EFFECT OF UNCERTAINTY AND USEFULNESS OF PRIOR INFORMATION ON STRATEGIC RISK ASSESSMENT AND PLANNING

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Abstract

This paper presents game theoretic models to analyze the effect of uncertainty about the auditee on the auditor's risk assessment and planning in an internal audit in which both the auditor and the auditee make strategic moves. In addition, an internal audit is conceptualized as a sequential, information-gathering activity during which the auditor obtains prior information about the auditee to assess risk and to plan strategically. In this context, this paper also examines the usefulness and the role of prior information about the auditee in risk assessment and planning.

The analytic results indicate that, with incomplete information about the auditee type, an internal auditor takes more extreme actions, such as 100% testing or

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no testing. The analytic results offer theoretical support for professional standards (Standards for the Professional Practice of Internal Auditing, SAS No. 53, and SAS No. 55): in certain settings, prior information about the auditee, if utilized properly, allows the internal auditor to plan the audit tests more effectively. Also, in some cases, prior information of different accuracy about the auditee type can be a substitute for actual testing. Support is also found for Statement No. 1 on Quality Control Standards: gathering sufficient prior information about management integrity before client acceptance is important.

Key words: Strategic risk assessment and planning, Uncertainty, The usefulness and role of prior information, Internal audits.

Abbreviated title: Uncertainty and strategic risk assessment

1 Introduction

In this paper, we analyze the effects of uncertainty and usefulness of prior information on strategic planning and risk assessment by an internal auditor. Towards that end we formulate three game models between the auditor and the auditee: a game with complete information; a game with incomplete information; and a game with incomplete information and information asymmetry.

Existing audit game models in the literature (Fellingham and Newman, 1985; Shibano, 1990) typically assume complete information about the auditee and address the auditor's decision problems such as planning and risk assessment. In the real world, auditors do not have complete information about the auditee and gather such information by applying inquiries and observation. Professional standards (SAS No. 53, SAS No. 55, and Standards for the Professional Practice of Internal Auditing) suggest that the auditor assesses risk based upon background information about the auditee (hereafter, prior information about the auditee type). The auditor's assessment of prior information, in turn, determines the extent of audit tests. Prior information about the auditee, therefore, plays an important role in an audit; however, the literature has never addressed the effects of incomplete information or the usefulness of prior information about the auditee on an audit.

Uncertainty about the auditee seems to encourage product differentiation. The analytic results indicate that, with incomplete information about the auditee type, it is optimal for the internal auditor to take more extreme actions, such as 100% testing or no testing. The analytic results on the usefulness of prior information offer theoretical support for professional standards (Standards for the Professional Practice of Internal Auditing, SAS No. 53, and SAS No. 55); prior information about the auditee, if utilized properly, allows the internal auditor to plan the audit tests more effectively in certain settings. Prior information of different accuracy about the auditee type can even be a substitute for actual testing if certain conditions are met. The analytic results also support Statement No. 1 on Quality Control Standards. That is, more accurate information about the auditee does not necessarily mean higher payoff. For example, the internal auditor could face lower expected payoff as the information about the high risk auditee gets more accurate. In the external-audit setting, gathering sufficient prior information about management integrity before client acceptance, as required by Statement No. 1 on Quality Control Standards, would prevent this type of scenario.

The paper is organized in 4 parts. Section 2 presents the game model with complete information. The next section describes the game with incomplete information and analyzes the effects of uncertainty on planning. The following section derives and discusses the analytic results on the usefulness of prior information in strategic internal audits. In Section 5, the paper concludes by discussing contributions of this research and broaching possible future research.

2 The Game Model with Complete Information

Consider the situation where two types of internal auditors exist: experienced and less experienced. An experienced auditor is characterized by higher testing cost and a higher detection rate of fraud; a less experienced auditor is characterized by lower testing cost and a lower detection rate of fraud. Two different types of auditees (managers) represent two different levels of risks: a high risk of material fraud and a low risk of material fraud.[1] The penalty for material fraud when the auditee gets caught is a function of the fraud amount and is constrained to be more than the fraud amount. A high risk of fraud is attributed to the lower penalty for fraud, and a low risk of fraud is attributed to the higher penalty for fraud.

Each player knows the type of opponent with certainty. Thus, only one type of auditor and one type of auditee exists in each game. The decision variable (strategy) of the auditor is to determine the probability of 100% testing, and the decision variable (strategy) of the auditee is to determine the probability of material fraud.[2,3] The strategy space of each player is continuous from 0% to 100%. Exhibit 2.1 presents notation.

[1] If the auditee commits fraud, he commits fraud of the maximum possible amount, and, thus, the fraud is material. The maximum possible amount of fraud is the same for all auditees.

[2] The auditor is assigned to the audit unit after the auditee makes the move.

[3] According to SAS No. 22 (AU 311.05): "In planning the audit, the auditor should consider the nature, extent, and timing of work to be performed . . . ."

Exhibit 2.1: Notation

Auditor's strategy
"i" indexes type of auditor, $i \in \{1, 2\}$
1 = experienced
2 = inexperienced
"X_i" represents the strategy of type i auditor.
Auditee's strategy
"j" indexes the type of auditee, $j \in \{1, 2\}$
1 = low risk of material fraud
2 = high risk of material fraud
"Y_j" represents the strategy of type j auditee.
Auditor's cost elements
"M" represents the loss to company due to undetected material fraud.
"S_i" represents the cost of 100% testing to type i auditor.
"R_i" represents the detection rate of type i auditor given 100% testing.
Auditee's payoff elements
"F" represents the maximum possible amount of fraud.
"P_j" represents the penalty for material fraud to type j auditee ($P_1 > P_2$).

The cost function of each auditor has two elements: the expected cost of testing ($X_i S_i$) and the expected loss to the company due to undetected material fraud [$(1 - R_i X_i) Y_j M$]. For both types of auditors, the expected cost of testing is a linear function of the auditor's strategy, i.e., the probability of 100% testing. The loss to the company due to undetected material fraud (M), i.e., the cost of a type II error, is a constant regardless of the type of auditor. The probability that material fraud goes undetected, ($1 - R_i X_i$), given material fraud, is an inverse function of (i) the detection rate, given the type of auditor and 100% testing and (ii) the auditor's strategy. If the auditee gets caught, he pays the penalty ($P_j F$). Otherwise, the payoff to the auditee is the amount of the material fraud (F).

The game can be summarized as follows.

The expected cost to a type i auditor, given a type j auditee is:

$$X_iS_i + (1 - R_iX_i)Y_jM \tag{1}$$

The expected payoff to a type j auditee, given a type i auditor, is:

$$Y_jF - (R_iX_i)P_jY_jF = \{1 - R_iX_i)P_j\}Y_jF. \tag{2}$$

The decision problem of the type i auditor is to minimize equation (1) with respect to X_i, given that the type j auditee does not change strategy Y_j^*; and the decision problem of the type j auditee is to maximize equation (2) with respect to Y_j, given that the type i auditor does not change strategy X_i^*.

Theorem 1. *A non-cooperative solution to the game with complete information exists. If $R_jP_j > 1$ and $R_iM > S_i$, then the unique equilibrium strategies are as follows.*

The equilibrium strategy for a type i auditor: $X_i^* = 1/(R_iP_j)$

The equilibrium strategy for a type j auditee: $Y_j^* = S_i/(R_iM)$

Proof: See Appendix A. ■

The first hypothesis of the theorem ensures that, given 100% testing, the expected penalty for material fraud exceeds the fraud amount for any type of auditee. That is, it does not pay for any type of auditee to defraud given 100% testing. The second hypothesis of the theorem stipulates that, given material fraud, the cost of 100% testing is less than the benefit of 100% testing for any type of auditor. The benefit of 100% testing is the reduction in the expected loss to the company due to undetected material fraud.

The equilibrium strategy of a type i auditor indicates that the probability of 100% testing is an inverse function of the detection rate of fraud given 100% testing (R_i); accordingly, an experienced internal auditor works less than an inexperienced internal auditor. The penalty for material fraud (P_j) also affects, inversely, the probability of 100% testing; that is, any type of internal auditor audits less with a low risk auditee.

Each player chooses an equilibrium strategy such that the cost of the opponent's action is the same as the benefit of the opponent's action. For example, the type j auditee chooses the equilibrium strategy (Y_j^*) such that the cost of 100% testing (S_i) is the same as the benefit of 100% testing ($R_i M Y_j^*$) to a type i auditor. The type i auditor chooses the equilibrium strategy (X_i^*) such that the expected penalty for material fraud (cost) ($X_i^* R_i P_j F$) is the same as the fraud amount (benefit) (F) of a type j auditee.

3 The Game Model with Incomplete Information

3.1 The Model

Internal auditors of two different types – experienced or inexperienced – are randomly assigned to different audit units. Audit units represent either a low risk of material fraud or a high risk of material fraud. A player is assumed to have each type of opponent with probability $\frac{1}{2}$; i.e., the opponent of a player is equally likely to be of either type. Players derive the weighted average of payoff or cost functions contingent on the type of the opponent, weighted by the probability distribution over opponent types. Each auditor minimizes the weighted average of cost functions; and each auditee maximizes the weighted average of payoff functions.

The cost function of a type i auditor, when the auditee is type j, is $X_i S_i + (1 - R_i X_i) Y_j M$. Since the auditee is equally likely to be type 1 or type 2, the expected cost to the type i auditor is $\frac{1}{2} \sum_{j=1}^{2} \{X_i S_i + (1 - R_i X_i) Y_j M\}$. Thus, the game with incomplete information can be summarized as follows.[4]

The expected cost to a type 1 auditor is: $X_1 S_1 + \{(1 - R_1 X_1) M\}(\frac{1}{2} \sum_{j=1}^{2} Y_j)$.

The expected cost to a type 2 auditor is: $X_2 S_2 + \{(1 - R_2 X_2) M\}(\frac{1}{2} \sum_{j=1}^{2} Y_j)$.

The expected payoff to a type 1 auditee is: $\{1 - P_1(\frac{1}{2} \sum_{i=1}^{2} R_i X_i)\} Y_1 F$.

The expected payoff to a type 2 auditee is: $\{1 - P_2(\frac{1}{2} \sum_{i=1}^{2} R_i X_i)\} Y_2 F$.

The decision problem of each auditor, given the strategy of the auditee, is to minimize the expected cost with respect to X_1 or X_2; and the decision problem of each auditee is to

[4] In Section 2, each player knows the type of opponent with certainty, and four different games with complete information are possible depending upon the types of auditor and auditee. However, in Section 3, each opponent is equally likely for each player, and only one game with incomplete information occurs.

maximize the expected payoff with respect to Y_1 or Y_2.

3.2 Equilibrium Concept

When each type of opponent is equally likely, four different pairs of an auditor's expected cost and an auditee's expected payoff are possible; and, depending upon the values of the parameters, four different equilibria exist. Assume that $S_1/(R_1M) > S_2/(R_2M)$. That is, when both types of auditors perform the same routine task, the cost to benefit ratio is higher for the experienced auditor than for the inexperienced auditor. This is due to diminishing marginal returns for such tasks. However, for complex tasks, the experienced auditor can attain increasing marginal returns, and therefore, may have lower cost to benefit ratio than the inexperienced auditor.

Theorem 2. *If $S_1/(R_1M) > S_2/(R_2M)$, a non-cooperative solution to the game with incomplete information exists for the following cases:*

(i) $0 < S_2/(R_2M) \leq 0.5$, *and* $0 < 1/P_j \leq 0.5R_2$ *for* $j = 1, 2$.

(ii) $0.5 \leq S_i/(R_iM) < 1$ *for* $i = 1, 2$, *and* $0 < 1/P_1 \leq 0.5R_2$.

(iii) $0 < S_i/(R_iM) \leq 0.5$ *for* $i = 1, 2$, *and* $0.5R_2 < 1/P_2 \leq 0.5(R_1 + R_2)$.

(iv) $0.5 \leq S_1/(R_1M) < 1$, *and* $0.5R_2 < 1/P_j \leq 0.5(R_1 + R_2)$ *for* $j = 1, 2$.

If $0 < S_2/(R_2M) \leq 0.5$ *and* $0 < 1/P_2 \leq 0.5R_2$, *then the unique equilibrium strategies are as follows.*

The equilibrium strategy for a type 1 auditor: $X_1^* = 0$

The equilibrium strategy for a type 2 auditor: $X_2^* = 2/(R_2P_2)$

The equilibrium strategy for a type 1 auditee: $Y_1^* = 0$

The equilibrium strategy for a type 2 auditee: $Y_2^* = 2S_2/(R_2M)$

Proof: See Appendix A. ■

Although four different non-cooperative equilibria are possible depending upon the penalties for material fraud and the cost to benefit ratios, at all four equilibria, players exhibit the same behavior (strategies for the four different cases of Theorem 2 appear in Appendix B). That is, players go more extreme when the opponent is equally likely to be of either type. At each equilibrium, the benefit of 100% testing (or material fraud) is the same as the cost of 100% testing (or material fraud) only for one type of auditor (or auditee). For the other type, either the benefit exceeds the cost or the cost exceeds the benefit. Therefore, the other type of player takes the extreme strategy such as no testing (or no material fraud) or 100% testing (or material fraud). When the benefit of action is the same as the cost of action for a player, since the player can influence the behavior of the opponent with probability $\frac{1}{2}$, the player exaggerates the move, that is, the player doubles the probability of 100%testing or doubles the probability of material fraud over the strategy of the complete information game studied in Section 2.

4 The Game Model with Information Asymmetry

Although two different types of auditors and auditees are equally likely to be matched, the auditors have more information about the auditee than the auditees have about the auditor, and thus there is information asymmetry in this game model. In this section we propose a model to investigate this asymmetry.

4.1 The Model

Here we conceptualize an internal audit as a sequential, information-gathering activity in which the auditor gathers background information about the auditee to assess auditee's incentive for fraud; and to generate a strategic plan based upon this assessment. This section further hypothesizes that only the experienced auditor can utilize this background information. An auditee is, however, required to make a move before an auditor is assigned to an audit unit; an auditee does not have any specific information about the auditor type.

Information asymmetry is modeled as follows. Type 1, the experienced auditor, is modeled to observe a signal about the type of opponent. There are two signals: θ_1 and θ_2; and the accuracy of the signals, $\phi > 0.50$. If the signal is θ_1, the probability that the opponent is type 1 is ϕ; and if the signal is θ_2, the probability that the opponent is type 2 is ϕ. Exhibit

4.1 explains the necessary changes in notation to incorporate this information asymmetry.

Exhibit 4.1: Notation

ϕ represents signal accuracy and $\phi > 0.5$.
$X_{1\theta_1}$ represents the strategy of the type 1 auditor contingent on θ_1.
$X_{1\theta_2}$ represents the strategy of the type 1 auditor contingent on θ_2.
X_2 represents the strategy of the type 2 auditor.

The type 1 auditor, observes a signal which indicates the type of auditee with probability ϕ, and commits to a strategy contingent upon the signal observed. For example, given the signal is θ_1 and the auditee is type 1, the cost to the type 1 auditor is: $X_{1\theta_1}S_1 + \{(1 - R_1X_{1\theta_1})Y_1M\}$.

When the type 1 auditor observes the signal θ_1, the probability of a type 1 auditee as an opponent is ϕ, and the probability of a type 2 auditee is $(1-\phi)$. When type 1 auditor observes θ_2, the probability of a type 1 auditee as an opponent is $(1-\phi)$, and the probability of a type 2 auditee is ϕ. The opponent of a type 2 auditor is equally likely to be type 1 or type 2 auditee.

Thus, the expected cost to a type 1 auditor when the signal observed is θ_1 is:

$$\phi[X_{1\theta_1}S_1 + \{(1 - R_1X_{1\theta_1})Y_1M\}] + (1-\phi)[X_{1\theta_1}S_1 + \{(1 - R_1X_{1\theta_1})Y_2M\}]. \tag{3}$$

The expected cost to a type 1 auditor when the signal observed is θ_2 is:

$$(1-\phi)[X_{1\theta_2}S_1 + \{(1 - R_1X_{1\theta_2})Y_1M\}] + \phi[X_{1\theta_2}S_1 + \{(1 - R_1X_{1\theta_2})Y_2M\}]. \tag{4}$$

The expected cost to a type 2 auditor is:

$$X_2S_2 + \{(1 - R_2X_2)M\}(\frac{1}{2}\sum_{j=1}^{2} Y_j). \tag{5}$$

The opponent of each type of auditee is equally likely to be a type 1 auditor or a type 2 auditor. However, when a type 1 auditor is matched with a type 1 auditee, the probability that the type 1 auditor observes θ_1 is ϕ and the probability that the type 1 auditor observes θ_2 is $(1-\phi)$. Thus, the probability distribution of the type 1 auditee is as follows: the probability that the type 1 auditee is matched with a type 1 auditor and the type 1 auditor uses the strategy contingent on θ_1 is 0.5ϕ, the probability that the type 1 auditee is matched with a type 1 auditor and the type 1 auditor uses strategy contingent on θ_2 is $0.5(1-\phi)$. Also the probability that the opponent is a type 2 auditor is 0.5. The probability distribution of matching a type 2 auditee can be derived similarly. Thus, the expected payoff for the type 1 auditee is:

$$0.5\phi\{1 - P_1(R_1X_{1\theta_1})\}Y_1F + 0.5(1-\phi)\{1 - P_1(R_1X_{1\theta_2})\}Y_1F + 0.5\{1 - P_1(R_2X_2)\}Y_1F. \quad (6)$$

The expected payoff for the type 2 auditee is:

$$0.5(1-\phi)\{1 - P_2(R_1X_{1\theta_1})\}Y_2F + 0.5\phi\{1 - P_2(R_1X_{1\theta_2})\}Y_2F + 0.5\{1 - P_2(R_2X_2)\}Y_2F. \quad (7)$$

The game with information asymmetry can be described by Equations (3)-(7). The decision problem of the type 1 auditor is to minimize Equation (3) with respect to $X_{1\theta_1}$ and Equation (4) with respect to $X_{1\theta_2}$. The decision of the type 2 auditor is to minimize Equation (5) with respect to X_2. The decision problem of the type 1 and type 2 auditee is to maximize Equation (6) and (7) with respect to Y_1 and Y_2 respectively.

4.2 Equilibrium Concept

The availability of a signal about the type of auditee to a type 1 auditor changes the game with incomplete information in two different ways. The type 1 auditor commits to a strategy contingent upon the signal observed and incurs a different expected cost contingent upon the signal. Thus, the game with information asymmetry consists of five objective functions instead of four.

Theorem 3. *A non-cooperative solution to the game with information asymmetry exists. For the following six cases,*

(i) $(1-\phi) \leq S_1/(R_1M) < 1$, *and* $0.5\{(1-\phi)R_1 + R_2\} \leq 1/P_j \leq 0.5(R_1 + R_2)$ *for* $j = 1, 2$,

(ii) $0 < S_i/(R_iM) \leq (1-\phi)$ *for* $i = 1, 2$, *and* $0.5(\phi R_1 + R_2) \leq 1/p_2 \leq 0.5(R_1 + R_2)$,

(iii) $\phi \leq S_1/(R_1M) < 1$, *and* $0.5R_2 \leq 1/P_1 < 0.5\{(1-\phi)R_1 + R_2\}$,

(iv) $0 < S_i/(R_iM) \leq \phi$ *for* $i = 1, 2$, *and* $0.5R_2 \leq 1/P_2 < 0.5(\phi R_1 + R_2)$,

(v) $0.5 \leq S_i/(R_iM) < 1$ *for* $j = 1, 2$, *and* $1/P_1 < 0.5R_2$, *and*

(vi) $0 < S_2/(R_2M) \leq 0.5$, *and* $1/P_j < 0.5R_2$ *for* $j = 1, 2$,

the equilibrium strategies also satisfy

$$(\phi - 0.5)(Y_2^* - Y_1^*) < \{S_1/(R_1M) - S_2/(R_2M)\} \tag{8}$$

and

$$(\phi - 0.5)(X_{1\theta_2}^* - X_{1\theta_1}^*) < \{1/R_1P_2) - 1/(R_1P_1)\}. \tag{9}$$

If $0 \leq S_i/(R_iM) < \phi$ *for* $i = 1, 2$, *and* $0.5R_2 \leq 1/P_2 < 0.5(\phi R_1 + R_2)$, *then the unique equilibrium strategies are as follows.*

The equilibrium strategy for a type 1 auditor with $\theta_1 : X_{1\theta_1}^* = 0$

The equilibrium strategy for a type 1 auditor with $\theta_2 : X_{1\theta_2}^* = 2/(\phi R_2P_2) - R_2/(\phi R_1)$

The equilibrium strategy for a type 2 auditor: $X_2^* = 1$

The equilibrium strategy for a type 1 auditee: $Y_1^* = 0$

The equilibrium strategy for a type 2 auditee: $Y_2^* = S_1/(\phi R_1M)$

Proof: See Appendix A. ■

Equilibrium strategies for all six cases of Theorem 3 appear in Appendix B.

Condition (8) on equilibrium strategies of Theorem 3 imposes a limit on the accuracy of prior information. The right-hand side of the inequality represents incremental cost per dollar return for an experienced auditor assuming material fraud and 100% testing. The left-hand side of the inequality is the increase in the probability of material fraud due to prior information of accuracy ϕ. The six cases considered in this theorem limit the accuracy of prior information such that incremental cost for an experienced auditor is higher than expected incremental return due to prior information. In other words, it is more cost-effective to use inexperienced auditors for routine tasks.

Condition (9) on equilibrium strategies of Theorem 3 also limits the accuracy of prior information. The right-hand side of the inequality represents the difference between the probability of 100% testing for a type 1 auditor with a type 2 auditee and such probability of 100% testing with a type 1 auditee. The left-hand side of the inequality is the difference between the probability of 100% testing contingent upon θ_2 and such probability of 100% testing contingent upon θ_1. θ_1 is related to a type 1 auditee with accuracy ϕ, and θ_2 is related to a type 2 auditee with the same accuracy. Condition (9) on equilibrium strategies limits the value of ϕ such that, the difference between the probability of 100% testing contingent upon θ_2 of accuracy ϕ and such probability of 100% testing contingent upon θ_1, is less than the difference between the probability of 100% testing for a type 1 auditor with a type 2 auditee and such probability of 100% testing with a type 1 auditee.

When we compare the strategy of a type 2 auditee obtained in this model with the strategy obtained in Theorem 2, we note that prior information about the auditee type decreases the probability of material fraud by this auditee (which represents a high risk of material fraud). The probability of 100% testing by a type 1 auditor is also decreased when the signal observed is θ_2.

4.3 Usefulness of Prior Information about Auditee Type

FASB Statement of Financial Accounting Concepts No. 2 defines that the decision usefulness of accounting information depends upon relevance and reliability. Relevant accounting information is defined to the information that can make a difference in a decision by helping users to form predictions about the outcomes of past, present, and future events or to confirm or correct prior expectations.

Theorem 4. *Prior information about the auditee type is useful in the first four cases of Theorem 3. In the last two cases of Theorem 3, prior information about the auditee type plays no roles in the strategy of a type 1 auditor.*

Proof: See Appendix A. ■

In the first four cases of Theorem 3, the equilibrium strategy of an experienced auditor is contingent upon the signal observed. In the last two cases, the penalties for material fraud are the highest, and the probability of material fraud is the lowest. In these two cases, the cost of testing is always greater than the benefit of testing for an experienced auditor, and, thus, the equilibrium strategy is no testing regardless of the signal observed.

The risk analysis model for internal and external auditors suggests that auditors assess risk based upon prior information about the auditees. Based upon the assessment of risk, auditors determine the nature, the timing, and the extent of audit tests. The role of prior information is to enable auditors to plan more effectively. The higher the risk, the larger the extent of the tests should be.

In the game models of this study, the auditee's strategy (Y_j^*) is equivalent to risk. The extent of testing is, on the other hand, the strategy of a type 1 auditor contingent upon prior information ($X_{1\theta_i}^*$). Since θ_2 represents a higher risk of material fraud than θ_1, an effective audit would mean $X_{1\theta_2}^* \geq X_{1\theta_1}^*$. In the first four cases of Theorem 3, $X_{1\theta_2}^* > X_{1\theta_1}^*$; that is, prior information about the auditee type allows an experienced auditor to plan more effectively.

4.4 Prior Information of Different Accuracy about the Auditee Type

This subsection assumes that any internal auditor gathers the same amount of prior information as requested by professional standards. Thus, the cost of gathering prior information is irrelevant or negligible as prior information should be obtained in any circumstances. This subsection also hypothesizes that the accuracy of prior information is solely a function of experience. That is, the accuracy of prior information depends upon an auditor's ability to recognize red flags.

If prior information of different accuracy about the auditee type is a substitute for actual testing, prior information has two possible roles: (i) to reduce the extent of actual testing or (ii) to discourage material fraud. Thus, as the accuracy of prior information about the auditee type increases, the probability of 100% testing and the probability of material fraud will decrease provided they change.

Theorem 5. *Prior information of different accuracy about the auditee type is not a substitute for actual testing in the first three cases of Theorem 3. Prior information of different accuracy about the auditee type is a substitute for actual testing in the fourth case of Theorem 3. In other words, in the fourth case of Theorem 3,* $\partial Y_j^*/\partial \phi \leq 0$ *and* $\partial X_{1\theta_i}^*/\partial \phi \leq 0$.

Proof: See Appendix A. ■

In Case (iv) of Theorem 3, the penalties for material fraud are such that a type 1 auditee does not have any incentive to commit material fraud; however, a type 2 auditee clearly has the incentive to commit material fraud. The cost to benefit ratio of testing for an experienced auditor is such that the optimal strategy for an experienced auditor, contingent upon θ_1, is no testing; however contingent upon θ_2, an experienced auditor has the incentive to do testing. Thus, as the accuracy of prior information about the auditee type improves, a type 2 auditee decreases the probability of material fraud, and an experienced auditor decreases the extent of testing contingent upon θ_2. Thus, in Case 4 of Theorem 3, prior information of different accuracy about the auditee a type is a substitute for actual testing.

In the first two cases of Theorem 3, the penalties for material fraud are the lowest for both types of auditees, and the probability of material fraud is the highest. The equilibrium strategy of an experienced auditor, contingent upon θ_2, is 100% testing regardless of the accuracy of prior information about the auditee type. Thus, as an experienced auditor gets more accurate information that the auditee is a high risk of material fraud, the experienced auditor faces higher expected loss due to undetected material fraud and, thus, higher expected cost.

In Case (iii) of Theorem 3, testing is costly for an experienced auditor, and, thus, the probability of 100% testing by an experienced auditor, contingent upon θ_2, is a function of the accuracy of prior information about the auditee type. As θ_2 is more closely related to a higher risk of material fraud, and the accuracy of the signals improves, an experienced auditor increases the probability of 100% testing, contingent upon θ_2.

5 Conclusion and Future Research

5.1 Conclusion

This study presents an audit game model which conceptualizes an internal audit as a sequential, information-gathering activity. In the game model, the auditor gathers prior information about the auditee, assesses the risk of material fraud, and plans the audit test strategically based upon the assessment. Using the audit game model, this research also analyzes the effects of uncertainty on the strategic relationship between the auditor and the auditee and, further, addresses the role of prior information about the auditee in strategic audits.

The analytic results indicate that both the auditor and the auditee take such more extreme actions as 100% testing or no testing (material fraud or no material fraud) with uncertainty about the type of opponent. The scenario is similar to that where two policemen try to get the truth out of a suspect. It is more effective (or strategic) for them to exaggerate the difference between their moves in a given situation. That is, one of them plays a nice guy, and the other a bad guy. Even without formal coordination of strategies between two types of auditors or auditees, the same phenomenon seems to occur in the game with incomplete information.[5] One of them increases the extent of testing (or the probability of material fraud), and the other decreases the extent of testing (or the probability of material fraud) due to uncertainty about the type of opponent.

The usefulness of costless prior information seems to depend upon the control environment. If the penalty for material fraud for any type of auditee is so high that the probability of material fraud is not high enough to warrant any testing, then the optimal strategy is to do no testing regardless of the accuracy of prior information. In this case, prior information about any type of auditee plays no role in planning the audit test.

Another analytic result is that more accurate prior information, even if costless, does not guarantee higher payoff or lower expected cost. If the penalty for material fraud is so low to warrant 100% testing regardless of prior information, then more accurate prior information about the auditee which represents a high risk of material fraud merely increases the expected loss due to undetected material fraud. This increase in the expected loss in inevitable due to imperfect audit technology. In the external-audit setting, gathering sufficient information about management integrity before client acceptance would prevent this type of scenario.

The role of prior information of different accuracy about the auditee type depends upon the penalties for material fraud for different types of auditees and the cost to benefit ratio

[5] According to the Folk Theorem, outcomes usually associated with cooperation can be supported by non-cooperative equilibrium strategies (Friedman (1986), p. 103).

of testing for the auditor who can properly utilize the information. For example, prior information of different accuracy can be a substitute for actual testing if the following two conditions are met: first, one group of auditees have no incentive to commit material fraud; however, the other group of auditees have incentive to commit material fraud; second, the equilibrium strategy for experiences auditors is no testing if the opponent is more likely to be a low risk; and, the equilibrium strategy is to do testing if the opponent is more likely to be a high risk. In this scenario, as the accuracy of prior information about the auditee type improves, a high risk type auditee decreases the probability of material fraud, and, thus, an experienced auditor can also decrease the extent of testing, contingent upon θ_2.

5.2 Future Research

The strength of game theory as a research tool is its ability to address conflicts of interests among different interest groups. Accordingly, game theoretic results should be most useful to the policy makers. Despite its usefulness, no attempt was made to evaluate the efficacy of any auditing policy with game theory until Newman and Noel (1989) investigated the equilibrium impact of policies (e.g., the Foreign Corrupt Practices Act) on the auditor's risk assessment and planning.

The welfare implications of some accounting and/or auditing issues on auditors and auditees can be derived by means of the game model. For example, the implications of different degrees of auditing on different interest groups can be analyzed in the game theoretic context. Because of excessive litigation against auditors, questions have been raised about the adequacy of audit procedures or the quality of audits. A need exists to understand the welfare implications of different degrees of auditing on different interest groups.

The audit game model can be extended to multiple period scenarios. Since internal audits are typically done once every three or four years, an internal audit is likely to be a single-shot game in response to changes in personnel or some other environmental characteristics. However, an internal audit can also be a finitely repeated game. The literature on reputation (Kreps and Wilson, 1982; Kreps et al. 1982; Milgrom and Roberts, 1982a; Milgrom and Roberts, 1982b) predicts bluffing in a multiple-period game. That is, the weaker player tries to influence the probability assessment of the opponent over its type by mimicking the stronger player as the game is repeatedly played and the assessment of the opponent's type is dependent on the prior moves.

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Appendix A

Theorem 1. *A non-cooperative solution to the game with complete information exists. If $R_iP_j > 1$ and $R_iM > S_i$, then the unique equilibrium strategies are as follows.*

The equilibrium strategy for a type i auditor: $X_i^* = 1/(R_iP_j)$

The equilibrium strategy for a type j auditee: $Y_j^* = S_i/(R_iM)$

Proof: Let X_i^* and Y_j^* be the equilibrium strategies.

Then, $(1 - X_i^*R_iP_j)(Y_j^* - Y_j) \geq 0$ for every $0 \leq Y_j \leq 1$ and

$(S_i - R_iY_j^*M)(X_i - X_i^*) \geq 0$ for every $0 \leq X_i \leq 1$.

If $(1 - X_i^*R_iP_j) = 0$ and $(S_i - R_iY_j^*M) = 0$, the above two conditions are satisfied.

Thus, since $R_iP_j > 1$ and $R_iM > S_i$, $X_i^* = 1/(R_iP_j)$ and $Y_j^* = S_i/(R_iM)$.

To see the uniqueness, if $1 - X_i^*R_iP_j > 0$, then $Y_j^* = 1$,

and $S_i - R_iY_j^*M < 0$, so $X_i^* = 1$ and $1 > R_iP_j$ a contradiction.

Also, if $1 - X_i^*R_iP_j < 0$, then $Y_j^* = 0$ and $X_i^* = 0$ a contradiction.

Theorem 2. *If $S_1/(R_1M) > S_2/(R_2M)$, a non-cooperative solution to the game with incomplete information exists for the following cases:*

(i) $0 < S_2/(R_2M) \leq 0.5$, *and* $0 < 1/P_j \leq 0.5R_2$ *for* $j = 1, 2$.

(ii) $0.5 \leq S_i/(R_iM) < 1$ *for* $i = 1, 2$, *and* $0 < 1/P_1 \leq 0.5R_2$.

(iii) $0 < S_i/(R_iM) \leq 0.5$ *for* $i = 1, 2$, *and* $0.5R_2 < 1/P_2 \leq 0.5(R_1 + R_2)$.

(iv) $0.5 \leq S_1/(R_1M) < 1$, *and* $0.5R_2 < 1/P_j \leq 0.5(R_1 + R_2)$ *for* $j = 1, 2$.

If $0 < S_2/(R_2M) \leq 0.5$, and $0 < 1/P_j \leq 0.5R_2$, then the equilibrium strategies are as follows.

The equilibrium strategy for a type 1 auditor: $X_1^* = 0$

The equilibrium strategy for a type 2 auditor: $X_2^* = 2/(R_2P_2)$

The equilibrium strategy for a type 1 auditee: $Y_1^* = 0$

The equilibrium strategy for a type 2 auditee: $Y_2^* = 2S_2/(R_2M)$

Proof: Let X_1^*, X_2^*, Y_1^* and Y_2^* be equilibrium strategies of the four players. The equilibrium strategies of the four players must meet the following conditions.

$$\{S_1 - R_1M(Y_1^* + Y_2^*)/2\}\{X_1 - X_1^*\} \geq 0 \text{ for all } 0 \leq X_1 \leq 1$$

$$\{S_2 - R_2M(Y_1^* + Y_2^*)/2\}\{X_2 - X_2^*\} \geq 0 \text{ for all } 0 \leq X_2 \leq 1$$

$$\{1 - P_1(X_1^*R_1 + X_2^*R_2)/2\}\{Y_1^* - Y_1\} \geq 0 \text{ for all } 0 \leq Y_1 \leq 1$$

$$\{1 - P_2(X_1^*R_1 + X_2^*R_2)/2\}\{Y_2^* - Y_2\} \geq 0 \text{ for all } 1 \leq Y_2 \leq 1$$

$\hat{Y} = (Y_1^* + Y_2^*)/2$ and $\hat{X} = (X_1^*R_1 + X_2^*R_2)/2$

Using the result of Theorem 1, it can be shown that equilibrium strategies, for each case, satisfy:

Case	$\hat{X}$	$\hat{Y}$
i	$1/P_2$	$S_2/(R_2M)$
ii	$1/P_1$	$S_2/(R_2M)$
iii	$1/P_2$	$S_1/(R_1M)$
iv	$1/P_1$	$S_1/(R_1M)$

For case (i) $\hat{Y} = (Y_1^* + Y_2^*)/2 = S_2/(R_2M)$. Therefore, $\{S_2 - R_2M(Y_1^* + Y_2^*)/2\} = 0$.
Then, $\{S_1 - R_1M(Y_1^* + Y_2^*)/2\} > 0$ since $S_1/(R_1M) > S_2/(R_2M)$.
Thus, X_2^* is arbitrary and $X_1^* = 0$.
$\hat{X} = (X_1^*R_1 + X_2^*R_2)/2 = 1/P_2$. Therefore, $\{1 - P_2(X_1^*R_1 + X_2^*R_2)/2\} = 0$.
Then, $\{1 - P_1(X_1^*R_1 + X_2^*R_2)/2\} < 0$ since $P_1 > P_2$

Thus, $Y_1^* = 0$ and Y_2^* is arbitrary.

Since $\{1 - P_2(X_1^* R_1 + X_2^* R_2)/2\} = 0$ and $X_1^* = 0$, $X_2^* = 2/(P_2 R_2)$.

Since $\{S_2 - R_2 M(Y_1^* + Y_2^*)/2\} = 0$ and $Y_1^* = 0, Y_2^* = 2S_2/(R_2 M)$.

Since $0 < S_2/(R_2 M) \leq 0.5$, and $2 \leq R_2 P_2$, $0 \leq X_2^* \leq 1$ and $0 \leq Y_2^* \leq 1$. The equilibrium strategies for the other cases can be derived similarly.

Theorem 3. *A non-cooperative solution to the game with information asymmetry exists. For the following six cases,*

(i) $(1-\phi) \leq S_1/(R_1 M) < 1$, *and* $0.5\{(1-\phi)R_1 + R_2\} \leq 1/P_j \leq 0.5(R_1 + R_2)$ *for* $j = 1, 2$,

(ii) $0 < S_i/(R_i M) \leq (1-\phi)$ *for* $i = 1, 2$, *and* $0.5(\phi R_1 + R_2) \leq 1/P_2 \leq 0.5(R_1 + R_2)$,

(iii) $\phi \leq S_1/(R_1 M) < 1$, *and* $0.5R_2 \leq 1/P_1 < 0.5\{(1-\phi)R_1 + R_2\}$,

(iv) $0 < S_i/(R_i M) \leq \phi$ *for* $i = 1, 2$, *and* $0.5R_2 \leq 1/P_2 < 0.5(\phi R_1 + R_2)$,

(v) $0.5 \leq S_i/(R_i M) < 1$ *for* $i = 1, 2$, *and* $1/P_1 < 0.5R_2$, *and*

(vi) $0 < S_2/(R_2 M) \leq 0.5$, *and* $1/P_j < 0.5R_2$ *for* $j = 1, 2$,

the equilibrium strategies also satisfy

$$(\phi - 0.5)(Y_2^* - Y_1^*) < \{S_1/(R_1 M) - S_2/(R_2 M)\}$$

and

$$(\phi - 0.5)(X_{1\theta_2}^* - X_{1\theta_1}^*) < \{1/(R_1 P_2) - 1/(R_1 P_1)\}.$$

If $0 \leq S_i/(R_i M) < \phi$ *for* $i = 1, 2$, *and* $0.5R_2 \leq 1/P_2 < 0.5(\phi R_1 + R_2)$, *then the unique equilibrium strategies are as follows.*

The equilibrium strategy for a type 1 auditor with $\theta_1 : X_{1\theta_1}^* = 0$

The equilibrium strategy for a type 1 auditor with $\theta_2 : X_{1\theta_2}^* = 2/(\phi R_1 P_2) - R_2/(\phi R_1)$

The equilibrium strategy for a type 2 auditor: $X_2^* = 1$

The equilibrium strategy for a type 1 auditee: $Y_1^* = 0$

The equilibrium strategy for a type 2 auditee: $Y_2^* = S_1/(\phi R_1 M)$

Proof: Let $X^*_{1\theta_1}, X^*_{1\theta_2}, X^*_2, Y^*_1$ and Y^*_2 represent the equilibrium strategies of the four players. The equilibrium strategies of the four players must meet the following conditions.

$$[S_1 - R_1M\{\phi Y^*_1 + (1-\phi)Y^*_2\}][X_{1\theta_1} - X^*_{1\theta_1}] \geq 0 \text{ for all } 0 \leq X_{1\theta_1} \leq 1$$

$$[S_1 - R_1M\{(1-\phi)Y^*_1 + \phi Y^*_2\}][X_{1\theta_2} - X^*_{1\theta_2}] \geq 0 \text{ for all } 0 \leq X_{1\theta_2} \leq 1$$

$$[S_2 - R_2M(Y^*_1 + Y^*_2)/2][X_2 - X^*_2] \geq 0 \text{ for all } 0 \leq X_2 \leq 1$$

$$[1 - P_1\{\phi R_1X^*_{1\theta_1} + (1-\phi)R_1X^*_{1\theta_2} + R_2X_2\}/2][Y^*_1 - Y_1] \geq 0 \text{ for all } 0 \leq Y_1 \leq 1$$

$$[1 - P_2\{(1-\phi)R_1X^*_{1\theta_1} + \phi R_1X^*_{1\theta_2} + R_2X^*_2\}/2][Y^*_2 - Y_2] \geq 0 \text{ for all } 0 \leq Y_2 \leq 1$$

$$\hat{Y}_1 = \phi Y^*_1 + (1-\phi)Y^*_2,\ \hat{Y}_2 = (1-\phi)Y^*_1 + \phi Y^*_2, \text{ and } \hat{Y}_3 = 0.5(Y^*_1 + Y^*_2)$$

$$\hat{X}_1 = 0.5\{\phi R_1X^*_{1\theta_1} + (1-\phi)R_1X^*_{1\theta_2} + R_2X^*_2\}.$$

$$\hat{X}_2 = 0.5\{(1-\phi)R_1X^*_{1\theta_1} + \phi R_1X^*_{1\theta_2} + R_2X^*_2\}.$$

Using the result of Theorem 1, it can be shown that the equilibrium strategies, for each case, satisfy:

Case	$\hat{X}_1$ or $\hat{X}_2$	$\hat{Y}_1, \hat{Y}_2$ or $\hat{Y}_3$
i	$\hat{X}_1 = 1/P_1$	$\hat{Y}_1 = S_1/(R_1M)$
ii	$\hat{X}_2 = 1/P_2$	$\hat{Y}_1 = S_1/(R_1M)$
iii	$\hat{X}_1 = 1/P_1$	$\hat{Y}_2 = S_1/(R_1M)$
iv	$\hat{X}_2 = 1/P_2$	$\hat{Y}_2 = S_1/(R_1M)$
v	$\hat{X}_1 = 1/P_1$	$\hat{Y}_3 = S_2/(R_2M)$
vi	$\hat{X}_2 = 1/P_2$	$\hat{Y}_3 = S_2/(R_2M)$

For case (iv) $\hat{Y}_2 = (1-\phi)Y^*_1 + \phi Y^*_2 = S_1/(R_1M)$. Therefore, $[S_1 - R_1M\{(1-\phi)Y^*_1 + \phi Y^*_2\}] = 0$. Then, $[S_1 - R_1M\{\phi Y^*_1 + (1-\phi)Y^*_2\}] > 0$ since $Y^*_2 > Y^*_1$. $Y^*_2 = Y^*_1$ or $Y^*_2 < Y^*_1$ leads to a contradiction.

$[S_2 - R_2M(Y^*_1 + Y^*_2)/2] < 0$ since $S_2/(R_2M) < \hat{Y}_3 < S_1/(R_1M)$ due to condition (8) on the equilibrium strategies. Thus, $X^*_2 = 1$ and $X^*_{1\theta_1} = 0$. $X^*_{1\theta_2}$ is arbitrary.

$\hat{X}_2 = 0.5\{(1-\phi)R_1X^*_{1\theta_1} + \phi R_1X^*_{1\theta_2} + R_2X^*_2\} = 1/P_2$. Therefore, $[1 - P_2\{(1-\phi)R_1X^*_{1\theta_1} + \phi R_1X^*_{1\theta_2} + R_2X^*_2\}/2] = 0$. Then, $[1 - P_1\{\phi R_1X^*_{1\theta_1} + (1-\phi)R_1X^*_{1\theta_2} + R_2X^*_2\}/2] < 0$ since $P_2\hat{X}_2 < P_1\hat{X}_1$ due to condition (9) on the equilibrium strategies.

$[1 - P_1\{\phi R_1X^*_{1\theta_1} + (1-\phi)R_1X^*_{1\theta_2} + R_2X^*_2\}/2] \geq 0$ cannot happen. Thus, $Y^*_1 = 0$ and Y^*_2 is arbitrary.

Since $[1 - P_2\{(1-\phi)R_1X^*_{1\theta_1} + \phi R_1X^*_{1\theta_2} + R_2X^*_2\}] = 0$ and $X^*_{1\theta_1} = 0$ and $X^*_2 = 1, X^*_{1\theta_1} = 2/(\phi R_1P_2) - R_2/(\phi R_1)$.

Since $\hat{Y}_2 = (1-\phi)Y^*_1 + \phi Y^*_2 = S_1/(R_1M)$ and $Y^*_1 = 0, Y^*_2 = S_1/\phi R_1 M$

Since $0 < S_i/(R_iM) \leq \phi$ for $i = 1, 2$, and $0.5R_2 \leq 1/P_2 < 0.5(\phi R_1 + R_2), 0 \leq X^*_{1\theta_2} \leq 1$ and $0 \leq Y^*_2 \leq 1$.

The equilibrium strategies for the other cases can be derived similarly.

Theorem 4. *Prior information about the auditee type is useful in the first four cases of Theorem 3. In the last two cases of Theorem 3, prior information about the auditee type plays no roles in the strategy of a type 1 auditor.*

Proof: The equilibrium strategies of a type 1 auditor in each case are as follows.

case	$X^*_{1\theta_1}$	$X^*_{1\theta_2}$
i	$2/(\phi R_1P_1) - (1-\phi)/\phi - R_2/(\phi R_1)$	1
ii	$2/\{(1-\phi)R_1P_1\} - \phi/(1-\phi) - R_2/\{(1-\phi)R_1\}$	1
iii	0	$2/\{(1-\phi)R_1P_1\} - R_2/\{(1-\phi)R_1\}$
iv	0	$2/(\phi R_1P_2) - R_2/(\phi R_1)$
v	0	0
vi	0	0

As it can be seen from the above table, for the first four cases, $X^*_{1\theta_1} \neq X^*_{1\theta_2}$, and $X^*_{1\theta_1} < X^*_{1\theta_2}$. For the last two cases, $X^*_{1\theta_1} = X^*_{1\theta_2}$.

Theorem 5. *Prior information of different accuracy about the auditee type is not a substitute for actual testing in the first three cases of Theorem 3. Prior information of different accuracy about the auditee type is a substitute for actual testing in the fourth case of Theorem 3. In other words, in the fourth case of Theorem 3, $\partial Y^*_j/\partial\phi \leq 0$ and $\partial X^*_{1\theta_i}/\partial\phi \leq 0$.*

Proof: The equilibrium strategies of a type 1 auditor in each case are given in the proof for Theorem 4, and the equilibrium strategies of the auditees in each case are as follows.

case	Y_1^*	Y_2^*
i	$S_1/(\phi R_1 M) - (1-\phi)/\phi$	1
ii	0	$S_1/\{(1-\phi)R_1 M\}$
iii	$S_1/\{(1-\phi)R_1 M\} - \phi(1-\phi)$	1
iv	0	$S_1/(\phi R_1 M)$

The signs of the available derivatives of $X_{1\theta_1}^*$ or $X_{1\theta_2}^*$ and Y_1^* or Y_2^* with respect to ϕ are as follows.

case	signs of derivatives	
i	$\partial X_{1\theta_1}^*/\partial\phi \geq 0$	$\partial Y_1^*/\partial\phi > 0$
ii	$\partial X_{1\theta_1}^*/\partial\phi \leq 0$	$\partial Y_2^*/\partial\phi > 0$
iii	$\partial X_{1\theta_2}^*/\partial\phi \geq 0$	$\partial Y_1^*/\partial\phi < 0$
iv	$\partial X_{1\theta_2}^*/\partial\phi \leq 0$	$\partial Y_2^*/\partial\phi < 0$

In Case (iv),

$$\begin{aligned}
Y_2^* &= S_1/(\phi R_1 M) \\
\partial Y_2^*/\partial\phi &= \{S_1/(R_1 M)\}\{-1/\phi^2\} < 0 \\
\partial X_{1\theta_2}^*/\partial\phi &= \{2/(R_1 P_2)\}\{-1/\phi^2\} - \{R_2/R_1\}\{-1/\phi^2\} \\
&= 1/\phi^2\{R_2/R_1 - 2/(R_1 P_2)\} \\
&= 1/\phi^2(2/R_1)\{R_2/2 - 1/P_2\} \leq 0 \text{ because } 0.5R_2 \leq 1/P_2 < 0.5(\phi R_1 + R_2).
\end{aligned}$$

The signs of the derivatives in the other cases can be derived similarly.

Appendix B

For the four cases of Theorem 2, the equilibrium strategies of the players are:

(i) $X_1^* = 0, X_2^* = 2/(R_2 P_2)$ $Y_1^* = 0, Y_2^* = 2S_2/(R_2 M)$

(ii) $X_1^* = 0, X_2^* = 2/R_2 P_1$ $Y_1^* = 2S_2/(R_2 M) - 1, Y_2^* = 1$

(iii) $X_1^* = 1/R_1[1/P_2 - R_2], X_2^* = 1$ $Y_1^* = 0, Y_2^* = 2S_1/(R_1 M)$

(iv) $X_1^* = 1/R_1[2/P_1 - R_2], X_2^* = 1$ $Y_1^* = 2S_1/(R_1 M) - 1, Y_2^* = 1$

For the cases of Theorem 3, the equilibrium strategies of the players are:

(i) $X^*_{1\theta_1} = 2/(\phi R_1 P_1) - (1-\phi)/\phi - R_2/(\phi R_1), X^*_{1\theta_2} = 1, X^*_2 = 1$

$Y^*_1 = S_1/(\phi R_1 M) - (1-\phi)/\phi, Y^*_2 = 1$

(ii) $X^*_{1\theta_1} = 2/\{(1-\phi)R_1 P_2\} - \phi/(1-\phi) - R_2/\{(1-\phi)R_1\}, X^*_{1\theta_2} = 1, X^*_2 = 1$

$Y^*_1 = 0, Y^*_2 = S_1/\{(1-\phi)R_1 M\}$

(iii) $X^*_{1\theta_1} = 0, X^*_{1\theta_2} = 2/\{(1-\phi)P_1 R_1\} - R_2/\{R_1(1-\phi)\}, X^*_2 = 1$

$Y^*_1 = S_2/\{R_1 M(1-\phi)\} - \phi/(1-\phi), Y^*_2 = 1$

(iv) $X^*_{1\theta_1} = 0, X^*_{1\theta_2} = 2/(\phi P_2 R_1) - R_2/(\phi R_1), X^*_2 = 1$

$Y^*_1 = 0, Y^*_2 = S_1/(\phi R_1 M)$

(v) $X^*_{1\theta_1} = 0, X^*_{1\theta_2} = 0, X^*_2 = 2/(P_1 R_2)$

$Y^*_1 = 2S_2/(R_2 M) - 1, Y^*_2 = 1$

(vi) $X^*_{1\theta_1} = 0, X^*_{1\theta_2} = 0, X^*_2 = 2/(P_2 R_2)$

$Y^*_1 = 0, Y^*_2 = 2S_2/(R_2 M)$