SAMPLE SIZE PLANNING FOR PROFILE ERROR EVALUATION OF SURFACE WITH ARRAYS OF CONTIGUOUS PATCHES USING COORDINATE MEASURING MACHINE

Sung Ho Chang
Gary D. Herrin

Department of Industrial & Operations Engineering
University of Michigan
Ann Arbor, MI 48109-2117

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ABSTRACT

This paper develops samplsize planning for profile error evaluation of surface with arrays of contiguous patches using coordinate measuring machine. Theorical sample size to evaluate profile error for sculptured surface is too large to implement in practical purpose. Therefore, we determine the sample size based on heuristic simulation study by observing the relative error rate.

1. Introduction

Profile error evaluation of surface with arrays of contiguous patches is different when compared to the problems that we have discussed so far. We should estimate concerned features from measurements in the previous problems, however, we already know the representation of a concerned surface in this problem due to the existing CAD system. With profile of a surface tolerancing, basic untoleranced dimensions are used to define the desired shape and/or relation of the toleranced feature. Where the toleranced feature is curved or irregular shaped, sufficient basic dimensions are used to describe the exact contour. The profile of a surface tolerance establishes a tolerance zone of uniform width, within which all points of the considered feature must lie. The boundary of the tolerance zone follow the geometric shape of the true profile, except at sharp corners. Where the basic contour includes a sharp corner, the interpretation is that the tolerance zone will extend to the intersections of the tolerance boundary lines. If these intersections of the tolerance zone boundaries permit an unacceptable variation in corner contour, a maximum or minimum radius may be specified for the corner of the part.

Our concern is evaluation of profile error of the considered surface. When the surface shape is complicated it is represented by the combination of many patches which are represented by the parametric representation in current CAD system. Then, we cannot get explicit single representation of whole surface while we have explicit parametric representation of each patch. Therefore, at least one measurement should be collected from patch to have information about the variation of each patch profile. By comparing the variation of each patch profile to the given profile tolerance zone width, we can make a decision whether the considered surface satisfies the profile tolerance or not. When the considered surface does not satisfy the given tolerance, we can find which patch generates unexpected variation.

Then, the problem is that how many measurements are needed to obtain information about patch profile variation with a certain confidence level. Also the determination of profile error using these information is another problem. Modeling of surfaces using minimum number of points can be found in many literatures [Coons (1974), Faux and Pratt (1979), Forrest (1968, 1972), Peters (1974), Chasen (1978), Barsky (1982), Gould (1972), and Mortenson (1985)]. However, those literatures were only concerning about the construction of surfaces and were not concerning about the variation of given surfaces' profile. Even though sample size planing for estimation of each patch surface is discussed in previous paper, these sample sizes are tremendously large when the considered surface is constructed by many patches. Then, these sample size cannot be justified economically in using CMM which is a discrete measurement device.

Consequently, we should develop new sample size planning for a surface with arrays of contiguous patches for a CMM. To accomplish this task we need a requirement in a CMM measurement procedure. This requirement is that the CMM coordinate system must be the same as the surface coordinate system which is represented in CAD system.

2. Sample Size Planning and Evaluation of Profile Error

2.1 Sample Size Planning

We define that the profile error is 6σ in previous paper [1990] when we assume that the deviations from the true profile follow a normal distribution. When we obtain the deviations from the true profile of the considered surface by a CMM, the profile error for each patch can be estimated by sample standard deviation. However, sample standard deviation (s) is not an unbiased estimator of σ while sample variance (s^2) is an unbiased estimator as follow:

$$s^2 = \frac{\sum_{i=1}^{n} e_i^2}{n-1}$$

where e_i: deviations from true profile

n: # of measurements in each patch

$$s = \sqrt{\frac{\sum_{i=1}^{n} e_i^2}{n-1}}$$

$$E(s^2) = \sigma^2$$

$$E(s) = \left(\frac{2}{n-1}\right)^{1/2} \frac{\Gamma[n/2]}{\Gamma[(n-1)/2]} \sigma$$

where $\Gamma(n)$ = Gamma function

$$= \int_0^\infty x^{n-1} e^{-x} \, dx.$$

Therefore, we cannot simply estimate patch profile error by multiplying 6 to the sample standard deviation. Instead we multiply by 6 and divided by the constant, $(\frac{2}{n-1})^{1/2} \frac{\Gamma[n/2]}{\Gamma[(n-1)/2]}$, to estimate patch profile error. Then, we can obtain the estimated profile error without considering confidence level. However, when we consider the confidence level, the standard deviation of the sample standard deviation s. That is $\sqrt{1 - c^2}$, where $c = (\frac{2}{n-1})^{1/2} \frac{\Gamma[n/2]}{\Gamma[(n-1)/2]}$. Theoretically, the desired sample size with high confidence level is too large. As an example almost 200 measurements are needed for 99% confidence. Consequently, the theoretical sample size cannot be used as a useful practice for CMM users. As an alternative approach we propose heuristic method by simulation.

For simulation study we use normal random numbers as deviations from true profile. We generated 5000 different sets of normal random numbers with mean zero and standard deviation 0.001 for each sample size from 2 to 25. Then we assumed that the true profile error was 0.006. We estimate profile errors for each different sample sizes by

$$6*s/\{(\frac{2}{n-1})^{1/2} \frac{\Gamma[n/2]}{\Gamma[(n-1)/2]}\} \quad (1)$$

where s: sample standard deviation

n: sample size

Average of estimated profile errors was calculated for each sample size. Also relative error, $\frac{\text{estimated profile error - true profile error}}{\text{true profile error}}$, was calculated. These average values of the estimated profile errors and relative errors are shown in Table 1.

The estimated profile errors were decreased and close to real profile error when the sample size was increased (Fig. 1). From these results we can choose the desired sample size depending on the relative error rate. As an example when the desired error rate is not more than 5%, minimum sample size will be 10 for each patch. Therefore, total sample size for a surface with arrays of contiguous patches will be multiplication of minimum sample size for a patch by the number of patches on a surface. Actually this proposed sampling plan is based on random sample. However, it will be easy for CMM measuring procedure that the sampled points are equally distributed on a patch. Also we recommend 16 points for each patch because the estimation of 16 coefficients is required for the construction of a bi-cubic surface. A bi-cubic surface is a most widely used surface in practice.

When a surface with arrays of contiguous patches is constructed, the boundaries of each patch should satisfy the condition of continuity. Therefore, when we measure 16 points in a patch, 12 points on the patch boundaries can be shared with adjacent patches (Fig. 2). Consequently actual total measuring points are reduced.

Sample Size	Estimated Profile Error	Relative Error (%)
2	0.0096	60.0
3	0.0077	28.3
4	0.0071	18.3
5	0.0068	13.3
6	0.0066	10.0
7	0.0065	8.3
8	0.0064	6.7
9	0.0064	6.7
10	0.0063	5.0
11	0.0063	5.0
12	0.0063	5.0
13	0.0062	3.3
14	0.0062	3.3
15	0.0062	3.3
16	0.0062	3.3
17	0.0062	3.3
18	0.0062	3.3
19	0.0062	3.3
20	0.0061	1.7
21	0.0061	1.7
22	0.0061	1.7
23	0.0061	1.7
24	0.0061	1.7
25	0.0061	1.7

Table 1 Estimated Profile Error and Relative Error

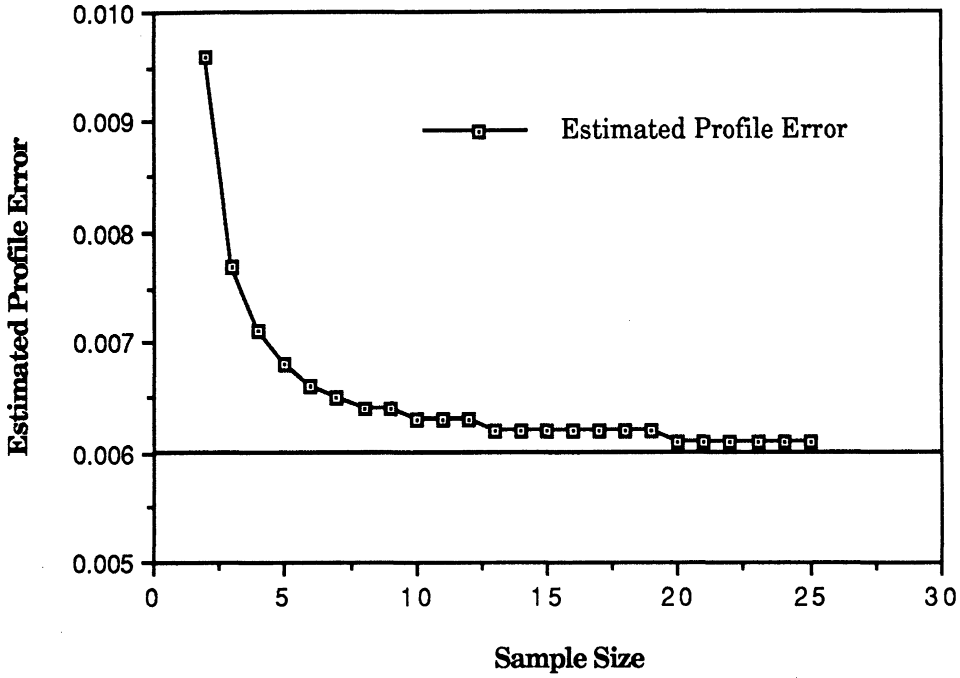

Figure 1 Estimate Profile Errors depending on Sample Size

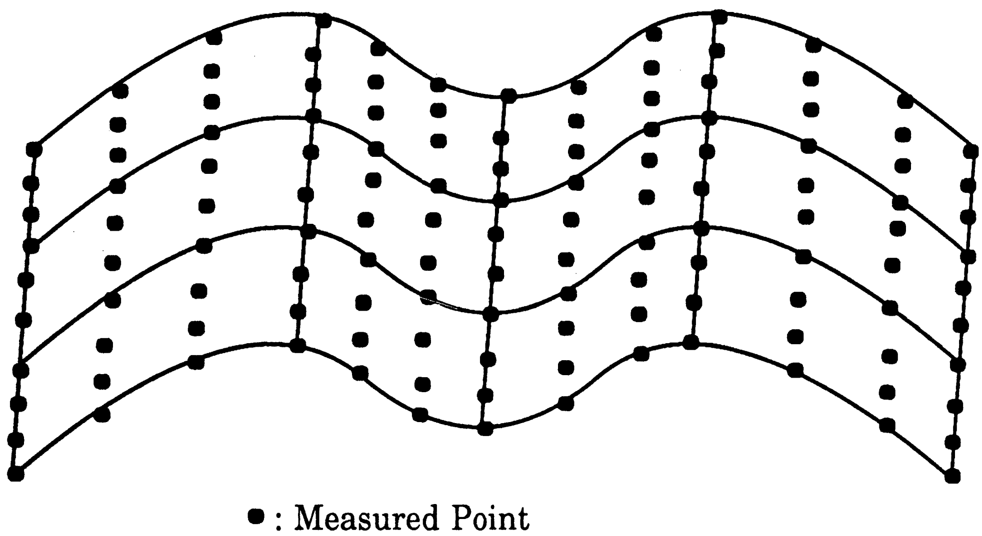

Figure 2 Recommended Measuring Points

2.2 Evaluation of Profile Error

The profile error of a surface can be determined based on the estimated profile errors of each patch. The profile tolerance specifies a uniform boundary along the true profile within which the elements of the surface must lie (ANSI Y14.5). Therefore, estimated profile errors of each patch should satisfy the surface profile tolerance because each patch is one element of the surface. We define the surface profile error as follow based on estimated profile errors of patches

The surface profile error is the largest value among the consisted patch profile errors.

When all the estimated profile errors of each patch are less than a surface profile tolerance, we can say that the surface profile error satisfies the tolerance. When any of the estimated profile errors of each patch is greater than a surface profile error, we can say that the surface profile error does not satisfy the tolerance. When the surface profile error does not satisfy the tolerance, we can find the cause area of out of specification by observing the estimated profile errors of each patch.

Evaluation of profile error of a surface was tested by simulation. Let a surface be consisted of four patches shown in Fig. 3 and its profile tolerance be given as 0.09 (Fig. 4). We used four different random normal number sets of size 16 for each patch. Sample size 16 was used because the desired relative error rate was assumed less than 5%. Random normal number sets with mean zero and standard deviations 0.010, 0.013, 0.012, and 0.011 for patch 1, patch 2, patch 3, and patch 4 respectively were generated. We assume these generated numbers were the deviations from each patch. From these numbers (Table 2) we estimated profile errors of each patch and compared with the surface tolerance (Table 3). From Table 3 results we can say that the surface profile error was 0.085 and it was in-tolerance. Even though surface profile error should be 0.078 (0.013*6) theoretically, when we consider the relative error rate as 5% (0.078*1.05 = 0.082), the obtained value is reasonable.

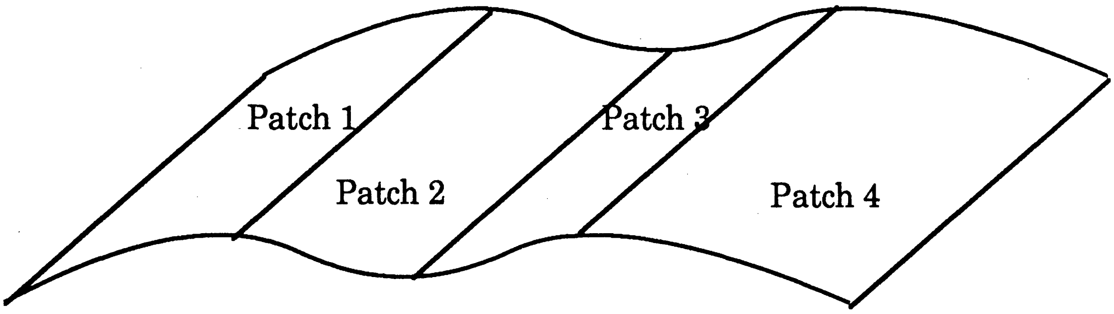

Figure 3 Simulated Surface consisted of Four Patches

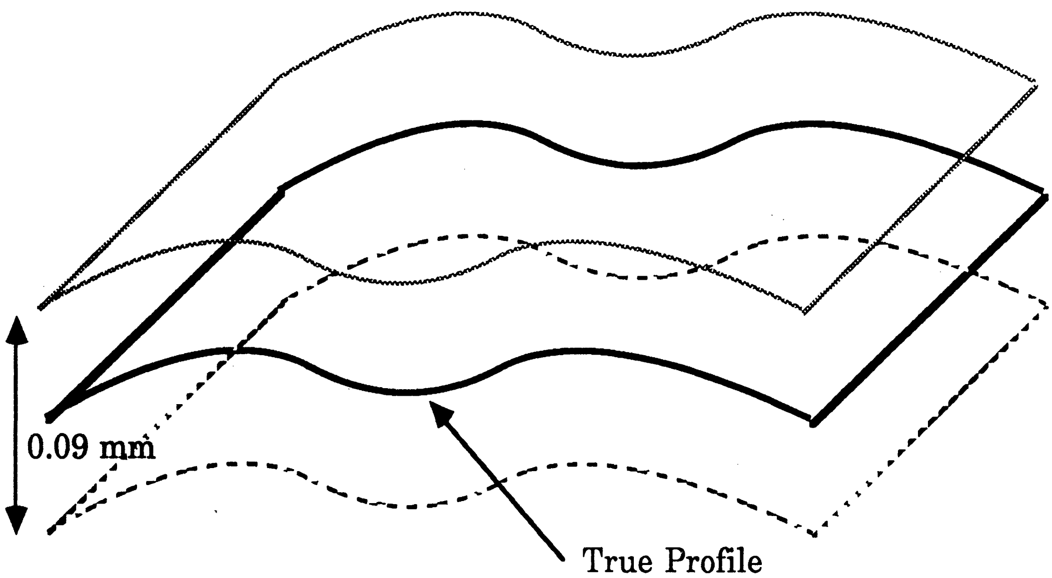

Figure 4 Profile Tolerance Boundaries of the Simulated surface

Measurement Number	Patch 1	Patch 2	Patch 3	Patch 4
1	0.004	-0.015	0.017	0.006
2	0.006	-0.013	0.023	0.002
3	-0.008	-0.007	0.003	-0.012
4	-0.010	-0.007	-0.009	0.006
5	-0.005	0.019	-0.007	-0.004
6	0.011	0.003	-0.013	0.003
7	-0.002	-0.001	-0.015	0.024
8	0.017	-0.031	0.025	0.012
9	0.032	0.007	0.011	-0.005
10	-0.006	0.022	0.021	0.004
11	0.009	-0.020	0.012	-0.009
12	0.006	-0.001	-0.005	0.021
13	0.006	-0.009	-0.001	0.003
14	-0.002	-0.011	-0.014	-0.009
15	0.005	-0.021	0.004	0.008
16	0.015	0.008	0.001	0.019
Standard Deviation	0.011	0.014	0.014	0.011

Table 2 Measured Values for each Patch and Standard Deviation

Surface Tolerance	Patch 1 Profile Error	Patch 1 Profile Error	Patch 1 Profile Error	Patch 1 Profile Error
0.09	0.067	0.085	0.085	0.067

Patch Profile Error = 6*Standard Deviation/ $\{(\frac{2}{15})^{1/2} \frac{\Gamma[8]}{\Gamma[15/2]}\}$

Table 3 Patch Profile Error Comparison to Surface Profile Tolerance for In-Tolerance Case

As a second simulation test, we changed the generated random normal numbers for patch 2 with mean zero and standard deviation 0.018 (Table 4). Estimated profile error of patch 2 was 0.122 and the surface profile was decided by 0.122. Because of this patch, the surface was decided as out-of-tolerance (Table 5).

Generated Values for Patch 2			
-0.021	0.026	0.010	-0.012
-0.018	0.004	0.033	-0.015
-0.009	-0.002	-0.031	-0.029
-0.010	-0.042	-0.001	0.011
Standard Deviation	0.020		

Table 4 Generated Values for Patch 2 in Out-of-Tolerance Case

Surface Tolerance	Patch 1 Profile Error	Patch 1 Profile Error	Patch 1 Profile Error	Patch 1 Profile Error
0.09	0.067	0.122	0.085	0.067

Patch Profile Error = 6*Standard Deviation/$\{(\frac{2}{15})^{1/2}\frac{\Gamma[8]}{\Gamma[15/2]}\}$

Table 5 Patch Profile Error Comparison to Surface Profile Tolerance for Out-of-Tolerance Case

For a reduced sample size case, we share the measurements with adjacent patches (Fig. 5). Random normal number sets with mean zero and standard deviations 0.10, 0.13, 0.12, and 0.11 for patch 1, patch 2, patch 3, and patch 4 respectively were generated. Shared measurements were generated with mean zero and average standard deviation value of adjacent standard deviations. In other words, 0.0115 for patch 1 and patch 2, 0.0125 for patch 2 and patch 3, and 0.0115 for patch 3 and patch 4 were used as standard deviation to generate random normal numbers. These values and estimated standard deviation values are shown in Table 6. Each patch profile error was estimated and was compared to surface profile tolerance (Table 7). From these results we can say that the surface profile error was 0.085 and it was in-tolerance.

Measurement Number	Patch 1	Patch 2	Patch 3	Patch 4
1	0.004	-0.015	0.017	0.006
2	0.002	-0.01	-0.006	0.003
3	-0.007	0.007	-0.006	0.016
4	0.002	-0.014	0.004	-0.002
5	0.017	-0.028	0.023	0.012
6	-0.005	-0.008	0.022	0.02
7	0.01	-0.011	0.008	0
8	-0.007	-0.001	0.003	-0.003
9	0.005	0.007	-0.005	0.008
10	0.017	-0.012	-0.023	0.014
11	0.002	-0.007	0.003	-0.002
12	0.005	-0.001	-0.003	-0.005
13	0.007	0.024	0.024	-0.005
14	-0.012	-0.015	-0.015	-0.023
15	-0.007	0.008	0.008	0.003
16	-0.001	0.024	0.024	-0.003
Standard Deviation	0.008	0.014	0.014	0.010

Table 6 Measured Values for each Patch shared with Adjacent Patches and Standard Deviation

Surface Tolerance	Patch 1 Profile Error	Patch 1 Profile Error	Patch 1 Profile Error	Patch 1 Profile Error
0.09	0.049	0.085	0.085	0.061

$$\text{Patch Profile Error} = 6*\text{Standard Deviation}/\{(\frac{2}{15})^{1/2}\frac{\Gamma[8]}{\Gamma[15/2]}\}$$

Table 7 Patch Profile Error Comparison to Surface Profile Tolerance for sharing measurements with Adjacent Patches

3. Conclusion

In this paper we proposed sampling planning and evaluation procedure of a surface profile error. A concerned surface was consisted of arrays of contiguous patches. Because of complex characteristic of this surface, theoretical sampling plan is not practical. Therefore, we proposed new sampling plan based on heuristic simulation results. Even though this sampling plan did not give the exact value of a surface profile error, it gave the reasonable estimation within the considered relative error rate. The results of this paper can be utilized for practical sampling plan in CMM users practice. Further investigation will be needed for more theoretical basis.

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