THE UNIVERSITY OF MICHIGAN

INDUSTRY PROGRAM OF THE COLLEGE OF ENGINEERING

PRELIMINARY STUDIES OF A HIGH TEMPERATURE,
GAS-COOLED NUCLEAR REACTOR

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SUMMARY

Parametric studies on a helium-cooled, hetero-homogeneous composite-type power reactor employing highly elevated temperature and gas pressure were carried out.

The fuel element is uranium carbide solid solution in carbon where an enrichment of 20% and a fuel-to-moderator volume ratio of $5 \times 10^{-3}$ and up were used for the preliminary design.

Maximum fuel element temperature of 2700°F and core pressure of 1000 psi were adopted, allowing improved characteristics of the reactor, although a series of unique approaches for both the nuclear and thermal designs were inevitably needed.

High power density and thermal efficiency can be incorporated with a fair degree of inherent safety of operation in this reactor, although some ambiguities still remain due to the lack of nuclear data at highly elevated temperatures.

This system, coupled with a closed-cycle gas turbine plant may be of use for application wherein extreme compactness and lightweight are necessary. Also such a system may be of interest for a rocket heat source, although in this case even higher temperatures may be required.
INTRODUCTION

Although the limitations imposed on the materials used in nuclear reactors restrict the maximum attainable temperature in the core and thus the thermodynamic efficiency in the associated power loop, attempts have been made to overcome the difficult task of obtaining new reactor core configurations as well as materials which can withstand highly elevated temperature and pressure. The formidable problem of cutting the nuclear power cost to the commercially competitive level may be solved partially by the enhanced thermal efficiency thus attained.

As found in other high temperature applications, metals in carbide form are among the possible materials on which attempts along this line can be based and thus uranium carbide or any fissionable or fertile material in the form of carbide compounds seems a very natural choice. Considering the fact that some of these fissionable materials, especially uranium, have phase-transients at medium-high temperature regions, any genuinely heterogeneous set-up of a fuel element should be avoided since it will most likely cause severe structural problems. One configuration, in which uranium carbides are dispersed homogeneously in graphite, can be considered suitable as a possible core material.

Apart from these material considerations, nuclear and heat transfer problems encountered under the specified conditions are
unique. All nuclear data obtained at room temperature or comparatively low temperature regions should now be modified for the new high temperature and certainly this can not be achieved without using several rather crucial assumptions. The temperature to be used in this study is far higher than those which have been employed by many investigators, as it will appear in the basic design parameters below.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average reactor pressure</td>
<td>1000 psi</td>
</tr>
<tr>
<td>Maximum fuel element temp.</td>
<td>2700°F</td>
</tr>
<tr>
<td>Fuel element</td>
<td>UC solid solution in carbon</td>
</tr>
<tr>
<td>Fuel element cladding</td>
<td>ZrC (0.3 mills thick)</td>
</tr>
<tr>
<td>Moderator</td>
<td>High density three-dimensional graphite</td>
</tr>
<tr>
<td>Reflector</td>
<td>Same as moderator</td>
</tr>
<tr>
<td>Thermal shield</td>
<td>5&quot; of iron outside of reflector</td>
</tr>
</tbody>
</table>

Once the materials for use are specified, the nuclear and heat transfer calculations can be carried out as usual. One may not have any particular difficulties in following the conventional approaches in heat transfer studies except that the figures used are very high. On the other hand, some bold assumptions must be made in nuclear calculations, thus any unnecessarily elaborate multigroup theory calculations will not be justified in the preliminary design studies. Rather, the use of a slightly modified two-group theory is thus recommended in the first effort to tackle these enhanced conditions, considering the subsequent ease and satisfactory accuracy in the calculation.
MATERIALS FOR HIGH TEMPERATURE REACTOR

General

In general, the materials to be used for nuclear reactors should fulfill certain physical and chemical requirements. In high temperature nuclear reactors, these requirements will be much more severe, since the enhanced conditions will not be sustained safely for any long periods of operation without extremely superior mechanical and chemical stabilities. The nuclear requirements, however, will not be very different from those for any conventional medium-temperature reactor.

At highly elevated temperatures, good thermal stabilities and chemical inertness are especially required. The answer of present-day technology to this problem may be the metal carbides, due to the fact that many carbides fulfill the above mentioned properties and, at the same time, carbon is a widely used moderator and reflector for thermal and intermediate type nuclear reactors. Possible carbides for each reactor component are summarized below.

Fuel

Thorium Carbides
ThC
ThC₂

Uranium Carbides
UC
U₂C₃
UC₃
Cladding Material

Titanium Carbide
TiC

Zirconium Carbide
ZrC

Uranium Carbides

As observed on the constitutional diagram of the Uranium-Carbon system in Figure 1, the composition for solid solution of uranium carbon ranges from 50 - 66.7 carbon atom percent or 4.8 - 9.2 weight percent. One can easily notice that the melting point of the solid solution is very high (2400°C in the vicinity of) throughout the range and the system can stand the proposed reactor core temperature. Any allotropic transformations such as those that appear in pure uranium at 650 - 770°C do not show up for the present composition and consequently the system has thermal stability. The homogeneous dispersion of the powdered uranium carbide into the reactor grade graphite may be easily incorporated during the formation process of the graphite blocks. Considering the fact that the volumetric ratio of uranium carbide to total graphite in the core should be very small, the overall physical properties of the core material will be those of pure graphite and thus the composition seems perfectly ideal for medium or low enrichment fuel systems.
Figure 1. The Constitutional Diagram of the System Uranium-Carbon.

Zirconium Carbide

For elevated temperature application, titanium carbide is outstanding for its thermal shock resistance and high tensile strength, but the absorption cross section for thermal neutrons is rather high. Zirconium and its alloys have somewhat better high-temperature strength but certainly their upper limit should be under 1000°F at the most.

Zirconium carbide, on the other hand, has a very high, melting point with a composition of carbon of around 50. In its optimum composition, temperatures of 1800 - 1900°C can be reached in solid phase. In case this material is to be used as a cladding material, however, the process of forming zirconium carbide at the surface of the fuel element will cause some technical problems. Although when graphite is used as the fuel-bearing core block material, the formation process will not be a very difficult one. Considering the fact that several different types of metal carbides have been successfully formed on the surface of graphite, it can be expected that similar experimental trials have been done for zirconium carbide by some investigator, though no further detailed descriptions are attempted here.

In a high-temperature reactor, the diffusion of fission products through the cladding material will be much more accelerated and the requirement must be more stringent. In these design studies, the thickness of zirconium carbide was tentatively set as 0.3 mills, which was believed to give enough thickness to withhold the fission products in the fuel element.
Figure 2. Tentative Constitutional Diagram of the System Zirconium-Carbon.

Figure 3. Geometric Buckling as a Function of Fuel-to-Moderator Volume Ratio with Enrichment as Parameter.

(1 group)
(1 region)
PRELIMINARY CONSIDERATIONS

To obtain a quick insight into the overall optimum conditions, bare one region and one group calculations were first carried out for various fuel-to-moderator ratios and enrichments, while the temperature was taken as 270^oK. Thus the inverse geometric buckling versus fuel-to-moderator volume ratio $V_F/V_M$ were plotted, the enrichment being one of the parameters. The result, in spite of itscrudeness caused by the over-simplified assumptions, gives some means of a quick check. The product of resonance escape probability and fast fission factor $p \cdot \epsilon$ was assumed to be unity which was later verified to be a fairly good assumption in the more elaborate one-group-two-region and modified two-group-two-region calculations.

As seen in Figure 3, as soon as the enrichment of 0.1 (10%) is attained, further increase in the enrichment does not affect the buckling considerably. On the other hand, the maximum $p$ value for the system is around 0.8 and the decrease in $p$ value will increase the fast fission factor $\epsilon$, thus changes the reactor into a faster one. The difficulty already encountered by the elevated temperature implies that the extremely fast reactor employing an elevated temperature, such as 2700^oF is not desirable in the design work as yet when the ambiguities still dominate several critical nuclear problems. Thus the enrichment of 20% and $V_F/V_M$ ratio of $5 \times 10^{-3}$ are found suitable for the core considered here. The trend observed for the cold
reactor should still be valid for a high-temperature reactor core in a primary evaluation, since the relative buckling will not change too much with changes in temperature. This can be clearly seen in Figure 4, which is the plot of the result of two-group calculations for the operating conditions at 2700°F where the core height and reflector thickness are held fixed as indicated therein.

Assumptions Made for Nuclear Calculations

A highly elevated temperature employed in the reactor necessitates a somewhat unique approach in the reactor calculations. Some difficulties encountered here are the uncertainty of cross sections and ambiguity of resonance region characteristics including Doppler broadening effect at elevated temperatures. The whole flux distribution will shift and little information is available on this critical problem. Neither core nor reflector expansion can be neglected. Thus the difficult task of choosing group constants becomes more complex, the situation still being such that the calculations based on the cold reactor can not be logically used.

Main assumptions as well as approximations which will be made are as follows:

1. Absorption cross sections follow $1/T$ laws. The product of non-$1/T$ factors and most probable-to-average neutron velocity correction factors for U-235 and U-238 were selected through the extrapolation of the data for considerably lower temperatures.
Cylindrical core, UC
Reflector graphite
Temperature 2700°F max.
Void 30%
H = 128 cm
Tg = 23 cm
Te = 84 cm
pe = ±1

Figure 4. Core Radius as a Function of Enrichment
2. The neutron cross section does not shift considerably at the epithermal energy region, thus the cross section data can be taken as they appear in BNL-325 for the region.

3. Thermal expansion of fuel and moderator is independent of change of neutron average energy due to the high temperature. The product of these two factors contributes to the correction of reactor material constants.

4. The effect of Doppler broadening in the resonance region is slight for the present system.

5. Voids for the cooling channel through core and reflector are homogenized in the nuclear and thermal calculation.
SLIGHTLY MODIFIED TWO-GROUP THEORY

The determination of minimum critical mass is the basic part of reactor analysis. Before going into the calculation, however, it would be beneficial to know the present reactor system. This is important in deciding the possible approaches. The fast fission factor was first examined from this viewpoint and from this, the approach was determined.

Fast Fission Factor

First, composite multiplication factor $K$ was defined as

$$K = K_T + K_F$$

where $K_T = K$-thermal

$K_F = K$-fast

Accordingly fast fission factor $\epsilon$ is

$$\epsilon = \frac{K}{K_T} = \frac{K_T + K_F}{K_T}$$

On the other hand, $K_F$ and $K_T$ are calculated as follows:

$$K_F = \nu \int_{E_T}^{\infty} f'dE' \frac{\Sigma_{f}}{J'}$$

$$+ \int_{E=E_T}^{\infty} \int_{E'=E}^{\infty} f'dE' \frac{\Sigma_{f}}{J'} P(E', E) \frac{dE}{\xi E} \frac{\Sigma_{f}}{J} \nu$$

$$K_T = \int_{E'=E}^{\infty} f'dE' \frac{\Sigma_{T}}{J'} P(E', E_T) \frac{\Sigma_{TT}}{\sum a_T + D_T B^2_T \sum a_T} \cdot \frac{\Sigma_{TT}}{\sum a_T} \cdot \nu$$
where \( E' \) = Initial neutron energy

\( E_T \) = Thermal threshold energy

\( f \) = Fast fission spectrum

\( \Sigma_f \) = Macroscopic fission cross section

\( \Sigma_s \) = Macroscopic scattering cross section

\( \Sigma_a \) = Macroscopic absorption cross section

\( J \) = Macroscopic total cross section

\( P_f \) = Probability of neutron slowing down from energy \( E' \) to \( E_T \).

\( \xi \) = Average logarithmic energy change per collision

\( v \) = Average number of fast neutrons generated per fission

\[ P_f (E',E) = e^{-\int_{E}^{E'} \left( \frac{\Sigma_a'' + \frac{2}{\xi} \Sigma_t'' + D \xi}{\Sigma_f'' + D \xi} \right) \frac{dE''}{E''}} \]

and

\[ f = 0.484 \ e^{-\frac{E}{h}} \sin h \sqrt{2E} \]

where \( E \) is in eV.

The first term of \( K_F \), the virgin fast fission, was found to be rather small, i.e., \( 1.67 \times 10^{-3} \) for the system, but the secondary fission part of \( K_F \) is appreciably larger although the double manipulation of graphical integration gives some difficulty. The value is computed to be \( 5.22 \times 10^{-1} \) and this, together with \( K_T = 1.58 \), gives the composite multiplication factor of \( K = 2.10 \), thus

\[ \epsilon = \frac{2.10}{1.58} = 1.33 \]
The reactor is slightly fast. The logical theory to be used in the critical mass calculation would be multigroup theory where several epithermal energy regions are specifically handled. However, the fact that there is little information on the neutron flux shift in epithermal regions due to the high temperature, makes the use of any multigroup handling somewhat awkward. Considering the saving of time and effort achieved by simple two-group calculation, a slightly modified two-group theory would be the proper compromise.

**Modified Two-Group Theory**

A slight modification can be made by using the fast fission factor with conventional neutron balance equation for a thermal reactor. The new equation will be

**Core:**

\[ D_1 \nabla^2 \phi_1 - \frac{\phi_1 D_1}{\tau} + \phi_2 \Sigma_2 f \eta e = 0 \]

\[ D_2 \nabla^2 \phi_2 - \phi_2 \Sigma_2 + \frac{\phi_1 D_1}{\tau} = 0 \]

**Reflector:**

\[ D_{1r} \nabla^2 \phi_{1r} - \frac{\phi_{1r} D_{1r}}{\tau_r} = 0 \]

\[ D_{2r} \nabla^2 \phi_{2r} - \phi_{2r} \Sigma_{2r} + \frac{\phi_{1r} D_{1r}}{\tau_r} = 0 \]

where

- \( D_1 \) = Fast diffusion coefficient of the core
- \( D_2 \) = Thermal diffusion coefficient of the core
- \( \phi_1 \) = Fast flux in the core
- \( \phi_2 \) = Thermal flux in the core
\[ \tau = \text{Age of the core} \]

\[ \Sigma_2 = \text{Macroscopic thermal cross section} \]

\[ f = \text{Thermal utilization} \]

\[ p = \text{Resonance escape probability} \]

\[ D_{1r} = \text{Fast diffusion coefficient of the reflector} \]

\[ D_{2r} = \text{Thermal diffusion coefficient of the reflector} \]

\[ \varphi_{1r} = \text{Fast flux in the reflector} \]

\[ \varphi_{2r} = \text{Thermal flux in the reflector} \]

\[ \tau_r = \text{Age of the reflector} \]

\[ \Sigma_{2r} = \text{Macroscopic thermal cross section of the reflector} \]
SEVERAL APPROACHES ADOPTED IN THE TREATMENT OF
A HIGH TEMPERATURE CONDITION

As described in the introduction, all reactor constants are
to be corrected for thermal effects.

Absorption Cross Sections

Temperature dependent absorption cross sections are, in
a simplified form,

\[ \frac{\bar{\sigma}_a(2)}{\bar{\sigma}_a(1)} = \sqrt{\frac{T_1}{T_2}} \]

where \( \bar{\sigma}_a \) = Average neutron absorption cross section
\( T \) = Temperature °K
\( 1 \) = Reference to 293°K
\( 2 \) = Reference to 1753°K

\( \bar{\sigma}_a(2) \), thus obtained, were further corrected for core void,
i.e., homogeneously diluted core composition to obtain the macroscopic
cross section. Thus

\[ \bar{\Sigma}_a(2) = \bar{\sigma}_a(2) \cdot [N \times \text{dilution factor}] \]

where \( N \) is Avogadro's Number.

Scattering Cross Sections

The scattering cross section for graphite is almost independ-
et of temperature.
C = Graphite

UC = Uranium carbide

Age

Temperature corrections for age were carried out using the relation

$$\tau = \int_{E_{th}}^{E_0} \frac{D}{\Sigma_s} \frac{dE}{E} = \int_{E_{th}}^{E_0} \frac{1}{3\Sigma_s\Sigma_t} \frac{dE}{E}$$

thus

$$\tau \approx \tau_0 - \int_{E_{th}}^{E_0} \frac{1}{3\Sigma_s\Sigma_t} \frac{dE}{E}$$

Specifically for the present system, this can be further approximated as

$$\tau \approx \tau_0 - \left( \frac{1}{3\Sigma_s\Sigma_t} \right) \ln \frac{T}{T_0}$$

since scattering cross sections are regarded temperature-independent, thus energy independent.

Diffusion Length

Diffusion length is temperature sensitive and a decisive factor which affects the criticality. From the original definition, it follows that

$$L^2 = \frac{1}{3\Sigma_s\Sigma_t} = \frac{1}{3\Sigma_{ao}t_0} \frac{1}{\omega - (n+1/2)}$$

$$= L_o^2 \omega^{n+1/2}$$
Thermal Expansion Contribution

For most group constants, the corrections which account for the thermal expansion of core and reflector should not be neglected.

Fast Diffusion Coefficient

As usual, flux averaged value of fast fission coefficients were used. Thus

\[
D_1 = \frac{\int_0^{u_t} D(u) \varphi(u,r) \, du}{\int_0^{u_t} \varphi(u,r) \, du}
\]

\[
= \frac{\int_{E_T}^{\infty} D(E) \varphi(E) \, dE}{\int_{E_T}^{\infty} \varphi(E) \, dE}
\]

\(E_T\) of 0.166 ev, which is the corrected value of thermal threshold energy 0.025 for room temperature, at 2700°F, was used.

Thermal Diffusion Coefficient

The relations

\[
D_{2c} = \frac{1}{3} \left[ \Sigma_{tr}^{'}(c) + \Sigma_{tr}^{'}(UC) \right]
\]

\[
D_{2r} = \frac{1}{3} \left[ \Sigma_{tr}^{''}(c) \right]
\]

were used to obtain the thermal diffusion coefficients,

where \(\Sigma_{tr}^{'}\) = Temperature-, void- and thermal expansion-corrected macroscopic transport cross section of core

\(\Sigma_{tr}^{''}\) = Temperature-, and thermal expansion corrected macroscopic transport cross section of reflector
where \( \Theta = \frac{T}{T_0} \)

\( L_0 = \) Diffusion length at 293°K

\( n \approx 0 \) for graphite.

**Resonance Escape Probability**

Lawrence-Dresner's data on the effective resonance integral gives the value of \( p = 0.820 \). On the other hand, Murray's formula gives approximately 0.781 which was considered safe from the viewpoint of criticality achievement and therefore adopted. Very little reliable data are available on the type of hetero-homogeneous core configuration similar to the one described later in the heat transfer studies.

**Doppler Effect**

Estimation of Doppler effect at an elevated temperature such as 2700°F for any intermediate type reactor is almost impossible at the present time. In this design calculation, the effect was simply neglected. The essential homogeneity of the present core may possibly allow this approximation. Nevertheless, a clearer understanding of this effect would certainly be helpful in the design of high-temperature reactors in the future.
GROUP CONSTANTS

General Approach

Group constants are computed according to the previously described approaches. The process is as follows: First a modified one-group calculation based on age theory is used to obtain the approximate critical mass of cold reactor, upon which the thermal distribution for core as well as reflector is determined. The maximum core temperature is set to be 2700°F, from which the side and end reflector temperatures are to be determined. The group constants are recalculated again at this point for the corresponding temperature in both core and reflector region. Ideally, an infinite number of repetitions of this process would give the best result but one or two trials are considered to be sufficient for practical purposes.

Calculation

Since the evaluation of group constants and general reactor constants are main problems in any high temperature reactor analysis, somewhat detailed computations are presented here for further reference.

Microscopic Cross Sections

Choosing 2700°F as the reference temperature of the core,

\[ \Theta = \frac{T}{T_0} = 5.99 \]
\[ \Theta^{1/2} = 2.44, \ \Theta^{-1/2} = 0.409 \]
For U-235:

\[
\text{non} \frac{1}{\nu} \text{ factor, } f_1(2700^\circ F) \approx 1.10
\]

most probable-to-average, \( f_2 = \sqrt{\pi}/2 \)

using these, one obtains

\[
\sigma_a = 274 \text{ b}, \quad \sigma_f = 231 \text{ b}
\]

\[
\sigma_c = \sigma_a - \sigma_f = 43 \text{ b}.
\]

For U-238:

\[
\sigma_a = 1.00 \text{ b}
\]

For Graphite:

\[
\sigma_a = 1.16 \times 10^{-3} \text{ b}.
\]

As can be seen, the figures are greatly reduced at the elevated tempera-
ture.

**Thermal Volume Expansion**

Using volume expansion coefficient \( \alpha = 75 \times 10^{-6} \) for U-235

and U-238 and \( \alpha = 13 \times 10^{-6} \) for graphite, the number of nuclei per unit

volume is calculated. The results are shown in Table I.

**TABLE I**

<table>
<thead>
<tr>
<th>Pure Substances at 2700°F</th>
<th>N</th>
<th>( \sigma_f )</th>
<th>( \sigma_a )</th>
<th>( \sigma_s )</th>
<th>( \Sigma_r )</th>
<th>( \Sigma_a )</th>
<th>( \Sigma_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>U-235</td>
<td>4.76 x 10^{22}</td>
<td>231</td>
<td>274</td>
<td>-</td>
<td>11.0</td>
<td>13.03</td>
<td>-</td>
</tr>
<tr>
<td>U-238</td>
<td>4.69 x 10^{22}</td>
<td>-</td>
<td>1.00</td>
<td>10.0</td>
<td>-</td>
<td>0.0469</td>
<td>0.469</td>
</tr>
<tr>
<td>C</td>
<td>1.01 x 10^{23}</td>
<td>-</td>
<td>1.16 x 10^{-3}</td>
<td>4.8</td>
<td>-</td>
<td>1.17 x 10^{-4}</td>
<td>0.485</td>
</tr>
</tbody>
</table>
Void Correction

As indicated previously, cooling channel voids were homogenized in the calculation. The ratio of moderator volume ratio of $5 \times 10^{-3}$ is used to obtain the final cross sections corrected for temperature and void as shown in Table II.

<table>
<thead>
<tr>
<th>Material</th>
<th>$\Sigma_t$</th>
<th>$\Sigma_b$</th>
<th>$\Sigma_s$</th>
<th>$\Sigma_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>U-235</td>
<td>$7.63 \times 10^{-3}$</td>
<td>$9.03 \times 10^{-3}$</td>
<td>-</td>
<td>$9.03 \times 10^{-3}$</td>
</tr>
<tr>
<td>U-238</td>
<td>-</td>
<td>$1.32 \times 10^{-4}$</td>
<td>$1.32 \times 10^{-3}$</td>
<td>$1.45 \times 10^{-3}$</td>
</tr>
<tr>
<td>C</td>
<td>-</td>
<td>$8.19 \times 10^{-5}$</td>
<td>$0.339$</td>
<td>$0.339$</td>
</tr>
<tr>
<td>Total</td>
<td>$7.63 \times 10^{-3}$</td>
<td>$9.24 \times 10^{-3}$</td>
<td>$0.340$</td>
<td>$0.350$</td>
</tr>
</tbody>
</table>

Thermal Diffusion Coefficient for Core, $D_{2c}$

$$D_{2c} = \frac{1}{3(\Sigma_t - \mu_0 \Sigma_s)}$$

but $\mu_0 = 0.0553$

Thus

$$D_{2c} = 1.01 \text{ cm}$$

Since $\Sigma_s$ or $\Sigma_t$ does not vary too much with temperature change, $D_{2c}$ is not very sensitive to temperature.

Thermal Diffusion Length for Core $L_{2c}$

$$L_{2c}^2 = \frac{D}{\Sigma_b}$$

Thus $L_{2c} = 10.45 \text{ cm}$

$L_{2c}$ is related to $\Sigma_b$ and thus very sensitively temperature dependent.

Thermal Utilization $\eta$ and $\eta$

Thermal utilization is temperature independent. Thus

$\eta = 0.977$ for this system at all temperatures. $\eta$ varies slightly
with internuclear relative velocities but the change is regarded negligible at the temperature range considered here. Calculation gives \( \eta = 2.08 \).

**Fast Diffusion Coefficient for Core and Reflectors, \( D_{1c} \) and \( D_{1r} \).**

These are almost temperature independent, but the void and thermal expansion corrections should be made.

**Age \( \tau \)**

Age \( \tau \) is slightly temperature dependent, and \( \tau \) decreases somewhat at elevated temperatures, but void correction gives increased \( \tau \) value.

**Cross Sections for Reflectors**

A separate heat transfer calculation which will be discussed in later chapters gives rough estimations of reflector temperatures.

<table>
<thead>
<tr>
<th>Portion</th>
<th>Average Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top reflector</td>
<td>1650°F</td>
</tr>
<tr>
<td>Bottom reflector</td>
<td>1050°F</td>
</tr>
<tr>
<td>Side reflector</td>
<td>1300°F</td>
</tr>
</tbody>
</table>

For convenience of calculations, it was assumed that the temperatures of all three reflectors are equal and 1300°F. Temperature and void corrections give the values shown in Table III.

**TABLE III**

**THERMAL CROSS SECTIONS OF DIFFERENT PORTIONS OF REACTOR**

<table>
<thead>
<tr>
<th></th>
<th>( \Sigma_5 )</th>
<th>( \Sigma_R )</th>
<th>( \Sigma_{tot} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>End Reflector</td>
<td>0.344</td>
<td>1.12 \times 10^{-4}</td>
<td>0.344</td>
</tr>
<tr>
<td>(( \alpha = 0.2965 ))</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Side Reflector</td>
<td>0.460</td>
<td>1.49 \times 10^{-4}</td>
<td>0.460</td>
</tr>
<tr>
<td>(( \alpha = 0.06 ))</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

where \( \alpha \) is void fraction.
Group Constants for Reflectors

Using the above calculated figures, $D_1$, $D_2$, $\tau$ and $L$ of side and end reflectors are obtained.

**Results**

Results of calculations are shown in Table IV.

**TABLE IV**

**SUMMARY OF GROUP CONSTANTS**

<table>
<thead>
<tr>
<th>Constants</th>
<th>Core $2700^\circ F \alpha = 0.2965$</th>
<th>End Reflector $1300^\circ F \alpha = 0.2965$</th>
<th>Side Reflector $1300^\circ F \alpha = 0.06$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau$</td>
<td>706</td>
<td>726</td>
<td>407</td>
</tr>
<tr>
<td>$D$</td>
<td>1.62</td>
<td>1.60</td>
<td>1.19</td>
</tr>
<tr>
<td></td>
<td>1.01</td>
<td>1.03</td>
<td>0.767</td>
</tr>
<tr>
<td>$L$</td>
<td>10.45</td>
<td>95.9</td>
<td>71.7</td>
</tr>
<tr>
<td>$\Sigma_a$</td>
<td>$9.24 \times 10^{-3}$</td>
<td>$1.12 \times 10^{-4}$</td>
<td>$1.49 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\Sigma_s$</td>
<td>0.340</td>
<td>0.344</td>
<td>0.460</td>
</tr>
<tr>
<td>$\sigma_f$</td>
<td>231 b</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\eta$</td>
<td>2.08</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r$</td>
<td>0.977</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td>0.781</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c$</td>
<td>1.33</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K_\infty$</td>
<td>2.10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
TREATMENT OF COMPLETELY REFLECTED CYLINDER CORE

At the present time, a completely reflected cylinder can not be analytically handled. Therefore as an alternative approach, the core was first considered to be a slab of thickness \( H' \), where \( H' \) includes the extrapolation distance with the top and bottom considered to be reflected.

![Diagram of a cylinder core and reflector](image)

Figure 5. Conversion of Completely Reflected Cylinder Core into Bare Slab System.

This is an approximation but it roughly provides the height of an equivalent bare reactor that can be used in solving the jacketed part of the cylinder by conventional two group calculation. The height which would have to be added to the reflected height \( H' \) to give the equivalent bare height is found by

\[
\delta = \frac{H_o'}{2} - \frac{H'}{2}
\]
$H'_0 = \text{New thickness of bare core}$

$H' = \text{Initial thickness of a reflected core}$

$\delta = \text{Reflector saving}$

The reflector saving $\delta$ was calculated from the expression for a slab reactor.

$$\delta = \frac{1}{B_c} \left[ \tan^{-1} \left( \frac{D_c B_0}{D_r \kappa_r} \tan h \kappa_r T_E \right) \right]$$

where $B_c = \text{Geometrical buckling}$

$D_c = \text{Diffusion coefficient for core}$

$D_r = \text{Diffusion coefficient for reflector}$

$\kappa_r = \text{Inverse diffusion length in the reflector}$

$T_E = \text{Reflector thickness}$

For the arbitrary value of $T_E = 84 \text{ cm}$, $\delta$ is computed as $\delta = 54 \text{ cm}$. The values of all material constants such as $D_c$, $D_r$ and $\kappa_r$ are based on the appropriate temperatures as previously calculated.
TWO-GROUP CALCULATION

General Approach

In obtaining the critical mass of the reactor, efforts have been made to make the final reactor shape the proper one under the high-temperature and high-pressure conditions. The usual approach of attempting to give it an approximately cubic shape is questionable under these conditions. Rather, a slim reactor which will facilitate the manufacture of the high-pressure vessel outside the reactor and which will cut down the pressure vessel cost accordingly was considered preferable. Considering the auxiliary reactor units to be attached, the overall core height-to-diameter ratio of around 3/2 was chosen. Thus after several trial calculations, the basic dimensions were assigned as given below:

Core height, \( H = 128 \text{ cm} \)

Side reflector thickness \( T_s = 25 \text{ cm} \)

End reflector thickness \( T_E = 84 \text{ cm} \)

Radius of Core \( R = \) To be determined.

Murray's Arrangement of Two-Group Calculation

Among the several computational set-ups for two-group calculations, Murray's arrangement seemed most convenient and was therefore adopted in this work. As described in the foregoing chapter, equivalent bare-core height \( H_0' \) was computed and corresponding buckling was found to be
1.78 x 10^{-4}. Material constants μ² and ν², which are the solutions of modified two group critical equation discussed previously, were calculated. These material constants are further corrected for geometric buckling β². Coupling coefficients S₁, S₂ and S₃ are calculated. Murray's handling of critical condition is interesting. A quantity α' in the critical equation varies slowly with reactor radius. On the other hand another quantity α, which is the counterpart of α' in the critical equation changes quickly with reactor dimensions. Both quantities are computed for different sets of reactor sizes and the critical radius is found.
The result for the system is as shown in Figure 6.

Results

The result of computations gives the following basic figures.

<table>
<thead>
<tr>
<th>Category</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core height</td>
<td>128 cm</td>
</tr>
<tr>
<td>Core radius</td>
<td>40 cm</td>
</tr>
<tr>
<td>Side reflector thickness</td>
<td>25 cm</td>
</tr>
<tr>
<td>End reflector thickness</td>
<td>84 cm</td>
</tr>
<tr>
<td>Actual core volume (less void)</td>
<td>4.5 x 10⁵ cc</td>
</tr>
<tr>
<td>Apparent core volume (with void)</td>
<td>6.5 x 10⁵ cc</td>
</tr>
<tr>
<td>Total graphite in the core</td>
<td>220 kg or 0.92 ton (wt.)</td>
</tr>
<tr>
<td>Total uranium</td>
<td>42.1 kg</td>
</tr>
<tr>
<td>Total U-235</td>
<td>8.42 kg</td>
</tr>
<tr>
<td>Average thermal flux</td>
<td>2.54 x 10^{14}</td>
</tr>
</tbody>
</table>

The reactor is rather compact and the power density is extremely high as will be shown in heat transfer considerations in a later discussion.
Figure 6. Determination of Critical Radius
Without proper core materials such as were proposed previously, the above features may not be easily attainable.
FLUX DISTRIBUTION

Flux distribution was evaluated and plotted in Figure 7. Fluxes were interpreted on the basis of reactor thermal power output of 50 megawatts. The thermal flux distribution is surprisingly flat as seen from the boundary values shown below.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central fast flux $\phi_{1c}$</td>
<td>$1.52 \times 10^{15}$</td>
</tr>
<tr>
<td>Central thermal flux $\phi_{2c}$</td>
<td>$2.69 \times 10^{14}$</td>
</tr>
<tr>
<td>Average fast flux $\bar{\phi}_1$</td>
<td>$1.15 \times 10^{15}$</td>
</tr>
<tr>
<td>Average thermal flux $\bar{\phi}_2$</td>
<td>$2.54 \times 10^{14}$</td>
</tr>
<tr>
<td>Core surface fast flux $\phi_{1s}$</td>
<td>$7.88 \times 10^{14}$</td>
</tr>
<tr>
<td>Core surface thermal flux $\phi_{2s}$</td>
<td>$2.70 \times 10^{14}$</td>
</tr>
<tr>
<td>Ratio, central thermal to aver. thermal $\phi_{2c}/\bar{\phi}_2$</td>
<td>$1.06$</td>
</tr>
<tr>
<td>Ratio, core surface thermal to aver. thermal $\phi_{2s}/\bar{\phi}_2$</td>
<td>$1.06$</td>
</tr>
</tbody>
</table>

The flat thermal distribution offers a great advantage in obtaining flat power density in the core. However, it should be noted that the actual reactor flux under excess loading conditions is different from the above unless a special effort is made in the actual fuel loading. The presence of control-, safety-, and shim rods will further distort the flux to a minor degree.
Figure 7. Flux Distribution
EVALUATION OF TEMPERATURE COEFFICIENTS

Temperature coefficients were found to be \(-1.85 \times 10^{-5}\), using a simple migration area model. The coefficient is for the operating temperature, i.e., 1480°C or 2700°F, whereas a rough calculation shows the overall temperature coefficient for 20°C to be around \(-5.9 \sim 6.0 \times 10^{-5}\), which indicates that the negativity of temperature coefficient is offset as the reactor temperature rises. Thus, the high temperature reactor is not as safe as low-temperature reactors.
Xe AND Sa POISONING

Xe and Sa poisoning at operating reactor flux were calculated from the equilibrium concentration of each nuclei. The diffusion of fission fragments through the zirconium carbide coating was tentatively assumed negligible, thus poison is not removed during the operation period. The assumption is obviously not correct, although the result thus calculated will give safe-side figures in the estimation of reactor reactivity due to the poisoning.

TABLE V

EQUILIBRIUM POISONING OF Xe AND Sa

<table>
<thead>
<tr>
<th>Nuclei</th>
<th>$\sigma_a$</th>
<th>Equilibrium Concentration</th>
<th>$\Sigma_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Xe$^{135}$</td>
<td>$2.7 \times 10^6$ b</td>
<td>$8.18 \times 10^{19}$</td>
<td>221</td>
</tr>
<tr>
<td>Sa$^{149}$</td>
<td>$5.3 \times 10^4$ b</td>
<td>$8.71 \times 10^{20}$</td>
<td>46.1</td>
</tr>
</tbody>
</table>
REACTIVITY SPECIFICATIONS FOR CONTROL DESIGN

Reactivity change due to fuel depletion and isotope production during the long operation should be taken care of by the enclosed shim rods. However, the control rods should be able to handle the considerable amount of excess reactivity above the level which the shim rods may override at any moment of the fuel cycle.

Approaches

If one defines

\[ \frac{\Sigma_u}{\Sigma_m} = Z \quad \text{and} \quad P = \frac{\Sigma_p}{\Sigma_u} \]

where \( \Sigma_u \) = Macroscopic cross section of uranium
\( \Sigma_m \) = Macroscopic cross section of moderator
\( \Sigma_p \) = Macroscopic cross section of poison
P = Poison

then reactivity becomes

\[ \rho = -\frac{PZ}{1+Z} \]

Respective reactivities for different origins will be briefly reviewed here.

Equilibrium Reactivity Due to Xe Poisoning

Equilibrium poisoning, \( P_0 \) can be expressed as

\[ P_0 = \frac{\sigma_X (\gamma_I + \gamma_X) \Sigma_f \phi}{\lambda_X + \alpha_X \phi} \]
where $X = \text{Xenon}$

$I = \text{Iodine}$

$\sigma = \text{Microscopic cross section}$

$\lambda = \text{Decay constant}$

$\gamma = \text{Yield fraction}.$

For the system,

$$P_0 = 4.74 \times 10^{-2} \quad , \quad Z = 1.02 \times 10^2$$

Hence

$$\rho = - \frac{PZ}{1+Z} = - 4.7 \times 10^{-2}$$

**Equilibrium Reactivity Due to $S_a$ Poisoning**

This is practically independent of thermal flux and

$-0.012.$

**Xenon Poisoning After Shut-Down and Reactivity Equivalent**

Amount of poison $P(t)$ after the reactor shut-down is expressed by;

$$P(t) = \frac{X(t) \sigma_X}{\Sigma_u}$$

where

$$X(t) = \frac{\lambda_{I}}{\lambda_{X} - \lambda_{I}} I_0 \left( e^{-\lambda_{I} t} - e^{-\lambda_{X} t} \right) + X_0 e^{-\lambda_{X} t}$$

At around eleven hours after reactor shut-down, poison reaches the maximum value, and

$$P_{\text{max}} \approx 0.60$$

Hence

$$\rho_{\text{max}} \approx 0.60$$
Samarium Poisoning After Shut-Down and Reactivity Equivalent

After a conventional consideration, the maximum poisoning \( P_\infty \) is expressed by

\[
P_\infty = 1.5 \times 10^{-16} \varphi + 0.012
\]

where \( \varphi \) is the thermal flux. The asymptotic value for the present system is

\[
P_\infty \approx 5.01 \times 10^{-2}
\]

Hence

\[
\rho \approx 5.01 \times 10^{-2}
\]

Reactivity Equivalent Due to Temperature Rise

Since

\[
\rho_{\text{total, 20°C}} = -6.0 \times 10^{-5}
\]

and

\[
\rho_{\text{total, 1460°C}} = -1.85 \times 10^{-5}
\]

Average value will be

\[
\rho_{\text{aver.}} = -3.92 \times 10^{-5} \text{ but } \Delta T = 1460°C
\]

Hence, reactivity change \( = -5.72 \times 10^{-2} \).

Reference Dead Time After the Shut-Down

At 30 minutes after shut-down, the reactivity will be decreased by about 0.05. This 30-minute basis seems a reasonable dead time between the reactor shut-down and reactor re-startup in
case any nuclear power plant has mechanical difficulties. Hence, the
evaluation of shut-down override is based on 30 minutes after the
shut-down event.

**Summary**

The above results are summarized below.

<table>
<thead>
<tr>
<th>Due-To</th>
<th>$\Delta K/K$ requirements for safety and control rods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>+ 0.057</td>
</tr>
<tr>
<td>Xe and Sm Equilibrium</td>
<td>- 0.059</td>
</tr>
<tr>
<td>Shut-Down override</td>
<td>- 0.050</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>- 0.052</td>
</tr>
</tbody>
</table>

It should be noted that the excess reactivity due to the
temperature change (from operating temperature to room temperature
for shut-down) takes a reverse sign and thus lessens the burden of control
rod and safety rod excess reactivity requirements. Thus, if one assumes
100% safety margin for rods, overall reactivity requirement will be
around 0.10 or 10%.
EVALUATION OF BASIC THERMAL CONSTANTS

Since elevated conditions are used in the present system, the task of determining physical constants related to heat transfer calculations is one of the major problems encountered here. The extrapolation of physical properties at any known temperature and pressure region seems to be the possible approach in many cases.

Conductivity, Viscosity and Specific Heat

At one atmosphere pressure, helium has the properties listed below.

<table>
<thead>
<tr>
<th>Density #/ft$^3$</th>
<th>Specific heat Btu/#°F</th>
<th>Viscosity #/(hr)(ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01191</td>
<td>0</td>
<td>1.240</td>
</tr>
<tr>
<td></td>
<td>all temp.</td>
<td>4.58 x 10$^{-2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>45</td>
</tr>
<tr>
<td>0.0087</td>
<td>212</td>
<td>5.81</td>
</tr>
<tr>
<td></td>
<td></td>
<td>405</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7.63</td>
</tr>
<tr>
<td></td>
<td></td>
<td>765</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9.26</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1125</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10.76</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1485</td>
</tr>
</tbody>
</table>

Thermal conductivity Btu/hr-ft$^2$°F/ft

<table>
<thead>
<tr>
<th>K</th>
<th>°F</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.03 x 10$^{-2}$</td>
<td>45</td>
</tr>
<tr>
<td>10.1</td>
<td>242</td>
</tr>
<tr>
<td>11.7</td>
<td>441</td>
</tr>
<tr>
<td>12.5</td>
<td>548</td>
</tr>
<tr>
<td>18.2</td>
<td>800</td>
</tr>
<tr>
<td>16.7</td>
<td>1000</td>
</tr>
</tbody>
</table>
Among these, the variation of $C_p$ is very slight and accordingly was set at 1.250. With reference to several previous investigations, one can further assume negligible effect of pressure on viscosity and thermal conductivity. Extrapolations of already existing data give for helium at 1200°F, 1000 psia,

$$K = 0.1870 \text{ Btu/(hr)(ft}^2)(^\circ\text{F}/\text{ft})$$
$$\rho = 0.224 \text{ lb/ft}^3$$
$$\mu = 0.0926 \text{ lb/(hr)(ft)}$$

**Mass Flow Rate**

If one sets the inlet temperature to core to be 900°F, outlet temperature 1500°F,

$$\Delta T = 600^\circ\text{F}$$

but $C_p = 1.25$ and $Q = wC_p\Delta T$

where $w$ = Helium flow rate

$Q$ = Rate of heat removal

Then

$$w = \frac{Q}{C_p \cdot \Delta T} = 4.55 \times 10^3 \text{ P lb/hr}$$

where $P$ = Power in MW.

For $P = 50 \text{ MW}$, $Q = 170.65 \times 10^6 \text{ Btu/hr}$

and $w = 227.50 \times 10^3 \text{ lb/hr}$.

**Heat Transfer Coefficient**

For turbulent flow in the tube, the Nusselt number is expressed by the correlation
\frac{hD}{K} = 0.023 \ (Re)^{0.8} \ (Pr)^{0.4}

where \( Re = \frac{DG}{\mu} = \frac{Dp\nu}{\mu} \) where \( v = \) velocity, ft/hr.

and \( Pr = \frac{C_p\mu}{K} \)

Hence,

\[ h = 0.023 \ \frac{K^{0.6}}{D^{0.2}} \ G^{0.8} \ \left( \frac{C_p}{\mu} \right)^{0.4} \]

where \( C_T = w_p \)

Using the values found in the previous section, one obtains

\[ h = 0.0252 \ \frac{K^{0.6}}{\mu^{0.4}} \ \frac{G^{0.8}}{D^{0.2}} \ \text{Btu/hr-ft}^2{^\circ\text{F}} \]

or

\[ h = 0.0239 \ \frac{G^{0.8}}{D^{0.2}} \ \text{Btu/hr-ft}^2{^\circ\text{F}}. \]

The plot of \( h \) values as a function of \( G \) and \( D \) are shown in Figure 8.
Figure 8. Helium Film Coefficients as a Function of Mass Flow Rate with Diameter of Channel as Parameter. (1200°F, 1000 psia).
GENERAL RELATIONSHIPS

Before proceeding further into any detailed specific discussions, more general relationships between the various reactor heat transfer parameters will be reviewed.

Local Gas Temperatures

Using the nomenclature shown in Table 6, basic relationships are expressed for total heat accumulated in distance traversed, $q_T$, as

$$ t_f - t_1 = \frac{q_T}{wC_p} \quad \text{and} \quad w = G(\pi a^2) $$

On the other hand,

$$ q_T = \int_0^Z A_c Q dz $$

If one approximates $Q$ by sine functions, i.e., $Q = Q_0 \sin \frac{\pi Z}{Z}$, then

$$ q_T = A_c \int_0^Z Q dz = A_c \int_0^Z Q_0 \sin \frac{\pi Z}{Z} dz $$

$$ = \frac{Q_0 A_c}{\pi} \left[ 1 - \cos \frac{\pi Z}{Z} \right] . $$

Therefore,

$$ t_f = t_1 + \frac{Q_0 A_c}{\pi^2 G \pi a^2 C_p} \left( 1 - \cos \frac{\pi Z}{Z} \right) $$

But

$$ t_f = t_2 \text{ at } z = Z, $$
TABLE VI
NOMENCLATURE FOR HEAT TRANSFER ANALYSIS

\[ t_1 = \text{Initial mixed mean fluid temperature, } ^\circ F \]
\[ t_f = \text{Local mixed mean fluid temperature, } ^\circ F \]
\[ t_s = \text{Cladding temperature, } ^\circ F \]
\[ t = \text{Fuel temperature at any point, } ^\circ F \]
\[ T = \text{Maximum fuel temperature at any given } z, ^\circ F \]
\[ T_M = \text{The maximum fuel temperature, } ^\circ F \]
\[ a = \text{Radius of channel, ft.} \]
\[ 2b = \text{Distance between channel centers (equilateral pitch), ft.} \]
\[ r = \text{Distance from center of channel, ft.} \]
\[ \Theta = \text{Angle between axis and vector } r, \text{ ft.} \]
\[ z = \text{Axial distance measured from entrance of channel} \]
\[ Z = \text{Total axial distance of free position, ft.} \]
\[ Q = \text{Local power density } \text{Btu/hr-ft}^3 \]
\[ D = \text{Diameter of channel, ft.} \]
\[ D_n = \text{Diameter of channel, in.} \]
\[ h = \text{Film coefficient, Btu/(hr)(ft^2)(^\circ F)} \]
\[ n = \text{Number of channels} \]
\[ \alpha = \text{Void fraction in core} \]
\[ L = \text{Length of core, ft.} \]
\[ R = \text{Radius of core, ft.} \]
\[ Q_A = \text{Average power density} \]
TABLE VI (CONT'D)

\( Q_0 \) = Maximum power density

\( A \) = Heat transfer area of free position

\( G \) = Mass flow rate \( \text{lb/hr-ft}^2 \)

\( t_2 \) = Final mixed mean fluid temperature, °F.
Thus
\[ t_2 = t_1 + \frac{2Q_0A_c Z}{\pi^2a^2c_pG} \]

On the other hand, \( n a^2 = \alpha \pi R^2 \)
\[ \therefore n a^2 = \alpha R^2 \quad (2) \]

But
\[ A_c = \frac{(1-\alpha) \pi R^2}{n} \]
\[ = \frac{(1-\alpha) \pi a^2}{\alpha} \]

Hence
\[ t_2 - t_1 = \frac{2(1-\alpha) Q_0Z}{\alpha \pi c_p G} \quad (3) \]

**Relationship Between the Total Thermal Power and the Maximum Power Density**

If one lets
\[ P_Q = \text{Total thermal power of reactor, Btu/hr} \]
\[ V = \text{Volume of core (ft}^3) \]

Then \( P_Q = V Q_A \int_0^Z Q \, dz \)

where
\[ Q_A = \frac{Q_0Z}{Z} \]

Substituting \( Q = Q_0 \sin \frac{KZ}{Z} \), one obtains
\[ Z Q_A = \int_0^Z Q_0 \sin \frac{KZ}{Z} \, dz \]
\[ = \frac{2Z Q_0}{\pi} \]

Thus \( Q_A = \frac{2}{\pi} Q_0 \quad (4) \)
Hence
\[ Q_0 = Q_A \cdot \frac{\pi}{2} \]
\[ = \frac{P_Q}{V} \cdot \frac{\pi}{2} \]

But,
\[ V = (1-\alpha) \pi R^2 Z \quad \text{and} \]
\[ P_Q = P(MW) \times 3.143 \times 10^6 \quad \text{Btu/hr} \]

\[ \therefore Q_0 = \frac{17.065 \times 10^5 P(MW)}{(1-\alpha) R^2 Z} \quad \text{Btu/hr} \quad (5) \]

**Fuel Surface Temperature and Inlet Gas Temperature**

The following basic relationship holds through the channel wall,
\[ dq = h \, dA \, \Delta T \]

where \( \Delta T = t_s - t_f \)

Furthermore,
\[ dq = QdV = Q A_c dz \]

and
\[ dA = 2\pi a \, dz \]

Then
\[ Q A_c dz = h \, (t_s - t_f) \, 2\pi a \, dz \]

or
\[ QA_c = Q_0 \sin \frac{\pi z}{\pi} \left( 1-\alpha \right) \frac{\pi a^2}{\alpha} \]

\[ = h \, (t_s - t_f) \, 2\pi a \]

thus
\[ 2h(t_s - t_f) = \frac{(1-\alpha) a}{\alpha} \, Q_0 \sin \frac{\pi z}{\pi} \quad (6) \]
At 1200°F and 1000 psi,

\[ h = 0.0239 \frac{q^{0.8}}{p^{0.2}} \]

On the other hand,

\[ G = \frac{w}{n \pi a^2} \quad \text{and} \quad q = 2.413 \times 10^6 \, P \]

\[ = w \, C_p \, (t_2 - t_1) \]

\[ \therefore w = \frac{3.413 \times 10^6 \, P}{C_p \, (t_2 - t_1)} \]

or

\[ G = \frac{3.413 \times 10^6 \, P}{n \pi a^2 C_p \, (t_2 - t_1)} \]

Thus

\[ h = 0.0239 \left[ \frac{3.413 \times 10^6 \, P}{n \pi a^2 C_p \, (t_2 - t_1)} \right]^{-0.8} \frac{1}{(2a)^{0.2}} \]

or

\[ h = 0.0239 \left[ \frac{3.413 \times 10^6}{\alpha \pi R^2 C_p \, (t_2 - t_1)} \right]^{-0.8} \left( \frac{1}{2} \right)^{0.2} \left( \frac{p^{0.8}}{a^{0.2}} \right) \]

Let us call

\[ 0.0239 \left[ \frac{3.413 \times 10^6}{\alpha \pi R^2 C_p \, (t_2 - t_1)} \right]^{-0.8} \left( \frac{1}{2} \right)^{0.2} = K \]

Then

\[ h = K \left( \frac{p^{0.8}}{a^{0.2}} \right) \]

Substituting this into equation (6) to obtain

\[ 2K \left( \frac{p^{0.8}}{a^{0.2}} \right) (t_s - t_f) = \frac{(1-\alpha) \, a}{\alpha} (q_0 \sin \frac{\pi z}{L}) \]
Hence,

\[ t_s = t_f + \frac{(1-\alpha) a}{2\alpha K} \left( Q_0 \sin \frac{\pi z}{Z} \right) \left( \frac{a^{0.2}}{p^{0.8}} \right) \]

or

\[ t_s = t_1 + \frac{Q_0 A_0 Z}{\pi a^2 c_p G} \left( 1 - \cos \frac{\pi z}{Z} \right) \]

\[ + \frac{(1-\alpha) a}{2\alpha K} \left( Q_0 \sin \frac{\pi z}{Z} \right) \left( \frac{a^{0.2}}{p^{0.8}} \right) \] from equation (1).
MAXIMUM CORE TEMPERATURE

Considering the fact that the temperature employed in the reactor system is very high, a safe estimation of the maximum temperature in the core is essential in order to prevent any possible fuel burn-up. Accordingly, a full chapter was devoted to the discussion of this problem.

Fend's Work

In a large solid reactor generating heat Q uniformly, with coolant channels placed on an equilateral pitch, the differential equation is

\[ k \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} \right) + Q = 0 \]

Figure 9. Cooling Channel and Neighborhood

Since the region has 30° symmetry, only the shaded area bounded by the adiabatic planes \( \varphi = 0 \), \( \varphi = 30^\circ \) and \( x = 0 \) need to be considered. The
temperature at the inner wall is assumed to be constant, \( T_a \). No exact solution can be found because all the boundary conditions can not be satisfied in both circular and rectangular coordinates. An approximate solution in circular harmonics has been obtained by Fend et al. The result is

\[
\frac{T - T_a}{\frac{Q}{b^2/K}} = \frac{1}{4} \left[ \frac{3\sqrt{3}}{\pi} \ln \left( \frac{r}{a} \right) \left( \frac{r}{b} \right)^2 + \left( \frac{a}{b} \right)^2 \right] \\
- 0.01484 \left( \frac{r}{b} \right)^6 \cos 6 \varphi - 0.00021 \left( \frac{r}{b} \right) \cos 12 \varphi
\]

**Derivation of Maximum Core Temperature for the Present Reactor System**

![Diagram of Cooling Channel and Vicinity](image)

*Figure 10. Geometry of Cooling Channel and Vicinity*

As indicated previously the system uses equilateral pitch for cooling channels, that is, each tube is enclosed in a hexagon of diagonal 2b. The total area of the hexagon is 6 x \( \Delta \). But \( \Delta = bh/2 \) and \( h = \frac{b \sqrt{3}}{2} \).

Thus the area of hexagon = \( \frac{3\sqrt{3}}{2} b^2 \)

\[ \therefore A_c = \frac{(1-\alpha) \pi a^2}{\alpha} = \frac{3\sqrt{3}}{2} b^2 - \pi a^2 \]

\[ \therefore \frac{3\sqrt{3}}{2} b^2 = \frac{1}{\alpha} \pi a^2 \]
In our new nomenclature, Fend's result can be written as

\[
\frac{(t - t_s) K}{Q b^2} = \frac{1}{4} \left[ \frac{3 \sqrt{3}}{\pi} \ln \left( \frac{a}{b} \right) - \left( \frac{a}{b} \right)^2 + \left( \frac{a}{b} \right)^3 \right] - 0.01484 \left( \frac{a}{b} \right)^6 \cos 6 \varphi - 0.00021 \left( \frac{a}{b} \right) \cos 12 \varphi
\]

The maximum fuel temperature at any given axial position will occur at \( r = b, \varphi = 30^\circ \). Then,

\[
\frac{(t_o - t_s) K}{Q b^2} = \frac{1}{4} \left[ \frac{3 \sqrt{3}}{2\pi} \ln \left( \frac{b}{a} \right)^2 + \left( \frac{a}{b} \right)^2 - 1 \right] + 0.01496 - 0.00028
\]

Substituting the relationship

\[ b^2 = \frac{2 \pi a^2}{3 \sqrt{3} \alpha} \]

into the above equation, one obtains

\[
\frac{3 \alpha \sqrt{3} (t_o - t_s) K}{2 \pi a^2 Q} = \frac{1}{4} \left[ \frac{\sqrt{3}}{2\pi} \ln \left( \frac{2 \pi}{3 \alpha \sqrt{3}} \right) + \frac{3 \alpha \sqrt{3}}{2\pi} - 1 \right] + 0.01468
\]

Hence

\[
\frac{t_o - t_s}{a^2 Q} = K_1 = \frac{2 \pi}{12 \alpha \sqrt{3} K} \left\{ \frac{3 \sqrt{3}}{2\pi} \ln \left( \frac{2 \pi}{3 \alpha \sqrt{3}} \right) + \frac{3 \alpha \sqrt{3}}{2\pi} - 0.94128 \right\}
\]

or

\[
t_o = t_s + a^2 Q K_1
\]

In a short form, the above relationship can be expressed by

\[
K_1 = \frac{1}{4} \frac{1}{\alpha K_c} \left\{ \ln \left( 1.21 / \alpha \right) + \alpha - 1.14 \right\}
\]
But as seen previously,

\[ Q = \frac{17.065 \times 10^5 P}{(1-\alpha) R^2 Z} \sin \frac{\pi Z}{Z} \]

Thus, after some rearrangements,

\[ t_o = t_s + a^2 Q k_1 \]

\[ = \left[ \frac{740 \ D^2 P \left\{ \ln \left(1.2 \frac{1}{\alpha} + \alpha - 1.14 \right) \right\}}{\alpha (1-\alpha) R^2 Z K_c} \right] \sin \frac{\pi Z}{Z} + t_s \]

or

\[ t_o = t_s + K_1 \sin \frac{\pi Z}{Z} \]

where

\[ K_1 = \frac{740 \ D^2 P \left\{ \ln \left(1.2 \frac{1}{\alpha} + \alpha - 1.14 \right) \right\}}{\alpha (1-\alpha) R^2 Z K_c} \]

On the other hand, we previously had

\[ t_s = t_f + \frac{(1-\alpha)a}{2\alpha K} \left( Q_o \sin \frac{\pi Z}{Z} \right) \frac{s^{0.2}}{p^{0.8}} \]

\[ = t_f + \left[ 14.3 \frac{0.2}{D^{1.2} \left\{ C_p \left( t_2 - t_1 \right) \right\}} Z \frac{t_0.4}{R} \left( \frac{P}{Z} \right)^{0.2} \right] \sin \frac{\pi Z}{Z} \]

or

\[ t_s = t_f + K_2 \sin \frac{\pi Z}{Z} \]

where

\[ K_2 = \left[ \text{above} \right] \]

Combining these expressions,

\[ t_o = t_f + K_2 \sin \frac{\mu Z}{Z} + K_1 \sin \frac{\mu Z}{Z} \]

where \( \mu = \frac{\pi}{Z} \)
Using the previously obtained value for $t_f$,

$$t_0 = t_1 + \frac{Q_0 A_c Z}{\pi a^2 c_p G} \left(1 - \cos \frac{\pi Z}{Z} \right) + (K_1 + K_2) \sin \mu Z$$

$$= t_1 + K_3 (1 - \cos \frac{\pi Z}{Z}) + K_4 \sin \mu Z$$

where

$$K_3 = \frac{Q_0 A_c Z}{\pi a^2 c_p G} \quad \text{and} \quad K_4 = K_1 + K_2$$

Now, to obtain the maximum core temperature, the condition is imposed so that

$$\frac{dT_0}{dz} = 0$$

$$\therefore \mu K_3 \sin \mu z + \mu K_4 \cos \mu z = 0$$

$$\therefore z = \frac{1}{\mu} \tan^{-1} \left( -\frac{K_4}{K_3} \right)$$

Thus $T_m$ takes the form of

$$T_m = t_1 + K_3 - K_3 \cos \left[\tan^{-1} \left( -\frac{K_4}{K_3} \right) \right] + K_4 \sin \left[\tan^{-1} \left( -\frac{K_4}{K_3} \right) \right]$$

$$= t_1 + K_3 + \sqrt{K_3^2 + K_4^2}$$

But $K_3 = \frac{t_2 - t_1}{2}$ \quad $t_1 + K_3 = \frac{t_2 + t_1}{2}$
one arrives at

\[ 18.85 \times 10^{-4} D^2 P_c + 0.529 D^{1.2} P_c^{0.2} = 1 \]

The trial and error method gives \( D \) and \( P_c \) values satisfying the equation as follows.

<table>
<thead>
<tr>
<th>( D )</th>
<th>( P_c )</th>
<th>( D )</th>
<th>( P_c )</th>
<th>( D )</th>
<th>( P_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 5/8 )</td>
<td>190.0</td>
<td>( 17/16 )</td>
<td>14.5</td>
<td>( 1 1/2 )</td>
<td>2.0</td>
</tr>
<tr>
<td>( 11/16 )</td>
<td>127.0</td>
<td>( 1 1/8 )</td>
<td>10.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 3/4 )</td>
<td>85.0</td>
<td>( 19/16 )</td>
<td>7.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 13/16 )</td>
<td>59.0</td>
<td>( 1 1/4 )</td>
<td>5.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 7/8 )</td>
<td>40.0</td>
<td>( 21/16 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 15/16 )</td>
<td>28.0</td>
<td>( 1 3/8 )</td>
<td>3.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 1 )</td>
<td>20.0</td>
<td>( 23/16 )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Thus, there exist distinctive maximum power limits for any cooling channel diameter of core under each set of reactor conditions. In this particular case, \( D \) should be around \( 13/16'' \) and less to keep the maximum core temperature \( 2700 \)\(^\circ\)F and below. Actually \( 3/4'' \) size was chosen for the present design.

Channel Diameter and Reflector Inlet Temperature

Setting \( T_m, \alpha, R, Z, K_c \) and \( P_c \) in the equation (8), one obtains

\[
(12.97 \times 10^6 - 7200 T)^{1/2} = 280 D^2 + 20.4 D^{1.2} T^{0.8}
\]

where \( T = t_2 - t_1 \)
Thus finally,
\[ T_m = \frac{t_2 + t_1}{2} \]
\[ + \sqrt{\frac{1}{4} (t_2 - t_1)^2 + \left[ \frac{740 D^2 P \left\{ \ln \left( \frac{1.21}{\alpha} \right) + \alpha - 1.14 \right\}}{\alpha (1-\alpha) R^2 Z K_c} \right] \left( \frac{14.3 d^{1.2} (C_p \Delta t)^{0.8} p^0.2}{Z R^{0.4} \alpha^{0.2}} \right)^2 } \]  
(7)

Channel Diameter and Power Output of Core

Substituting \( C_p = 1.20 \) into equation (7), one obtains
\[ T_m = \frac{t_2 - t_1}{2} \]
\[ + \left[ \frac{1}{4} (t_2 - t_1)^2 + \left[ \frac{740 D^2 P c \left\{ \ln \left( \frac{1.21}{\alpha} \right) + \alpha - 1.14 \right\}}{\alpha (1-\alpha) R^2 Z K_c} \right] \left( \frac{17.1 D^{1.2} (t_2 - t_1)^{0.8} P_c^0.2}{Z R^{0.4} \alpha^{0.2}} \right)^2 \right]^{1/2} \]  
(8)

Setting the temperature and critical conditions as previously observed,
\[ t_1 = 900°F \]
\[ t_2 = 1500°F \]
\[ Z = 128 \text{ cm} = 4.20 \text{ ft} \]
\[ R = 40 \text{ cm} = 1.31 \text{ ft} \]
\[ \alpha = 0.2965 \]
\[ T_m = 2700°F \]
\[ K_c = 100 \]
The result of calculation shows

<table>
<thead>
<tr>
<th>t2</th>
<th>D</th>
<th>t2</th>
<th>D</th>
<th>t2</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1400</td>
<td>1.03</td>
<td>1900</td>
<td>0.53</td>
<td>2400</td>
<td>0.27</td>
</tr>
<tr>
<td>1500</td>
<td>0.88</td>
<td>2000</td>
<td>0.47</td>
<td>2500</td>
<td>0.22</td>
</tr>
<tr>
<td>1600</td>
<td>0.77</td>
<td>2100</td>
<td>0.42</td>
<td>2600</td>
<td>0.16</td>
</tr>
<tr>
<td>1700</td>
<td>0.68</td>
<td>2200</td>
<td>0.37</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1800</td>
<td>0.60</td>
<td>2300</td>
<td>0.32</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For convenience in further calculations, the values of cooling channel film coefficient h for a different set of n and D, were calculated. The total heat transfer area A is then calculated from the relationship of $A = n x DL$. The products hA as a function of channel number n are then plotted for different reactor portions, the channel diameter D being a parameter. One set of such plots which is for thermal shield is shown as an example in the Appendix. Similar plots were used for core and side reflectors, though they are not shown.
PRESSURE AND TEMPERATURE DISTRIBUTION IN THE REACTOR

In the analysis of a high-pressure and high-temperature reactor, the distribution of temperature and pressure patterns is especially important in connection with the selection of structural materials and thermal and pressure shock evaluations. A general approach will be observed here.

**Power Generation and Coolant Temperature Drop in Thermal Shield.**

If one lets

\[ P_c = \text{Power released in the core} \]
\[ P_T = \text{Total power released in the core reflector and shield,} \]

and assume

\[ P_c = 0.85 \, P_T, \quad \text{then} \]
\[ P_c = 0.85 \, 50 \, \text{MW} \]
\[ \therefore P_c = 58.8 \, \text{MW} \]
\[ \approx 2 \times 10^8 \, \text{Btu/Hr} \]

If one further assumes 5% heat generation of the total in the thermal shield (abbrev. T.S.).

\[ Q_{TS} = 10^7 \, \text{Btu/Hr} \]
\[ \therefore hA(\Delta T) = 10^7 \, \text{Btu/Hr} \]

Assume again the average temperature of thermal shield to be 1100°F.

Furthermore, from

\[ \alpha C_p \, (T_2 - T_1) = 10^7 \, \text{Btu/Hr}, \]

one can evaluate

\[ T_2 - T_1 = \frac{10^7}{\alpha C_p} \]
\[ = 35.2 \, ^\circ \text{F}. \]
This is the coolant temperature drop in the thermal shield under the conditions assumed.

**Power Generation in Reflector**

Let us set

\[
\begin{align*}
\text{Inlet temperature to side reflector} & = T_2 \\
\text{Outlet temperature from side reflector} & = T_3 \\
\text{Inlet temperature to bottom reflector} & = T_3 \\
\text{Outlet temperature from top reflector} & = T_4 \\
\text{Heat generated in side reflector} & = H_S \\
\text{Heat generated in end reflector} & = H_E
\end{align*}
\]

Due to the additional gamma heating of reflector and thermal shields, it is not easy to assign any definite power generation to the different portion of the reactor core through purely analytical means. As has been the practice, a rough assumption is made on the basis of estimated power flux and relative gamma emission effect. Thus it is assumed that

\[
\frac{(H_S + H_E)}{P_T} = 0.10
\]

Further, \( \frac{H_S}{H_E} = \text{Side area of core/End area of core} \)

\[
= \frac{Z}{(1-\alpha)R}
\]

\[
= 2.16
\]

Then,

\[
H_S + H_E = 0.10 \times P_T = 3.16 \times H_E
\]

∴ \( H_S = 13.68 \times 10^6 \text{ Btu/Hr} \)

\( H_E = 6.32 \times 10^6 \text{ Btu/Hr} \)
Local Coolant Temperatures in Reflector and Thermal Shield

From the relationship

\[ \omega C_p (T_3 - T_2) = 13.68 \times 10^6 \text{ Btu/Hr} \]

\[ T_3 - T_2 = 48.1^\circ F \]

and,

\[ T_4 - T_3 = 622.2^\circ F \]

\[ T_4 - T_1 = 704^\circ F \]

\[ T_4 + T_3 = 2400^\circ F \]

From these relationships, local temperatures are found to be

\[ T_4 = 1511.1^\circ F \]

\[ T_3 = 888.9^\circ F \]

\[ T_2 = 840.8^\circ F \]

\[ T_1 = 805.6^\circ F \]

Furthermore, the average temperature for thermal shield is

\[ T_{av,TS} = \frac{T_1 + T_2}{2} = 823.2^\circ F. \]

The coolant flow pattern for the reactor core is shown in Figure 11, together with the local coolant temperatures evaluated thus far.

Pressure Drops Through the Cooling Channel: Number of Channels

For flow of fluids in channel, the pressure drop gradient can be expressed by

\[ \frac{dp}{dx} = \frac{2fG}{Dg_c} = \frac{2fG^2}{Dg_c f} \]

which is the accepted Fanning's equation, where the notations are conventional. But

\[ f = 0.046 \left( \frac{Dg_c}{\mu} \right)^{-0.2} \]
Figure 11. Coolant Flow Pattern (Helium at 1000 psia)
Thus

\[
\frac{dp}{dx} = \frac{0.992 \mu^{0.2} g^{0.8}}{\rho g_e b^{1.2}}
\]

\[
\Delta P = \frac{2.27 \times 10^{-14}}{(19.7)^{-2}} \frac{\mu_0 1.8 L}{\rho D_n^{1.2}} \text{ psi}
\]

where \( \mu_0 = \text{In cp} \)

\( G = \text{In lb-mass/ft}^2\text{-Hr} \)

\( L = \text{In feet} \)

\( D_n = \text{In inch} \)

\( \rho = \text{In lb-mass/ft}^3 \)

But for the system \( L = 9.85 \text{ ft} \)

\( \omega = 22.75 \times 10^4 \text{ lb/Hr} \)

\[
\therefore \Delta P = 1.85 \times 10^4 \frac{\mu_0^{0.2}}{\rho n^{1.8} D_n^{4.8}} \text{ psi}
\]

\( \mu_0 \) and \( \rho \) are temperature dependent and thus the pressure drop is a function of temperature, number of channels and channel diameters.

**TABLE VII**

CHANNEL PRESSURE DROP AS A FUNCTION OF TEMPERATURE, CHANNEL NUMBER AND DIAMETER

<table>
<thead>
<tr>
<th>Temperature (°F)</th>
<th>( \Delta P ) (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>825</td>
<td>3.29 \times 10^4/n^{1.8} D_n^{4.8}</td>
</tr>
<tr>
<td>870</td>
<td>3.59 \times 10^4/n^{1.8} D_n^{4.8}</td>
</tr>
<tr>
<td>900</td>
<td>3.46 \times 10^4/n^{1.8} D_n^{4.8}</td>
</tr>
<tr>
<td>1200</td>
<td>4.22 \times 10^4/n^{1.8} D_n^{4.8}</td>
</tr>
<tr>
<td>1500</td>
<td>5.05 \times 10^4/n^{1.8} D_n^{4.8}</td>
</tr>
</tbody>
</table>
There can be two approaches in finding the optimum cooling channel
diameter and numbers as follows.

i) Specify $n$ and $D$ arbitrarily and find pressure drop through
the channel.

ii) Specify pressure drops and find $n$ and $D$.

From economic and operational viewpoints, pressure drop is
the controlling factor, hence the latter approach was adopted in the
work.

Let us require that $\Delta p$ through the thermal shield and side reflector/
are both 2 psi.

**Thermal Shield**

$$\Delta p = 3.29 \times 10^4/n^{1.8}D^{4.8}$$  psi

Since $T_{av}\|_{He} = 825^\circ F$,

$$n = 220 \text{ for } D = 1''$$

As previously seen, $hA(\Delta T) = 10^7$  Btu/HR

But from the plot $hA = 60 \times 10^3$ for $n = 220$, $D = 1''$

Thus

$$T_{TS} \approx 990^\circ$$

This is the thermal shield channel surface temperature.

**Side Reflector**

From $\Delta p = 3.39 \times 10^4/n^{1.8}D^{4.8}$  for $T_{av}\|_{He} \approx 870^\circ F$, one

obtains

$$n = 76$$

$$D = 1 1/2''$$

$$T_{sr} = 1265^\circ F$$
Summary of Reactor Component Temperatures and Powers

It would be interesting to observe the overall temperature
distributions for different reactor components as calculated previously.
Results are shown in Table VIII.

<table>
<thead>
<tr>
<th>TABLE VIII</th>
</tr>
</thead>
<tbody>
<tr>
<td>SUMMARY OF REACTOR LOCAL TEMPERATURES AND POWERS</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reactor:</th>
<th>Average Temp. °F</th>
<th>Average Helium Temp. °F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Component</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thermal Shield</td>
<td>990</td>
<td>323</td>
</tr>
<tr>
<td>Side Reflector</td>
<td>1265</td>
<td>865</td>
</tr>
<tr>
<td>Bottom End Reflector</td>
<td>1026</td>
<td>894</td>
</tr>
<tr>
<td>Top End Reflector</td>
<td>1637</td>
<td>1506</td>
</tr>
<tr>
<td>Core</td>
<td>--</td>
<td>1200</td>
</tr>
</tbody>
</table>

Helium, Local:

| Inlet to Thermal Shield | 806°F |
| Inlet to Side Reflector | 841°F |
| Inlet to Bottom Reflector | 889°F |
| Inlet to Core Reflector | 900°F |
| Inlet to Top Reflector | 1500°F |
| Outlet from Reactor System | 1511°F |

Local Power:

<table>
<thead>
<tr>
<th>Component</th>
<th>Power MW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core</td>
<td>50</td>
</tr>
<tr>
<td>Thermal Shield</td>
<td>2.94</td>
</tr>
<tr>
<td>Side Reflector</td>
<td>4.00</td>
</tr>
<tr>
<td>Bottom Reflector</td>
<td>0.93</td>
</tr>
<tr>
<td>Total</td>
<td>58.80</td>
</tr>
</tbody>
</table>

Pressure Drops Through Core and End Reflectors

After setting the channel diameter $D = \frac{3}{4}$, the number of channels through the core, the end reflectors were determined to be $n = 521$. Using the pressure drop equations as previously done for side reflectors, pressure drops were then calculated. Results are

$$\Delta P_{\text{core}} = 0.93 \text{ psi}$$
$$\Delta P_{\text{TR}} = 0.74 \text{ psi}$$
$$\Delta P_{\text{BR}} = 0.51 \text{ psi}$$

$$\Delta P_{\text{total}} = 2.18 \text{ psi}$$

Pressure Drops Due to Entrance and Exit Losses in The Reactor

Through the coolant path, enlargements, contractions and 180° bends are experienced by helium as shown below.

**Enlargements**

$\Delta P_1 = \text{Entrance to plenum chamber}$

$\Delta P_3 = \text{Exit from thermal shield}$

$\Delta P_5 = \text{Exit from side reflector}$

$\Delta P_7 = \text{Exit from top reflector}$

**Contraction**

$\Delta P_2 = \text{Entrance to thermal shield}$

$\Delta P_4 = \text{Entrance to side reflector}$

$\Delta P_6 = \text{Entrance to bottom reflector}$

$\Delta P_8 = \text{Exit from top plenum chamber}$

**Bend (180°)**

$\Delta P_9 = \text{180° bend at top of thermal shield and side reflector}$

$\Delta P_{10} = \text{180° bend at bottom of side reflector and bottom reflector}$
Using the conventional approach, pressure drops and gains were calculated. Results are shown in Table IX.

**TABLE IX**

**PRESSURE LOSS OF COOLANT FLOW DUE TO ENTRANCE AND EXIT**

<table>
<thead>
<tr>
<th>Portions</th>
<th>Pressure Drops (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta P_1$</td>
<td>- 0.17</td>
</tr>
<tr>
<td>$\Delta P_2$</td>
<td>1.47</td>
</tr>
<tr>
<td>$\Delta P_3$</td>
<td>- 0.22</td>
</tr>
<tr>
<td>$\Delta P_4$</td>
<td>2.59</td>
</tr>
<tr>
<td>$\Delta P_5$</td>
<td>- 0.19</td>
</tr>
<tr>
<td>$\Delta P_6$</td>
<td>0.04</td>
</tr>
<tr>
<td>$\Delta P_7$</td>
<td>- 0.02</td>
</tr>
<tr>
<td>$\Delta P_8$</td>
<td>1.8</td>
</tr>
<tr>
<td>$\Delta P_9$</td>
<td>0.05</td>
</tr>
<tr>
<td>$\Delta P_{10}$</td>
<td>0.02</td>
</tr>
<tr>
<td>$\Delta_{Total}$</td>
<td>5.37</td>
</tr>
</tbody>
</table>
SUMMARY OF HEAT TRANSFER DATA

The contribution of radiant heat transfer to the overall reactor heat transfer was separately investigated but found to be negligible for the present system. Some additional thermal and coolant flow data for the reactor core are summarized as shown below.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Heat Transfer Area</td>
<td>429 ft²</td>
</tr>
<tr>
<td>Average Heat Flux</td>
<td>$3.98 \times 10^5$ Btu/HR-ft²</td>
</tr>
<tr>
<td>Maximum Heat Flux</td>
<td>$6.25 \times 10^5$</td>
</tr>
<tr>
<td>Average Power Density</td>
<td>$1.51 \times 10^5 P_e (Btu/HR)$ Btu/HR-ft³</td>
</tr>
<tr>
<td>Specific Power</td>
<td>34.15 $P_e$ Btu/HR-ft³</td>
</tr>
<tr>
<td>Film Coefficient</td>
<td>1238 Btu/HR-ft² -°F</td>
</tr>
<tr>
<td>Helium Flow Rate</td>
<td>$22.75 \times 10^4$ lb/HR</td>
</tr>
<tr>
<td>Total Reactor Pressure Drop</td>
<td>11.55 psi</td>
</tr>
</tbody>
</table>
CONCLUSION

As has been observed in the foregoing discussions, the application of high-temperature nuclear reactors to power generation seems quite feasible with some safety margins in material, nuclear and thermal characteristics.

Crude reactor design and analysis with accuracy of first approximation can be performed using presently available technical information and techniques. Thermal analysis can be as conventional and straightforward as it has been for the usual low- or medium-temperature reactors designed, although a rather careful treatment will be essential in the estimation of the maximum core temperature to secure the ultimate reactor safety. The problem of handling nuclear properties at the highly elevated temperatures is not an easy one at the present time however, and several basic assumptions are inevitably needed.

Further studies on high-temperature reactor materials as well as nuclear properties at elevated conditions will certainly be helpful in a more elaborate analysis of reactors of this type.


Helium film coefficient as function of number of channels with diameter of channel as parameter.

\[ h = \text{Btu/hr/ft}^2/\text{°F} \]

Based on reactor power level of 58.8 mev heat to shield \( \approx 5\% \) of reactor power.

Helium flow rate \( w = 22.75 \times 10^4 \text{#/hr} \)

**THERMAL SHIELD HEAT REMOVAL**
Heat transfer area as function of number of channels with diameter of channel as parameter.

\[ A = \text{n} \times d^2 \]

- \( D = 1 \frac{1}{2} '' \)
- \( D = 1 '' \)
- \( D = 3/4 '' \)
- \( D = 1/2 '' \)

THERMAL SHIELD HEAT REMOVAL
Product of helium film coefficient and heat transfer as function of number of channels with diameter of channel as parameter.

\[ hA = \text{Btu/hr/°F} \]

Helium flow rate = \( w = 22.75 \times 10^4 \text{#/hr} \) based on reactor level of 58.8 mw heat to shield = \( 1 \times 10^7 \text{Btu/hr} \)

\( D = 1/2'' \)
\( D = 3/4'' \)
\( D = 1'' \)
\( D = 1\frac{1}{2}'' \)

THERMAL SHIELD HEAT REMOVAL