4563-39-T

INTERACTION OF A RADIATING SOURCE WITH A PLASMA

EFFECT OF AN ELECTROACOUSTIC WAVE

KUN-MU CHEN

July 1963

Radar Laboratory
Institute of Science and Technology
THE UNIVERSITY OF MICHIGAN
Ann Arbor, Michigan

NOTICES

Sponsorship. The work reported herein was conducted by the Institute of Science and Technology for the U. S. Air Force, under Contract AF 33(616)-8365. Contracts and grants to The University of Michigan for the support of sponsored research by the Institute of Science and Technology are administered through the Office of the Vice-President for Research.

Acknowledgments. The author is grateful to Mr. Ralph E. Hiatt, Professor C-M Chu, and Dr. R. F. Goodrich for reading the manuscript.

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ABSTRACT

When a radiating source is immersed in a homogeneous plasma of infinite extent, an electroacoustic wave may be excited in addition to the usual electromagnetic wave. The electroacoustic wave becomes a longitudinal plasma wave in the far zone of the source.

The case of a Hertzian dipole in a lossless plasma is considered first. The fields of both the EM and plasma modes excited by the dipole are explicitly obtained, and the EM and plasma components of the radiation resistance of a Hertzian dipole are determined. The case of a cylindrical dipole antenna in a lossless plasma is investigated next. The far zone fields of both the EM and plasma modes are explicitly obtained, and the EM and plasma components of the radiation resistance are then derived as functions of the antenna dimension and the plasma parameters. Finally, the case of a Hertzian dipole in a slightly lossy plasma is studied; the effect of collisions in a plasma on the radiation resistance of the immersed antenna is discussed.

1
INTRODUCTION

The interaction of an antenna with a plasma is one of the most interesting and important topics in physics and engineering, since communication between a satellite and the ground involves the ionosphere. The performance of an antenna in an ionized gas or in a plasma is entirely different from the performance in the case of free space. The behavior of an antenna in a plasma medium has been widely investigated, but in most cases a plasma has been treated as a dissipative medium with $\epsilon = \epsilon_0\left(1 - \frac{\omega_p^2}{\omega^2 + \nu^2}\right)$, $\mu = \mu_0$, and $\sigma = \frac{n_0 e^2}{m_e}\left(\frac{\nu}{\omega^2 + \nu^2}\right)$ where ω_p, ω, and ν are the plasma frequency, the antenna frequency, and the plasma collision frequency, respectively, and n_0 is the ambient electron density. This treatment has completely ignored a plasma wave or an electroacoustic wave which may be excited by an antenna in a plasma.

Cohen has shown in a series papers [1, 2, 3] that an electric source immersed in a homogeneous plasma of infinite extent can excite an electroacoustic wave in addition to the usual electromagnetic wave. In the absence of a static electric or magnetic field, Cohen [1] and Field [4] have shown that the electroacoustic wave and the electromagnetic wave are uncoupled and can be separated into two independent modes. The electromagnetic wave becomes an ordinary transverse electromagnetic wave, and the electroacoustic wave becomes a longitudinal plasma wave at great distances from the source. Furthermore, Whale [5] has observed the existence of an electroacoustic wave in a rocket flight in the ionosphere. These studies tend to predict that the radiation resistance of an antenna can be seriously affected by this electroacoustic wave.

The purpose of the present study is to investigate specifically the interaction of a plasma with a dipole antenna and the effect of an electroacoustic wave on the radiation of a dipole antenna. We shall concentrate on tracing the effects of an electroacoustic wave on the input resistance and the fields of a dipole antenna in a plasma. The case of a Hertzian dipole in a lossless plasma is considered first. The fields of both the EM and the plasma mode excited by the dipole are explicitly obtained. The radiation resistance of a Hertzian dipole is determined as the sum of the EM and plasma components, where the former is due to the excitation of an EM wave and the latter is due to the excitation of an electroacoustic wave. The case of a cylindrical dipole antenna in a lossless plasma is investigated next. The far zone fields of both the EM and plasma modes are explicitly obtained, and the EM and plasma components of the radiation resistance are derived as functions of the antenna dimension and the plasma parameters. Finally, the case of a Hertzian dipole in a slightly lossy plasma is studied; the effect of collisions in a plasma on the radiation resistance of the immersed antenna is discussed.

In this study MKS rationalized units are used and the time variation for the radiating source is assumed to be $e^{j\omega t}$. The plasma involved is assumed to be a weakly ionized gas type.

2
BASIC EQUATIONS

We assume that a radiating source with current, $\vec{J}^s$, and charge, ρ^s, is immersed in a homogeneous plasma of infinite extent. No static electric or magnetic field is present. The current and charge of the radiating source are related by the equation of continuity as

$$\nabla \cdot \vec{J}^s + \frac{\partial \rho^s}{\partial t} = 0 \tag{1}$$

The plasma is assumed to be a weakly ionized gas having an ambient electron density, n_0. The collision frequency of electrons with neutral particles of the gas is ν.

In its unperturbed state the plasma is assumed to be homogeneous and neutral and the perturbation of the plasma due to the source is assumed to be small, so that the linearized equations are applicable. This assumption may be poor in the very vicinity of the source, where the field can be very strong and the perturbation in the plasma may not be small.

The basic equations which govern this system are Maxwell's equations,

$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \tag{2}$$

$$\nabla \times \vec{H} = \vec{J}^s - en_0\vec{V} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \tag{3}$$

$$\nabla \cdot \vec{E} = \rho^s/\epsilon_0 - en_1/\epsilon_0 \tag{4}$$

$$\nabla \cdot \vec{H} = 0 \tag{5}$$

and the linearized Euler equations,

$$n_0(\nabla \cdot \vec{V}) + \frac{\partial n_1}{\partial t} = 0 \tag{6}$$

$$n_0 m_e \left(\frac{\partial \vec{V}}{\partial t}\right) + n_0 m_e \nu \vec{V} = -n_0 e\vec{E} - m_e v_0^2 \nabla n_1 \tag{7}$$

In the above notation, n_1 is the deviation of the electron density from its ambient density, n_0. m_e and e are the mass and charge of an electron. $\vec{V}$ is the mean induced velocity of electrons. v_0 is the rms velocity of electrons and can be expressed as

$$v_0 = \sqrt{\frac{3kT}{m_e}} \tag{8}$$

where k is the Boltzmann constant and T is the electron temperature.

The power relation, which can be governed by Poynting's theorem as shown by Field [4], is

$$\nabla \cdot \left(\vec{E} \times \vec{H} + m_e v_0^2 n_1 \vec{V}\right) + \frac{\partial}{\partial t}\left[\frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\mu_0 H^2 + \frac{1}{2} n_0 m_e V^2 + \frac{1}{2} n_0 m_e v_0^2 (n_1/n_0)^2\right] = -\vec{E} \cdot \vec{J}^s \tag{9}$$

If we assume that the current and the charge of the radiating source, $\vec{J}^s$ and ρ^s, vary with time as $e^{j\omega t}$, the basic equations become

$$\nabla \times \vec{E} = -j\omega\mu_0\vec{H} \tag{10}$$

$$\nabla \times \vec{H} = \vec{J}^s - en_0\vec{V} + j\omega\epsilon_0\vec{E} \tag{11}$$

$$\nabla \cdot \vec{E} = \rho^s/\epsilon_0 - en_1/\epsilon_0 \tag{12}$$

$$\nabla \cdot \vec{H} = 0 \tag{13}$$

and

$$n_0(\nabla \cdot \vec{V}) + j\omega n_1 = 0 \tag{14}$$

$$(j\omega + \nu)\vec{V} = -e/m_e\vec{E} - v_0{}^2/n_0\nabla n_1 \tag{15}$$

3
SEPARATION OF THE FIELD INTO EM AND P MODES

In our formulation of the problem, there are four unknowns, $\vec{E}$, $\vec{H}$, $\vec{V}$, and n_1, and two given quantities, $\vec{J}^s$ and ρ^s. In the absence of a static electric or a static magnetic field and under the constant temperature, these fields can be separated into two groups when the perturbation in a plasma due to the electric source is small. The first group consists of three vector fields, $\vec{E}_e$, $\vec{H}$, and $\vec{V}_e$, and is called the electromagnetic (EM) mode. The second group consists of two vector fields, $\vec{E}_p$, $\vec{V}_p$, and a scalar field, n_1, and is called the plasma (P) mode. The EM mode is the usual electromagnetic mode with an electric and a magnetic field but no charge accumulation; this mode gives a transverse electromagnetic wave in the far zone of the source. The P mode has the charge accumulation and an electric field but no magnetic field; this mode gives a longitudinal plasma wave at great distances from the source. Physically, the separations of the fields into two modes means that the electromagnetic wave and the plasma wave excited by the radiating source are uncoupled.

After the separation of the field the following relations hold:

$$\vec{E} = \vec{E}_e + \vec{E}_p \tag{16}$$

$$\vec{V} = \vec{V}_e + \vec{V}_p \tag{17}$$

$$\nabla \times \vec{E}_P = 0 \tag{18}$$

Equation 18 holds because there is no magnetic field in the P mode.

The substitution of Equations 16 to 18 in the basic Equations 10 to 15 gives the following two groups of equations.

<u>For the EM mode ($\vec{E}_e$, $\vec{H}$, $\vec{V}_e$):</u>

$$\nabla \times \vec{E}_e = -j\omega\mu_o\vec{H} \tag{19}$$

$$\nabla \times \vec{H} = \vec{J}^s - en_0\vec{V}_e + j\omega\,{}_0\vec{E}_e \tag{20}$$

$$\nabla \cdot \vec{H} = 0 \tag{21}$$

$$(j\omega + \nu)\vec{V}_e = -\frac{e}{m_e}\vec{E}_e \tag{22}$$

One more equation for $\nabla \cdot \vec{E}_e$ can be obtained by using Equations 20, 22, and 1. The result is

$$\nabla \cdot \vec{E}_e = \rho^s/\epsilon_0\xi \tag{23}$$

where

$$\xi = \left(1 - \frac{\omega_p^2}{\omega^2 + \nu^2}\right) - j \frac{\omega_p^2 \nu}{\omega(\omega^2 + \nu^2)} \tag{24}$$

$$\omega_p^2 = \frac{n_0 e^2}{\epsilon_0 m_e} \tag{25}$$

<u>For the P mode ($\vec{E}_p$, $\vec{V}_p$, n_1):</u>

$$\nabla \times E_p = 0 \tag{26}$$

$$j\omega\epsilon_0 \vec{E}_p - en_0 \vec{V}_p = 0 \tag{27}$$

$$(j\omega + \nu)\vec{V}_p = -\frac{e}{m_e} \vec{E}_p - \frac{v_0^2}{n_0} \nabla n_1 \tag{28}$$

These vector equations may be solved for some particular cases, but a more general approach is to find some potential functions which are derived from those equations.

4
POTENTIAL EQUATIONS FOR EM AND P MODES

For the EM mode a vector potential can be derived conventionally. For the P mode, n_1 is found to act as a scalar potential.

<u>For the EM mode:</u>

Let us assume that

$$\mu_0 \vec{H} = \nabla \times \vec{A} \tag{29}$$

$$\vec{E}_e = -\nabla\phi - j\omega\vec{A} \tag{30}$$

An equation for the vector potential, $\vec{A}$, can be obtained from Equation 20 as

$$(\nabla^2 + \beta_e^2)\,\vec{A} = -\mu_0 \vec{J}^s \tag{31}$$

where

$$\beta_e^2 = \omega^2 \epsilon_0 \mu_0 \xi \tag{32}$$

and Equation 31 is subject to a condition of

$$\nabla \cdot \vec{A} + j\omega\epsilon_0\mu_0\xi\phi = 0 \tag{33}$$

ϕ is the conventional scalar potential of the EM field.

Thus from Equation 31, $\vec{A}$ can be determined if $\vec{J}^s$ is given. After $\vec{A}$ is obtained, all fields, $\vec{E}_e$, $\vec{H}$, and $\vec{V}_e$, can be completely determined.

Physically, this mode is an EM wave set up in a dissipative medium with a complex dielectric constant, $\epsilon = \epsilon_0\xi$. This EM wave propagates with a complex propagation constant of $\omega\sqrt{\xi}/c_0$ in the medium.

For the P mode:

From Equations 27 and 28 we have

$$E_p = \frac{ev_0^2}{\left(\omega^2 - \omega_p^2 - j\omega\nu\right)\epsilon_0}\nabla n_1 \tag{34}$$

The substitution of Equations 23 and 34 in Equation 12 leads to

$$\left(\nabla^2 + \beta_p^2\right)n_1 = -\frac{\omega_p^2}{ev_0^2}\rho^s \tag{35}$$

where

$$\beta_p^2 = \left(\omega^2 - \omega_p^2\right)\Big/v_0^2 - j\omega\nu\Big/v_0^2 \tag{36}$$

Thus, if the charge source, ρ^s, is given, n_1 can be obtained by solving Equation 35. After n_1 is determined, $\vec{E}_p$ and $\vec{V}_p$ can be obtained from Equations 34 and 27. It is noted at this point that the deviation of the electron density from its ambient density, n_1, in this case acts as a scalar potential for $\vec{E}_p$.

Physically, n_1 behaves like an acoustic wave and propagates roughly with the rms velocity of electrons, v_0. We call this an electroacoustic wave because it is excited by the electric charge, ρ^s.

Up to this point, we are theoretically able to solve for all fields which are excited by a radiating source ($\vec{J}^s$, ρ^s) in a plasma medium. It is interesting to observe that the EM wave is excited by both $\vec{J}^s$ and ρ^s, but the plasma wave or the electroacoustic wave is excited solely by ρ^s. The electroacoustic wave affects the radiation property of an antenna to a great extent, as we shall see in the following sections.

5 RADIATION RESISTANCE AND FIELDS OF A HERTZIAN DIPOLE IN A LOSSLESS PLASMA

The first and simplest case to be investigated is a Hertzian dipole in a lossless plasma. We shall attempt to find the EM and plasma waves which are excited by a Hertzian dipole in a lossless plasma, and, after the fields are determined, to obtain the dipole's radiation resistance.

A Hertzian dipole is an oscillating electric dipole with a uniform current in a short wire, $d\ell$, and the concentration of charges at the two ends, as shown in Figure 1. The current and charge of a Hertzian dipole can be symbolically represented as

$$\vec{J}^s = I\,d\ell\,\delta(x)\delta(y)\delta(z)\,e^{j\omega t}\,\hat{z} \tag{37}$$

$$\rho^s = -\frac{j}{\omega} I\left[\delta\left(z - \frac{1}{2}d\ell\right) - \delta\left(z + \frac{1}{2}d\ell\right)\right]\delta(x)\delta(y)\,e^{j\omega t} \tag{38}$$

where δ is the delta function.

The EM and plasma waves excited by $\vec{J}^s$ and ρ^s are analyzed separately below.

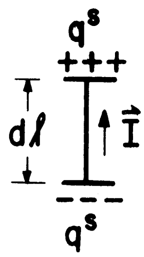

FIGURE 1. HERTZIAN DIPOLE

5.1. THE EM MODE ($\vec{E}_e$, $\vec{H}$, $\vec{V}_e$)

We assume that the plasma is lossless and $\nu = 0$. With $\vec{J}^s$ as expressed in Equation 37, the vector potential $\vec{A}$ can be found from Equation 31 to be

$$\vec{A} = \frac{\mu_0}{4\pi}\int \frac{e^{-j\beta_e r}}{r}\vec{J}^s\,dv$$

$$= \frac{\mu_0}{4\pi} I d\ell\,\frac{e^{-j\beta_e r}}{r}\,e^{j\omega t}\,\hat{z} \tag{39}$$

where $r = \sqrt{x^2 + y^2 + z^2}$ = the distance between the observation point and the source point. In spherical coordinates, $\vec{A}$ can be expressed as

$$\vec{A} = \frac{\mu_0}{4\pi} I d\ell \frac{e^{j(\omega t - \beta_e r)}}{r} (\cos\theta\, \hat{r} - \sin\theta\, \hat{\theta}) \tag{40}$$

The scalar potential, ϕ, can be determined from Equations 34 and 40 to be

$$\phi = \frac{j}{\omega \epsilon_0 \mu_0 \xi} \nabla \cdot \vec{A}$$

$$= \frac{-jId\ell}{4\pi\epsilon_0 \xi \omega} e^{j(\omega t - \beta_e r)} \left(j\frac{\beta_e}{r} + \frac{1}{r^2} \right) \cos\theta \tag{41}$$

From Equations 30, 40, and 41, the electric field $\vec{E}_e$ can be determined as

$$\vec{E}_e = -\nabla\phi - j\omega\vec{A}$$

$$= \frac{-jId\ell}{2\pi\epsilon_0 \xi \omega} \left(\frac{1}{r^3} + j\frac{\beta_e}{r^2} \right) \cos\theta\, e^{j(\omega t - \beta_e r)} \hat{r}$$

$$+ \frac{-jId\ell}{4\pi\epsilon_0 \xi \omega} \left(\frac{1}{r^3} + j\frac{\beta_e}{r^2} - \frac{\beta_e^2}{r} \right) \sin\theta\, e^{j(\omega t - \beta_e r)} \hat{\theta} \tag{42}$$

The magnetic field can be obtained from Equations 29 and 40 as

$$\vec{H} = \frac{1}{\mu_0} \nabla \times \vec{A}$$

$$= \frac{-jId\ell}{4\pi} \left(j\frac{1}{r^2} - \frac{\beta_e}{r} \right) \sin\theta\, e^{j(\omega t - \beta_e r)} \hat{\phi} \tag{43}$$

Equations 42 and 43 give the complete expressions for the electric and magnetic fields of the EM mode, which is excited by a Hertzian dipole in a plasma. $\vec{E}_e$ has a radiation term $(1/r)$, an induction term $(1/r^2)$, and an electrostatic term $(1/r^3)$. $\vec{H}$ has a radiation and an induction term. Both $\vec{E}_e$ and $\vec{H}$ fields are exponentially attenuating when β_e has an imaginary component, if the plasma is assumed to be lossy. In this section β_e is assumed to be a real number.

To determine the EM component of radiation resistance of a Hertzian dipole, we use a Poynting vector method. This method, without modification, fails if β_e is assumed to be a complex number. (This is discussed in Section 7.)

The radiation terms of $\vec{E}_e$ and $\vec{H}$ are

$$\vec{E}_e^{\,r} = \frac{jId\ell}{4\pi\epsilon_0 \xi \omega} \frac{\beta_e^2}{r} \sin\theta\, e^{j(\omega t - \beta_e r)} \hat{\theta} \tag{44}$$

$$\vec{H}^r = \frac{jId\ell}{4\pi} \frac{\beta_e}{r} \sin\theta \, e^{j(\omega t - \beta_e r)} \hat{\phi} \tag{45}$$

It is assumed above that

$$\beta_e^2 = \omega^2 \epsilon_0 \mu_0 \left(1 - \frac{\omega_p^2}{\omega^2}\right) \quad \text{and} \quad \xi = 1 - \frac{\omega_p^2}{\omega^2} \tag{46}$$

as the result of $\nu = 0$.

The Poynting vector of the EM wave is

$$\vec{p}_e = \frac{1}{2}(\vec{E}_e^{\,r} \times \vec{H}^{r*})$$

$$= \frac{I^2 d\ell^2}{32\pi^2 \epsilon_0 \xi \omega} \frac{\beta_e^3}{r^2} \sin^2\theta \, \hat{r} \tag{47}$$

The total power radiated as an EM wave in a lossless plasma by a Hertzian dipole is obtained by integrating the Poynting vector over a large sphere. That is,

$$P_e = \int_0^{2\pi} d\phi \int_0^{\pi} d\theta r^2 \sin\theta \, \vec{p}_e \cdot \hat{r}$$

$$= \frac{I^2 d\ell^2}{12\pi\epsilon_0 \xi \omega} \beta_e^3$$

$$= 40\pi^2 \left(\frac{d\ell}{\lambda_0}\right)^2 \sqrt{1 - \omega_p^2/\omega^2} \, I^2 \tag{48}$$

The corresponding radiation resistance can be obtained by dividing P_e by $1/2\ I^2$, which gives

$$R_e = 80\pi^2 \left(\frac{d\ell}{\lambda_0}\right)^2 \sqrt{1 - \omega_p^2/\omega^2} \text{ ohms} \tag{49}$$

where λ_0 is the wavelength in free space. Equation 49 gives the expression for the EM component of the radiation resistance of a Hertzian dipole in a lossless plasma. In free space or when $\omega_p = 0$, Equation 49 reduces to a well known value for the radiation resistance of a Hertzian dipole. The behavior of R_e as a function of ω_p^2/ω^2 is shown graphically in Figure 2.

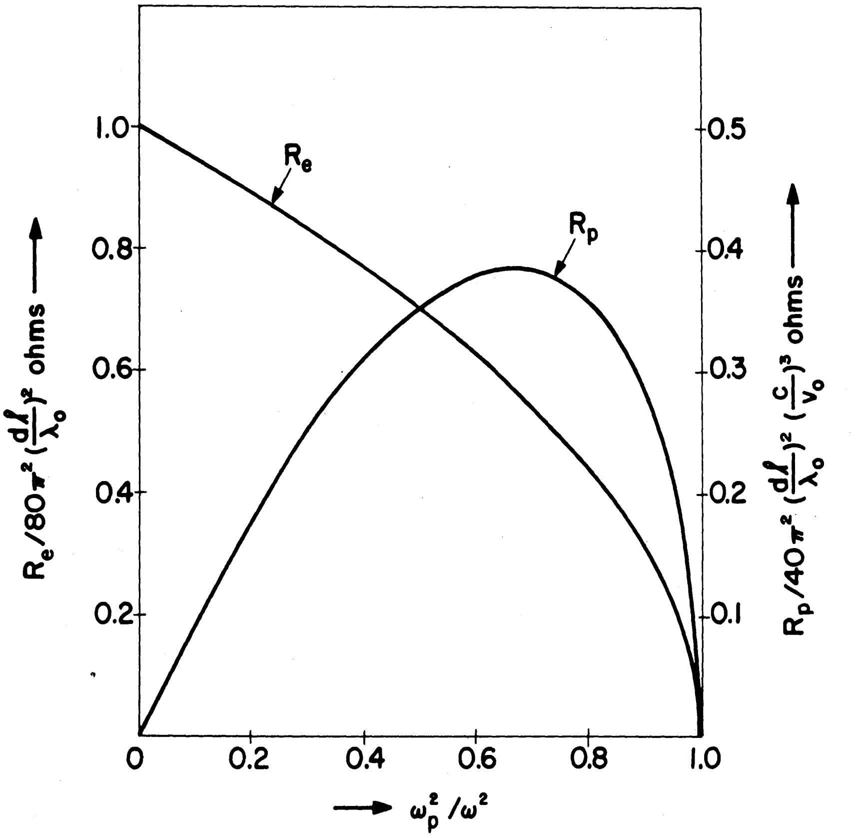

FIGURE 2. RADIATION RESISTANCE OF A HERTZIAN DIPOLE IN A PLASMA. R_e = EM component of radiation resistance; R_p = plasma component of radiation resistance.

5.2. THE PLASMA MODE ($\vec{E}_p$, $\vec{V}_p$, n_1)

If the charge source, ρ^s, is expressed as in Equation 38, the deviation of the electron density from its ambient density, n_1, can be determined as

$$n_1 = \frac{1}{4\pi}\frac{\omega_p^2}{ev_0^2}\int \frac{e^{-j\beta_p r}}{r}\rho^s dv$$

$$= \frac{-jId\ell}{4\pi\omega}\frac{\omega_p^2}{ev_0^2}\left(\frac{1}{r^2}+j\frac{\beta_p}{r}\right)\cos\theta\, e^{j(\omega t-\beta_p r)} \tag{50}$$

It is noted that n_1 has been transformed from a rectangular coordinate form to a spherical coordinate form.

From Equations 34 and 50 the electric field fo the plasma mode, $\vec{E}_p$, is evaluated as

$$\vec{E}_p = \frac{j\omega_p^2 I d\ell}{4\pi\epsilon_0\omega\left(\omega^2 - \omega_p^2\right)}\left[\left(\frac{2}{r^3} + j\frac{2\beta_p}{r^2} - \frac{\beta_p^2}{r}\right)\cos\theta\,\hat{r} + \left(\frac{1}{r^3} + j\frac{\beta_p}{r^2}\right)\sin\theta\,\hat{\theta}\right] e^{j(\omega t - \beta_p r)} \tag{51}$$

Equation 51 shows that $\vec{E}_p$ has a radiation term (1/r) in the radial direction.

The induced velocity of electrons, $\vec{V}_p$, in a plasma is easily obtained from Equations 27 and 51 as

$$\vec{V}_p = \frac{-\omega_p^2 I d\ell}{4\pi e n_0\left(\omega^2 - \omega_p^2\right)}\left[\left(\frac{2}{r^3} + j\frac{2\beta_p}{r^2} - \frac{\beta_p^2}{r}\right)\cos\theta\,\hat{r} + \left(\frac{1}{r^3} + j\frac{\beta_p}{r^2}\right)\sin\theta\,\hat{\theta}\right] e^{j(\omega t - \beta_p r)} \tag{52}$$

$\vec{V}_p$ also has a radiation term in the radial direction.

From a Poynting theorem in a plasma, as expressed in Equation 9, it can be shown that there is a net flow of power from the source. The power density flowing out of the source as a form of plasma wave is

$$\vec{p}_p = m_e v_0^2\, n_1^r \vec{V}_p^r \tag{53}$$

where n_1^r and $\vec{V}_p^r$ are the radiation terms of n_1 and $\vec{V}_p$ fields. Since

$$n_1^r = \frac{Id\ell}{4\pi\omega}\frac{\omega_p^2}{ev_0^2}\frac{\beta_p}{r}\cos\theta\, e^{j(\omega t - \beta_p r)} \tag{54}$$

and

$$\vec{V}_p^r = \frac{\omega_p^2 I d\ell}{4\pi e n_0\left(\omega^2 - \omega_p^2\right)}\frac{\beta_p^2}{r}\cos\theta\, e^{j(\omega t - \beta_p r)}\,\hat{r} \tag{55}$$

The time average value of $\vec{p}_p$ can be found to be

$$(\vec{p}_p)\,\mathrm{av} = \frac{I^2 d\ell^2 \omega_p^2}{32\pi^2\omega\epsilon_0 v_0^3}\sqrt{\omega^2 - \omega_p^2}\,\frac{1}{r^2}\cos^2\theta\,\hat{r} \tag{56}$$

The total power radiated by a Hertzian dipole as a form of a plasma wave is then

$$P_p = \int_0^{2\pi} d\phi \int_0^{\pi} d\theta\, r^2 \sin\theta\, (\vec{p}_p)\,\mathrm{av}\cdot\hat{r}$$

$$= \frac{1}{24\pi} \frac{\omega_p^2}{\epsilon_0 v_0^3} I^2 d\ell^2 \sqrt{1 - \omega_p^2/\omega^2}$$

$$= 20\pi^2 \left(\frac{d\ell}{\lambda_0}\right)^2 \left(\frac{c_0}{v_0}\right)^3 \frac{\omega_p^2}{\omega^2} \sqrt{1 - \omega_p^2/\omega^2}\, I^2 \tag{57}$$

The corresponding radiation resistance is obtained by dividing P_p by $1/2\ I^2$. This gives

$$R_p = 40\pi^2 \left(\frac{d\ell}{\lambda_0}\right)^2 \left(\frac{c_0}{v_0}\right)^3 \frac{\omega_p^2}{\omega^2} \sqrt{1 - \omega_p^2/\omega^2} \text{ ohms} \tag{58}$$

where c_0 is the velocity of light and v_0 is the rms velocity of electrons in a plasma. Equation 58 gives the expression for the plasma component of the radiation resistance of a Hertzian dipole in a lossless plasma. For a typical case of ionosphere, R_p can be much greater than R_e if ω_p is not very small compared to ω. This may cause a serious change of the input impedance of a Hertzian dipole in the ionosphere. The behavior of R_p as a function of ω_p^2/ω^2 is shown graphically in Figure 2.

5.3. INPUT RESISTANCE OF A HERTZIAN DIPOLE IN A LOSSLESS PLASMA

The input resistance of a Hertzian dipole in a lossless plasma is the sum of the EM and plasma components of the radiation resistance of a Hertzian dipole. That is,

$$R_0 = R_e + R_p \tag{59}$$

The total input power to a Hertzian dipole is $1/2\ I^2 R_0$, out of which $1/2\ I^2 R_e$ is radiated as an electromagnetic wave and $1/2\ I^2 R_p$ is radiated as a plasma wave.

It is important to note that R_p appears as a part of input resistance of a Hertzian dipole only when an electroacoustic wave is excited by the dipole and propagates away from it. If a radiating source is surrounded by a plasma of limited size, an electroacoustic wave may be excited but a part of this wave may be reflected at the plasma boundary so that R_p in that case can be considerably smaller than the value expressed in Equation 58.

6
RADIATION RESISTANCE AND FIELDS OF A CYLINDRICAL DIPOLE ANTENNA IN A LOSSLESS PLASMA

The second case to be studied is that of a cylindrical dipole antenna in a lossless plasma. This type of antenna has a great practical importance and also can be analyzed quite accurately.

Our objective is to find the far zone fields set up by a cylindrical dipole antenna in a lossless plasma. The near zone fields are not considered in this study for the sake of simplicity. The radiation resistance of a cylindrical dipole antenna can be determined after the far zone fields are obtained.

A dipole antenna as shown in Figure 3 can be assumed to have the following current and charge distribution.

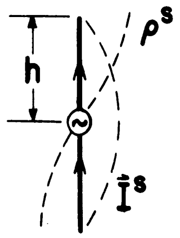

FIGURE 3. A CYLINDRICAL DIPOLE ANTENNA

$$\vec{I}^s = I_m \sin \beta_e(h - |z|)\, e^{j\omega t}\, \hat{z} \tag{60}$$

$$\rho^s = \pm j\sqrt{\epsilon_0 \mu_0 \xi}\; I_m \cos \beta_e(h - |z|)\, e^{j\omega t} \tag{61}$$

where $\beta_e = \omega\sqrt{\epsilon_0\mu_0\xi}$ and $\xi = \left(1 - \frac{\omega_p^2}{\omega^2}\right)$. In this section we also assume that $\nu = 0$. These distributions of current and charge along the antenna are approximate but are quite sufficient for our study (see Reference 6).

With the current and the charge distribution specified in Equations 60 and 61, the far zone fields for the EM mode and plasma mode can be obtained separately.

6.1. THE EM MODE (E_e, H, V_e)

The vector potential $\vec{A}$ maintained by the antenna current $\vec{I}^s$ is

$$\vec{A} = \frac{\mu_0}{4\pi} \int_{-h}^{h} I_m \sin \beta_e(h - |z|)\, e^{j\omega t} \frac{e^{-j\beta_e r}}{r}\, dz\, \hat{z} \tag{62}$$

where $r = |\vec{r}_0 - \vec{z}|$ is the distance between an observation point in space and a point on the antenna, and r_0 is the distance from the center of the antenna to the observation point in space.

To avoid further complication in analysis, we evaluate only the far zone field of $\vec{A}$. At the far zone of the cylindrical dipole antenna,

$$r = r_0 - z\cos\theta \tag{63}$$

and $\vec{A}$ is simplified to

$$\vec{A} = \frac{\mu_0 I_m e^{j(\omega t - \beta_e r_0)}}{4\pi r_0} \int_{-h}^{h} \sin\beta_e(h - |z|)\, e^{j\beta_e z\cos\theta}\, dz\, \hat{z}$$

$$= \frac{\mu_0 I_m e^{j(\omega t - \beta_e r_0)}}{2\pi\beta_e r_0} \left[\frac{\cos(\beta_e h\cos\theta) - \cos(\beta_e h)}{\sin^2\theta}\right]\hat{z} \tag{64}$$

If $\vec{A}$ is expressed in terms of spherical coordinates, A_θ can be represented as

$$A_\theta = -A_z \sin\theta \tag{65}$$

The electric field in the far zone of the antenna is directly related to A_θ as follows:

$$\vec{E}_e^{\,r} = -j\omega\vec{A}_\theta = j\omega\sin\theta\, A_z\,\hat{\theta} \tag{66}$$

where the superscript r stands for the radiation field and subscript e stands for the EM mode.

The far zone magnetic field can be obtained from Maxwell equations as

$$\vec{H}^r = \frac{\sqrt{\xi}}{\zeta_0} E^r\,\hat{\phi} \tag{67}$$

where $\zeta_0 = 120\pi$ ohms and $\sqrt{\xi} = \sqrt{1 - \frac{\omega_p^2}{\omega^2}}$.

The Poynting vector of the EM wave is

$$\vec{P}_e = \frac{1}{2}\left(\vec{E}_e^{\,r} \times \vec{H}^{r*}\right)$$

$$= \frac{\zeta_0 I_m^2}{8\pi^2\sqrt{\xi}}\,\frac{1}{r_0^2}\left[\frac{\cos(\beta_e h\cos\theta) - \cos(\beta_e h)}{\sin\theta}\right]^2 \hat{r} \tag{68}$$

The total power radiated as an EM wave in a lossless plasma is obtained by integrating the Poynting vector over a large sphere; that is,

$$P_e = \int_0^{2\pi} d\phi \int_0^{\pi} d\theta\, r_0^2 \sin\theta\, \vec{p}_e \cdot \vec{r}$$

$$= \frac{30}{\sqrt{\xi}} I_m^2 \int_0^{\pi} \frac{[\cos(\beta_e h\cos\theta) - \cos(\beta_e h)]^2}{\sin\theta}\, d\theta \tag{69}$$

The integral in Equation 69 has been evaluated [7], and the final form of P_e can be written as

$$P_e = \frac{15}{\sqrt{\xi}} I_m^2 \Big\{ -\cos(2\beta_e h)\,\mathrm{Cin}(4\beta_e h) + 2\,[1 + \cos(2\beta_e h)]\,\mathrm{Cin}(2\beta_e h)$$

$$+ \sin(2\beta_e h)\,[\mathrm{Si}(4\beta_e h) - 2\mathrm{Si}(2\beta_e h)]\Big\} \qquad (70)$$

The electromagnetic component of the radiation resistance of a cylindrical dipole antenna is obtained by dividing P_e by $1/2\ I_m^2$. This gives

$$R_e^m = \frac{30}{\sqrt{1 - \omega_p^2/\omega^2}} \Big\{ -\cos(2\beta_e h)\,\mathrm{Cin}(4\beta_e h) + 2\,[1 + \cos(2\beta_e h)]\,\mathrm{Cin}(2\beta_e h)$$

$$+ \sin(2\beta_e h)\,[\mathrm{Si}(4\beta_e h) - 2\mathrm{Si}(2\beta_e h)]\Big\} \qquad (71)$$

where $\beta_e = \omega\sqrt{\mu_0\epsilon_0}\sqrt{1 - \omega_p^2/\omega^2}$. It is noted that R_e^m symbolizes the radiation resistance referred to the maximum antenna current.

6.2. THE PLASMA MODE ($E_p \cdot V_p$, n_1)

Following a procedure similar to that of the previous section, we can determine the deviation of the electron density from its ambient density, n_1, as

$$n_1 = \frac{1}{4\pi} \frac{\omega_p^2}{ev_0^2} \int_{-h}^{h} \rho^s \frac{e^{-j\beta_p r}}{r} dz \qquad (72)$$

where $\beta_p^2 = \left(\omega^2 - \omega_p^2\right)/v_0^2$ (because in this section ν is also assumed to be zero) and v_0 = rms velocity of electrons.

Again, for simplicity, only n_1 at the far zone of the antenna is determined. With Equation 63, Equation 72 leads to

$$n_1 = \frac{-j\omega_p^2 \sqrt{\epsilon_0\mu_0}\,\xi\, I_m}{4\pi e v_0^2 r_0} e^{j(\omega t - \beta_p r_0)} \Bigg[\int_0^h \cos\beta_e(h - z)\, e^{j\beta_p z\cos\theta}\, dz$$

$$- \int_{-h}^{0} \cos\beta_e(h + z)\, e^{j\beta_p z\cos\theta}\, dz\Bigg] \qquad (73)$$

That is,

$$n_1 = \frac{-\omega_p^2 \sqrt{\epsilon_0\mu_0}\,\xi\, I_m}{4\pi e v_0^2} \frac{e^{j(\omega t - \beta_p r_0)}}{r_0} \left\{\frac{\beta_p \cos\theta[\cos(\beta_p h\cos\theta) - (\beta_e h)]}{\beta_p^2\cos^2\theta - \beta_e^2}\right\} \qquad (74)$$

Since

$$\vec{E}_p = \frac{ev_0^2}{\left(\omega^2 - \omega_p^2\right)\epsilon_0} \nabla n_1 \tag{75}$$

the electric field of the plasma mode, $\vec{E}_p$, is determined as

$$\vec{E}_p = \frac{\omega_p^2 \sqrt{\xi}\, \zeta_0 I_m}{4\pi\left(\omega^2 - \omega_p^2\right)} \left(\left[\frac{j\beta_p}{r_0} + \frac{1}{r_0^2}\right] \left\{ \frac{\beta_p \cos\theta \left[\cos\left(\beta_p h \cos\theta\right) - \cos\left(\beta_e h\right)\right]}{\beta_p^2 \cos^2\theta - \beta_e^2} \right\} \hat{r} \right.$$

$$- \frac{\beta_p \sin\theta}{r_0^2} \left\{ \frac{\left(\beta_p^2 \cos^2\theta + \beta_e^2\right)\left[\cos\left(\beta_p h \cos\theta\right) - \cos\left(\beta_e h\right)\right]}{\left(\beta_p^2 \cos^2\theta - \beta_e^2\right)^2} \right.$$

$$\left. \left. + \frac{\beta_p h \cos\theta \sin\left(\beta_p h \cos\theta\right)}{\left(\beta_p^2 \cos^2\theta - \beta_e^2\right)} \right\} \hat{\theta} \right) e^{j(\omega t - \beta_p r_0)} \tag{76}$$

The radiation term of $\vec{E}_p$ which is significant in the far zone of the antenna is

$$\vec{E}_p^{\,r} = \frac{j\omega_p^2 \sqrt{\xi}\, \zeta_0 I_m}{4\pi v_0^2} \frac{e^{j(\omega t - \beta_p r_0)}}{r_0} \left\{ \frac{\cos\theta \left[\cos\left(\beta_p h \cos\theta\right) - \cos\left(\beta_e h\right)\right]}{\beta_p^2 \cos^2\theta - \beta_e^2} \right\} \hat{r} \tag{77}$$

The radiation term of the induced velocity of electrons, $\vec{V}_p^{\,r}$, can be determined simply as

$$\vec{V}_p^{\,r} = \frac{j\omega\epsilon_0}{en_0} \vec{E}_p^{\,r}$$

$$= \frac{-\omega\,\omega_p^2 \sqrt{\xi}\, I_m}{4\pi v_0^2 c_0 e n_0} \frac{e^{j(\omega t - \beta_p r_0)}}{r_0} \left\{ \frac{\cos\theta \left[\cos\left(\beta_p h \cos\theta\right) - \cos\left(\beta_e h\right)\right]}{\beta_p^2 \cos^2\theta - \beta_e^2} \right\} \hat{r} \tag{78}$$

The radiation term of n_1 is exactly the same as Equation 74.

The power density flowing out of the antenna as a form of plasma wave is

$$\vec{p}_p = m_e v_0^2 n_1^{\,r} \vec{V}_p^{\,r} \tag{79}$$

The substitution of Equations 74 and 78 in Equation 79 gives

$$(\vec{p}_p)_{av} = \frac{1}{32\pi^2}\left(\frac{c_0}{v_0}\right)^3\left(\frac{\omega_p^2}{\omega^2}\right)\frac{I_m^2\zeta_0}{\sqrt{1-\omega_p^2/\omega^2}}\frac{1}{r_0^2}\left\{\frac{\cos\theta\,[\cos(\beta_p h\cos\theta)-\cos(\beta_e h)]}{1-c_0^2/v_0^2\cos^2\theta}\right\}^2\hat{r} \tag{80}$$

In the evaluation of Equation 80, the relation

$$\frac{\beta_p^2}{\beta_e^2} = \frac{\left(\omega^2-\omega_p^2\right)}{v_0^2}\cdot\frac{1}{\omega^2\mu_0\epsilon_0\left(1-\omega_p^2/\omega^2\right)} = \frac{c_0^2}{v_0^2} \tag{81}$$

is used.

The total power radiated by a cylindrical dipole antenna as a form of plasma wave is then

$$P_p = \int_0^{2\pi} d\theta \int_0^{\pi} d\theta r^2 \sin\theta\,(\vec{p}_p)_{av}\cdot\hat{r}$$

$$= \frac{15}{2}\left(\frac{c_0}{v_0}\right)^3\left(\frac{\omega_p^2}{\omega^2}\right)\frac{I_m^2}{\sqrt{1-\omega_p^2/\omega^2}}\int_0^{\pi}\frac{\sin\theta\cos^2\theta[\cos(\beta_p h\cos\theta)-\cos(\beta_e h)]^2}{\left(1-c_0^2/v_0^2\cos^2\theta\right)^2}d\theta \tag{82}$$

The integral in Equation 82 can be evaluated by a contour integration for the case of $c_0/v_0 >> 1$ in the appendix. The final expression for P_p is

$$P_p = \frac{15\pi}{8}\left(\frac{\omega_p^2}{\omega^2}\right)\frac{1}{\sqrt{1-\omega_p^2/\omega^2}} I_m^2 (2\beta_e h + \sin 2\beta_e h) \tag{83}$$

for $c_0/v_0 >> 1$. The plasma component of radiation resistance referred to the maximum antenna current is obtained by dividing P_p by $1/2\, I_m^2$.

$$R_p^m = \frac{15\pi}{4}\left(\frac{\omega_p^2}{\omega^2}\right)\frac{1}{\sqrt{1-\omega_p^2/\omega^2}}(2\beta_e h + \sin 2\beta_e h) \text{ ohms} \tag{84}$$

for $c_0/v_0 >> 1$, where $\beta_e = \omega\sqrt{\epsilon_0\mu_0}\ \sqrt{1-\omega_p^2/\omega^2}$.

6.3. INPUT RESISTANCE OF A CYLINDRICAL DIPOLE ANTENNA IN A LOSSLESS PLASMA

The total radiation resistance of a cylindrical dipole antenna referred to the maximum antenna current is

$$R^m = R_e^m + R_p^m \tag{85}$$

Since the power relation of

$$\frac{1}{2} I_m^2 R^m = \frac{1}{2} I_0^2 R_0$$

with I_0 as the input antenna current and R_0 as the input antenna resistance is satisfied, the input resistance of a dipole antenna in a plasma is obtained as

$$R_0 = \left(\frac{I_m}{I_0}\right)^2 \left(R_e^m + R_p^m\right) \tag{86}$$

For short antenna $(I_m/I_0) \sim 1/\sin \beta_e h$.

R_e^m and R_p^m as functions of ω_p^2/ω^2 are shown graphically for the cases of $h = \lambda_0/4$ and $h = \lambda_0/10$ in Figures 4 and 5. R_e^m and R_p^m as functions of h/λ_0 are shown graphically for the cases of $\omega_p^2/\omega^2 = 0.5$ and $\omega_p^2/\omega^2 = 0.8$ in Figures 6 and 7.

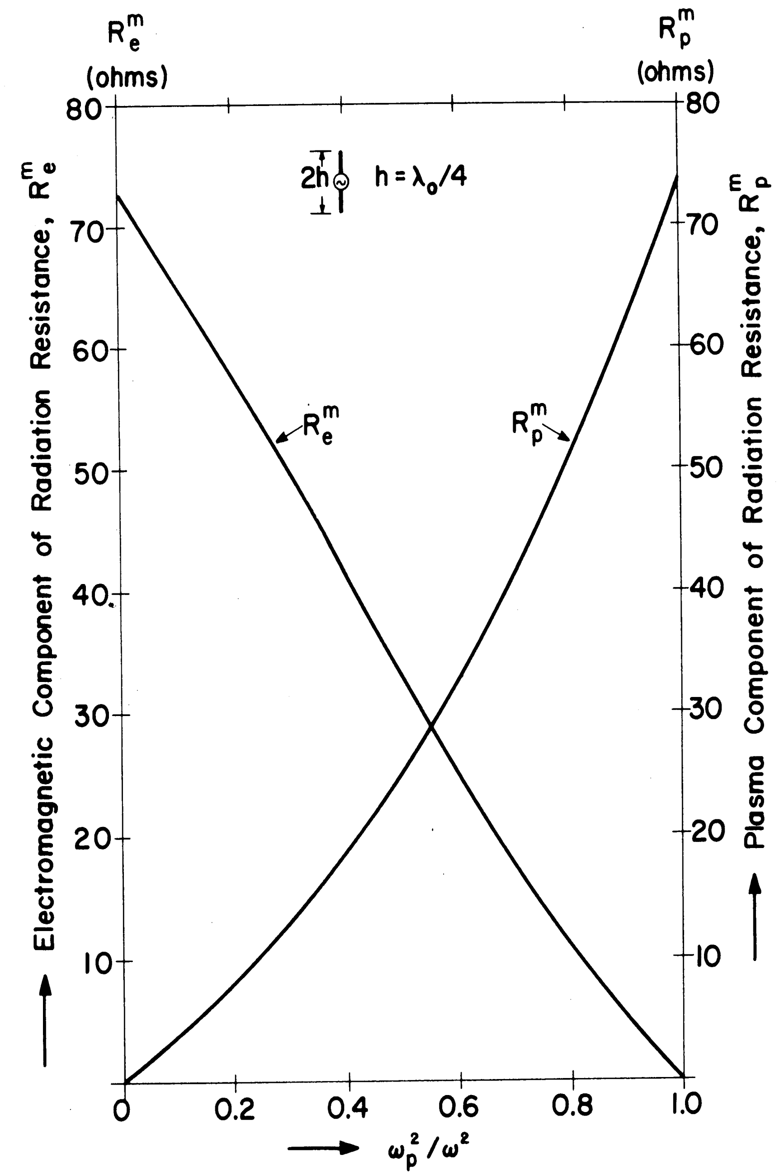

FIGURE 4. RADIATION RESISTANCE OF A HALF-WAVE DIPOLE ($h = \lambda_0/4$) IN A PLASMA VS. ω_p^2/ω^2

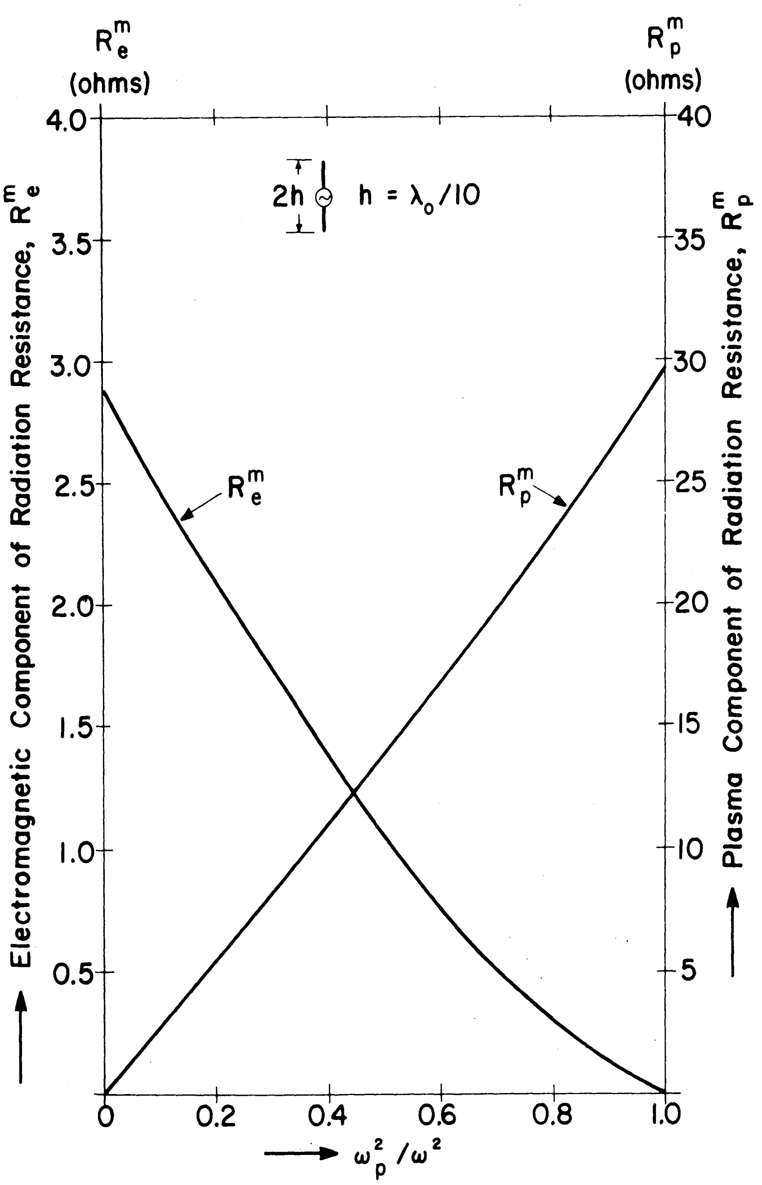

FIGURE 5. RADIATION RESISTANCE OF A SHORT DIPOLE ($h = \lambda_0/10$) IN A PLASMA VS. $\omega_p^{\ 2}/\omega^2$

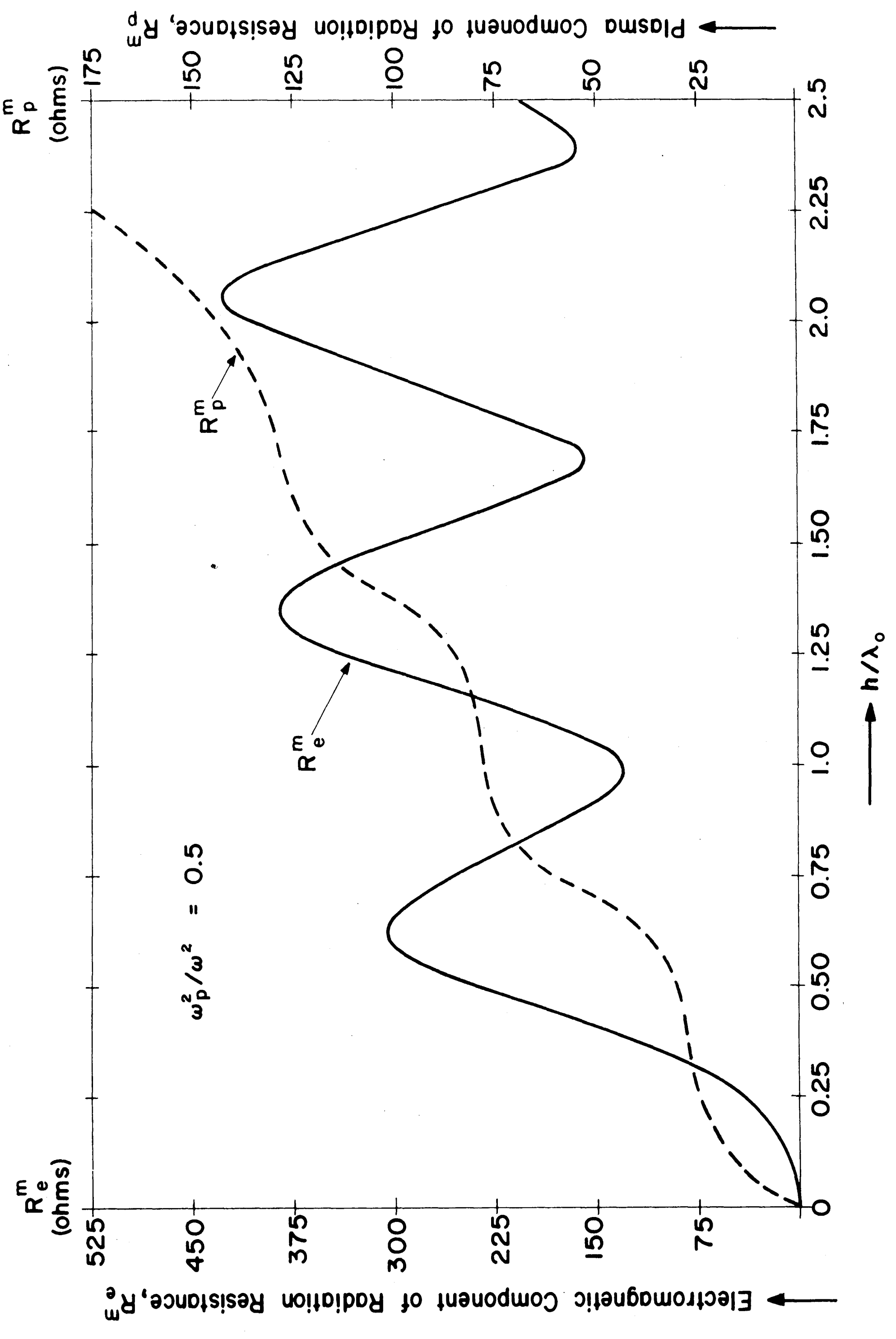

FIGURE 6. RADIATION RESISTANCE OF A DIPOLE ANTENNA VS. ANTENNA LENGTH AT $\omega_p^2/\omega^2 = 0.5$

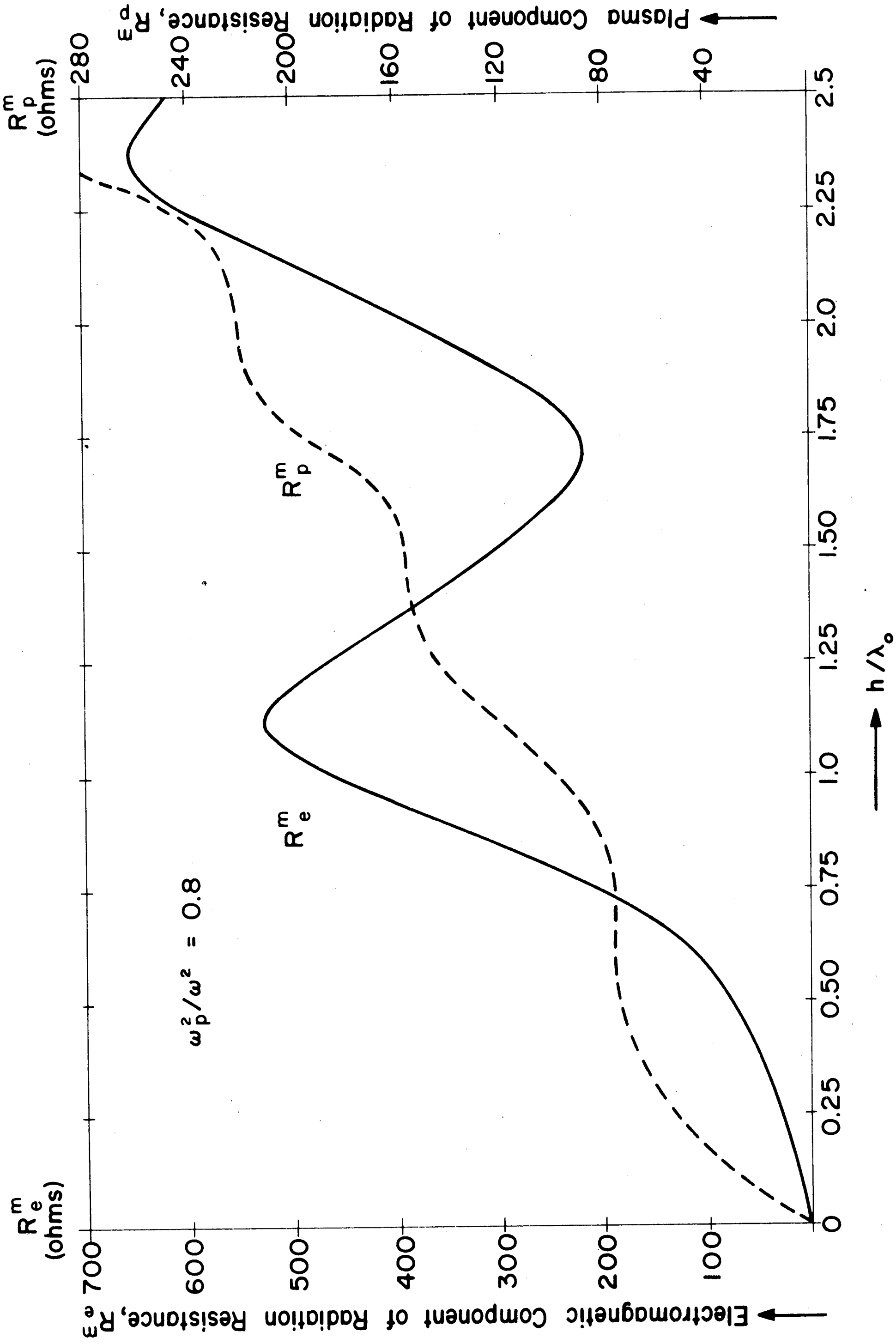

FIGURE 7. RADIATION RESISTANCE OF A DIPOLE ANTENNA VS. ANTENNA LENGTH AT $\omega_p^2/\omega^2 = 0.8$

7
A HERTZIAN DIPOLE IN A LOSSY PLASMA

In the previous sections on the interaction of an antenna with a plasma, the plasma has been assumed to be lossless. It is the purpose of this section to observe the effect of loss in plasma on the radiation of an antenna. In restricting our study to a weakly ionized gas type of plasma, we assume that the loss in a plasma is merely due to the collision between electrons and neutral particles. We will find the radiation resistance of a Hertzian dipole as a function of the collision frequency of the plasma, among other parameters. Actually, when the medium is lossy, the radiated power at the far zone of an antenna is zero, but the power radiated by an antenna into the medium is finite. The power radiated by an antenna in a lossy medium can be obtained approximately by integrating the power radiating from a small sphere which surrounds the antenna. The radiation resistance can then be determined after the radiated power is obtained.

The geometry of the problem is shown in Figure 8. A Hertzian dipole is immersed in a lossy plasma and surrounded by an imaginary sphere of radius r_1.

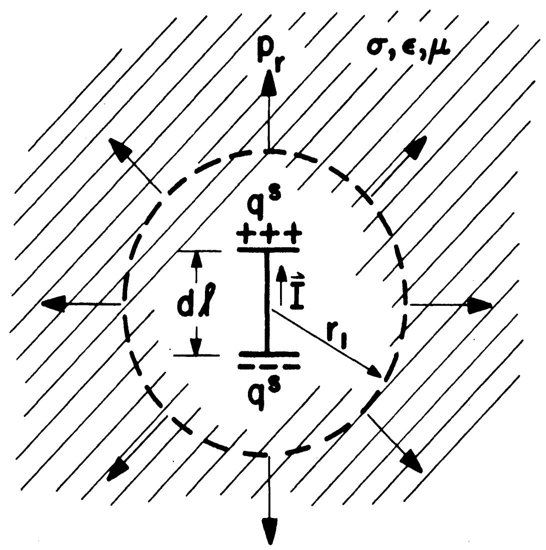

FIGURE 8. A HERTZIAN DIPOLE IN A LOSSY PLASMA SURROUNDED BY AN IMAGINARY SPHERE OF RADIUS r_1

The current and the charge on the Hertzian dipole are assumed to be

$$\vec{J}^s = I\ d\ell\ \delta(x)\delta(y)\delta(z)e^{j\omega t}\hat{z} \tag{87}$$

$$\rho^s = -\frac{j}{\omega} I \left[\delta\left(z - \frac{1}{2}d\ell\right) - \delta\left(z + \frac{1}{2}d\ell\right)\right] \delta(x)\delta(y)e^{j\omega t} \tag{88}$$

The EM and the plasma mode excited by $\vec{J}^s$ and ρ^s are analyzed separately as follows.

7.1. THE EM MODE

The propagation constant of the EM mode, $\bar{\beta}_e$, in this case is complex and can be expressed as

$$\bar{\beta}_e^2 = \omega^2 \epsilon_0 \mu_0 \bar{\xi} = \omega^2 \epsilon_0 \mu_0 \left[\left(1 - \frac{\omega_p^2}{\omega^2 + \nu^2}\right) - j \frac{\omega_p^2 \nu}{\omega(\omega^2 + \nu^2)}\right] \tag{89}$$

If we write

$$\bar{\beta}_e = \beta_e - j\alpha_e \tag{90}$$

we have

$$\beta_e^2 - \alpha_e^2 = \omega^2 \epsilon_0 \mu_0 \left(1 - \frac{\omega_p^2}{\omega^2 + \nu^2}\right) \tag{91}$$

$$2\beta_e \alpha_e = \omega \nu \epsilon_0 \mu_0 \left(\frac{\omega_p^2}{\omega^2 + \nu^2}\right) \tag{92}$$

From Equations 91 and 92, β_e and α_e can be expressed as

$$\beta_e = \frac{\omega\sqrt{\mu_0 \epsilon_0}}{\sqrt{2}} \left[1 - \frac{\omega_p^2}{\omega^2 + \nu^2} + \sqrt{1 - \frac{2\omega_p^2}{\omega^2 + \nu^2} + \frac{\omega_p^4}{\omega^2(\omega^2 + \nu^2)}}\right]^{1/2} \tag{93}$$

$$\alpha_e = \frac{\omega\sqrt{\mu_0 \epsilon_0}}{\sqrt{2}} \left[-1 + \frac{\omega_p^2}{\omega^2 + \nu^2} + \sqrt{1 - \frac{2\omega_p^2}{\omega^2 + \nu^2} + \frac{\omega_p^4}{\omega^2(\omega^2 + \nu^2)}}\right]^{1/2} \tag{94}$$

The electric field of the EM mode, $\vec{E}_e$, set up by a Hertzian dipole in a lossy plasma can be found to be

$$\vec{E}_e = \frac{-jI\,d\ell}{2\pi\epsilon_0 \omega \bar{\xi}} \left(\frac{1}{r^3} + j\frac{\bar{\beta}_e}{r^2}\right) \cos\theta \, e^{j(\omega t - \bar{\beta}_e r)} \hat{r}$$

$$+ \frac{-jI\,d\ell}{4\pi\epsilon_0 \omega \bar{\xi}} \left(\frac{1}{r^3} + j\frac{\bar{\beta}_e}{r^2} - \frac{\bar{\beta}_e^2}{r}\right) \sin\theta \, e^{j(\omega t - \bar{\beta}_e r)} \hat{\theta} \tag{95}$$

The magnetic field of the EM mode, $\vec{H}$, set up by a Hertzian dipole in a lossy plasma is

$$\vec{H} = \frac{-jI\,d\ell}{4\pi}\left(j\frac{1}{r^2} - \frac{\bar{\beta}_e}{r}\right)\sin\theta\, e^{j(\omega t - \bar{\beta}_e r)}\,\hat{\phi} \tag{96}$$

The radial component of the complex Poynting vector is given by

$$(\bar{p}_e)_r = \frac{1}{2}(E_{e\theta}H_\phi^*) \tag{97}$$

and the real part of $(\bar{p}_e)_r$ is obtained from Equations 95 and 96 as

$$\text{Real}\,(\bar{p}_e)_r = \frac{I^2 d\ell^2\,\beta_e\alpha_e}{16\pi^2\epsilon_0^2\omega^3\mu_0|\bar{\xi}|^2}\left[\frac{1}{r^5} + \frac{2\alpha_e}{r^4} + \frac{\beta_e^2 + \alpha_e^2}{r^3} + \frac{\left(\beta_e^2 + \alpha_e^2\right)^2}{2\alpha_e}\,\frac{1}{r^2}\right]\sin^2\theta\, e^{-2\alpha_e r} \tag{98}$$

The total power radiated by the dipole into the plasma is the sum of the power radiated outward from the sphere of radius r_1 and the power dissipated within the sphere. Symbolically,

$$P_e = \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta\, r_1^2\, \text{Real}\,(\bar{p}_e)_r + [W_e]_0^{r_1} \tag{99}$$

The first term of the right-hand side (RHS) of Equation 99 can be evaluated, but the second term, the power dissipated within the sphere of radius r_1, cannot be evaluated unless the exact shape of the dipole and the exact expression of its near zone field are known. The relative values of the first and the second terms of Equation 99 depend on the conductivity of the medium. King [8] has found the following results.

The following observations are based on a short dipole of $\beta_e h = 0.3$, and $\beta_e r_1 = 1$, radian sphere. (1) If $\alpha_e/\beta_e \leq 10^{-3}$, 90% or more of the power supplied to the antenna is transferred beyond the radian sphere; in other words, the first term dominates the RHS of Equation 99 in this case. (2) If $\alpha_e/\beta_e \leq 10^{-1}$, only about 8% of the power is dissipated outside and 92% of the power is used to heat the medium inside the radian sphere; that is to say, the second term dominates the RHS of Equation 99 in this case. (3) When $\alpha_e/\beta_e \simeq 1$, virtually all of the power is dissipated as heat within the radian sphere.

Because of this difficulty, only the case of a slightly lossy plasma is studied. We shall try to predict the effect of the collision in a plasma on the radiation of the dipole from the first term of the RHS of Equation 99. Let us write the first term of the RHS of Equation 99, or the power radiated beyond the sphere of radius r_1, as

$$P'_e = \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta\, r_1^2\, \text{Real}\,(\bar{p}_e)_r \tag{100}$$

By using Equation 98, P'_e can be determined as

$$P'_e = \frac{I^2 d\ell^2 \beta_e \alpha_e}{12\pi\epsilon_0 \omega^2 \mu_0^2 |\bar{\xi}|^2} \left[\frac{2}{r_1^3} + \frac{4\alpha_e}{r_1^2} + \frac{2}{r_1}\left(\beta_e^2 + \alpha_e^2\right) + \frac{1}{\alpha_e}\left(\beta_e^2 + \alpha_e^2\right)^2 \right] e^{-2\alpha_e r_1} \tag{101}$$

Equation 101 gives the expression for the power radiated outward from the sphere of radius r_1. It is interesting to show that this amount of power is actually dissipated as heat in the medium outside the sphere of radius r_1. To prove this, we will first consider the power dissipated in the unit volume of the medium. If we assume that the conductivity of the medium is σ and the electric field in the medium is $\vec{E}_e$, as given in Equation 95, the power dissipated in the unit volume of the medium is

$$w = \sigma E_e^2 \tag{102}$$

With Equation 95, the above equation leads to

$$\sigma E_e^2 = \frac{\sigma I^2 d\ell^2}{32\pi^2 \epsilon_0^2 \omega^2 |\bar{\xi}|^2} \left\{ \left[\left(\frac{1}{r^3} + \frac{\alpha_e}{r^2} - \frac{\beta_e^2 - \alpha_e^2}{r} \right)^2 + \left(\frac{\beta_e}{r^2} + \frac{2\beta_e \alpha_e}{r} \right)^2 \right] \sin^2\theta \right.$$

$$\left. + \left[\left(\frac{2}{r^3} + \frac{2\alpha_e}{r^2} \right)^2 + \left(\frac{2\beta_e}{r^2} \right)^2 \right] \cos^2\theta \right\} e^{-2\alpha_e r} \tag{103}$$

The total power dissipated in the medium outside the sphere of radius r_1 can be obtained as

$$[W_e]_{r_1}^{\infty} = \int_{r_1}^{\infty} dr \int_0^{\pi} d\theta \int_0^{2\pi} d\phi \; r^2 \sin\theta \, \sigma E_e^2$$

$$= \frac{\sigma I^2 d\ell^2}{24\pi\epsilon_0^2 \omega^2 |\bar{\xi}|^2} \left[\frac{2}{r_1^3} + \frac{4\alpha_e}{r_1^2} + \frac{2}{r_1}\left(\beta_e^2 + \alpha_e^2\right) \right.$$

$$\left. + \frac{1}{\alpha_e}\left(\beta_e^2 + \alpha_e^2\right)^2 \right] e^{-2\alpha_e r_1} \tag{104}$$

In a plasma of weakly ionized gas type, σ can be expressed as

$$\sigma = \frac{\omega_p^2 \nu}{\omega^2 + \nu^2} \epsilon_0 \tag{105}$$

It is now very easy to prove that Equations 101 and 104 are identical.

The EM component of the radiation resistance of a Hertzian dipole in a lossy plasma is defined as

$$R_e = \frac{P_e}{1/2\, I^2} = \frac{2}{I^2}\left\{P'_e + [W_e]_0^{r_1}\right\} = \frac{2}{I^2}\left\{[W]_{r_1}^{\infty} + [W_e]_0^{r_1}\right\} \tag{106}$$

Since the power, $[W_e]_0^{r_1}$, dissipated within the sphere of radius r_1 cannot be evaluated, we evaluate the first term of the RHS of Equation 106 only and write R_e as

$$R_e = R'_e + R_{eh} \tag{107}$$

where R'_e is that part of the radiation resistance which is responsible for radiating an EM wave out of the sphere of radius r_1, and R_{eh} is that part of the radiation resistance which takes into account the power dissipated as heat within the sphere of radius r_1.

With Equation 101 or 104, R'_e can be determined as

$$R'_e = 80\pi^2\left(\frac{d\ell}{\lambda_0}\right)^2 \sqrt{\xi}\left(1 + \frac{\alpha_e^2}{\beta_e^2}\right)^2\left(1 + \frac{\alpha_e}{\beta_e}L\right)e^{-2\alpha_e r_1} \tag{108}$$

where

$$\xi = \sqrt{1 - \frac{2\omega_p^2}{\omega^2 + \nu^2} + \frac{\omega_p^4}{\omega^2(\omega^2 + \nu^2)}} \tag{109}$$

$$L = \frac{2}{\left(1 + \frac{\alpha_e^2}{\beta_e^2}\right)\beta_e r_1} + \frac{4\,\alpha_e/\beta_e}{\left(1 + \frac{\alpha_e^2}{\beta_e^2}\right)^2 \beta_e^2 r_1^2} + \frac{2}{\left(1 + \frac{\alpha_e^2}{\beta_e^2}\right)^2 \beta_e^3 r_1^3} \tag{110}$$

β_e and α_e are expressed in Equations 93 and 94.

R'_e, expressed in Equation 108, reduces to the free space value of the radiation resistance of a Hertzian dipole when α_e is zero. In general, R'_e is a function of α_e/β_e and r_1, a fact which implies that the value of R'_e is dependent on the size of the sphere on which the integration of the Poynting vector is performed. R'_e, therefore, can not give a very accurate and unique estimation of the EM component of the radiation resistance except for very small α_e/β_e or a slightly lossy plasma. R'_e is plotted as a function of ν/ω in Figure 9 for the case of a Hertzian dipole

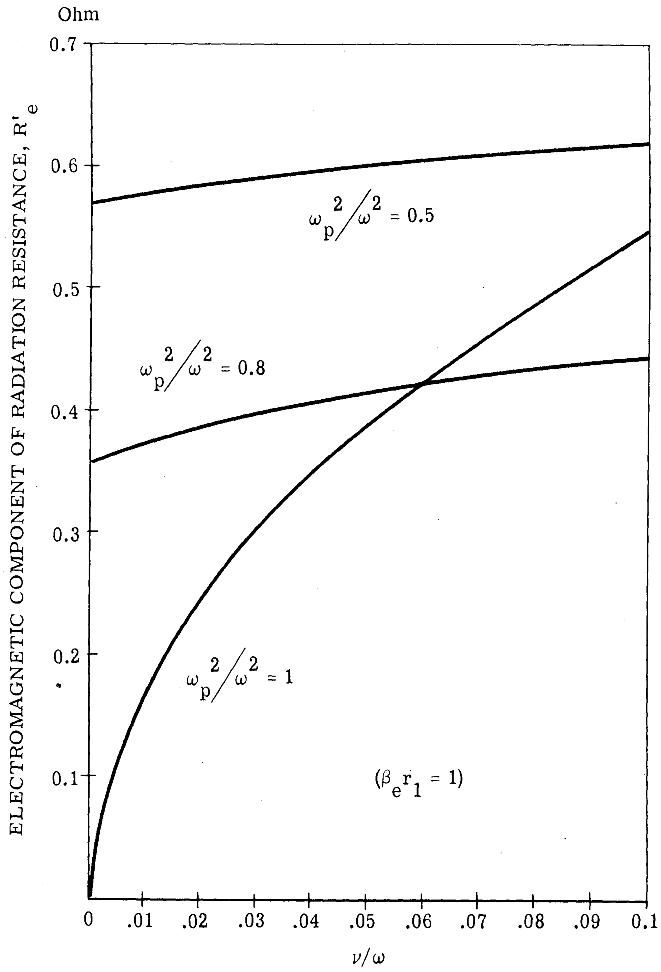

FIGURE 9. EM COMPONENT OF RADIATION RESISTANCE OF A SHORT DIPOLE ($d\ell = \lambda_0/10\pi$) IN A PLASMA AS A FUNCTION OF PLASMA COLLECTION FREQUENCY, ν/ω

with $d\ell = \lambda_0/10\pi$ and $\beta_e r_1 = 1$. R'_e is seen to increase with ν/ω, and this effect becomes very significant when ω is close to ω_p. From this behavior, we may conclude that the collision in a plasma tends to increase the radiation resistance of an antenna.

It is noted that the Poynting vector method used in this section is not adequate to find the radiation resistance of an antenna in a lossy plasma. King [8] has obtained the EM component of the radiation resistance in a lossy medium by using an integral equation method. In his results, R_e becomes much greater than R'_e when ν/ω is greater than 10^{-2}. This tends to show that for ν/ω greater than 10^{-2}, R_{eh} can be much greater than R'_e; we may therefore be forced to conclude that the collision in a plasma may greatly increase the radiation resistance of an antenna.

7.2. THE PLASMA MODE

The propagation constant of the plasma mode, $\bar{\beta}_p$, in this case is complex and can be expressed as follows:

$$\bar{\beta}_p^{\,2} = \frac{1}{v_0^2}\left[\left(\omega^2 - \omega_p^2\right) - j\omega\nu\right] \tag{111}$$

If we write

$$\bar{\beta}_p = \beta_p - j\alpha_p \tag{112}$$

we have

$$\beta_p^2 - \alpha_p^2 = \frac{1}{v_0^2}\left(\omega^2 - \omega_p^2\right) \tag{113}$$

$$2\beta_p\alpha_p = \frac{1}{v_0^2}\omega\nu \tag{114}$$

From Equations 113 and 114 we get

$$\beta_p = \frac{1}{\sqrt{2}\,v_0}\left[\omega^2 - \omega_p^2 + \sqrt{\left(\omega^2 - \omega_p^2\right)^2 + \omega^2\nu^2}\right]^{1/2} \tag{115}$$

$$\alpha_p = \frac{1}{\sqrt{2}\,v_0}\left[-\omega^2 + \omega_p^2 + \sqrt{\left(\omega^2 - \omega_p^2\right)^2 + \omega^2\nu^2}\right]^{1/2} \tag{116}$$

The deviation of the electron density from its ambient density, n_1, caused by a Hertzian dipole in a lossy plasma can be found to be

$$n_1 = \frac{Id\ell}{4\pi\omega}\,\frac{\omega_p^2}{e\,v_0^2}\left(\frac{\bar{\beta}_p}{r} - j\frac{1}{r^2}\right)\cos\theta\, e^{j(\omega t - \bar{\beta}_p r)} \tag{117}$$

and the radial component of induced velocity of the electrons of the plasma mode, $(V_p)_r$, in this case is

$$(V_p)_r = \frac{\omega_p^2\, Id\ell}{4\pi e n_0\left(\omega^2 - \omega_p^2 - j\omega\nu\right)}\left(\frac{\bar{\beta}_p^{\,2}}{r} - \frac{2}{r^3} - j\frac{2\bar{\beta}_p}{r^2}\right)\cos\theta\, e^{j(\omega t - \beta_p r)} \tag{118}$$

The power density flowing from the dipole as a form of an electroacoustic wave is

$$\text{Real}\,(\bar{p}_p)_r = \text{Real}\left[\frac{1}{2} m_e v_0^2 n_1^*(V_p)_r\right]$$

$$= \frac{\omega_p^2 I^2 d\ell^2}{32\pi^2 \omega \epsilon_0 \left[\left(\omega^2 - \omega_p^2\right)^2 + \omega^2 \nu^2\right]} \left[\beta_p\left(\omega^2 - \omega_p^2\right)\left(\frac{\beta_p^2 + \alpha_p^2}{r^2} + \frac{2\alpha_p}{r^3}\right)\right.$$

$$\left. + \omega\nu\left(\frac{2}{r^5} + \frac{4\alpha_p}{r^4} + \frac{\beta_p^2 + 3\alpha_p^2}{r^3} + \frac{\beta_p^2 \alpha_p + \alpha_p^3}{r^2}\right)\right] \cos\theta \, e^{-2\alpha_p r} \tag{119}$$

The total power flowing from a sphere of radius r_1 as a form of an electroacoustic wave is

$$P'_p = \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta \, r_1^2 \, \text{Real}\,(\bar{p}_p)_r \tag{120}$$

The total power transferred from the Hertzian dipole into the lossy plasma in a form of an electroacoustic wave is then the sum of P'_p and the power dissipated within the sphere of radius r_1 by the electroacoustic wave, $[W_p]_0^{r_1}$. Symbolically,

$$P_p = P'_p + [W_p]_0^{r_1} \tag{121}$$

Again, we are only able to evaluate P'_p, for the same reason as in the EM mode case.

The plasma component of the radiation resistance of a Hertzian dipole in a lossy plasma is

$$R_p = \frac{1}{1/2\, I^2}\left\{P'_p + [W_p]_0^{r_1}\right\} \tag{122}$$

Let us define

$$R_p = R'_p + R_{ph} \tag{123}$$

with R'_p as that part of the radiation resistance which is responsible for radiating a plasma wave out of the sphere of radius r_1, and R_{ph} as that part of the radiation resistance which is responsible for the power dissipated as heat within the sphere of radius r_1. We can then express R'_p as follows:

$$R'_p = 40\pi^2\left(\frac{d\ell}{\lambda_0}\right)^2\left(\frac{c_0}{v_0}\right)^3 \frac{\omega_p^2}{\omega^2}\sqrt{1 - \omega_p^2/\omega^2}\, f(\alpha_p/\beta_p, r_1)\, e^{-2\alpha_p r_1} \tag{124}$$

where

$$f(\alpha_p/\beta_p, r_1) = \frac{1}{\sqrt{1 - x^2}} \left[1 + \frac{4x}{1 + x^2}\left(\frac{1}{\beta_p r_1}\right) + \frac{8x^2}{(1 + x^2)^2}\left(\frac{1}{\beta_p^2 r_1^2}\right) + \frac{4x}{(1 + x^2)^2}\left(\frac{1}{\beta_p^3 r_1^3}\right)\right] \tag{125}$$

and

$$x = \alpha_p/\beta_p$$

R'_p expressed in Equation 124 reduces to the free space value when α_p is zero. To observe the effect of the collision in a plasma on R'_p, Equations 124 and 125 are carefully studied. Let us consider the case of a Hertzian dipole with $d\ell = \lambda_0/10\pi$. To have a sphere of reasonable size surround the dipole, we let $\beta_e r_1 = 1$, consistent with the EM mode case. If r_1 is set to be $1/\beta_e$, $\alpha_p r_1$ and $\beta_p r_1$ become very large, even for a very small ν/ω. On the other hand, $f(\alpha_p/\beta_p, r_1)$ remains quite close to unity. This implies that R'_p is very small, compared to its free space value, when α_p is not zero. This also implies that R_{ph} is much larger than R'_p when α_p is not zero. We then conclude that the major part of the power of the plasma mode is dissipated in the very vicinity of the antenna if the plasma is lossy. Unfortunately, we cannot observe the effect of the collision in a plasma on the change of R_p in Equation 124. However, the effect of the collision in a plasma on R_p may not be significant as in the case of the EM mode since in the plasma mode case the major part of the power is dissipated in the very vicinity of the antenna. It may also be fair to state that R_p does not change very rapidly with ν/ω, as does R_e.

Since the total radiation resistance of an antenna is the sum of R_e and R_p, the effect of the collision in a plasma on the total radiation resistance may be very significant. Therefore, when ν/ω is not equal to or smaller than 10^{-3}, the results of R_e and R_p for the case of lossless plasma cannot be applied to the lossy plasma case. We can only say that when ν/ω is larger than 10^{-3}, both R_e and R_p increase with ν/ω, and R_e increases very rapidly with ν/ω when ω is very close to ω_p.

In a wide range of communications involving the ionosphere, ν/ω is smaller than 10^{-3} and ω is not close to ω_p. For this case R_e can be approximately calculated from Equation 108, and R_p may be approximated with fair accuracy by the free space value of R_p.

Appendix A
EVALUATION OF THE INTEGRAL IN EQUATION 82

The integral is

$$I = \int_0^{\pi} \frac{\sin\theta \cos^2\theta \left[\cos(\beta_p h \cos\theta) - \cos(\beta_e h)\right]^2}{\left(1 - c_0^{\,2}/v_0^{\,2} \cos^2\theta\right)^2} d\theta \tag{126}$$

If we let $c_0/v_0 \cos\theta = x$ and $\beta_e h = \alpha$, then I becomes

$$I = 2\left(\frac{v_0}{c_0}\right)^3 \int_0^{c_0/v_0} \frac{x^2[\cos(\alpha x) - \cos(\alpha)]^2}{(1 - x^2)^2} dx \tag{127}$$

For the case of $c_0/v_0 >> 1$, I can be represented as

$$I = 2\left(\frac{v_0}{c_0}\right)^3 \int_0^{\infty} \frac{x^2[\cos(\alpha x) - \cos(\alpha)]^2}{(1 - x^2)^2} dx \tag{128}$$

The integral

$$I_1 = \int_0^{\infty} \frac{x^2[\cos(\alpha x) - \cos(\alpha)]^2}{(1 - x^2)^2} dx$$

can be evaluated by contour integration as follows:

$$I_1 = \frac{1}{2}\int_{-\infty}^{\infty} \frac{x^2\cos^2(\alpha x)}{(1 - x^2)^2} dx - \int_{-\infty}^{\infty} \frac{\cos(\alpha) x^2 \cos(\alpha x)}{(1 - x^2)^2} dx + \frac{1}{2}\int_{-\infty}^{\infty} \frac{\cos^2(\alpha)\, x^2}{(1 - x^2)^2} dx \tag{129}$$

The first integral in Equation 129 can be expressed as

$$\int_{-\infty}^{\infty} \frac{x^2\cos^2(\alpha x)}{(1 - x^2)^2} dx = \frac{1}{4}\left[\int_{-\infty}^{\infty} f_1(z)\,dz + \int_{-\infty}^{\infty} f_2(z)\,dz + \int_{-\infty}^{\infty} f_3(z)\,dz\right] \tag{130}$$

where

$$f_1(z) = \frac{z^2 e^{2i\alpha z}}{(1 - z)^2 (1 + z)^2}$$

$$f_2(z) = \frac{2z^2}{(1 - z)^2 (1 + z)^2}$$

$$f_3(z) = \frac{z^2 e^{-2i\alpha z}}{(1 - z)^2 (1 + z)^2}$$

By integrating along the closed paths as indicated in Figure 10, we can obtain Equation 130 as

$$\int_{-\infty}^{\infty} \frac{x^2 \cos^2 (\alpha x)}{(1 - x^2)^2} dx = \frac{-2\pi i}{4} \left\{ \text{Res} \left[f_3(z), z = -1\right] + \text{Res} \left[f_3(z), z = 1\right] \right\}$$

$$= -\frac{\pi\alpha}{2} \cos 2\alpha - \frac{\pi}{4} \sin 2\alpha \qquad (131)$$

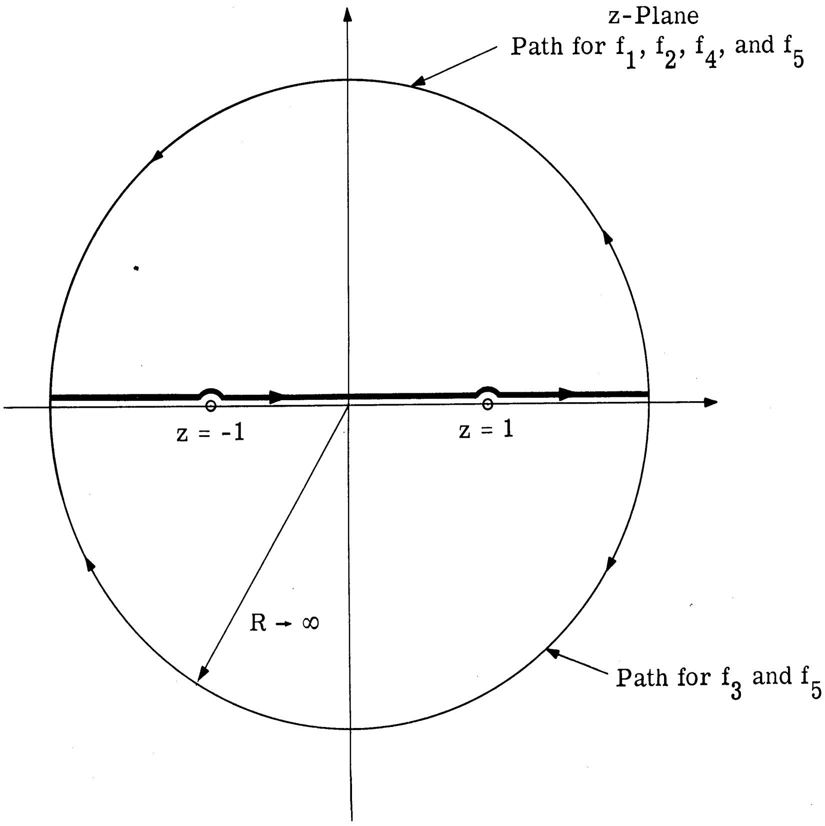

FIGURE 10. PATHS FOR CONTOUR INTEGRATIONS

The second integral in Equation 129 can be expressed as

$$\int_{-\infty}^{\infty} \frac{\cos (\alpha) x^2 \cos (\alpha x)}{(1 - x^2)^2} dx = \frac{\cos (\alpha)}{2} \left[\int_{-\infty}^{\infty} f_4(z) dz + \int_{-\infty}^{\infty} f_5(z) dz \right] \qquad (132)$$

where

$$f_4(z) = \frac{z^2 e^{i\alpha z}}{(1 - z)^2 (1 + z)^2}$$

$$f_5(z) = \frac{z^2 e^{-i\alpha z}}{(1 - z)^2 (1 + z)^2}$$

By integrating along the closed paths as indicated in Figure 10, we can express Equation 132 as

$$\int_{-\infty}^{\infty} \frac{\cos(\alpha)\, x^2 \cos(\alpha x)}{(1 - x^2)^2} dx = -\pi i \cos(\alpha) \left\{ \text{Res}\,[f_5(z), z = -1] + \text{Res}\,[f_5(z), z = 1] \right\}$$

$$= -\cos\alpha \left[\frac{\pi\alpha}{2} \cos\alpha + \frac{\pi}{2} \sin\alpha \right] \tag{133}$$

The third integral in Equation 129 can be expressed as

$$\int_{-\infty}^{\infty} \frac{\cos^2(\alpha)\, x^2}{(1 - x^2)^2} dx = \cos^2(\alpha) \int_{-\infty}^{\infty} f_6(z)\, dz = 0 \tag{134}$$

where

$$f_6(z) = \frac{z^2}{(1 - z)^2 (1 + z)^2}$$

Equation 134 is found to be zero if the integration is performed along the closed path indicated in Figure 10.

By summing up Equations 131, 133, and 134, we obtain the following result:

$$I = \left(\frac{v_0}{c_0}\right)^3 \left(-\frac{\pi\alpha}{2} \cos 2\alpha - \frac{\pi}{4} \sin 2\alpha + \pi\alpha \cos^2\alpha + \pi \sin\alpha \cos\alpha\right)$$

$$= \left(\frac{v_0}{c_0}\right)^3 \left(\frac{\pi\alpha}{2} + \frac{\pi}{4} \sin 2\alpha\right)$$

$$= \left(\frac{v_0}{c_0}\right)^3 \left[\frac{\pi}{4} (2\beta_e h + \sin 2\beta_e h)\right] \tag{135}$$

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The case of a Hertzian dipole in a lossless plasma is considered first. The fields of both the EM and plasma modes excited by the dipole are explicitly obtained, and the EM and plasma components of the radiation resist-ance of a Hertzian dipole are determined. The case of a cylindrical dipole antenna in a lossless plasma

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Inst. of Science and Technology, U. of Mich., Ann Arbor
INTERACTION OF A RADIATING SOURCE WITH A PLASMA: Effect of an Electroacoustic Wave by Kun-Mu Chen. July 63. 35 p., incl. illus., 8 refs.
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