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INDUSTRY PROGRAM OF THE COLLEGE OF ENGINEERING

THE RADIAL HEAT FLUX

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## LIST OF FIGURES

<u>Figure</u>		<u>Page</u>
1	Radial Heat Flux for Uniform Heat Flux at the Wall...	10
2	Radial Heat Flux Density for Uniform Heat Flux at the Wall.....	11
3	Radial Heat Flux for Laminar Flow and Constant Wall Temperature.....	12
4	Radial Heat Flux for Slug Flow and Constant Wall Temperature.....	13
5	Radial Heat Flux for Turbulent Flow and Constant Wall Temperature.....	14
6	Radial Heat Flux for Fully Developed Flow and Constant Wall Temperature.....	15

## INTRODUCTION

The shear stress is known to decrease linearly from the wall to the axis of a pipe. The radial heat flux density which is in some ways an analogous quantity varies much more complexly as will be shown.

An energy balance over a differential ring within a fluid in fully developed flow in a tube can be written in terms of local, time-mean variables as follows:

$$\frac{1}{r} \frac{\partial}{\partial r} (rq) = \rho c \frac{\partial T}{\partial z} - \frac{\partial}{\partial z} \left( k_z \frac{\partial T}{\partial z} \right) \quad (1)$$

where  $q$  is the radial heat flux density in BTU/(hr)(sq ft). The other variables are defined at the end of the paper.

For simplicity physical properties will be assumed constant and  $k_z$  will be assumed to be negligibly small in all subsequent derivations. The latter assumption has been shown by Schneider<sup>(1)</sup> to be reasonable for most circumstances.

Integration of Equation (1) from the axis to any radius gives

$$qr = \rho c \int_0^r u \frac{\partial T}{\partial z} r dr \quad (2)$$

and

$$\frac{Q}{Q_w} = \frac{qr}{q_w r_w} = \frac{\int_0^r u \frac{\partial T}{\partial z} d(r^2)}{\int_0^{r_w} u \frac{\partial T}{\partial z} d(r^2)} = 1 - \frac{\int_w^r u \frac{\partial T}{\partial z} d(r^2)}{\int_0^{r_w} u \frac{\partial T}{\partial z} d(r^2)} \quad (3)$$

where  $Q$  is the radial heat flux, in BTU/hr, and the subscript  $w$  designates the wall. The first integral form is more convenient near the center of the tube and the second form near the wall.

The radial heat flux, the radial heat flux density and the above ratio can therefore be obtained by integration if only the local longitudinal temperature gradient and the local velocity are known as functions of radius.

If the rate of radial transfer of energy by diffusion and eddy motion is expressed in terms of an effective radial conductivity

$$k_t = q / \left( \frac{\partial T}{\partial r} \right) \quad (4)$$

the temperature distribution can be obtained by one integration

$$T_w - T = \int_r^{r_w} \frac{q \, dr}{k_t} = \frac{q_w r_w}{k} \int_{\frac{r}{r_w}}^1 \left( \frac{q}{q_w} \right) \left( \frac{k}{k_t} \right) d\left( \frac{r}{r_w} \right) \quad (5)$$

and the mixed mean temperature,  $T_m$ , and the Nusselt number by another:

$$T_w - T_m = \int_0^1 \left( \frac{u}{u_m} \right) (T_w - T) d\left( \frac{r}{r_w} \right)^2 = \frac{q_w r_w}{k} \int_0^1 \left( \frac{u}{u_m} \right) \left[ \int_{\frac{r}{r_w}}^1 \left( \frac{q}{q_w} \right) \left( \frac{k}{k_t} \right) d\left( \frac{r}{r_w} \right) \right] d\left( \frac{r}{r_w} \right)^2 \quad (6)$$

$$Nu = \frac{2hr_w}{k} = \frac{2r_w q_w}{(T_w - T_m)k} = \frac{2}{\int_0^1 \left( \frac{u}{u_m} \right) \left[ \int_{\frac{r}{r_w}}^1 \left( \frac{q}{q_w} \right) \left( \frac{k}{k_t} \right) d\left( \frac{r}{r_w} \right) \right] d\left( \frac{r}{r_w} \right)^2} \quad (7)$$

The local Nusselt number can thus be determined from the radial heat flux and velocity distribution if the effective conductivity is also known as a function of radius.

Analytical solutions and detailed experimental data for heat transfer are primarily restricted to two boundary conditions: uniform wall temperature and uniform heat flux density. These two cases will be considered in detail.

Uniform Heat Flux Density at the Wall

Seban and Shimazaki<sup>(2)</sup> asserted that for a fully developed thermal boundary layer,  $h$  and  $(T - T_m)/(T_w - T_m)$  are invariant with length. For a uniform heat flux density at the wall, it follows that  $(T_w - T_m)$  is invariant, and that  $\frac{\partial T}{\partial z} = \frac{\partial T_m}{\partial z}$  and is also invariant. For this case Equation (3) reduces to

$$\frac{Q}{Q_w} = \int_0^{\frac{r}{r_w}} \left(\frac{u}{u_m}\right) d\left(\frac{r}{r_w}\right)^2 \quad (8)$$

and the integral depends only on the velocity distribution. The results obtained for several important cases follow:

1. Slug Flow:  $\frac{u}{u_m} = 1$

$$\frac{Q}{Q_w} = \left(\frac{r}{r_w}\right)^2, \quad \frac{q}{q_w} = \frac{r}{r_w} \quad (9)$$

For  $k/k_t = 1$  integration of Equation (7) then yields the well known result,  $Nu = 8$ .

2. Laminar Flow:  $\frac{u}{u_m} = 2 \left[ 1 - \left(\frac{r}{r_w}\right)^2 \right]$

$$\frac{Q}{Q_w} = 2 \left(\frac{r}{r_w}\right)^2 \left[ 1 - \frac{1}{2} \left(\frac{r}{r_w}\right)^2 \right], \quad \frac{q}{q_w} = 2 \frac{r}{r_w} \left[ 1 - \frac{1}{2} \left(\frac{r}{r_w}\right)^2 \right] \quad (10)$$

For  $\frac{k}{k_t} = 1$ , Equation (7) yields another well known result,  $Nu = 48/11$ .

3. Turbulent flow:  $\frac{u}{u_m} = f \left[ Re, \frac{e}{r_w}, \frac{r}{r_w} \right]$ .

The integral in Equation (8) can be evaluated graphically using the generalized graphical correlations which have been developed for the turbulent velocity distribution<sup>(3)</sup>. The results for Re of  $4.0 \times 10^3$  and  $2.35 \times 10^6$  in smooth pipe and Re of  $2.5 \times 10^6$  in rough pipe ( $r_w/e = 15$ ) are illustrated in Figure 1. The curves for slug and laminar flow are also included. As could be anticipated the curves for turbulent flow lie between those for laminar and slug flow. Since these curves depend only on the velocity distribution they should be valid for any fluid.

Although the heat flux ratio is a well behaved function for all of the illustrated cases, the heat flux density ratio as illustrated in Figure 2 actually goes through a maximum near the wall for laminar and turbulent flow. This maximum results from the competing effects of decreasing area and longitudinal transfer which increase and decrease, respectively, the heat flux density as the radius decreases.

The integration for Nu has been carried out for smooth pipe by a number of investigators, all of whom assumed  $k/k_t = 1 + \alpha Pr \left( \frac{\epsilon}{\nu} \right)$  with a constant  $\alpha$  and used values of  $\epsilon/\nu$  obtained directly or indirectly from experimental velocity distribution. Empirical equations were used for  $\epsilon/\nu$  and analytical solutions were obtained by von Karman<sup>(4)</sup> and Deissler<sup>(5)</sup> who assumed  $q/q_w$  constant, and by Martinelli<sup>(6)</sup> who assumed  $q/q_w = r/r_w$ . A graphical representation for the velocity distribution was used by

Lyon<sup>(7)</sup>, who carried out the equivalent to the integrations in Equation (7) numerically for several values of Pr less than 0.1, and obtained a graphical correlation which can be represented approximately by the equation

$$\text{Nu} = 7.0 + 0.25 (\alpha \text{ Pr Re})^{0.8} \quad (11)$$

Equation (7) could be integrated rigorously for the entire range of conditions for which experimental velocity and  $\alpha$  distributions are available but the results would not be expected to differ significantly from the above approximate results since the integrations are an effective smoothing operation.

By an alternative procedure utilizing an analogue computer and not explicitly involving the radial heat flux, Sleicher<sup>(8)</sup> obtained the first three coefficients, eigenvalues and eigenfunctions in series solution for the temperature field as well as for Nu. The ratio of  $\alpha/\alpha_{\text{air}}$  derived by Jenkins and experimental values of  $\alpha \epsilon/\nu$  for air were utilized in the calculations. Sleicher's results could be utilized to integrate Equation (3) for a developing thermal boundary layer insofar as the series converges in three terms. However the radial heat flux in the inlet region will instead be illustrated for uniform wall temperature for which more extensive experimental data are available.

#### Uniform Wall Temperature

For uniform wall temperature, invariance of  $(T_w - T)/(T_w - T_m)$  requires that

$$\frac{\partial T}{\partial z} = \left( \frac{T_w - T}{T_w - T_m} \right) \left( \frac{\partial T_m}{\partial z} \right) \quad (12)$$



Hence for a fully developed thermal boundary layer Equation (3) can be reduced to

$$\frac{Q}{Q_w} = \frac{\int_0^r (T_w - T) u \, d(r^2)}{\int_0^{r_w} (T_w - T) u \, d(r^2)} \quad (13)$$

Thus the radial distribution of the longitudinal temperature gradient is required for a developing thermal boundary layer but only the radial temperature distribution for a fully developed thermal boundary layer. Again several important cases will be considered in detail:

1. Slug Flow with  $k/k_t = 1.0$

An analytical solution for the temperature distribution was obtained by Graetz<sup>(9)</sup> as a function of  $r/r_w$  and  $wc/kL$  only in the form of an infinite series. Since only the first several eigenvalues and eigenfunctions in the series have been computed the usefulness of the solution is limited to values of  $wc/kL$  for which only the corresponding values of the series contribute. Equation (3) was integrated graphically for several values of  $wc/kL$  in this range and Equation (13) for  $wc/kL = 0$ . The results are shown in Figure 4.

2. Laminar Flow with  $k/k_t = 1.0$

A series solution analogous to that for slug flow was also accomplished for parabolic flow by Graetz. The results computed from this solution are shown in Figure 3.

3. Turbulent Flow

The temperature field in turbulent flow is a function of  $Re$ ,  $Pr$  and  $e/r_w$ . Experimental values for a developing temperature field have apparently been reported only by Abbrecht<sup>(10)</sup> for air in smooth pipe at

Re of 15,000 and 65,000. The values of the radial heat flux ratio computed from the temperature data at Re = 15,000 are shown in Figure 5. The data of Sleicher<sup>(8)</sup> for the radial temperature distribution for the same conditions but at  $L/r_w$  of 62 were utilized in Equation (13) and the results are included in Figure 5. His theoretical results for  $L/r_w \rightarrow \infty$  match the experimental curve for  $L/r_w = 62$  very closely.

Seban and Shimazaki<sup>(2)</sup> integrated the equivalent of Equation (5) and (13) by reiteration assuming  $\alpha = 1$  and using experimental values of  $\epsilon/\nu$  for air in smooth pipe. They reported values of Nu for a range of conditions but the temperature distribution itself only for Re = 10,000 and Pr = 0.01. The radial heat flux ratio calculated from this distribution is compared with the values obtained from the Graetz solutions for  $w_c/kL = 0$  and with the values obtained from the theoretical results of Sleicher for  $L/r_w \rightarrow \infty$  in Figure 6. Contrary to Figure 1 for uniform wall flux, the heat flux ratios for turbulent flow fall below the curve for slug flow since eddy transfer of heat enters Equation (2) indirectly via  $\partial T/\partial z$ .

In principle the series solution of Sleicher could be used to carry out the integrations for the radial heat flux for any  $L/r_w$ ,  $\epsilon/r_w$ , Pr and Re, but such integrations are limited practically by the eigenvalues and eigenfunctions which he has computed.

Plots of  $q/q_w$  versus  $r/r_w$  (not shown) reveal a maximum near the wall for all  $L/r_w$  for laminar and turbulent flow.

### Conclusions

The radial heat flux provides a new basis for the interpretation and computation of heat transfer. The values of the heat flux ratio which

were computed for a variety of conditions do not yield to precise generalization but do permit interpolation with reasonable confidence. The heat flux density goes through a maximum near the wall for most conditions due to the competing effects of longitudinal transfer and changing radius, but the heat flux itself decreases monotonically from the wall to zero at the axis. The results provide a more accurate basis for calculations with eddy diffusivities than any of the idealizations previously utilized.

## NOMENCLATURE

- c = heat capacity, BTU/(lb)(°F)
- e = height of pipe roughness, feet
- h = local heat transfer coefficient, BTU/(hr)(sq.ft.)(°F)
- k = thermal conductivity, BTU/(hr)(ft)(°F)
- L = distance down pipe from start of heating, feet.
- $Nu = 2h r_w/k$
- $Pr = c\mu/k$
- Q = heat flux, BTU/(hr)
- q = heat flux density, BTU/(hr) (sq. ft.)
- $Re = 2 r_w \rho u_m/\mu$
- r = radial distance from axis, ft.
- T = temperature, °F
- u = local velocity, ft/hr
- w = mass flow rate, lb/hr
- z = distance down pipe, feet
- $\alpha$  = ratio of eddy diffusivities for heat and momentum transfer
- $\epsilon$  = eddy diffusivity for momentum transfer, ft<sup>2</sup>/hr
- $\nu = \mu/\rho$
- $\mu$  = viscosity, lb/(ft)(hr)
- $\rho$  = density, lb/(ft)<sup>3</sup>

## Subscripts

- m = mixed mean
- t = total
- w = wall
- z = longitudinal

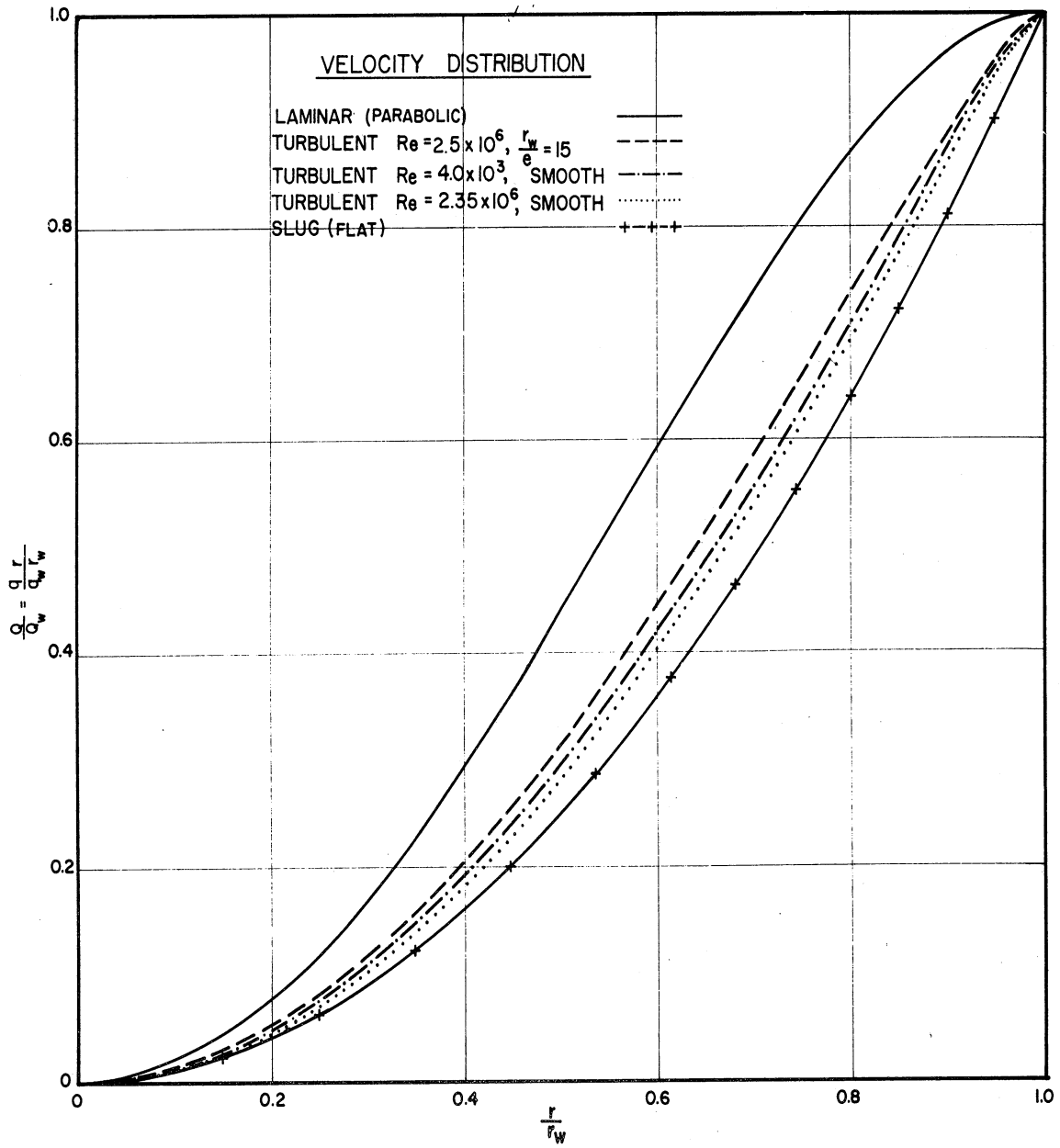


Figure 1. Radial Heat Flux for Uniform Heat Flux at the Wall.

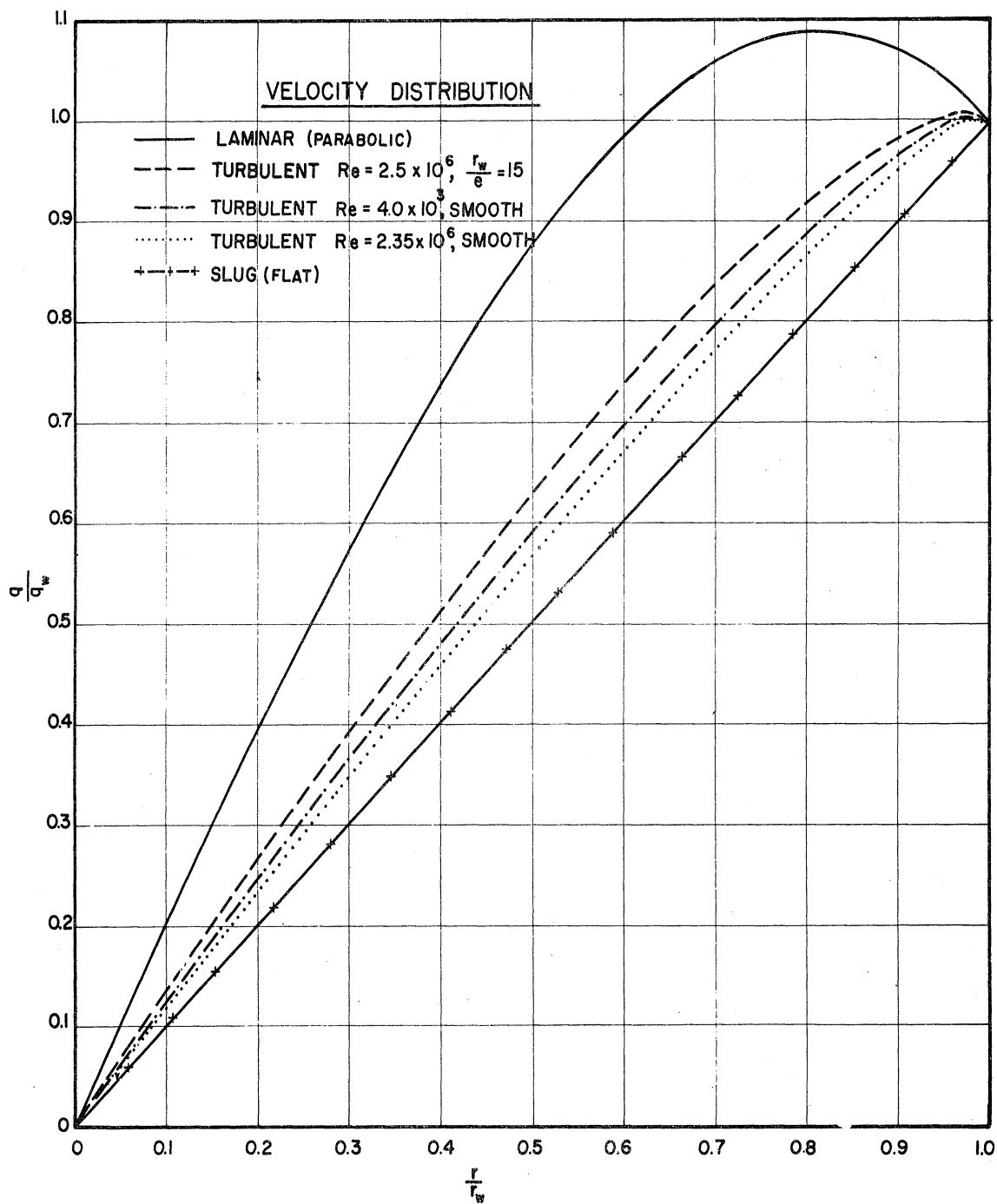


Figure 2. Radial Heat Flux Density for Uniform Heat Flux at the Wall.

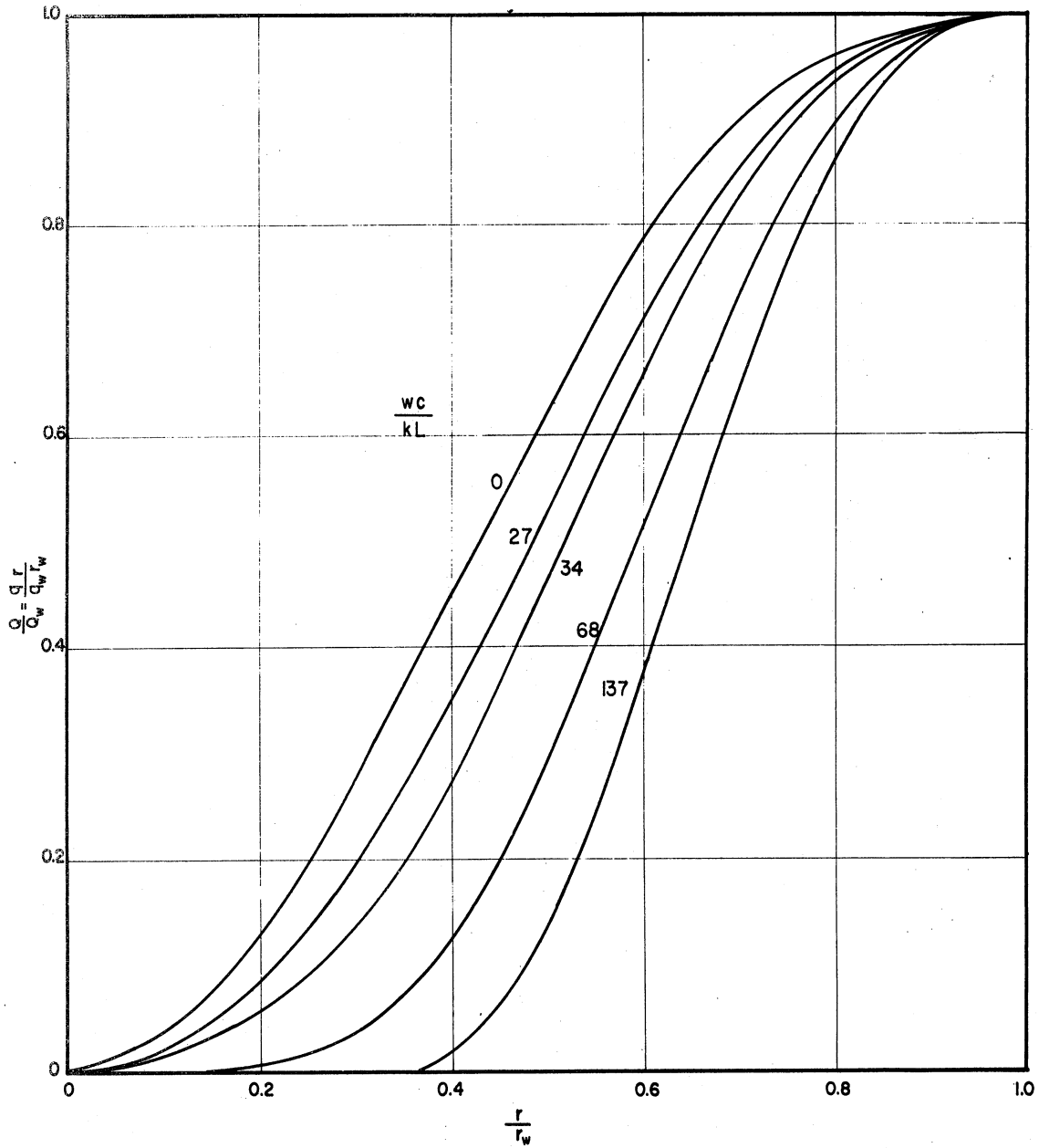


Figure 3. Radial Heat Flux for Laminar Flow and Constant Wall Temperature.

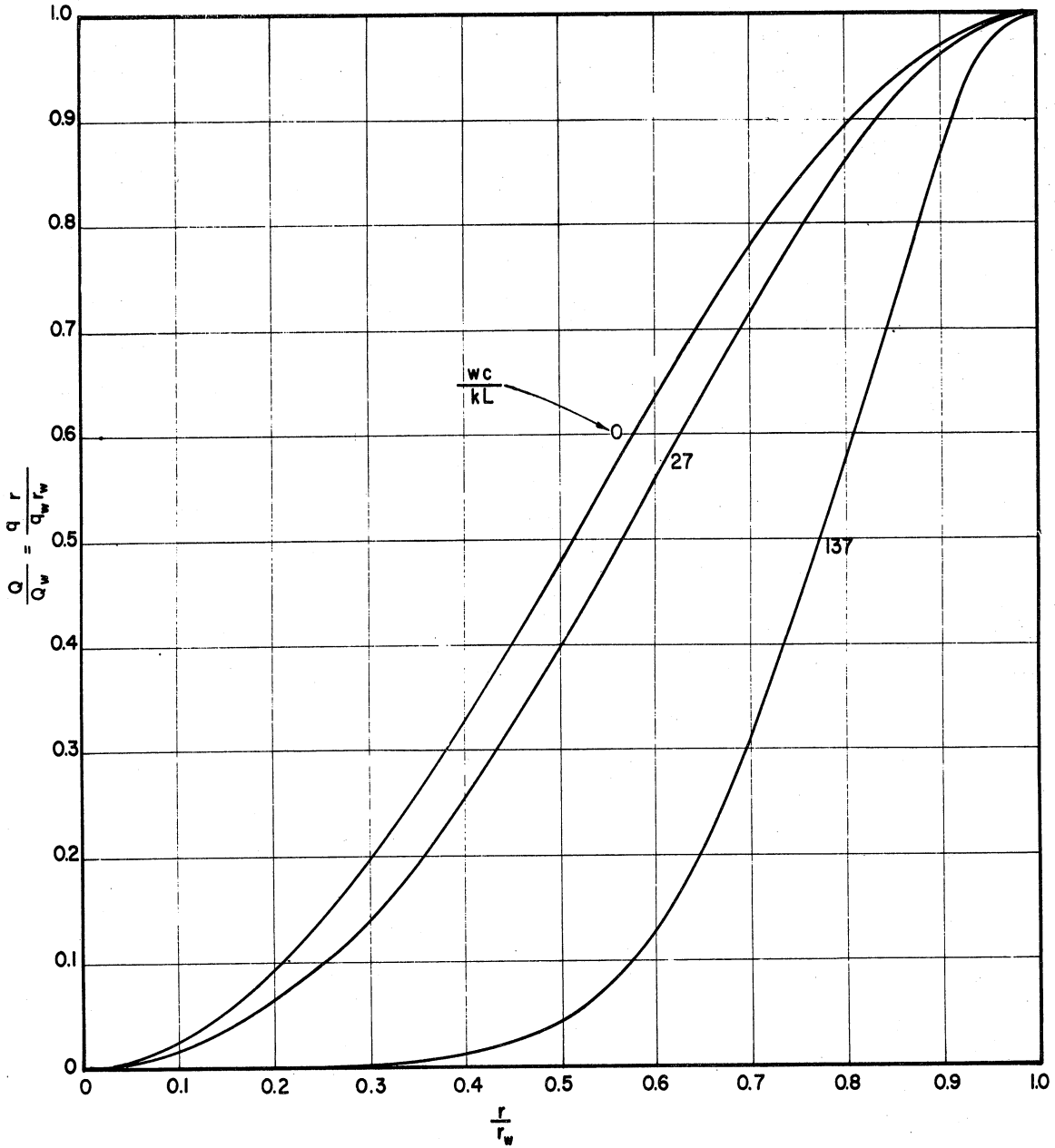


Figure 4. Radial Heat Flux for Slug Flow and Constant Wall Temperature.



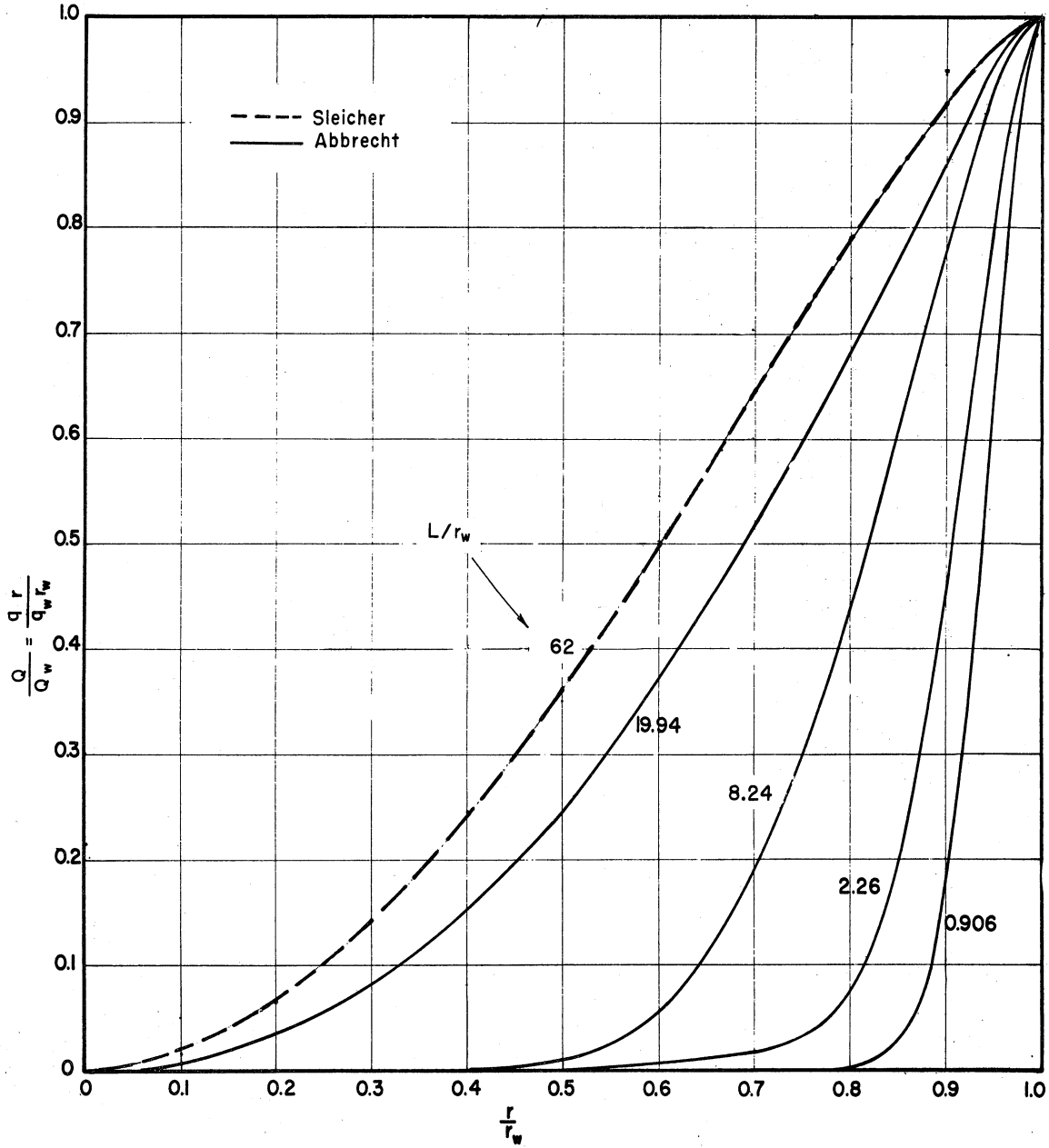


Figure 5. Radial Heat Flux for Turbulent Flow and Constant Wall Temperature.

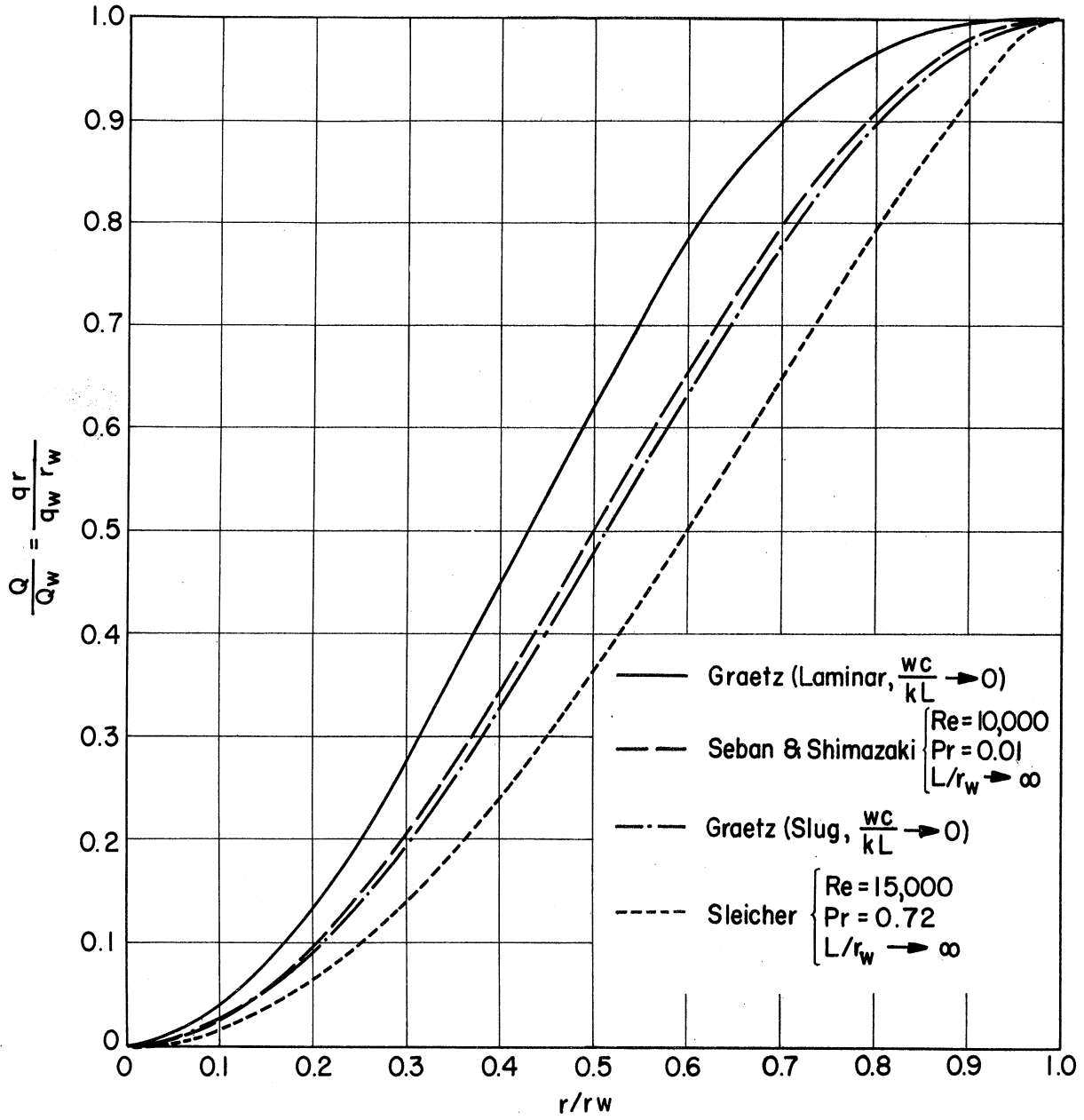


Figure 6. Radial Heat Flux for Fully Developed Flow and Constant Wall Temperature.

#### REFERENCES

1. Schneider, P. J., Trans. Amer. Soc. Mech. Engrs. 79, 765 (1957).
2. Seban, R. A., and Shimazaki, T. T., Trans. Amer. Soc. Mech. Engrs. 73, 803 (1951).
3. Bakhmeteff, B. A., The Mechanics of Turbulent Flow, Princeton University Press, Princeton, N. J. (1941).
4. von Karman, T., Trans. Amer. Soc. Mech. Engrs. 61, 705 (1939).
5. Deissler, R. G., Natl. Adv. Comm. Aeronaut. Tech Note 3145 (1954).
6. Martinelli, R. C., Trans. Amer. Soc. Mech. Engrs. 69, 947 (1947).
7. Lyon, R. N., Chem. Eng. Prog. 47, 75 (1951).
8. Sleicher, C. A., Ph.D. Thesis, University of Michigan, Ann Arbor (1955) and Trans. Amer. Soc. Mech. Engrs. 79, 789 (1957).
9. Jakob, Max, Heat Transfer, Vol. I, John Wiley and Sons, New York (1949).
10. Abbrecht, P. H., Ph.D. Thesis, University of Michigan, Ann Arbor (1956).

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