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TURBULENT NATURAL CONVECTION
FROM A VERTICAL PLANE SURFACE

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ABSTRACT

The paper reports the results of an experimental investigation which was intended to clarify the present uncertain position with regard to the distributions of mean temperature and mean velocity in a turbulent natural convection boundary layer. Data reported for the turbulent boundary layer for Grashof numbers between 10^{10} and 10^{11} includes local heat-transfer coefficients as well as temperatures and velocities. Local heat-transfer coefficients and temperature distributions are also reported for the laminar and transitional boundary layer regions. Results are compared with other experimental data and with theoretical predictions.

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NOMENCLATURE

a, b, c	constants
Gr	$(= g \beta (\theta_w - \theta_\infty) x^3/\nu^2)$ Grashof Number
g	specific gravitational force
k	thermal conductivity
Nu	$(= \dot{Q}'' x/k(\theta_w - \theta_\infty))$ Nusselt number
$\dot{Q}''$	heat-transfer rate per area
u	velocity in the x direction
u^*	$(= (g \beta (\theta_w - \theta_\infty) x)^{0.5})$ a characteristic velocity
x	distance from the leading edge of the plate
y	distance normally away from the plate
β	coefficient of cubical expansion
λ	$\left(= \int_0^\theta k(\theta) d\theta\right)$
η	dimensionless distance normal to the plate
θ	temperature
ν	kinematic viscosity
Φ	dimensionless temperature

Subscripts

w	conditions on the surface of the plate
∞	conditions outside the boundary layer

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Introduction

A number of authors have remarked on the lack of any reliable experimental data pertaining to turbulent natural convection, other than overall heat-transfer rates. This paper presents the results of an investigation aimed at providing such data.

The majority of the data previously available were measured by Griffiths and Davis (1)* in 1922. They measured local heat-transfer rates, temperature profiles and velocity profiles. No other measurements of velocity profiles exist and the poor agreement between the data and the semi-theoretical result of Eckert and Jackson (2) has led to suggestions that the data are very inaccurate. Although slightly more data exist (12, 13) concerning local heat-transfer rates and temperature profiles, in neither case are they adequate to provide a description of the effects.

In the present work, profiles of mean velocity and mean temperature, together with local heat-transfer coefficients, are reported over a range of Grashof numbers from 10^4 to 1.5×10^{11}. The data for $10^4 < Gr < 2 \times 10^9$ pertain to a laminar boundary layer and were taken as a check on the measuring techniques. The data for $2 \times 10^9 < Gr < 10^{10}$ do not include any velocity profiles, but the heat-transfer rates and

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*Numbers in parenthesis refer to similarly numbered references in bibliography at end of paper.

the temperature profiles show important details of the development and of the transition process. For Gr $> 10^{10}$ the boundary layer was turbulent.

Apparatus and Measuring Techniques

The apparatus, which has been described in detail in (3), consisted of a pair of aluminum plates, each 0.6 m by 2.75 m, bolted together with electrical heater pads between them. The heaters could be adjusted independently so as to obtain a uniform plate surface temperature. Side screens were fitted to the plate in order to prevent inflow into the boundary layer from the sides. It was realized that the side screens could cause a lack of two-dimensionality in the flow because they would create additional boundary layers, but it was felt that the plate was sufficiently wide so that measurement made down the center line of the plate would not be significantly affected (the plate width was approximately 25 times the boundary layer thickness in the laminar region and 4 times the boundary layer thickness in the turbulent region).

The method adopted for the measurement of the local heat-transfer rate, was that of measuring the temperature gradient normal to the plate and calculating the heat-transfer rate from the value of this gradient at the plate surface. This method had previously been employed, in laminar natural convection by Goldstein and Eckert (4), who used an interferometer to measure the temperature. In the present investigation, the requirements for averaging over long periods of time led to the adoption of a fine wire, platinum resistance thermometer which was also used as a hot wire anemometer. The measuring head of this instrument was designed so that errors due to operation in regions of very large temperature gradient were made as small as possible. (See (3) for details.)

In the measurement of local heat-transfer rates, local mean temperatures were measured at six points distant 0.05, 0.1, 0.15, 0.2, 0.25 and 0.3 cm, respectively, from the heated plate. In addition, the local temperature outside the boundary layer (at the same horizontal level) was measured. The temperature of the heated plate was obtained by extrapolation from the measurements close to the plate. In the determination of the local heat-transfer rate, account was taken of the variation of the thermal conductivity of air with temperature. This was achieved by working in terms of $\lambda \left(= \int_0^\theta k(\theta) d\theta \right)$ instead of simply θ. Thus $(\partial\lambda/\partial y)_{y=0}$ is the local heat-transfer rate at the wall.

For data taken in the laminar boundary layer it was found that a graph of λ vs y, for the six points near the wall, was linear in almost all cases. The only exception was the point at 0.3 cm in tests at low Grashof numbers. A graphical determination of $(\partial\lambda/\partial y)_{y=0}$ and θ_0 was used for the laminar data.

For data taken in the transitional and turbulent boundary layers, λ vs y was not linear. Because of the difficulty in obtaining consistency in any graphical curve fitting procedure, an analytical curve of the type $\lambda = a + by + cy^3$ (where a, b and c are constants) was fitted to each set of data by means of the "least squares" procedure, using a digital computer. This particular analytical curve was used because it was the simplest which satisfied the required boundary condition and fitted the data within the experimental accuracy. The fact that this curve is of a type often encountered in boundary layer integral work with laminar boundary layers is not surprising when it is appreciated that at least half of the points to which it is applied in this case would lie within the viscous sublayer if such a concept is tenable in a turbulent natural convection flow.

For the temperature measurements associated with the local heat-transfer data and the mean temperature profiles, the platinum resistance thermometer was used in conjunction with a simple Wheatstone bridge circuit. In order to facilitate averaging over periods of the order of five minutes (required in the turbulent boundary layer), the out-of-balance voltage from the bridge was converted to a pulse signal having a frequency proportional to the voltage. The conversion was achieved by means of a digital voltmeter,* which also provided a second pulse signal for timing purposes. The two pulse signals were counted on scalers. This arrangement, which is described in detail in (3), as well as providing a wide range of averaging periods (10 s to 1 h), had the advantage that the digital voltmeter could be used in its normal operating mode with steady signals. The system had the same accuracy ($\pm$ 1.5μV) regardless of whether the averaging arrangement was being used or not. This represented ± 0.04 deg C. The measuring element of the resistance thermometer consisted of a platinum wire 0.00127 cm in diameter and approximately 1.4 cm long. The ice point resistance, (approximately 10.9 Ω) and the temperature coefficient of resistance

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*Solartron. Type LM 1420

of the wire (39.1×10^{-4} deg C^{-1}) were determined by calibration against a standard thermocouple. A current through the wire of about 1 mA was normally used for temperature measurements.

The same measuring instrument was also used for velocity measurement. In that case it was necessary to measure first the local temperature (as above). Then the wire current was increased to about 70 mA and the temperature of the heated wire was measured together with the wire current. These data enabled the Nusselt number for heat-transfer from the wire to be calculated. The corresponding Reynolds number for flow past the wire was determined from a calibration graph, thus permitting the flow velocity to be calculated. In these calculations the physical properties were evaluated in the manner of Davies and Fisher (5) (i.e., k at the hot wire temperature and all other properties at the local ambient temperature).

The calibration graph was produced by making measurements in the laminar boundary layer under similar conditions of wire current, etc., and assuming that the velocity distribution there was that given by the theoretical solution of Ostrach (6). This procedure was necessitated by the absence of any other calibration mechanism at the time when this work was carried out. However, a number of features of the calibration may be combined to give an estimate of the accuracy of the procedure. Firstly, previous measurements of velocity in laminar natural convection (Eichhorn (7), Schmidt and Beckmann as reported in (6)) had shown reasonable agreement (5 - 10%) with the theoretical result except at $Gr \simeq 10^9$. Secondly, calibrations at different positions, i.e., different values of Gr, yielded consistent results except at $Gr \simeq 10^9$

where the discrepancy was in agreement with that noted in the results of Schmidt and Beckmann as reported in (6). Finally, the resulting calibration curve agreed with the empirical result of Collis and Williams (8) within 7% when account was taken of end losses from the wire and the different basis for the evaluation of fluid properties.

All the above considerations tend to suggest that the velocities reported in this work are accurate to ± 10%. It should be noted, however, that the day to day repeatability of the present velocity measurements was considerably better than this, being ± 3%.

Results and Discussion

Figure 1 shows, in dimensionless form, the results of local heat-transfer measurements made in the laminar boundary layer. The agreement with the theoretical results of Ostrach (6) is seen to be generally satisfactory, but not as good as that reported by Goldstien and Eckert (4). The discrepancies can be explained by differences in experimental conditions. For $10^7 < Gr < 2 \times 10^9$, the discrepancy is due to the existence of a small vertical temperature gradient in the laboratory during the experiments (Cheesewright (3, 10)). For $10^4 < Gr < 10^6$ the discrepancy is due to a slight non-uniformity in the plate surface temperature.

Figures 2 and 3 show the local heat-transfer data measured in the transitional and turbulent parts of the boundary layer. The arbitrary nature of attempts to specify the beginning and end of transition is shown in Figure 2. $Gr = 2 \times 10^9$ has been taken as the beginning of transition because it was the condition at which significant fluctuations first appeared in the boundary layer, but no significant change occurred in the mean temperature profiles or in the trend of the heat-transfer data until $Gr \simeq 5 \times 10^9$. (The first appearance of significant fluctuations at $Gr \simeq 2 \times 10^9$ is in agreement with the experimental work of Szewczyk (11).) Similarly, major changes in mean temperature profiles and heat-transfer rates appear to have ended by $Gr \simeq 8 \times 10^9$, but another change in the trend of the heat-transfer data is apparent at $Gr \simeq 2 \times 10^{10}$ (see Figure 3) which is often quoted as the end of transition. $Gr \simeq 2 \times 10^{10}$ was also associated with a change in the characteristics of the turbulent fluctuations. For $8 \times 10^9 < Gr < 2 \times 10^{10}$ the amplitude of the temperature fluctuations decreased with increasing Gr (Cheesewright (3)), while for $Gr > 2 \times 10^{10}$ it remained approximately constant.

It may be seen from Figure 3 that for $Gr > 2 \times 10^{10}$ the data is in reasonable agreement with the semi-empirical result of Eckert and Jackson (2) while for $8 \times 10^9 < Gr < 2 \times 10^{10}$ the data agrees more closely with the result of Bayley (9). The scatter of the data is in accord with the result of attempts to assess experimental accuracy, but recent improvements in the experimental techniques suggest that it may be possible to improve on this in future experiments. It is noted on Figure 3 that some data points may contain errors due to trailing edge effects. These data were measured within 5 cm of the square trailing edge of the plate which was 2.5 cm thick. The presence of the trailing edge may be expected to enhance the local heat-transfer rate, but it is not possible to estimate the magnitude or the extent of the effect.

Figure 4 shows temperature profiles in the laminar boundary layer. Two theoretical results are shown; the classic result (from (6)) for isothermal conditions outside the boundary layer, and also the result (from (10)) for conditions approximating to the temperature distribution outside the boundary layer, which was actually present in the experiments. Again the agreement with theory is satisfactory, being comparable with the agreement between the classical result and the experimental data of Schmidt and Beckmann as reported in (6).

The development of the profiles of mean temperature through the transition region is shown in Figure 5 and is seen to be in general accord with the variation of local heat-transfer rate in this region as described above. In view of the agreement between the turbulent heat-transfer data and the semi-empirical result of Eckert and Jackson, the present data concerning distributions of mean temperature and mean velocity, in the turbulent boundary layer, are presented in Figures 6, 7, 8, and 9 using the form of independent variable suggested by that work. Figures 6 and 8 show the present temperature and velocity data while Figures 7 and 9 show corresponding comparisons with the results of other investigators.

It is apparent from Figures 6 and 8 that the use of $\eta = (y/x)\ Gr^{0.1}$ as the independent variable does not achieve good correlation of the results. $\eta = (y/x)\ Gr^{0.4}$ would give much better correlation over the inner part of the boundary layer, but would give the wrong behavior near the outer edge. The use of the boundary layer thickness as a normalizing parameter was not possible, even if it had been desirable, because it could not be determined with sufficient accuracy.

The above considerations suggest that one parameter similarity, in the sense in which it applies to the laminar boundary layer, cannot be used to describe the whole turbulent boundary layer. Such a conclusion would be in keeping with the results of experiments on turbulent boundary layers in forced flows. It is possible that different one parameter similarity descriptions could be applied to different parts of the boundary layer, but this has not been investigated in detail at the present time.

Figures 7 and 9 show that the present results are in reasonably good agreement with those of Griffiths and Davis (1). Agreement with the predictions of Eckert and Jackson (2) is, however, much less satisfactory, even when the difficulty mentioned above concerning the choice of independent variable is ignored. In making this second comparison, it must be borne in mind that the predictions are the result of a boundary layer integral type of solution so that exact agreement would not be expected. However, the discrepancy with regard to maximum velocity and boundary layer thickness is very much greater than that occurring in the analogous laminar flow problem. The use of experimental values for the boundary layer thickness and the maximum velocity in the equations used by Eckert and Jackson to describe the profiles suggests that these equations do not provide a good description of the shapes of these profiles.

A detailed comparison of the present data with the distributions of temperature and velocity predicted by Bayley (9) is not given because of the close similarity between that work and the work of

Eckert and Jackson; it suffices to say that agreement is even less satisfactory than with the predictions of those authors. This result is not surprising in view of the trends in the heat-transfer data.

Conclusions

New experimental measurements of temperature profiles, velocity profiles and local heat-transfer rates in natural convection on a vertical plane surface have been presented. For the important turbulent region these have been shown to be in reasonable agreement with the only existing experimental results. The heat-transfer data is also in agreement with theoretical predictions, but this is not the case with regard to the temperature and velocity profiles.

For the laminar part of the boundary layer, the results are in good agreement with existing theory, thus giving some indication of the accuracy of the experimental techniques.

No previous data exists for the transition region and the present results show important details of the transition process in terms of the development of profiles of mean properties.

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BIBLIOGRAPHY

(1) E. Griffiths and A. H. Davis, "The Transmission of Heat by Radiation and Convection," British Food Investigation Board Spec. Dept. 9, D.S.I.R. London, 1922 (1931).

(2) E. R. G. Eckert and T. W. Jackson, "Analysis of Turbulent Free-Convection Boundary Layer on Flat Plate," NACA TN 2207, 1950.

(3) R. Cheesewright, "Natural Convection From a Vertical Plane Surface," Ph.D. Thesis, Mechanical Engineering, University of London, Oct. 1966.

(4) R. J. Goldstien and E. R. G. Eckert, "The Steady and Transient Free Convection Boundary Layer on a Uniformly Heated Vertical Plate," International Journal of Heat and Mass Transfer, 4, 1960, 208.

(5) P. A. O. L. Davies and M. J. Fisher, "Heat Transfer from Electrically Heated Cylinders," Proceedings of the Royal Society, A. 280, 1964, pp. 486-527.

(6) S. Ostrach, "An Analysis of Laminar Natural-Convection Flow and Heat Transfer about a Flat Plate Parallel to the Direction of the Generating Body Force," NACA TN 2635, 1952.

(7) R. Eichhorn, "Measurement of Low Speed Gas Flows by Particle Trajectories: A New Determination of Free Convection Velocity Profiles," International Journal of Heat and Mass Transfer 5, 915, Oct. 1962.

(8) D. G. Collis and M. J. Williams, "Two Dimensional Convection from Heated Wires at Low Reynolds Numbers," Journal of Fluid Mechanics, 6, 1959, 359.

(9) F. J. Bayley, "An Analysis of Turbulent Free-Convection Heat Transfer," Proceedings of the Institute of Mechanical Engineers, 169, (20), 1955, 361.

(10) R. Cheesewright, "Natural Convection from a Plane Vertical Surface in Non-Isothermal Surroundings," to be published in Int. Journ. of Heat and Mass Trans.

(11) A. A. Szewczyk, "Stability and Transition of the Free Convection Boundary Layer Along a Flat Plate," Int. Journ. of Heat and Mass Trans., 5, 1962, 903.

(12) E. R. G. Eckert and A. J. Diagula, "Experimental Investigation of Free Convection Heat Transfer in a Vertical Tube at Large Grashof Numbers," NACA TR 1211, 1955.

(13) R. J. Goldstien, "Interferometric Study of Steady and Transient Free Convection Thermal Boundary Layers in Air and Water," Univ. of Minnesota, Ph.D. Thesis, 1959.

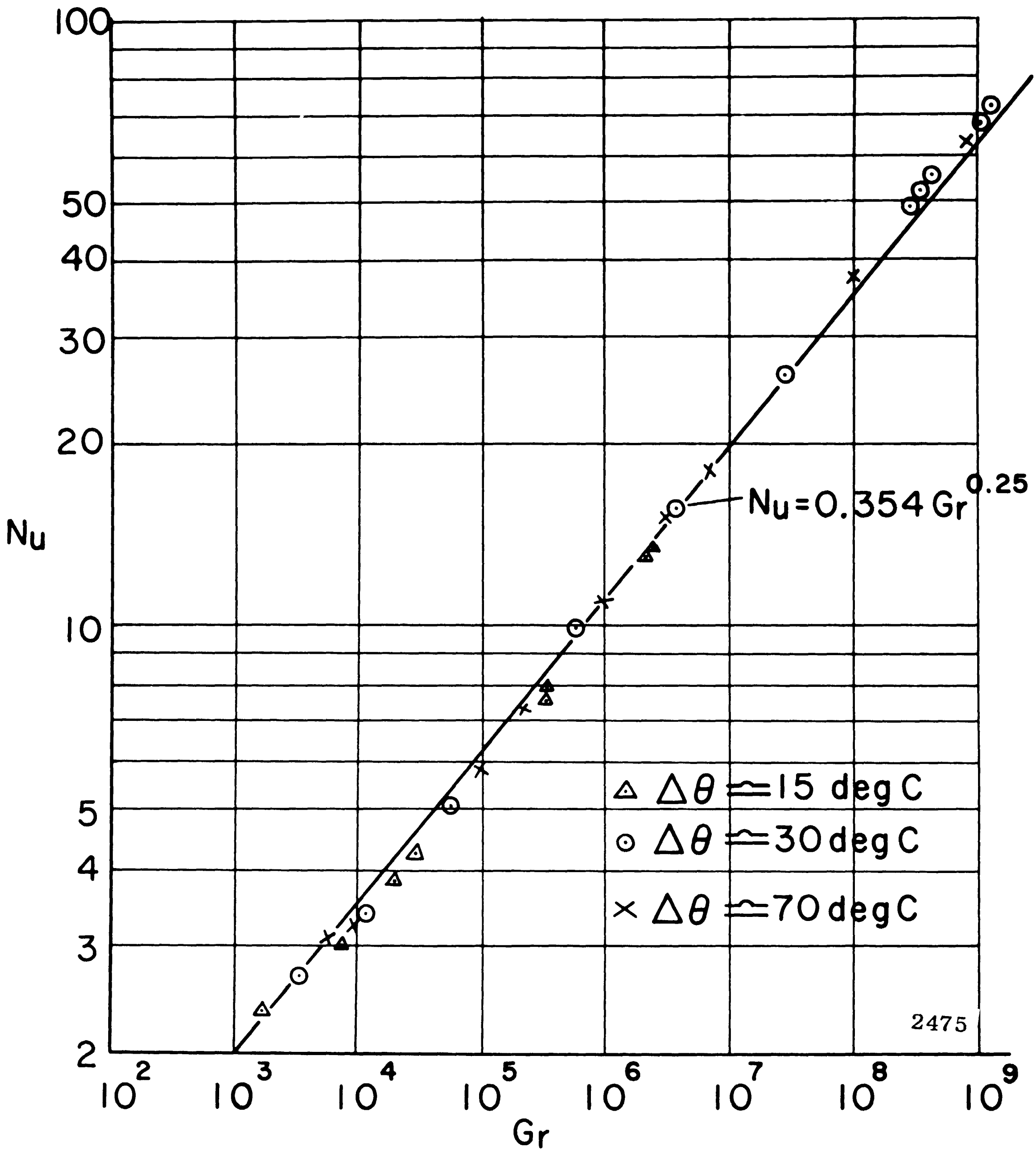

Figure 1 Local Heat Transfer Rates in the Laminar Boundary Layer

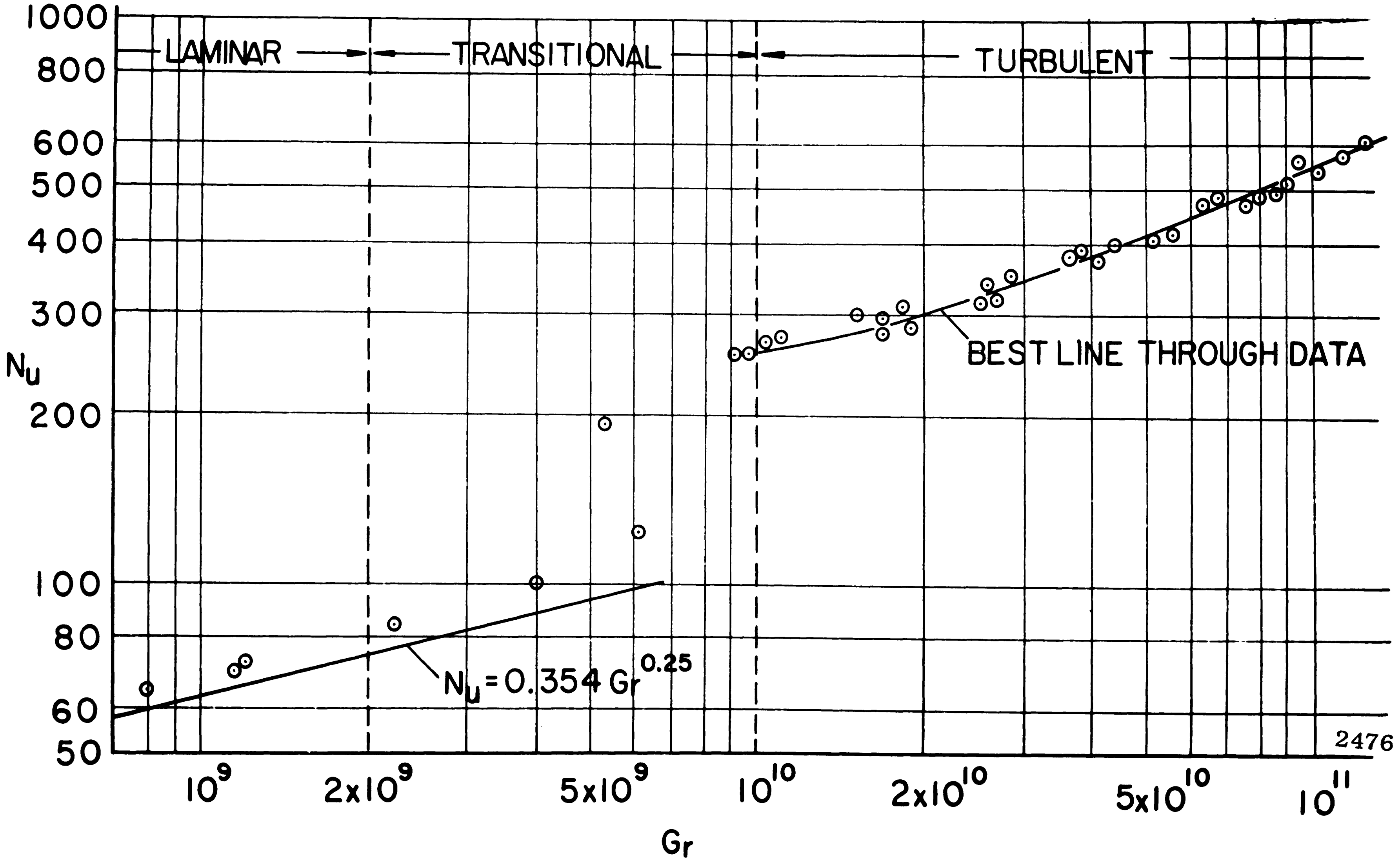

Figure 2 Local Heat Transfer Rates in the Transitional and Turbulent Boundary Layers

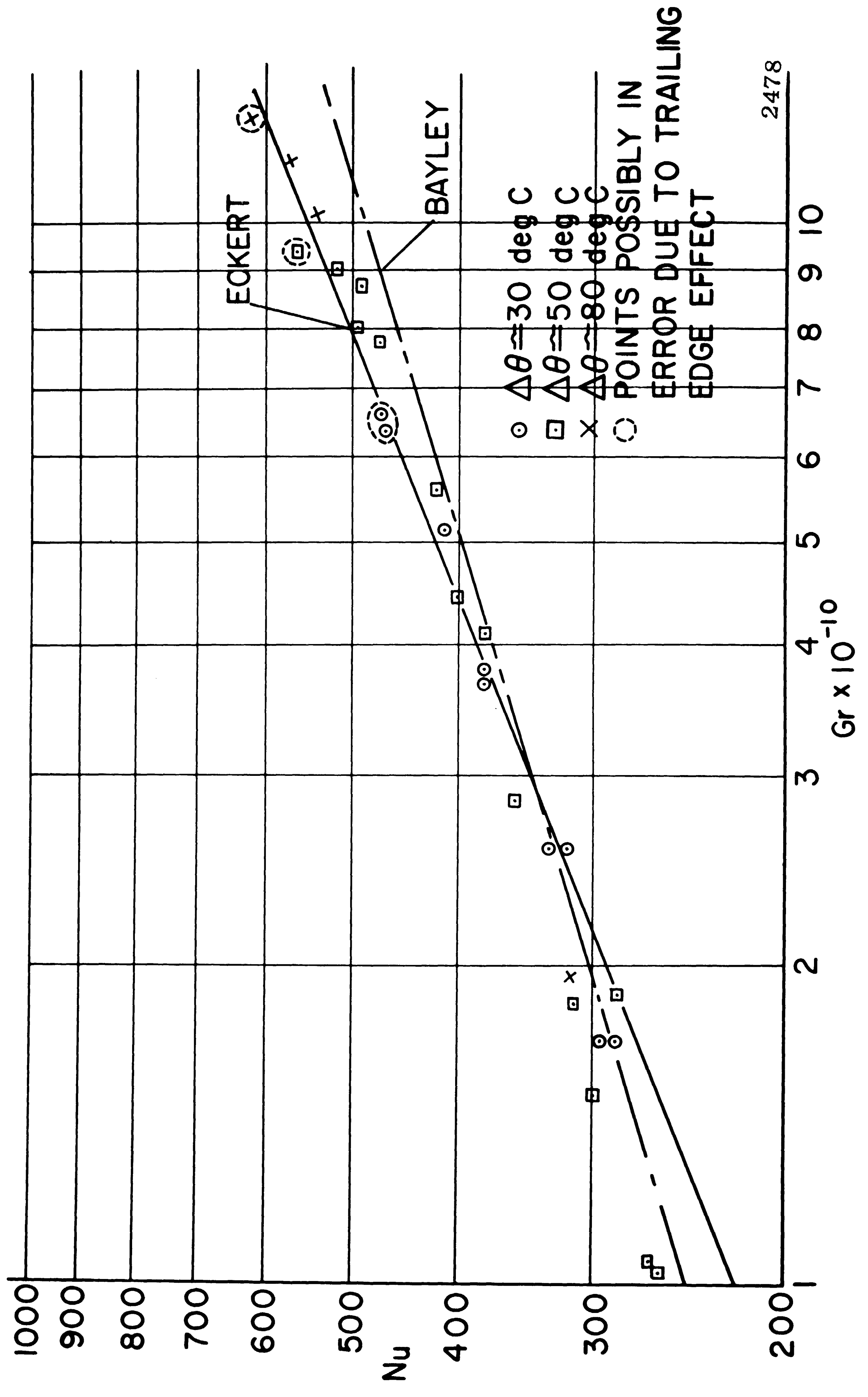

Figure 3 Local Heat Transfer Rates in the Turbulent Boundary Layer--Comparison with Theory

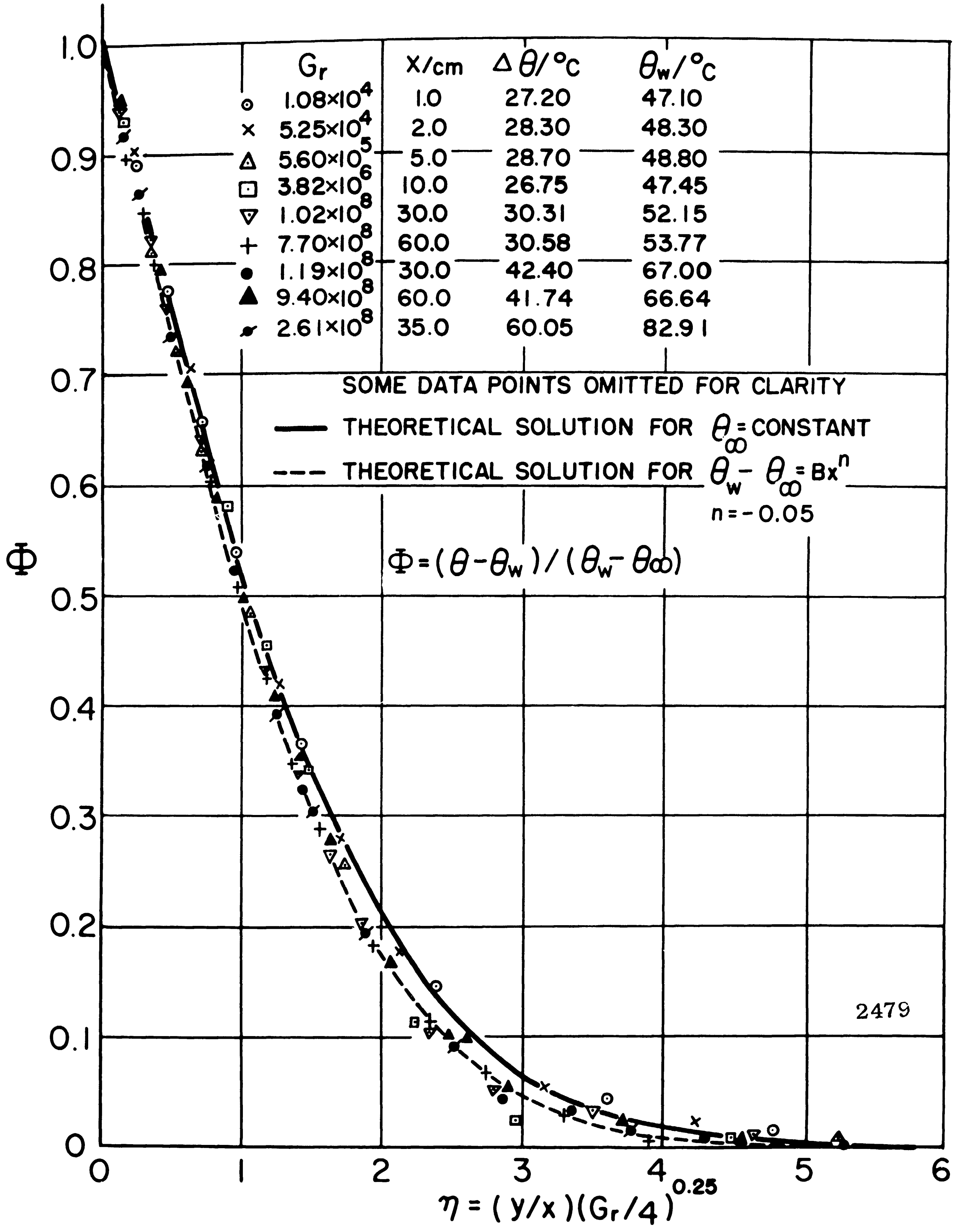

Figure 4 Temperature Profiles in the Laminar Boundary Layer

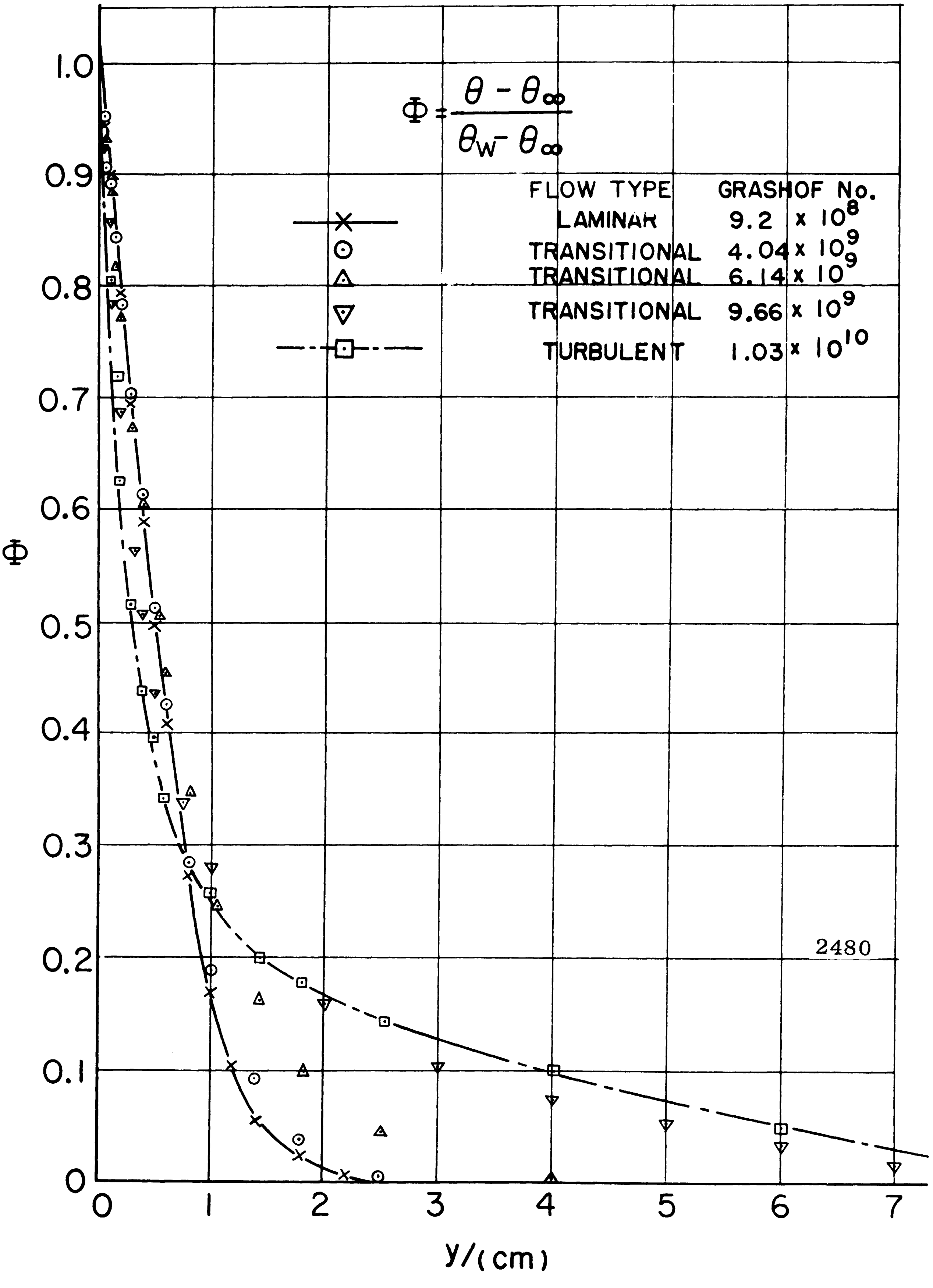

Figure 5 Profiles of Mean Temperature in the Transitional Boundary Layer

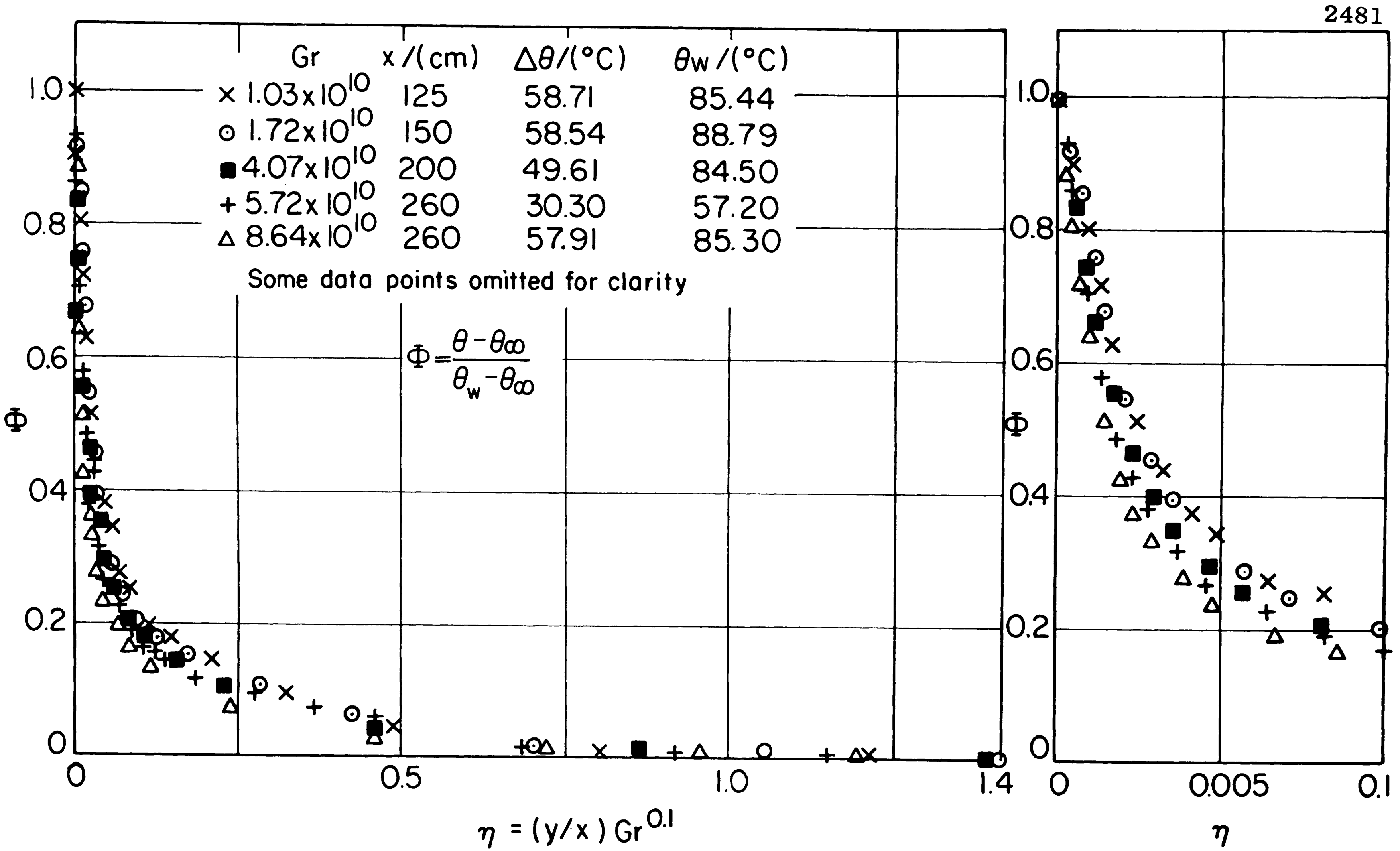

Figure 6 Profiles of Mean Temperature in the Turbulent Boundary Layer--Present Data

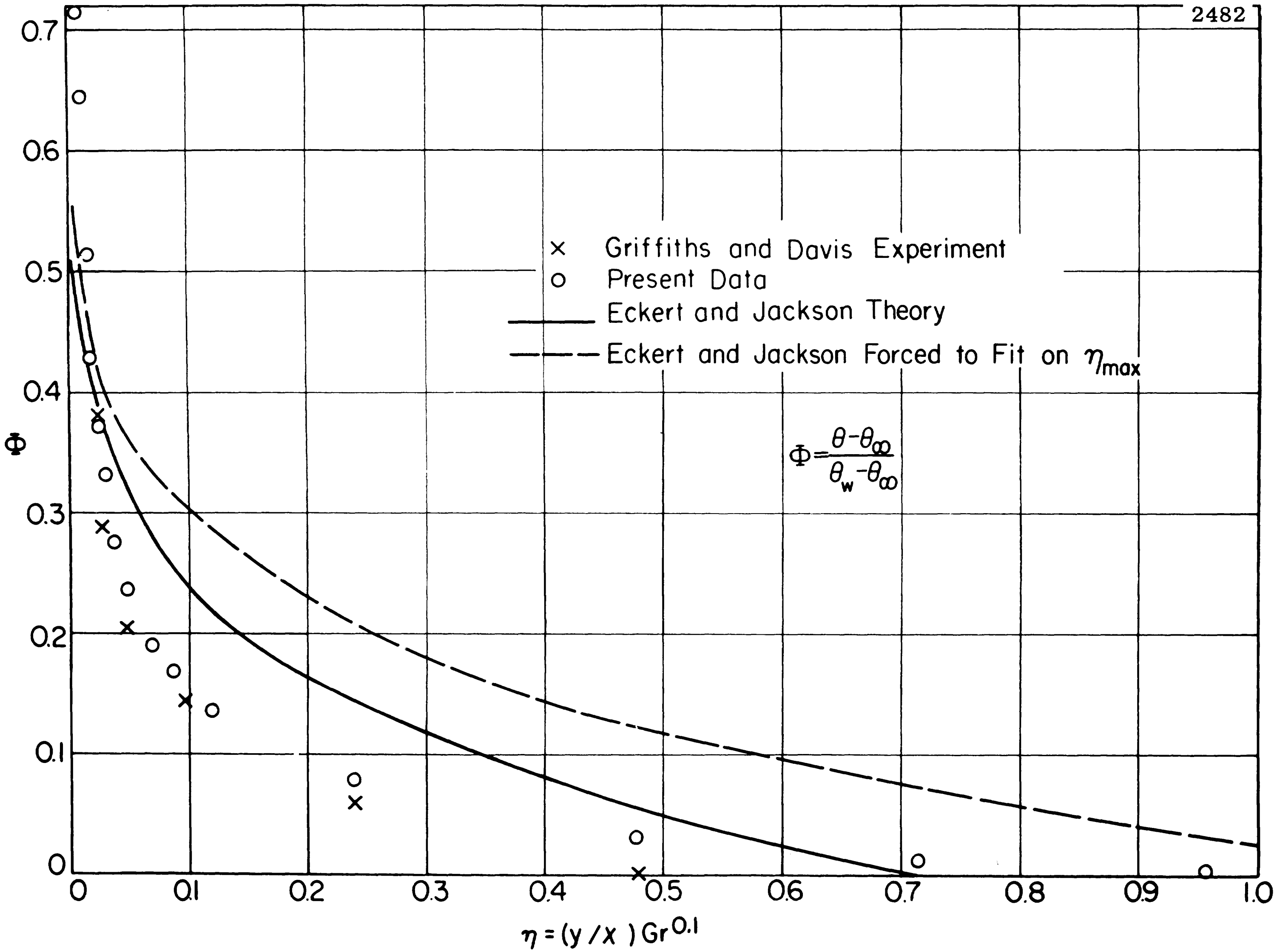

Figure 7 Profiles of Mean Temperature in the Turbulent Boundary Layer--Comparison with other Data

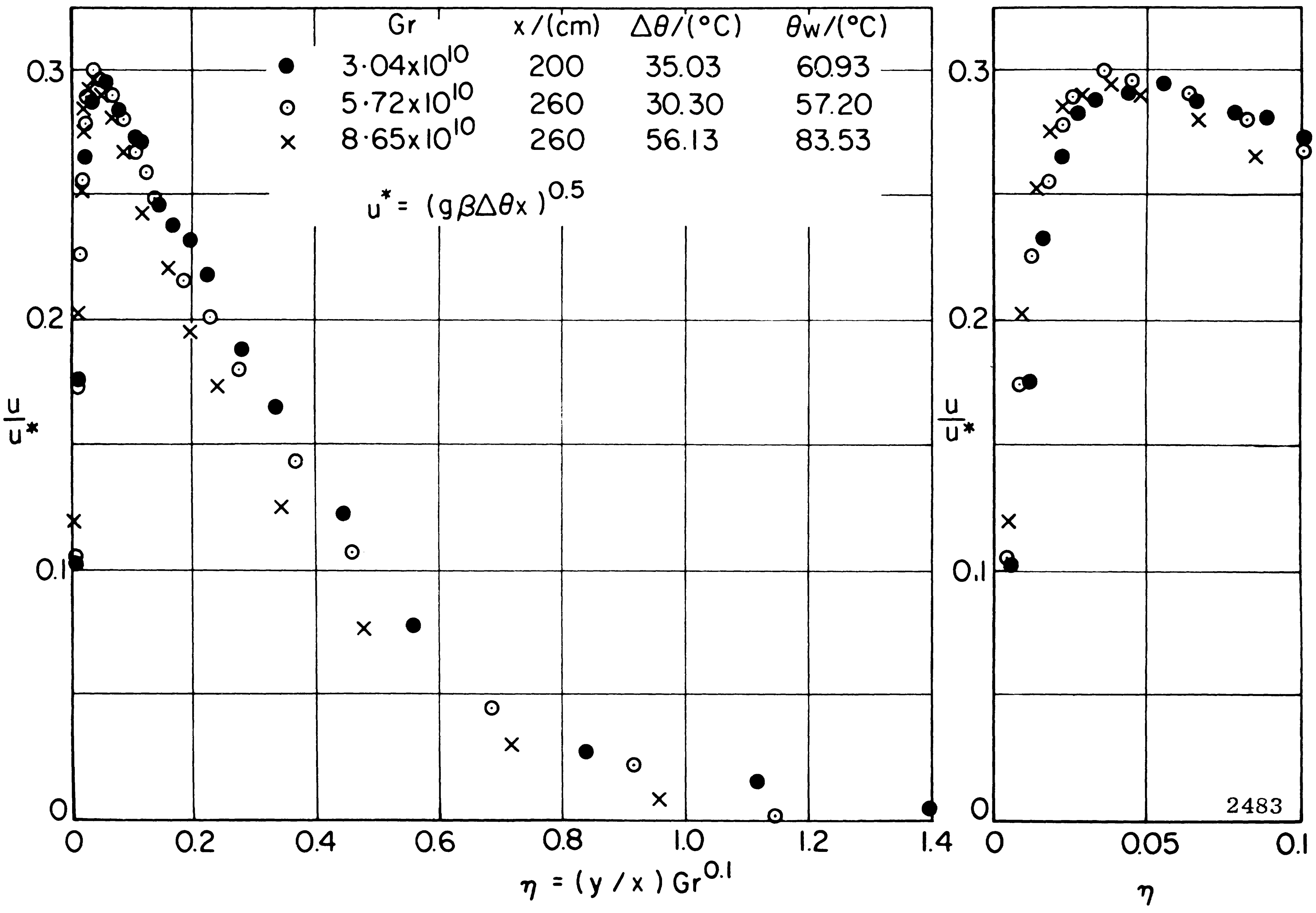

Figure 8 Profiles of Mean Velocity in the Turbulent Boundary Layer--Present Data

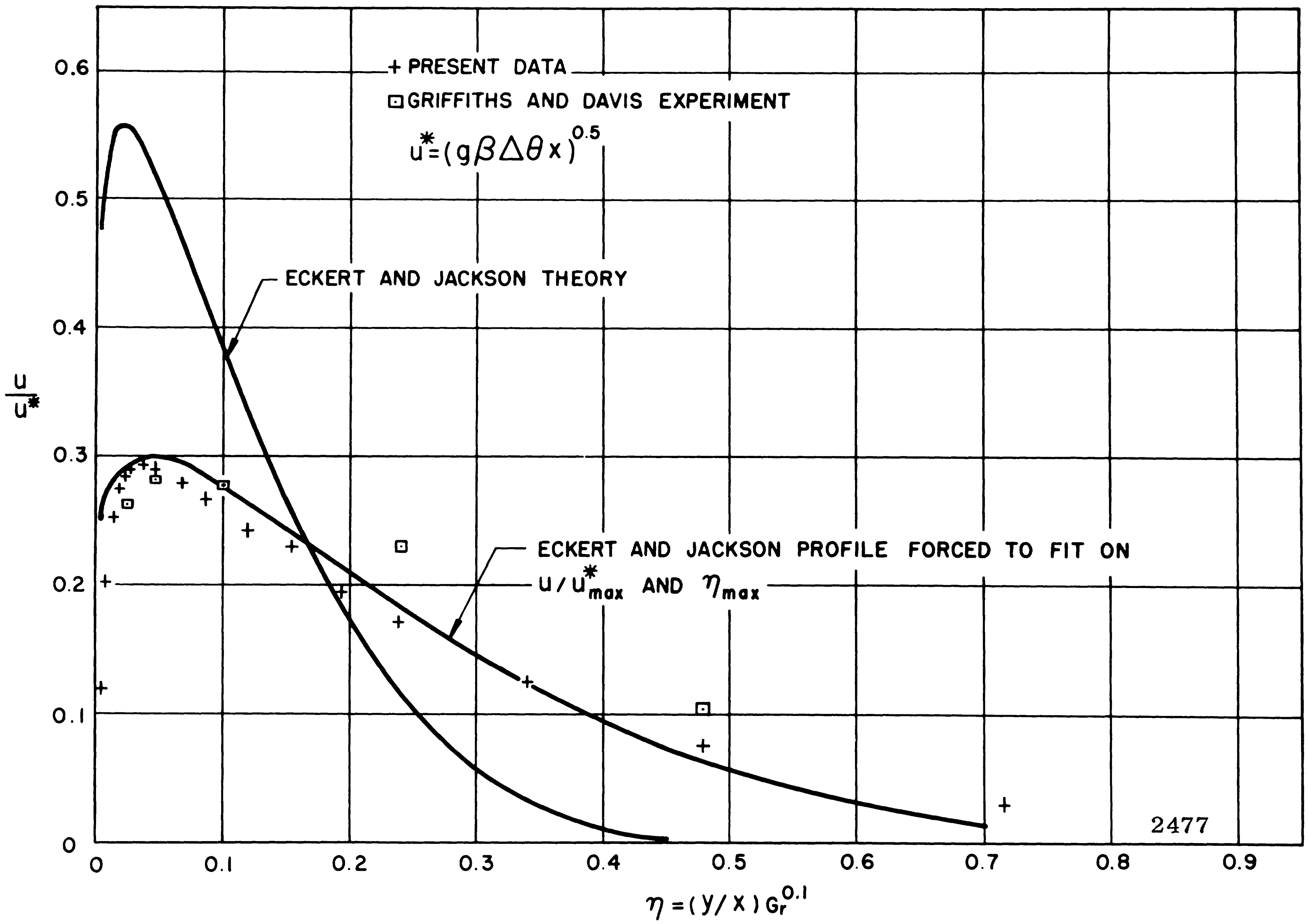

Figure 9 Profiles of Mean Velocity in the Turbulent Boundary Layer--Comparison with other Data