABLE
J=K
CALL ENC
ADCHN=0
CALL ADC
VI(K)=ADCVAL
ADCHN=1
CALL ADC
PV(K)=ADCVAL
P(K)=ENC2V
V(K)=(P(K)-POS)*FREQ
E(K)=P(K)-ICL
EI(K)=E(K)*DT+ERROR
POS=P(K)
ERROR=EI(K)
COMPUTATION OF THE CONTROL SIGNAL
RM = -AK1*(P(K)-POST)-AK2*V(K)-AK3*EI(K)
M(K) = IFIX(RM)
IF(M(K),GE.127) M(K) = 127
IF(M(K),LE.-128) M(K) = -128
SEND OUT THE CONTROL SIGNAL
DACVAL = M(K)
CALL DAC
WAIT FOR THE INTERRUPT SERVICE ROUTINE TO INCREMENT K.
17 IF(J.EQ.K) GO TO 17
18 IF(K.LE.N) GO TO 18
68 J = K
ADCHIN = 0
CALL ADC
VI(K) = ADCVAL
ADCHIN = 1
CALL ADC
PV(K) = ADCVAL
SHUT OFF THE MOTOR.
DACVAL = 0
CALL DAC
IF(J.EQ.K) GO TO 67
IF(K.LE.N) GO TO 68
DISABLE THE INTERRUPT.
CALL DISABL
OUTPUT THE RESULTS.
DO 70 I = 1, N
P(I) = P(I)/31.25
V(I) = V(I)/31.25
E(I) = E(I)/31.25
RM1(I) = M(I)*0.039055118
T(I) = (I-1)*DT
CONTINUE
WRITE(*,19)
Microsoft FORTRAN77 V3.31

> 153 C 19 FORMAT(/4X,'TIME',5X,'POSITION',4X,'VELOCITY',6X,'ERROR',
> 154 C 15X,'CONTROL',9X,'(DEGREES)',3X,'(DEG/SEC)',3X,'(DEGREES)',
> 155 C 13X,(VOLTS'),/)
> 156 C     DO 20 I=1,N
> 157 C     WRITE(*,21)T(I),P(I),V(I),E(I),RM1(I)
> 158 C 21 FORMAT(2X,F8.5,2X,F10.5,2X,F10.5,2X,E10.5,2X,F10.5,2X,F10.5)
> 159 C 20 CONTINUE
> 160 C
> 161 C     WRITE(*,22)
> 162 C 22 FORMAT(/4X,'TIME',4X,'VIBRATION MEASUREMENT',/17X
> 163 C 1,'(MM/SECOND')',/)
> 164 C     DO 94 I=1,NF
> 165 C 94     T(I)=(I-1)*DT
> 166 C     CONTINUE
> 167 C
> 168 C     DO 89 I=1,NF
> 169 C     WRITE(*,23)T(I),VI(I)
> 170 C 23 FORMAT(2X,F8.5,2X,F10.5)
> 171 C 89 CONTINUE
> 172 C
> 173 C     DO 111 I=1,N
> 174 C     WRITE(17,112)T(I),P(I),V(I),E(I),RM1(I)
> 175 C 112 FORMAT(2X,F8.5,2X,F10.5,2X,F10.5,2X,F10.5,2X,F10.5)
> 176 C 111 CONTINUE
> 177 C
> 178 C     DO 113 I=1,NF
> 179 C     WRITE(17,114)T(I),VI(I),PV(I)
> 180 C 114 FORMAT(2X,F8.5,2X,F10.2,2X,F10.2)
> 181 C 113 CONTINUE
> 182 C
> 183 C     WRITE(*,24)
> 184 C 24 FORMAT(2X,'WOULDN YOU LIKE TO MAKE ANOTHER RUN ? (YN)')
> 185 C     READ(*,25)AT2
> 186 C 25 FORMAT(A1)
> 187 C     IF(AT2.EQ.'Y')GO TO 16
> 188 C
> 189 C     RETURN
> 190 C     END
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Microsoft FORTRAN77 V3.31

COMMON
SUBROUTINE
SUBROUTINE
COMMON
SUBROUTINE
SUBROUTINE
COMMON
SUBROUTINE
COMMON
COMMON

> >ADCBL 4
> >CLSĽP
> >DAC
> >DACBL 4
> >DISABL
> >ENABLE
> >ENC
> >ENCBL 6
> >FRQ1 10
> >STEP 2
>
>
>Pass One No Errors Detected
> 191 Source Lines
#

The IBM Personal Computer Assembler 01-01-80 PAGE 1-1

EXPERIMENTAL WORK ASSEMBLY CODE

TITLE EXPERIMENTAL WORK ASSEMBLY CODE

PAGE, 132 ; Set page width to 132 characters.

; This routine initializes the digital I/O port and
; sets up the IRQ4 interrupt vector.
; FORTRAN "CALL INIT" calls this procedure.

DATA SEGMENT PUBLIC 'DATA'
; For local assembler program data storage.
; Not used, but required for linking
DATA ENDS

STEP$A SEGMENT COMMON '$STEP$
; This segment is equivalent to FORTRAN's
; COMMON/STEP/K. The $ and A are
; required to match the segment name and
; class that the linker uses for the FORTRAN
; common. Eleven variables must be declared
; INTEGER*2 in the FORTRAN calling routine
; before the common block.

STEP$ ???? K DW ?

STEP$A ENDS

DGROUP GROUP DATA,STEP$A
; The DATA,STEP$A segment will be linked into
; the group called DGROUP to match Microsoft
; FORTRAN convention.

CODE SEGMENT PUBLIC 'CODE'

ASSUME CS:CODE,DS:DGROUP,SS:DGROUP
PUBLIC INIT ; Make the INIT label available
; to other segments.

; Labtender Address, in IBM PC, for SETDDA routine.
>= 033F
IOPCOM EQU 831 ; Parallel Port Control Register.

INIT PROC FAR

55 PUSH BP ; Save calling framepointer.
8B EC MOV BP,SP
B2 49 MOV DL,73 ; Print an 'I' to the screen
B4 02 MOV AH,2 ; for debugging
CD 21 INT 21H
The IBM Personal Computer Assembler 01-01-80

> Experimental Work Code

> INTERRUPT SERVICE ROUTINE

> ; Load the IRQ4 interrupt vector (address to
goto interrupt 4).

> 0009 FA CLI ; Disable interrupts while we are
modifying them to prevent any
catastrophes.

> 000A 06 PUSH ES ; Save Extra Segment

> 000B B8 0000 MOV AX,0000H

> 000E 8E C0 MOV ES,AX ; Point Extra Segment at the ISR address

> 0010 BF 0030 MOV DI,30H ; Table. Offset of entry for IRQ 4 in the

> 0013 B8 0020 R MOV AX,OFFSET ISR ; address table. Get offset of DDASER,
our interrupt service routine.

> 0016 AB STOSW ; Load offset in AX into address in DI,
and add 2 to DI to prepare to store CS.

> 0017 8C C8 MOV AX,CS ; Get Code Segment for assembly code.

> 0019 AB STOSW ; Load code segment in AX into address
in DI. Now the 4-byte address of our

> ; DDASER begins at memory location 30H.

> ; An IRQ4 Interrupt (if enabled) will
cause the microprocessor to immedi-
ately go to that address, and start
performing our DDASER.

> 001A 07 POP ES ; Restore Extra Segment

> 001B FB STI ; Enable interrupts

> 001C 8B E5 MOV SP,BP ; Restore frame pointer

> 001E 5D POP BP ; Return

> 0020 INT ENDP

; : SUBTTL INTERRUPT SERVICE ROUTINE
; :---------------------------------------------------------------------

; : Labtender Addresses, in IBM PC, for DDASER routine.

= 0332 INTC1R EQU 818 ; Timer Interrupt Clear Address.

= 8259 Programmable Interrupt Controller Addresses

= in IBM PC.

= 0020 ICR EQU 20H ; Interrupt Command Register.

= 0021 IMR EQU 21H ; Interrupt Mask Register.

= Constants for DDASER routine.

= 0020 EOI EQU 20H ; End of Interrupt signal for
; 8259 Interrupt Controller.
The IBM Personal Computer Assembler 01-01-80
>Experimental Work Code
>
INTERRUPT SERVICE ROUTINE
>
> 0020 ISR PROC FAR
> 0020 55 PUSH BP ; Save calling framepointer.
> 0021 8B EC MOV BP, SP
>
> 0023 50 PUSH AX ; Save all the registers
> 0024 53 PUSH BX ; we will be modifying.
> 0025 51 PUSH CX ; This is done automatically
> 0026 52 PUSH DX ; only during FORTRAN CALLs,
>
> ; not interrupts.
>
> 0027 FF 06 0000 R INC K ; K = K + 1, count another
> ; time step.
> 002B CLKEOI:
> 002B BA 0332 MOV DX, INTCLR ; Timer Interrupt Clear
> 002E B0 00 MOV AL, 0 ; Data could be anything
> 0030 EE OUT DX, AL ; Write to clear the interrupt.
>
> 0031 B0 20 MOV ALEOI ; Get end-of interrupt signal
> 0033 BA 0020 MOV DX, ICR ; Send it to the 8259's
>
> 0036 EE OUT DX, AL ; Interrupt Command Register
>
> 0037 5A POP DX ; Restore registers
> 0038 59 POP CX
> 0039 5B POP BX
> 003A 58 POP AX
>
> 003B 8B E5 MOV SP, BP ; Restore framepointer
> 003D 5D POP BP
>
> 003E CF IRET ; Interrupt return.
> 003F ISR ENDP
>
> 003F CODE ENDS
>
> END
The IBM Personal Computer Assembler 01-01-80

> Experimental Work Code

> Segments and groups:

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<td>PARA</td>
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> Symbols:

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<td>LWORD</td>
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<td>STEP$A</td>
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</table>

> Warning Severe
> Errors Errors

> 0

#
TITLE CLOCK - ROUTINE TO INITIALIZE COUNTERS 1 AND 2

PAGE 132 ;Set page width to 132 characters

CLOCK is a routine which initializes Counter 1 and Counter 2 in the Labtender. Counters can be used to count and/or generate various signals. The frequency sources, signal outputs, and maximum counts are all programmable. Counter 1 will be used as a clock, generating pulses at a desired frequency. Counter 2 will count the number of pulses that Counter 1 generates. FORTRAN "CALL CLOCK" calls this procedure.

COUNTER 1:
TERMCT and SOURCF control the frequency generated by counter 1. TERMCT, the terminal count for Counter 1, is the number of pulses Counter 1 will count from the source frequency before generating a pulse. Counter 1 counts down from TERMCT to zero repetitively.
SOURCF selects the source frequency for Counter 1 from the Labtender. NOTE THAT SOURCF ISN'T THE ACTUAL SOURCE frequency; it is used to SELECT A FREQUENCY SOURCE ON THE LABTENDER.
The Labtender has a 1 MHz crystal which generates 5 possible source frequencies, from 1 MHz (10^6) to 100 Hz (10^2). To choose a source frequency, we define SOURCF as the power of 10 of the desired source frequency from the Labtender for Counter 1. We generally need frequencies below 5 kHz, and higher frequencies have occasionally caused problems, so we will limit the source frequency to 100 kHz and the clock frequency to 5 kHz. SOURCF must then be between 2 and 5.
TERMCT must be between 2 and 32767. A program called CLKFRQ.FOR is available to select TERMCT and SOURCF, and can be copied into your program and modified as desired.
Counter 1 will generate a pulse on pin OUT 1 every time it counts TERMCT pulses. The output frequency of Counter 1 (the 'clock' frequency) is (1*10**SOURCF)/TERMCT. To generate a 500 Hz clock, choose 500 =10000/20 so SOURCF = 4 and TERMCT = 20, for example.
The OUT1 pin can be used to generate timer interrupts. Enabling the IRQ4 interrupt will cause the timer interrupts to be sent to the processor. If we don't enable IRQ4 then the interrupts will be ignored.

COUNTER 2:
Counter 2 counts the pulses generated from Counter 1. The
The IBM Personal Computer Assembler 01-01-80 PAGE 1-2
> CLOCK - ROUTINE TO INITIALIZE COUNTERS 1 AND 2
>
> assembler routine CLKOUT(CLKTIC) can then be used to read
> the value in Counter 2 and return it to FORTRAN as CLKTIC,
> the number of clock counts.
>0000 DATA SEGMENT PUBLIC 'DATA'
> ; For local assembler program data storage.
> ; Not used, but required for linking
>0000 DATA ENDS
>
>0000 COUNT$A SEGMENT COMMON $COUNT'
> ; This segment is equivalent to COMMON/COUNT/
> ; TERMCT,SOURCF in FORTRAN.
> ; The $ and A are required to match the segment name
> ; and class that the linker uses for the FORTRAN common.
> ; TERMCT and SOURCF must be declared INTEGER*2 in the
> ; FORTRAN calling routine before the common block.
>
>0000 ???? TERMCT DW ?
>0002 ???? SOURCF DW ?
>
>0004 COUNT$A ENDS
>
>DGROUP GROUP DATA,COUNT$A
> ; The DATA and COUNT$A segments will be linked into
> ; the group called DGROUP, to match Microsoft
> ; FORTRAN convention.
>
>0000 CODE SEGMENT PUBLIC 'CODE'
>
> ASSUME CS:CODE,DS:DGROUP,SS:DGROUP
> PUBLIC CLOCK ; Make the CLOCK label available
> ; to other segments.
> ; Labtender Addresses, in IBM PC, for the programmable timer.
>
>= 0338 CLKDATA EQU 824 ; Data Port for 9513 Timer.
>= 0339 CLKCOM EQU 825 ; Command Port for 9513 timer.
>
>0000 CLOCK PROC FAR
>
>0000 55 PUSH BP ; Save calling framepointer.
>0001 8B EC MOV BP,SP
>
>0003 B2 43 MOV DL,67 ; Print a 'C' to the screen
>0005 B4 02 MOV AH,2 ; for debugging
>0007 CD 21 INT 21H
> ; First reset all counters to zero.
>0009 BA 0339 MOV DX,CLKCOM ; Master Reset command.
The IBM Personal Computer Assembler 01-01-80 PAGE 1-3

; CLOCK - ROUTINE TO INITIALIZE COUNTERS 1 AND 2

>000C B0 FF  MOV AL, 255
>000E EE  OUT DX, AL

; Set up the Master Mode Register.

>000F BA 0339  MOV DX, CLKCOM ; Set Data Pointer to Master mode register
>0012 B0 17  MOV AL, 23
>0014 EE  OUT DX, AL

>0015 BA 0338  MOV DX, CLKDATA ; Write data to Master Mode
>0018 B0 00  MOV AL, 0 ; Register, lo-byte:
>001A EE  OUT DX, AL ; Use default values

>001B B0 80  MOV AL, 128 ; Write data to Master Mode

>001D EE  OUT DX, AL ; Register, hi-byte:

; Use BCD division when dividing frequencies. Ex: 1 MHz
crystal/1000 (base 10) = 1000 Hz

; Set up Counter 1.

>001E BA 0339  MOV DX, CLKCOM ; Disarm Counter 1, so we can
>0021 B0 C1  MOV AL, 193 ; write to its control port
>0023 EE  OUT DX, AL

>0024 B0 01  MOV AL, 1 ; Set Data Pointer to Counter 1's
>0026 EE  OUT DX, AL ; Mode Register.

>0027 BA 0338  MOV DX, CLKDATA ; Write Data to Counter 1 Mode Register
>002A B0 21  MOV AL, 33 ; lo-byte, Count repetitively, reload from load register,
>002C EE  OUT DX, AL ; Count down, Terminal Count pulse high.

; Manipulate SOURCF to create command to select source
; frequency. By coincidence, (17 - SOURCF) will be correct
; command.

>002D BB 0E 0002 R MOV CX, SOURCF ; Put SOURCF in CX.
>0031 BB 0011  MOV AX, 17 ; AX = 17
>0034 2B C1  SUB AX, CX ; AX <-- AX - CX. Now AL has command.
>0036 EE  OUT DX, AL ; Write Data to Counter 1 Mode Register

; hi-byte: Count on rising edge, source frequency = 10**SOURCF

; Put TERMCT in Counter 1's Load Register. TERMCT is the value
; from which Counter 1 will start counting down.

>0037 BA 0339  MOV DX, CLKCOM ; Set Data Pointer to Counter 1's Load
>003A B0 09  MOV AL, 9 ; Register.
>003C EE  OUT DX, AL
The IBM Personal Computer Assembler 01-01-80 PAGE 1-4

>CLOCK - ROUTINE TO INITIALIZE COUNTERS 1 AND 2

>003D BA 0338 MOV DX,CLKDATA ; Write data to Counter 1's Load Register.
>0040 A1 0000 R MOV AX,TERMCT ; Put TERMCT in AX.
>0043 EE OUT DX,AL ; Send lo-byte to Load Register.
>0044 8A C4 MOV AL,AH ; Put hi-byte of TERMCT into AL.
>0046 EE OUT DX,AL ; Send hi-byte to Load Register.
>
> ; Set up Counter 2.
>
>0047 BA 0339 MOV DX,CLKCOM ; Disarm Counter 2, so we can
>004A B0 C2 MOV AL,194 ; write to its control port.
>004C EE OUT DX,AL
>
>004D B0 02 MOV AL,2 ; Set Data Pointer to Counter 2's
>004F EE OUT DX,AL ; Mode Register.
>
>0050 BA 0338 MOV DX,CLKDATA ; Write Data to Counter 2 Mode
>0053 B0 2A MOV AL,42 ; Register, lo-byte: Count repetitively,
>0055 EE OUT DX,AL ; reload from load register,
>0057 8A C4 MOV AL,AH ; Count up, square wave output.
>
>0056 B0 00 MOV AL,0 ; Write Data to Counter 2 Mode
>0058 EE OUT DX,AL ; Register, hi-byte:
> ; Count on rising edge, source
> ; frequency = Counter 1. Each time
> ; Counter 1 =0 we get a pulse.
>
> ; Put zero in Counter 2's Load Register.
>
>0059 BA 0339 MOV DX,CLKCOM ; Set Data Pointer to Counter
>005C B0 0A MOV AL,10 ; 1's Load Register.
>005E EE OUT DX,AL
>
>005F B8 0000 MOV AX,0 ; Put zero in AX.
>0062 EE OUT DX,AL ; Send lo-byte to Load Register.
>0063 8A C4 MOV AL,AH ; Put hi-byte of AX into AL.
>0065 EE OUT DX,AL ; Send hi-byte to Load Register.
>
> ; Turn on Counters 1 & 2.
>
>0066 B0 63 MOV AL,99 ; Command to Load and Arm
>0068 EE OUT DX,AL ; Counters 1 & 2
>
>0069 8B E5 MOV SP,BP ; Restore framepointer
>006B 5D POP BP
>006C CB RET ; Return
>
>006D CLOCKENDP
>006D CODEENDS
>
> END
The IBM Personal Computer Assembler 01-01-80  PAGE  Symbols-1

>CLOCK - ROUTINE TO INITIALIZE COUNTERS 1 AND 2

>Segments and groups:

>   Name  Size  align  combine  class

>CODE .................. 006D  PARA  PUBLIC  'CODE'
>DGROUP ................. GROUP
>DATA .................. 0000  PARA  PUBLIC  'DATA'
>COUNT$A ............... 0004  PARA  WORD  '$COUNT'

>Symbols:

>   Name  Type  Value  Attr

>CLKCOM ................ Number  0339
>CLKDATA ............... Number  0338
>CLOCK ................. FPROC  0000  CODE  Global Length=006D
>SOURCEF ............... LWORD  0002  COUNT$A
>TERMCT ................. LWORD  0000  COUNT$A

>Warning Severe
>Errors  Errors

   0
TITLE DISABL - ROUTINE TO DISABLE INTERRUPTS

; Set page width to 132 characters.

; FORTRAN "CALL DISABL" calls this procedure.

DATA SEGMENT PUBLIC 'DATA'

; For local assembler program data storage.
; Not used, but required for linking

DATA ENDS

GROUP GROUP DATA

; The DATA segment will be
; linked into the group called
; DGROUP to match Microsoft
; FORTRAN convention.

CODE SEGMENT PUBLIC 'CODE'

ASSUME CS:CODE, DS:GROUP, SS:GROUP
PUBLIC DISABL

; Make the DISABL label available to other segments.

; Labtender Addresses, in IBM PC, for initialization routine.

= 0332
INTCLR EQU 818
= 033D
IOPORTB EQU 829

; 8259 Programmable Interrupt Controller Addresses in IBM PC.

= 0021
IMR EQU 021H

; Constants:
= 0000
TRAIL EQU 0H

; DISABL PROC FAR

= 0000 55
PUSH BP
= 0001 8B EC
MOV BP, SP

= 0003 BA 0021
MOV DX, IMR
= 0006 EC
IN AL, DX
= 0007 0C 10
OR AL, 000010000B
= 0009 EE
OUT DX, AL

= 000A BA 0332
MOV DX, INTCLR
= 000D B0 00
MOV AL, 0
= 000F FA
CLI
= 0010 EE
OUT DX, AL
= 0011 FB
STI

; To allow the stepper motor to be
The IBM Personal Computer Assembler 01-01-80

>DISABL - ROUTINE TO DISABLE INTERRUPTS
>
>0015 B0 00 MOV AL,TRAIL ; manually reset, we must leave the output to it high, so we don't complement (NOT) the low trail here.
>0017 EE OUT DX,AL
>
>0018 B2 44 MOV DL,68 ; Print a 'D' to the screen
>001A B4 02 MOV AH,2 ; for debugging
>001C CD 21 INT 21H
>
>001E 88 E5 MOV SP,BP ; Restore framepointer
>0020 5D POP BP
>0021 CB RET ; Return
>
>0022 DISABLEN DP
>
>0022 CODEENDS
>
>END
>
>Segments and groups:
>
>   Name   Size  align  combine  class
>
>CODE ............... 0022  PARA  PUBLIC  'CODE'
>DGROUP .............. GROUP
>DATA ................. 0000  PARA  PUBLIC  'DATA'
>
>Symbols:
>
>   Name   Type  Value  Attr
>
>DISABL .............. FPROC  0000  CODE  Global Length=0022
>IMR ................. Number  0021
>INTCLR .............. Number  0332
>IOPORTB ............. Number  033D
>TRAIL ............... Number  0000
>
>Warning Severe
>Errors Errors
>
>0
#

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> ENABLE - ROUTINE TO ENABLE INTERRUPTS

TITLE ENABLE - ROUTINE TO ENABLE INTERRUPTS

PAGE 1,132 ; Set page width to 132 characters.

; FORTRAN "CALL ENABLE" calls this procedure.

>0000
DATA SEGMENT PUBLIC 'DATA'
; For local assembler program data storage.
; Not used, but required for linking
>0000
DATA ENDS

>0000
DGROUP GROUP DATA ; The DATA segment will be
; linked into the group called
; DGROUP to match Microsoft
; FORTRAN convention.
>0000
CODE SEGMENT PUBLIC 'CODE'

ASSUME CS:CODE, DS:DGROUP, SS:DGROUP
PUBLIC ENABLE ; Make the ENABLE label avail-
; able to other segments.

; Labtender Address, in IBM PC.
>= 0332
INTCLR EQU 818 ; Timer Interrupt Clear address.

; 8259 Programmable Interrupt Controller Addresses in IBM PC.
>= 0021
IMR EQU 021H ; Interrupt Mask Register.

>0000
ENABLE PROC FAR

>0000 55
PUSH BP ; Save calling framepointer.
>0001 8B EC
MOV BP, SP

>0003 BA 0332
MOV DX, INTCLR ; Clear timer interrupt,
>0006 B0 00
MOV AL, 0 ; just in case it's not already.
>0008 EE
OUT DX, AL ; Write to clear the interrupt

>0009 B2 45
MOV DL, 69 ; Print an 'E' to the screen.
>000B B4 02
MOV AH, 2 ; for debugging
>000D CD 21
INT 21H

>000F BA 0021
MOV DX, IMR ; Get contents of IMR.
>0012 EC
IN AL, DX
>0013 24 EF
AND AL, 11101111B ; Mask to include bit 4.
>0015 FA
CLI ; Clear interrupt flag
>0016 EE
OUT DX, AL ; Reset the IMR.
>0017 FB
STI ; Reset the interrupt flag
The IBM Personal Computer Assembler 01-01-80

>ENABLE - ROUTINE TO ENABLE INTERRUPTS
>
>0018 8B E5 MOV SP,BP ; Restore framepointer
>001A 5D POP BP
>001B CB RET ; Return
>
>001C ENABLE ENDP
>001C CODE ENDS
>
>END
>
>Segments and groups:
>
> Name    Size  align  combine  class
>
>CODE         001C  PARA  PUBLIC  'CODE'
>DGROUP       GROUP
>DATA          0000  PARA  PUBLIC  'DATA'
>
>Symbols:
>
> Name    Type  Value  Attr
>
>ENABLE      FPROC  0000  CODE  Global Length=001C
>IMR          Number 0021
>INTCLR       Number  0332
>
>Warning Severe
>Errors Errors
>
> 0
TITLE ENC - ENCODER READING ROUTINE.
; This routine reads the contents of the counters
; of the second joints and sends their value in the
; variable 'enc2V' to the fortran code.

PAGE ,132 ;Set page width to 132 characters.

>0000 DATA SEGMENT PUBLIC 'DATA'
; For local assembler program data storage
; not used here, but required for linking purposes.
>0000 DATA ENDS

>0000 ENCBLS$A SEGMENT COMMON '$ENCBL'
; This segment is equivalent to COMMON/
ENCBLK/CJ1,CJ2,ENC2V
; in FORTRAN. The $ sign and A are required to
; match the segment name and class that the linker
; uses for the FORTRAN common statement. ENC2V,
; CJ1 and CJ2 must be declared INTEGER*2 in the
; FORTRAN calling routine before the common block.

>0000 ???? CJ1 DW ?
>0002 ???? CJ2 DW ?
>0004 ???? ENC2V DW ?
>0006 ENCBLS$A ENDS

DGROUP GROUP DATA,ENCBLS$A
; The data and ENCBLK$A segments will be
; linked in the group called DGROUP, to match
; MICROSOFT FORTRAN convention.

>0000 CODE SEGMENT PUBLIC 'CODE'

ASSUME CS:CODE,DS:DGROUP,SS:DGROUP
PUBLIC LP
PUBLIC ENC
; make ENC label available to other segments

SUBTLT. READING OF THE ENCODER

>= 0330 SLOTNO EQU 816
>= 033F ENCMT EQU SLOTNO+15
>= 033D OUTPORT EQU SLOTNO+13

>0000 ENC PROC FAR

>0000 55 PUSH BP ; save calling framepointer
>0001 8B EC MOV BP,SP ; on the stack.
>0003 50 PUSH AX ; save AX register.
>0004 53 PUSH BX ; save BX register.
The IBM Personal Computer Assembler 01-01-80

ENC-ENCODER READING ROUTINE

READING OF THE ENCODER

> 0005 52 PUSH DX ; save DX register.
> 0006 BA 033F LP: MOV DX, ENCM ; setting the i/o ports by writing
> 0009 B0 90 MOV AL, 090H ; a value to AL according to the
> 000B EE OUT DX, AL ; latching specifications and
> ; sending the latter to the control
> ; port through the DX register.
> ; This will determine which port
> ; is to be used as input port and
> ; which to be used as output port.
> ;
> 000C B0 08 MOV AL, 08H ; strobing the counters by sending
> 000E BA 033D MOV DX, OUTPORT ; a high signal to bit 4 through the
> 0011 EE OUT DX, AL ; output port. Physically, this will
> ; lock all the counters, mount their
> ; contents on the latches and wait.
> ;
> 0012 8B 1E 0002 R MOV BX, CJ2 ; get the address of the counter to
> 0016 8A C3 MOV AL, BL ; be read. Mask off high byte since
> 0018 EE OUT DX, AL ; only low byte is used. Send it
> 0019 4A DEC DX ; through the DX register. Move to
> 001A EC IN AL, DX ; the input port. Read the counter.
> ;
> 001B 24 0F AND AL, 0FH ; Since only 12 bits are assigned to
> ; joints 2. Therefore, the 4 most
> ; significant bits of the upper byte
> ; of joint 2 word must be masked.
> ;
> 001D 8A E0 MOV AH, AL ; Save the counter reading in AH.
> 001F 42 INC DX ; Move back to the output port.
> 0020 8B 1E 0000 R MOV BX, CJ1 ; Read the address of the low byte
> ; in joint 2 counter word.
> 0024 8A C3 MOV AL, BL ; Mask off high byte since only
> ; low byte is used.
> ;
> 0026 EE OUT DX, AL ; Send it.
> 0027 4A DEC DX ; Move to the input port.
> 0028 EC IN AL, DX ; Read the content of the counter in AL.
> 0029 2D 0412 SUB AX, 0412H ; Set up the origin of the theta motion.
> 002C A3 0004 R MOV ENC2V, AX ; Transfer the data to FORTRAN.
> ;
> 002F 58 POP AX ; restore registers
> 0030 5B POP BX ;
> 0031 5A POP DX ;
> ;
> 0032 8B E5 MOV SP, BP ; restore framepointer
> 0034 5D POP BP ;
> ;
> 0035 CB RET ; return
> 0036 ENCBNDP
> 0036 CODEENDS
> ;
> END
The IBM Personal Computer Assembler 01-01-80

>ENC-ENCODER READING ROUTINE

>Segments and groups:

>  Name      Size  align  combine  class
>  CODE      0036  PARA  PUBLIC  'CODE'
>  DGROUP    GROUP
>  DATA      0000  PARA  PUBLIC  'DATA'
>  ENCBLSA   0006  PARA  WORD   '$ENCBL'

>Symbols:

>  Name      Type  Value  Attr
>  C1         LWORD 0000  ENCBLSA
>  C2         LWORD 0002  ENCBLSA
>  ENC        FPROC 0000  CODE   Global Length
>  ENC2V      LWORD 0004  ENCBLSA
>  ENCM       Number 033F
>  LP         LNEAR 0006  CODE   Global
>  OUTPUT     Number 033D
>  SLOTNO     Number 0330

>Warning Severe
>Errors Errors

>  0

#
; ANALOG TO DIGITAL CONVERSION ROUTINE

>0000 DATA SEGMENT PUBLIC 'DATA'
 ; For local assembler program data storage
 ; Not used here, but required for linking
>0000 DATA ENDS

>0000 ADCBL$A SEGMENT COMMON '$ADCBL'
 ; This segment is equivalent to COMMON/ADCBL
 ; /ADCCHN, ADCVAL
 ; in FORTRAN. The $ and A are required to match the segment
 ; name and class that the linker uses for the FORTRAN common.
 ; ADCCHN and ADCVAL must be declared INTEGER*2 in the
 ; FORTRAN calling routine before the common block.

>0000 ???? ADCCHN DW ?
>0002 ???? ADCVAL DW ?

>0004 ADCBL$A ENDS

> DGROUP GROUP DATA,ADCBL$A
 ; The DATA and ADCBL$A segments will be
 ; linked into the group called DGROUP,
 ; to match Microsoft FORTRAN convention.

>0000 CODE SEGMENT PUBLIC 'CODE'

 ASSUME CS:CODE, DS: DGROUP, SS: DGROUP
 PUBLIC ADC ; Make ADC label available
 ; to other segments.

;-------------------------------
SUBTTL A/D CONVERSION
;-------------------------------
 ; ADCCHN is passed to ADC from the calling
 ; program, and ADCVAL
 ; is returned through the COMMON/ADCBL/ADCCHN,
 ; ADCVAL block.
 ; FORTRAN "CALL ADC" calls this A/D procedure

:= 0330 LABTDR EQU 816 ; Address of Labtender in
 ; IBM-PC memory
:= 0331 ADDATA EQU 817 ; A/D Converter data register
The IBM Personal Computer Assembler 01-01-80

A/D CONVERSION

> > > 0000   ADC PROC FAR
> > > > 0000 55  PUSH BP  ;  Save calling framepointer
> > > > 0001 8B EC  MOV  BP,SP  ;  on the stack.
> > > > 0003 BA 0330  MOV  DX,LABTDR  ;  Indirect addressing using
> > > >        ;  DX will have to
> > > >        ;  be used when using IN
> > > >        ;  and OUT commands.
> > > > 0006 A1 0000 R  MOV  AX,ADCHN  ;  Get channel number
> > > > 0009 B4 00  MOV  AH,0  ;  Mask off high byte, we
> > > >        ;  only use AL
> > > >        ;  Now create the proper command byte for the Labtender's A/D
> > > >        ;  routine, based on the channel number. Use the AL register.
> > > > 000B D0 E0  SHL  AL,1  ;  Shift ADCHN left 1 bit
> > > >        ;  (from bits 0-2 to 1-3)
> > > > 000D 24 3F  AND  AL,3FH  ;  Mask off bits 6 and 7
> > > > 000F 0C 01  OR  AL,1  ;  Put a 1 in bit 0 to start conversion
> > > > 0011 EE  OUT  DX,AL  ;  Send command to ADC command port
> > > > > 0012 24 FE  AND  AL,0FEH  ;  Mask off error bit (bit 0),
> > > >        ;  leave bits 1-6
> > > > 0014 0C 80  OR  AL,80H  ;  the same so we're still
> > > >        ;  looking at the same
> > > >        ;  channel, and put a 1 in bit 7
> > > >        ;  (status bit -a 1 indicates that con-
> > > >        ;  version is complete).
> > > > 0016 8A E0  MOV  AH,AL  ;  Save the reference for
> > > >        ;  'conversion complete' in AH.
> > > > > 0018  CYCLE:
> > > > 0018 EC  IN  AL,DX  ;  Read status port.
> > > > 0019 3A C4  CMP  AL,AH  ;  Conversion complete?
> > > > 001B 75 FB  JNE  CYCLE  ;  Wait until it is.
> > > > 001D BA 0331  MOV  DX,ADDATA  ;  Point to data port.
> > > > 0020 EC  IN  AL,DX  ;  Read data.
> > > > 0021 B4 00  MOV  AH,0  ;  Clear hi-byte of AX
> > > > 0023 2D 0080  SUB  AX,128  ;  Convert from unsigned
> > > >        ;  TO SIGNED
A/D CONVERSION

0026 A3 0002 R    MOV ADCVAL,AX ; 0 to 255 becomes -128 to 127

0029 8B E5       MOV SP,BP    ; Return A/D value to
002B 5D          POP BP     ; calling program.
002C CB          RET       ; Restore framepointer

002D              ADCENDP  ; Return

002D              CODEENDS

002D              END
The IBM Personal Computer Assembler 01-01-80

Segments and groups:

<table>
<thead>
<tr>
<th>Name</th>
<th>Size</th>
<th>align</th>
<th>combine</th>
<th>class</th>
</tr>
</thead>
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<td>CODE</td>
<td>002D</td>
<td>PARA</td>
<td>PUBLIC</td>
<td>CODE</td>
</tr>
<tr>
<td>DATA</td>
<td>0000</td>
<td>PARA</td>
<td>PUBLIC</td>
<td>'DATA'</td>
</tr>
<tr>
<td>ADCBLSA</td>
<td>0004</td>
<td>PARA</td>
<td>WORD</td>
<td>'$ADCBLSA'</td>
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</tbody>
</table>

Symbols:

<table>
<thead>
<tr>
<th>Name</th>
<th>Type</th>
<th>Value</th>
<th>Attr</th>
</tr>
</thead>
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<td>ADC</td>
<td>FPROC</td>
<td>0000</td>
<td>CODE Global Length=002D</td>
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<tr>
<td>ADCCHN</td>
<td>LWORDB</td>
<td>0000</td>
<td>ADCBLSA</td>
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<tr>
<td>ADCVAL</td>
<td>LWORDB</td>
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<td>ADCBLSA</td>
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<tr>
<td>ADDATA</td>
<td>Number</td>
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<tr>
<td>CYCLE</td>
<td>LINEAR</td>
<td>0018</td>
<td>CODE</td>
</tr>
<tr>
<td>LABTDR</td>
<td>Number</td>
<td>0330</td>
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</tr>
</tbody>
</table>

> Warning Severe
> Errors Errors

> 0

#
The IBM Personal Computer Assembler 01-01-80 PAGE 1-1

> DIGITAL TO ANALOG CONVERSION ROUTINE

> TITLE DIGITAL TO ANALOG CONVERSION ROUTINE

> PAGE 132 ; Set page width to 132 characters.

> ; This routine sends the digital to analog conversion value
> ; (DACVAL) for a specified channel (DACHN).
> ; FORTRAN *CALL DAC* calls this D/A procedure.

>0000 DATA SEGMENT PUBLIC 'DATA'

> ; For local assembler program data storage
> ; Not used here, but required for linking
>0000 DATA ENDS

>0000 DACBL$A SEGMENT COMMON '$DACBL$

> ; This segment is equivalent to COMMON/DACBL/DACHN,
> ; DACVAL in FORTRAN. The $ and A are required to match
> ; the segment name and class that the linker uses for the
> ; FORTRAN common. DACHN and DACVAL must be declared
> ; INTEGER*2 in the FORTRAN calling routine before the
> ; common block.

>0000 DACHN DW ?
>0002 DACVAL DW ?

>0004 DACBL$A ENDS

> DGROUP GROUP DATA,DACBL$A

> ; The DATA and DACBL$A segments will be
> ; linked into the group called DGROUP,
> ; to match Microsoft FORTRAN convention.

>0000 CODE SEGMENT PUBLIC 'CODE'

> ASSUME CS:CODE,DS:DGROUP,SS:DGROUP

> PUBLIC DAC ; Make DAC label available to other segments.

>0334 DACNTL EQU 820 ; D/A control port location in memory
>0335 DADATA EQU 821 ; D/A converter data register

>0000 DAC PROC FAR

>0000 55 PUSH BP ; Save calling framepointer
>0001 8B EC MOV BP,SP ; on the stack.

>0003 BA 0335 MOV DX,DADATA ; Indirect addressing using DX will have to

> ; be used when using IN and OUT commands.

>0006 8B 1E 0002 R MOV BX,DACVAL ; Get D/A value
>000A B7 00 MOV BH,0 ; Mask off hi byte
The IBM Personal Computer Assembler 01-01-80

DIGITAL TO ANALOG CONVERSION ROUTINE

> 000C 81 C3 0080 ADD BX,128 ;Convert from signed to unsigned
> ;128 to 127 becomes 0 to 255
> >
> 0010 A1 0000 R MOV AX,DACHN ;Get channel number
> 0013 B4 00 MOV AH,0 ;Mask off high byte, we only use AL
> ; Now create the proper command byte for the Labtender's D/A
> ; routine based on the channel number. Use the AL register
> 0015 24 0F AND AL,0FH ;Mask of bits 4-7
> 0017 0C 08 OR AL,8 ;Set bit 3 hi
> 0019 8A E0 MOV AH,AL ;Save D/A command byte in AH
> >
> 001B 8A C3 MOV AL,BL ;Get DACVAL (only lower byte)
> 001D EE OUT DX,AL ;Load value into D/A data register
> 001E BA 0334 MOV DX,DACNTL ;Point to D/A control word
> 0021 8A C4 MOV AL,AH ;Get D/A command byte
> 0023 EE OUT DX,AL ;Start conversion
> >
> 0024 8B E5 MOV SP,BP ;Restore framepointer
> 0026 5D POP BP
> 0027 CB RET BP ;Return
> >
> 0028 DACENDP
> >
> 0028 CODEENDS
> >
> END
The IBM Personal Computer Assembler 01-01-80

DIGITAL TO ANALOG CONVERSION ROUTINE

Segments and groups:

<table>
<thead>
<tr>
<th>Name</th>
<th>Size</th>
<th>Align</th>
<th>Combine</th>
<th>Class</th>
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<tbody>
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<td>CODE</td>
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<td>PARA</td>
<td>PUBLIC</td>
<td>'CODE'</td>
</tr>
<tr>
<td>DGROUP</td>
<td></td>
<td>GROUP</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DATA</td>
<td>0000</td>
<td>PARA</td>
<td>PUBLIC</td>
<td>'DATA'</td>
</tr>
<tr>
<td>DACBL$A</td>
<td>0004</td>
<td>PARA</td>
<td>WORD $</td>
<td>$DACBL$</td>
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Symbols:

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<th>Attr</th>
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<td>DAC</td>
<td>FPROC</td>
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<td>CODE Global Length= 0028</td>
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<tr>
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<tr>
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</tr>
<tr>
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</tr>
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</table>

Warning Severe
Errors Errors

0
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