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THE MINIMIZATION OF BACK SCATTERING OF A CYLINDER BY DOUBLE LOADING

by

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ABSTRACT

A theory on the minimization of the back scattering of a cylinder by loading a cylinder at two points with lumped impedances is presented. The induced current on a doubly loaded cylinder when illuminated by a plane electromagnetic wave at an arbitrary angle is obtained. The optimum loading to eliminate the broadside back scattering which is caused by the symmetrical component of the induced current is established. A suitable loading to reduce the off-broadside back scattering, due to the antisymmetrical component of the induced current, is also determined. A proper choice of the impedance and the position of double loading can lead to the reduction of the back scattering over a wide aspect range. The useful formulas for optimum loading and some graphical illustrations are included. The advantages of double loading over central loading are discussed.

I
INTRODUCTION

The first use of reactive loading to reduce the back scattering of a metallic body was by Iams (1950). The idea of using the technique to decrease the radar cross section of objects in space was suggested and employed by Sletten (1962) in 1950. The method of modifying the back scattering cross section of a cylinder by loading techniques has been studied by others (Hu, 1958; As and Schmitt, 1958; Harrington, 1963). The method of minimizing the back scattering of a cylinder by central loading was studied by Chen and Liepa (1964b) from the viewpoint of the surface current. In their study, it was found that a cylinder center-loaded with an optimum impedance which has both reactive and resistive components can lead to zero broadside back scattering. The optimum impedance is passive for a cylinder shorter than a wavelength but an active impedance is required when a cylinder is longer than a wavelength. They also found that central loading can not reduce the off-broadside scattering which is due to the antisymmetrical component of the induced current and is very large in the case of an anti-resonant cylinder (Chen and Liepa, 1964a)(a cylinder with an anti-resonant length).

The purpose of this study is to investigate the double loading method which overcomes the shortcomings of the central loading method.

In general, when a conducting cylinder is illuminated by a plane wave the induced current on the cylinder can be divided into symmetrican and antisymmetrical components. In a resonant cylinder, the symmetrical component is predominant and it gives a large broadside return. In an anti-resonant cylinder, the antisymmetrical component can be very large resulting in a large off-broadside return. Central loading can modify greatly the symmetrical component of the induced current but it does not affect the antisymmetrical component. The double loading which loads the cylinder at two symmetrical points with two identical lumped impedances is designed to modify both components of the induced current. In this report we show that a properly designed double loading can eliminate the broadside back scatter or reduce the off-broadside back scatter. Furthermore, we show that with the freedom to choose the impedance and the position of double loading, we can reduce the back scattering over a wide aspect range.

The scheme of the analysis is (1) to find the induced current on a doubly loaded cylinder as a function of cylinder dimensions, the impedance and the position of the double loading, and the parameters of the incident plane wave; (2) to determine an optimum loading for zero broadside back scattering; (3) to determine a suitable loading to reduce the off-broadside back scattering; and (4) to obtain an optimum loading which functions to reduce the back scattering over a wide aspect range.

The accuracy of the results in this report should be quite adequate in practical applications since a similar analysis of the central loading case has been completely checked by experiment (Chen and Liepa, 1964a, b).

II
INTEGRAL EQUATION FOR INDUCED CURRENT

The geometry of the problem is as shown in Fig. 1. A cylinder with a radius a and length 2h is assumed to be perfectly conducting. Two identical impedances Z_L are loaded at $z = \pm d$ on the cylinder. A plane electromagnetic wave is incident upon the cylinder at an angle θ. The dimensions of interest are $1/4\lambda < 2h < 2\lambda$ and $\beta_o^2 a^2 \ll 1$, where λ is the wavelength and β_o the wave number. We assume that the cylinder is thin enough so that only the axial current is induced.

The tangential component of the incident electric field on the surface of the cylinder is

$$E_z^{in} = E_o \cos\theta \, e^{-j\beta_o \sin\theta z} \quad . \tag{1}$$

The tangential component of the electric field maintained by the current and the charge on the cylinder at the surface of the cylinder is

$$E_z^a = -j \frac{\omega}{\beta_o^2}\left(\frac{\partial^2}{\partial z^2} + \beta_o^2\right) A_z \ , \tag{2}$$

where A_z is the tangential component of the vector potential maintained by the current on the cylinder.

The total tangential component of the electric field on the surface of the cylinder is zero except at $z = \pm d$ where potential differences exist across the lumped impedances. The electric fields across the gaps at $z = \pm d$ can be expressed as

$$E_z^g = Z_L I(d)\delta(z-d) + Z_L I(-d)\,\delta(z+d) \ , \tag{3}$$

where $I(d)$ is the induced current at $z = d$ on the cylinder and $\delta(z)$ is a delta function.

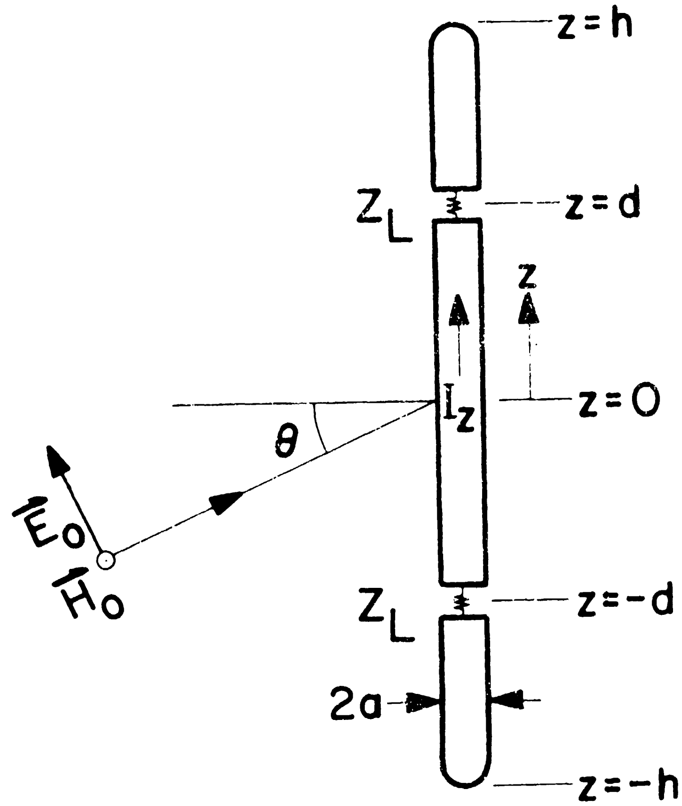

FIG. 1: DOUBLY LOADED CYLINDER ILLUMINATED OBLIQUELY BY A PLANE WAVE

The combination of (1) to (3) gives a differential equation for A_z as

$$\left(\frac{\partial^2}{\partial z^2} + \beta_o^2\right) A_z = -j \frac{\beta_o^2}{\omega} \left[E_o \cos\theta \, e^{-j\beta_o \sin\theta z} - Z_L \left[I(d)\,\delta(z-d) + I(-d)\delta(z+d) \right] \right] . \tag{4}$$

The solution for A_z can be expressed as

$$A_z = \frac{-j}{v_o} \left[C_1 \cos\beta_o z + C_2 \sin\beta_o z + \theta(z) \right] , \tag{5}$$

where $v_o = 1/\sqrt{\mu_o \epsilon_o}$, C_1 and C_2 are arbitrary constants and $\theta(z)$ is a particular integral. $\theta(z)$ can be found to be

$$\theta(z) = \frac{E_o}{\beta_o \cos\theta} e^{-j\beta_o \sin\theta z} - \frac{Z_L}{2} \left[I(d) \sin\beta_o |z-d| + I(-d) \sin\beta_o |z+d| \right] . \tag{6}$$

For convenience, A_z is divided into symmetrical and antisymmetrical components as follows:

$$A_z^s(z) = \frac{-j}{v_o} \left[C_1 \cos\beta_o z + \frac{E_o}{\beta_o \cos\theta} \cos(\beta_o \sin\theta z) - \frac{Z_L}{4} \left[I(d) + I(-d) \right] \cdot \right.$$
$$\left. \cdot \left[\sin\beta_o |z-d| + \sin\beta_o |z+d| \right] \right] , \tag{7}$$

$$A_z^a(z) = \frac{-j}{v_o} \left[C_2 \sin\beta_o z - \frac{jE_o}{\beta_o \cos\theta} \sin(\beta_o \sin\theta z) - \frac{Z_L}{4} \left[I(d) - I(-d) \right] \cdot \right.$$
$$\left. \cdot \left[\sin\beta_o |z-d| - \sin\beta_o |z+d| \right] \right] . \tag{8}$$

After some modification we obtain the following relations

$$A_z^s(z) - A_z^s(h) = \frac{-j}{v_o} \sec\beta_o h \left\{ \left[jv_o A_z^s(h) - \frac{E_o}{\beta_o \cos\theta} \cos(\beta_o h \sin\theta) \right] (\cos\beta_o z - \cos\beta_o h) \right.$$

$$+ \frac{E_o \cos\beta_o h}{\beta_o \cos\theta} \left[\cos(\beta_o z \sin\theta) - \cos(\beta_o h \sin\theta) \right]$$

$$\left. + \frac{Z_L}{4} \left[I(d) + I(-d) \right] \left[2 \sin\beta_o h \cos\beta_o d \cos\beta_o z - \cos\beta_o h (\sin\beta_o |z-d| + \sin\beta_o |z+d|) \right] \right\} \tag{9}$$

$$A_z^a(z) = \frac{-j}{v_o} \csc\beta_o h \left\{ jv_o A_z^a(h) \sin\beta_o z \right.$$

$$+ \frac{jE_o}{\beta_o \cos\theta} \left[\sin(\beta_o h \sin\theta) \sin\beta_o z - \sin\beta_o h \sin(\beta_o z \sin\theta) \right]$$

$$\left. - \frac{Z_L}{4} \left[I(d) - I(-d) \right] \left[2 \cos\beta_o h \sin\beta_o d \sin\beta_o z + \sin\beta_o h (\sin\beta_o |z-d| - \sin\beta_o |z+d|) \right] \right\} . \tag{10}$$

According to the definitions of the vector potential, $A_z^s(z)$ and $A_z^a(z)$ can be expressed in terms of the symmetrical component of the induced current $I_z^s(z)$ and the antisymmetrical component of the induced current $I_z^a(z)$ as follows:

$$A_z^s(z) = \frac{\mu_o}{4\pi} \int_{-h}^{h} I_z^s(z') K_a(z, z') dz' , \tag{11}$$

$$A_z^a(z) = \frac{\mu_o}{4\pi} \int_{-h}^{h} I_z^a(z') K_a(z, z') dz' , \tag{12}$$

where

$$K_a(z,z') = \frac{\exp\left\{-j\beta_o\sqrt{(z-z')^2+a^2}\right\}}{\sqrt{(z-z')^2+a^2}} \quad . \tag{13}$$

An integral equation for $I_z^s(z)$ can be obtained from (9) and (11) as

$$\begin{aligned}\int_{-h}^{h} I_z^s(z')K_d(z,z')dz' = \frac{-j4\pi}{\zeta_o} \sec\beta_o h \Bigg\{ \Bigg[jv_o A_z^s(h) \\ - \frac{E_o}{\beta_o\cos\theta}\cos(\beta_o h\sin\theta)\Bigg](\cos\beta_o z - \cos\beta_o h) \\ + \frac{E_o\cos\beta_o h}{\beta_o\cos\theta}\Big[\cos(\beta_o z\sin\theta) - \cos(\beta_o h\sin\theta)\Big] \\ + \frac{Z_L}{4}\Big[I(d)+I(-d)\Big]\Big[2\sin\beta_o h\cos\beta_o d\cos\beta_o z - \cos\beta_o h(\sin\beta_o|z-d| \\ + \sin\beta_o|z+d|)\Big]\Bigg\} \quad ,\end{aligned} \tag{14}$$

where

$$K_d(z,z') = K_a(z,z') - K_a(h,z') \ , \tag{15}$$

and $\zeta_o = 120\pi$ ohms. Notice that $A_z^s(h)$, I(d) and I(-d) are still unknown in the right hand side of (14).

An integral equation for $I_z^a(z)$ can be obtained from (10) and (12) to be

$$\int_{-h}^{h} I_z^a(z')K_a(z,z')dz' = \frac{-j4\pi}{\zeta_o} \csc\beta_o h \Big\{ jv_o A_z^a(h)\sin\beta_o z$$

$$+ \frac{jE_o}{\beta_o\cos\theta}\Big[\sin(\beta_o h\sin\theta)\sin\beta_o z - \sin\beta_o h\sin(\beta_o z\sin\theta)\Big]$$

$$- \frac{Z_L}{4}\Big[I(d) - I(-d)\Big]\Big[2\cos\beta_o h\sin\beta_o d\sin\beta_o z + \sin\beta_o h(\sin\beta_o|z-d|$$

$$- \sin\beta_o|z+d|)\Big]\Big\} . \qquad (16)$$

Equations (14) and (16) are solved separately in the following sections.

III
SYMMETRICAL COMPONENT OF INDUCED CURRENT

In this section, $I_z^s(z)$ is determined approximately from (14). In view of the complexity of the integral equation (14), it is very difficult to obtain a solution which is simple enough for further theoretical development and with the accuracy higher than the first order approximation. In general, this type of integral equation can be solved very accurately by using King-Middleton's iterative method (King, 1956) or Tai's variation method (Tai, 1950) but the solutions would be too complicated to make further theoretical development tractable. A compromise technique which is similar to King's recent method (1961) is used here.

From the nature of the kernal $K_d(z, z')$ which has a sharp peak at $z' = z$ and from the form of the right hand side of (14), we assume $I_z^s(z)$ as

$$\begin{aligned} I_z^s(z) &= C_c(\cos\beta_o z - \cos\beta_o h) + C_\theta\left[\cos(\beta_o z\sin\theta) - \cos(\beta_o h\sin\theta)\right] \\ &\quad + C_i\left[2\sin\beta_o h\cos\beta_o d\cos\beta_o z - \cos\beta_o h(\sin\beta_o|z-d| + \sin\beta_o|z+d|)\right] . \end{aligned} \tag{17}$$

This distribution has the advantage of satisfying the boundary conditions at $z = h$ and $z = -h$. In other words, with this distribution (14) is satisfied at $z = h$ and $z = -h$.

Furthermore, the problem of finding $I_z^s(z)$ is simplified due to the fortunate relation of

$$I(d) + I(-d) = I_z^s(d) + I_z^s(-d) + I_z^a(d) + I_z^a(-d) = I_z^s(d) + I_z^s(-d) \tag{18}$$

because $I_z^a(d) + I_z^a(-d) = 0$ by definition.

The constants, C_c, C_θ and C_i can be determined by the substitution of (17) in (14) and letting $z = 0$. By doing so the current in (17) has been made to satisfy the boundary conditions at three points, i.e. $z = 0$, $z = h$ and $z = -h$. It is noted that this method yields a quite satisfactory solution as discussed by King (1961) and Chen and Liepa (1964a, b).

After C_c, C_θ and C_i are determined, $I_z^s(z)$ can be expressed as

$$I_z^s(z) = \frac{-j4\pi}{\zeta_o T_{cd}} \sec\beta_o h \left[jv_o A_z^s(h) - \frac{E_o}{\beta_o \cos\theta} \cos(\beta_o h \cos\theta) \right] (1 - \cos\beta_o h)(\cos\beta_o z - \cos\beta_o h)$$

$$+ \frac{-j4\pi}{\zeta_o T_{\theta d}} \frac{E_o}{\beta_o \cos\theta} \left[1 - \cos(\beta_o h \sin\theta) \right] \left[\cos(\beta_o z \sin\theta) - \cos(\beta_o h \sin\theta) \right]$$

$$+ \frac{-j2\pi}{\zeta_o T_{id}} \sec\beta_o h \, Z_L \left[I_z^s(d) + I_z^s(-d) \right] \sin\beta_o (h-d) \left[2 \sin\beta_o h \cos\beta_o d \cos\beta_o z - \cos\beta_o h (\sin\beta_o |z-d| + \sin\beta_o |z+d|) \right], \quad (19)$$

where

$$T_{cd} = \int_{-h}^{h} (\cos\beta_o z' - \cos\beta_o h) K_d(0, z') dz' \, , \quad (20)$$

$$T_{\theta d} = \int_{-h}^{h} \left[\cos(\beta_o z' \sin\theta) - \cos(\beta_o h \sin\theta) \right] K_d(0, z') dz' \, , \quad (21)$$

$$T_{id} = \int_{-h}^{h} \left[2 \sin\beta_o h \cos\beta_o d \cos\beta_o z' - \cos\beta_o h (\sin\beta_o |z'-d| + \sin\beta_o |z'+d|) \right] K_d(0, z') dz' \, . \quad (22)$$

Equation (19) is not the final form for $I_z^s(z)$ because $A_z^s(h)$ and $\left[I_z^s(d) + I_z^s(-d) \right]$ in the right hand side of (19) are still unknown.

$\left[I_z^s(d) + I_z^s(-d) \right]$ can be determined directly from (19) as

$$\left[I_z^s(d)+I_z^s(-d)\right] = \frac{-j8\pi}{D_1 \zeta_o T_{cd}} \sec\beta_o h \left[jv_o A_z^s(h) \right.$$

$$\left. - \frac{E_o}{\beta_o \cos\theta} \cos(\beta_o h \sin\theta) \right] \left[(1-\cos\beta_o h)(\cos\beta_o d - \cos\beta_o h) \right]$$

$$+ \frac{-j8\pi}{D_1 \zeta_o T_{\theta d}} \frac{E_o}{\beta_o \cos\theta} \left[1-\cos(\beta_o h \sin\theta) \right] \left[\cos(\beta_o d \sin\theta) - \cos(\beta_o h \sin\theta) \right] , \qquad (23)$$

where

$$D_1 = 1 + \frac{jZ_L}{15T_{id}} \sec\beta_o h \cos\beta_o d \sin^2\beta_o (h-d) \ . \qquad (24)$$

$A_z^s(h)$ can be determined from (11), (19) and (23) as follows:

$$A_z^s(h) = \frac{j\mu_o E_o}{D_2 120\pi\beta_o \cos\theta} \left\{ \cos(\beta_o h \sin\theta)(\sec\beta_o h - 1) \left[\frac{T_{ca}}{T_{cd}} \right. \right.$$

$$\left. - \frac{jZ_L}{30 D_1 T_{cd}} \frac{T_{ia}}{T_{id}} \sin\beta_o (h-d)(\sec\beta_o h \cos\beta_o d - 1) \right]$$

$$- \left[1-\cos(\beta_o h \sin\theta) \right] \left[\frac{T_{\theta a}}{T_{\theta d}} \right.$$

$$\left. \left. - \frac{jZ_L}{30 D_1 T_{\theta d}} \frac{T_{ia}}{T_{id}} \sec\beta_o h \sin\beta_o (h-d) \left[\cos(\beta_o d \sin\theta) - \cos(\beta_o h \sin\theta) \right] \right] \right\} , \qquad (25)$$

where

$$D_2 = 1 - (\sec\beta_o h - 1) \frac{T_{ca}}{T_{cd}} + \frac{jZ_L}{30 D_1 T_{cd}} \frac{T_{ia}}{T_{id}} \sin\beta_o (h-d)(\sec\beta_o h - 1) \cdot$$

$$\cdot (\sec\beta_o h \cos\beta_o d - 1) , \qquad (26)$$

$$T_{ca} = \int_{-h}^{h} (\cos\beta_o z' - \cos\beta_o h) K_a(h, z') dz' \quad , \tag{27}$$

$$T_{\theta a} = \int_{-h}^{h} \Big[\cos(\beta_o z' \sin\theta) - \cos(\beta_o h \sin\theta)\Big] K_a(h, z') dz' \quad , \tag{28}$$

$$T_{ia} = \int_{-h}^{h} \Big[2\sin\beta_o h \cos\beta_o d \cos\beta_o z' - \cos\beta_o h(\sin\beta_o|z'-d| + \sin\beta_o|z'+d|)\Big] K_a(h, z') dz' \tag{29}$$

With (19), (23) and (25), the final solution for $I_z^s(z)$ can be summarized as follows:

$$I_z^s(z) = \frac{jE_o}{30\beta_o \cos\theta} \Big\{ F_c(\cos\beta_o z - \cos\beta_o h) + F_\theta\Big[\cos(\beta_o z \sin\theta) - \cos(\beta_o h \sin\theta)\Big] + F_i\Big[2\sin\beta_o h \cos\beta_o d \cos\beta_o z - \cos\beta_o h(\sin\beta_o|z-d| + \sin\beta_o|z+d|)\Big]\Big\} \quad , \tag{30}$$

where

$$F_c = \frac{1}{T_{cd}}(\sec\beta_o h - 1)\Big\{\cos(\beta_o h \sin\theta) + \frac{1}{D_2}\Big[\cos(\beta_o h \sin\theta)(\sec\beta_o h - 1)\frac{T_{ca}}{T_{cd}} - \Big[1 - \cos(\beta_o h \sin\theta)\Big]\frac{T_{\theta a}}{T_{\theta d}}\Big] - \frac{jZ_L}{30D_1D_2}\frac{T_{ia}}{T_{id}}\sin\beta_o(h-d)\Big[\cos(\beta_o h \sin\theta)(\sec\beta_o h - 1)(\sec\beta_o h \cos\beta_o d - 1)\frac{1}{T_{cd}} - \sec\beta_o h\Big[1 - \cos(\beta_o h \sin\theta)\Big]\Big[\cos(\beta_o d \sin\theta) - \cos(\beta_o h \sin\theta)\Big]\frac{1}{T_{\theta d}}\Big]\Big\} , \tag{31}$$

$$F_\theta = \frac{-1}{T_{\theta d}} \left[1 - \cos(\beta_o h \sin\theta) \right] \quad , \tag{32}$$

$$F_i = \frac{-jZ_L}{30D_1 T_{id}} \sin\beta_o(h-d) \left\{ F_c(\sec\beta_o h \cos\beta_o d - 1) - \frac{1}{T_{\theta d}} \sec\beta_o h \left[1 - \cos(\beta_o h \sin\theta) \right] \cdot \right.$$

$$\left. \cdot \left[\cos(\beta_o d \sin\theta) - \cos(\beta_o h \sin\theta) \right] \right\} . \tag{33}$$

We see that the symmetrical component of the induced current $I_z^s(z)$ is a function of the cylinder dimensions (h, a), the impedance (Z_L) and the position (d) of the loading, and the incidence angle (θ) and the magnitude (E_o) of the incident plane wave.

IV
ANTISYMMETRICAL COMPONENT OF INDUCED CURRENT

In this section, $I_z^a(z)$ is determined from (16). By the same reasoning as in the preceding section we can assume $I_z^a(z)$ as

$$I_z^a(z) = C_m\left[\sin(\beta_o h\sin\theta)\sin\beta_o z - \sin\beta_o h\sin(\beta_o z\sin\theta)\right]$$
$$+ C_n\left[2\cos\beta_o h\sin\beta_o d\sin\beta_o z + \sin\beta_o h(\sin\beta_o|z-d| - \sin\beta_o|z+d|)\right] . \tag{34}$$

An important relation,

$$I(d) - I(-d) = I_z^s(d) - I_z^s(-d) + I_z^a(d) - I_z^a(-d) = I_z^a(d) - I_z^a(-d) \quad , \tag{35}$$

is also needed in the analysis.

The constants, C_m and C_n, can be determined approximately by substituting (34) in (16) and letting $z = h/2$. It is also assumed that the first term of the right hand side of (16) is negligible compared with the other terms. This assumption is not rigorously justified but it gives a reasonable answer (King, 1956). The final results for C_m and C_n are:

$$C_m = \frac{E_o}{30\beta_o\cos\theta}\sin\left(\frac{\beta_o h}{2}\sin\theta\right)\left[\cos\left(\frac{\beta_o h}{2}\sin\theta\right)\sec\left(\frac{\beta_o h}{2}\right) - 1\right]\frac{1}{T_{ma}} \quad , \tag{36}$$

$$C_n = \frac{-j}{120}Z_L\left[I_z^a(d) - I_z^a(-d)\right]\sec\frac{\beta_o h}{2}\begin{bmatrix}-\sin\beta_o d, \text{ for } h/2 > d \\ \\ \sin\beta_o(h-d), \text{ for } d > h/2\end{bmatrix}\frac{1}{T_{na}} \quad , \tag{37}$$

where

$$T_{ma} = \int_{-h}^{h}\left[\sin(\beta_o h\sin\theta)\sin\beta_o z' - \sin\beta_o h\sin(\beta_o z'\sin\theta)\right]K_a(h/2, z')dz' \quad , \tag{38}$$

$$T_{na} = \int_{-h}^{h} \Big[2\cos\beta_o h \sin\beta_o d \sin\beta_o z' + \sin\beta_o h(\sin\beta_o |z'-d| - \sin\beta_o |z'+d|)\Big] K_z(h/2, z')dz' \quad . \qquad (39)$$

After (36) and (37) are substituted in (34), $\left[I_z^a(d) - I_z^a(-d)\right]$ can be obtained as

$$\left[I_z^a(d) - I_z^a(-d)\right] = \frac{E_o}{30\beta_o \cos\theta} \sin\left(\frac{\beta_o h}{2}\sin\theta\right)\left[\cos\left(\frac{\beta_o h}{2}\sin\theta\right)\sec\frac{\beta_o h}{2} - 1\right]\frac{2}{T_{ma}D_3} \cdot$$

$$\cdot \Big[\sin(\beta_o h \sin\theta)\sin\beta_o d - \sin\beta_o h \sin(\beta_o d \sin\theta)\Big] \, , \qquad (40)$$

where

$$D_3 = 1 - \frac{jZ_L}{30T_{na}} \sec\frac{\beta_o h}{2} \sin\beta_o d \sin\beta_o (h-d) \begin{bmatrix} \sin\beta_o d \, , & \text{for } h/2 > d \\ \sin\beta_o (h-d), & \text{for } d > h/2 \end{bmatrix} . \qquad (41)$$

With (34), (36), (37) and (40), the antisymmetrical component of the induced current $I_z^a(z)$ can be expressed as follows:

$$I_z^a(z) = \frac{E_o}{30\beta_o \cos\theta} \Big\{F_m \Big[\sin(\beta_o h \sin\theta)\sin\beta_o z - \sin\beta_o h \sin(\beta_o z \sin\theta)\Big]$$

$$+ F_n \Big[2\cos\beta_o h \sin\beta_o d \sin\beta_o z + \sin\beta_o h(\sin\beta_o |z-d| - \sin\beta_o |z+d|)\Big]\Big\} \, , \qquad (42)$$

where

$$F_m = \sin\left(\frac{\beta_o h}{2}\sin\theta\right)\left[\cos\left(\frac{\beta_o h}{2}\sin\theta\right)\sec\frac{\beta_o h}{2} - 1\right]\frac{1}{T_{ma}} \quad , \qquad (43)$$

$$F_n = F_m\left(\frac{-jZ_L}{60D_3}\right)\frac{1}{T_{na}}\Big[\sin(\beta_o h\sin\theta)\sin\beta_o d$$

$$-\sin\beta_o h\sin(\beta_o d\sin\theta)\Big]\sec\frac{\beta_o h}{2}\begin{bmatrix}\sin\beta_o d\,, & \text{for } h/2 > d\\ \sin\beta_o(h-d), & \text{for } d > h/2\end{bmatrix}. \quad (44)$$

$I_z^a(z)$ is also determined as a function of the cylinder dimensions, the impedance and location of the loading, and the incident angle and magnitude of the plane wave.

V
OPTIMUM LOADING FOR ZERO BROADSIDE BACK SCATTERING

The total induced current on the cylinder is

$$I_z(z) = I_z^s(z) + I_z^a(z) \ . \tag{45}$$

This total current maintains the scattered field. As far as the back scattered field is concerned, $I_z^s(z)$ maintains a large broadside back scatter and $I_z^a(z)$ maintains a large off-broadside back scatter at $\theta \simeq 40^o$ when the cylinder has an anti-resonant length.

In this section, we shall determine an optimum loading to achieve zero broadside back scattering. With this optimum loading, $I_z^s(z)$ has a minimum amplitude and an appropriate phase distribution along the cylinder.

When the plane wave is incident broadside ($\theta = 0^o$) on the cylinder, the induced current on the cylinder is

$$I_z^s(z) = \frac{jE_o}{30\beta_o}\left\{F_c(\cos\beta_o z - \cos\beta_o h) + F_i\Big[2\sin\beta_o h\cos\beta_o d\cos\beta_o z - \cos\beta_o h(\sin\beta_o|z-d| + \sin\beta_o|z+d|)\Big]\right\} , \tag{46}$$

where

$$F_c(\theta = 0^o) = \frac{1}{T_{cd}D_2}(\sec\beta_o h - 1) , \tag{47}$$

$$F_i(\theta = 0^o) = \frac{-jZ_L}{30D_1D_2T_{cd}T_{id}}\sin\beta_o(h-d)(\sec\beta_o h - 1)(\sec\beta_o h\cos\beta_o d - 1) , \tag{48}$$

$$F_\theta(\theta = 0^o) = 0 \ . \tag{49}$$

The antisymmetrical component of the induced current $I_z^a(z) = 0$ when $\theta = 0^o$.

The back scattered field in the broadside direction can be found as

$$E_{scat}(\theta=0^o) = \frac{j\omega\mu_o}{4\pi R_o} e^{-j\beta_o R_o} \int_{-h}^{h} I_z^s(z)dz$$

$$= \frac{2E_o}{\beta_o R_o} e^{-j\beta_o R_o} \Big[F_c(\theta=0^o)(\sin\beta_o h - \beta_o h\cos\beta_o h)$$

$$+ F_i(\theta=0^o)(2\cos\beta_o d - 2\cos\beta_o h) \Big] . \qquad (50)$$

To make $E_{scat}(\theta=0^o) = 0$, we let

$$\frac{F_c(\theta=0^o)}{F_i(\theta=0^o)} = \frac{\sin\beta_o h - \beta_o h\cos\beta_o h}{2(\cos\beta_o d - \cos\beta_o h)} . \qquad (51)$$

The optimum impedance for zero broadside back scattering $\left[Z_L\right]_o^s$ can be determined from (51), (47) and (48) as follows:

$$\left[Z_L\right]_o^s = \frac{-j15T_{id}\cos\beta_o h(\sin\beta_o h - \beta_o h\cos\beta_o h)}{\sin\beta_o(h-d)(\cos\beta_o d - \cos\beta_o h)^2 - \cos\beta_o d\sin^2\beta_o(h-d)(\sin\beta_o h - \beta_o h\cos\beta_o h)} . \qquad (52)$$

$\left[Z_L\right]_o^s$ is thus determined as a function of the cylinder dimensions (h, d) and the position (d) of the loading. The form for $\left[Z_L\right]_o^s$ is quite simple and should prove useful in pracitcal design. The numerical value of $\left[Z_L\right]_o^s$ is readily obtained once T_{id} as defined in (22) is calculated by a digital computer. It is noted that $\left[Z_L\right]_o^s$ reduces to the corresponding value for central loading (Chen and Leipa, 1964b) when $d=0$.

Two numerical examples for $\left[Z_L\right]_o^s$ are given in Figs. 2 and 3. In Fig. 2 we show $\left[Z_L\right]_o^s$ for the case of $a=0.0173\lambda$ and $d=h/2$ as a function of the cylinder length h/λ. In this case two impedances are loaded at the centers of two halves of

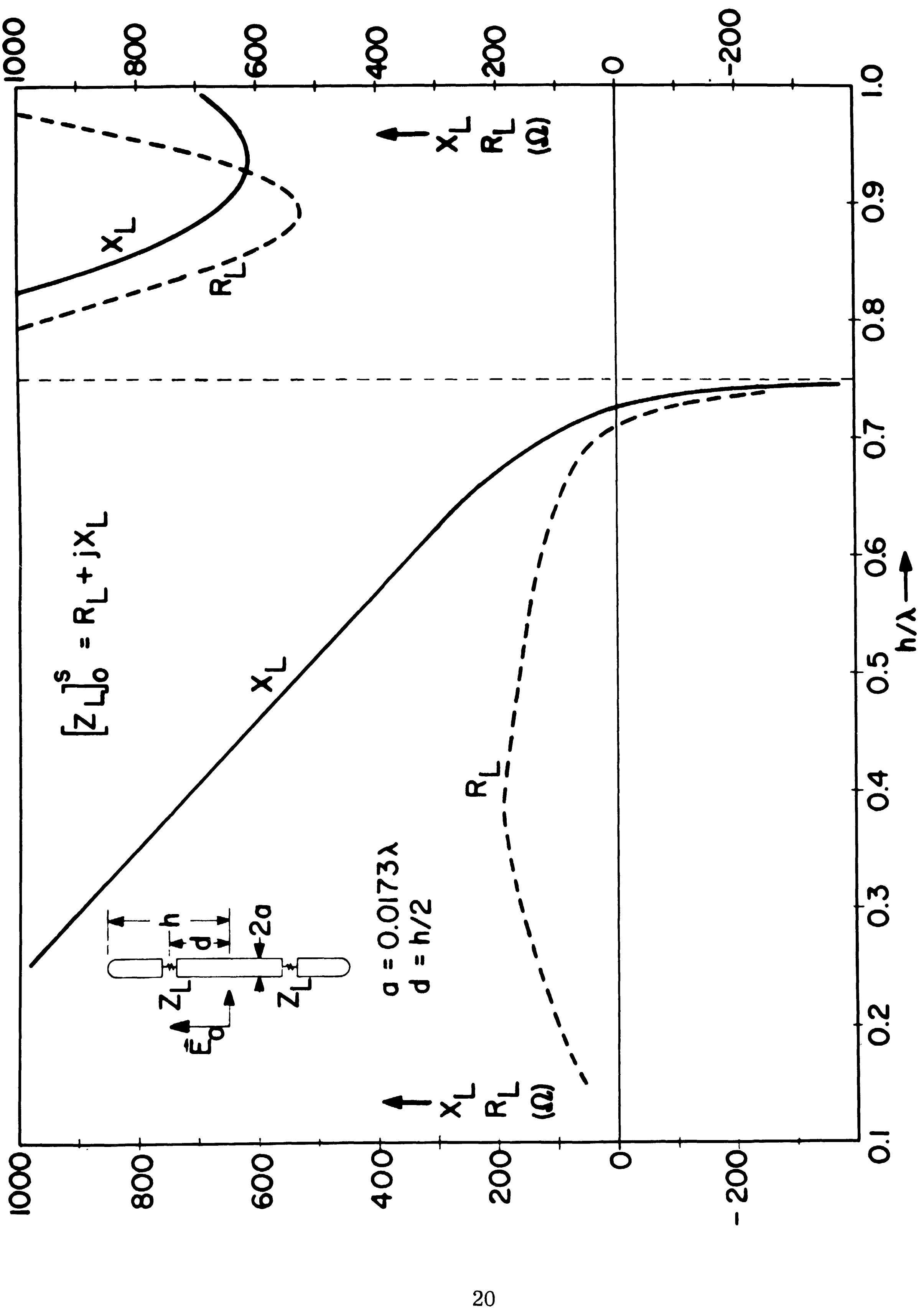

FIG. 2: OPTIMUM IMPEDANCE FOR ZERO BROADSIDE BACK SCATTERING VS CYLINDER LENGTH WITH d = h/2.

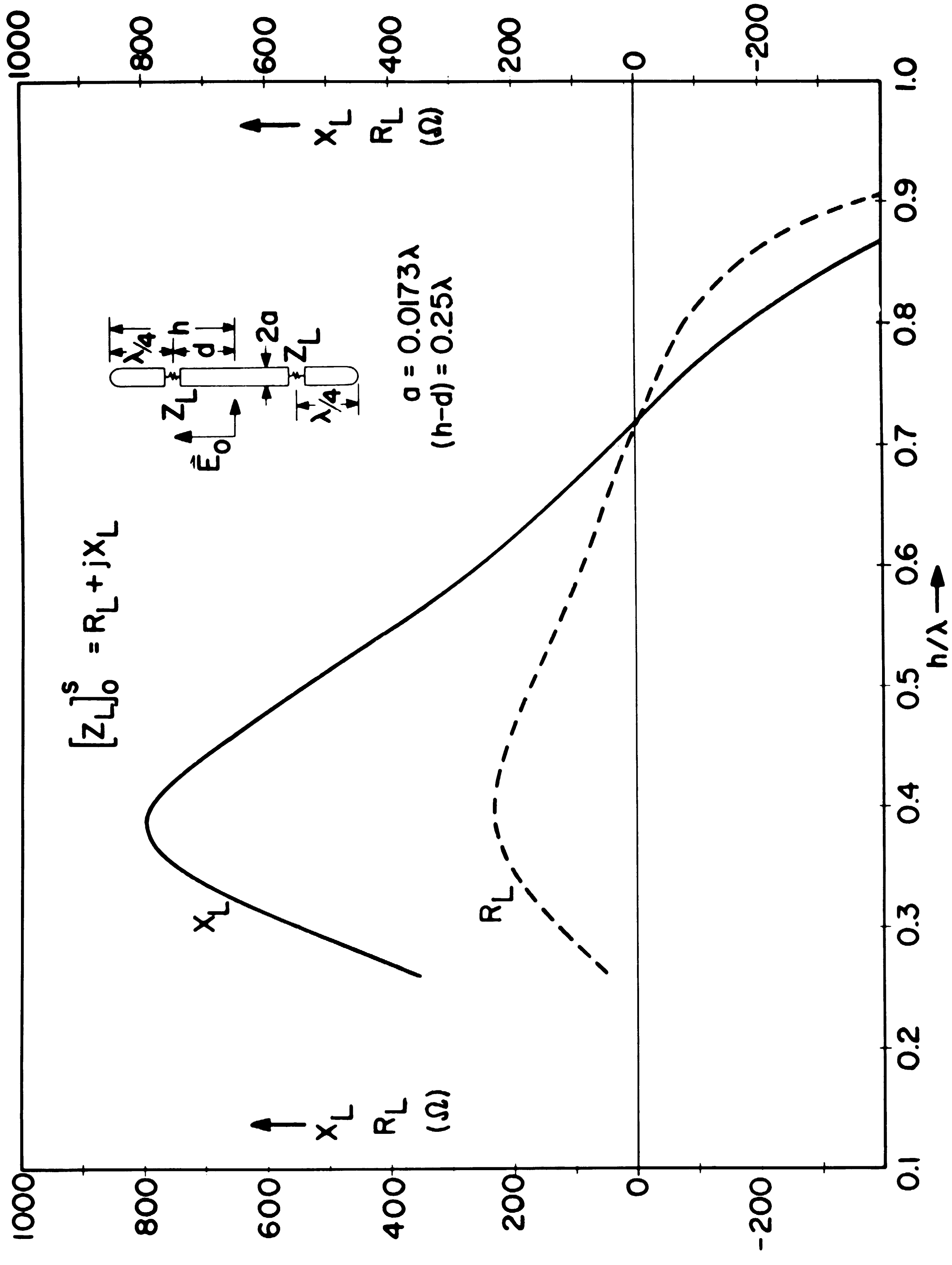

FIG. 3: OPTIMUM IMPEDANCE FOR ZERO BROADSIDE BACK SCATTERING VS CYLINDER LENGTH WITH h - d = λ/4.

the cylinder and $\left[Z_L\right]_o^s$ behaves very smoothly up to $h = 0.7\lambda$. In Fig. 3, $\left[Z_L\right]_o^s$ for the case of $a = 0.0173\lambda$ and $h - d = \lambda/4$ is shown graphically as a function of the cylinder length h/λ. In this case two impedances are loaded at the points a quarter-wavelength from the ends of the cylinder and $\left[Z_L\right]_o^s$ requires a negative resistance for $h > 0.7\lambda$. From these two examples we see that $\left[Z_L\right]_o^s$ is dependent on the position of the loading and it appears possible to have a passive impedance for $\left[Z_L\right]_o^s$ up to $h = \lambda$ by a proper choice of d. In this way we can avoid using an active impedance which is necessary for central loading (Chen and Liepa, 1964b).

VI
OPTIMUM LOADING FOR LOW OFF-BROADSIDE BACK-SCATTERING

When a cylinder has a resonant length $(2h \simeq (2n+1)\lambda/2)$, the symmetrical component of the induced current $I_z^s(z)$ is the predominant component and $\left[Z_L\right]_o^s$ is adequate to minimize the scattered field in the broadside direction and other directions as well (Chen and Liepa, 1964a). But when a Cylinder has an anti-resonant length $(2h \simeq n\lambda)$, the antisymmetrical component of the induced current $I_z^a(z)$ can be very large and it gives a large off-broadside return. In this section we seek an optimum impedance which keeps $I_z^a(z)$ small along the cylinder, resulting in a small off-broadside back scatter.

For a cylinder shorter than two wavelengths and with two impedances loaded at $z = \pm d$, the distribution of $I_z^a(z)$ along one half of the cylinder can be shown from (42) to be the sum of two shifted sinusoidal curves. After the distribution of $I_z^a(z)$ is studied graphically, one would write the simplest criterion for small $I_z^a(z)$ and low off-broadside back scattering as

$$\int_0^h I_z^a(z)\, dz = 0 \qquad \text{(at a particular } \theta) \quad . \tag{53a}$$

Notice that the lower limit of the integral is 0 instead of -h. The condition imposed in (53a) usually requires $I_z^a(z)$ to be small and causes cancellation between two shifted sinusoidal curves. This criterion is rather arbitrary but quite sufficient, as we can see in the Appendix that $I_z^a(z)$ under (53a) becomes very small indeed. An alternative and perhaps more accurate criterion, such as demanding that

$$\int_{-h}^h I_z^a(z)\, e^{j\beta_o z \sin\theta}\, dz = 0 \qquad \text{(at a particular } \theta) \tag{53b}$$

or requiring that the maximum off-broadside back scatter due to $I_z^a(z)$ equal zero in the θ direction, would be more rigorous, but the algebraic evaluation is more involved. In case more rigorous treatment is desired, (53b) may be used.

For the sake of simplicity, (53a) is used as the criterion in this report and the corresponding optimum impedance is obtained as follows.

The substitution of (42) in (53a) gives

$$\frac{F_n}{F_m} = -\frac{\left[\sin\beta_o h\cos(\beta_o h\sin\theta) - \sin\beta_o h + \sin(\beta_o h\sin\theta)\sin\theta - \sin(\beta_o h\sin\theta)\sin\theta\cos\beta_o h\right]}{2\sin\theta\left[\sin\beta_o h - \sin\beta_o d - \sin\beta_o(h-d)\right]}. \tag{54}$$

An optimum impedance $\left[Z_L\right]_o^a$ which gives small $I_z^a(z)$ and low off-broadside back scattering can be solved from (54) after the substitution of (43) and (44). The final expression for $\left[Z_L\right]_o^a$ is

$$\left[Z_L\right]_o^a = \frac{-j15T_{na}\left[\sin\beta_o h\cos(\beta_o h\sin\theta) - \sin\beta_o h + \sin(\beta_o h\sin\theta)\sin\theta - \sin(\beta_o h\sin\theta)\sin\theta\cos\beta_o h\right]}{\sin\frac{\beta_o h}{2}\begin{bmatrix}\sin(\beta_o h\sin\theta)\sin\beta_o d\sin\theta\left[1-\cos\beta_o(h-d)\right] \\ +\sin(\beta_o d\sin\theta)\sin\theta\left[\sin\beta_o(h-d) - \sin\beta_o h + \sin\beta_o d\right] \\ +\sin\beta_o d\sin\beta_o(h-d)\left[\cos(\beta_o h\sin\theta) - 1\right]\end{bmatrix}\begin{bmatrix}\sin\beta_o d, \text{ for } \frac{h}{2} > d \\ \sin\beta_o(h-d), \text{for } d > \frac{h}{2}\end{bmatrix}} \tag{55}$$

where T_{na} was defined in (39). $\left[Z_L\right]_o^a$ is a function of the cylinder dimensions (h, a), the location d of the loading and the incident angle θ of the plane wave.

In the practical application, $\left[Z_L\right]_o^a$ is calculated from (55) by assigning θ as the angle where the maximum off-broadside back scatter occurs. Usually this angle is about 40^o off the broadside direction. A numerical example is shown in Fig. 4. We consider a cylinder with $a = 0.0173\lambda$, $d = h/2$ and $\theta = 45^o$. $\left[Z_L\right]_o^a$ for this case is passive and well behaved up to $h = 0.95\lambda$ as shown in Fig. 4.

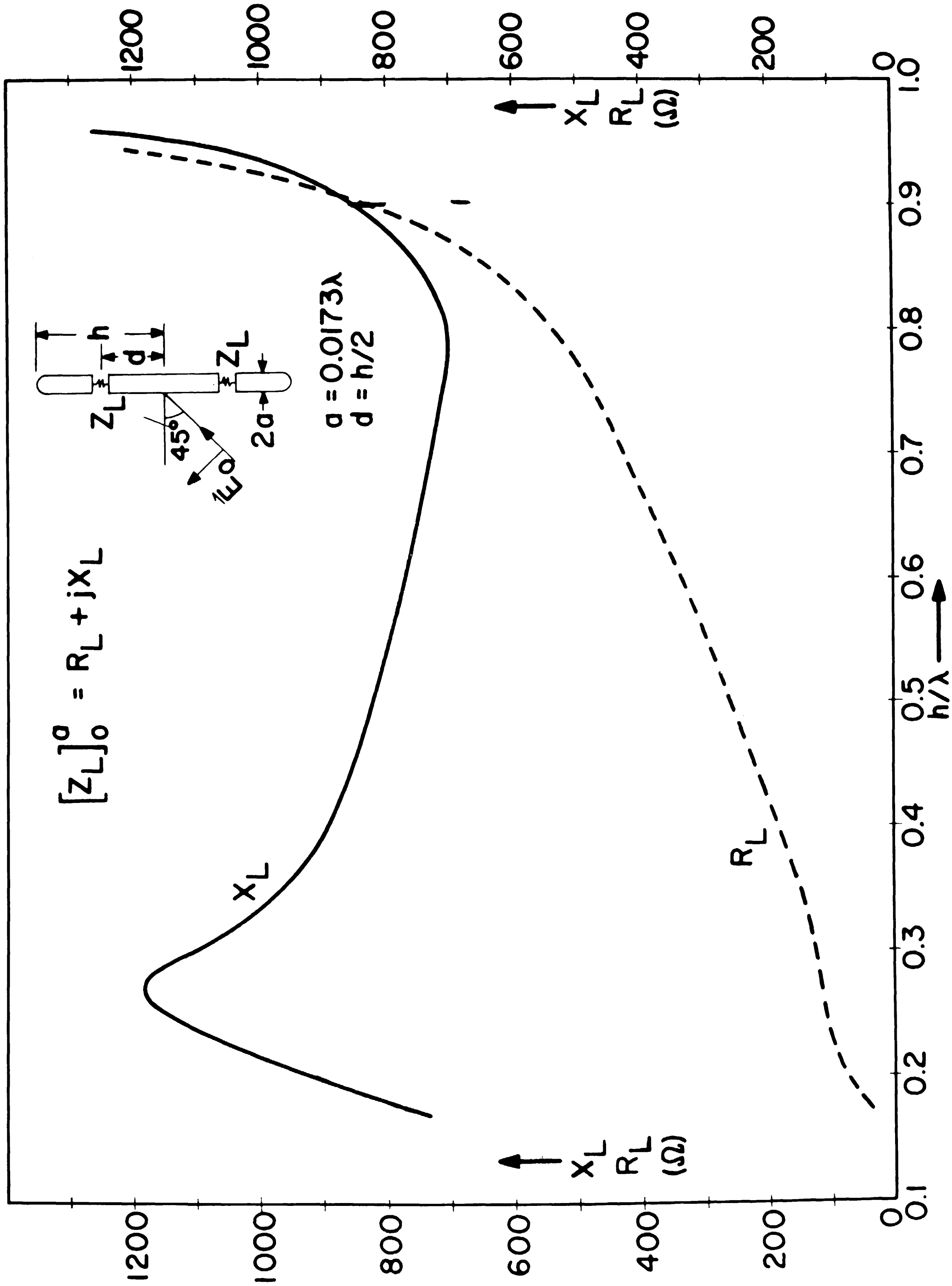

FIG. 4: OPTIMUM IMPEDANCE FOR MINIMUM ANTISYMMETRICAL CURRENT WITH 45^o INCIDENCE ANGLE, d = h/2.

VII
OPTIMUM LOADING FOR LOW BACK SCATTERING OVER A WIDE ASPECT RANGE

If the back scattering cross section of a cylinder with $2h < 2\lambda$ is plotted as a function of the aspect angle θ, there are three possible peaks. The first peak occurs at the broadside direction ($\theta = 0^o$) for a cylinder with any length between $\lambda/4 \leqslant 2h \leqslant 2\lambda$. The two other peaks occur at each side of the first peak and about 40^o off the broadside direction. The off-broadside back scatter is insignificant for a resonant cylinder but it is very large for an anti-resonant cylinder. In the preceding sections, $\left[Z_L\right]_o^s$ is designed to eliminate the broadside back scatter and $\left[Z_L\right]_o^a$ is designed to reduce the off-broadside scatter separately. It is then desirable to design an omptimum impedance $\left[Z_L\right]_o$ to function as $\left[Z_L\right]_o^s$ and $\left[Z_L\right]_o^a$ simultaneously. Actually $\left[Z_L\right]_o^s$ and $\left[Z_L\right]_o^a$ are both functions of d and it is possible to adjust the value of d to make $\left[Z_L\right]_o^s$ and $\left[Z_L\right]_o^a$ equal or nearly equal. If such an empedance $\left[Z_L\right]_o$ and the position d are obtainable, the back scattering of a cylinder can be reduced greatly over a wide aspect range.

To seek an optimum impedance $\left[Z_L\right]_o$ such as

$$\left[Z_L\right]_o \doteq \left[Z_L\right]_o^s \doteq \left[Z_L\right]_o^a , \tag{56}$$

it is necessary to find an optimum value of d from the following equation.

$$\frac{T_{id}(d)}{T_{na}(d)} \doteq$$

$$\frac{\left[\sin\beta_o(h-d)(\cos\beta_o d - \cos\beta_o h)^2 - \cos\beta_o d \sin^2\beta_o(h-d)(\sin\beta_o h - \beta_o h\cos\beta_o h)\right] \cdot \left[\sin\beta_o h\cos(\beta_o h\sin\theta) - \sin\beta_o h + \sin(\beta_o h\sin\theta)\sin\theta - \sin(\beta_o h\sin\theta)\sin\theta\cos\beta_o h\right]}{\cos\beta_o h\sin\frac{\beta_o h}{2}(\sin\beta_o h - \beta_o h\cos\beta_o h)\begin{bmatrix}\sin\beta_o d & \text{for } h/2 > d\\ \sin\beta_o(h-d) & \text{for } d > h/2\end{bmatrix} \cdot \begin{bmatrix}\sin(\beta_o h\sin\theta)\sin\beta_o d\sin\theta\left[1-\cos\beta_o(h-d)\right]\\ +\sin(\beta_o d\sin\theta)\sin\theta\left[\sin\beta_o(h-d) - \sin\beta_o h + \sin\beta_o d\right]\\ +\sin\beta_o d\sin\beta_o(h-d)\left[\cos(\beta_o h\sin\theta) - 1\right]\end{bmatrix}} \cdot \qquad (57)$$

Actually, T_{id} and T_{na} are both complex numbers and it is only possible to find an optimum d which makes both sides of (57) nearly equal. After the optimum d is determined from (57), $\left[Z_L\right]_o$ can be obtained easily from (52) or (55).

A numerical example is given in Fig. 5. $\left[Z_L\right]_o$ and d for a cylinder of $a = 0.0173\lambda$ is shown graphically as functions of the cylinder length h/λ. We observe that d varies almost linearly with h and $\left[Z_L\right]_o$ is passive and well-behaved in the entire range of $\lambda/4 \leqslant 2h \leqslant 2\lambda$. This $\left[Z_L\right]_o$ appears to be obtainable by a simple network synthesis.

One remaining question is whether one can design an appropriate double loading for a fixed cylinder to minimize its back scattering cross section over a wide frequency range. The answer is self evident. We can calculate $\left[Z_L\right]_o$ and d for a fixed cylinder as a function of frequency by the same technique employed in this section. Of course, the calculation will be somewhat complicated because $\beta_o h$ and $\beta_o a$ both vary with frequency in this case and we can expect similar sets of curves for $\left[Z_L\right]_o$ and d as those shown in Fig. 5.

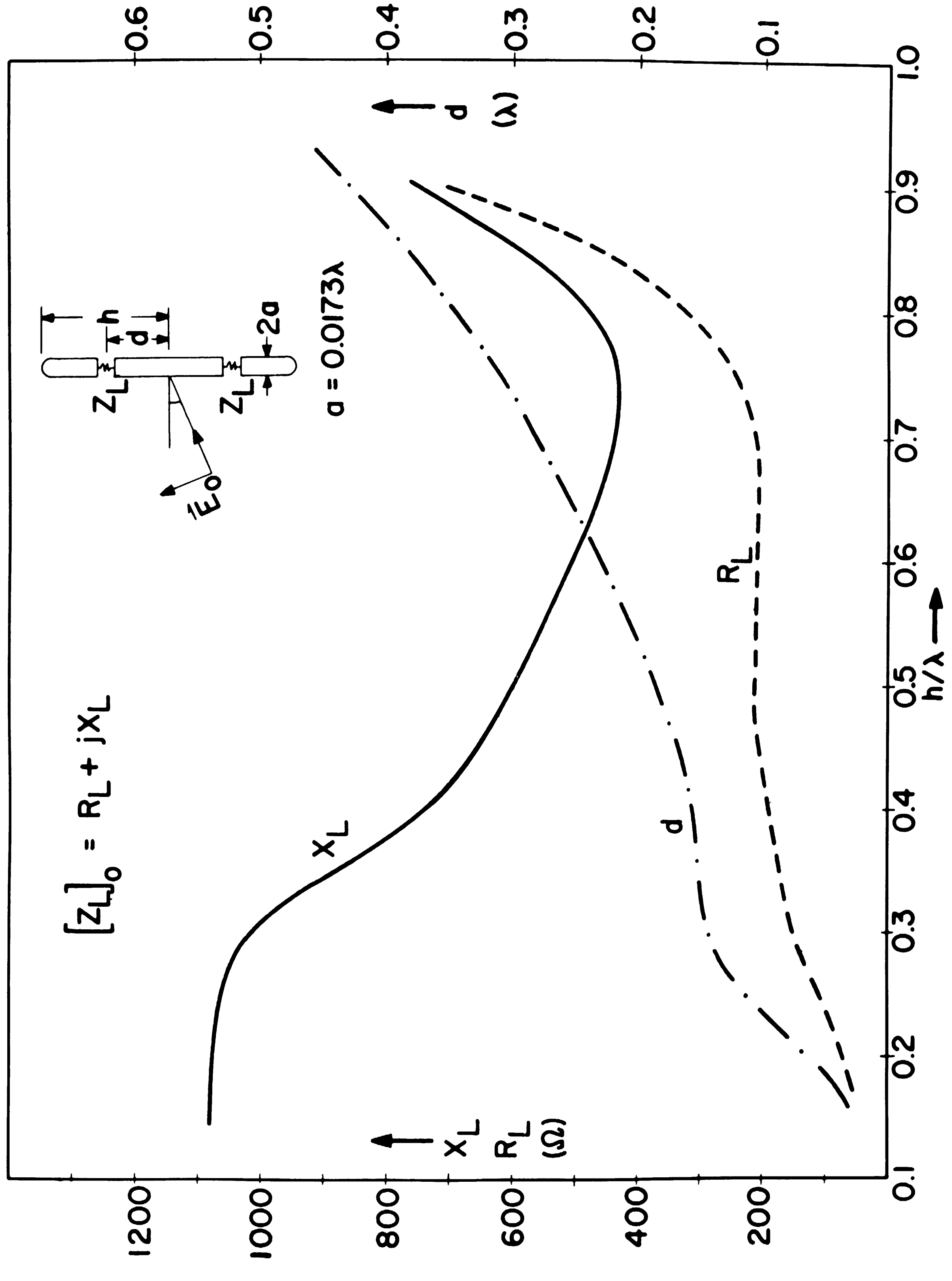

FIG. 5: OPTIMUM IMPEDANCE AND OPTIMUM POSITION FOR DOUBLE LOADING TO MINIMIZE BACK SCATTERING OVER A WIDE ASPECT RANGE.

VIII
CONCLUSION

The induced current on a doubly-loaded cylinder when illuminated by a plane wave at an arbitrary angle is obtained as a function of the cylinder dimensions, the impedance and the position of the double loading and the amplitude and the incidence angle of the plane wave. The optimum loading to eliminate the broadside back scattering is established. A suitable loading to reduce the off-broadside back scattering is also determined. The combination of these two techniques enables one to design an optimum loading to reduce the back scattering over a wide aspect range. Some reasonalby simple formulas for optimum loading derived in this paper should prove useful in practical design. The accuracy of this theory has been justified in a central loading case (Chen and Liepa, 1964a) by experiment and we feel that no additional experimental check is needed for the double loading case.

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APPENDIX
ANTISYMMETRICAL COMPONENT OF INDUCED CURRENT WITH AND WITHOUT OPTIMUM LOADING

To show the effectiveness of $\left[Z_L\right]_o^a$ which is derived under the condition of (53a) in reducing $I_z^a(z)$ and off-broadside back scattering, $I_z^a(z)$ on some cylinders are calculated for the cases with and without $\left[Z_L\right]_o^a$. We consider the cylinders of various lengths and with $A = 0.0173\lambda$, $d = h/2$ and $\theta = 45^o$. The numerical results are shown graphically in Fig. 6. The solid lines represent $I_z^a(z)$ without $\left[Z_L\right]_o^a$ and the dashed lines represent $I_z^a(z)$ with $\left[Z_L\right]_o^a$. From Fig. 6, it is evident that with $\left[Z_L\right]_o^a$ the antisymmetrical component of the induced current is greatly reduced in amplitude and the phase is reversed so that the off-broadside back scattering is reduced to a very low value. An interesting point in Fig. 6 is also worth mentioning. For a cylinder with $h = 0.5\lambda$, $I_z^a(z)$ can be eliminated theoretically with $\left[Z_L\right]_o^a$ while without $\left[Z_L\right]_o^a$, $I_z^a(z)$ would be very large on this cylinder. It is noted that $I_z^a(z)$ without $\left[Z_L\right]_o^a$ for various cylinders are drawn in comparable scale for the sake of clarity in Fig. 6. Actually, $I_z^a(z)$ on a cylinder of $h = 0.5\lambda$ would be much larger than that on a cylinder of $h = 0.25\lambda$.

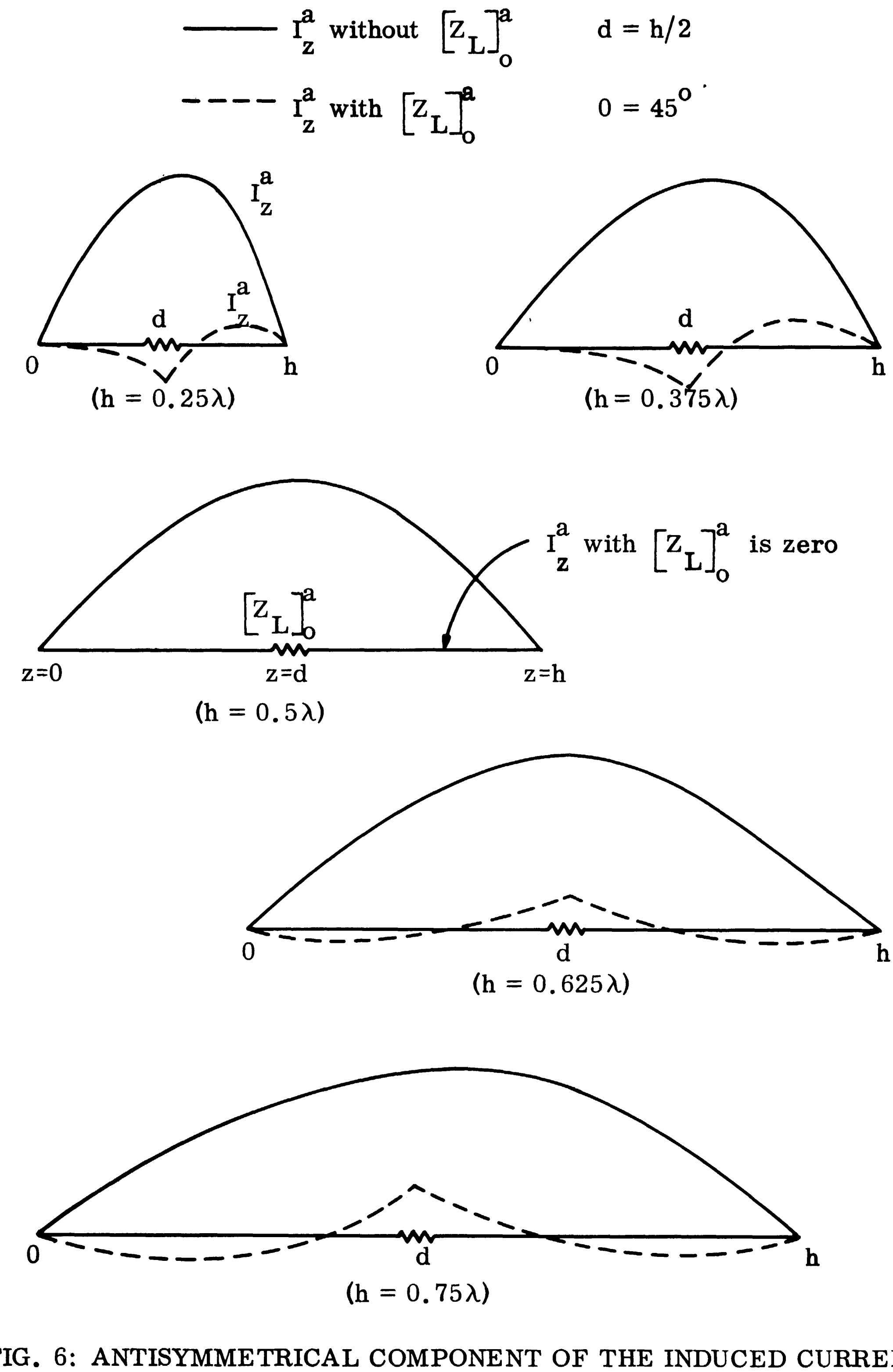

FIG. 6: ANTISYMMETRICAL COMPONENT OF THE INDUCED CURRENT WITH AND WITHOUT OPTIMUM LOADING $[Z_L]_o^a$.

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A theory on the minimization of the back scattering of a cylinder by loading a cylinder at two points with lumped impedances is presented. The induced current on a doubly loaded cylinder when illuminated by a plane electromagnetic wave at an arbitrary angle is obtained. The optimum loading to eliminate the broadside back scattering which is caused by the symmetrical component of the induced current is established. A suitable loading to reduce the off-broadside back scattering, due to the antisymmetrical component of the induced current, is also determined. A proper choice of the impedance and the position of double loading can lead to the reduction of the back scattering over a wide aspect range. The useful formulas for optimum loading and some graphical illustrations are included. The advantages of double loading over central loading are discussed.

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