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OCCUPANT DYNAMICS SIMULATION WITH USER-SPECIFIED NUMBER OF BODY SEGMENTS (VARISEG)

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OCCUPANT DYNAMICS SIMULATION WITH USER-SPECIFIED NUMBER OF BODY SEGMENTS (VARISEG)

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CONTENTS

FIGURES

TABLES

1.0 INTRODUCTION

The Variseg analysis which is presented in this report lays the foundation for a new class of occupant dynamics simulation models. It follows the lead established in the MVMA 2-D Model (1), the Calspan CVS (2), and the UM-VCS-1 vehicle crashworthiness simulation (3). The analytical concepts which are used make possible the simulation of various problems in human protection such as:

- deformable joint structures (e.g., knee joint for use in design studies of restraint bolsters)
- addition of an ankle joint for more accurate prediction of lower extremity kinematics in frontal impact
- loose coupling of flesh mass (e.g., the leg) with the skeletal linkage to compare cadaver, human, and dummy responses measured in experiments
- breakage of bones and joints in pedestrian impacts with the front of a vehicle
- the inclusion of a multijoint neck and spine to provide a more flexible torso linkage.

In addition, they bring the user one step closer to simultaneously tapping the power of rigid body dynamic and finite element analytical tools. Variseg, as the name implies, is intended to allow the user to specify an arbitrary number of segments connected in an arbitrary manner with joints of recognized characteristics. As a program, Variseg will be Version Five of the MVMA Two Dimensional Model.

Section 1.1 The MVMA Two Dimensional Model

The MVMA Two Dimensional Model is the current descendant of a long chain of simulation programs dating from 1966. The MVMA Two Dimensional Model itself has been through four previous versions and is now a five processor program which expands or contracts to fit the complexity of the current data set. This feature essentially eliminates lost runs due to data deck size and minimizes the storage capabilities necessary for simple runs.

Section 1.2 The Generalizations To Be Made

Yet just as in its earliest predecessor, the MVMA Two Dimensional Model is a fixed linkage, eight mass model. This configuration is more than adequate for most applications, but makes all but impossible the modeling of breaking limbs, knee transverse displacements, an ankle joint, loose leg flesh, etc. The generalizations included in Variseg are designed to offer maximum increased flexibility without making the model usable only by specialists.

1. Instead of eight segments, each of which participate in the equations of motion, the user may specify as many or as few as is needed. Not all of the segments are required to be in the equations of motion but rather can be determined from constraining relationships at the user's option.

2. Instead of a fixed linkage, the user can connect or leave unconnected each specified segment as suits his purpose. Four basic types of connectors and six specialized variants are formulated.

3. Instead of a fixed vehicle for which acceleration profiles in three degrees of freedom may be specified, Variseg allows acceleration profiles or velocity profiles, or position profiles to be specified for any component of any one point of any segment.

4. Instead of allowing applied forces only to the head, Variseg will allow applied forces at any point of any segment.

5. For the user interested in doing so, it will be possible to specify arbitrary coordinate systems and units for each segment for both input and output. The problem will, of course, be solved in consistent coordinate systems and units. One conversion will be made before solution and another after.

6. Ellipses will be generalized to allow both position and orientation as a function of time with respect to any segment.

7. Lines will be generalized to allow attachment to any segment.

8. Where angulation occurs in joints and connectors, one or more of five types of resistance to angulation will be specifiable for both flexion and extension.

9. Where elongation occurs in joints and connectors, one or more of two types of resistance to elongation will be specifiable for both extension and contraction.

10. Other model features such as belts, etc. will be modified minimally to obtain needed information from the new equations.

11. A restart procedure will be added to facilitate continuing runs.

Section 1.3 The Implications of the Generalizations

With the generalizations outlined in the previous section each of the modeling problems mentioned can be attacked. In general, multiple vehicles and/or multiple occupants/pedestrians of varying complexity can be modelled. Certain kinds of simple mechanisms can also be modelled.

All this is achieved without any additional complexity; it is intrinsic in the approach. Also a simple problem can be described by a simple data set. The MVMA 2-D system of defaults will be adjusted so that the user will need to supply only non-standard information. The program will also be modified to accept a free-format input more suitable to current interactive systems but will still accept the keypunch-based previous input formats.

Section 1.4 Report Layout

Section 2 will discuss the basic layout of the analysis. Section 3 will present the types of joints and connectors and develop the equations governing them. Section 4 will deal with the other constraints and their equations. Section 5 is technically not analysis but is included because discussion of analysis by itself would seem incomplete. In a program as dependent on the techniques of implementation as Variseg, the complete analysis can not be understood without consideration of how and when the equations are used.

2.0 COORDINATE SYSTEMS, UNITS, AND EQUATIONS OF MOTION

Coordinate systems abound in Variseg for purposes of user convenience. The coordinate systems are divided into three classes:

1. The External Input Systems, defined by the user for describing the problem,
2. The Internal Systems, in which the problem is solved, and
3. The External Output Systems, defined by the user and used to record the results for printing and postprocessing.

The external coordinate systems are used only for input and output and therefore can be whatever the user wishes them to be. This includes such unusual features as measuring angles positive clockwise and non-orthogonal systems. The default external system definition will be Z positive downward, X positive forward, and angles measured positive counterclockwise as in Version Four of the MVMA 2-D. It is presumed that users will almost always prefer this definition; however, situations have arisen in which unusual systems could have been used to cut down the labor in setting up data sets.

The Internal Systems are used for the solution of the problem and are chosen to yield the simplest equations. After input is complete, the run description is converted from External Input Systems to Internal Systems. After solution at each time point, the results are converted from the Internal Systems to the External Output Systems and recorded for printing and postprocessing. The user can therefore describe the problem in the most convenient manner, have the problem solved as efficiently as possible, and then see the results in the manner he desires.

The External Systems are defined in terms of each other but ultimately in terms of the basic Internal System which is the only predefined system.

The Inertial System has the X-axis pointing forward and the Z-axis pointing upward, a right-hand system. Angulation is measured counterclockwise from the X-axis.

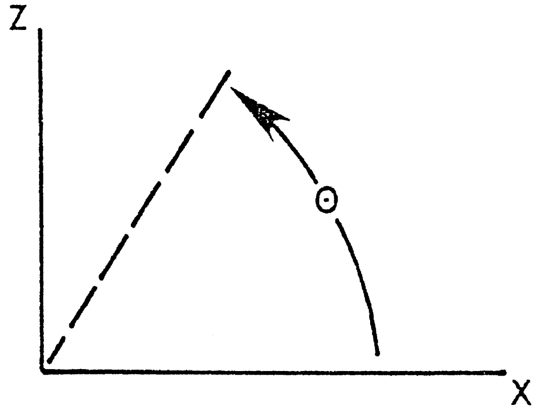

Fig. 1. The Inertial System

Section 2.1 Relationship to Inertial System

All other coordinate systems are specified by supplying:

1. the counterclockwise angle from the positive x-axis of the Inertial System to the positive x-axis of that system,
2. the counterclockwise angle from the positive x-axis to the positive z-axis of that system,
3. the direction indicator for positive angulation in that system, and
4. the x and z coordinates of the system origin with respect to the Inertial System.

(Each User Defined System can actually be defined in terms of another User Defined System or directly in terms of the Inertial System. If the User Defined System is defined recursively, it must be possible to relate the system ultimately to the Inertial system.)

Corresponding to each User Defined System is a Model Defined Internal System which has its x-axis coincident with the User Defined x-axis and has a z-axis completing a right hand, orthogonal system. Angulation of the Internal System is always positive counterclockwise. Figure 2 shows the most general case, a User Defined System, q, which is non-orthogonal and measures angles positive clockwise. Figure 2 also shows the corresponding Internal System and the relationship of both systems to the Inertial System as well as the position of a Point P in space.

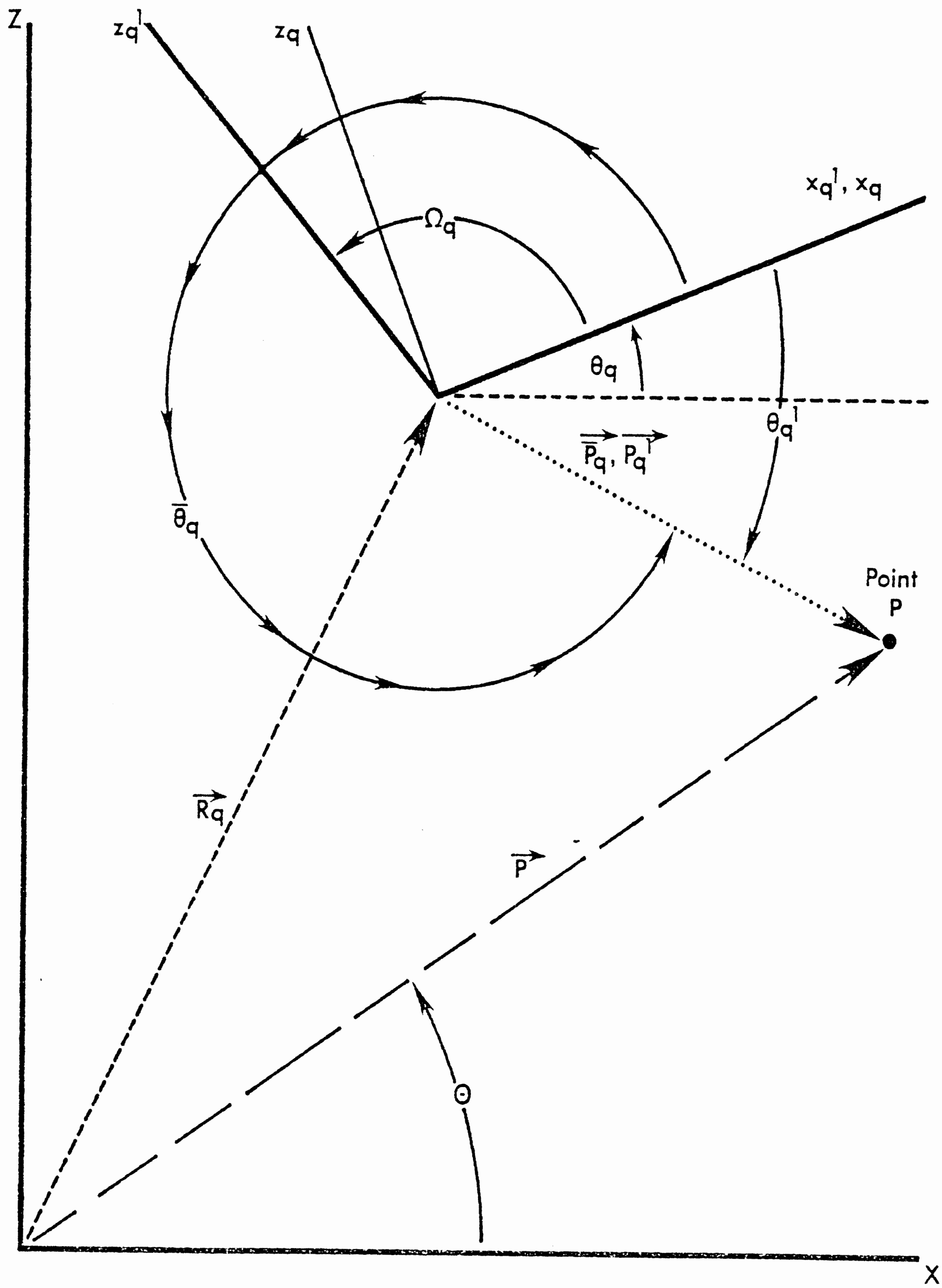

Fig. 2. A User Defined System and its Relationships

In Figure 2,

$\vec{R}_q$ is the position vector for the System q Origin with respect to the Inertial System. This vector is valid for both User Defined System q and Internal System q.

X, Z are the x and z axes of the Inertial System (with unit vectors $\vec{I}, \vec{K}$)

x'_q, z'_q are the x and z axes of User Defined System q (with unit vectors $\vec{i}'_q, \vec{k}'_q$).

Ω_q is the counterclockwise angle between $\vec{i}'_q$ and $\vec{k}'_q$ which must satisfy one of the following:

$$0 < \Omega_q < 180° \text{ or } 0 > \Omega_q > -180°.$$

Θ_q is the Inertial angle between $\vec{I}$ and $\vec{i}'_q$

x_q, z_q are the x and z axes of the Internal System q (with unit vectors $\vec{i}_q, \vec{k}_q$). This is always right-handed and orthogonal.

$\vec{P}'_q, \theta'_q$ are the position vector of Point P and its direction with respect to the User Defined System q.

$\vec{\bar{P}}_q, \bar{\theta}_q$ are the position vector of Point P and its direction with respect to the Internal System q.

$\vec{P}, \Theta$ are the position vector of Point P and its direction with respect to the Inertial System.

Let

$$\vec{P}'_q = p'_{1,q}\,\vec{i}'_q + p'_{2,q}\,\vec{k}'_q \tag{1}$$

$$\vec{\bar{P}}_q = \bar{p}_{1,q}\,\vec{i}_q + \bar{p}_{2,q}\,\vec{k}_q \tag{2}$$

$$\vec{P} = p_1\,\vec{I} + p_2\,\vec{K} \tag{3}$$

$$\vec{R}_q = r_{1,q}\,\vec{I} + r_{2,q}\,\vec{K} \tag{4}$$

We can relate the Internal System and the User Defined System q as follows:

$$\bar{\rho}_{1,q} = \rho'_{1,q} + \rho'_{2,q} \cos \Omega_q$$

$$\bar{\rho}_{2,q} = \rho'_{2,q} \sin \Omega_q \tag{5}$$

and the inverse relationships

$$\rho'_{1,q} = \bar{\rho}_{1,q} - \bar{\rho}_{2,q} \cot \Omega_q$$

$$\rho'_{2,q} = \bar{\rho}_{2,q} \csc \Omega_q \tag{6}$$

The direction of angulation for the User Defined System q is defined by the Angulation Direction Indicator (λ_q)

where $\lambda_q = \begin{cases} 1 & \text{for counterclockwise positive} \\ -1 & \text{for clockwise positive} \end{cases}$

then

$$\bar{\theta}_q = \lambda_q \theta'_q$$

$$\cos \bar{\theta}_q = \cos \theta'_q \tag{7}$$

$$\sin \bar{\theta}_q = \lambda_q \sin \theta'_q$$

The above relationships are used to convert from User Defined Systems to the Internal Systems before problem solution and then to put the computed results in the appropriate User Defined System for postprocessing. The analysis from this point on will deal exclusively with problem solution and hence with the Inertial System and the Internal System.

For any body segment, n, $\vec{P} = \vec{R}_n + \vec{\bar{P}}_n$

where $\vec{\bar{P}}_n$ is the position vector to a point P defined with respect to the body segment, n.

If   If $e = \begin{pmatrix} \vec{I} \\ \vec{K} \end{pmatrix}$ and $e_m = \begin{pmatrix} \vec{i}_m \\ \vec{k}_m \end{pmatrix}$

then

$$e_m = d_m\, e = \begin{pmatrix} \vec{i}_m \cdot \vec{I} & \vec{i}_m \cdot \vec{K} \\ \vec{k}_m \cdot \vec{I} & \vec{k}_m \cdot \vec{K} \end{pmatrix} e \qquad (9)$$

$$= \begin{pmatrix} \cos\theta_m & \sin\theta_m \\ -\sin\theta_m & \cos\theta_m \end{pmatrix} e \qquad (10)$$

or inversely

$$e = d_m^{-1}\, e_m = d_m^{T}\, e_m \qquad (11)$$

If we adopt the convention that a vector equation such as (3) or (4) can be written in matrix form as a column vector by

$\vec{p} \equiv \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}$ that is by listing the coefficients of the unit vectors. The set of unit vectors hidden by this notation is generally supplied by context but where the underlying system needs to be made explicit we will add a subscript with the number of the internal system and further we will understand the Inertial System as system zero.

Then equation (8) can be written

$$\begin{pmatrix} p_1 \\ p_2 \end{pmatrix}_0 = \begin{pmatrix} r_{1,m} \\ r_{2,m} \end{pmatrix}_0 + \begin{pmatrix} \bar{p}_{1,m} \\ \bar{p}_{2,m} \end{pmatrix} \qquad (12)$$

normally matrix equations will be written all in one frame of reference and using the vector symbol without the arrow to stand for the corresponding matrix.

So we would say $\quad P = R_m + d_m^{T}\, \bar{P}_m \qquad (13)$

or $\quad P - R_m = d_m^{T}\, \bar{P}_m \qquad (14)$

and inversely (15)

$$\bar{P}_r = d_m (P - R_r)$$

Section 2.2 Units

The user is free to use whatever units that are convenient both for input and output. Units will be defined in terms of conversion factors relating the units used to one consistent set of units. More than one system of User Defined Units can be defined and used as convenient, but as with User Defined Coordinate Systems, the User Defined Input Units are converted to one consistent system before problem solution and then converted to the User Defined Output Units for postprocessing. It is suggested that the inches, pounds, seconds system generally be used for the internal consistent units because experience has shown this system causes fewer numerical problems in solution. (Some years ago after we first incorporated SI units as an option in the MVMA 2-D and were running parallel runs in English and SI, we found that for some data sets, the SI runs required smaller time steps for solution. The reason was not discovered.) Whenever the old airbag algorithm is used, it is mandatory that the internal system be the American Standard. If airbag is not used, then any consistent system of units will be possible as the internal system. The reason for this restriction is that the old algorithm has constants imbedded which were not easy to compensate and which are good only for American Standard.

Section 2.3 Basic Equations of Motion

Each segment is considered to be either a rigid two dimensional body if mass and inertial properties are provided, or to be a frame of reference which can not be included in the equations of motion if this information is not provided. In the latter case, the position, orientation, etc., must be completely definable from constraint equations.

Each segment is represented by three degrees of freedom. Some of these degrees of freedom are not really free but are controlled by implicity or explicity constraints.

For the purpose of this analysis an <u>implicit</u> constraint is one that <u>can not be changed</u> during the course of a run, e.g., an unlockable, unbreakable pin joint. These will be handled by setting up the equation of motion omitting these degrees of freedom and then computing them from the constraint equations from the results of solving the equations of motion. The "implicit" implies an indirect use of the constrain ing equations in solution of the problem.

The explicit constraint is one that can be modified during the run, e.g., a lockable, breakable pin joint or a rolling-sliding constraint. These are handled by leaving the "bound" degrees of freedom in the equations of motion, using constraint forces to cause the desired motion, and adding equations to compute constraint forces. The "explicit" implies direct use of the constraint equations in solution of the problem.

If a group of segments is joined by implicit constraints, it forms a chain. A chain must be singly connected in the sense that only one path of implicit constraints can exist between any two links of the chain. The chain, however, can be joined to itself with an explicit constraint.

Experience has shown that shorter equations are better than longer equations both from the point of view of numerical stability as well as computational speed. Length of equations can be minimized by basing chains on a properly chosen interior segment. Each chain will then be analyzed for the optimal choice of base segment. Starting in each case with the base segment, the chain will be organized by the model into branches in such a way that all chain segments are in at least one contiguous line of segments, the total number of all such appearances of segments is a minimum, and the base segment is interior to the chain. This chain and branch structure is used throughout the run after being set up in the input processor.

Each chain will have at least two branches. Every segment will appear in at least one branch of one chain even if a chain of one. Consider Fig. 3, the traditional eight segment man. There is clearly only one chain.

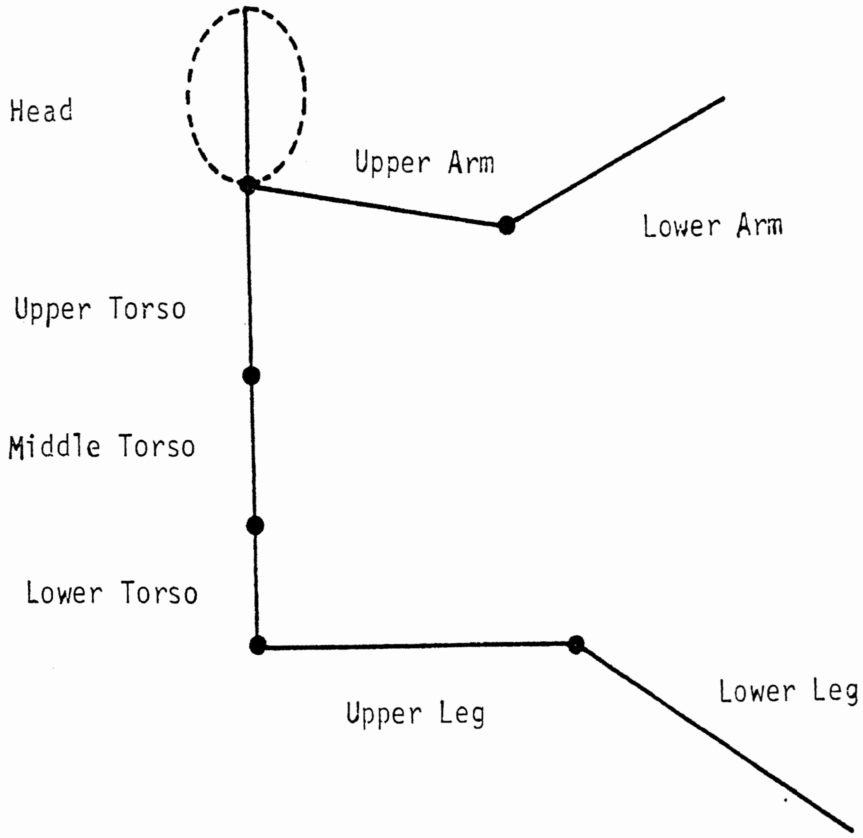

Fig. 3. The Eight Segment Man

Contrast the effect of taking various body segments as the base segment of the chain.

Base Segment	Branches	Total Appearances
Head	2	3 + 6 = 9
Upper Torso	3	2 + 3 + 5 = 10
Middle Torso	3	3 + 4 + 4 = 11
Lower Torso	3	4 + 5 + 3 = 12
Lower Leg	2	6 + 7 = 13

Table 1. Effect of Base Segment Selection

Here the head would appear to be the optimal base segment, but another factor comes into play. Even though the equations have one extra term, Upper Torso is a much better choice because the Head is usually the lightest body segment of all and numerical stability is affected by the relative heaviness of the base segment. The Upper Torso is the optimal choice for this chain. This is the basis for the interior selection rule outlined above. This organization is done automatically when the tables for execution are set up.

As in the case of the previous versions of the MVMA Two Dimensional Model, the Lagrangian approach is used.

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} + \frac{\partial D}{\partial q_i} = Q_i \tag{16}$$

Kinetic energy for all rigid segments is expressible as

$$T = \frac{1}{2} \sum_{i=1}^{N} \left\{ m_i \left[\dot{x}_i^2 + \dot{z}_i^2 \right] + I_i \dot{\theta}_i^2 \right\} \tag{17}$$

and the kinetic energy terms reduce to

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} = \sum_{j=1}^{N} \left\{ m_j \left(\ddot{x}_j \frac{\partial x_j}{\partial q_i} + \ddot{z}_j \frac{\partial z_j}{\partial q_i} \right) + I_j \ddot{\theta}_j \frac{\partial \theta_j}{\partial q_i} \right\} \tag{18}$$

so the total kinetic contribution to the equations of motion for segment n and the i th equation is

$$T_{n,i} = m_n \left(\ddot{x}_n \frac{\partial x_n}{\partial q_i} + \ddot{z}_n \frac{\partial z_n}{\partial q_i} \right) + I_n \ddot{\theta}_n \frac{\partial \theta_n}{\partial q_i} \tag{19}$$

Of course the partials in equation (19) will be zero for all generalized coordinates not associated with any branch of which the segment is a member. Since these contributions appear in equation (18) exactly once for each segment, the segments held in common by more than one branch are skipped in all but the first branch taken up.

The procedure for generating the kinetic energy terms will be as follows. Each segment will be taken in turn. Using the tables in which branch organization has been stored by the Input Processor, the branch and link number on the branch will be looked up and the contribution to each of the generalized coordinates will be added in starting with the base segment and working out to the segment in question. The generalized coordinates will depend on the types of connectors along the branch. A recursion relationship will be developed for each connector and these will be used to build up the contributions along the appropriate branch to each segment.

Section 2.4 Handling of Constraints

In keeping with the Lagrangian approach, constraints are handled by the Langranian multiplier method.

If the constraints are of the form:

$$f_\ell (q_1, q_2, \ldots, q_n, t) = 0 \quad \text{for } \ell = 1, M$$

then the equations of motion will be changed by setting

$$Q_i = \sum_{\ell=1}^{M} C_\ell \frac{\partial f_\ell}{\partial q_i} \tag{20}$$

in equation (16) and moving to left hand side. In addition, constraint equations are added to cover the new unknowns C_ℓ for $\ell = 1, M$. These are put in terms of accelerations.

$$\sum_{i=1}^{N} \frac{\partial f_\ell}{\partial q_i} \ddot{q}_i = -\frac{d}{dt}\left(\frac{\partial f_\ell}{\partial t}\right) - \sum_{i=1}^{N} \frac{d}{dt}\left(\frac{\partial f_\ell}{\partial q_i}\right) \dot{q}_i \tag{21}$$

for $\ell = 1, M$.

In this form, these equations can be solved along with equation (16) conveniently. The new unknowns are simply the constraint forces necessary to keep the corresponding constraint satisified. These constraint forces will be used for determining whether joints unlock and/or break where applicable. Explicit constraints then are handled in terms of augmenting the equations of motion by constraint equations in terms of accelerations which are solved with the equations of motion. For brevity, we will call this augmented set of equations, the equations of motion.

Implicit constraints are those for which the affected degrees of freedom are eliminated from the equations of motion and for which no constraint forces are computed. Constraint forces are not needed for an unbreakable, unlockable joint, and therefore represent needless computation. In this case, the constrained degrees of freedom are computed after the equations of motion are solved using expressions in terms of the "true" degrees of freedom. These expressions are presented in the sections dealing with the joints and are generated functionally along with the kinetic energy terms.

Any massless coordinate systems are either rigidly attached to another coordinate system (for special input/output purposes) or are controlled by position, velocity, or acceleration profiles as a function of time (e.g., to simulate a vehicle). In the first case, the degrees of freedom are computed by transformation equations from the degrees of freedom of the attached coordinate system. In the second case, the degrees of freedom are computed from the profiles supplied, the current time, and the degrees of freedom of the system with respect to which the profiles are specified.

This feature can be used to model one or more vehicles or vehicle parts which move as a function of time. It could also be used for obtaining output with respect to a moving observation point without affecting the equations of motion.

3.0 JOINTS AND CONNECTORS

This section describes the available ways in which two body segments can be joined together in the sense of affecting each others motion without contact forces. The mechanisms which are used for this purpose are called joints and connectors. The distinguishing character of a joint is that it represents a usual model for the human body part whereas the connectors are more general and correspond to mechanical connectors usually found in more general structural simulations.

Joints and connectors effectively remove degrees of freedom from the equations of motion. The preceding sections have discussed the general approach taken for elimination of degrees of freedom. The following sections will describe in detail the types of joints and their impact on the equations.

3.1 General Types of Connectors

Connectors fall into two classes, those that constrain implicitly by removing degrees of freedom from the equations and those that constrain explicitly by leaving the degrees of freedom alone and adding constraint equations to bind some of them. Table 2 lists the connector types, the types of constraint, a description of the connector, the net effect on the degrees of freedom and the net effect on the number of the equations of motion. In interpreting these latter two columns, assume that there are three degrees of freedom and three equations for every body segment. These fifteen connectors will be investigated in the following sections. Each of the connectors will be thought of in terms as "states" of motion and "transitions" between states. Each of the implicit connectors have one state. Each of the explicit connectors can take on variable states. Each state will be described followed by the tests for transition to another state and finally by the transition equation itself.

Section 3.2 Unbreakable Pin Joints and the Equations of Motion

Fig. 4 shows body segment n connected to body segment m by means of a pin joint. The notation is the same as before except that two subscripts are used; the first one is the defined point number at which the

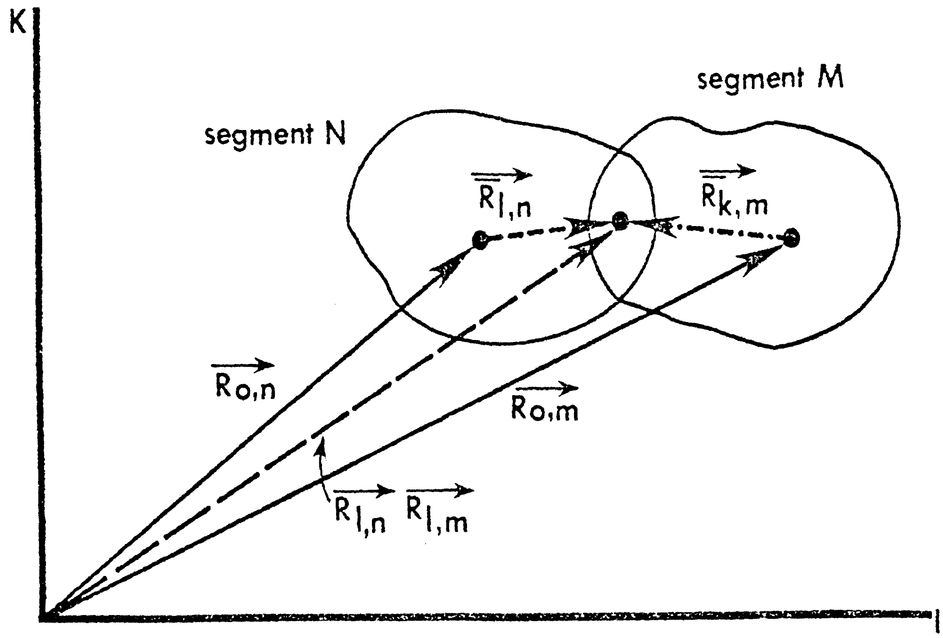

Fig. 4. Pin Joint Geometry

connection takes place and the second is the body segment number. The bar still indicates a specification with respect to the body segment system and the lack of a bar still indicates an inertial relative specification. If the defined point subscript is zero, then the center of gravity is always indicated. Hence, $\vec{\bar{R}}_{o,j} \equiv 0$ for all j.

So in matrix form

$$R_{o,m} = R_{o,n} + d_n^T \bar{R}_{l,n} - d_m^T \bar{R}_{k,m} \tag{22}$$

Without loss of generality, let us assume that segment n lies closer to the base segment on the branch. Let us also assume that these two segments have link numbers on the branch of a and a+1 respectively. Define a function $\eta_{i,j}(a)$ such that for branch j and link number a, the function takes on the value of the true segment number (n) and the point subscripts l and k at which connections in the chain occur for i = 1, 2, and 3, respectively. Let us simplify the

TABLE 2. Connector Types

Number	Type Name	Constraint Type	Description	Figure Number	Change in Degrees of Freedom	Change in Equations of Motion
1	Pin	Implicit	Unbreakable, unlockable	4	-2	-2
2	Extensional no rotation	Implicit	Unbreakable, unlockable, no rotation at either end. Redefines one degree of freedom as distance.	5	-2	-2
3	Extensional with 2 rotation	Implicit	Unbreakable, unlockable effectively a ball joint at both ends. Redefines one degree of freedom as distance and another as second angle.	5	0	0
4	With 1 rotation			5	-1	-1
5	Rigid no rotation	Implicit	Unbreakable, unlockable, no rotation at either end. Redefines two segments as one.	5	-3	-3
6	Rigid with single rotation	Implicit	Unbreakable, unlockable, effectively a ball joint at one end.	5	-2	-2
7	Rigid with double rotation	Implicit	Unbreakable, unlockable. effectively a ball joint at both ends.	5	-1	-1
8	Lockable pin	Explicit	Unbreakable, will unlock when constraint torque exceeds specified value.	4	-3 when locked	-2 + 1 = 1
9	Lockable breakable pin	Explicit	Will unlock as with 7 and will break when constraint forces exceed specified.	4	0 when broke -2 when unlocked -3 when locked	+3
10	Lockable breakable extensional with 2 rotations	Explicit	Will unlock and/or break much as 7 and 8 but enlongation as well.	5	0 when broke -1 locked rotation on locked elongation -2 unlocked rotation at one end or unlocked -3 all locked	+3
11	Extensional with 1 rotation					
12	Extensional with 0 rotation					

TABLE 2. Connector Types

Number	Type Name	Constraint Type	Description	Figure Number	Degrees of Freedom	Equations of Motion
13	Rolling and/ or sliding	Explicit	Rolls or slides along specified perimeter	7	-3 all locked	+3
14	Neck model	Implicit	Special case of 3 for compatibility	5	0	0
15	Shoulder model	Implicit	Special case of 3 for compatibility	6	0	0

Note: These Connector Types represent variations of only three basic connectors.

notation as follows:

$$R_{0,a+1} = R_{0,a} + d_a^T \bar{R}_{2,a} - d_{a+1}^T \bar{R}_{1,a+1} \tag{23}$$

where $\bar{R}_{2,a} \equiv \bar{R}_{\ell,m}$ where $m = \eta_{1,j}(a)$ and

$\ell = \eta_{3,j}(a)$ the existing defined point

$\bar{R}_{1,a+1} \equiv \bar{R}_{k,m}$ where $m = \eta_{1,j}(a+1)$ and

$k = \eta_{2,j}(a+1)$ the entering defined point

and $d_a \equiv d_m$ where $m = \eta_{1,j}(a)$

Looking back at equation (19), we see that $\ddot{y}_m$, $\ddot{z}_m$, $\ddot{\theta}_m$, $\frac{\partial y_m}{\partial q_i}$, $\frac{\partial z_m}{\partial q_i}$, and the $\frac{\partial \theta_m}{\partial q_i}$ are necessary in order to compute the contributions to the equations of motion. Each of these quantities are related by recursion equations.

$$\frac{\partial R_{0,a+1}}{\partial q_i} = \frac{\partial R_{0,a}}{\partial q_i} + \left(\frac{\partial d_a}{\partial q_i}\right)^T \bar{R}_{2,a} - \left(\frac{\partial d_{a+1}}{\partial q_i}\right)^T \bar{R}_{1,a+1} \tag{24}$$

where $\left(\frac{\partial d_a}{\partial q_i}\right)^T = \begin{pmatrix} -\sin\theta_a & -\cos\theta_a \\ \cos\theta_a & -\sin\theta_a \end{pmatrix}$ if $q_i = \theta_a$

and zero otherwise.

$$\dot{R}_{1,a+1} = \dot{R}_{0,a} + \left(\frac{\partial d_a}{\partial \theta_a}\dot{\theta}_a\right)^T \bar{R}_{2,a} - \left(\frac{\partial d_{a+1}}{\partial \theta_{a+1}}\dot{\theta}_{a+1}\right)^T \bar{R}_{1,a+1} \tag{25}$$

$$\ddot{R}_{0,a+1} = \ddot{R}_{0,a} + \left(\frac{\partial d_a}{\partial \theta_a}\right)^T \bar{R}_{2,a}\ddot{\theta}_a - \left(\frac{\partial d_{a+1}}{\partial \theta_{a+1}}\right)^T \bar{R}_{1,a+1}\ddot{\theta}_{a+1}$$

$$- d_a^T \dot{\theta}_a^2 \bar{R}_{2,a} + d_{a+1}^T \dot{\theta}_{a+1}^2 \bar{R}_{1,a+1} \tag{26}$$

Note that $\left(\frac{\partial d_a}{\partial \theta_a}\dot{\theta}_a\right)^T = \left(\frac{\partial d_a}{\partial \theta_a}\right)^T \dot{\theta}_a = \frac{\partial}{\partial \theta_a}\left(d_a^T\right)\dot{\theta}_a$

So the procedure for each branch involves starting with the base segment, computing the partials and time derivatives and then the kinetic energy contribution for each branch segment in turn. For each new segment, one non-zero partial will be modified and one will be non-zero for the first time.

Section 3.3 Breakable Pin Joints and the Equations of Motion

This type of joint is a variation of the pin joint in Section 3.2. The variant is that the joint may become unconnected or it may become immobile. The approach taken is to treat each of those possibilities as a "state of motion" and to provide a separate analysis for each of the states expressed in common terms so that the when and how of transitions between states can be specified. Toward this end, each state is defined in terms of its operational constraint equations and each transition is defined in terms of constraint equations which are a function of time.

State 1 (When the joint is free to move).

The motion in this state is governed by two constraint equations:

$$R_{0,m} + d_m^T \bar{R}_{j,m} - R_{0,m} - d_m^T \bar{R}_{e,m} = 0 \tag{27}$$

State 2 (When the joint is locked so that it cannot move).

The motion in this state is governed by three constraint equations.

$$R_{o,n} + d_n^T \bar{R}_{j,n} - R_{o,m} - d_m^T \bar{R}_{\ell,m} = 0$$

and (28)

$$\theta_n - \theta_m - \theta_c = 0$$

State 3 (When the joint is broken apart)

The motion in this state has no constraint equations. The two body segments are not connected.

All three states can be described by the following equations

$$R_{o,n} + d_n^T \bar{R}_{j,n} - R_{o,m} - d_m^T \bar{R}_{\ell,m} = \lambda$$

$$\theta_n - \theta_m = \lambda_\theta \qquad (29)$$

where the three new variables take on values which define the states so when

$\lambda_x = \lambda_z = 0$ and λ_θ free, we have State 1,

when $\lambda_x = \lambda_z = 0$ and $\lambda_\theta = \theta_c$, we have State 2, and when all three are free, we have State 3.

How C_x, C_z and C_θ are the forces (or torques) necessary to maintain the constraints. When these exceed certain specified values, the joint begins transition to unlocking or breaking. This is accomplished by treating the constraint forces not as independent variables, but as constraints or as determined as a function of time until transition is complete. In order to complete the equations of motion, the equations replace the constraint equations.

For the unlocking joint, C_θ remains a constant with the value of the unlocking torque. When the sign of the λ acceleration changes, the joint torque must be less than the unlocking torque in magnitude and the equations again compute constraint forces for the locked state.

For the breaking joint, the process is irreversible when complete. When the breaking force is exceeded in either component, the λ equations replace the constraint equations and both C_x and C_z are ramped from the breaking force down to zero as a function of the maximum magnitude of the vector $\vec{\lambda}$. This means that further reduction in the C's are tied to further increase in $|\lambda|$ only, not to time or to reversals. If $|\lambda|$ becomes less than a prescribed quantity, the equations are put back as before except that the breaking forces are set to the current values of the C's. Once the C's go to zero, the constraint equations are removed and the model behaves as if the segments were never joined.

Section 3.4 The Extensional Connector and Variants

The Unbreakable Extensional Connector and the Rigid Connector both have four variations -no angulation, angulation at either end, and angulation at both ends. Figure 5 illustrates the situation.

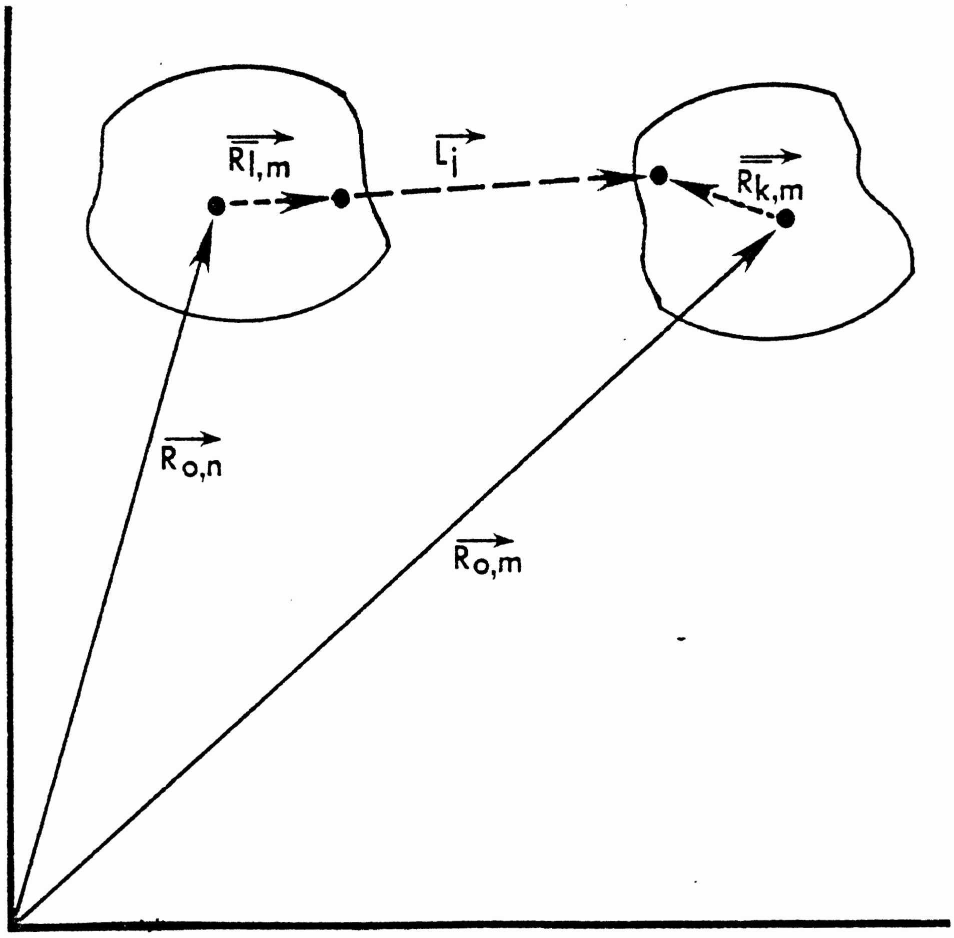

Fig. 5. Rigid and Extensional Connector Geometry

The basic matrix equation is

$$R_{o,m} + d_m^T \bar{R}_{k,m} = R_{o,n} + d_n^T \bar{R}_{l,n} + \begin{pmatrix} \cos\theta_j \\ \sin\theta_j \end{pmatrix} |L_j| \qquad (30)$$

The variations in these connectors depend entirely on what is included in the degrees of freedom and what is held constant. Table 3 summarizes each connector.

The contributions to the kinetic energy are

$$\frac{\partial R_{0,m}}{\partial q_i} = \frac{\partial R_{0,m}}{\partial q_i} + \left(\frac{\partial d_m}{\partial q_i}\right)^T \bar{R}_{\ell,m} + \begin{pmatrix} -\sin\theta_j \\ \cos\theta_j \end{pmatrix} \frac{\partial \theta_j}{\partial q_i} |L_j| + \begin{pmatrix} \cos\theta_j \\ \sin\theta_j \end{pmatrix} \left|\frac{\partial L_j}{\partial q_i}\right| - \left(\frac{\partial d_m}{\partial q_i}\right)^T \bar{R}_{k,m} \tag{31}$$

and

$$\begin{aligned} \ddot{R}_{0,m} = {} & \ddot{R}_{0,m} + \left(\frac{\partial d_m}{\partial \theta_m}\right)^T \bar{R}_{\ell,m} \ddot{\theta}_m - \left(\frac{\partial d_m}{\partial \theta_m}\right)^T \bar{R}_{k,m} \ddot{\theta}_m \\ & + \begin{pmatrix} -\sin\theta_j \\ \cos\theta_j \end{pmatrix} |L_j| \ddot{\theta}_j + \begin{pmatrix} \cos\theta_j \\ \sin\theta_j \end{pmatrix} |\ddot{L}_j| - d_m^T \vec{R}_{\ell,m} \dot{\theta}_m^2 \\ & + d_m^T \bar{R}_{k,m} \dot{\theta}_m^2 - \begin{pmatrix} \cos\theta_j \\ \sin\theta_j \end{pmatrix} |L_j| \dot{\theta}_j^2 \end{aligned} \tag{32}$$

The proper interpretation of equations (31) and (32) depends on the applicable line of Table 3. In equation (31), the corresponding partial is zero if a "No" is listed under the potential degree of freedom. The time derivative (equation (32)) necessitates determining the dependence relationship if motion is pōssible for the "No" entries in Table 3. If $|L_j|$ is not a degree of freedom, then $|L_j| \equiv 0$.

However, since all angles are with respect to the Inertial System if the θ_j column is "No", then $\theta_m - \theta_j = \theta_c$ which is a constant, and

$$\tag{33}$$

if the θ_m column is "No", then $\theta_m - \theta_j = \hat{\theta}_c$ which is another constant.

The time derivatives are obtained from the equations (33) in case of angular dependences and are used in equation (32).

Table 3 The Family of Extensional Connectors

Connector	q_i (Degrees of freedom) include					
	$R_{0,m}$	θ_m	$R_{0,n}$	θ_n	θ_j	$\lvert L_j \rvert$
Extensional no rotation	No	No	Yes?	Yes	No	Yes
Extensional 1 rotation in base segment end	No	No	Yes?	Yes	Yes	Yes
Extensional 1 rotation on other end	No	Yes	Yes?	Yes	No	Yes
Extensional 2 rotations	No	Yes	Yes?	Yes	Yes	Yes
Rigid no rotation	No	No	Yes?	Yes	No	No
Rigid 1 rotation near end	No	No	Yes?	Yes	Yes	No
Rigid 1 rotation far end	No	Yes	Yes?	Yes	No	No
Rigid 2 rotations	No	Yes	Yes?	Yes	Yes	No
Number of degrees of freedom	always dependent	1	2	1	1	1

Note that while it is not known whether $R_{0,n}$ is free or not, the chain as a whole will effectively provide two degrees of freedom at this point.

For compatibility sake, the "neck" and "shoulder" joint models of the MVMA 2-D, Version 4 are available wherever the user specifies. These are both special variations on the double rotation flexible joint and differ in choice of degrees of freedom.

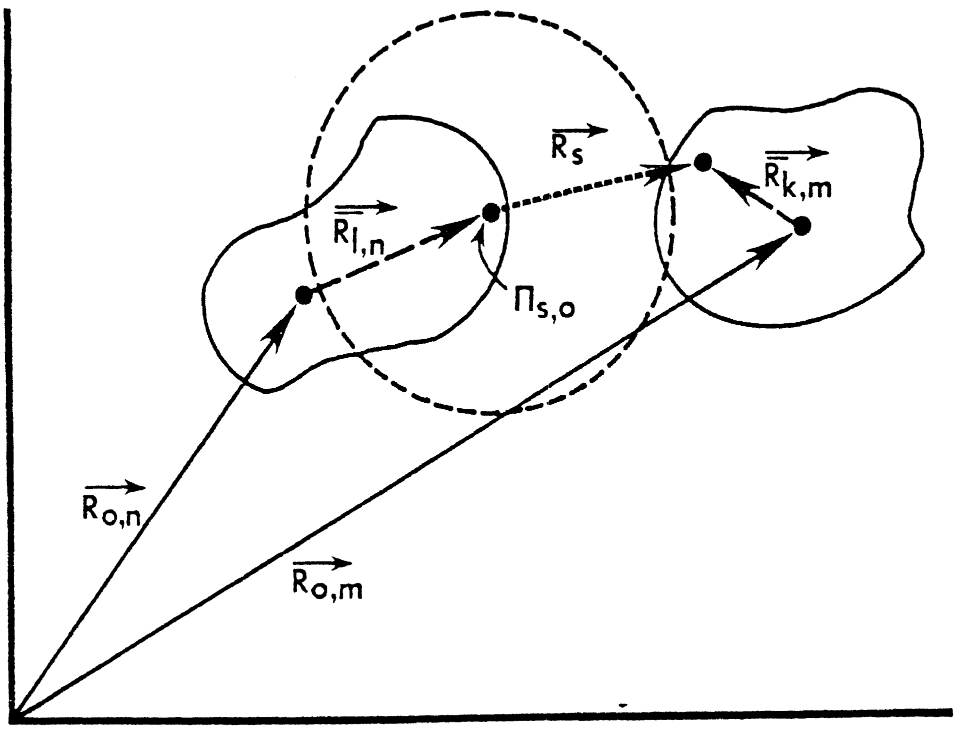

Fig. 6. The "Shoulder" Joint

Here the degrees of freedom chosen are the inertial components of R_s and a circle of radius $\vec{R}_s$ is set up about the near joint end which is supposed to represent the area of free motion for the shoulder. This will supercede usual joint force production and is discussed more fully in Reference 1. The "neck" model looks just like Fig. 5 and differs only in the particular joint forces available.

Breakable and lockable extensional connectors are treated just the same way as described in Section 3.3 except that the basic constraint equations have one more term (see equation (27)).

$$R_{o,m} + d_m^T \bar{R}_{j,m} - R_{o,m} - d_m^T \bar{R}_{\ell,m} + \begin{pmatrix} \cos\theta_k \\ \sin\theta_k \end{pmatrix} |L_k| = 0 \qquad (34)$$

and there is no need of equation (29). When $|L_k|$ is locked, an equation $|L_k| = L$, a constant,is added.

There are eight Locked States plus the Broken State for the Extensional joint with two rotations. The lesser degree of freedom possibilities mentioned in Table 3 lead to a similar reduction in locked states.

3.5 Rolling and Sliding Constraints

This type of constraint is considered essentially to be a connector and is pictured in Figure 7.

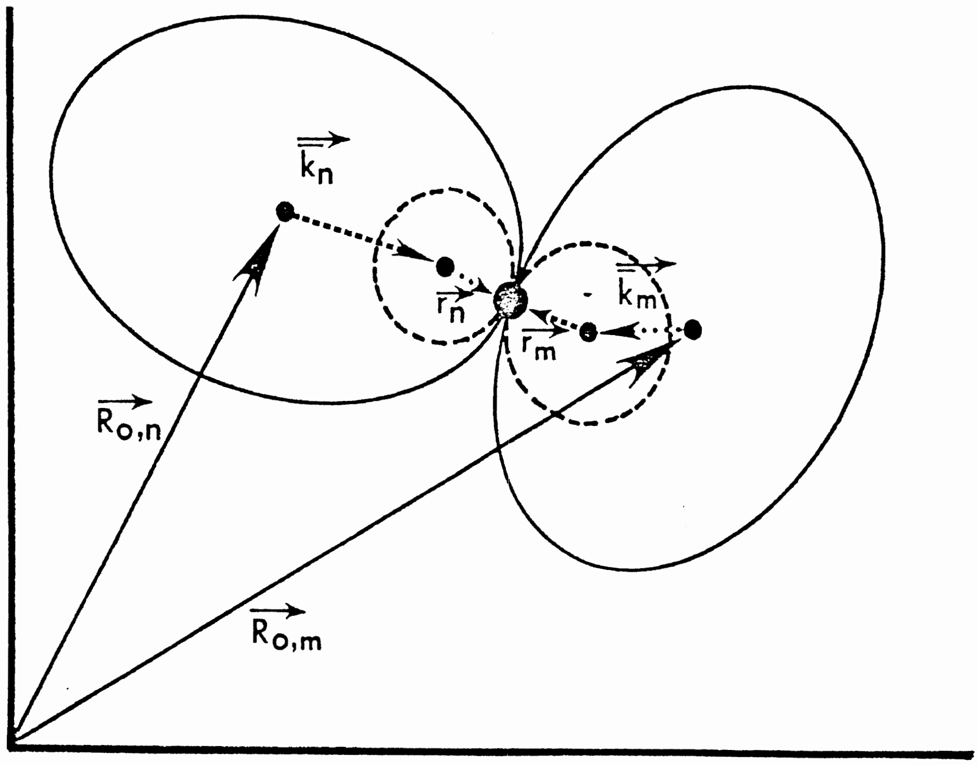

Fig. 7. The Rolling-Sliding Constraint

where $\vec{k}_i$ is the vector from the Center of Gravity to the Instantaneous Center of Rotation and $\vec{r}_i$ is the vector from the center of rotation to the point of contact.

If $$T_n = R_{0,n} + d_n^T k_n + d^T_n r_n \tag{35}$$

and $$T_m = R_{0,m} + d_m^T k_m + d_m^T r_m , \tag{36}$$

then the connection equation is

$T_n = T_m$ at any given instant.

For either rolling or sliding, the normal component of velocity at the point of contact must be the same. For rolling, in addition, the tangential component must be the same as well.

So for rolling

$$\dot{T}_n = \dot{T}_m \tag{37}$$

and for sliding

$$\left(\dot{T}_n\right)^T \frac{r_n}{|r_n|} = \left(\dot{T}_m\right)^T \frac{r_m}{|r_m|} \tag{38}$$

For general shapes of segments the $\vec{k}_n$ and $\vec{r}_n$ vectors can be difficult to compute. If a two-dimensional curve is defined parametrically as

$$x = \phi(t)$$
$$y = \psi(t)$$

then the radius of curvature is $$|r| = \left| \frac{(\dot{\phi}^2 + \dot{\psi}^2)^{\frac{3}{2}}}{\dot{\phi}\ddot{\psi} - \ddot{\phi}\dot{\psi}} \right| , \tag{39}^*$$

the center of curvature is

$$x_c = \phi - \dot{\psi}\left(\frac{\dot{\phi}^2 + \dot{\psi}^2}{\dot{\phi}\ddot{\psi} - \ddot{\phi}\dot{\psi}}\right)$$

$$y_c = \psi - \dot{\phi}\left(\frac{\dot{\phi}^2 + \dot{\psi}^2}{\dot{\phi}\ddot{\psi} - \ddot{\phi}\dot{\psi}}\right) , \tag{40}^*$$

* See note next page.

the normal to the curve is

$$\left\{x-\phi\right\}\dot{\phi}+\left\{z-\psi\right\}\dot{\psi}=0, \tag{41}*$$

and the tangent to the curve is

$$\left\{x-\phi\right\}\dot{\psi}-\left\{z-\psi\right\}\dot{\phi}=0. \tag{42}*$$

Two curves are tangent at a point if

$$\dot{x}\,\dot{z}_1-\dot{x}_1\,\dot{z}=0 \tag{43}*$$

We will be dealing with only two shapes, the ellipse and the line. Of the four combinations of these shapes, line on line rolling is not allowed so that a line can either be sliding or be held in place when interacting with another line.

For an ellipse, the equations

$$\begin{aligned} x &= a\cos\beta\cos\alpha - b\sin\beta\sin\alpha + x_{e,0} \\ z &= a\sin\beta\cos\alpha + b\cos\beta\sin\alpha + z_{e,0} \end{aligned} \tag{44}$$

where α is the parameter and β is the inertial orientation angle of the ellipse. a and b are the semi-major axes along the x and z axes of the ellipse system. $x_{e,0}$, $z_{e,0}$ are the inertial coordinates of the center of the ellipse and the origin of the ellipse system.

*Equations (39) through (43) are taken from page 546, Volume I, Reference (4).

Then comparing with above, we see

$$
\begin{aligned}
\phi(t) &= a\cos\beta\cos\alpha - b\sin\beta\sin\alpha + x_{c,0} \\
\dot{\phi}(t) &= -a\cos\beta\sin\alpha - b\sin\beta\cos\alpha \\
\ddot{\phi}(t) &= -a\cos\beta\cos\alpha + b\sin\beta\sin\alpha = x_{c,0} - \phi \\
\psi(t) &= a\sin\beta\cos\alpha + b\cos\beta\sin\alpha + z_{c,0} \\
\dot{\psi}(t) &= -a\sin\beta\sin\alpha + b\cos\beta\cos\alpha \\
\ddot{\psi}(t) &= -a\sin\beta\cos\alpha - b\cos\beta\sin\alpha = z_{c,0} - \psi
\end{aligned}
\tag{45}
$$

When these are substituted back in (39) through (42), we have everything necessary for computing $\vec{R}_m$ and $\vec{n}_m$ for any given value of α which depends on the other shape.

For a line

$$
\begin{aligned}
x = \phi(t) &= x_{L,0} + \eta(x_{L,1} - x_{L,0}) \\
&= x_{L,0} + s\cos\alpha_x \\
z = \psi(t) &= z_{L,0} + \eta(z_{L,1} - z_{L,0}) \\
&= z_{L,0} + s\cos\alpha_z
\end{aligned}
\tag{46}
$$

In the latter form s is the distance along the line.

Since curvature is infinite for a line, equations (39) and (40) will not be used for a line. $\vec{R}_m$ will be taken as the vector to the tangent point and $\vec{n}_m$ will be defined as a unit normal at the tangent point for any given value of s.

For any of the combinations of shapes, the two parameters describing the tangent point on each of the two shapes are obtained from the "touching" equations.

$$\phi_1(t_1) = \phi_2(t_2)$$
$$\psi_1(t_1) = \psi_2(t_2) \tag{47}$$

Once the equations are solved for the two parameters, equations (37) are completely determined and there remains only to put this vector in terms of its normal and tangential components rather than its inertial components as in (37). Equation (38) is the magnitude of normal component and

$$\left| \, |\dot{T}_m| - \left((\dot{T}_m)^T \frac{n_m}{|n_m|} \right) \frac{n_m}{|n_m|} \right|$$

is the tangential component. The corresponding components of equations (37) then form the constraint equations which are added to the equations of motion. The two constraint forces will be the normal and tangential constraint forces. These two forces in comparison with user specifed values determine if the constraint is broken or sliding.

When sliding, t_1 and t_2 are invariant in time. The normal constraint equation is used as before. The tangential constraint force is held constant and the tangential constraint equation is given an additional unknown, the relative tangential velocity which is solved for. When the tangential acceleration changes signs, the sliding will cease and rolling recommences. The equations are put back for rolling.

When the normal constraint forces exceed the user specified breaking force, both normal and tangential constraint forces are ramped to zero, the t_1 and t_2 parameters are held constant, and both the normal and tangential relative velocities are solved. When the ramping is complete, the constraint equations are removed altogether.

Section 3.6 Joint Forces

In the preceding sections, the handling of connections from the point of view of their impact on the degrees of freedom in the equations of motion has been discussed. In addition, each of the connectors has the capability for resisting motion in each of the allowable degrees of freedom. This section will examine force production by connectors and joints. These forces are of the nature of $F(\delta)$ where δ is a positional deflection and a function of certain of the degrees of freedom. The potential energy for this force is

$$V = \int_0^{\delta} F(\mu)\, d\mu \tag{48}$$

and the contributions in the equations of motion (see equation (16)) are

$$Q \cong -\frac{\partial V}{\partial q_i} = -\frac{\partial V}{\partial \delta}\frac{\partial \delta}{\partial q_i} = -F(\delta)\frac{\partial \delta}{\partial q_i} \tag{49}$$

Here the symbol Q_i is used to indicate not an applied force but rather what will be called a "generalized force". The factor $F(\delta)$ is the force and the factor $\frac{\partial \delta}{\partial q_i}$ is called the lever arm because of strong resemblence to the relation

Torque = Force x Lever Arm

and it serves the same purpose although it is in general not a distance.

Table 4 presents a description of the types of force resistance available to both the pin joint and the extensional connector. Sliding-rolling connectors do not offer resistance of this sort.

TABLE 4 Types of Joint Forces

Type	Description	
1. General Material From Rest	If $\delta - \delta_0 > 0$, $F_1(\delta - \delta_0)$	Used
	If $\delta - \delta_0 < 0$, $F_2(\delta_0 - \delta)$	Used
2. General Material From Stop	If $\delta > \delta_u$, $F_3(\delta - \delta_u)$	Used
	If $\delta_\ell \leq \delta \leq \delta_u$, zero	Used
	If $\delta < \delta_\ell$, $F_4(\delta_\ell - \delta)$	Used
3. Friction	If $\lvert\dot{\delta}\rvert \geq \dot{\delta}_E$, $F_5 \operatorname{sgn} \dot{\delta}$	Used
	If $\lvert\dot{\delta}\rvert < \dot{\delta}_E$, $\frac{\dot{\delta}}{\dot{\delta}_E} F_5$	Used
4. Viscous Damping	If $\delta \geq \delta_e$, $c_v \dot{\delta}$	Used
	If $\delta < \delta_e$, $\frac{\delta}{\delta_e} c_v \dot{\delta}$	Used
5. Muscle Tension	F is used where F satisfies: $\dot{F} + \frac{k}{c} F = -k\dot{\delta}$ where $k = a_1 + a_2 \lvert M(t) \rvert$ and $c = a_3 \lvert M(t) \rvert$ See Section 2.3.2, Vol. 1 of MVMA Report (Ref. 1)	

In the above, each constant and function indicated is inputted to the program for each usage of each type of joint force. That is to say that each joint or connector can have any desired combination of the above types of joint restraints. For each such usage, the force is applied to the equations of motion via equation (49).

4.0 FORCED OR APPLIED VARIABLES

It is often very useful to cause a particular sequence of events on one part of a system and then to simulate the reactions of the rest of the system to these events. To aid in this endeavor, Variseg permits the specification of four types of prescribed events, the position of any point as a function of time, the velocity of any point as a function of time, the acceleration of any point as a function of time, and a force as a function of time applied to any point. The following sections deal with these events.

It will be noted that no sophistication has been employed in the numerical differentiation formulas used in the following sections whereas the numerical integration formulas are exact. The reasons for which these sections were set up this way are as follows:

1. The model is position dependent and almost rate independent (dependent on rate sign); the accuracy is where it is needed, and
2. More sophistication would be misleading since the user is very unlikely to supply the necessary information to do a proper job to obtain accurate derivatives.

The user is warned that if rate information is important to a particular run, the user must obtain good velocity or acceleration information and submit it to the model. The position option will be used mainly for special coordinate system placement.

Section 4.1 Forced Position

When a displacement is specified as a table of points in time, straightforward curve fitting is used in order to approximate velocity and acceleration. If there are four or more table points, each four points are taken in turn and equations (50) are used.

$$\begin{aligned} s &= \beta_1 t^3 + \beta_2 t^2 + \beta_3 t + \beta_4 \\ v &= 3\beta_1 t^2 + 2\beta_2 t + \beta_3 \\ a &= 6\beta_1 t + 2\beta_2 \end{aligned} \tag{50}$$

where

$$\beta_1 = \frac{1}{\alpha}\left[\alpha_4 - \alpha_1 - \alpha_2 - \alpha_3\right]$$

$$\beta_2 = \frac{1}{\alpha}\left[\alpha_1(t_1+t_2+t_3) + \alpha_2(t_0+t_2+t_3) + \alpha_3(t_0+t_1+t_3) - \alpha_4(t_0+t_1+t_2)\right]$$

$$\beta_3 = \frac{1}{\alpha}\left[-\alpha_1(t_1t_2+t_2t_3+t_3t_1) - \alpha_2(t_0t_2+t_2t_3+t_3t_0) - \alpha_3(t_0t_1+t_1t_3+t_3t_0) + \alpha_4(t_0t_1+t_1t_2+t_2t_0)\right]$$

$$\beta_4 = \frac{1}{\alpha}\left[t_1t_2t_3\alpha_1 + t_0t_2t_3\alpha_2 + t_0t_1t_3\alpha_3 - t_0t_1t_2\alpha_4\right]$$

and further

$$\alpha_1 = (t_3-t_2)(t_3-t_1)(t_2-t_1)S_0$$

$$\alpha_2 = (t_3-t_2)(t_3-t_0)(t_2-t_0)S_1$$

$$\alpha_3 = (t_3-t_1)(t_3-t_0)(t_1-t_0)S_2$$

$$\alpha_4 = (t_2-t_1)(t_2-t_0)(t_1-t_0)S_3$$

$$\alpha = (t_3-t_2)(t_3-t_1)(t_3-t_0)(t_2-t_1)(t_2-t_0)(t_1-t_0)$$

given that the four table points are labelled

$$(t_0, S_0),\ (t_1, S_1),\ (t_2, S_2),\ \text{and}\ (t_3, S_3)$$

If there are only three table points available the equation (50) simplifies to equation (51).

For three point tables,

$$\begin{aligned}
\beta_1 &= 0 \\
\beta_2 &= \frac{1}{\hat{\alpha}}\left[S_0 t_1 + t_0 S_2 + S_1 t_2 - S_2 t_1 - S_0 t_2 - S_1 t_0\right] \\
\beta_3 &= \frac{1}{\hat{\alpha}}\left[S_1 t_0^2 + S_0 t_2^2 + S_2 t_1^2 - S_1 t_2^2 - S_2 t_0^2 - S_0 t_1^2\right] \\
\beta_4 &= \frac{1}{\hat{\alpha}}\left[S_2 t_1 t_0^2 + S_1 t_0 t_2^2 + S_0 t_2 t_1^2 \right. \\
&\qquad \left. - S_0 t_1 t_2^2 - S_1 t_2 t_0^2 - S_2 t_0 t_1^2\right]
\end{aligned} \tag{51}$$

where

$$\hat{\alpha} = t_1 t_0^2 + t_0 t_2^2 + t_2 t_1^2 - t_2^2 t_1 - t_2 t_0^2 - t_1^2 t_0$$

For two point tables,

$$\begin{aligned}
\beta_1 &= 0 \\
\beta_2 &= 0 \\
\beta_3 &= \frac{S_1 - S_0}{t_1 - t_0} \\
\beta_4 &= S_0
\end{aligned} \tag{52}$$

For one point tables

$$\begin{aligned}
\beta_1 &= 0 \\
\beta_2 &= 0 \\
\beta_3 &= 0 \\
\beta_4 &= S_0
\end{aligned} \tag{53}$$

Section 4.2 Forced Velocities

$$S = \frac{1}{3}\beta_2 t^3 + \frac{1}{2}\beta_3 t^2 + \beta_4 t + S(t_0)$$

$$V = \beta_2 t^2 + \beta_3 t + \beta_4 \tag{54}$$

$$a = 2\beta_2 t + \beta_3$$

where β_2, β_3, and β_4 are exactly similar to the definitions given in equations (51), (52) and (53) of the preceding section except the V_i replaces S_i wherever it occurs. $S(t_0)$ is the computed value for that point or in the case of the first group of points it is initial conditions specified by the user.

Section 4.3 Forced Accelerations

$$S = \frac{1}{6}\beta_3 t^3 + \frac{1}{2}\beta_4 t^2 + V(t_0)t + S(t_0)$$

$$V = \frac{1}{2}\beta_3 t^2 + \beta_4 t + V(t_0) \tag{55}$$

$$a = \beta_3 t + \beta_4$$

where β_3 and β_4 are defined as in (52) and (53), except that now a_i replaces S_i and the $V(t_0)$ and $S(t_0)$ are both related ultimately back to initial conditions.

We have yet to discuss how these impressed accelerations, velocities, and displacements are related back to the body segment for which the point in question is defined. Now since the forced variables can be specified with respect to any coordinate system, Fig. 8 illustrates the situation.

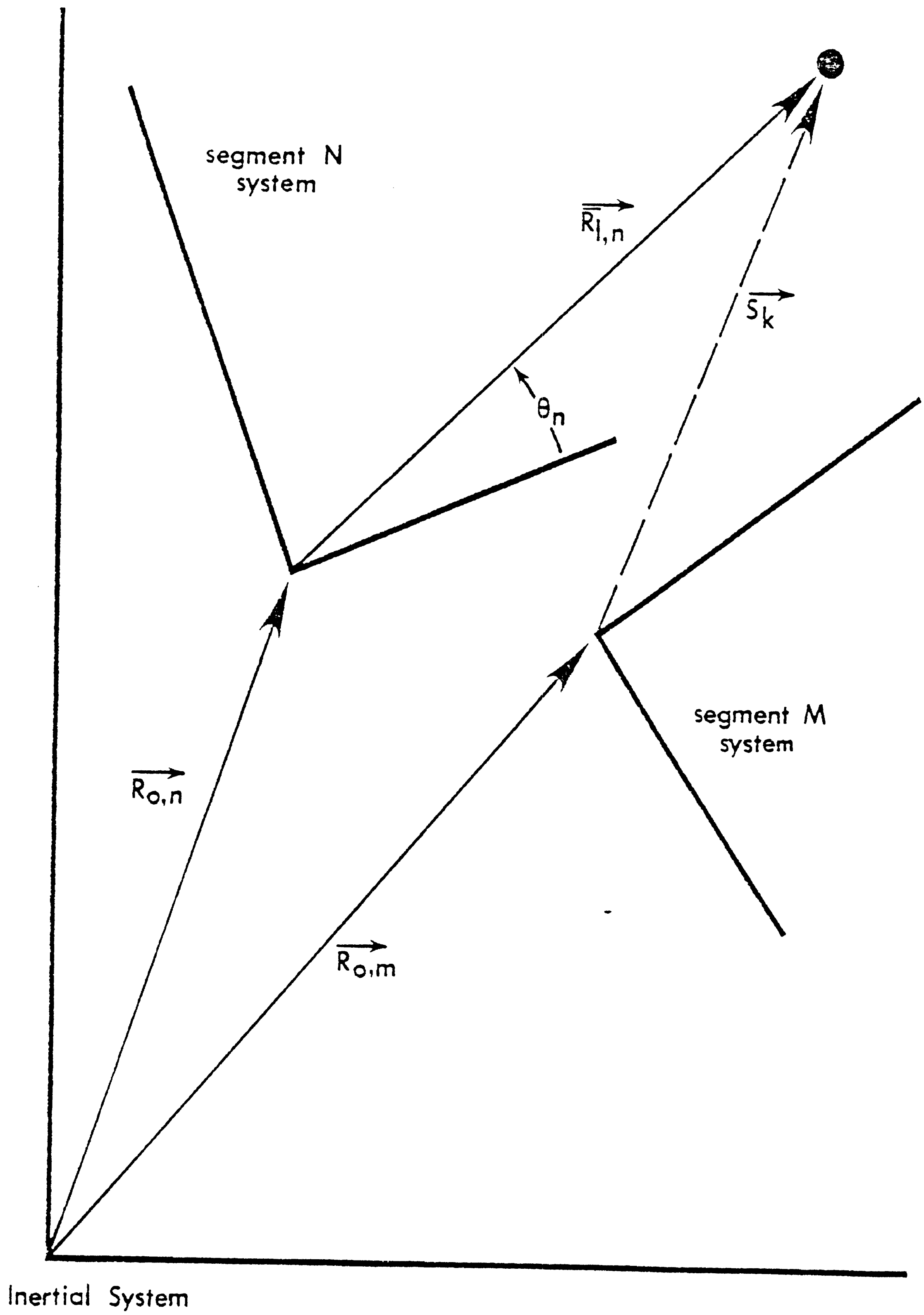

Fig. 8. Forced Acceleration

$$R_{0,n} = R_{0,m} + d_m^T S_k - d_n^T \bar{R}_{\ell,m} \tag{56}$$

are the ingredients to the contributions to the equations of motion specified in equation (19) or

$$\frac{\partial R_{0,n}}{\partial q_i} = \frac{\partial R_{0,m}}{\partial q_i} + \left(\frac{\partial d_m}{\partial q_i}\right)^T S_k + d_m^T \frac{\partial S_k}{\partial q_i} - \left(\frac{\partial d_n}{\partial q_i}\right)^T \bar{R}_{\ell,m} \tag{57}$$

and

$$\ddot{R}_{0,n} = \ddot{R}_{0,m} - d_m^T \ddot{S}_k + \left(\frac{\partial d_m}{\partial \theta_m}\right)^T \ddot{\theta}_m S_k - \left(\frac{\partial d_n}{\partial \theta_n}\right)^T \ddot{\theta}_n \bar{R}_{\ell,m}$$
$$- d_m^T \dot{\theta}_m^2 S_k + d_n^T \dot{\theta}_n^2 \bar{R}_{\ell,m} + 2\left(\frac{\partial d_m}{\partial \theta_m}\right)^T \dot{\theta}_m \dot{S}_k$$

Note that not only time derivatives but also the partials of the specified vector $\vec{S}_k$ are included because if one component of $\vec{S}_k$ is left unspecified, it is considered to be left free. In this case, the appropriate portion of equation (56) is added to the equations of motion and the missing component is solved for. If there are no unknowns which are not generalized coordinates in equation (56), then $R_{0,m}$ is eliminated from the equations of motion and (56) is used to compute $R_{0,n}$. This effectively makes the forced variable into an unlockable and unbreakable but movable pin joint without joint force capability.

Then to complete the usefulness of this feature, if the component specified is angular, it is always taken to specify the angle from the body segment system to the point.

Section 4.4 Applied Forces

Forces can be applied to any points in terms of either Cartesian or polar components specified with respect to any system. If the point is specified with respect to Segment n and the force with respect to Segment m then the components of the force vector in the segment n system are

$$d_n d_m^T F_{Sk} \tag{58}$$

The normal component is the dot product of the force vector and the unit vector to the point from the origin. The tangential component is the vector difference of the vector and its normal component, hence

$$\begin{aligned} Q_{y_m} &= \cos\theta_n F_T \\ Q_{z_m} &= \sin\theta_n F_T \\ Q_{\theta_m} &= \left|\bar{R}_{\ell,m}\right| F_N \end{aligned} \tag{59}$$

where F_T and F_N are the tangential and normal components obtained above.

5.0 PREVIEW OF INPUT TO VARISEG

Variseg will accept numbered input cards of fixed format as described in volume 2 of the MVMA 2-D Model (Ref. 1). Some of these cards must be generalized somewhat. This older format of input was keypunch oriented with fixed column ranges and identification field in columns 73 through 80. The approach has been found inconvenient with the advent of terminal systems and direct submittal of data decks into disk files.

Variseg will also accept a free format symbolic form of input designed for user ease. The following rules control the free format input:

1. Card identification is always first and is a specified three or four character designator. The card terminator is a period which replaces the last comma.
2. Usually one name is required immediately following the card identification and before the field terminator (a comma).
3. Certain specific fields are expected with each card. Each field consists of an optional symbolic designator, an appropriate value or name, and field terminator (comma).
4. The user elects to use default values by failing to specify a field or a whole card. A field can be left unspecified by including only the field terminator for the field or by skipping the field by use of designators.
5. The fields for each card are expected in a specific order. Any field without its designator will be taken as the next field in the expected sequence. If the designator is present, it will cause the pointer which keeps track of position in the expected sequence to be reset to the designated field. The implications are that the user can work in sequence or in arbitrary order and specify only the fields for which the default value is not adequate. Both techniques can be used interchangeably.

Table 5 contains a sampling of the free format cards with their card designators, expected sequence of fields, and their field designators. Table 5 shows only two of many cards to illustrate the type of input. Details are not yet complete for many of the cards.

TABLE 5 VARISEG FREE FORMAT INPUT

Card or Field	Contents	Defaults	Purpose and Description of Card or Field
1. Segment Specification Card			
SEG	Segment-Name,	Error if absent	Definition of external body segment system.
(MASS)	Mass,	Zero	Mass of segment. If zero, motion must be computable from applied positions, velocities, or accelerations
(MMI)	Moment of inertia,	Zero	Segment moment of inertia.
(FLAM)	Angulation Indicator,	Plus One	Plus one if positive counterclockwise; minus one if positive clockwise.
(OGME)	Counterclockwise angle from x-axis to z-axis,	270 degrees	Must not be zero, +180, or -180 degrees.
(UNIT)	Unit-name.	SI (Metric)	Name of unit system used for above values, SI (Metric and ANSI (American Standard) predefined, others defined by REF input cards.
2. Segment Initial Conditions			
RSEG	Segment-Name,	Error if absent	Segment-Name must be defined by SEG card.
(X)	Center of Gravity x Value,	Zero	Position of Center of Gravity, x-coordinate with respect to reference system.
(Z)	Center of Gravity z Value,	Zero	Position of Center of Gravity, z-coordinate with respect to reference system.
(TH)	Angle of x-axis from x-axis of reference system,	Zero	Orientation of Segment system with respect to reference system.
(XDOT)	x-velocity of center of gravity,	Zero	Velocity of Center of Gravity, x-coordinate with respect to reference system.
ZDOT)	z-velocity of center of gravity,	Zero	Velocity of Center of Gravity, z-coordinate with respect to reference system.
(THDOT)	Angular velocity,	Zero	Velocity of orientation angle with respect to reference system.
(COORD)	Segment-Name or INTERL,	INERTL	Reference system name. INERTL (Inertial) predefined, others defined with SEG cards. Path to INERTL must exist
(UNIT)	Unit-name.	Same as corresponding SEG card.	Name of unit system for above. See note under SEG card above.

6.0 REFERENCES

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