

*Note***MHD flow over a moving flat plate
with a step change in the magnetic field**

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Summary. The boundary layer flow over a moving continuous flat plate in an electrically conducting ambient fluid with a step change in the applied magnetic field is considered. The governing equations are solved numerically using the Keller box method. It is shown that the skin friction decreases as the magnetic parameter increases.

1 Introduction

Boundary layer behavior over a moving continuous surface is an important type of flow occurring in several engineering processes. For example, many metallurgical processes involve the cooling of continuous strips or filaments by drawing them through a quiescent fluid. However, by drawing such strips in an electrically conducting fluid subject to a magnetic field, the rate of cooling can be controlled and a final product of desired characteristics can be achieved. Another interesting application of hydromagnetics to metallurgy lies in the purification of molten metals from non-metallic inclusions by the application of a magnetic field.

Kumari et al. [1], Andersson [2], Vajravelu [3], and Watanabe and Pop [4] have recently studied the laminar boundary layer flow over a flat plate which issues from a slot at $x = 0$ and moves with a constant velocity in an electrically conducting fluid in the presence of a transverse magnetic field which acts over the whole flow region $x \geq 0$. However, an important practical situation is that of an external magnetic field of the form (see Chiam [5])

$$B = \begin{cases} 0, & x \leq x_0 \\ B_0, & x > x_0 \end{cases} \quad (1)$$

where B_0 is a constant. The physical problem is that of the entrance of a plane boundary layer, which has already been formed over a moving flat plate, into a magnetic field, as shown in Fig. 1.

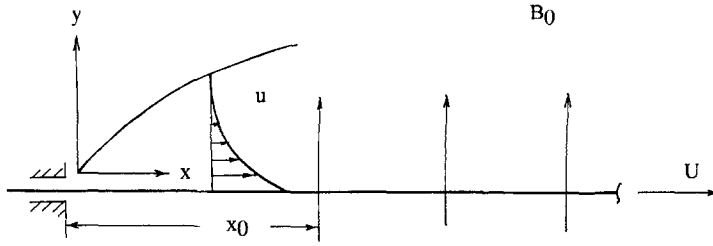


Fig. 1. Physical model and coordinate system

2 Basic equations

Consider the boundary layer over a flat plate which moves with a constant velocity U in an electrically conducting fluid (with electric conductivity σ) in the presence of a transverse magnetic field of the form given by (1). The boundary layer equations, in the usual notation, are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u \tag{3}$$

subject to the boundary conditions

$$\begin{aligned} v = 0, \quad u = 0 & \quad \text{at } y = 0 \\ u = 0 & \quad \text{as } y \rightarrow \infty \\ u = u_0(y) & \quad \text{at } x = x_0 \end{aligned} \tag{4}$$

where $u_0(y)$ is the Sakiadis [6] velocity profile at $x = x_0$. Here u and v are the velocity components along x and y axes, respectively, ν is the kinematic viscosity, and ρ is the density of the fluid.

If we introduce the variables

$$\psi = (\nu U x)^{1/2} f(x, \eta), \quad \eta = \left(\frac{U}{\nu x}\right)^{1/2} \tag{5}$$

where ψ is the stream function, Eq. (3) becomes

$$f''' + \frac{1}{2} f f'' - m x f' = x \left(f' \frac{\partial f'}{\partial x} - f'' \frac{\partial f}{\partial x} \right) \tag{6}$$

with $m = \sigma B_0^2 / (\rho U)$ being the magnetic parameter. The boundary conditions of Eq. (6) are

$$f(x, 0) = 0, \quad f'(x, 0) = 1, \quad f'(x, \infty) = 0. \tag{7}$$

It is worth mentioning that for $m x \leq m x_0$, Eq. (6) reduces to that of Sakiadis [6], namely,

$$f''' + \frac{1}{2} f f'' = 0 \tag{8}$$

subject to the boundary conditions

$$f(0) = 0, \quad f'(0) = 1, \quad f'(\infty) = 0. \tag{9}$$

The coefficient of the skin friction is defined by

$$C_{fx} = \frac{\tau_w}{\frac{1}{2} \rho U_0^2} = \frac{2}{\text{Re}_x^{1/2}} f''(x, 0) \tag{10}$$

where τ_w is given by

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}. \tag{11}$$

From (10), we have

$$\frac{C_{fx} \text{Re}_x^{1/2}}{2} = f''(x, 0). \tag{12}$$

The quantity $-C_{fx} \text{Re}_x^{1/2}/2$, i.e., $[-f''(x, 0)]$, is calculated for $mx > mx_0$. As in Chiam [5], we have taken $x_0 = 1$ and $m = 0.05, 0.15, 0.5$ and 1.0 , respectively.

3 Results and discussion

The partial differential equation (6) subject to the boundary conditions (7) has been solved numerically using the Keller box method [7]. The details are essentially the same as those described in Na [8], and Na and Pop [9].

Figure 2 represents the variation of the skin friction coefficient given by (12) with the streamwise distance x and the magnetic parameter m . From this figure it can be seen that the effect of the magnetic field is to reduce the skin friction coefficient. Thus, the higher magnetic parameter corresponds to the skin friction at a closer distance to $x_0 (= 1.0)$. This appears reasonable in view of the thinner boundary layer in the higher-magnetic fluid.

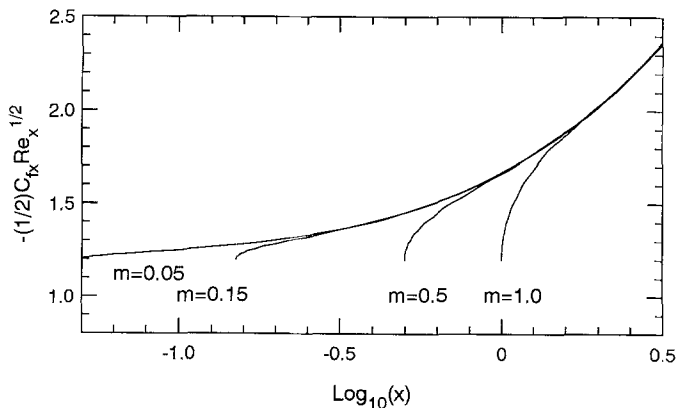


Fig. 2. Variation of local skin friction coefficient

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