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Damian R. Beil
Stephen M. Ross School of Business
at the University of Michigan

Amy Cohn
University of Michigan
College of Engineering

Amitabh Sinha
Stephen M. Ross School of Business
at the University of Michigan

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Damian R. Beil, Amy Cohn, Amitabh Sinha
University of Michigan, Ann Arbor, MI 48109
dbeil@umich.edu, amycohn@umich.edu, amitabh@umich.edu

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Abstract

Combinatorial auctions are very useful in theory, but their applicability in practice has been limited by the need for bidders to bid on an exponential number of bundles and for the auctioneer to solve an exponentially large winner-determination problem. We present a new auction mechanism to eliminate these challenges for a broad class of VCG combinatorial auctions. This mechanism, which yields equivalent results to a fully-enumerated combinatorial auction, eliminates the need for the bidder to explicitly compute and communicate bids on each bundle by exploiting the fact that true-cost bidding is a dominant strategy for these auctions. It also eliminates the need for the auctioneer to solve an exponentially large combinatorial optimization problem to select the optimal bids. Instead, the bidders’ true-cost types are explicitly incorporated within a mathematical program that is used to solve the winner- and payment- determination problems. A detailed example based on truckload procurement auctions is provided and several other applicable classes of problems are discussed as well.

Keywords: Combinatorial Auction; Procurement; Mathematical Programming; Truthful Bidding; Mechanism Design


1 Introduction

With the ever-increasing prevalence of electronic markets and auctions for commercial transactions, combinatorial auctions worth billions of dollars are conducted every year. Perhaps the most famous is the series of Federal Communications Commission auctions, surveyed early by Cramton (1997), with other surveys of those and related auctions including Cramton (2002), Binmore and Klemperer (2002) and Klemperer (2002). Other applications of combinatorial auctions include transportation services (at Sears Logistics, as documented by Ledyard et al. (2002)), airline landing slots at airports (Rassenti et al. 1982), and operations procurement at GE (worth $6 billion in 2000 (General Electric Corporation 2001)).

Nevertheless, two main hurdles remain in the efficient implementation of combinatorial auctions. First, there are an exponential number of bundles for which bidders must construct valuations and submit bids. Second, the auctioneer must solve a set partitioning problem over the corresponding exponential number of variables, which is highly intractable. The implications of this are pointed out, for instance, by de Vries and Vohra (2003) and Pekec and Rothkopf (2003). Although there has been much recent research into the amelioration of these challenges, which we survey in §1.1, they continue to present a significant obstacle to the practical use of combinatorial auctions.

In this paper, we present a mechanism that overcomes both of these hurdles whenever the underlying cost structure of the bidders can be compactly represented and efficiently computed (defined formally in §2). We focus on procurement auctions guided by our motivating examples, but our results apply to forward auctions as well. We formally prove that our proposed mechanism has all the desirable properties of combinatorial auctions, including individual rationality, incentive compatibility, and efficient allocation. We show, by means of examples, several situations where such a compact representation and efficient computability of bidder cost functions is possible, thus demonstrating a wide range of applicability of our mechanism.

We begin with a review of the literature, which provides appropriate context for our work. Our model and mechanism are described in §2, followed by applications including transportation services (§3), wireless spectra, energy markets, and general procurement (all in §4).
1.1 Literature Review

Auctions have been used for millennia to leverage competition and find market clearing prices; the reader interested in general auction theory is referred to a recent text by Krishna (2002). In this paper, our focus is on *combinatorial procurement auctions*. Combinatorial auctions were surveyed by de Vries and Vohra (2003) as well as Pekec and Rothkopf (2003), while a recent book by Cramton et al. (2006) provides a comprehensive examination of the theory and applications of combinatorial auctions in various domains.

Applications of combinatorial auctions in practice include long-term contracts for truck-load shipments (Caplice and Sheffi 2006, Sheffi 2004, Song and Regan 2003), airline landing slot allocation (Ball et al. 2006, Rassenti et al. 1982), wireless spectra (McMillan 1994, Cramton 1997, 2002, Binmore and Klemperer 2002), and even school lunch programs (Epstein et al. 2002). These applications have spawned an entire industry specializing in combinatorial auctions, with some of the bigger names being CombineNet, Logistics.com, and Manhattan Associates. This area is also related to “smart markets”, as studied by McCabe et al. (1991), and the more general field of electronic markets (Anandalingam et al. 2005, Wu and Kleindorfer 2005).

Procurement auctions, while in theory identical to the traditional auctions where items are sold, nonetheless hold specific contextual challenges and have received a lot of attention. In addition to the transportation procurement papers mentioned above, for example, Chen et al. (2005) study multi-unit auctions for supply chains, Che (1993) studies multi-dimensional auctions, and Hohner et al. (2003) study the combinatorial auctions for strategic goods conducted by Mars, Inc.

Despite the widespread use of combinatorial auctions, the problems we alluded to earlier due to the combinatorial explosion in the number of bundles remain. A number of approaches have been suggested in the literature to tackle this issue. One stream of research restricts the bidding language, so that bidders may only bid on a smaller set of specific bundles (Nisan 2000). An example of this is the XOR-of-OR bidding language, which was used by the FCC in their 2000 spectrum auction (Günlük et al. 2005). Another approach is preference elicitation, where the auctioneer proactively asks bidders to submit bids for specific bundles, with the aim of quickly uncovering a “good” set of bundles (Sandholm and Boutilier 2006). Many
auction mechanisms (including the ones cited above) for combinatorial auctions are iterative, where bidders submit bids in multiple rounds. Since bidders submit only a small number of bids in each round, convergence of the iterations is sufficient to overcome the combinatorial explosion. Parkes (2006) is a recent survey of iterative combinatorial auctions, with some other work including that of Kwon et al. (2005) and Ausubel et al. (2006); in fact, the FCC auctions are typically such iterative auctions (Cramton 1997).

Even if iterative auctions are used to reduce the number of bundles considered in each round, the auctioneer still must solve a set partitioning problem in each round in order to determine the optimal allocation. Set partitioning is a notoriously hard \( \mathcal{NP} \)-complete problem (Balas and Padberg 1976). For special cases (some of which are outlined in de Vries and Vohra (2003)), this problem can be solved with ease; however, most practical applications (such as the ones cited above) do not fall into these special cases. Various heuristics and algorithms have been suggested to solve these problems (Rothkopf et al. 1998, Bichler and Kalagnanam 2005, Günlük et al. 2005, Sandholm et al. 2005). However, all these approaches are essentially solving the set partitioning problem in different ways, and are not generalizable to more complex valuation functions of the bidders for different bundles.

1.2 Our Approach: Implicit Valuations

Our approach addresses precisely these two limitations of combinatorial auctions: the impact of the combinatorial explosion on the number of bids and the winner determination problem. Although the complexity of bidding on bundles arises from the fact that the valuation of a bundle is not simply the sum of valuations of the individual items, often there is an underlying structure which dictates the valuation of the bundle. For example, in transportation auctions, economies of scope are achieved when routes can be combined effectively to construct round-trips (Caplice and Sheffi 2006). In spectrum auctions, firms achieve synergy by winning licenses in adjacent locations (Günlük et al. 2005). In auctions for routes to deliver milk, efficiencies are obtained from contiguity of regions (Marshall et al. 2006). In energy markets, efficiencies are achieved based on scale (Hobbs et al. 2000). Our main point is this: Very often, the firms involved use sophisticated mathematical models to achieve these synergies and optimize their profits. We propose, therefore, to incorporate the mathematical model
used by the firms to achieve these synergies within the auction mechanism. In this way, in a single round, the synergies of all participants is simultaneously optimized and the optimal allocation is determined.

The crux of our approach is the following: If each bidder’s valuation of bundles can be compactly represented and efficiently computed by means of a mathematical program, and if bidders can be induced to reveal their parameters for these programs, then the auctioneer can embed these mathematical programs within a single winner determination problem which will identify the optimal allocation. In §2.1, we discuss how Vickrey-Clarke-Groves (VCG) auctions can present this incentive for bidders to truthfully reveal their valuation structures. We then use numerous examples to show practical uses of our approach. Our model and formulation is described in the next section.

2 Preliminaries

We consider multi-item, single round auctions. Since most of our motivating examples comprise procurement (or reverse) auctions, our model is defined in terms of procurement auctions. However, all our results apply to forward auctions where objects are being sold; in particular, the wireless spectrum auction we discuss in §4.1 is such an auction.

The auction is conducted by a single buyer, and a set $\mathcal{N} = \{1, 2, \ldots, N\}$ of prospective sellers (bidders). The set $\mathcal{M} = \{1, 2, \ldots, M\}$ is the set of objects/services being auctioned. A bundle or combination is a subset of $\mathcal{M}$, and different bidders are interested in selling different such bundles at different prices. There are $2^M - 1$ such bundles. The auctioneer’s objective is to procure one item of each of the objects in $\mathcal{M}$ at the lowest possible total price.

Let bidder $i$’s valuation for bundle $S$ be given by $c_i(S)$. In theory, bidders must compute the function $c_i(S)$ for each of the $2^M - 1$ bundles $S$. This combinatorial explosion is what dooms most traditional combinatorial auctions, and is precisely what we are trying to avoid. We do so by focusing on the true cost type $\theta_i$ of bidder $i$, which captures all the determinants of bidder $i$’s costs. In our primary example of trucking services procurement, $\theta_i$ may capture, among other things, the fuel costs, driver costs, existing contracts, potential future contracts, location of hubs and relay stations, and lease and maintenance costs of the fleet. If the cost
to any bidder $i$ for providing any bundle $S$ can be represented as a common function $c(S, \theta_i)$, where $c$ is compact and computable, then this function can be leveraged directly in solving the winner determination problem, bypassing the combinatorial enumerating of bids. As our examples will demonstrate, there is a wide variety of situations where the combinatorial cost function $c_i$ can in fact be well represented (or well approximated) by a common cost function $c$ and a true cost type $\theta_i$.

2.1 VCG Auctions

In this paper, we focus on VCG mechanisms (Vickrey 1961, Clarke 1971, Groves 1973). For the single-item case, this mechanism is the familiar second-price auction, wherein the buyer procures the object from the lowest bidder but pays the second-lowest bid. Such auctions are elegant and popular because it is a dominant strategy for bidders to truthfully reveal their true cost—overbidding does not increase the winner’s payment but decreases the likelihood of being awarded the item, whereas underbidding can lead to being awarded the item at a value less than its cost to provide. This truthful-bidding property eliminates the need for modeling strategic, competitive behaviors and greatly simplifies analysis. The single-item VCG mechanism extends in a fairly straightforward manner to a combinatorial auction. In the next section, we describe the canonical VCG combinatorial auction, followed by our proposed mechanism.

2.2 Canonical Mechanism: Basic Combinatorial Auction

Recall that bidder $i$ has a specific cost function $c_i$, and the cost to bidder $i$ of providing bundle $S$ is given by $c_i(S)$. In its most elementary form, the auction proceeds as follows:

BCA (Basic Combinatorial Auction)

1. Each bidder provides valuations $c_i(S)$ for every bundle $S \in 2^M$. We require the cost function to be non-negative (a natural assumption). Initially, we also require the cost function to be sub-additive; that is, for every $S \subset M$ and $j \in M \setminus S$, and for every bidder $i$, we have $c_i(S \cup \{j\}) \leq c_i(S) + c_i(\{j\})$. Sub-additivity is a natural assumption.
stating that the union of two bundles costs no more than the sum of the two bundles taken separately. We remark on the implications of sub-additivity later in this section.

2. The auctioneer solves a winner-determination problem, where she partitions the objects in $\mathcal{M}$ and awards bundles to bidders in such a way that the total cost is minimized. That is, the auctioneer solves the following mathematical program, denoted $E-WDP$, where the $E$ stands for “enumeration” and the binary variable $x_i(S)$ takes the value 1 to indicate that bidder $i$ is awarded bundle $S$, and 0 otherwise.

$$z^*(\mathcal{N}) = \min \sum_{i \in \mathcal{N}} \sum_{S \subseteq \mathcal{M}} c_i(S)x_i(S) \quad (E-WDP)$$

subject to:

$$\sum_{i \in \mathcal{N}} \sum_{S \subseteq \mathcal{M}: j \in S} x_i(S) = 1 \quad \forall j \in \mathcal{M}$$

$$x_i(S) \in \{0, 1\} \quad \forall i \in \mathcal{N}, \ S \subseteq \mathcal{M}.$$  

Sub-additivity ensures the existence of an optimal solution awarding no more than one bundle per bidder. For notational concision, we denote the cost incurred due to bidder $i$ as $z_i^*$. That is, $z_i^* = \sum_{S \subseteq \mathcal{M}} c_i(S)x_i^*(S)$.

3. In order to determine the payment to bidder $i$, the auctioneer computes the surplus added to the system by bidder $i$. That is, the auctioneer begins by computing $z^*(\mathcal{N} \setminus i)$, which is the total cost of providing all the bundles had bidder $i$ been absent. The difference $z^*(\mathcal{N} \setminus i) - z^*(\mathcal{N})$ is the premium awarded to bidder $i$, so that his total payment is $z_i^* + z^*(\mathcal{N} \setminus i) - z^*(\mathcal{N})$.

For the rest of this paper, we use the abbreviation BCA to refer to the auction mechanism described above. This traditional VCG combinatorial auction suffers due to the combinatorial explosion in the size of the set $\mathcal{M}$ being auctioned. Resultantly, this type of combinatorial auctions are rarely, if ever, conducted in practice.

Researchers have suggested several mechanisms to ameliorate this hurdle, as surveyed in §1.1. All of these suggested mechanisms either fail to capture the entire cost function $c_i$ (losing cost optimality), or require several rounds (resulting in increased procurement cycle times, of the order of months). Our proposed mechanism solves both these problems: it
provides the exact optimal allocation and payments in a single round. We formally specify and compare our proposed mechanism to BCA in the following section.

Note that even if non-VCG mechanisms are considered, the problems caused by the combinatorial explosion remain. The use of VCG as opposed to some other payment mechanism is not the cause of these hurdles.

Proposition 1 (Krishna (2002), ch.16) The BCA auction mechanism is incentive compatible (induces truthful bidding) and individually rational (induces bidders to participate).

2.3 Proposed Mechanism: Implicit Cost Combinatorial Auction

Our proposed mechanism eliminates the problems caused by the combinatorial explosion for certain classes of problems where the cost functions of bidders for bundles can be represented by a set of compact, efficiently computable functions. We require that the cost of bidder \( i \) providing bundle \( S \) be given by a common cost function \( c(S, \theta_i) \), where \( \theta_i \) is the type of bidder \( i \). Furthermore, the compactness and efficient computability of \( c \) is defined as follows.

There is a matrix \( A_i = A(\theta_i) \) and vectors \( \tilde{c}_i = \tilde{c}(\theta_i) \) and \( b_i = b(\theta_i) \), each of size polynomial in \( M \), such that \( c(S, \theta_i) \) can be computed using the following math program, where the \( j^{th} \) component of the binary vector \( x_S \) takes the value 1 if \( j \in S \) and 0 otherwise:

\[
c(S, \theta_i) = \min_{y \in P} \tilde{c}_i^T(x_S, y) \quad \text{s.t.} \quad A_i(x_S, y) \leq b_i.
\]

Here \((x_S, y)\) is just the concatenation of \( x_S \) and \( y \). \( P \) describes the feasible region of \( y \) (which may include integrality constraints). As will be made clear by examples, the vector \( y \) describes any decisions supplier \( i \) must make when computing his cost for bundle \( S \), for instance, routing decisions in the truckload procurement example of §3. These decisions are made subject to the supplier’s specific business context as described by \( \tilde{c}_i, A_i \) and \( b_i \); once again relating to truckload procurement, these could describe loaded and unloaded costs per mile as well as existing loads. The functions \( \tilde{c}(\cdot), A(\cdot), \) and \( b(\cdot) \) are application-specific, but not bidder-specific. That is, once we know the bidder’s type \( \theta_i \), we (the auctioneer) can construct \( \tilde{c}_i, A_i \) and \( b_i \). With this formulation, our proposed auction mechanism is defined as follows:

Implicit Cost Combinatorial Auction (ICCA)
1. Each bidder provides its true cost type $\theta_i$.

2. The auctioneer solves the winner determination problem (now denoted $T-WDP$, for cost Type) defined as follows. Here the variable $x_{ij}$ takes the value 1 if bidder $i$ is awarded object $j \in M$ and 0 otherwise.

$$z^*(N) = \min \sum_{i \in N} \tilde{c}_i^T(x_i, y_i) \quad (T-WDP)$$

subject to: \[ \sum_{i \in N} x_{ij} = 1 \quad \forall j \in M \]

$$A_i(x_i, y_i) \leq b_i \quad \forall i \in N$$

$$x_{ij} \in \{0, 1\} \quad \forall i \in N, \ j \in M.$$ 

As before, we use $z_i^* = \tilde{c}_i^T(x_i^*, y_i^*)$ to denote the cost incurred by bidder $i$ in the optimal solution.

3. The payment to bidder $i$, as before, is $z_i^* + z^*(N \setminus i) - z^*(N)$.

The constraints $A_i(x_i, y_i) \leq b_i$ are sometimes referred to as linking constraints in the sequel, and will be elaborated further in the specific applications we consider. Compared to the traditional combinatorial auctions, our mechanism has several significant benefits:

**Benefit 1.** The cognitive burden on the bidder is greatly reduced - rather than computing and communicating an exponential number of bids, only the true cost type must be relayed.

**Benefit 2.** The computational burden on the auctioneer is greatly reduced by eliminating the need to solve an exponentially large integer program, and instead solving a much smaller program. While the resulting problem is still an integer program (solving which is an NP-hard problem in general), the special structure of the program (arising out of the specific application) often results in efficiently computable formulations.

**Benefit 3.** ICCA is equivalent to the fully-enumerated VCG auction BCA (as proved below in Proposition 2) and yields all of the corresponding benefits.
Proposition 2 Consider a combinatorial VCG auction for the set of objects $\mathcal{M}$, where for each bidder $i \in \mathcal{N}$ the cost to provide the bundle $S \subseteq \mathcal{M}$ is given by $c_i(S)$. Suppose there exists a type $\theta_i$ for each bidder and functions $\tilde{c}_i = \tilde{c}(\theta_i)$, $b_i = b(\theta_i)$ and a matrix $A_i = A(\theta_i)$, all of which are polynomial in $\mathcal{M}$, such that for every $S \subseteq \mathcal{M}$, we have $c(S, \theta_i) = \min_{y \in \mathcal{Y}} \tilde{c}_i^T(x, y)$ s.t. $A_i(x, y) \leq b_i$, where $x$ is the incidence vector\footnote{That is, $x$ is a binary vector with $M$ components, with the $j$th component being 1 if $j \in S$ and 0 otherwise.} for $S$. Then, for any set of types $\{\theta_i\}_{i \in \mathcal{N}}$, the optimal solutions of the integer programs $(E - WDP)$ and $(T - WDP)$ have the same values.

Proof: Let $x^*(E)$ be an optimal solution to the formulation $(E - WDP)$ of cost $z^*(E)$, and let $(x^*(T), y^*(T))$ be an optimal solution to $(T - WDP)$ of cost $z^*(T)$. Observe that $z^*(T) = \sum_{i \in \mathcal{N}} z_i^*(T)$, and likewise $z^*(E) = \sum_{i \in \mathcal{N}} z_i^*(E)$.

First, consider the solution $x^*(E)$. Let $S_i^*(E)$ be the set of goods allocated to bidder $i$ in this solution. Therefore, $z^*(E) = \sum_{i \in \mathcal{N}} c_i(S_i^*(E))$. Define the vector $\hat{x}_i(T)$ to be the incidence vector of $S_i^*(E)$; that is, $\hat{x}_{ij}(T) = 1$ if $j \in S_i^*(E)$ and 0 otherwise. By definition, there must exist a vector $\hat{y}_i$ such that $c_i(S_i^*(E)) = \tilde{c}_i^T(\hat{x}_i(T), \hat{y}_i)$ and $A_i(\hat{x}_i(T), \hat{y}_i) \leq b_i$. Let the vector $\hat{x}(T)$ be defined as the concatenation of the $\hat{x}_i(T)$ vectors for each bidder, and similarly define $\hat{y}$ as the concatenation of the $\hat{y}_i$ vectors. The covering constraint in $(T - WDP)$ continues to hold, so that $(\hat{x}(T), \hat{y})$ is a feasible solution to $(T - WDP)$, of cost $z^*(E)$. Since the optimal solution to $(T - WDP)$ can only do better, we must have $z^*(T) \leq z^*(E)$.

Conversely, consider the solution $(x^*(T), y^*(T))$. For any bidder $i$, let $S_i^*(T)$ be the bundle allocated to bidder $i$ in ICCA, and define the vector $x(E)$ appropriately where $x_i(S) = 1$ for $S = S_i^*(T)$ and 0 otherwise. By definition, we have $c_i(S_i^*(T)) \leq \tilde{c}_i^T(x_i^*(T), y^*_i(T)) = z_i^*(T)$. Therefore, the solution given by $(x(E))$ is feasible for $(E - WDP)$, and has cost no more than $\sum_{i \in \mathcal{N}} z_i^*(T) = z^*(T)$. The optimal solution to $(E - WDP)$ can only do better, so $z^*(E) \leq z^*(T)$.

This proves that $z^*(T) = z^*(E)$, which is the claim of the proposition. \qed

Corollary 1 The premium paid to each bidder is the same under both BCA and ICCA auctions.
Proof: Recall that the premium paid to bidder $i$ is $z^*(\mathcal{N} \setminus i) - z^*(\mathcal{N})$. Proposition 2 guarantees that $z^*(\mathcal{N})$ is the same under both BCA and ICCA, and so is $z^*(\mathcal{N} \setminus i)$. The corollary now follows.

Corollary 2 If there is a unique optimal allocation in one of the two mechanisms (BCA or ICCA), then the same allocation is also the unique optimal allocation in the other mechanism, and each bidder gets the same total payment in both auctions.

Proof: Observe that the proof of Proposition 2 relies on showing that the optimal allocation of one mechanism is a feasible allocation in the other mechanism of same or lower cost. Therefore, if the optimal allocation is unique in one mechanism, the same allocation must be the unique optimal allocation in the other. The cost incurred by bidder $i$ in both auctions is therefore the same, $z_i^*$. Since the payment to each bidder is his cost plus his premium, Proposition 2 and Corollary 1 imply that the payment to each bidder is the same under both auctions.

Proposition 3 ICCA is an individually rational and incentive compatible auction mechanism.

Proof: The cost incurred in $(T - WDP)$ by agent $i$ is $z_i^*$; by truthful bidding, $z_i^*$ is also $i$’s true cost for the bundle he is awarded. Hence the payment minus cost (profit) of agent $i$ is simply $z_i^* + z^*(\mathcal{N} \setminus i) - z^*(\mathcal{N}) - z_i^*$, or $z^*(\mathcal{N} \setminus i) - z^*(\mathcal{N})$. The value $z^*(\mathcal{N} \setminus i)$ must be no less than $z^*(\mathcal{N})$ since the optimal allocation in $\mathcal{N} \setminus i$ is feasible and has the same cost even with bidder $i$ included. Hence $z^*(\mathcal{N} \setminus i) - z^*(\mathcal{N})$ is always non-negative, implying individual rationality.

We next prove incentive compatibility. Let $z^*(\mathcal{N})$ be the optimal value of $(T - WDP)$ over agent set $\mathcal{N}$ with truthful reporting of types, and let $\hat{S}_j$ be the optimal allocation to agent $j$ specified by $(T - WDP)$ when agent $j$ reports type $\hat{\theta}_j$, for $j \in \mathcal{N}$. Note that the cost incurred to the auctioneer by agent $j$ in this optimal allocation is $z_j^* = c(\hat{S}_j, \hat{\theta}_j)$, and $j$’s true cost for his bundle is $c(\hat{S}_j, \theta_j)$. Suppose that $\hat{\theta}_j = \theta_j$ for all $j$ except possibly $i$. Then agent $i$’s profit (payment minus cost) is $c(\hat{S}_i, \hat{\theta}_i) + z^*(\mathcal{N} \setminus i) - \sum_{j \in \mathcal{N} \setminus i} c(\hat{S}_j, \theta_j) - c(\hat{S}_i, \hat{\theta}_i) - c(\hat{S}_i, \theta_i)$, which equals $z^*(\mathcal{N} \setminus i) - \sum_{j \in \mathcal{N}} c(\hat{S}_j, \theta_i)$. The first term is independent of $\hat{\theta}_i$. The second term
is the objective function of \((T - WDP)\) with truthful reporting evaluated at some feasible allocation \(\hat{S}_j, \ j \in \mathcal{N}\), and therefore is no less than the optimal value \(z^*(\mathcal{N})\). Hence, if all bidders \(j \neq i\) report their true cost, \(i\)’s profit is no greater than \(z^*(\mathcal{N} \setminus i) - z^*(\mathcal{N})\). Since this profit is achieved when he reports his true cost \(\theta_i\), we are done.

Therefore, it is a dominant strategy for every bidder to reveal their type vector \(\theta_i\) truthfully. We discuss other issues regarding the practical implementation of ICCA in §2.5.

### 2.4 Sub-Additivity

Recall that we initially assumed that our cost function is sub-additive; that is, the cost of a combination of two bundles is no more than the sum of costs of the bundles taken individually. While this is often a natural assumption, it need not always hold. For example, if a bidder has a capacity constraint and can provide no more than 3 items, then the cost of a bundle of 4 items is effectively infinity, which is greater than the sum of costs of two bundles of 2 items each.

However, all is not lost for combinatorial auctions if the cost function is not sub-additive. In BCA, one would simply add the following constraint:

\[
\sum_{S \subseteq M} x_i(S) \leq 1 \quad \forall i \in \mathcal{N}.
\]

It is undesirable to add such a constraint in ICCA because the constraint has an exponential number of variables. Depending on the situation, however, one could add constraints to the matrix \(A_i(x_i, y_i) \leq b_i\) to enforce these requirements. For example, if sub-additivity is violated due to a capacity constraint of the form that the bidder can provide no more than 3 items, then the simple constraint \(\sum_{j \in M} x_{ij} \leq 3\) can be added to \(A_i(x_i, y_i) \leq b_i\) and will do the job. Indeed, we consider some such constraints in our formulations in the subsequent sections. The upshot is that one must examine the source of violation of sub-additivity, and see if that can be modeled by a compact set of constraints in the ICCA framework.
2.5 Implementation of ICCA

Since the ICCA mechanism is incentive compatible and individually rational (Proposition 3), it is in a bidder’s best interests to bid honestly. However, there are a few other hurdles which prevent bidders from honestly revealing their true types in practice. Bidders might be concerned about giving up private information, which may be used against them in other arenas. As Sakurai et al. (2000) point out, bidders may indulge in false-name bidding and other forms of collusion. However, in practice, when trusted third-party companies like CombineNet or Manhattan Associates conduct auctions, they already pre-qualify bidders and protect the confidentiality of all transactions. Pre-qualification prevents the problem of false name bidding, and the auctioning firm can be made to guarantee that the true-cost vectors $\theta_i$ are not revealed to anyone, not even the procuring firm. There is an emerging stream of literature in cryptographically-secured auctions, which achieve precisely this desired data privacy (Kudo 1998, Franklin and Reiter 1996). We believe that with a trusted third-party auction service provider and these appropriate layers of cryptography and pre-qualification, there is no significant hurdle to firms revealing their true types as required by ICCA.

3 Truckload Procurement: A Demonstrative Example

Freight transportation plays a critical role in the U.S. economy. The Bureau of Economic Analysis (2005) reported over $300 billion in U.S. transportation expenditures in 2004. Only housing, health-care, and food account for greater shares of the GDP than transportation-related goods and services (Department of Transportation 2003). More than eighty percent of the U.S. transportation costs are for trucking, with over half of this made up of truckload carriers (Sheffi 2004).

These carriers move full loads (i.e. trailers) directly from origin to destination, sometimes according to long-term contracts and other times as contracted through a spot market. The auction marketplace for single loads appears to be quite robust in practice, especially for small “mom-and-pop” trucking companies (Internet Truckstop 2006, Huff 2006). This success has not been fully realized for large carriers, however, nor for large shippers who want to place large numbers of loads up for bid concurrently. Although a number of factors play
into this, one of the most critical is the need to take into account the interaction between loads (Caplice and Sheffi 2006, Ledyard et al. 2002, Sheffi 2004), for example concatenating loads to create continuous moves, thereby reducing empty mileage. While a combinatorial auction addresses this concern in theory, in practice it is impossible for a carrier to bid on each possible bundle of loads, especially since each individual bidding decision itself requires the solution of a complex optimization problem - how to best sequence the loads in the bundle, along with the carrier’s existing loads, so as to minimize empty miles. Similarly, even if the bidders were to provide bids for all bundles, the resulting winner-determination problem would be intractable for the auctioneer for all but a very small set of loads. In practice, it is sometimes the case that combinatorial auctions are run for truckload procurement in which only some subsets of the bundles are bid upon. Such auctions, however, are not guaranteed to possess properties of efficiency or truthfulness, and in practice many carriers have expressed skepticism about participating in such combinatorial auctions (Huff 2006).

3.1 Problem Definition

Consider a large consumer-goods retailer, such as Target. According to their 2005 Annual Report (Target Corporation 2006), Target had a total of 1397 stores in the United States. They also had 23 distribution centers and 3 import warehouses. A large part of Target’s expenses, therefore, is the shipment of goods from vendors and import warehouses, via the distribution centers, to the retail stores. Target has three main options for this shipping:

1. Vendors: Some vendors are contracted to do their own shipping. This may be limited to delivery to the distribution centers, or may include delivery to the retail stores.

2. Internal Operations: The company may maintain its own fleet of vehicles to do the shipping.

3. Outsourced Shipping: The company contracts with one or several freight companies which provide the transportation services.

Our focus in this paper is on the third item, when shipping is outsourced to third-party providers. While we are not aware of the precise amount of shipping outsourced by
Target, the vibrancy of the shipping industry and anecdotal evidence indicate that for several companies, this is the dominant method of freight transportation.

Typically, such companies like to enter into contracts of length 1 to 3 years for this type of transportation services (Sheffi 2004). The company forecasts the shipping requirements, consisting of a list of (origin, destination, weekly loads) triples. This list specifies the anticipated number of weekly loads on each origin-destination lane. While there are seasonal spikes in demand (such as in December for retailers), these seasonal effects can either be incorporated into the main list of shipping requirements or procured separately when more accurate forecasts are available. For the purpose of this paper, we assume that the requirements consist of a list of triples which is stable every period (week) for the length of the planning horizon.

In such a situation, the company now contracts with a third-party auction provider to conduct a procurement auction, where truckers must bid on combinations of lanes. This presents a problem to the truckers: they would like to bid on lanes which either complement each other or complement their existing contracts, so as to construct repeatable round trips and minimize empty miles. This is where the combinatorial aspect of the exercise comes in, since this construction of round-trips has major implications on the valuation of any bundle (of lanes). However, observe that once a trucker knows what lanes it must serve, it can use a simple multi-commodity flow problem (Ahuja et al. 1993) to figure out the optimal construction of tours and coverage by trucks. This computation is precisely what we implicitly use in the ICCA version of this auction, as discussed in the rest of this section.

Auctions for truckload transportation services have also received some attention in the literature. Caplice and Sheffi (2006) provide a comprehensive exposition of the current state-of-the-art for procurement auctions for trucking services. Other such studies include those by Ledyard et al. (2002), Song and Regan (2003), Sheffi (2004) and Figliozzi et al. (2003).

### 3.2 Model Formulation

The underlying network structure of the truckload procurement problem and the nature of the carrier cost function makes this problem well-suited to ICCA. For the (simplified) version of the problem considered here, a carrier’s true type $\theta_i$ can be described simply by his/her
loaded and empty costs for each pair of cities, as well as existing contracted loads. The cost
function $\tilde{c}_i$ for a set of loads is then just these costs, while the constraints given by $A_i$, $b_i$
enforce the construction of continuous moves - sequences of loads, assigned the loaded cost
if they are contracted loads and zero if they are existing loads, with the segments connecting
these loads charged the empty cost. Thus, the formulation becomes equivalent to an instance

Our model is based on the following notation. The set of bidders is simply the set of
carriers, and is denoted $\mathcal{N}$ as before. The set of objects up for auction are the loads $\mathcal{M}$. The
true type $\theta_i$ for carrier $i$ is a triple $(\mathcal{M}^i, m^i, e^i)$ of vectors, defined as follows. The set $\mathcal{M}^i$
consists of the existing loads for carrier $i$, which it can leverage in constructing its continuous
moves. The vector $m^i = (m^i_1, m^i_2, \ldots, m^i_M)$ is the vector of loaded costs for carrier $i$; that is,
the cost to carrier $i$ of carrying load $j$ is $m^i_j$. The vector $e^i = \{e^i_{jk}\}_{j,k \in \mathcal{M} \cup \mathcal{M}^i}$ is the vector of
empty costs; the cost to move empty from the end of load $j$ to the origin of load $k$ is $e^i_{jk}$.

While we have defined the type vector $\theta_i$, for clearer exposition we first define the mathematical
programming formulation of the problem before elaborating on the components $\tilde{c}_i$, $A_i$ and $b_i$. Our formulation has two sets of binary variables. The variable $x^i_j$ takes the
value 1 if carrier $i$ is assigned load $j$, and 0 otherwise; this variable is defined for loads in
$\mathcal{M} \cup \mathcal{M}^i$; in other words, not only are we assigning new loads (from $\mathcal{M}$), but we are also
allowing carries to leverage existing loads (from $\mathcal{M}^i$) in constructing continuous moves). The
variable $y^i_{jk}$ is also a binary variable which takes the value 1 if carrier $i$ moves empty from
the destination of load $j$ to the origin of load $k$, and is also defined for $j, k \in \mathcal{M} \cup \mathcal{M}^i$. The
The integer programming formulation now follows.

\[
\min \sum_{i \in N} \sum_{j \in M} m^i_j x^i_j + \sum_{i \in N} \sum_{j \in M, k \in M} e^i_{jk} y^i_{jk} \tag{2}
\]

subject to

\[
x^i_j - \sum_{k \in M, M^i} y^i_{kj} = 0 \quad \forall i \in N, j \in M \cup M^i \tag{3}
\]

\[
x^i_j - \sum_{k \in M, M^i} y^i_{jk} = 0 \quad \forall i \in N, j \in M \cup M^i \tag{4}
\]

\[
\sum_{i \in N} x^i_j = 1 \quad \forall j \in M \tag{5}
\]

\[
x^i_j \in \{0, 1\} \quad \forall i \in N, j \in M \cup M^i \tag{6}
\]

\[
y^i_{jk} \in \{0, 1\} \quad \forall i \in N, j, k \in M \cup M^i \tag{7}
\]

In the formulation above, the constraints (5) ensure that each load up for bid is covered. The constraints (3) and (4) ensure “conservation”: that is, if a truck is covering load \( j \), it must move empty from the destination of that load to the origin of some other load. Notice that if the origin of the subsequent load is the same as the destination of the first load \( j \), then this empty move is effectively non-existent and is purely a modeling aid. These constraints together ensure that all bid loads in \( M \) are covered as parts of “continuous moves” or “loops” by carriers, so that the truck returns to its starting point. This results in a solution which can be repeated every period (week), as is required in our problem statement, and indeed is critical in the trucking industry.

The objective function (2) accounts for the cost of all the loaded moves up for bid, as well as all empty moves incurred in the process. Note that the cost of moves in \( M^i \) is not accounted for, since the trucking firms are already receiving payment for those loads from the respective clients. Note also that we do not require this formulation to cover all the loads in \( M^i \), allowing the truckers latitude in how they cover those contracts. Indeed, the purpose of this formulation is to figure out how truckers can best leverage their existing loads while covering all the loads that are up for bid, although more elaborate formulations can be conceived (for example, see §3.4).

We now cast our formulation in the framework of ICCA. The variables \( x \) have the same
meaning in both: they indicate whether or not a particular bidder provides a particular item, although in the transportation model the set of items (loads) is expanded to include existing loads not up for bid. The constraints (3-4,7) constitute the constraint set \( A_i(x_i, y_i) \leq b_i \) for each bidder \( i \), where the \( y \) of ICCA corresponds to the use of empty moves in the transportation model. The objective function consists of the two vectors \( m \) and \( e \), specified for each bidder; that is, \( \tilde{c}_i = (m^i, 0, e^i) \), where \( \tilde{0} \) is the vector of costs applied to loads in \( M^i \). This completely specifies the type \( \theta_i \) for each bidder \( i \). Furthermore, the constraints and objective function are polynomial in \( M \). Thus,

**Proposition 4** The program (2-7) is an instance of ICCA where \( \tilde{c}_i, b_i \) and \( A_i \) are all polynomial in \( M \).

Proposition 4 shows that our formulation follows the ICCA framework and is therefore compactly representable. We next discuss computational issues. Although the formulation above has two sets of integer variables, the \( y \) variables can be relaxed and assumed to be linear, since enforcing integrality of the \( x \) variables guarantees that any optimal solution with fractional \( y \) values can be converted to an integer solution of the same cost.
Table 1: Type vectors (cost characteristics) of 3 truckers

<table>
<thead>
<tr>
<th>Carrier $i \in \mathcal{N}$</th>
<th>Loaded cost /mile $\hat{m}^i$</th>
<th>Empty cost /mile $\hat{e}^i$</th>
<th>Existing loads (#: from, to) $\mathcal{M}^i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td>8</td>
<td>(1; 4, 5)</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>6</td>
<td>(2; 2, 1), (3; 4, 3)</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
<td>5</td>
<td>none</td>
</tr>
</tbody>
</table>

3.2.1 Illustration of Mechanism for Trucking Services Procurement

Consider a network of five cities represented by nodes $N_1$ through $N_5$ in Figure 1. The distances between the cities are given by the Euclidean distance between their locations in the two-dimensional plane. There are four loads up for auction, indicated by the solid lines; this constitutes the set $\mathcal{M}$. In addition, some carriers have pre-existing contracts ($\mathcal{M}^i$) on some lanes, indicated by the dashed lines. There are three carriers ($\mathcal{N} = \{1, 2, 3\}$) in all participating in this auction. Their characteristics (type vectors $\theta_i$) are detailed in Table 1. Note that for simplicity, we have assumed that each carrier has a constant loaded cost $\hat{m}^i$ per mile and empty cost $\hat{e}^i$ per mile, rather than lane-specific costs. That is, for a lane $j$ of length $d_j$ miles, the loaded cost to carrier $i$ is given by $m_{ij} = \hat{m}^id_j$, and likewise if the distance between the destination of lane $j$ and the origin of lane $k$ is $d_{jk}$, we have $e_{jk} = \hat{e}^id_{jk}$.

ICCA begins by computing the optimal allocation by solving the program (2-7). This allocation is described below, and is shown in Figure 2. (This is not the unique solution to the problem.)

- Carrier 1 is awarded $BL_2$. This carrier is effectively able to combine $BL_2$ with their pre-existing load $EL_1$, resulting in a round-trip. The cost to the shipper is therefore only the one-way cost of $BL_2$, which is $12 \times 60 = 720$. Despite being the highest per-mile cost bidder, Carrier 1 is awarded one lane due to the round-trip created.

- Carrier 2 is awarded loads $BL_3$ and $BL_4$, resulting in a continuous move ($N_2 \rightarrow N_1 \rightarrow N_4 \rightarrow N_3 \rightarrow N_2$) with the two bid loads, one pre-existing load ($EL_3$), and one empty move (from $N_3$ to $N_2$) for which the auctioneer is charged. Note that Carrier 2 has
another pre-existing load \((EL_2)\) which does not provide any benefit, and is not part of any continuous move.

- Carrier 3 is awarded load \(BL_1\). This carrier has to make an empty backhaul from the end of \(BL_1\) back to its origin. The net cost to the shipper on this lane is therefore \((11+5) \times 60 = 960\). The solution is not unique since Carrier 2 could also have obtained the same load at the same price.

The payment determination is as in any VCG auction; each carrier is awarded their cost in the optimal allocation plus the increase in cost due to their removal. For example, consider the payment to Carrier 1. Recall that the cost incurred by Carrier 1 in the optimal solution is 720. When Carrier 1 is removed, the optimal allocation is now to award \(BL_2\) to Carrier 2, resulting in a net increase in cost of 240. Therefore, the net payment to Carrier 1 is \(720 + 240 = 960\). The costs, premiums and net payments to each carrier are shown in Table 2.

The cost of the optimal allocation (with all carriers included) is 3,629.9. The overall cost to the shipper is the sum of payments made to the carriers, and is 4,263.1. The carriers, taken together, therefore earn a premium of 633.2, constituting a 17.45% increase over the net cost.
Table 2: Payments in trucking auction example

<table>
<thead>
<tr>
<th>Carrier</th>
<th>Cost</th>
<th>Premium</th>
<th>Payment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>720.0</td>
<td>240.0</td>
<td>960.0</td>
</tr>
<tr>
<td>2</td>
<td>1949.9</td>
<td>393.2</td>
<td>2343.1</td>
</tr>
<tr>
<td>3</td>
<td>960.0</td>
<td>0.0</td>
<td>960.0</td>
</tr>
<tr>
<td>Total</td>
<td>3629.9</td>
<td>633.2</td>
<td>4263.1</td>
</tr>
</tbody>
</table>

Table 3: Comparison of BCA and ICCA for trucking services procurement

<table>
<thead>
<tr>
<th></th>
<th>BCA formulation</th>
<th>ICCA formulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Integer variables</td>
<td>$N \times (2^M - 1)$</td>
<td>$N \times M$</td>
</tr>
<tr>
<td>No. of Continuous variables</td>
<td>0</td>
<td>$\sum_{i=1}^{N} (M +</td>
</tr>
<tr>
<td>No. of Constraints</td>
<td>$M$</td>
<td>$M + 2 \sum_{i=1}^{N} (M +</td>
</tr>
</tbody>
</table>

Some salient observations about ICCA as applied to trucking services procurement are illustrated by this example. First, the lowest-cost carrier may not win all the loads; high-cost carriers may be able to take advantage of pre-existing loads to compete effectively. Second, a carrier may end up with no premium at all if competition is intense. Third, two (or more) carriers may win loads on the same lane, depending on their cost structures and pre-existing loads. Fourth, pre-existing loads do not always bring added value to a carrier’s bids.

3.3 Implications for Larger Scale Problems

One of the principal advantages of ICCA is computational tractability. Table 3 compares the size of our ICCA formulation for this problem with the corresponding BCA formulation. Recall that $N = |\mathcal{N}|$ is the number of bidders, while $M = |\mathcal{M}|$ is the number of loads.

More importantly, the basic formulation involves solving a set partitioning problem which is notoriously hard (de Vries and Vohra 2003, Pekec and Rothkopf 2003). Our formulation, on the other hand, is a multi-commodity flow problem, which is much more amenable to
solution despite having some integer variables (Nemhauser and Wolsey 1999). Given that typical such problems in industry have hundreds or thousands of loads (Caplice and Sheffi 2006), the computational advantage of our mechanism becomes vast.

3.4 Additional Constraints and Requirements

Elaborations of the simplified transportation model discussed above can benefit the practicality of auctions for both shippers and carriers. Business requirements from the shipper’s side could include min and max limits on the number of service providers, enforced through additional constraints on the math program formulation. Truck availability and labor limits set by work rules could be modeled from the carrier’s side. Stochasticity in existing loads could be introduced to reflect the fact that some loads can be filled on an ongoing spot market basis. Compactness of the model and solvable formulations would be the research challenge in taking these next steps, but nonetheless the potential for significant advantages over the BCA approach remain. We save these and other extensions of our current ICCA transportation model for future research.

4 Other Application Areas

The key to implementing ICCA in other applications is the existence of the compactly representable and efficiently computable cost function; i.e., the existence of the type vector \( \theta_i \) which satisfies (1). In this section, we show how these vectors can be constructed for three other well-known applications of combinatorial auctions: wireless spectra, energy markets, and operations procurement. Observe that the energy market auction is actually a traditional auction where items are sold, thus demonstrating that our mechanism is applicable in both types of auctions.

4.1 Wireless Spectrum Auctions

The auction of frequencies for wireless communication has been one of the most dramatic and insightful applications of combinatorial auctions. They have been held in many countries
for different industry segments, and continue to be held: In 2006, the United States Federal
Communications Commission will auction frequencies to allow in-flight cellphone usage in
passenger aviation. Wireless spectrum auctions provide ample scope for combinatorial bids:
Bidders gain from scale, as well as contiguity of regions where they win presence and conti-
guity of frequencies within a single region (Günlük et al. 2005). Currently, the mechanism
most favored by the FCC is an iterative combinatorial auction (Federal Communications
Commission 2006), where bidding goes on for several rounds determined by a specific set
of rules. Our ICCA mechanism has the advantage of making the auction a single-round
mechanism, while at the same time capturing all the economic efficiencies of a enumerative
combinatorial auction.

For illustrative purposes, in this section we will focus on the auction of frequencies
for wireless cellphone communications (others include in-flight communications, land-lines,
broadcast spectra, etc.). In such an auction, the unit commodity is a frequency band in
one specific location (such as metropolitan area or state). Several locations are up for bid
(typically encompassing the entire nation), and multiple frequency bands are also offered for
auction in each location.

While the valuation by each bidder of a package of licenses could be a fairly complex
function in general, the literature (Cramton 1997, Günlük et al. 2005) suggests that the
two biggest combinatorial components of the valuation are synergies in locations allocated
and frequencies allocated. Although other synergies can also be modeled in our framework,
in order to keep the discussion straightforward and brief we choose to focus on locational
synergies.

Locational synergies occur when the same bidder wins licenses for a set of locations which
together give the bidder more value than the sum of the individual valuations. For example,
if a bidder wins licenses for the metro areas surrounding New York, Boston, Philadelphia and
Washington, DC, then that bidder can become a major player in the “Northeast corridor”
region with contiguous coverage throughout the region. Therefore the bidder may be willing
to pay more for the package of these four locations than the sum of the bidder’s valuations
for winning each of these in isolation. Locational synergies may also occur if bidders already
have licenses in some areas and only want to cover their gaps, or they may want to focus on
large metropolitan areas, etc.

4.1.1 Mathematical Formulation

Note that although our initial model in §2 is in terms of a reverse auction, this application is a regular auction where bundles are sold. However, the notation and formulation carry over with minimal changes. Let $\mathcal{N}$ denote the set of bidders (wireless providers), indexed by $i$. Let $\mathcal{L}$ denote the set of locations, indexed by $l$. Thus the individual items being auctioned are the locations in $\mathcal{L}$. The problem can easily be formulated as $|\mathcal{L}|$ single-item auctions. We now show one possible formulation as a multi-item combinatorial auction which allows bidders to leverage the locational synergies.

Since this is a forward auction, the locational synergies actually imply that super-additivity holds across the units being auctioned, which is precisely what we desire. That is, if $L' \subset \mathcal{L}$ represents a set of locations and $v(i, L')$ represents bidder $i$’s valuation for the package $L'$, then $v(i, L') \geq \sum_{l \in L'} v(i, l)$. In general, the valuation function $v$ for any bidder $i$ must now be specified over all subsets of $\mathcal{L}$, which is precisely the combinatorial explosion ICCA enables us to avoid.

The key concept to avoid the combinatorial explosion is to associate, for each bidder $i$ and each location $l$, a synergy term $w(i, l, l')$ which is the added value to bidder $i$ in location $l$ from having both locations $l$ and $l'$. Therefore, if a bidder wins both locations $l$ and $l'$, his net valuation for the package $(l, l')$ is $v(i, l) + v(i, l') + w(i, l, l') + w(i, l', l)$. In general, if a bidder wins a set of locations $L'$, his valuation is $\sum_{l \in L'} v(i, l) + \sum_{l \in L'} \sum_{l' \in L'} w(i, l, l')$.

We therefore associate a variable $y(i, l, l')$ indicating that bidder $i$ has won both locations $l$ and $l'$, and can realize the associated synergy. For the formulation to work, however, we require that $w(i, l, l') \geq 0$ for all $i$, $l$, and $l'$. That is, there should be no disadvantage to winning two locations, which is a fairly mild assumption (particularly if we assume an

\[^2\text{This might seem like double-counting, but that problem can easily be avoided by dividing the second term by 2, or several such factors.} \]
efficient after-market). The ICCA formulation is now as follows:

\[
\max \sum_{i \in \mathcal{N}, l \in \mathcal{L}} v(i, l) x(i, l) + \sum_{i \in \mathcal{N}, l \in \mathcal{L}, l' \in \mathcal{L}} w(i, l, l') y(i, l, l')
\]

\[
\sum_{i \in \mathcal{N}} x(i, l) = 1 \quad \forall l \in \mathcal{L}
\]

\[
y(i, l, l') \leq x(i, l) \quad \forall i \in \mathcal{N}, l, l' \in \mathcal{L}
\]

\[
y(i, l, l') \leq x(i, l') \quad \forall i \in \mathcal{N}, l, l' \in \mathcal{L}
\]

\[
x(i, l), y(i, l, l') \in \{0, 1\} \quad \forall i \in \mathcal{N}, \forall l, l' \in \mathcal{L}
\]

Observe that constraints (10) and (11) prevent the program from realizing the synergies unless both locations are in fact awarded to the same bidder; these are the linking constraints for this program. Since this is a maximization program and we require \( w(i, l, l') \geq 0 \), the program will always realize the synergies whenever two synergistic locations are allocated to the same bidder.

**Proposition 5** The program (8-12) is an instance of the forward-auction version of ICCA, where the sizes of \( \tilde{c}, A \) and \( b \) are polynomial in the number of bidders and locations awarded.

Proof: The value function \( \tilde{c} \) is simply the concatenation of the vectors \( v \) and \( w \), while the constraints (10-11) show the construction of the matrices \( A_i \) and \( b_i \). Each bidder’s type vector \( \theta_i \) is simply the concatenation of the vectors \( v \) and \( w \), capturing the valuation of any package of bids.

Locational synergies in general may involve much more than pair-wise synergistic terms. For example, the number of contiguous blocks awarded may have an impact on the value of the total package, as may other requirements such as the proportion of blocks which are large metropolitan areas (as opposed to rural areas), whether or not an entire state is covered, and etc. While we believe that most of these issues can be compactly modeled within the ICCA framework, we defer them to future research due to space constraints.

### 4.1.2 Bandwidth and Other Synergies

Bandwidth synergies occur when the same bidder in the same location is awarded to adjacent frequency bands, enabling that bidder to utilize the dead zone band between the two allocated
bands at no further cost. A simple extension of our model allows us to incorporate these bandwidth synergies within the ICCA framework.

One can conceive of several other synergistic effects which dictate the value of a bundle of several location-bandwidth pairs. For example, existing equipment and partnerships with existing providers may make a very specific bundle particularly attractive to a bidder. A comprehensive study of incorporating all such synergies into the ICCA framework is left open for future research.

### 4.2 Commodities with Non-Linear Value Functions

#### 4.2.1 Combinatorial Auctions for Single Commodities

Combinatorial auctions are typically thought of as auctions in which many different items are being auctioned simultaneously and the cost of providing a group of these items is not simply the sum of the individual items’ costs. It is also possible, however, to think of combinatorial auctions in the context of a single commodity, in the case where the cost associated with that commodity is not linearly dependent on the quantity provided. For example, consider the cost function in Figure 3(a). If the bidder provides one unit the cost is twenty, but if two units are provided, the cost is only thirty (i.e. fifteen per unit), and when the provision increases to three units, the cost is thirty-five (i.e. $11\frac{2}{3}$ per unit). Thus, if bidders are only

![Figure 3: Examples of non-linear cost functions.](image)
allowed to submit a single per-unit bid cost, they may over- or under- bid, depending upon the quantity of the commodity that they are awarded.

Conversely, the non-linearity of a cost function can be captured (or closely approximated) by a combinatorial auction mechanism, in which bidders are allowed to place multiple bids. Here, a “bundle” is really an upper and lower bound within which the commodity’s cost is either fixed or linearly dependent on the quantity. The cost structure may be a step function (e.g., Figure 3(a)), in which case the bid for any quantity within a given range is constant. The bid is twenty if the quantity is in the range \([0, 1]\], thirty for the range \((1, 2]\], and thirty-five for the range \((2, 3]\]. Alternatively, for the cost function in Figure 3(b), a bid with no fixed base cost and a marginal cost of twenty per unit would be placed for range \([0, 1]\], a bid with fixed base cost of ten plus marginal cost ten per unit would be placed for range \([1, 2]\], etc.

4.2.2 A Math-Programming Formulation for Single-Commodity Auctions

Single-commodity auctions with non-linear cost functions, such as those described in the preceding section, are a natural fit for ICCA. In this section, we limit our presentation to formulating the WDP. Our formulation is based on the use of binary variables to limit each bidder to a specific range. Once that range is specified, the cost function becomes linearized.

We start by presenting the formulation for those cases where the value is constant within a given range, then extend this model to the case where the value is linear relative to the award within a given range.

Let \(N\) be the set of bidders, each of whom specifies a set of ranges \(R_i\). Each range \(r \in R_i\) is defined by an upper bound \(u_r\), a lower bound \(l_r\), and a constant cost for that range \(c_r\). Let \(Q\) be the quantity of the commodity to be auctioned. For each bidder \(i\) and range \(r \in R_i\), we define the binary variable \(z_r\) that takes value one if the quantity awarded to bidder \(i\) falls within the range \(r\) and zero otherwise. For each bidder \(i\) and range \(r \in R_i\), we also define the continuous, non-negative variable \(y_r\), which represents the quantity of the commodity within range \(r\) awarded to bidder \(i\). If the award does not fall within range \(r\), then \(y_r\) is
The formulation is then:

$$\min \sum_{i \in \mathcal{N}} \sum_{r \in R^i} c^i_r z^i_r$$

subject to:

$$\sum_{r \in R^i} z^i_r = 1 \quad \forall i \in \mathcal{N}$$

$$u^i_r z^i_r \geq y^i_r \geq l^i_r z^i_r \quad \forall i \in \mathcal{N}, r \in R^i$$

$$\sum_{i \in \mathcal{N}} \sum_{r \in R^i} y^i_r = Q$$

$$z^i_r \in \{0, 1\} \quad \forall i \in \mathcal{N}, r \in R^i$$

$$y^i_r \geq 0 \quad \forall i \in \mathcal{N}, r \in R^i.$$

The first set of constraints ensures that each bidder’s award is within a single range; the objective function can be computed from these ranges, as each range corresponds to a constant fixed cost. The second and third set of constraints force the bidder’s award to be within his/her designated range. The fourth constraint allocates the total quantity of the commodity across the set of bidders. The final constraints ensure the integrality, where appropriate, and non-negativity of the variables.

To accommodate linear cost functions within a given range, the bidders instead specify for each range $r \in R^i$ a fixed base cost $f^i_r$ and a marginal per-unit cost $m^i_r$. The constraints above continue to define the feasible region of the problem. The objective function is modified to:

$$\min \sum_{i \in \mathcal{N}} \sum_{r \in R^i} (f^i_r z^i_r + m^i_r y^i_r).$$

This captures both the fixed cost associated with the designated range and the marginal cost associated with the size of the award. The following proposition is straightforward.

**Proposition 6** The programs (13-18) and (19, 14-18) are instances of ICCA where $\bar{c}_i$, $b_i$ and $A_i$ are all polynomial in $Q$, the number of items being procured.

Note that here we use $Q$ in lieu of $M$ since we are procuring multiple items of the same object, rather than multiple objects.
4.2.3 Energy Auctions

Energy auctions provide a number of examples of single commodity auctions where the cost has a non-linear dependence on the quantity. Such auctions can be single-seller/multiple buyer, single-buyer/multiple seller, or multiple-buyer/multiple-seller. Examples of all three cases and further references can be found in Hobbs et al. (2000). In particular, energy production is often characterized by alternating fixed costs and linear marginal costs. As certain thresholds are reached, it may become necessary to utilize an additional generator, with a fixed start-up cost. Such cost functions provide natural breakpoints for linearization as shown above.

4.3 Procurement Auctions with Capacity Constrained Suppliers

Purchasing agents often use procurement auctions to lower prices paid to suppliers, particularly when the item or contract is standard or well-specified and multiple suppliers can compete. When suppliers’ production costs include significant fixed costs, a supplier’s lowest price usually coincides with excess production capacity. Realizing that supplier bid prices are often directly tied to capacity constraints, it is important whether or not suppliers are able to estimate capacity. As we discuss below, this estimation problem is easily handled in the ICCA framework.

Due to the capacity constraints, a supplier who offers to produce item $j$ must anticipate how the offered production of $j$ will affect his ability to produce items other than $j$. Capacity forecasts are required if items are auctioned off sequentially, since when bidding on items auctioned earliest the supplier must try to anticipate what his capacity position will be during subsequent auctions. This creates an exposure problem for the supplier, as committing his capacity through aggressive bidding in early auctions might backfire with lost opportunities for greater profit in later auctions (Elmaghraby 2003, Gallien and Wein 2005). This exposure problem can be avoided by auctioning off all contracts simultaneously.

We consider a setting in which a purchasing agent (buyer) seeks to purchase items in the set $\mathcal{M}$, where $d_j$ is the buyer’s demand for item $j \in \mathcal{M}$. We let $\mathcal{N}$ be the set of suppliers, and to keep the exposition concise we assume that for each supplier $i \in \mathcal{N}$, production quantity
vector \( x_i = (x_{i1}, x_{i2}, \ldots, x_{iM}) \) costs the supplier \( c_i^T x_i \) to produce and must satisfy the linear constraints \( A_i x_i \leq b_i \), where \((a_{ij})_i \in A_i\) represents the amount of resource \( l \) consumed by one unit of item \( j \) and \( b_l \) represents the total amount of resource \( l \) available. For instance, the resources could be labor hours, machine time availability, or possibly production emission limits.

The traditional way to conduct this auction would be the BCA auction described in §2.2. While it removes the need for suppliers to make detailed, complex estimates of their capacity exposure problem (by auctioning all contracts simultaneously) or the capacity and costs of their bidder counterparts (by truthful bidding), its enumerative approach means every vector and associated cost for the feasible region must be communicated explicitly to the buyer by every supplier \( i \).

Continuing with our theme of a cost structure bidding approach alternative, we propose that in lieu of the above combinatorial program, the buyer simply ask each supplier \( i \) to reveal \( \theta_i = (c_i, A_i, b_i) \), a much simpler task. Then, the buyer’s winner determination problem is just a linear program as follows, which is much easier to solve than the enumerative combinatorial problem.

\[
\begin{align*}
\min & \sum_{i \in \mathcal{N}} c_i x_i \\
\text{subject to: } & A_i x_i \leq b_i \quad \forall i \in \mathcal{N} \\
& \sum_{i \in \mathcal{N}} x_{ij} \geq d_j \quad \forall j \in \mathcal{M} \\
& x_i \geq 0 \quad \forall i \in \mathcal{N}.
\end{align*}
\]

The following proposition is straightforward and ties the above formulation to the ICCA approach.

**Proposition 7** The program (20-23) is an instance of ICCA where \( \bar{c}_i, b_i \) and \( A_i \) are all polynomial in \( M \).

While for brevity we have assumed linear capacity constraints and costs, a richer cost and capacity structure (e.g., that described in §4.2) could be incorporated with an analogous approach, which we leave for future research.
5 Conclusions and Future Research

In this paper, we address two problems of traditional combinatorial auctions: the cognitive burden on bidders wrought by enumerative bidding, and the computational burden imposed on the auctioneer by an exponentially large winner determination problem. Both problems are addressed simultaneously by a novel approach in which each bidder’s optimization model is incorporated implicitly into the winner determination problem, rather than through enumerative bidding as in traditional models. This greatly reduces the cognitive burden on bidders, who simply reveal to the auctioneer parameters of a mathematical program. With the resulting compact formulation (polynomial in the number of objects auctioned), the auctioneer’s winner determination problem becomes far simpler than that with enumerative bidding, e.g., a multi-commodity flow problem in the transportation example and a linear program in the operations procurement example.

This approach creates many future directions for research. One could consider more elaborate bidder optimization models that better capture bidder costs, while maintaining tractability. Also, while we have focused on VCG auctions, other auction mechanisms that have desirable properties and an approach similar to ours could be studied. In fact, a first-price version of ICCA can be conducted in a fairly straightforward manner: Bidders submit their vectors $\theta_i$, the auctioneer solves the program ($T - WDP$), and each bidder is paid $z_i^*$. However, bidders have an incentive to strategically provide a vector $\theta_i$ which may not be the true cost but which maximizes their expected revenue. While this strategic behavior is well-described for simpler auctions (e.g., single-item), it is much harder to analyze in combinatorial auctions. For instance, how would a trucking company construct priors on other bidders and then strategically quote different values for their costs and existing contracts? While the computational efficiency of our mechanism would continue to hold in a first-price auction, the cognitive efficiency would be lost, leading to inefficient outcomes if the participants aren’t able to construct bids optimally. It is an open question if there are applications where the strategic behavior for first-price auctions can be described for our framework. Finally, the possible scope of applicability for our approach is essentially the scope of combinatorial auctions themselves, and work remains to be done to better understand where our approach will work well.
References


