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PRESSURIZATION OF LIQUID OXYGEN CONTAINERS

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ABSTRACT

Three phases of the work are covered in this report, namely, heat transfer, self-pressurization, and rapid pressurization studies. The heat transfer may be quite accurately estimated by assuming an infinitely thin wall. The inside coefficient decreases with an increase in pressure. During self-pressurization the rate of increase of liquid temperature decreases after the saturation pressure is reached. Boil-off begins soon after rapid pressurization, and continues to increase until equilibrium is restored. The fraction of heat transfer to the liquid varies from 90 to 80% during this period.

I. INTRODUCTION

During the period November 15 to December 31 approximately 350 man hours were spent on this project. The heat-transfer studies were completed and their results are included here, as are the results of the completed study of the rate of self pressurization and heat transfer after rapid pressurization.

To expedite the work, one full-time employee, Mr. Philip Jackson, was added to the staff on November 10. The three students who are working part-time on this project have proved to be very satisfactory, and it is anticipated that, with Mr. Jackson working full-time, good progress will be achieved in the next six weeks. The apparatus is now being modified to permit a study of the amount of pressurizing gas required for actual discharge of the liquid nitrogen.

One significant change has been made in the apparatus since the last detailed report. A new tank was constructed with walls thinner than those of the original tank. This was done to approximate more closely the actual heat-transfer conditions.

One additional piece of apparatus has been utilized. A 14-point Consolidated engineering recording galvanometer, loaned to this project by the Aeronautical Laboratory, has been used to measure the liquid temperatures. Reference to this instrument is made in the body of the report. It appears that this instrument will be particularly useful in measuring temperatures during the emptying process. These tests were initiated January 1.

The sketch of the apparatus is shown in Fig. A. Figures B and C show the location of the various thermocouples for the two tanks. Several of the graphs which are included in this report refer to thermocouples by number, and these numbers are in accordance with Figs. B and C.

II. HEAT-TRANSFER STUDIES

ANALYTICAL STUDIES

In Progress Report No. 3 the results of an analytical study of the heat transfer to the liquid oxygen were reported. The appendix of the present report includes the derivation of the relations presented in Progress Report No. 3.

Since this report covers the relation of experimental studies on heat transfer to the analytical work, a brief summary of the analytical work is presented here. This study of heat transfer is considered to be basic to an under-

standing of the other phenomena associated with the pressurization of the LOX tank.

The attempt was made in the analytical studies to determine how the heat is transferred to the liquid, and three analyses were made. In Analysis I, it was assumed that the wall had no thermal resistance, and that the only resistance to heat transfer is the outside film coefficient. This analysis gives the maximum heat-transfer rate, and assumes that the heat transfer is independent of the level of the liquid in the tank. Quite obviously, this represents the maximum extreme. In this case,

$$q = q_o = h_o (t_a - t_w) \pi D_o L_o = \lambda \omega_e ,$$

where

- q = heat-transfer rate at any liquid level L ,
- q_o = heat-transfer rate when tank is full,
- h_o = outside film coefficient,
- t_a = ambient temperature,
- t_w = wall temperature,
- D_o = outside diameter of tank,
- L_o = length of tank,
- λ = enthalpy of evaporation, and
- ω_e = rate at which liquid is evaporated.

In Analysis II, the other extreme case was studied. This involved the assumption of an infinitely thin wall, and thus there could be no longitudinal conduction of heat down the wall. That is, heat could only be transferred to the liquid radially, and the heat-transfer area was the wall area in contact with the liquid. In this case,

$$q = h_o (t_a - t_w) \pi D_o L .$$

Analysis III is a more realistic approach to this problem, and this analysis takes into account what might be called the fin effect. That is, consideration is given to the fact that heat which is transferred from the ambient air to that portion of the outside wall which is above the liquid level can be transferred longitudinally down the tube wall and then to the liquid. Thus that portion of the wall above the liquid level acts as an extended surface or fin. In this calculation the thermal conductivity of the tank material and the wall thickness must be considered. In this case,

$$q = h_o \left(t_a - \frac{C}{N^2} \right) \left\{ \frac{\frac{N}{n^2} - \frac{1}{N}}{\coth NL + \frac{N}{n} \coth [n (L_o - L)]} + L \right\} \pi D_o ,$$

where

$$\frac{h_o}{KA} = n^2,$$

A = heat-transfer area for longitudinal conduction,

K = thermal conductivity of the wall material,

$$C = \frac{h_o t_a + h_i t_L}{KA}, \text{ and}$$

$$N^2 = \frac{h_o + h_i}{KA}.$$

The derivation for this equation is given in the appendix.

In Analysis III, it was assumed that there is no heat transfer from the wall above the liquid to vapor above the liquid, that there is no heat transfer from the vapor to the liquid, and that the outside coefficient is constant.

EXPERIMENTAL WORK

The experimental heat-transfer studies involved boil-off tests at atmospheric pressure and at 35 psig. In these tests the tank was filled with liquid nitrogen and both wall temperatures and liquid temperatures were measured. The boil-off rate was measured, and from these measurements the heat-transfer rate was calculated. At atmospheric pressure the boil-off rate was determined from the readings with the load cell, but at 35 psig the boil-off rate was determined from both the load cell and gas meter readings. (These readings were in good agreement.) The gas meter was not used on the atmospheric pressure runs because the drop in pressure across the gas meter caused the tank pressure to be somewhat above atmospheric.

Some data were obtained on the original tank, which had a wall thickness of 3/8 in., and these are also included in these results. The results are shown in the graphs which follow. In all runs the bottom of the tank was insulated with a 3 in. thickness of styrofoam. The tests for which graphs are shown are tests 5 and 6, and 17 and 18. The significant variables in these tests are:

<u>Test No.</u>	<u>Tank Wall Thickness, in.</u>	<u>Tank Pressure</u>
5	3/8	Atmospheric
6	3/8	35 psig
17	1/8	35 psig
18	1/8	Atmospheric

Figure 1 shows the boil-off rate, L/L_o , as a function of time for these four tests. Since the ambient conditions varied on the days on which these tests were run, an exact comparison cannot be made from this plot. However, it is significant that the liquid level drops more slowly at 35 psig (tests 6 and 17) than the comparable tests at atmospheric pressure. In drawing these curves

the starting point was arbitrarily taken as $L/L_0 = .6$, and the curves are relative to this point.

Figures 2 through 5 show the various heat-transfer coefficients as a function of liquid level. In these graphs the various heat-transfer coefficients are defined as follows:

$$h_o = \frac{Q}{A_o (t_a - t_w)}$$

Q = heat transfer at a given liquid level

$$U_o = \frac{Q}{A_o (t_a - t_L)}$$

A_o = outside area of tank

$$h_i = \frac{Q}{A_i (t_w - t_L)}$$

A_i = wetted wall area on the inside of the tank

$$U_i = \frac{Q}{A_i (t_a - t_L)}$$

t_a = ambient temperature

t_w = wall temperature below the level of the liquid

t_L = liquid temperature

The heat-transfer rates tend to increase at lower liquid levels primarily because of the end effects. Since the ambient conditions vary from run to run, the precise values of U are not particularly significant.

Perhaps the most significant point to be made regarding these graphs is the difference in h_i for the various runs. These values are approximately as follows:

Wall Thickness, in.	Pressure, psia	h_i , BTU/(ft ²)(hr)(°F)
3/8	14.7	360
3/8	50	220
1/8	14.7	260
1/8	50	125

Note that as the pressure increases from 14.7 to 50 psia the inside coefficient decreases significantly. The difference in the inside coefficient for the two wall thicknesses is believed to be primarily a matter of accuracy in measuring the difference between the wall temperature and the liquid temperature and therefore no conclusions are drawn regarding this point.

Figures 6 through 9 show q/q_o vs L/L_0 for the four tests. The results of the analytical analysis are also shown on these graphs. Note that the heat transfer is somewhat greater than is predicted by the thin wall analysis, but less than is predicted by the finned tube analysis.

Figures 10 through 13 show the difference between the wall temperature and the reference temperature as a function of liquid level. The rapid increase in

wall temperature as the liquid level drops is particularly significant. This indicates that the heat transfer from the ambient air to the wall above the liquid level is significantly less than to that portion of the wall which is below the liquid level, and also that there is a significant longitudinal heat transfer.

One other point to be made regarding Figs. 10 through 13 is that the longitudinal conduction down the tube wall may be calculated, because the slope of this curve represents the longitudinal temperature gradient. The points marked A on Fig. 6 were found by calculating the longitudinal conduction from Fig. 10, and adding this longitudinal heat transfer to that calculated by the thin-wall analysis. This indicates that the reason the curve of experimental points in Fig. 6 lies above the thin-wall analysis is because of longitudinal heat conduction down the tube wall.

To summarize, the significant conclusions from the heat-transfer studies are:

- 1) The heat-transfer rate decreases with an increase in pressure. This is primarily due to the decrease in the inside coefficient at the higher pressures.
- 2) A very significant temperature gradient exists in the tube wall at the liquid level. From these measurements the longitudinal heat transfer may be calculated, and this is found to be a fairly significant item.
- 3) The heat-transfer rate may be quite accurately predicted from the thin-wall analysis.

III. SELF-PRESSURIZATION TESTS

Tests were conducted to study the temperature, pressure, and boil-off rate as a function of time during self-pressurization. In this test the vent valve on the tank was closed. When the tank pressure reached 35 psig, the pressure regulator valve opened, and the pressure was maintained at 35 psig. Figures 14 and 15 show the results of test 11. It is significant to note from Fig. 14 that the liquid temperature is -307.6°F (compared to an equilibrium temperature of -297°F) when the tank reaches a pressure of 35 psig. It is also interesting to note how the rate of increase in liquid temperature decreases after the pressure of 35 psig is reached. This is apparently because after the pressure reaches 35 psig all the vapor formed leaves the tank. Therefore, that fraction of heat input utilized in evaporating the liquid is not available for increasing the liquid temperature. Referring to Fig. 15, it should be noted that three distinct regions of the total weight vs time curve can be identified. For the first 4.8 minutes, the weight remains constant, for there is no vapor leaving.

Between 4.8 and 12.2 minutes the mass in the tank decreases because all the vapor formed leaves the tank. However, this rate of decrease is much less than after the equilibrium temperature is reached at 12.2 minutes. The reason is that after the equilibrium temperature is reached all the heat input is used for evaporation, whereas between 4.8 and 12.2 minutes some of the heat input is used in increasing the energy of the liquid, which causes the increase in liquid temperature. Figure 15 also shows the fraction of the total heat input which is utilized in heating the liquid.

It should be noted that the time required to reach 35 psig is a function of the amount of ullage. The results shown in Figs. 14 and 15 are for 90 lb of liquid N² in the tank which corresponds to about 20% ullage.

IV. RAPID PRESSURIZATION TESTS

The next tests conducted involved the rapid pressurization of the tank with nitrogen and helium at room temperature. These tests were done at three different liquid levels in the tank. The purpose of these tests was to determine boil-off rate and temperatures as a function of time. The results of these tests are shown in Figs. 16 through 18. In the tests represented by Figs. 17 and 18, the temperatures were measured with the 14-point recording galvanometer which was referred to in the introduction. Figures 16 and 17 represent identical tests, but the Sanborn was used to record the gas space temperature in Fig. 16. There is a significant difference in the gas space temperature vs time for these two runs. The exact reason for this is not clear, and will be investigated further. Figures 17 and 18 also show two liquid temperatures, the wall temperature and the gas space temperature.

The following points are particularly significant regarding these tests.

- 1) The boil-off rate increases with time at a fairly steady rate until just before equilibrium is reached, at which point the boil-off rate increases very rapidly.
- 2) The liquid temperature increases at a steady rate until equilibrium is reached.
- 3) The wall temperature increases at a steady rate until just before equilibrium is reached. At this point the wall temperature drops, no doubt because the inside heat-transfer coefficient is much greater with boiling than with the convection associated with the period before equilibrium is reached.
- 4) The difference between the liquid temperature at the wall and at the center increases with time, apparently because the larger convection currents are set up as the boil-off rate increases. Although the larger convection cur-

rents tend to make the temperature more homogeneous, they cause a greater heat transfer and this effect apparently predominates as far as the difference in temperature of the liquid at the wall and at the center is concerned.

5) It is interesting to note that in every case (and also in some preliminary tests which are not reported in detail) the time for equilibrium is greater with helium pressurization. Although the outside conditions were not accurately controlled, and therefore limited significance should be attached to this point (for example, the amount of frost build-up differed from run to run), it is intended to keep this point in mind and to study its significance further.

6) Figure 16 shows the fraction of heat going to the liquid. At least 80% of the heat transferred goes to the liquid (resulting in an increase in liquid temperature), until just before equilibrium is reached.

V. FUTURE WORK

Work has been essentially completed in modifying the apparatus to study the actual discharge process. A propeller-type flow meter has been installed and calibrated to measure the rate of liquid discharge. The 14-point recording galvanometer will be used to measure temperatures. The piping is being changed so that the gas meter can be utilized to measure the actual amount of pressurizing gas used to discharge the tank. It is anticipated that this phase of the work will be completed by January 31, 1958.

APPENDIX

In Progress Report No. 3, a number of relations and curves were presented for a steady-state analysis of the processes which occur when liquid oxygen is discharged from a tank by means of a pressurizing gas. The purpose of this analysis was to study the influence of the rate of discharge and pressurizing gas temperature on the amount of pressurizing gas required and on the weight of gas present in the tank when all the liquid has been discharged. All the assumptions involved in this analysis were given in Progress Report No. 3. The main ones are that saturated liquid is always present in the tank, that all the heat transfer goes to evaporating the liquid, and that there is no heat transfer between the liquid and the gas.

From the vantage point of a few more months' experience, we realize that these assumptions are rather unrealistic, and that therefore the results are of limited quantitative value. However, the derivations of these relations are presented here, both for the sake of completeness and because this approach may be useful on other occasions.

Also included in this appendix is the "finned-tube" heat-transfer analysis, the results of which were presented earlier in this report.

The physical significance of the symbols used in this analysis are shown in Fig. 19. This analysis involves the emptying of a tank of liquid by pressurizing gas while heat transfer to the contents of the tank is taking place.

ANALYSIS I. ZERO THERMAL RESISTANCE WALL

This analysis assumed a wall material of zero thermal resistance. Thus the heat-transfer rate is constant.

$$q = \text{const.} = (q/A)A_{\text{total}} = \lambda w_e \quad (1)$$

But

$$w_e + w_o = -w \quad (2)$$

(That is, the rate of decrease of liquid in the tank is equal to the rate of evaporation plus the rate of liquid outflow from the bottom of the tank.)

Let us first find the rate of liquid outflow with no pressurizing gas; i.e., utilizing self-pressurization.

Then

$$w_g = w_e$$

$$w_e = w_g = -\frac{\pi D^2}{4} \rho_g \frac{dL}{d\theta} ; w_L = \frac{\pi D^2}{4} \rho_L \frac{dL}{d\theta}$$

Therefore,

$$w_L = -\frac{\rho_L}{\rho_g} w_g = -\frac{\rho_L}{\rho_g} w_e = -\frac{\rho_L}{\rho_g} \frac{q}{\lambda}$$

$$w_o = -w_L - w_R = \frac{\rho_L q}{\rho_g \lambda} - \frac{q}{\lambda} = \frac{q}{\lambda} \left(\frac{\rho_L}{\rho_g} - 1 \right)$$

But

$$\rho_g = \frac{P}{RT_g}$$

$$q = U\Delta t \left[\frac{\pi D^2}{4} + \pi DL_o \right]$$

$$\frac{w_o}{mL_o} = \frac{U\Delta t \left[\frac{\pi D^2}{4} + \pi DL \right]}{\lambda \rho_g \pi D^2 4L_o} \left(\frac{\rho_L}{\rho_g} - 1 \right)$$

$$\frac{w_o}{mL_o} = \frac{U\Delta t}{\lambda} \left[\frac{1}{L_o} + \frac{4}{D} \right] \left(\frac{1}{\rho_g} - \frac{1}{\rho_L} \right) = \frac{q/A}{\lambda} \left[\frac{1}{L_o} + \frac{4}{D} \right] \left(\frac{1}{\rho_g} - \frac{1}{\rho_L} \right) \quad (3)$$

$$w_o = \frac{q}{\lambda} \left(\frac{\rho_L}{\rho_g} - 1 \right)$$

$$w_o = \frac{q}{\lambda} \left(\frac{\rho_L}{\rho_g} - 1 \right)$$

$$= \frac{q/A}{\lambda} \left[\frac{\pi D^2}{4} + \frac{4\pi}{D} DL_o \frac{D}{4} \right] \left[\frac{\rho_L}{\rho_g^*} - 1 \right]$$

$$= \frac{\pi D^2}{4} \left[\frac{q/A}{\lambda} \right] \left[1 + \frac{4L_o}{D} \right] \left[\frac{\rho_L}{\rho_g^*} - 1 \right] \quad (4)$$

This equation gives the mass rate of flow out of the tank, assuming no pressurizing gas is used.

Next, let us assume that pressurizing gas is used to empty the tank. First, we find $(m_i/m_g)_f$ as a function of the time θ_f required to empty the tank.

When the liquid gas been expelled from the tank, a fraction of the gas in the tank comes in as pressurizing gas and the rest comes from the liquid phase

as a result of evaporation. The quantity $(m_i/m_g)_f$ represents the mass fraction of the gas finally in the tank which was pressurizing gas.

From a mass balance,

$$\begin{aligned} w_i - w_o &= w_L + w_g \\ w_g &= w_e + w_i \\ w_o + w_e &= -w_L \end{aligned} \quad (5)$$

From a first-law analysis,

$$\begin{aligned} dE_g &= h_e dm_e + h_i dm_i - PdV_g \\ dE_g + PdV_g + V_g dP - V_g dP &= h_e dm_e + h_i dm_i \\ dH_g - V_g dP &= h_e dm_e + h_i dm_i \end{aligned}$$

But

$$dH_g = d(C_p m T) \text{ and } V_g dP = 0$$

Then

$$\begin{aligned} d \left\{ C_p m_g T_g \right\} &= C_p T_L dm_e + C_p T_i dm_i \\ \frac{d(m_g T_g)}{d\theta} &= T_L w_e + T_i w_i \end{aligned} \quad (6)$$

But

$$P = \rho_g R T_g \text{ and } m_g = \rho_g V_g = \frac{P}{R T_g} V_g$$

Then

$$\frac{P}{R} \frac{dV_g}{d\theta} = T_L w_e + T_i w_i$$

But

$$\frac{dV_g}{d\theta} = \frac{w_e + w_o}{\rho_L}$$

Therefore

$$\frac{P}{R} \left(\frac{w_e + w_o}{\rho_L} \right) = T_L w_e + T_i w_i$$

$$T_i w_i = \left(\frac{P}{R\rho_L} - T_L \right) w_e + \frac{P}{R\rho_L} w_o \quad (7)$$

$$w_g = w_e + w_i$$

$$\frac{w_i}{w_g} = \frac{w_i}{w_e + w_i} = \frac{1}{w_e/w_i + 1}$$

$$\frac{w_i}{w_g} = \frac{1}{\left[\frac{T_i}{\frac{P}{R\rho_L} - T_L + \frac{P}{R\rho_L} \frac{w_o}{w_e}} + 1 \right]} \quad (8)$$

But

$$w_e = q/\lambda$$

$$\frac{w_i}{w_g} = \frac{m_i}{m_g} = \left\{ \frac{1}{\left[\frac{T_i}{\frac{P}{R\rho_L} - T_L + \left(\frac{P\lambda}{R\rho_L q} \right) w_o} + 1 \right]} \right\}$$

But

$$q = q/A \left(\pi DL + \frac{\pi D^2}{4} \right)$$

$$\frac{m_i}{m_g} = \frac{1}{\frac{T_i}{\left(\frac{P}{R\rho_L} - T_L \right) + \frac{P\lambda w_o}{R\rho_L q/A \left(\frac{\pi D^2}{4} + \pi DL \right)}} + 1}$$

$$\frac{m_i}{m_g} = \left\{ \frac{1}{\left(\frac{P}{R\rho_L} - T_L \right) \left[\frac{\lambda P}{Rq \left(\frac{1}{L} + \frac{4}{D} \right) mL_o} \right] + 1} \right\} \quad (9)$$

The final relation to be derived involves (m_{g_f}/m_g^*). Physically this relation represents the final mass of gas in the tank, m_{g_f} , as a function of m_g^* , which represents the mass of gas in the tank at the saturation pressure and temperature. Thus, if no pressurization gas were used, the vapor would all be saturated vapor and this ratio would be 1.0. On the other hand, if some pressurization gas were used which is at a higher temperature than the saturation temperature, this ratio would be less than 1.0. The derivation is as follows.

$$w_g = w_e + w_i$$

$$m_g = (w_e + w_i)\theta ; m_g)_{final} = m_{g_f} = (w_e - w_i)\theta_f$$

But

$$- w_L\theta_f = mL_o = (w_o + w_e)\theta_f$$

$$\theta_f = \frac{mL_o}{w_o + w_e}$$

$$m_{g_f} = \left(\frac{w_e + w_i}{w_e + w_o} \right) mL_o = \left(\frac{1 + w_i/w_e}{1 + w_o/w_e} \right) mL_o \quad (10)$$

Substituting Eq. 7,

$$m_{g_f} = \left\{ \frac{1 + \frac{1}{T_i} \left[\left(\frac{P}{R\rho_L} - T_L \right) + \frac{P}{R\rho_L} \frac{w_o}{w_e} \right]}{1 + w_o/w_e} \right\} mL_o$$

But

$$\frac{w_o}{w_e} = \frac{w_o P_L \pi D^2 / 4 L_o \lambda}{q/A \left(\frac{\pi D^2}{4} + \pi D L_o \right)} = \frac{\lambda \rho_L w_o / mL_o}{q/A \left(\frac{1}{L_o} + \frac{4}{D} \right)}$$

$$m_{g_f} = \left\{ \frac{1 + \frac{1}{T_i} \left[\left(\frac{P}{R\rho_L} - T_L \right) + \frac{P\lambda}{Rq/A} \frac{w_o / mL_o}{\left(\frac{1}{L_o} + \frac{4}{D} \right)} \right]}{1 + \frac{\lambda \rho_L w_o / mL_o}{q/A \left(\frac{1}{L_o} + \frac{4}{D} \right)}} \right\} mL_o$$

$$m_g^* = \frac{P \frac{\pi D^2}{4} L_o}{RT_L}$$

$$\frac{m_g)_{final}}{m_g^*} = \frac{\rho_L}{\rho_g^*} \frac{1 + \frac{1}{T_i} \left[\left(\frac{P}{R\rho_L} - T_L \right) \frac{P\lambda w_o/mL_o}{q/A \left(\frac{1}{L_o} + \frac{4}{D} \right)} \right]}{1 + \frac{\lambda\rho_L w_o/mL_o}{q/A \left(\frac{1}{L_o} + \frac{4}{D} \right)}} \quad (11)$$

ANALYSIS II. INFINITELY THIN WALL

This analysis assumes an infinitely thin wall. Thus there is no longitudinal heat transfer down the tube, and the area for heat transfer is the area wetted by the liquid. We will proceed to derive the same three equations derived for Analysis I.

$$q = U\Delta t \left[\frac{\pi D^2}{4} + \pi DL \right] = -\lambda\rho_g \frac{\pi D^2}{4} \frac{dL}{d\theta} \quad (12)$$

Therefore,

$$\frac{dL}{d\theta} + \left(\frac{4U\Delta t}{\lambda\rho_g D} \right) L = -\frac{U\Delta t}{\lambda\rho_g}$$

Let

$$\frac{4U\Delta t}{\lambda\rho_g D} = \beta$$

Integrating

$$L = \left(L_i + \frac{D}{4} \right) e^{-\beta\theta} - D/4$$

$$\frac{dL}{d\theta} = - (L_i + D/4) \beta e^{-\beta\theta} \quad (13)$$

From the conservation of mass.

$$w_L = \rho_L \frac{\pi D^2}{4} \frac{dL}{d\theta}$$

But

$$w_o + w_e = w_L ; w_e = w_g$$

Therefore

$$w_o = -w_L - w_e = -w_L - w_g$$

$$w_o = -\frac{\pi D^2}{4} \rho_L \frac{dL}{d\theta} + \frac{\pi D^2}{4} \rho_g \frac{dL}{d\theta} = -\frac{\pi D^2}{4} (\rho_L - \rho_g) \frac{dL}{d\theta}$$

$$w_o = \frac{\pi D^2}{4} \left(\frac{4U\Delta t}{\lambda \rho_g D} \right) \left(L + \frac{D}{4} \right) (\rho_L - \rho_g) e^{-\beta\theta}$$

$$w_o = \left(\frac{\pi D U \Delta t}{\lambda} \right) \left(L_i + \frac{D}{4} \right) \left(\frac{\rho_L}{\rho_g} - 1 \right) e^{-\beta\theta}$$

If $\beta\theta \ll 1$

$$w_o = \left(\frac{\pi D U \Delta t}{\lambda} \right) \left(L_i + \frac{D}{4} \right) \left(\frac{\rho_L}{\rho_g} - 1 \right) (1 - \beta\theta)$$

$$\frac{w_o}{mL_o} = \frac{\frac{\pi D U \Delta t}{\lambda} \left(L + \frac{D}{4} \right) \left(\frac{\rho_L}{\rho_g} - 1 \right) e^{-\beta\theta}}{\frac{\pi D^2}{4} L_o \rho_L}$$

$$\frac{w_o}{mL_o} = \frac{U\Delta t}{\lambda} \left(\frac{1}{L_o} + \frac{D}{4} \right) \left(\frac{1}{\rho_g} - \frac{1}{\rho_L} \right) e^{-\beta\theta} \quad (15)$$

Next we derive the expression for $(m_i/m_g)_f$. The physical significance has already been discussed.

$$w_i + w_o = w_L + w_g$$

$$w_g = w_e - w_i ; -w_L = w_e + w_o \quad (16)$$

$$q = U\Delta t \left(\frac{\pi D^2}{4} + \pi D L \right) = \lambda w_e = -\lambda (w_L + w_o)$$

$$q = -\lambda \left[\frac{\pi D^2}{4} \rho_L \frac{dL}{d\theta} + w_o \right] = U\Delta t \left[\frac{\pi D^2}{4} + \pi D L \right]$$

$$\frac{dL}{d\theta} + \frac{4U\Delta t}{\lambda\rho_L D} L = - \left[\frac{4w_o}{\pi D^2 \rho_L} + \frac{U\Delta t}{\lambda\rho_L} \right] \quad (17)$$

Let

$$\frac{4U\Delta t}{\lambda\rho_L D} = \alpha ; \quad - \left[\frac{4w_o}{\pi D^2 \rho_L} + \frac{U\Delta t}{\lambda\rho_L} \right] = \gamma$$

Then

$$\frac{dL}{d\theta} + \alpha L = \gamma \quad (18)$$

This integrates to

$$L = C e^{-\alpha\theta} - \frac{\gamma}{\alpha}$$

But at $\theta = 0 ; L = L_o = C - \frac{\gamma}{\alpha}$

$$C = L_o + \frac{\gamma}{\alpha}$$

$$L = \left(L_o + \frac{\gamma}{\alpha} \right) e^{-\alpha\theta} - \frac{\gamma}{\alpha} \quad (19)$$

When

$$L = 0 , \alpha\theta_f = \ln \left(1 + \frac{L_o}{\gamma/\alpha} \right)$$

The time to empty the tank, θ_f , is

$$\theta_f = \frac{1}{\alpha} \ln \left(1 + \frac{L_o}{\gamma/\alpha} \right) \quad (20)$$

Considering now a first-law analysis,

$$dE_g = h_e dm_e + h_i dm_i - PdV_g = dH_g - PdV_g - V_g dP$$

But

$$V_g dP = 0 .$$

Therefore

$$h_e dm_e + h_i dm_i = dH_g$$

$$dH_g = d(m_g C_p T_g) = d(\rho_g V_g C_p T_g) = d \left\{ \frac{P}{RT_g} V_g C_p T_g \right\}$$

$$\frac{P}{R} dV_g = T_L dm_e + T_i dm_i$$

$$\frac{PdV_g}{Rd\theta} = T_L w_w + T_i w_i \quad (21)$$

But

$$\frac{dV_g}{d\theta} = - \frac{\pi D^2}{4} \frac{dL}{d\theta} \text{ and } w_e = - w_L - w_o$$

$$\frac{PdV_g}{Rd\theta} = - T_L (w_L + w_o) + T_i w_i$$

$$\left(\frac{P}{R} - \rho_L T_L \right) \frac{dV_g}{d\theta} + T_L w_o = T_i w_i = \left(\rho_L T_L - \frac{P}{R} \right) \frac{\pi D^2}{4} \frac{dL}{d\theta} + T_L w_o = T_i w_i$$

$$w_i = \left[\left(\rho_L T_L - \frac{P}{R} \right) \frac{\pi D^2}{4} \left(L_o + \frac{\gamma}{\alpha} \right) (-\alpha) e^{-\alpha\theta} + T_L w_o \right] \quad (22)$$

$$m_e = \frac{\pi D^2}{4} \rho_{L_o} - w_o \theta_f = m_{L_o} - w_o \theta_f$$

$$m_e = m_{L_o} - \frac{w_o}{\alpha} \ln (L_o + \gamma/\alpha) \quad (23)$$

Integrating Eq. 22 to the final time θ_f ,

$$m_i = \frac{1}{T_i} \left(\rho_L T_L - \frac{P}{R} \right) \frac{\pi D^2}{4} L_o + \frac{\gamma}{\alpha} \left[e^{-\alpha\theta} \right]_o^{\theta_f} + \frac{T_L}{T_i} w_o \theta_f$$

$$m_i = \frac{1}{T_i} \left(\frac{P}{R} - \rho_L T_L \right) \frac{\pi D^2}{4} \left(L_o + \frac{\gamma}{\alpha} \right) \left[1 - \frac{\gamma}{T_o + \frac{\gamma}{\alpha}} \right] + \frac{T_L}{T_i} w_o \theta_f$$

$$m_i = \left[\frac{1}{T_i} \frac{P}{R\rho_L} - T_L \right] \frac{\rho_L \pi D^2}{4} L_o + \frac{T_L}{T_i} w_o \theta_f$$

$$m_i = \frac{1}{T_i} \left[\left(\frac{P}{R\rho_L} - T_L \right) m_{L_o} + T_L \theta_f w_o \right] \quad (24)$$

But

$$m_g = m_i + m_e$$

Therefore

$$\frac{m_i}{m_g} = \frac{m_i}{m_i + m_e} = \frac{1}{1 + m_e/m_i}$$

$$\frac{m_e}{m_i} = \frac{m_{L_o} - w_o \theta_f}{\frac{1}{T_i} \left(\frac{P}{R\rho_L} - T_L \right) m_{L_o} + \frac{T_L}{T_i} \theta_f w_o}$$

$$\frac{m_e}{m_i} = \frac{- \left(\frac{w_o}{m_{L_o}} \theta_f \right)}{\frac{1}{T_i R \rho_L} - \frac{T_L}{T_i} + \frac{T_L}{T_i} \theta_f \frac{w_o}{m_{L_o}}}$$

But

$$\theta_f = \frac{1}{\alpha} \ln \left[1 + \frac{L_o}{\gamma/\alpha} \right]$$

$$\theta_f = \frac{\lambda \rho_L D}{4U\Delta t} \ln \left\{ 1 + \frac{L_o}{D/4 + \frac{\lambda w_o}{\pi D U \Delta t}} \right\}$$

But

$$\frac{\lambda w_o}{\pi D U \Delta t} = \frac{\lambda}{\pi D U \Delta t} \frac{\pi D^2}{4} L_o \rho_L \frac{w_o}{m_{L_o}} = \frac{\lambda \rho_L}{4U\Delta t} L_o \frac{w_o}{m_{L_o}}$$

$$\theta_f = \frac{\lambda \rho_L D}{4U\Delta t} \ln \left\{ 1 + \frac{L_o}{D/4 + \left(\frac{\lambda \rho_L D}{4U\Delta t} L_o \right) \frac{w_o}{m_{L_o}}} \right\}$$

$$\theta_f = \frac{1}{\alpha} \ln \left\{ 1 + \frac{L_o}{D/4 + \frac{L_o}{\alpha} \frac{w_o}{m_{L_o}}} \right\} \quad (25)$$

$$\frac{m_e}{m_i} = \left\{ \frac{1 - \frac{1}{\alpha} \left[\ln \left(1 + \frac{L_o}{D/4 + \frac{L_o}{\alpha} \frac{w_o}{m_{L_o}}} \right) \right] \frac{w_o}{m_{L_o}}}{\frac{P}{\rho_L R T_i} - \frac{T_L}{T_i} + \frac{1}{\alpha} \left[\ln \left(1 + \frac{L_o}{D/4 + \frac{L_o}{\alpha} \frac{w_o}{m_{L_o}}} \right) \right] \frac{T_L}{T_i} \frac{w_o}{m_{L_o}}} \right\} \quad (26)$$

But

$$\frac{m_i}{m_g} = \frac{m_i}{m_i + m_e} = \frac{1}{1 + \frac{m_e}{m_i}}$$

$$\left(\frac{m_i}{m_g} \right)_f = \left\{ \frac{1}{1 + \left\{ \frac{1 - \frac{1}{\alpha} \left[\ln \left(1 + \frac{1}{\frac{D}{4L_o} + \frac{1}{\alpha} \frac{w_o}{m_{L_o}}} \right) \right] \frac{w_o}{m_{L_o}}}{\frac{P}{\rho_L R T_i} - \frac{T_L}{T_i} + \frac{1}{\alpha} \left[\ln \left(1 + \frac{1}{\frac{D}{4L_o} + \frac{1}{\alpha} \frac{w_o}{m_{L_o}}} \right) \right] \frac{T_L}{T_i} \frac{w_o}{m_{L_o}}} \right\}} \right\} \quad (27)$$

To derive the relation for $\frac{m_g \text{ final}}{m_g^*}$ we proceed as follows.

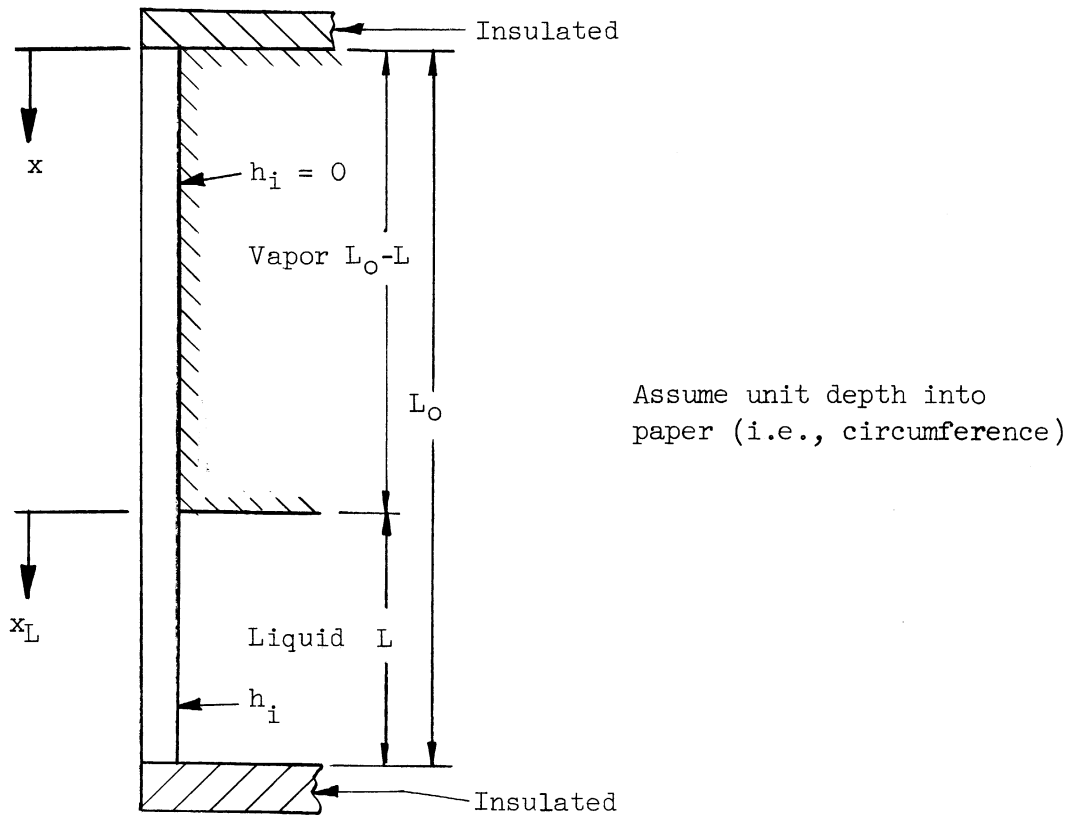
$$m_g = m_i + m_e = \frac{1}{T_i} \left[\left(\frac{P}{\rho_L R} - T_L \right) m_{L_o} + T_L \theta_f w_o \right] + m_{L_o} - w_o \theta_f$$

$$m_g = \left(\frac{\rho}{P_L R T_i} - \frac{T_L}{T_i} + 1 \right) m_{L_o} + \left(\frac{T_L}{T_i} - 1 \right) \theta_f w_o$$

$$\begin{aligned}
 \frac{m_g \text{ final}}{m_g^*} &= \left(1 - \frac{T_L}{T_i} + \frac{P}{\rho_L R T_i}\right) \frac{\rho_L V}{(PV/RT_L)} + \left(1 - \frac{T_L}{T_i}\right) \theta_f \frac{w_o}{(PV/RT_L)} \\
 &= \left(1 - \frac{T_L}{T_i} + \frac{P}{\rho_L R T_i}\right) \frac{\rho_L R T_L}{P} - \left(1 - \frac{T_L}{T_i}\right) \theta_f \frac{R T_L}{PV} \frac{w_o}{m_{L_o}} \rho_L V \\
 &= \frac{\rho_L R T_L}{P} \left\{ \left[1 - \frac{T_L}{T_i} + \frac{P}{\rho_L R T_i}\right] - \frac{1}{\alpha} \left[\ln \left(1 + \frac{1}{\frac{D}{4L_o} + \frac{w_o}{\alpha m_L}}\right) \right] \left(1 + \frac{L}{T_i}\right) \frac{w_o}{m_{L_o}} \right\}
 \end{aligned}$$

(28)

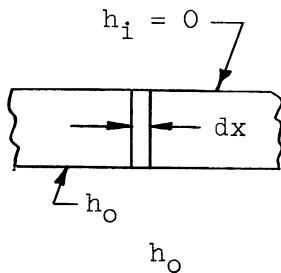
ANALYSIS III. FINNED TUBE—CONSTANT LIQUID-LEVEL ANALYSIS



The purpose of this analysis is to determine the heat transfer to the liquid when the longitudinal heat transfer is taken into account. Since the wall above the liquid acts as an extended surface, this analysis has been termed the finned tube analysis. The main assumptions in this analysis are that neither heat transfer from the wall to the vapor above the liquid nor heat transfer between the vapor and the liquid takes place, and that the outside coefficient is constant.

1) From $x = 0$ to $x = L_0 - L$

$$h_o (t_a - t) dx = -d \left(KA \frac{dt}{dx} \right) \quad (29)$$



$$\frac{d^2 t}{dx^2} + \frac{h_o}{KA} t_a - \frac{h_o}{KA} t = 0$$

Call $h_o/KA = n^2$

(Since unit circumference of 1 ft is assumed, all dimensions involving h_o or h_i should be multiplied by unit length.)

$$\therefore \frac{d^2 t}{dx^2} - n^2 t = -n^2 t_a \quad (30)$$

Solution:

$$t = t_a + \frac{c_1}{2} e^{nx} + \frac{c_2}{2} e^{-nx}$$

$$\frac{dt}{dx} = \frac{c_1}{2} n e^{nx} + \frac{c_2}{2} (-n) e^{-nx} \quad (31)$$

when $x = 0$, $\frac{dt}{dx} = 0$

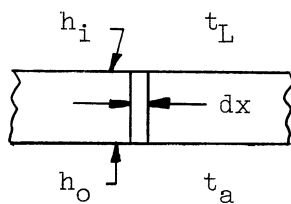
$$\therefore c_1 = c_2$$

$$t = \frac{c_1}{2} (e^{nx} + e^{-nx}) + t_a$$

$$t = c_1 \cosh nx + t_a \quad (0 < x < L_0 - L) \quad (32)$$

$$\frac{dt}{dx} = c_1 n \sinh nx \quad (0 < x < L_0 - L) \quad (33)$$

2) From $x = L_0 - L$ to $x = L_0$ ($x_L = 0$ to L)



$$\{h_o (t_a - t) - h_i (t - t_L)\} dx = -d \left\{ KA \frac{dt}{dx} \right\}$$

$$h_o (t_a - t) - h_i (t - t_L) = -KA \frac{d^2 t}{dx^2}$$

$$\frac{d^2 t}{dx^2} - \frac{h_o t + h_i t}{KA} = - \frac{h_o t_a + h_i t_L}{KA} \quad (34)$$

$$N^2 = \frac{h_o + h_i}{KA} \quad C = \frac{h_o t_a + h_i t_L}{KA}$$

$$\frac{d^2 t}{dx^2} - N^2 t = -C \quad (35)$$

Solution:

$$t = C_3 e^{nx_L} + C_4 e^{-nx_L} + \frac{C}{N^2} \quad (0 < x_L < L)$$

$$\frac{dt}{dx_L} = N (C_3 e^{Nx_L} - C_4 e^{-Nx_L}) \quad (0 < x_L < L)$$

$$\text{at } x_L = L, \quad \frac{dt}{dx} = 0$$

$$\therefore C_4 = C_3 e^{2NL}$$

$$\text{at } x_L = 0, \quad t = t_a + \frac{C_1}{2} [e^{n(L_o-L)} + e^{-n(L_o-L)}]$$

At $x_L = 0$: t from 1) = t from 2)

$$\frac{dt}{dx} \text{ from 1) } = \frac{dt}{dx} \text{ from 2)}$$

Equating t :

$$C_1 \cosh [n(L_o-L)] + t_a = C_3 + C_4 + \frac{C}{N^2} \quad (36)$$

Equating $\frac{dt}{dx}$:

$$N(C_3 - C_4) = C_1 n \sinh [n(L_o-L)] \quad (37)$$

Equations to evaluate C_1, C_3, C_4

$$1) \quad C_4 = C_3 e^{2NL} \quad (38)$$

$$2) \quad C_3 + C_4 = C_1 \cosh [n(L_o-L)] + \left(t_a - \frac{C}{N^2} \right) \quad (39)$$

$$3) C_3 - C_4 = C_1 \frac{n}{N} \sinh [n(L_0-L)] \quad (40)$$

Solving:

$$C_1 = \frac{-\frac{N}{n} \left\{ t_a - \frac{C}{N^2} \right\} \frac{\sinh NL}{\sinh [n(L_0-L)]}}{\cosh NL + \frac{N}{n} (\sinh NL) \coth [n(L_0-L)]} \quad (41)$$

$$C_3 = \frac{\frac{1}{2} \left\{ t_a - \frac{C}{N^2} \right\} e^{-NL}}{\cosh NL + \frac{N}{n} (\sinh NL) \coth [n(L_0-L)]} \quad (42)$$

$$C_4 = \frac{\frac{1}{2} \left\{ t_a - \frac{C}{N^2} \right\} e^{NL}}{\cosh NL + \frac{N}{n} (\sinh NL) \coth [n(L_0-L)]} \quad (43)$$

From $x = 0$ to $x = L_0 - L$

$$q = \int_0^{L_0-L} h_0 (t_a - t) dx = -h_0 \int_0^{L_0-L} \cosh nxdx$$

$$q = -h_0 \frac{C_1}{n} \sinh [n(L_0-L)]$$

$$q = \frac{\frac{h_0 N}{n^2} \left\{ t_a - \frac{C}{N^2} \right\}}{\coth NL + \frac{N}{n} \coth [n(L_0-L)]} \quad (44)$$

From $x_L = 0$ to $x_L = L$

$$q = \int_0^L h_0 (t_a - t) dx_L = h_0 \int_0^L \left(t_a - C_3 e^{Nx_L} - C_4 e^{-Nx_L} - \frac{C}{N^2} \right) dx_L$$

$$q = h_0 \left(t_a - \frac{C}{N^2} \right) \left\{ L - \frac{1/N}{\coth NL + \frac{N}{n} \coth [n(L_0-L)]} \right\} \quad (45)$$

Total q ($x = 0$ to $x = L_0$) Add "q" for 1) and 2)

$$q = h_0 \left\{ t_a - \frac{c}{N^2} \right\} \left\{ \frac{\frac{N}{n^2} - \frac{1}{N}}{\coth NL + \frac{N}{n} \coth [n(L_0-L)]} + L \right\} \pi D_0 \quad (46)$$

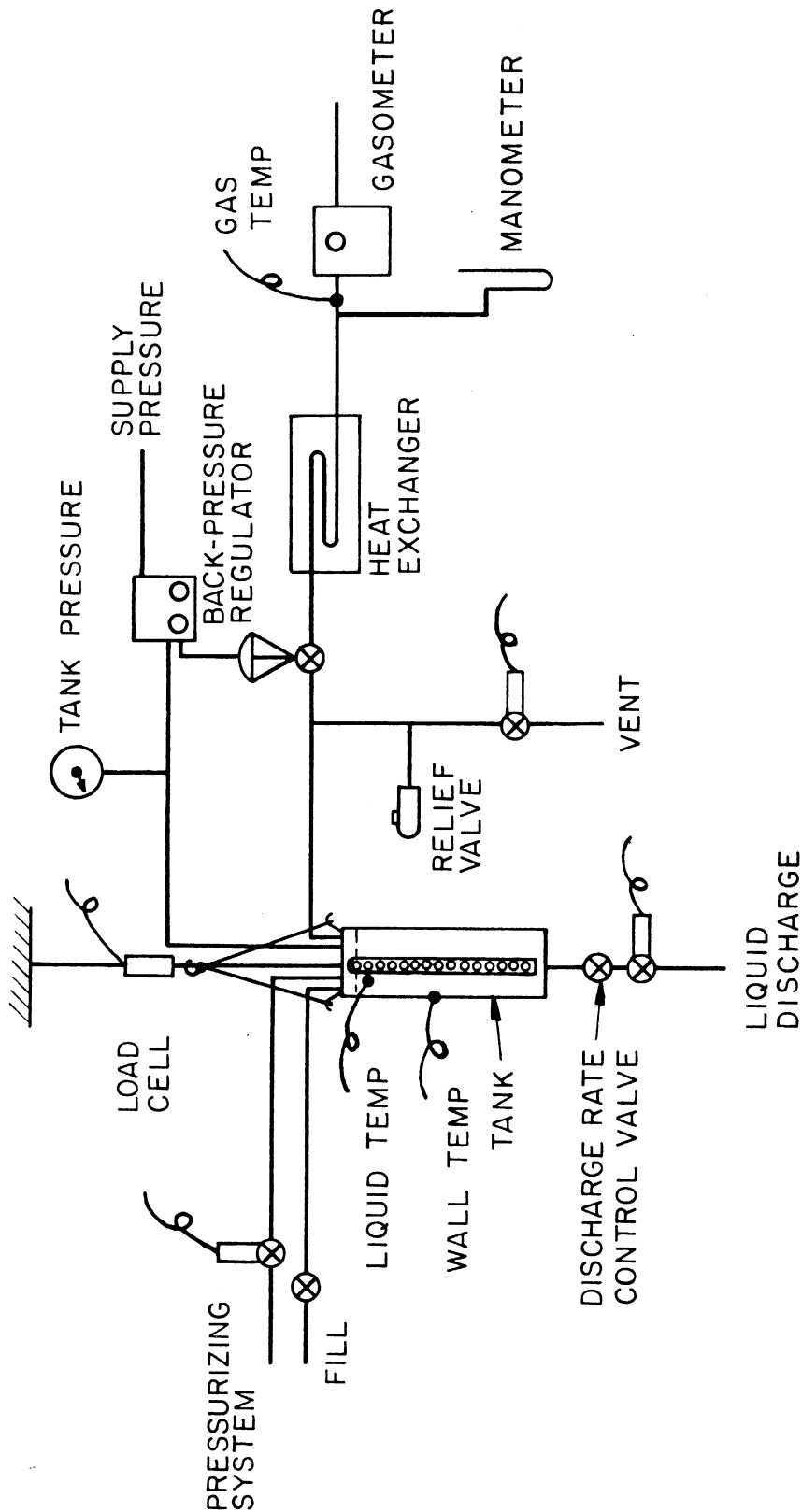
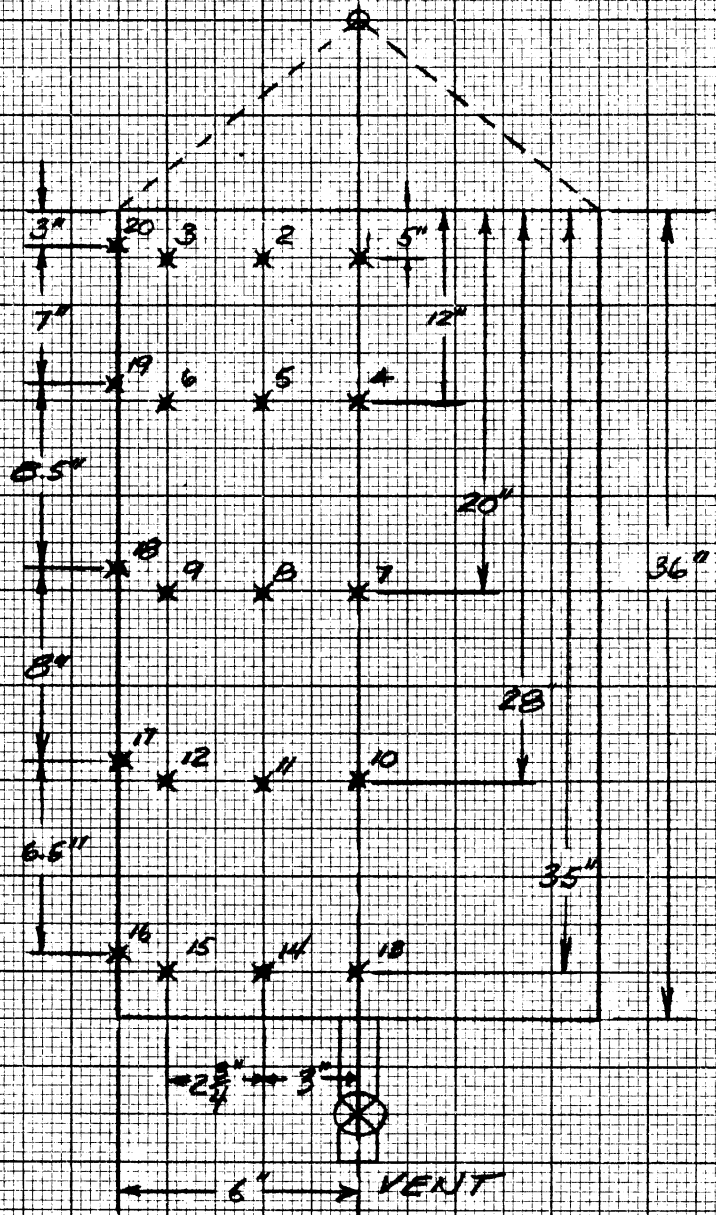


FIG. A

FIGURE B
PROJECT 2646

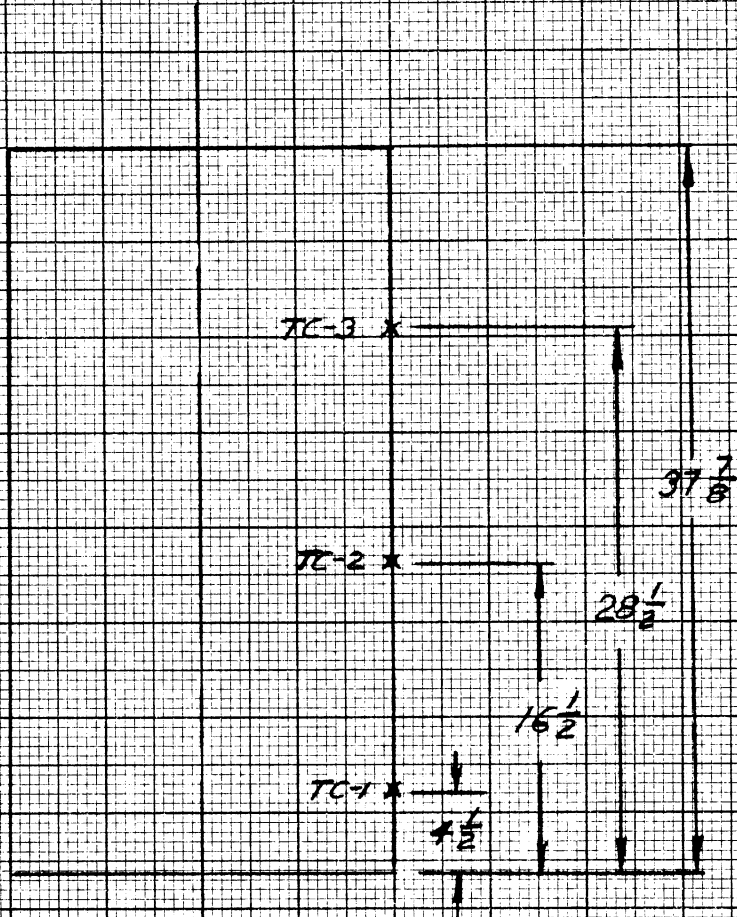
POSITION OF THERMOCOUPLES FOR $\frac{1}{8}$ " WALL
LIQUID NITROGEN BOIL-OFF TANK



NOT TO SCALE

FIGURE C
PROJECT 2696

POSITION OF THERMOCOUPLES IN THE $3\frac{1}{2}$ "
WALL LIQUID NITROGEN BOILOFF TANK
(EXTERNAL DIMENSIONS)



NOT TO SCALE

FIG. 1

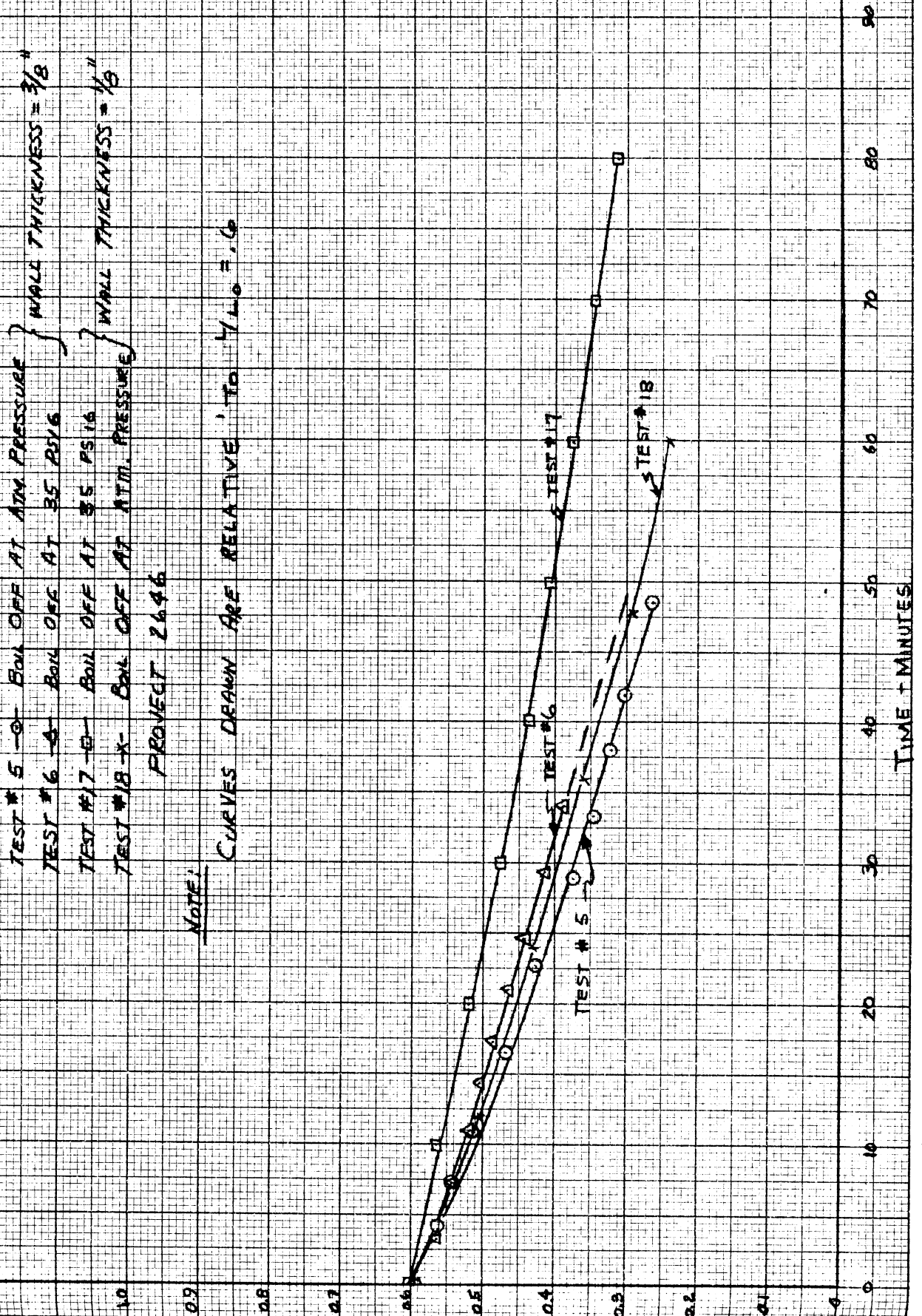
TEST # 5 - O - BOIL OFF AT ATM. PRESSURE } WALL THICKNESS = 3/8"
 TEST # 6 - A - BOIL OFF AT 35 PSIA }
 TEST # 7 - G - BOIL OFF AT 35 PSIA } WALL THICKNESS = 1/8"
 TEST # 18 - X - BOIL OFF AT ATM. PRESSURE }

PROJECT 2646

NOTE:

CURVES DRAWN ARE RELATIVE TO $V_{L0} = 7.6$

CALCULATED LIQUID LEVEL / LIQUID LEVEL OF FULL TANK - V_L



TIME - MINUTES

HEAT TRANSFER COEFFICIENT IN $\frac{BTU}{HR-ft^2-OF}$

400

350

300

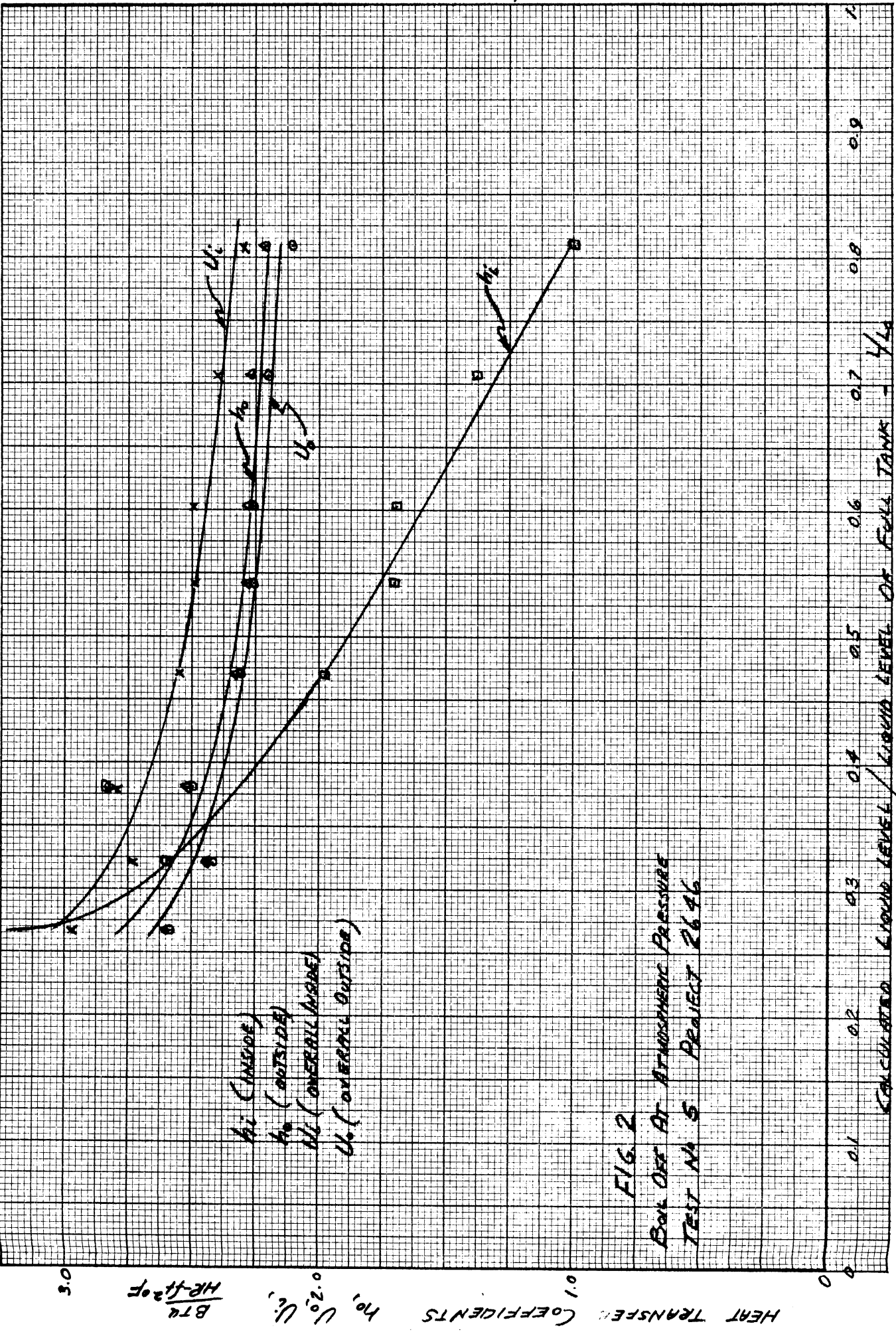


FIG. 2
 BOIL OFF AT ATMOSPHERIC PRESSURE
 TEST NO. 5 PROJECT 2646

HEAT TRANSFER COEFFICIENTS h_o, U_o, U_i

AT $L/L_o = 1.0$

$U_i = 1.76$ (OVERALL INSIDE)
 $h_o = 1.62$ (OUTSIDE)
 $U_o = 1.60$ (OVERALL OUTSIDE)
 $h_i = 2.15$ (INSIDE)

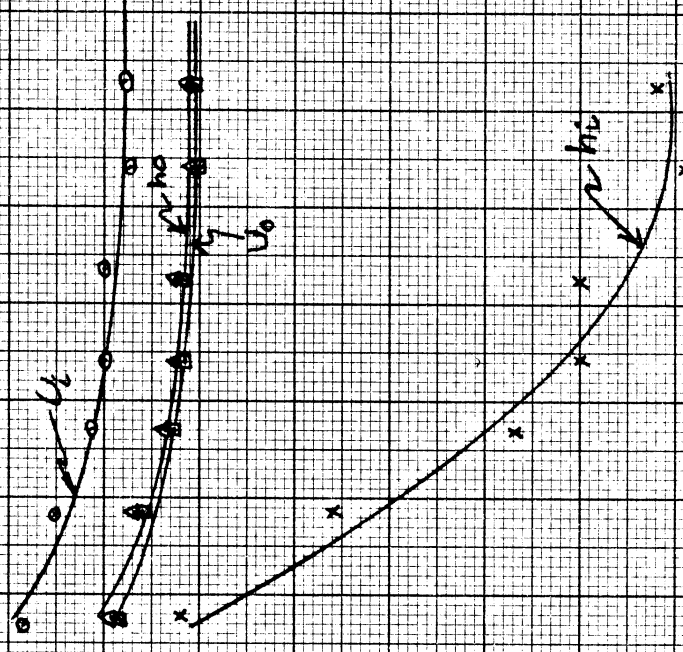


FIG. 3

BOW OFF AT 35 PSIG
 TEST No. 6. PROBERT 2646

0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
CALCULATED LIQUID LEVEL / LIQUID LEVEL OF FULL TANK - L/L_o

260

HEAT TRANSFER

240

COEFFICIENT h_i
 $\frac{BTU}{HR \cdot ft^2 \cdot ^\circ F}$

220

200

10

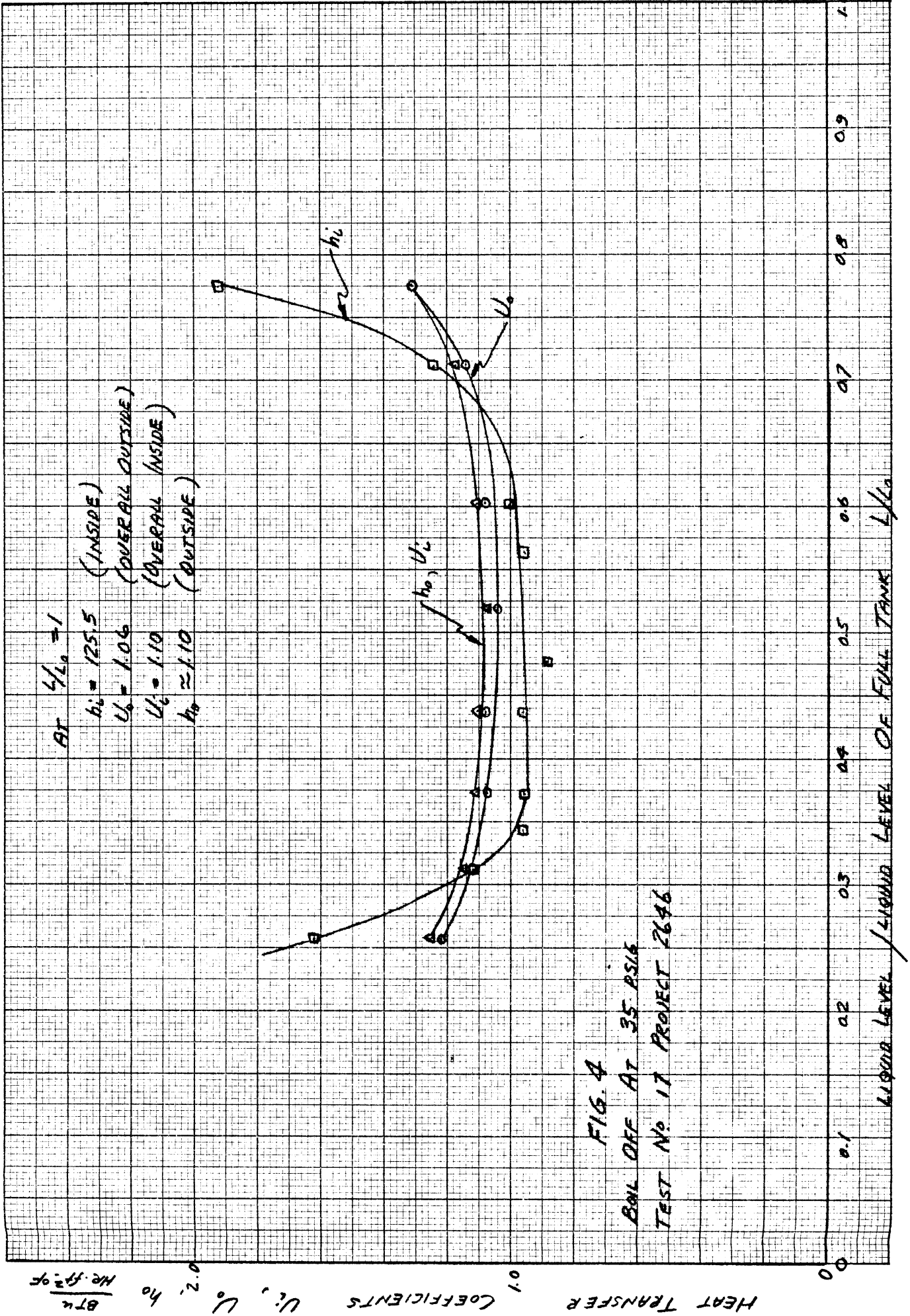


FIG. 4
 BOILER AT 35 PSIG
 TEST No. 17 PROJECT 2646

$\frac{BTU}{HR-F^2-OF}$

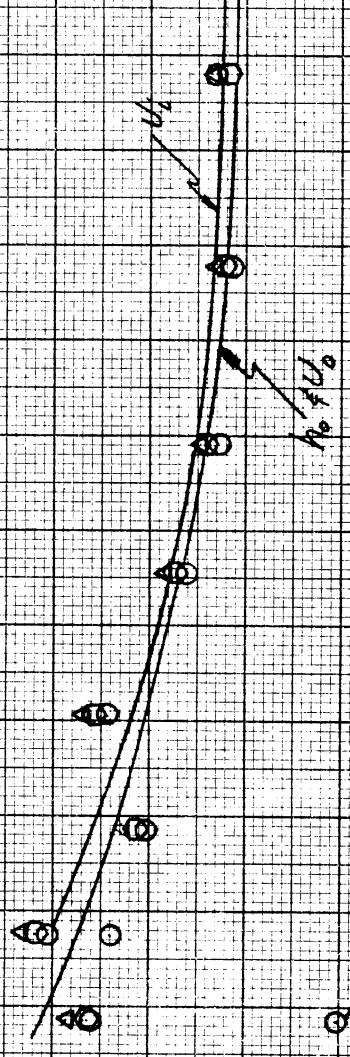
2.0

h_o, U_o, U_i

HEAT TRANSFER COEFFICIENTS

1.0

0



AT $\frac{U_o}{U_i} = 1$

- $h_o = 2.00$ (OUTSIDE)
- $U_o = 2.00$ (OVERALL OUTSIDE)
- $U_i = 2.03$ (OVERALL INSIDE)
- $h_i = 246.5$ (INSIDE)

FIG. 5
 BOIL OFF AT ATMOSPHERIC PRESSURE
 TEST No. 18 PROVEST 2646

LIQUID LEVEL / LIQUID LEVEL OF FULL TANK = $\frac{U_o}{U_i}$

0.1

0.2

0.3

0.4

0.5

0.6

0.7

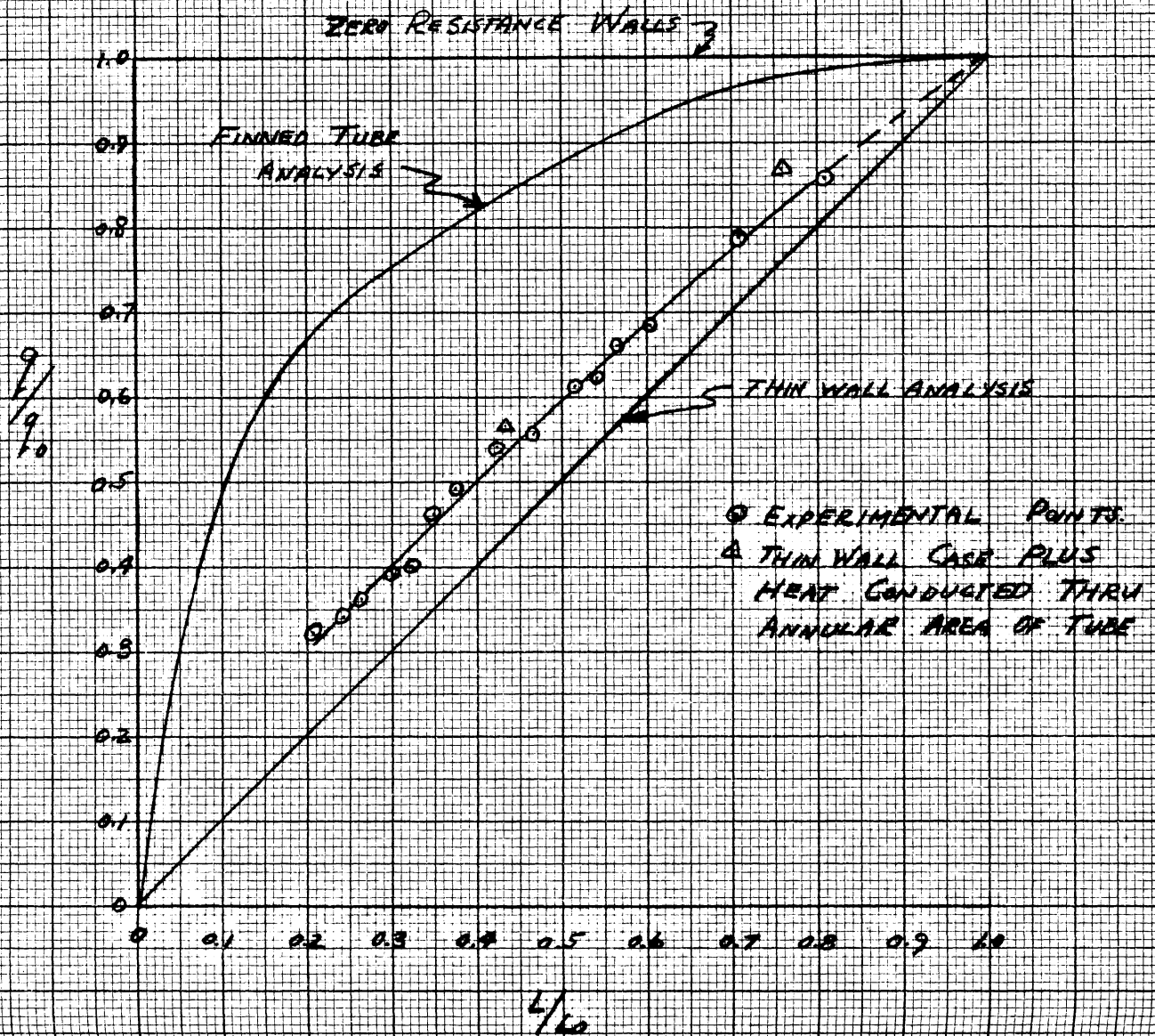
0.8

0.9

1.0

FIG. 6
 UNDER ATMOSPHERIC PRESSURE
 TEST N.O.5 PROJECT 2646

q = HEAT TRANSFER THRU WALL AT ANY SPECIFIC "L"
 L = LIQUID LEVEL IN TANK
 q_0 = HEAT TRANSFER AT "L"
 L_0 = LIQUID LEVEL OF FULL TANK = 37"



EPL PROJECT NO. 2646

FIG. 7

TEST NO. 6

35 PSIG

$L_0 = 37\text{ in.}$

⊙ EXPERIMENTAL POINTS

△ THIN WALL CASE PLUS LONGITUDINALLY CONDUCTED HEAT RATIO (q/q_0)

q = HEAT TRANSFER RATE, BTU/HR

q_0 = FULL TANK HEAT TRANSFER RATE, BTU/HR ($L/L_0 = 1$)

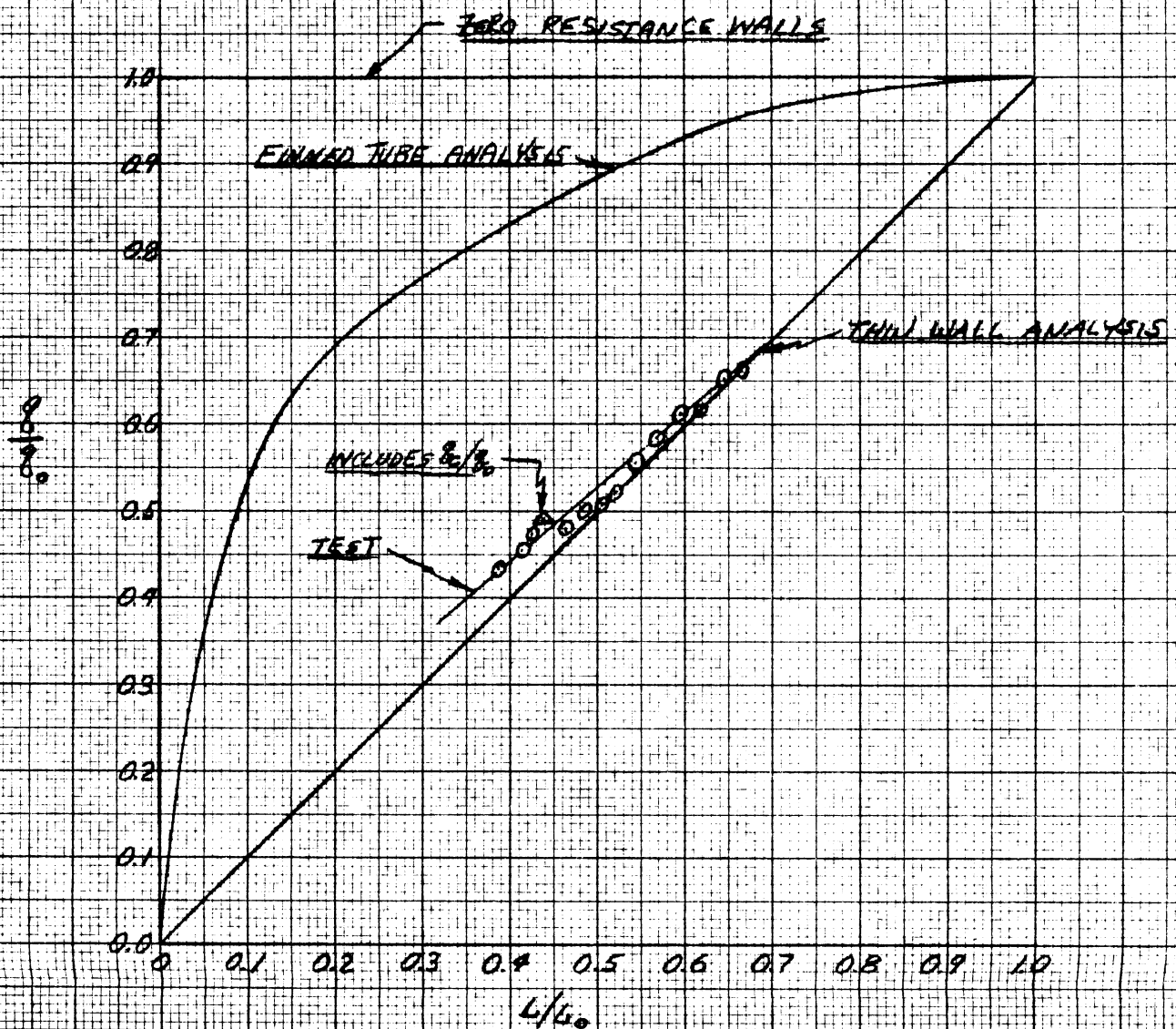


FIG. 8

UNDER 35 P.S.I.G.

TEST NO 17 PROJECT 2646

q = HEAT TRANSFER THRU WALL AT ANY SPECIFIC "L"

L = LIQUID LEVEL IN TANK.

q_0 = HEAT TRANSFER AT "L"

L_0 = LIQUID LEVEL OF FULL TANK = 36"

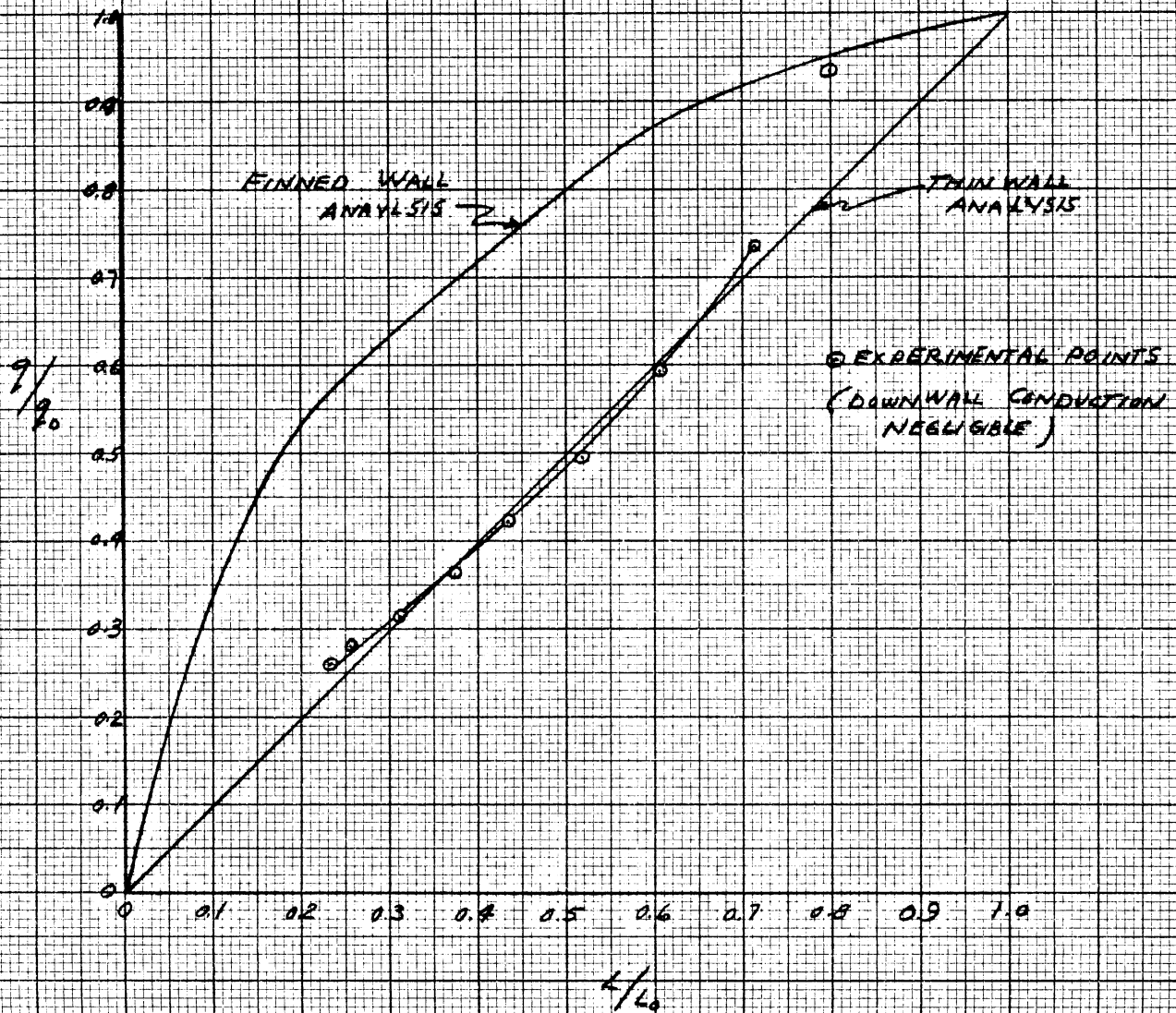


FIG. 9

ATMOSPHERIC PRESSURE
TEST NO. 10 PROJECT 2646

q = HEAT TRANSFER THRU WALL AT ANY SPECIFIC L'
 L = LIQUID LEVEL IN TANK
 q_0 = HEAT TRANSFER AT L_0
 L_0 = LIQUID LEVEL OF FULL TANK = 36"

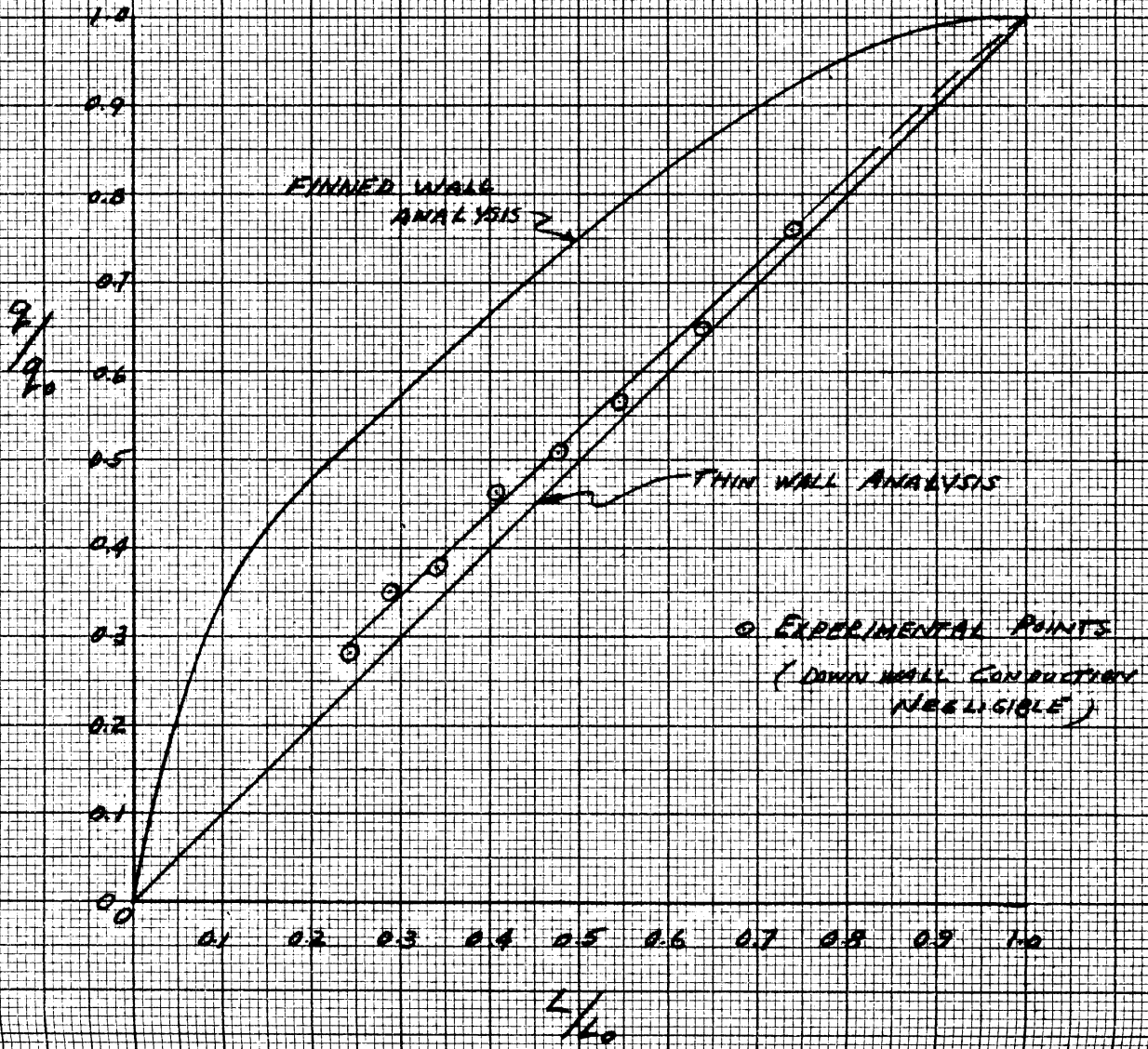
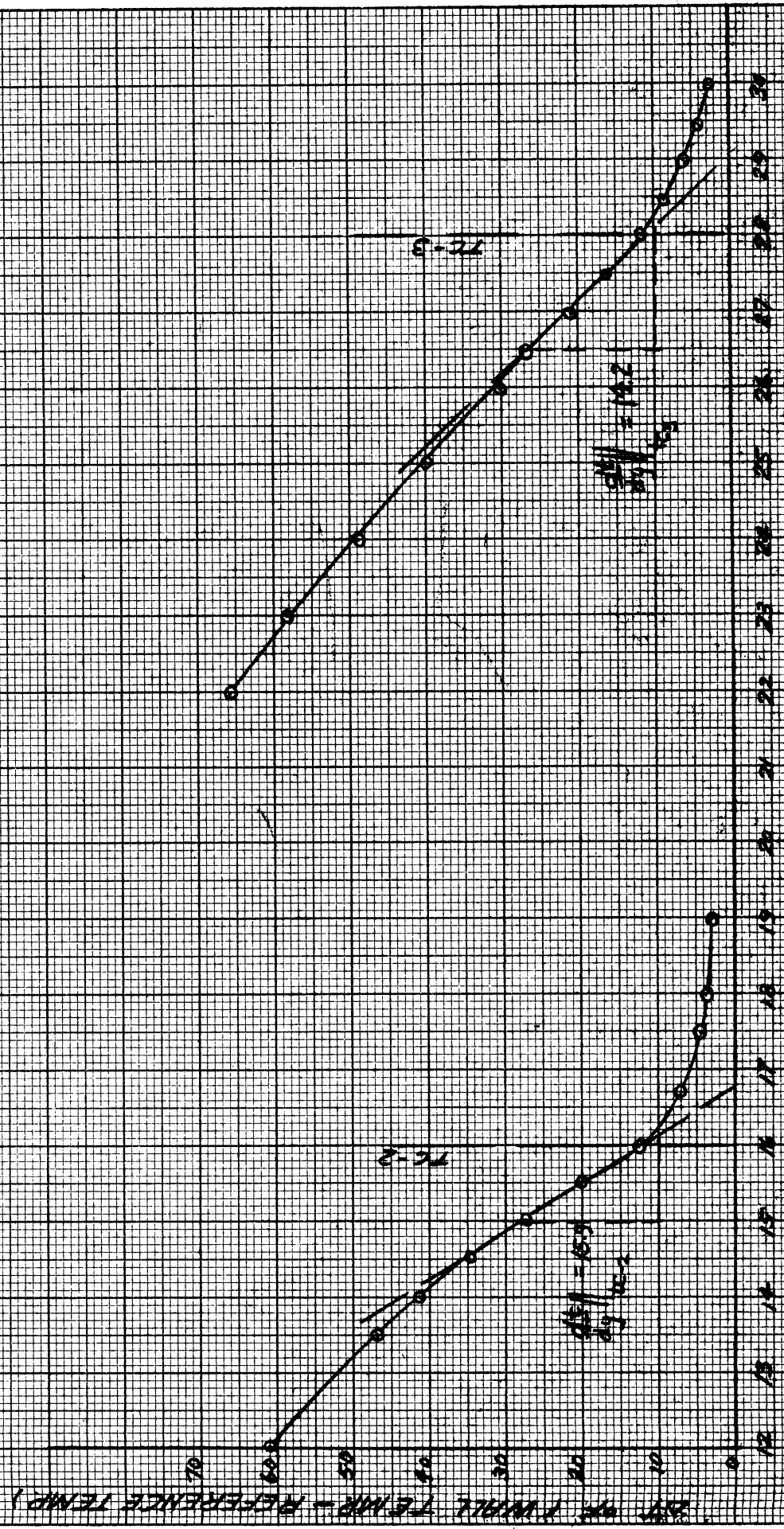


FIG. 10
 UNDER ATMOSPHERIC PRESSURE
 TEST NO. 5 PROJECT 2.6.46



CALCULATED LENGTH IN INCHES

FIG. 11

UNDER 35 PSIG.
TEST NR 6 PROJECT 2846

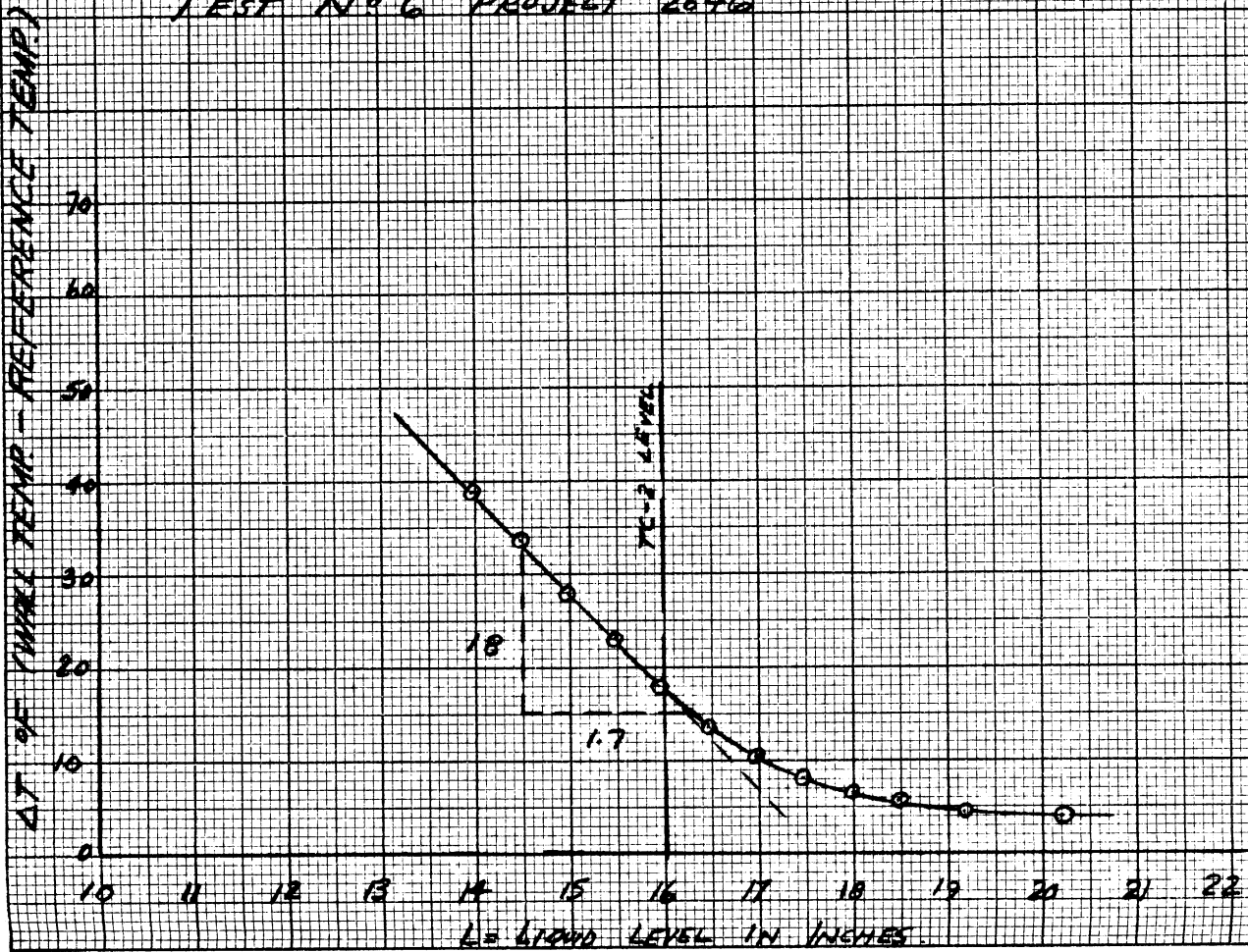
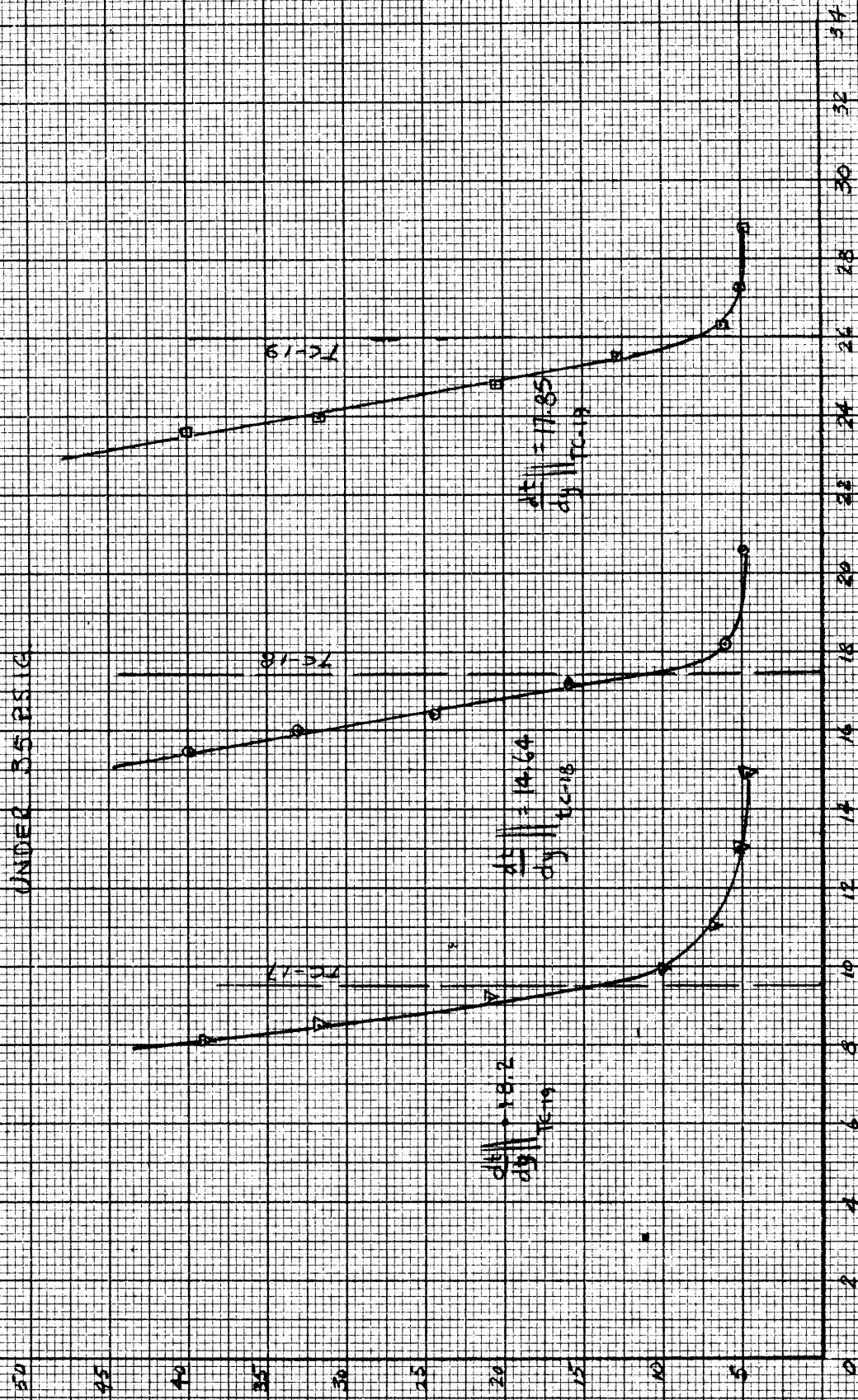


FIG. 12

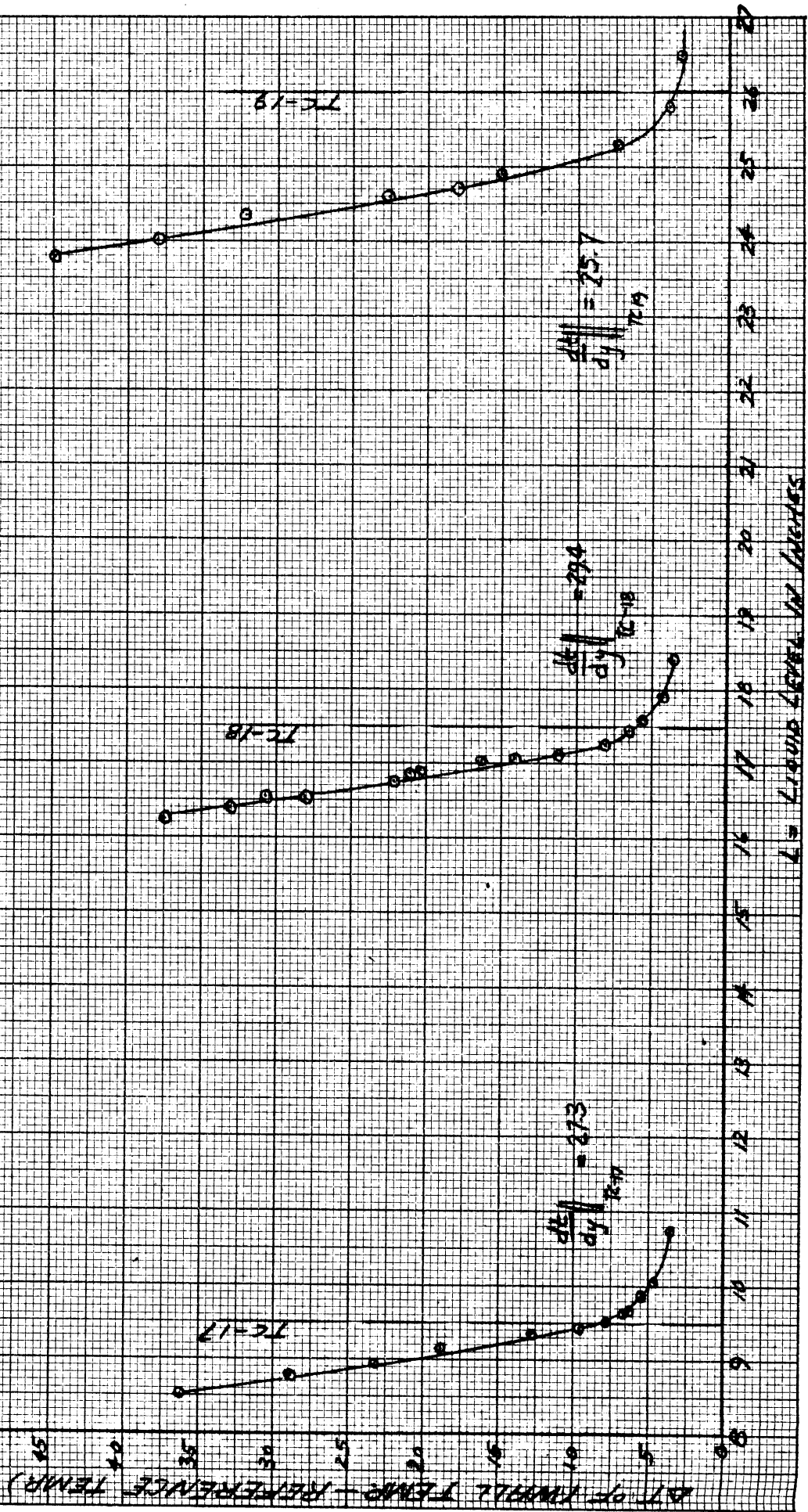
TEST NO. 17 PROJECT 2646
UNDER 35 PSIG

ΔT °F (WALL TEMPERATURE - REFERENCE TEMP)



L - LIQUID LEVEL IN INCHES

FIG. 13
 UNDER ATMOSPHERIC PRESSURE
 TEST N° 18 PROTECT 2646



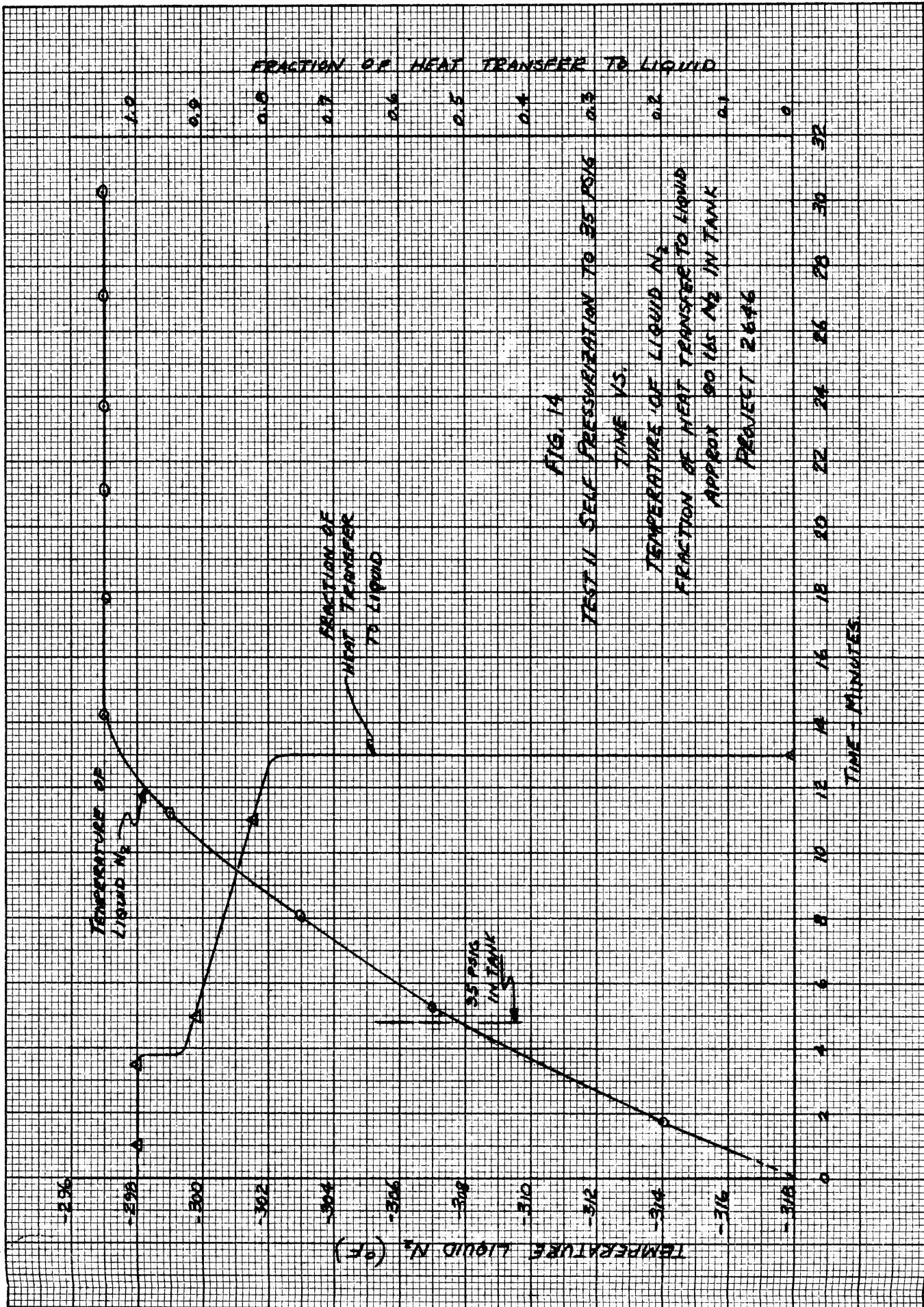


FIG. 14

TEST II SELF PRESSURIZATION TO 35 PSIG
 TIME VS.

TEMPERATURE OF LIQUID N₂
 FRACTION OF HEAT TRANSFER TO LIQUID
 APPROX 90 LBS N₂ IN TANK
 PROJECT 2646

TIME - MINUTES

FIG. 15

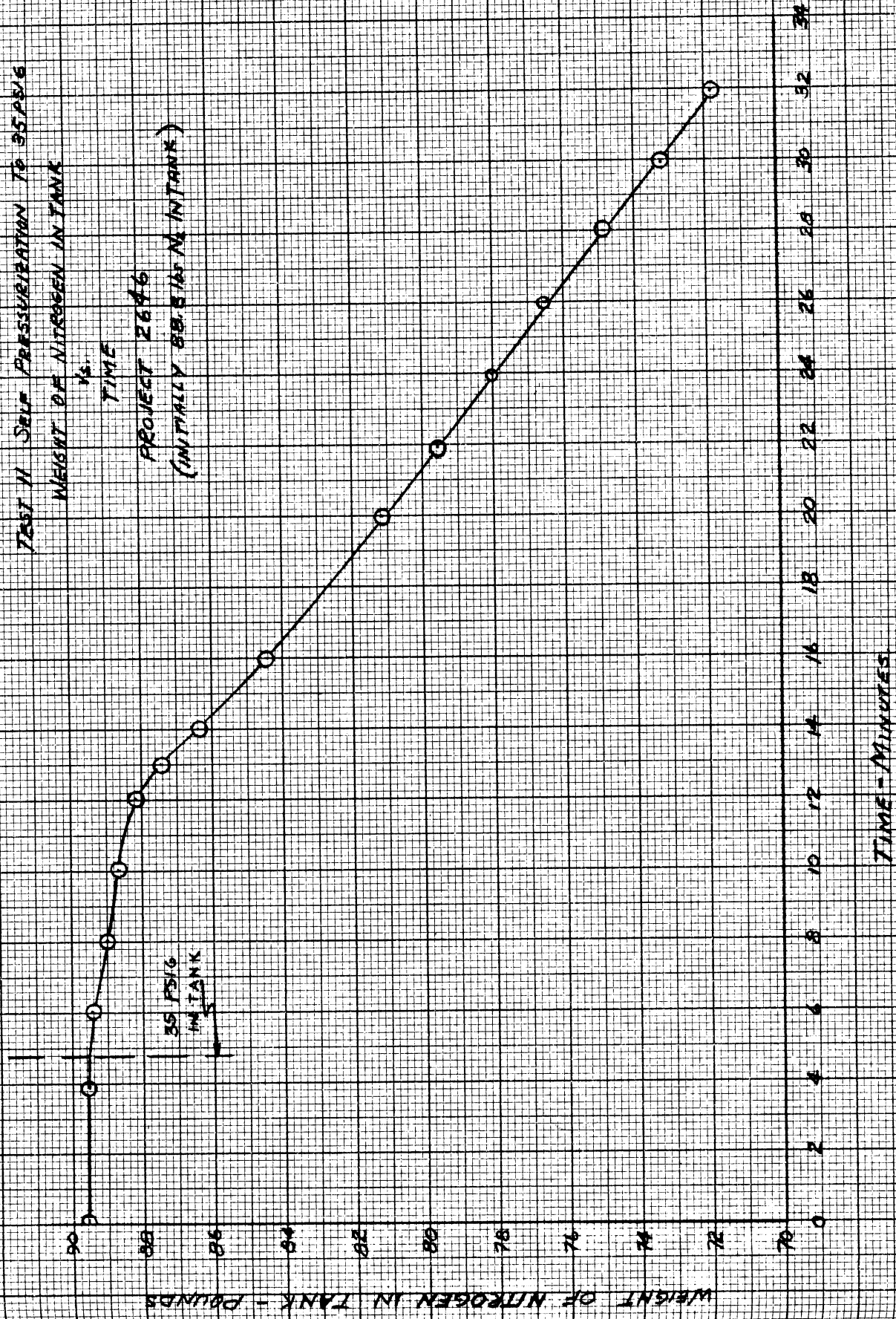
TEST II SELF PRESSURIZATION TO 95 PSIG

WEIGHT OF NITROGEN IN TANK

vs
TIME

PROJECT 2646

(INITIALLY 88.5 LBS N₂ IN TANK)



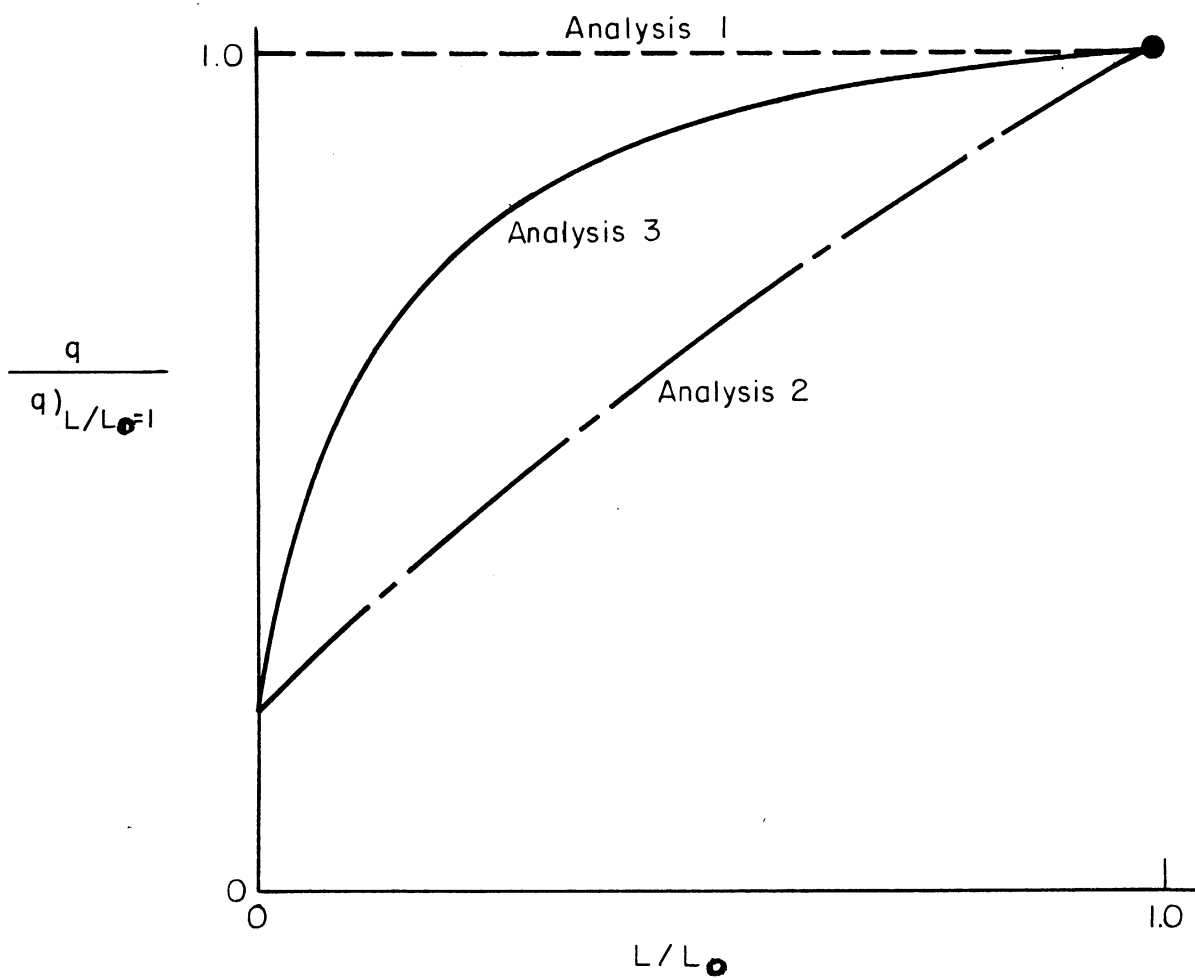
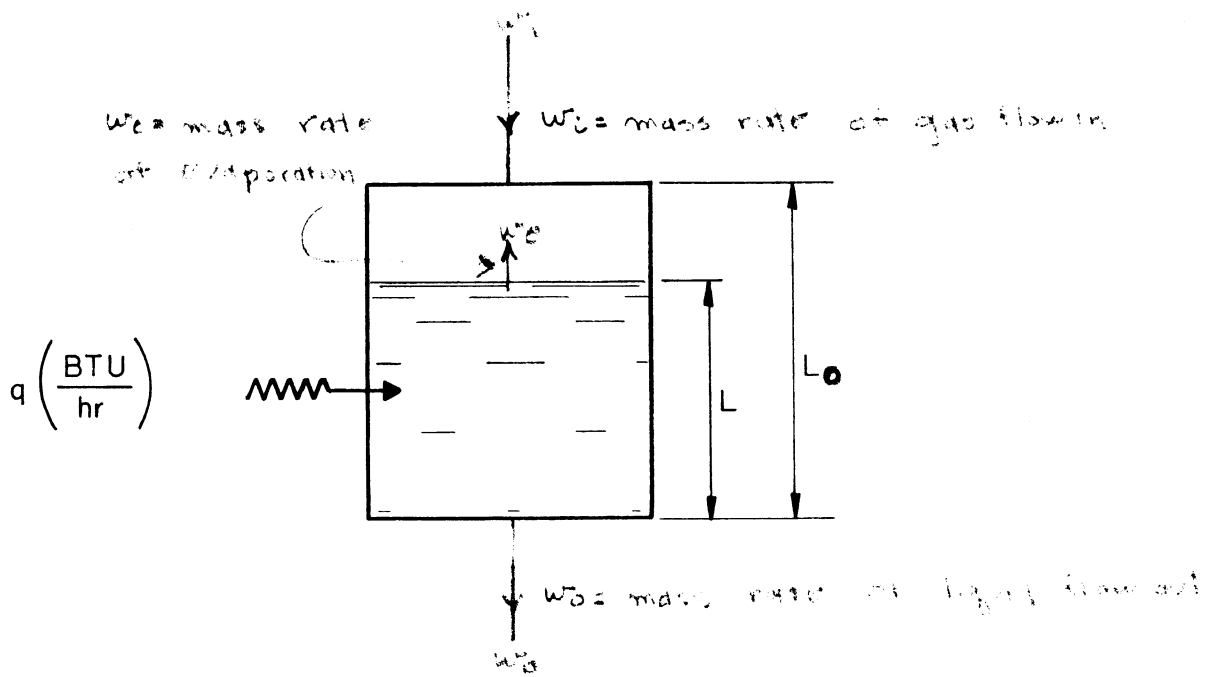


FIG. 19

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