

Natural Convection from a Wavy Cone

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Abstract. The problem of natural convection flow and heat transfer induced by a vertically oriented cone with constant surface temperature is treated in this paper. The cone is assumed to have transverse wavy configurations. The resulting boundary layer flow is described by two coupled parabolic partial differential equations. These equations are solved numerically using the Keller-box method for a sinusoidal wavy cone. The effect of sinusoidal waves on the local Nusselt number is determined and presented on graphs. The local Nusselt number is found to be lower than that of the corresponding flat cone.

Nomenclature

a = amplitude of the wavy surface of the cone
 f = reduced stream function, Eq. (23)
 g = acceleration due to gravity
 Gr = Grashof number based on l
 h = reduced temperature function, Eq. (23)
 k = heat conductivity
 l = half-wavelength, or length scale, of the wavy surface of the cone
 \mathbf{n} = unit vector normal to the wavy surface of the cone
 Nu = local Nusselt number
 p = pressure
 Pr = Prandtl number
 q = heat flux
 r = local radius of the flat cone
 T = temperature
 u, v = velocity component in x - and y -directions
 u_c = reference velocity
 x, y = rectangular coordinates
 Y = boundary layer variable, Eq. (15)

Greek symbols

ρ = density
 ν = kinematic viscosity
 β = bulk modulus
 θ = dimensionless temperature
 η = pseudo-similarity variable, Eq. (23)
 σ = function associated with the wavy surface of the cone
 Φ = cone half angle
 ψ = stream function

Subscripts

x = differentiation with respect to x
 ∞ = condition at infinity in the y -direction
 w = condition at the wall

Superscripts

$\bar{}$ = dimensional variables
 $\hat{}$ = transformed variables, Eq. (9), or boundary layer variables, Eq. (15)
 $'$ = differentiation with respect to η

1. Introduction

Convection problems associated with irregular surfaces have received less attention than the cases with regular surfaces. Surfaces are sometimes roughened to disturb the flow and alter the rate of heat transfer on such surfaces. Thus, it is clear that convection problems associated with wavy surfaces occur frequently in practice. Typical applications are flat-plate solar collectors and flat-plate condensers in refrigerators. Among the few papers to date which study the effects of roughness elements on natural convection of a viscous fluid, we mention those of Yao [1], and Moulic and Yao [2], while for a fluid-saturated porous medium we mention the recent papers by Rees and Pop [3, 4].

In the present paper, we consider the effects of a transverse wavy surface on the basic boundary layer flow induced by an isothermal vertical cone for which the resulting flow remains two-dimensional. The Grashof number, Gr , is based on the wavelength of the surface waves and it is assumed to be large in order that the boundary layer approximation may be invoked. It is found that the non-dimensional amplitude of the waves, a , must be within $O(Gr^{-1/4})$ range in order to balance direct and indirect buoyancy forces. The resulting boundary layer equations cannot be transformed to ordinary differential equations by means of a similarity transformation. However, the form of the usual similarity transformation can be used to transform the partial differential equations into a form which can be conveniently solved numerically using the Keller-box method [5]. The analysis is carried out for the natural convection along a vertical cone with arbitrary surface waves. Then, a numerical solution is presented for a sinusoidal wavy cone in order to show the effects of roughness on natural convection. The distribution of the local Nusselt number is determined and illustrated in figures.

2. Analysis

The physical model is a vertical cone with a transversal wavy surface at a constant wall temperature, T_w , which is higher than the ambient temperature, T_∞ . The geometry and coordinate system are illustrated in Fig. 1. The wavy surfaces are described by

$$\bar{y} = \bar{\sigma}(\bar{x}) = \bar{a} \sin\left(\pi \frac{\bar{x}}{l}\right), \quad (1)$$

where \bar{a} is the amplitude of the wavy surface and l is the characteristic length scale associated with the waves. Overbars denote dimensional quantities. The flow is considered to be steady, laminar, and the Boussinesq approximation is employed. Another assumption is that the cone angles under consideration are large so that the transverse curvature effects are negligible. This assumption has been extensively analyzed in the past, see Hering and Grosh [6], and Kuiken [7]. It means that the distance to a point in the boundary layer from the cone axis is approximated by the local radius of the flat cone ($\bar{a} = 0$).

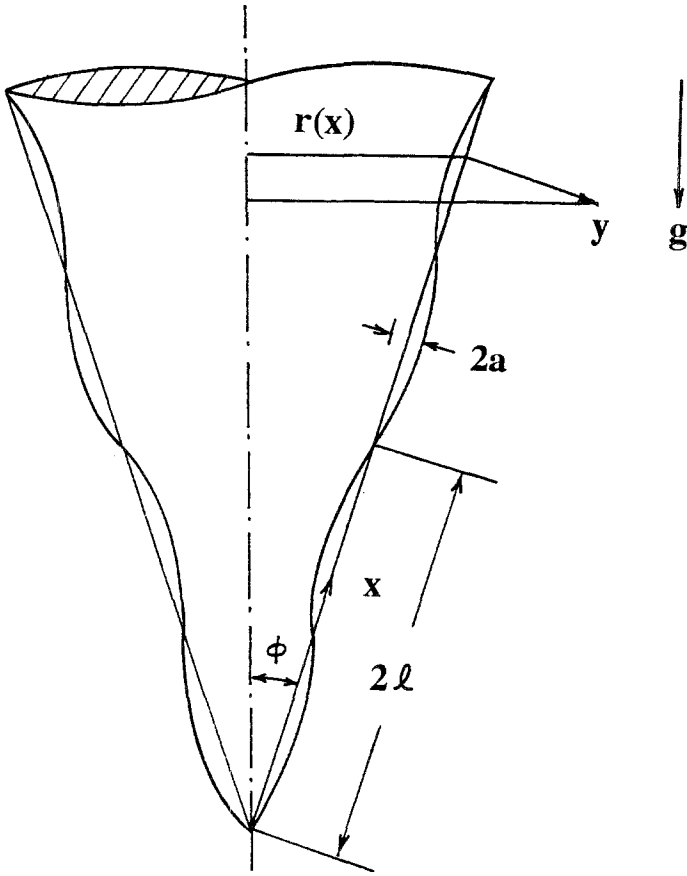


Fig. 1. Physical model and coordinate system.

Under these assumptions, the equations describing the complete description of the convective flow over the wavy cone can be written in non-dimensional form as

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0, \tag{2}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{\sqrt{Gr}} \nabla^2 u + \theta, \tag{3}$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \frac{1}{\sqrt{Gr}} \nabla^2 v - \tan \phi \theta, \tag{4}$$

$$u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{1}{\sqrt{Gr}} \nabla^2 \theta. \tag{5}$$

Here the dimensionless variables are related to their dimensional variables by

$$x = \frac{\bar{x}}{l}, \quad y = \frac{\bar{y}}{l}, \quad r = \frac{\bar{r}}{l}, \quad \sigma = \frac{\bar{\sigma}(\bar{x})}{l},$$

$$u = \frac{\bar{u}}{u_c}, \quad v = \frac{\bar{v}}{u_c}, \quad p = \frac{\bar{p}}{\rho u_c^2}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty},$$

where \bar{r} is the local radius of the flat cone ($\bar{a} = 0$) and

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \quad \text{Gr} = \frac{g\beta(T_w - T_\infty)l^3 \cos \phi}{\nu^2},$$

$$u_c = \sqrt{g\beta(T_w - T_\infty)l \cos \phi}.$$

It should be remarked that for the present problem we used the operator ∇^2 instead of

$$\frac{\partial^2}{\partial y^2} + \frac{1}{r} \frac{\partial}{\partial y} + \frac{\partial^2}{\partial x^2}.$$

The neglect of the second term does not seriously affect solution, apart from a small region near the vertex of the cone.

The boundary conditions are

$$y = \sigma(x) : \quad u = v = 0; \quad \theta = 1; \quad (6)$$

$$y = \infty : \quad u = v = \theta = 0; \quad p = p_\infty. \quad (7)$$

Next, defining the stream function, ψ , such that

$$u = \frac{1}{r} \frac{\partial \psi}{\partial y}, \quad v = -\frac{1}{r} \frac{\partial \psi}{\partial x}, \quad (8)$$

where $r(x) = x \sin \phi$ and a transformation of the independent variables given by

$$\hat{x} = x, \quad \hat{y} = y - \sigma(x). \quad (9)$$

Equations (3)–(5) become

$$\frac{1}{r^2} \left\{ \frac{\partial \psi}{\partial \hat{y}} \frac{\partial^2 \psi}{\partial x \partial \hat{y}} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial \hat{y}^2} - \frac{r_x}{r} \left(\frac{\partial \psi}{\partial \hat{y}} \right)^2 \right\} = -\frac{\partial p}{\partial x} + \sigma_x \frac{\partial p}{\partial \hat{y}} + \frac{1}{\sqrt{\text{Gr}}} \frac{1}{r}$$

$$\times \left\{ (1 + \sigma_x^2) \frac{\partial^3 \psi}{\partial \hat{y}^3} + \frac{\partial^3 \psi}{\partial x^2 \partial \hat{y}} - 2\sigma_x \frac{\partial^3 \psi}{\partial x \partial \hat{y}^2} - \sigma_{xx} \frac{\partial^2 \psi}{\partial \hat{y}^2} \right.$$

$$\left. + 2 \frac{r_x}{r} \left(\frac{r_x}{r} \frac{\partial \psi}{\partial \hat{y}} - \frac{\partial^2 \psi}{\partial x \partial \hat{y}} + \sigma_x \frac{\partial^2 \psi}{\partial \hat{y}^2} \right) \right\} + \theta, \quad (10)$$

$$\begin{aligned}
 & \frac{1}{r^2} \left\{ \frac{\partial\psi}{\partial\hat{y}} \frac{\partial^2\psi}{\partial x^2} - \sigma_{xx} \left(\frac{\partial\psi}{\partial\hat{y}} \right)^2 - \frac{\partial\psi}{\partial x} \frac{\partial^2\psi}{\partial x\partial\hat{y}} \right. \\
 & \quad \left. + \sigma_x \left\{ \frac{\partial\psi}{\partial x} \frac{\partial^2\psi}{\partial\hat{y}^2} - \frac{\partial\psi}{\partial\hat{y}} \frac{\partial^2\psi}{\partial x\partial\hat{y}} \right\} - \frac{r_x}{r} \left[\frac{\partial\psi}{\partial x} \frac{\partial\psi}{\partial\hat{y}} - \sigma_x \left(\frac{\partial\psi}{\partial\hat{y}} \right)^2 \right] \right\} \\
 & = \frac{\partial p}{\partial\hat{y}} + \frac{1}{\sqrt{\text{Gr}}} \frac{1}{r} \left\{ \frac{\partial^3\psi}{\partial x^3} - 3\sigma_x \frac{\partial^3\psi}{\partial x^2\partial\hat{y}} + 3\sigma_x^2 \frac{\partial^3\psi}{\partial x\partial\hat{y}^2} \right. \\
 & \quad - 3\sigma_{xx} \frac{\partial^2\psi}{\partial x\partial\hat{y}} - \sigma_{xxx} \frac{\partial\psi}{\partial\hat{y}} + 3\sigma_x\sigma_{xx} \frac{\partial^2\psi}{\partial\hat{y}^2} \\
 & \quad - \sigma_x(1 + \sigma_x^2) \frac{\partial^3\psi}{\partial\hat{y}^3} + \frac{\partial^3\psi}{\partial x\partial\hat{y}^2} + 2\frac{r_x^2}{r^2} \left(\frac{\partial\psi}{\partial x} - \sigma_x \frac{\partial\psi}{\partial\hat{y}} \right) \\
 & \quad \left. - 2\frac{r_x}{r} \left(\frac{\partial^2\psi}{\partial x^2} - 2\sigma_x \frac{\partial^2\psi}{\partial x\partial\hat{y}} - \sigma_{xx} \frac{\partial\psi}{\partial\hat{y}} + \sigma_x^2 \frac{\partial^2\psi}{\partial\hat{y}^2} \right) \right\} + \tan\phi\theta, \tag{11}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{r} \left\{ \frac{\partial\psi}{\partial\hat{y}} \frac{\partial\theta}{\partial x} - \frac{\partial\psi}{\partial x} \frac{\partial\theta}{\partial\hat{y}} \right\} \\
 & = \frac{1}{\text{Pr}} \frac{1}{\sqrt{\text{Gr}}} \left\{ (1 + \sigma_x^2) \frac{\partial^2\theta}{\partial\hat{y}^2} + \frac{\partial^2\theta}{\partial x^2} - 2\sigma_x \frac{\partial^2\theta}{\partial x\partial\hat{y}} - \sigma_{xx} \frac{\partial\theta}{\partial\hat{y}} \right\}. \tag{12}
 \end{aligned}$$

The boundary conditions, equations (6) and (7), become

$$\hat{y} = 0 : \quad \psi = 0; \quad \frac{\partial\psi}{\partial\hat{y}} = 0; \quad \theta = 1; \tag{13}$$

$$\hat{y} = \infty : \quad \frac{\partial\psi}{\partial\hat{y}} = 0; \quad \frac{\partial\psi}{\partial x} = 0; \quad \theta = 0; \quad p = p_\infty. \tag{14}$$

If we now introduce the boundary layer variables,

$$\psi = \frac{1}{\text{Gr}^{1/4}} \hat{\psi}; \quad Y = \text{Gr}^{1/4} \hat{y}, \tag{15}$$

and taking the limit as Gr approaches to infinity, i.e., we apply the boundary layer approximation, equations (10)–(12) become

$$\begin{aligned}
 & \frac{1}{r^2} \left\{ \frac{\partial\hat{\psi}}{\partial Y} \frac{\partial^2\hat{\psi}}{\partial x\partial Y} - \frac{\partial\hat{\psi}}{\partial x} \frac{\partial^2\hat{\psi}}{\partial Y^2} - \frac{r_x}{r} \left(\frac{\partial\hat{\psi}}{\partial Y} \right)^2 \right\} \\
 & = -\frac{\partial p}{\partial x} + \text{Gr}^{1/4} \sigma_x \frac{\partial x}{\partial Y} + \frac{1}{r} (1 + \sigma_x^2) \frac{\partial^3\hat{\psi}}{\partial Y^3} + \theta, \tag{16}
 \end{aligned}$$

$$\begin{aligned} & \frac{1}{r^2} \left\{ \sigma_x \left(\frac{\partial \hat{\psi}}{\partial x} \frac{\partial^2 \hat{\psi}}{\partial Y^2} - \frac{\partial \hat{\psi}}{\partial Y} \frac{\partial^2 \hat{\psi}}{\partial x \partial Y} \right) - \sigma_{xx} \left(\frac{\partial \hat{\psi}}{\partial Y} \right)^2 + \frac{r^2}{r} \sigma_x \left(\frac{\partial \hat{\psi}}{\partial Y} \right)^2 \right\} \\ & = \text{Gr}^{1/4} \frac{\partial p}{\partial Y} - \sigma_x (1 + \sigma_x^2) \frac{1}{r} \frac{\partial^3 \hat{\psi}}{\partial Y^3} + \tan \phi \theta, \end{aligned} \quad (17)$$

$$\frac{1}{r} \left\{ \frac{\partial \hat{\psi}}{\partial Y} \frac{\partial \theta}{\partial x} - \frac{\partial \hat{\psi}}{\partial x} \frac{\partial \theta}{\partial Y} \right\} = \frac{1}{\text{Pr}} (1 + \sigma_x^2) \frac{\partial^2 \theta}{\partial Y^2}. \quad (18)$$

Strictly speaking, this asymptotic analysis is valid only within the framework of the boundary layer scalings, $r = O(\text{Gr}^{-1/4})$ and $a = O(\text{Gr}^{-1/4})$ as $\text{Gr} \rightarrow \infty$, which resulted from (9) and (15).

In order to eliminate the term $\text{Gr}^{1/4} \sigma_x \partial p / \partial Y$, we multiply equation (17) by $-\sigma_x$ and we add the resulting equation to equation (16), which gives

$$\begin{aligned} & \frac{1}{r^2} (1 + \sigma_x^2) \left\{ \frac{\partial \hat{\psi}}{\partial Y} \frac{\partial^2 \hat{\psi}}{\partial x \partial Y} - \frac{\partial \hat{\psi}}{\partial x} \frac{\partial^2 \hat{\psi}}{\partial Y^2} \right\} \\ & + \frac{1}{r^2} \sigma_x \sigma_{xx} \left(\frac{\partial \hat{\psi}}{\partial Y} \right)^2 - \frac{r_x}{r^3} (1 + \sigma_x^2) \left(\frac{\partial \hat{\psi}}{\partial Y} \right)^2 \\ & = -\frac{\partial p}{\partial x} + \frac{1}{r} (1 + \sigma_x^2)^2 \frac{\partial^3 \hat{\psi}}{\partial Y^3} + (1 - \sigma_x \tan \phi) \theta. \end{aligned}$$

The lowest-order pressure gradient along the x -direction is determined from the inviscid solution and can be seen to give $\partial p / \partial x = 0$. Thus, we finally get

$$\begin{aligned} & \frac{1}{r^2} \left\{ \frac{\partial \hat{\psi}}{\partial Y} \frac{\partial^2 \hat{\psi}}{\partial x \partial Y} - \frac{\partial \hat{\psi}}{\partial x} \frac{\partial^2 \hat{\psi}}{\partial Y^2} + \frac{\sigma_x \sigma_{xx}}{1 + \sigma_x^2} \left(\frac{\partial \hat{\psi}}{\partial Y} \right)^2 - \frac{r_x}{r} \left(\frac{\partial \hat{\psi}}{\partial Y} \right)^2 \right\} \\ & = \frac{1}{r} (1 + \sigma_x^2) \frac{\partial^3 \hat{\psi}}{\partial Y^3} + \frac{1 - \sigma_x \tan \phi}{1 + \sigma_x^2} \theta, \end{aligned} \quad (19)$$

$$\frac{1}{r} \left\{ \frac{\partial \hat{\psi}}{\partial Y} \frac{\partial \theta}{\partial x} - \frac{\partial \hat{\psi}}{\partial x} \frac{\partial \theta}{\partial Y} \right\} = \frac{1}{\text{Pr}} (1 + \sigma_x^2) \frac{\partial^2 \theta}{\partial Y^2}, \quad (20)$$

with the boundary conditions

$$Y = 0 : \quad \hat{\psi} = 0; \quad \frac{\partial \hat{\psi}}{\partial Y} = 0; \quad \theta = 1; \quad (21)$$

$$Y = \infty : \quad \frac{\partial \hat{\psi}}{\partial Y} = 0; \quad \theta = 0. \quad (22)$$

It is important to notice that we have restricted our attention in this paper to values of x , which take $O(1)$ values as Gr approaches infinity. For this range of

values of x , the boundary layer thickness is $O(\text{Gr}^{-1/4})$, which is much smaller than the $O(1)$ length scale associated with the transverse waves placed on the flat surface of the cone.

To reduce the equations to a form suitable for numerical solution, let us introduce the transformation

$$\eta = \frac{Y}{x^{1/4}}, \quad \hat{\psi} = x^{3/4} r f(x, \eta), \quad \theta = h(x, \eta), \quad r(x) = x \sin \phi. \quad (23)$$

Equations (19) and (20) then become

$$(1 + \sigma_x^2) f''' + \frac{7}{4} f f'' - \frac{1}{2} f'^2 + \frac{1 - \sigma_x \tan \phi}{1 + \sigma_x^2} h = x \left\{ f' \frac{\partial f'}{\partial x} - f'' \frac{\partial f}{\partial x} + \frac{\sigma_x \sigma_{xx}}{1 + \sigma_x^2} f'^2 \right\}, \quad (24)$$

$$\frac{1}{\text{Pr}} (1 + \sigma_x^2) h'' + \frac{7}{4} f h' = x \left\{ f' \frac{\partial h}{\partial x} - h' \frac{\partial f}{\partial x} \right\}, \quad (25)$$

and the boundary conditions, equations (21) and (22), now reduce to

$$\eta = 0 : \quad f = 0; \quad f' = 0; \quad h = 1; \quad (26)$$

$$\eta = \infty : \quad f' = 0; \quad h = 0. \quad (27)$$

The physical quantity of interest is the local Nusselt number defined as

$$\text{Nu} = \frac{\bar{x} q_w}{k(T_w - T_\infty)}, \quad (28)$$

where

$$q_w = -k \mathbf{n} \cdot \nabla T$$

and the vector

$$\mathbf{n} = \left\{ -\frac{\sigma_x}{\sqrt{1 + \sigma_x^2}}, \frac{1}{\sqrt{1 + \sigma_x^2}} \right\}$$

is the unit vector normal to the wavy surface of the cone. Thus, we have

$$\begin{aligned} q_w &= -k \left\{ -\frac{\sigma_x}{\sqrt{1 + \sigma_x^2}} \frac{\partial T}{\partial \bar{x}} + \frac{1}{\sqrt{1 + \sigma_x^2}} \frac{\partial T}{\partial \bar{y}} \right\}_{\bar{y}=\bar{\sigma}(\bar{x})} \\ &= -\frac{k(T_w - T_\infty)}{l} \left\{ -\frac{\sigma_x}{\sqrt{1 + \sigma_x^2}} \frac{\partial \theta}{\partial x} + \frac{1}{\sqrt{1 + \sigma_x^2}} \frac{\partial \theta}{\partial y} \right\}_{y=\sigma(x)} \end{aligned} \quad (29)$$

Since

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial \hat{x}} - \sigma_x \frac{\partial}{\partial \hat{y}},$$

one gets

$$\begin{aligned} q_w &= -\frac{k(T_w - T_\infty)}{l} \left\{ -\frac{\sigma_x}{\sqrt{1 + \sigma_x^2}} \frac{\partial \theta}{\partial \hat{x}} + \sqrt{1 + \sigma_x^2} \frac{\partial \theta}{\partial \hat{y}} \right\}_{\hat{y}=0} \\ &= -\frac{k(T_w - T_\infty)}{l} \sqrt{1 + \sigma_x^2} \left(\frac{\partial \theta}{\partial \hat{y}} \right)_{\hat{y}=0}, \end{aligned} \quad (30)$$

where $\partial \theta / \partial \hat{x} = 0$ at $\hat{y} = 0$. Using boundary layer variables, we get

$$q_w = -\frac{k(T_w - T_\infty)}{l} \sqrt{1 + \sigma_x^2} \left(\frac{\text{Gr}}{x} \right)^{1/4} \left(\frac{\partial h}{\partial \eta} \right)_{\eta=0}. \quad (31)$$

Then, the local Nusselt number defined by equation (28) becomes

$$\frac{\text{Nu}}{\text{Gr}^{1/4}} = -x^{3/4} \sqrt{1 + \sigma_x^2} \left(\frac{\partial h}{\partial \eta} \right)_{\eta=0}. \quad (32)$$

3. Results and discussion

Equations (24) and (25), subject to the boundary conditions (26) and (27), have been solved numerically using an implicit finite-difference scheme developed by Keller and Cebeci [5]. Since a good description of this method and its application to boundary layer flow problems are given in [8–10], it will not be presented here. For the problem under consideration, i.e., $\sigma(x) = a \sin \pi x$, the differential equations for the solution of the starting computation at $x = 0$ can be obtained by putting $x = 0$ in equations (24) and (25), which become

$$(1 + a^2 \pi^2) f''' + \frac{7}{4} f f'' - \frac{1}{2} f'^2 + \frac{1 - a \pi \tan \Phi}{1 + a^2 \pi^2} h = 0, \quad (33)$$

$$\frac{1}{\text{Pr}} (1 + a^2 \pi^2) h'' + \frac{7}{4} f h' = 0 \quad (34)$$

subject to the boundary conditions

$$f(0) = f'(0) = 0; \quad h(0) = 1, \quad (35)$$

$$f'(\infty) = h(\infty) = 0, \quad (36)$$

where primes denote differentiation with respect to η .

Solutions are generated with a variable step-size in the η -direction

$$\Delta\eta_0 = 0.01; \quad \eta_j = \eta_{j-1} * m; \quad m = 1.01.$$

The edge of the boundary layer is reached when both values of f' and h approach to zero asymptotically (i.e., not only their values but also their derivatives approach zero). For other values of x , a constant step-size Δx of 0.05 is used.

The variation of the local Nusselt number, Nu , with x is presented in Figs 2 to 5 for the values of the Prandtl number $Pr = 0.72$ (air) and 6.7 (water), cone apex half-angle $\Phi = 10^\circ, 20^\circ$ and 40° and the amplitude of the sinusoidal wave surfaces $a = 0$ (flat cone), 0.15 and 0.20, respectively. The solutions are presented for x values from 0 to 4, which correspond to two complete cycles of the sinusoidal wave surfaces, as is illustrated in Fig. 1.

Figures 2 and 3 show the effect of the amplitude a on the local Nusselt number for two values of the Prandtl number $Pr = 0.72$ and 6.7, respectively. It is seen that in general the value of the local Nusselt number is lower for a wavy cone ($a \neq 0$) than that of the corresponding flat cone ($a = 0$); this may be explained as follows. When the heated surface of the cone is not plane, the component of the buoyancy force along the cone is reduced by a factor $(1 - \sigma_x \tan \Phi)/(1 + \sigma_x^2)$, as shown in equation (24), from its maximum value of the flat cone. Consequently, the boundary layer thickness is locally thicker, and hence local rates of heat transfer at the cone surface are reduced. It is also seen that the variation of the local Nusselt number is periodic in the direction of x and increases with x . Further, we notice that the change in the local Nusselt number is more pronounced for larger values of the amplitude a .

The effect of the cone half-angle Φ is shown in Figs 4 and 5 for $a = 0.15$ and for the same values of Pr . Inspection of these figures shows that the effect of Φ on the local Nusselt number distribution alternates in the direction of x . Also, as expected the values of the local Nusselt number are higher for larger values of Pr , as is seen from Figs 2–5.

4. Conclusions

The non-similar boundary layer analysis of steady laminar free convection along a vertical wavy cone is studied numerically. Numerical results have been obtained for a sinusoidal wavy surface of the cone. This is a model problem for the investigation of heat transfer from a roughened surface in order to understand heat transfer enhancement.

Based on this investigation of the laminar boundary layer behavior, the following conclusions can be drawn:

- The local Nusselt number varies periodically in the direction of x and increases with x .

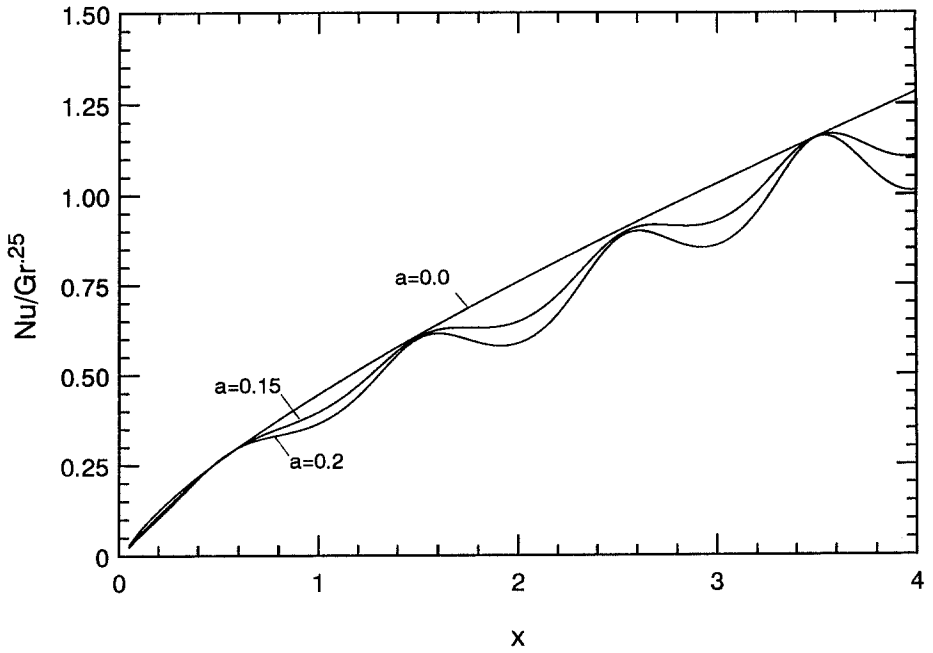


Fig. 2. Variation of the local Nusselt number with x for $a = 0, 0.15$ and 0.2 , $\Phi = 10^\circ$ and $Pr = 0.72$.

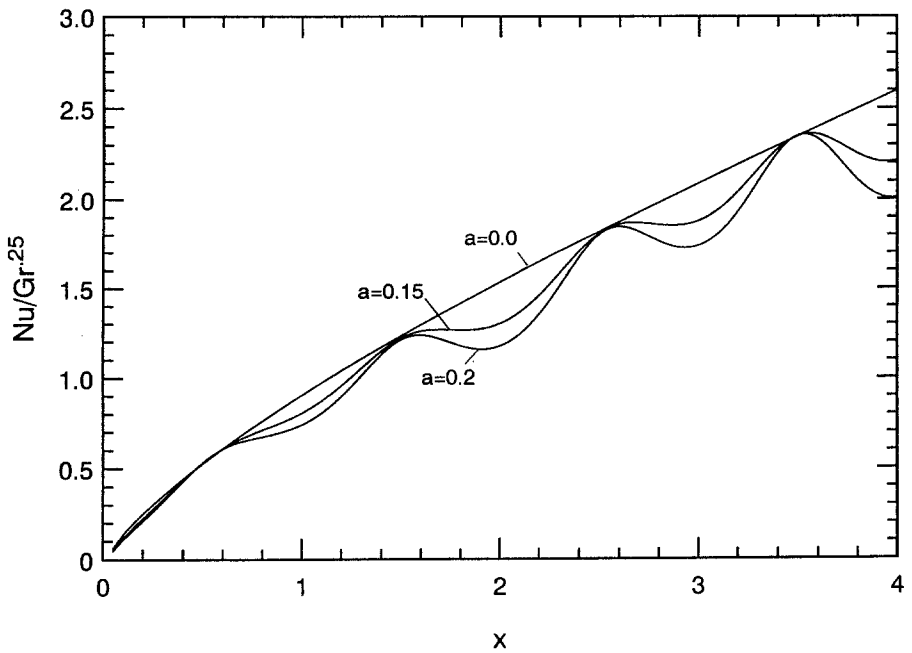


Fig. 3. Variation of the local Nusselt number with x for $a = 0, 0.15$ and 0.2 , $\Phi = 10^\circ$ and $Pr = 6.7$.

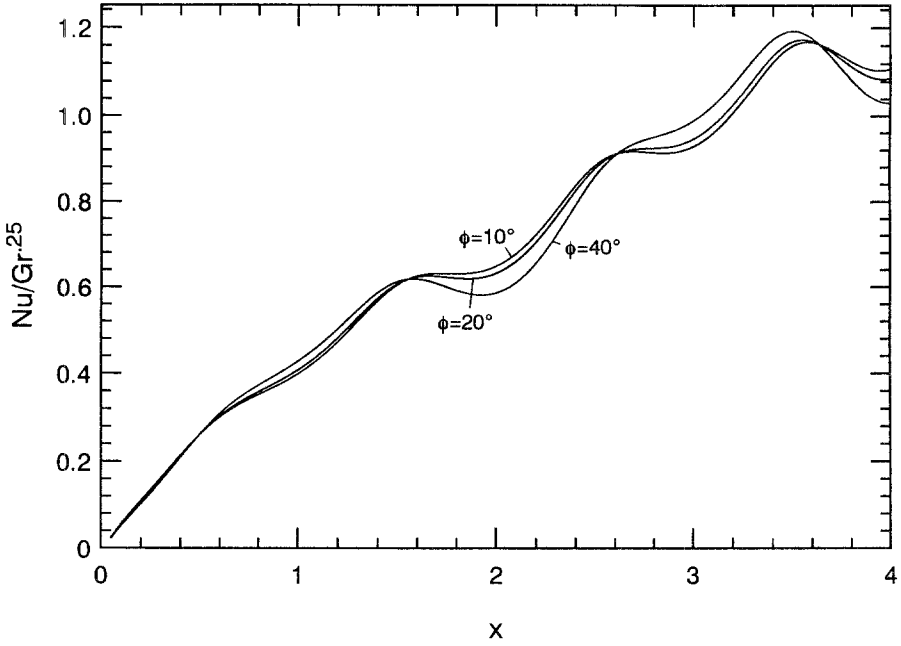


Fig. 4. Variation of the local Nusselt number with x for $\Phi = 10^\circ, 20^\circ$ and 40° ; $a = 0.15$ and $Pr = 0.72$.

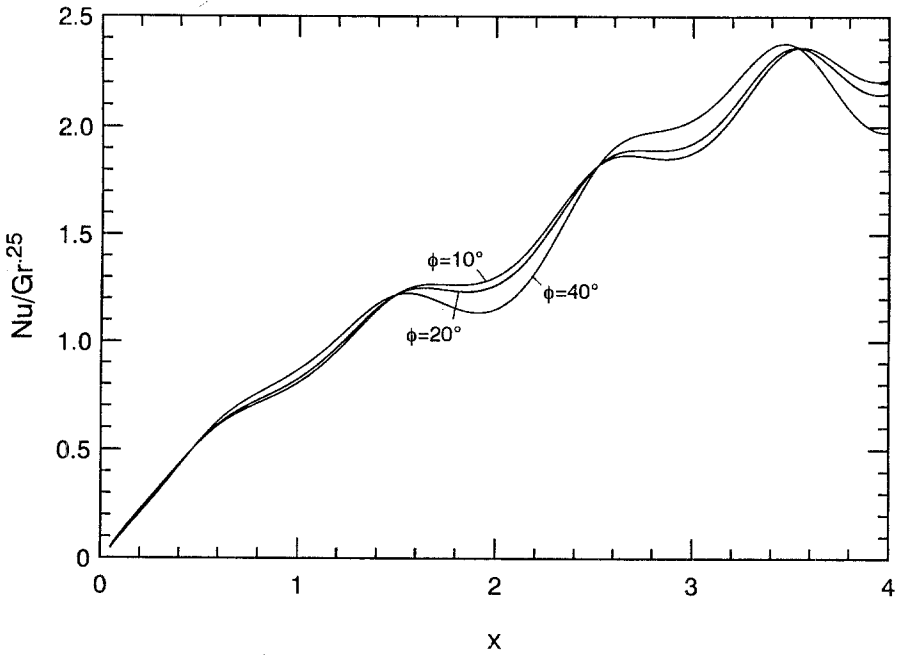


Fig. 5. Variation of the local Nusselt number with x for $\Phi = 10^\circ, 20^\circ$ and 40° ; $a = 0.15$ and $Pr = 6.7$.

- The local Nusselt number is smaller for a wavy cone ($a \neq 0$) than that corresponding to a flat cone ($a = 0$).
- The changes of the local Nusselt number are more pronounced for larger values of the amplitude of the wavy surfaces of the cone.
- The values of the local Nusselt number are higher for larger values of the Prandtl number.

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