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PRESSURIZATION OF LIQUID OXYGEN CONTAINERS

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ABSTRACT

A new tank design which permits a more accurate determination of residual mass is described. This design also allows for measurement of liquid-vapor hold-up in a boiling system using a floating piston.

An analysis of the thermodynamic behavior of a pressurizing gas in a closed container having one receding boundary is given. It is shown that the thermodynamic behavior of the pressurizing gas can be expressed as a function of four dimensionless moduli:

$$\frac{\bar{t}(\theta) - t_i}{t_o - t_i}, \quad a\theta, \quad \frac{\bar{h}_g}{(w_i/A)C_p}, \quad \text{and} \quad \frac{y_p}{D}.$$

The result includes the effects of transient heat flow interaction between the gas and all the surfaces which it wets, including the receding boundary.

Further results are given on the study of pool boiling in an accelerating system.

NOMENCLATURE

a	$H^2\alpha$, 1/hr
A	Cross-sectional area of container = $(\pi/4)D^2$, ft^2
C_p	Constant pressure specific heat of pressurizing gas, Btu/lbm °F
C_v	Constant volume specific of pressurizing gas, Btu/lbm °F
C	Inside circumference of container, ft
D	Inside diameter of container, ft
e	Internal energy of gas, Btu/lbm
$\bar{h}_g$	Convection heat-transfer coefficient, Btu/sq ft-°F-hr
h	Enthalpy of gas, Btu/lbm
H	$\bar{h}_g/k$, ft^{-1}
k	Thermal conductivity, Btu-ft/hr-sq ft-°F
m	Mass of gas in control volume, lbm
n	Outward normal to the control surface with n = 0 at control surface, ft
P_x	Rate of work transfer, Btu/hr
p	pressure of gas, lbf/sq ft
q	Rate of heat transfer, Btu/hr
R	Gas constant
$\bar{t}$	Spacially mean temperature of pressurizing gas in control volume, °F
v	Specific volume of gas, ft^3/lbm
V	Velocity, ft/hr
Y	Coordinate, ft; see Fig. 4
w	Mass flow rate, lbm/hr

NOMENCLATURE
(Concluded)

t_i Inlet temperature of pressurizing gas, °F

t_o Initial temperature of container and receding boundary, °F

t Temperature of surroundings, function of n and θ, °F

α Thermal diffusivity, $k/\rho C_p$, ft^2/hr

β Angle between velocity of flow across the control surfaces and the outward drawn normal, n

∂ Partial differential operator, 1

ρ Density of gas, lbm/ft^3

θ Time, hr

θ^* Wetted time, hr

T Temperature of gas, degrees absolute, °R

Subscripts

T Top of container

w Wall of container

p Receding boundary

i Inlet stream of pressurizing gas

I. STUDY OF RESIDUAL GAS MASS

A. EXPERIMENTAL APPARATUS

To study the effect of a variation in heat flux on the quantity of residual gas mass remaining in the tank after discharge, a reliable and accurately monitored electrical system (Fig. 1) has been introduced for heating the tank. The tank has been assembled as shown in Fig. 2. Next to the tank is a 1/32-in.-thick layer of asbestos as a secondary insulation. The Fiberglas-covered Midohm high-resistance wire is wound over the asbestos. This wire has been wound to provide four independently controllable heating sections, each section having 17.4 ohms ± 0.1 ohm resistance. Between the heating wire and a 2-in. layer of Fiberglas blanket is a second layer of asbestos 1/32 in. thick as a fire preventative in case of an electrical short-circuit. This whole assembly is then encased in a 3-in.-thick styrofoam jacket. This system has been designed to minimize the amount of heat leakage.

The power requirements are controlled by four individual 'Variac' transformers. The input power is monitored by a voltmeter and ammeter for each heater section. The power that is recorded, however, is obtained by means of a suitable switching arrangement (Fig. 1), connecting each section to a single Weston Wattmeter. The maximum heat flux that the present system will operate at using 220 v and 12.7 amp is approximately 4500 Btu/hr-ft^2 which is approximately 4-1/2 times the heat flux caused by heat transfer with the ambient. This system can be run at 2-1/2 times the above maximum heat flux or, at a level of 11,250 Btu/hr-ft^2, by increasing the input voltage to 440 v.

To improve the accuracy of the residual-gas-mass calculations, eight levels of thermocouples are being utilized instead of the five levels previously used.

This system has been designed to function with or without a floating piston. Without a piston the liquid level indicator is utilized by means of a very small bob which floats on the interface and provides the necessary electrical contacts. With a piston the system operates as before. This system will effectively isolate effects caused by the liquid-vapor interface on the residual gas mass. It will also enable a more complete study of vapor-liquid hold-up measured as a function of liquid fill and heat flux.

B. AN ANALYSIS OF THE THERMODYNAMIC BEHAVIOR OF A PRESSURIZING GAS IN A CLOSED CONTAINER HAVING ONE RECEDING BOUNDARY

As indicated in Progress Report No. 15, an analysis of the thermodynamic behavior of the pressurizing gas used in discharging the liquid from a container is reported. This analysis considers the effects of heat transfer between the pressurizing gas and the surfaces which it wets, including the effects of the receding boundary. The analysis will be for the general case and the results obtained will be applicable to the pressurized discharge of liquid oxygen from a liquid-fueled rocket. In this application the main consideration will be the determination of the principal quantities which govern the residual mass of pressurizing gas.

The analytical model studied and the "Control Volume"* used are shown in Fig. 3. The analytical model is characterized by the following:

(1) the mass of gas initially in the control volume is zero;
(2) the mass rate of gas (w_i) introduced into the control volume is constant;
(3) the walls and top of the container and the receding boundary behave as semi-infinite solids exchanging heat with a gas at the inlet-gas temperature through a coefficient of heat transfer, $\bar{h}_g$; and
(4) the temperature of the gas in the control volume is a spatially mean temperature which changes with time.

The control volume used is the volume containing the pressurizing gas in the container. This volume is changing with time as the receding boundary moves towards the bottom of the container. Further refinements in the model will be made when the results of this analysis are compared with the experimental data to be obtained from the physical system now being prepared.

For transient conditions of the control volume, the energy and mass relationships may be expressed by the First Law of Thermodynamics, Eq. (1), and the Law of Conservation of Mass, Eq. (2), both in instantaneous rate form.

$$\frac{\partial}{\partial \theta} \int_V e dm + \int_A h \rho V \cos \beta \, dA = q - P_x \quad (1)$$

$$\frac{\partial}{\partial \theta} \int_V dm + \int_A \rho V \cos \beta \, dA = 0 \quad (2)$$

The integrals of Eqs. (1) and (2) are over the volume and area, respectively, of the control volume. In Eq. (1) the term q is the net heat-transfer rate between the control volume and the surroundings, and the term P_x is the net work-flow rate between the control volume and the receding boundary.

For the model described above, Eq. (2) becomes

*Shapiro, A. H., Mechanics and Thermodynamics of Compressible Fluids, Vol. I, Roanald Press, 1953.

$$\frac{\partial m}{\partial \theta} - w_i = 0$$

or (3)

$$\frac{\partial m}{\partial \theta} = w_i \quad .$$

In Eq. (1),

$$\frac{\partial}{\partial \theta} \int_V edm = \frac{\partial}{\partial \theta}(em) = e\frac{\partial m}{\partial \theta} + m\frac{\partial e}{\partial \theta} = e\frac{\partial m}{\partial \theta} + m\frac{\partial e}{\partial \bar{T}}\left(\frac{\partial \bar{t}}{\partial \theta}\right) \quad . \qquad (4)^*$$

And using the Law of Conservation of Mass in the form of Eq. (3), and assuming the gas to behave as a perfect gas with constant specific heat at constant volume, Eq. (4) becomes

$$\frac{\partial}{\partial \theta} \int_V edm = ew_i + m\, C_v \frac{\partial \bar{t}}{\partial \theta} \quad . \qquad (5)$$

Also in Eq. (1)

$$\int_A h\rho\, V \cos \beta\, dA = -\, w_i\, h_i \quad , \qquad (6)$$

and

$$P_x = p\, V_p\, A_p = p\frac{dV}{d\theta} \quad . \qquad (7)$$

But noting that

$$\frac{dm}{d\theta} = w_i = \frac{d}{d\theta}(\rho V) = \rho\frac{\partial V}{\partial \theta} + V\frac{\partial \rho}{\partial \theta} \quad ,$$

is, when rearranged,

$$\frac{\partial V}{\partial \theta} = \frac{1}{\rho} w_i - \frac{V}{\rho}\frac{\partial \rho}{\partial \theta} \quad . \qquad (7a)$$

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*Actually

$$\frac{\partial e}{\partial \theta} = \left(\frac{\partial e}{\partial t}\right)_v \frac{\partial \bar{t}}{\partial \theta} + \left(\frac{\partial e}{\partial v}\right)_{\bar{t}} \frac{\partial v}{\partial \theta} \quad ,$$

but for a gas at low pressure the term $(\partial e/\partial v)_{\bar{t}}$ is vanishingly small and is neglected.

Also,

$$\rho = p/RT$$

or

$$\frac{\partial \rho}{\partial \theta} = \frac{\partial}{\partial \theta}\left(\frac{p}{RT}\right) = -\frac{p}{RT^2}\frac{\partial \bar{t}}{\partial \theta}$$

Now Eq. (7a) becomes

$$\frac{\partial V}{\partial \theta} = \frac{1}{\rho} w_i + \frac{V}{\rho} \cdot \frac{p}{RT} \cdot \frac{1}{T} \frac{\partial \bar{t}}{\partial \theta} = \frac{1}{\rho} w_i + \frac{V}{T}\frac{\partial \bar{t}}{\partial \theta} .$$

With this result, Eq. (7) becomes

$$P_x = \frac{p}{\rho} w_i + \frac{pV}{T}\frac{\partial \bar{t}}{\partial \theta} = pv\ w_i + mR\frac{\partial \bar{t}}{\partial \theta} . \qquad (8)$$

In Eq. (8) P_x is the rate of work being done by the gas in the control volume on the receding boundary. Hence Eq. (1) may be written

$$e\ w_i + m\ C_v \frac{\partial \bar{t}}{\partial \theta} - w_i\ h_i + pv\ w_i + mR\frac{\partial \bar{t}}{\partial \theta} = q$$

or

$$w_i\ (e + pv - h_i) + m\ (C_v + R)\frac{\partial \bar{t}}{\partial \theta} = q . \qquad (9)$$

Now,

$$h \equiv e + pv , \qquad (10)$$

So Eq. (9) becomes,

$$w_i(h-h_i) + m(C_v+R)\frac{\partial \bar{t}}{\partial \theta} = q . \qquad (11)$$

Again using perfect gas relations, Eq. (11) becomes

$$w_i\ C_p(\bar{t}-t_i) + m\ C_p\frac{\partial \bar{t}}{\partial \theta} = q . \qquad (12)$$

Upon integrating Eq. (3), the Law of Conservation of Mass, and applying the initial condition of zero mass, we get

$$m = w_i \theta \quad . \tag{13}$$

Now Eq. (12) may be written

$$w_i \; C_p(\bar{t}-t_i) + w_i\theta \; C_p \frac{\partial \bar{t}}{\partial \theta} = q$$

or

$$w_i C_p \left[(\bar{t}-t_i) + \theta \frac{\partial \bar{t}}{\partial \theta} \right] = q \quad . \tag{14}$$

Now letting

$$\zeta \equiv \bar{t} - t_i \quad ,$$

Eq. (14) becomes

$$w_i C_p \left[\zeta + \theta \frac{\partial \zeta}{\partial \theta} \right] = q$$

or (15)

$$w_i C_p \frac{\partial}{\partial \theta} (\zeta \theta) = q \quad .$$

Upon integration, Eq. (15) becomes

$$\zeta\theta - (\zeta\theta)_o = \frac{1}{w_i C_p} \int_o^\theta q \, d\theta \quad ,$$

or, since $(\zeta\theta)_o = 0$,

$$w_i C_p \theta (\bar{t}-t_i) = \int_o^\theta q \, d\theta \quad . \tag{16}$$

The problem which remains is to find an expression for q or q/A. Using the heat-conduction equation for the control surface as a thermodynamic system, we have

$$\frac{q}{A} = k \left(\frac{\partial t}{\partial n} \right)_o \quad , \tag{17}$$

where q/A is the heat flux from the surroundings to the control volume, k is the thermal conductivity of the surrounding material, and $(\partial t/\partial n)_o$ is the gradient of the temperature in the surroundings at the control surface where n is the outward normal to the control surface. The coordinate n has a zero value at the control surface.

From Eq. (16) the heat quantity shown as the integral may be written for all surfaces wetted by the gas, as:

$$\int_0^\theta q\,d\theta = \int_0^\theta q_p d\theta + \int_0^\theta q_T d\theta + \int_0^\theta q_w d\theta \quad , \tag{18}$$

where

$\int_0^\theta q_p d\theta \equiv$ Heat transferred in time θ between the pressurizing gas and the receding boundary,

$\int_0^\theta q_T d\theta \equiv$ Heat transferred in time θ between the pressurizing gas and the top of the container,

and $\int_0^\theta q_w d\theta \equiv$ Heat transferred in time θ between the pressurizing gas and the side walls of the container, the wetted areas of which vary with time.

Equation (18) can also be written as

$$\int_0^\theta q\,d\theta = \left[\int_0^\theta kA\left(\frac{\partial t}{\partial n}\right)_0 d\theta\right]_p + \left[\int_0^\theta kA\left(\frac{\partial t}{\partial n}\right)_0 d\theta\right]_T + \int_0^\theta q_w d\theta \quad . \tag{19}$$

In Eq. (19) the gradients are functions of time. In this type of transient the temperature gradient $(\partial t/\partial n)_0$ is taken as that at the surface of a semi-infinite homogeneous solid exchanging heat with a fluid through a coefficient of heat transfer $\bar{h}_g$. The fluid is assumed to undergo a step-change in temperature at zero time. The temperature profile for this case is given by Carslaw and Jaeger* to be

$$\frac{t-t_0}{t_i-t_0} = \mathrm{erfc}\left(\frac{n}{2\sqrt{\alpha\theta}}\right) - e^{Hn+H^2\alpha\theta}\,\mathrm{erfc}\left(\frac{n}{2\sqrt{\alpha\theta}} + H\sqrt{\alpha\theta}\right) \quad . \tag{20}$$

Hence the gradient at n = 0 becomes

$$\left(\frac{\partial t}{\partial n}\right)_0 = \left[\frac{\partial}{\partial n}(t-t_0)\right]_0$$

$$= (t_i-t_0)\frac{\partial}{\partial n}\left[\mathrm{erfc}\left(\frac{n}{2\sqrt{\alpha\theta}}\right) - e^{Hn+H^2\alpha\theta}\,\mathrm{erfc}\left(\frac{n}{2\sqrt{\alpha\theta}} + H\sqrt{\alpha\theta}\right)\right]_0$$

$$\left(\frac{\partial t}{\partial n}\right)_0 = -(t_i-t_0)\,[H\,e^{H^2\alpha\theta}\,\mathrm{erfc}\,(H\sqrt{\alpha\theta})] \quad . \tag{21}$$

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*Carslaw, H. S., and Jaeger, J. C., Conduction of Heat in Solids, Oxford Univ. Press, 1950, p. 53, Eq. (5).

Therefore, in Eq. (19), the heat transferred at the receding boundary is written*

$$\left[\int_0^\theta kA\left(\frac{\partial t}{\partial n}\right)_0 d\theta\right]_p = -k_pA_p(t_i-t_o)\int_0^\theta [H e^{H^2\alpha\theta} \operatorname{erfc}(H\sqrt{\alpha\theta})]d\theta \quad (22)$$

$$= -(t_i-t_o)A_p\left[\frac{\bar{h}_g e^{a\theta}}{a}\operatorname{erfc}(\sqrt{a\theta}) - \frac{\bar{h}_g}{a} + 2\bar{h}_g\sqrt{\frac{\theta}{\pi a}}\right]_p \quad (23)^{**}$$

Also in Eq. (19), the heat transferred at the top of the container is written*

$$\left[\int_0^\theta kA\left(\frac{\partial t}{\partial n}\right)_0 d\theta\right]_T = -k_TA_T(t_i-t_o)\int_0^\theta [H e^{H^2\alpha\theta} \operatorname{erfc}(H\sqrt{\alpha\theta}]_T d\theta \quad (24)$$

$$= -(t_i-t_o)A_T\left[\frac{\bar{h}_g e^{a\theta}}{a}\operatorname{erfc}(\sqrt{a\theta}) - \frac{\bar{h}_g}{a} + 2\bar{h}_g\sqrt{\frac{\theta}{\pi a}}\right]_T \quad (25)^{***}$$

The last integral in Eq. (19) represents the total heat quantity exchanged between the pressurizing gas and the container walls in time θ. However, all portions of the wall surfaces wetted at time θ have not equally contributed to the total heat quantity, simply because all portions of the surface have been exposed to the gas for differing periods of time. This variable heat-exchange characteristic is a direct consequence of the receding boundary continually exposing new surface. The evaluation of this process is carried out with an integration on time using the wetted time, θ*, as the time variable. The wetted time, θ*, is not the same as the discharge time θ, except for the surface at the very top of the container. At all other locations along the container, $\theta^* < \theta$ and is identically equal to zero at the position corresponding to the level of the receding boundary. This may be seen with reference to Fig. 4. Thus,

$$\theta^* \equiv \frac{y_p - y}{V_p}, \quad y \leqq y_p, \quad (26)$$

and

$$\theta^* \equiv 0, \quad y \geqq y_p. \quad (27)$$

*This assumes that no gas condenses at this interface.

**Integration of Eq. (22) to obtain Eq. (23) may be found in the Appendix, p. 12.

***Integration of Eq. (24) to obtain Eq. (25) is identical to that for Eq. (22) to obtain Eq. (23), which may be found in the Appendix, p. 12.

For this reason the last integral in Eq. (19) may be written

$$\int_0^{\theta} q_w \, d\theta \;=\; \int_0^{\theta^*} q_w \, d\theta \;=\; \int_0^{A} \int_0^{\theta^*} k \left(\frac{\partial t_w}{\partial n}\right)_0 d\theta^* \, dA \quad . \tag{28}$$

The differential area, dA, for heat transfer at any position $y \leqq y_p$ is C dy, where C is the circumference of the container. Hence, Eq. (28) is written

$$\int_0^{\theta} q_w \, d\theta \;=\; k_w \, C \int_0^{y_p} \left[\int_0^{\theta^*} \left(\frac{\partial t}{\partial n}\right)_0 d\theta^* \right]_w dy \quad . \tag{29}$$

The gradient $(\partial t/\partial n)_0$ is given by Eq. (21). Equation (29) becomes

$$\int_0^{\theta} q_w \, d\theta \;=\; -\,(t_i - t_o) k_w \, C \int_0^{y_p} \left\{ \int_0^{\theta^*} [H \, e^{H^2 \alpha \theta^*} \operatorname{erfc} (H \sqrt{\alpha\theta^*})] d\theta^* \right\} dy \quad . \tag{30}$$

It should be noted that the right-hand side of Eq. (29) is analogous to the double integral obtained when determining the volume of a solid. In this case the volume of the solid shown in Fig. 5 is the total heat quantity transferred to the walls from the control volume as is seen from Eq. (31) or Eq. (33). For the plane y equal to zero the time θ^* is equal to the discharge time θ

Upon integration, Eq. (30)* becomes,

$$\int_0^{\theta} q_w \, d\theta \;=\; -\,(t_i - t_o) C \, V_p \left[\frac{\bar{h}_g \, e^{a(Y_p/V_p)}}{a^2} \operatorname{erfc} \sqrt{a(Y_p/V_p)} \;-\; \frac{\bar{h}_g}{a^2} \right.$$

$$\left. +\; \frac{2\,\bar{h}_g}{a} \sqrt{\frac{Y_p}{a\pi V_p}} \;-\; \frac{\bar{h}_g \, Y_p}{a \, V_p} \;+\; \frac{4a \, \bar{h}_g \, Y_p^{3/2}}{3a \sqrt{a\pi V_p^3}} \right]_w \tag{31}$$

In the above determination of the heat-transfer interaction between the control volume and the wall a relation is established between the position of the receding boundary and time θ, which is

$$Y_p \;=\; V_p \, \theta \quad . \tag{32}$$

Using Eq. (32), Eq. (31) becomes

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*Integration of Eq. (30) to obtain Eq. (31) may be found in the Appendix, p. 13.

$$\int_0^\theta q_w \, d\theta = -(t_i - t_o) \frac{C\,V_p}{a_w} \left[\frac{\bar{h}_g \, e^{a\theta}}{a} \operatorname{erfc}(\sqrt{a\theta}) - \frac{\bar{h}_g}{a} \right.$$

$$\left. + 2\,\bar{h}_g \sqrt{\theta/a\pi} - \bar{h}_g\theta + 4/3\,\bar{h}_g \sqrt{a\theta^3/\pi} \right]_w . \qquad (33)$$

Equation (16) may now be written, with the use of Eqs. (19), (23), (25), and (33), as

$$w_i C_p \theta(\bar{t} - t_i) = (t_o - t_i) A_p \left[\frac{\bar{h}_g \, e^{a\theta}}{a} \operatorname{erfc}(\sqrt{a\theta}) - \frac{\bar{h}_g}{a} \right.$$

$$\left. + 2\,\bar{h}_g \sqrt{\theta/\pi a} \right]_p + (t_o - t_i) A_T \left[\frac{\bar{h}_g \, e^{a\theta}}{a} \operatorname{erfc}(\sqrt{a\theta}) \right.$$

$$\left. - \frac{\bar{h}_g}{a} + 2\,\bar{h}_g \sqrt{\theta/\pi a} \right]_T + (t_o - t_i) \frac{C\,V_p}{a_w} \left[2\,\bar{h}_g \sqrt{\theta/a\pi} \right.$$

$$\left. + \frac{\bar{h}_g \, e^{a\theta}}{a} \operatorname{erfc}(\sqrt{a\theta}) - \frac{\bar{h}_g}{a} - \bar{h}_g\theta + 4/3\,\bar{h}_g \sqrt{a\theta^3/\pi} \right]_w \qquad (34)$$

or, upon algebraic rearrangement, and assuming $\bar{h}_g$ is the same for all surfaces,

$$\frac{\bar{t}(\theta) - t_i}{t_o - t_i} = \frac{\bar{h}_g}{(w_i/A) C_p} \left\{ \frac{e^{a_p\theta}}{a_p\theta} \operatorname{erfc}(\sqrt{a_p\theta}) - \frac{1}{a_p\theta} + \frac{2}{\sqrt{a_p\theta\pi}} \right.$$

$$+ \frac{e^{a_T\theta}}{a_T\theta} \operatorname{erfc}(\sqrt{a_T\theta}) - \frac{1}{a_T\theta} + \frac{2}{\sqrt{a_T\theta\pi}}$$

$$+ \frac{4(Y_p/D)}{(a_w\theta)^2} \left[e^{a_w\theta} \operatorname{erfc}(\sqrt{a_w\theta}) + 2\sqrt{a_w\theta/\pi} \right.$$

$$\left.\left. + 4/3\sqrt{(a_w\theta)^3/\pi} - a_w\theta - 1 \right] \right\} , \qquad (35)$$

where $a = H^2\alpha$. Equation (35) is in a dimensionless form with the significant dimensionless moduli;

$$\frac{\bar{t}(\theta) - t_i}{t_o - t_i} , \quad \frac{\bar{h}_g}{(w_i/A) C_p} , \quad a\theta , \quad \text{and} \quad Y_p/D .$$

Here the quantity $\overline{h}_g/(w_i/A)C_p$ is a modified Stanton Number and the quantity $a\Theta = \alpha\Theta/(k/\overline{h}_g)^2$ is a modified Fourier Number.

Equation (35) describes the thermodynamic response of the pressurizing gas during the process of pressurized discharge of a liquid from a closed container. This result includes the effects of transient heat-flow interaction between the gas and all the surfaces which it wets, including the receding interface. It has been assumed that all surfaces respond thermally as semi-infinite solids, with no gas condensation. Refinements in this analysis to include departure from these conditions will be made subsequently.

Of importance in Eq. (35) is the mean gas temperature shown as a function of time, the thermal properties of the container and the gas, the system geometry, and the conditions of the discharge process. Hence this result permits the computation of the final mean temperature of the pressurizing gas, or its final mean density which is a direct measure of the mass of the residual gas.

It can be seen from Eq. (35) that the temperature of the pressurizing gas remains constant at the inlet temperature if the thermal conductivity of the surroundings is zero. This is to be expected because under such a circumstance there is no interaction between the pressurizing gas and the surroundings.

During the next period, further conclusions will be drawn from this analysis to show the effects of the various system parameters.

C. WORK DURING THE NEXT PERIOD

During the next period difficulties encountered in the design of a new piston level indicator will be eliminated and the piston will be incorporated into the new system. Experimental work will begin on the optimization of residual gas mass for a discharging system with the main emphasis on the effects caused by variations in heat flux. Also, the results of this analysis [i.e., Eq. (35)] will be plotted employing the dimensionless moduli given. These graphs will show the significance of the various system parameters. The graphs will be presented in the next progress report.

II. A STUDY OF POOL BOILING IN AN ACCELERATING SYSTEM

A. CURRENT STATUS OF THE WORK

The experimental work on the present phase of this study has been virtually completed. This consists of measurements made under the following conditions.

(1) Boiling of saturated water from a flat, chrome-plated heating surface 3 in. in diameter.

(2) The heat fluxes of 10,000, 25,000, 50,000, 75,000, and 100,000 Btu/hr-ft^2.

(3) A flow guide was installed over the heating surface to simulate the condition of having the entire bottom surface of the container as the heating surface.

(4) Accelerations were applied normal to the heating surface over the range of one standard gravitational acceleration to 21.15 standard gravitational acceleration.

In addition, at the low values of heat flux it was possible to obtain data on the effect of small degrees of subcooling on the boiling process in an accelerating field.

Figure 6 is a preliminary smoothed plot of the raw data showing the effect of total acceleration upon $t_{wall} - t_{sat}$. The saturation temperature is that at the heating surface.

Figure 7 is a log-log plot of heat flux versus $t_{wall} - t_{sat}$ for representative test runs with the system subjected to one standard gravitational acceleration only. The consistency of these data is in part a measure of the reliability of the power and temperature measuring apparatus.

B. WORK DURING THE NEXT PERIOD

The available data will be thoroughly analyzed. Attempts will be made to put them into a generalized correlation. Reruns will be made as found necessary to check various points.

APPENDIX

INTEGRATION OF EQ. (22) TO OBTAIN EQ. (23) FROM p. 7

$$\left[\int_0^\theta kA\left(\frac{\partial t}{\partial n}\right)_o d\theta\right]_p = -k_pA_p(t_i-t_o)\int_0^\theta [H\, e^{H^2\alpha\theta}\ \mathrm{erfc}\ (H\sqrt{\alpha\theta})]_p\, d\theta \qquad (22)$$

By employing $a = \alpha H^2 = \alpha(\bar{h}_g/k)^2$, Eq. (22) becomes

$$\left[\int_0^\theta kA\left(\frac{\partial t}{\partial n}\right)_o d\theta\right]_p = -A_p\bar{h}_g\ (t_i-t_o)\int_0^\theta e^{a_p\theta}\ \mathrm{erfc}\ (\sqrt{a_p\theta})d\theta\ , \qquad (22a)$$

where the integral can be integrated by parts:

$$\int_0^\theta e^{a_p\theta}\ \mathrm{erfc}\ (\sqrt{a_p\theta})d\theta = \left[\mathrm{erfc}\ (\sqrt{a_p\theta})\cdot\frac{e^{a_p\theta}}{a_p}\right]_o^\theta + \int_0^\theta \frac{d\theta}{\sqrt{a_p\theta\pi}} \qquad (22b)$$

or

$$\int_0^\theta e^{a_p\theta}\ \mathrm{erfc}\ (\sqrt{a_p\theta})d\theta = \left[\mathrm{erfc}\ (\sqrt{a_p\theta})\cdot\frac{e^{a_p\theta}}{a_p} + 2\sqrt{\theta/a_p\pi}\right]_o^\theta \qquad (22c)$$

$$= \frac{e^{a_p\theta}}{a_p}\ \mathrm{erfc}\ (\sqrt{a_p\theta}) - \frac{1}{a_p} + 2\sqrt{\theta/a_p\pi} \qquad (22d)$$

Equation (22) now becomes

$$\left[\int_0^\theta kA\left(\frac{\partial t}{\partial n}\right)_o d\theta\right]_p = -\bar{h}_gA_p(t_i-t_o)\left[\frac{e^{a\theta}}{a}\ \mathrm{erfc}\ (\sqrt{a\theta}) - \frac{1}{a} + 2\sqrt{\theta/a\pi}\right]_p$$

or

$$\left[\int_0^\theta kA\left(\frac{\partial t}{\partial n}\right)_o d\theta\right]_p = -(t_i-t_o)A_p\left[\frac{\bar{h}_g\ e^{a\theta}}{a}\ \mathrm{erfc}\ (\sqrt{a\theta}) - \frac{\bar{h}_g}{a} + 2\ \bar{h}_g\sqrt{\theta/a\pi}\right]_p \qquad (22e)$$

Equation (22e) is identical to Eq. (23), p. 7.

INTEGRATION OF EQ. (30) TO OBTAIN EQ. (31) FROM p. 8

$$\int_0^{\theta} q_w \, d\theta = -(t_i - t_o) k_w C \int_0^{y_p} \left[\int_0^{\theta^*} H \, e^{H^2 \alpha \theta^*} \operatorname{erfc}(H\sqrt{\alpha\theta^*}) d\theta^*\right]_w dy \qquad (30)$$

where

$$\theta^* = \frac{y_p - y}{V_p}, \quad H^2\alpha = a, \quad \text{and} \quad H = \frac{\bar{h}_g}{k}.$$

The inside integral becomes:

$$\int_0^{\theta^*} H \, e^{a_w \theta^*} \operatorname{erfc}(\sqrt{a_w \theta^*}) d\theta^* = H\left[\frac{e^{a_w\theta^*}}{a_w} \operatorname{erfc}(\sqrt{a_w\theta^*}) - \frac{1}{a_w} + 2\sqrt{\frac{\theta^*}{a_w \pi}}\right], \qquad (30a)$$

which was shown in Eqs. (22b,c,d). Now Eq. (30) becomes

$$\int_0^{\theta} q_w d\theta = -(t_i - t_o)\bar{h}_g C \int_0^{y_p} \left[\frac{e^{a_w \frac{y_p - y}{V_p}}}{a_w} \operatorname{erfc}\left(\sqrt{a_w \frac{y_p - y}{V_p}}\right) - \frac{1}{a_w} + 2\sqrt{\frac{y_p - y}{a_w V_p \pi}}\right] dy \qquad (30b)$$

$$= -(t_i - t_o)\bar{h}_g C \left[\frac{e^{\frac{a_w y_p}{V_p}}}{a_w} \int_0^{y_p} \frac{\operatorname{erfc}\left(\sqrt{a_w \frac{y_p - y}{V_p}}\right)}{e^{\frac{a_w y}{V_p}}} dy - \frac{1}{a_w}\int_0^{y_p} dy \right.$$

$$\left. + \frac{2}{\sqrt{a_w V_p \pi}} \int_0^{y_p} \sqrt{y_p - y}\, dy \right] \qquad (30c)$$

In Eq. (30c) (integration by parts),

$$\int_0^{y_p} \frac{\operatorname{erfc}\left(\sqrt{a_w \frac{y_p - y}{V_p}}\right)}{e^{\frac{a_w y}{V_p}}} dy = \left[-\operatorname{erfc}\sqrt{a_w \frac{y_p - y}{V_p}} \cdot \frac{V_p}{a_w} e^{-\frac{a_w y}{V_p}}\right]_0^{y_p}$$

$$+ \frac{e^{-\frac{a_w y_p}{V_p}}}{\sqrt{a_w(\pi/V_p)}} \int_0^{y_p} \frac{dy}{\sqrt{y_p - y}}$$

$$= \left[-\frac{V_p}{a_w} e^{-\frac{a_w y}{V_p}} \operatorname{erfc}\left(\sqrt{a_w \frac{y_p - y}{V_p}}\right) - \frac{2\, e^{-\frac{a_w y_p}{V_p}}}{\sqrt{a_w(\pi/V_p)}} \sqrt{y_p - y} \right]_o^{y_p}$$

$$= -\frac{V_p}{a_w} e^{-\frac{a_w y_p}{V_p}} + \frac{V_p}{a_w} \operatorname{erfc}\left(\sqrt{a_w(y_p/V_p)}\right) + 2\, e^{-\frac{a_w y_p}{V_p}} \sqrt{\frac{y_p V_p}{a_w \pi}} \quad ,$$

and

$$\frac{1}{a_w} \int_o^{y_p} dy = \frac{y_p}{a_w} \quad ,$$

and

$$\int_o^{y_p} \sqrt{y_p - y}\, dy = -\left[\frac{2}{3} (y_p - y)^{3/2} \right]_o^{y_p} = \frac{2}{3} y_p^{3/2} \quad .$$

Now Eq. (30c) becomes

$$\int_o^{\Theta} q_w d\Theta = -(t_i - t_o)\bar{h}_g C \left[-\frac{V_p}{a_w^2} + \frac{V_p\, e^{\frac{a_w y_p}{V_p}}}{a_w^2} \operatorname{erfc}\left(\sqrt{a_w(y_p/V_p)}\right) + \frac{2}{a_w} \sqrt{\frac{y_p V_p}{a_w \pi}} - \frac{y_p}{a_w} + \frac{4}{3} \frac{y_p^3}{a_w V_p \pi} \right] \tag{30d}$$

or

$$\int_o^{\Theta} q_w\, d\Theta = -(t_i - t_o) C V_p \left[\frac{\bar{h}_g\, e^{\frac{a_w y_p}{V_p}}}{a_w^2} \operatorname{erfc}\left[\sqrt{a_w(y_p/V_p)}\right] - \frac{\bar{h}_g}{a_w^2} + \frac{2\bar{h}_g}{a_w} \sqrt{\frac{y_p}{a_w V_p \pi}} - \frac{\bar{h}_g y_p}{a_w V_p} + \frac{4}{3} \cdot \frac{a_w \bar{h}_g\, y_p^{3/2}}{a_w \sqrt{a_w \pi V_p^3}} \right] \tag{30e}$$

Equation (30e) is identical to Eq. (31), p. 8.

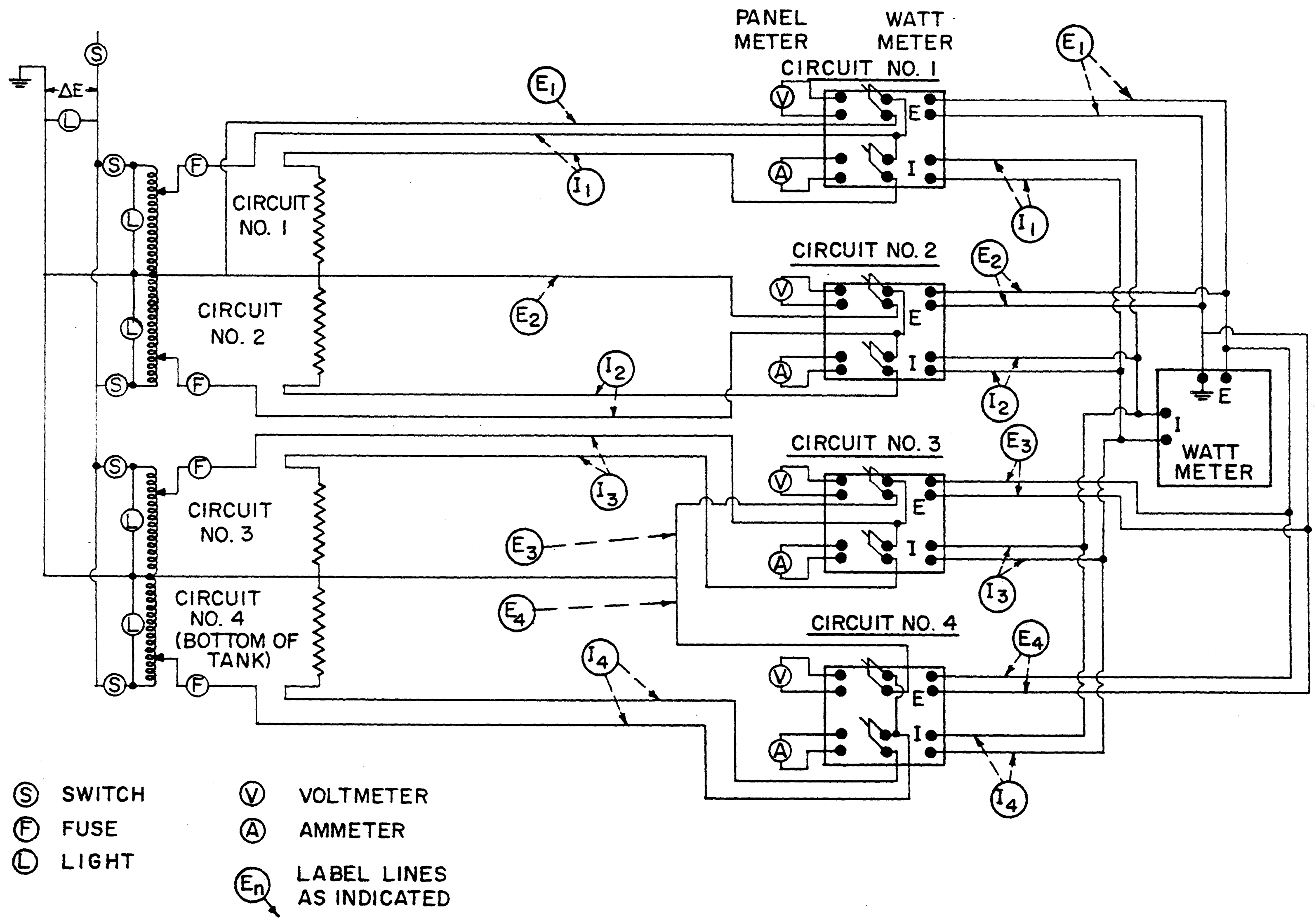


Fig. 1-Resistance heating wiring diagram

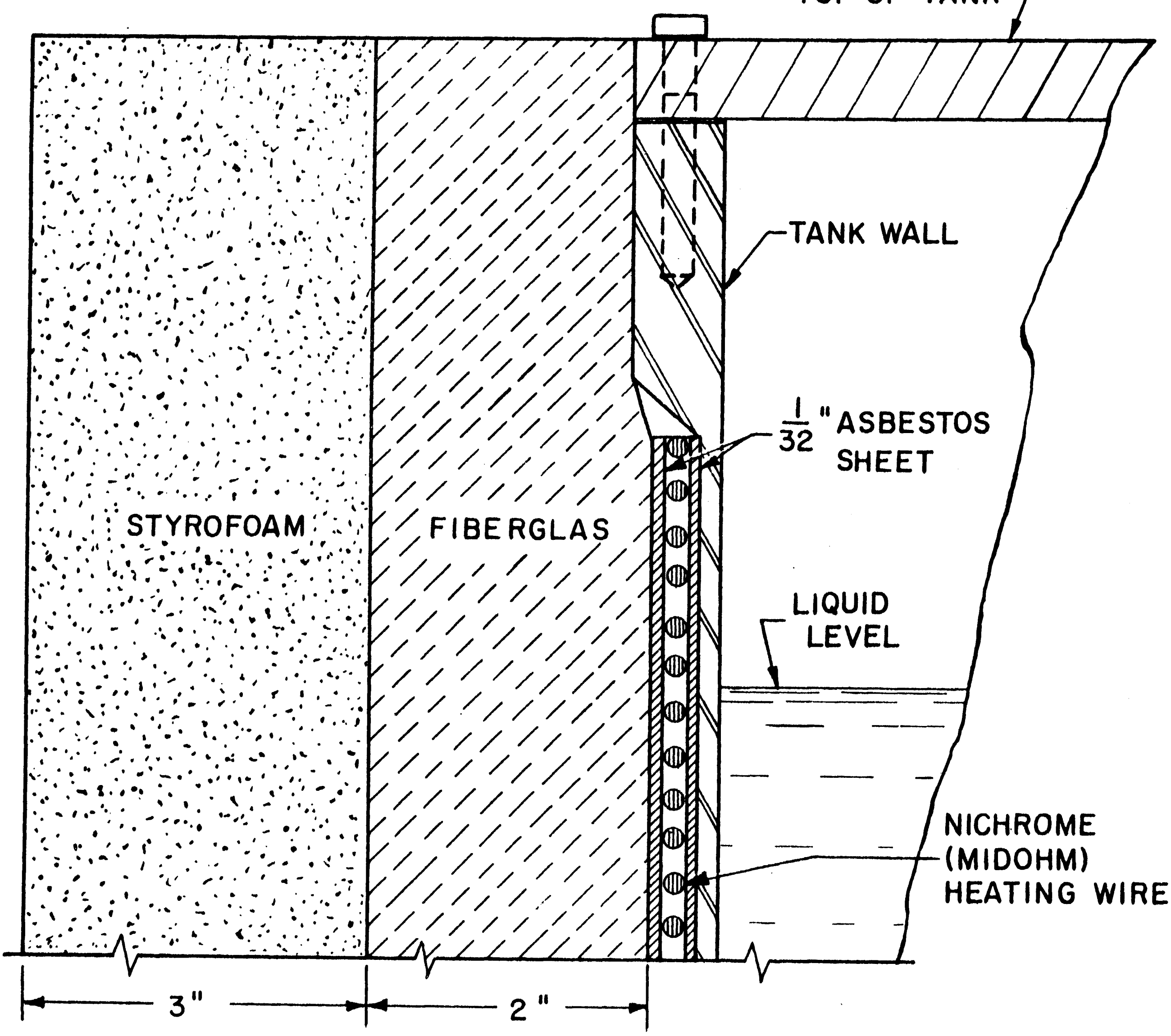


Fig. 2 - Heater wire and insulation assembly

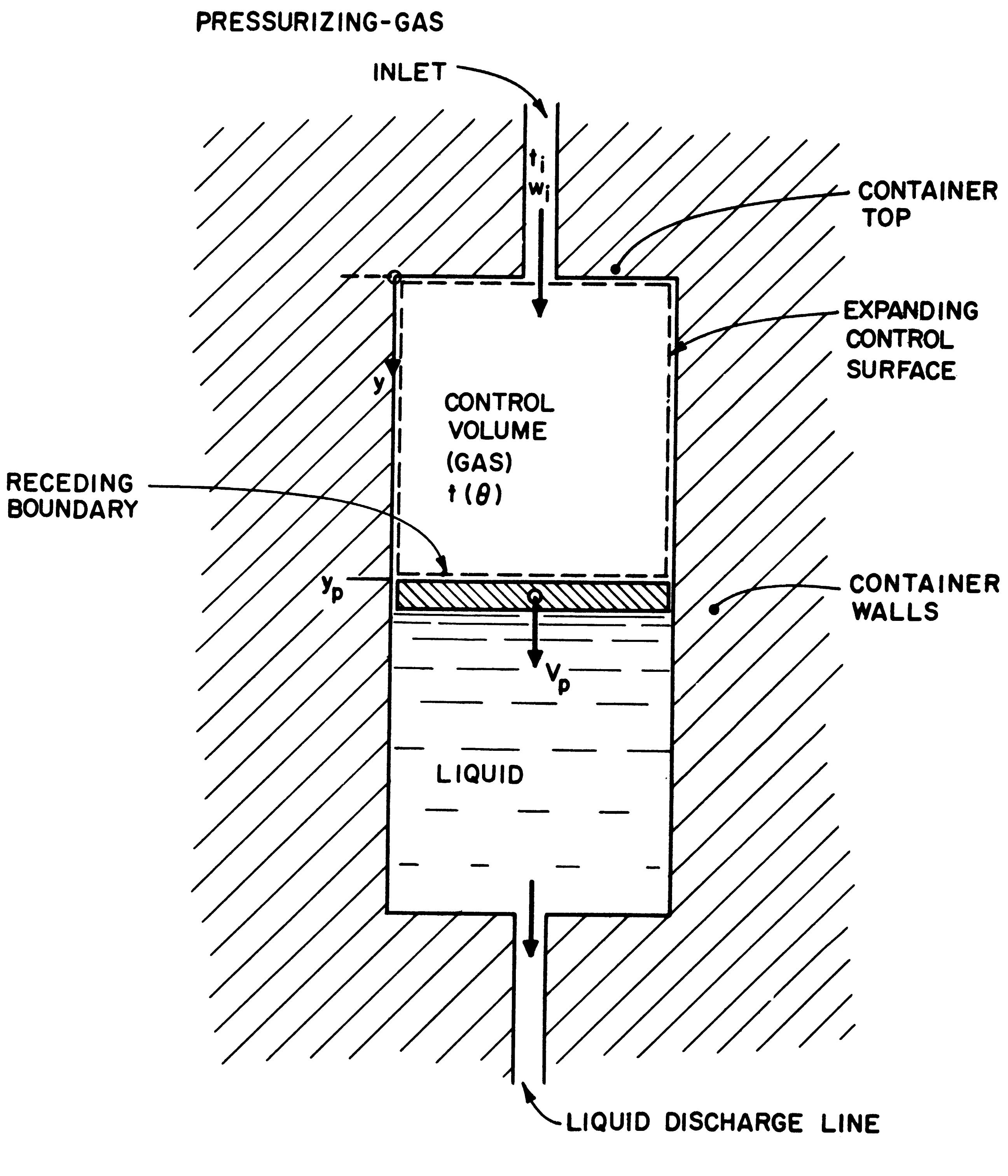


Fig. 3 - Control volume and system for thermodynamic analysis

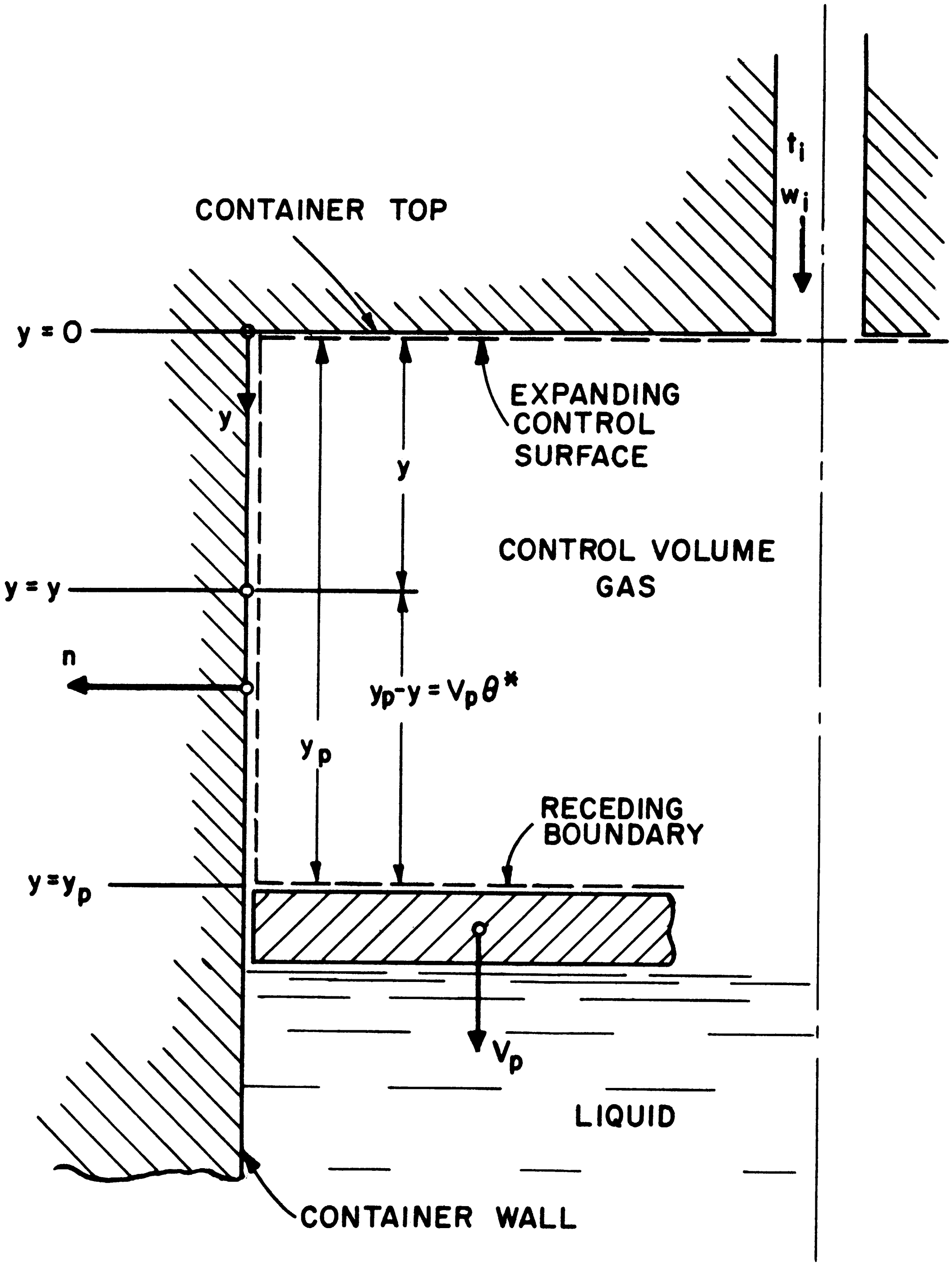


Fig. 4 - Control volume for thermodynamic analysis

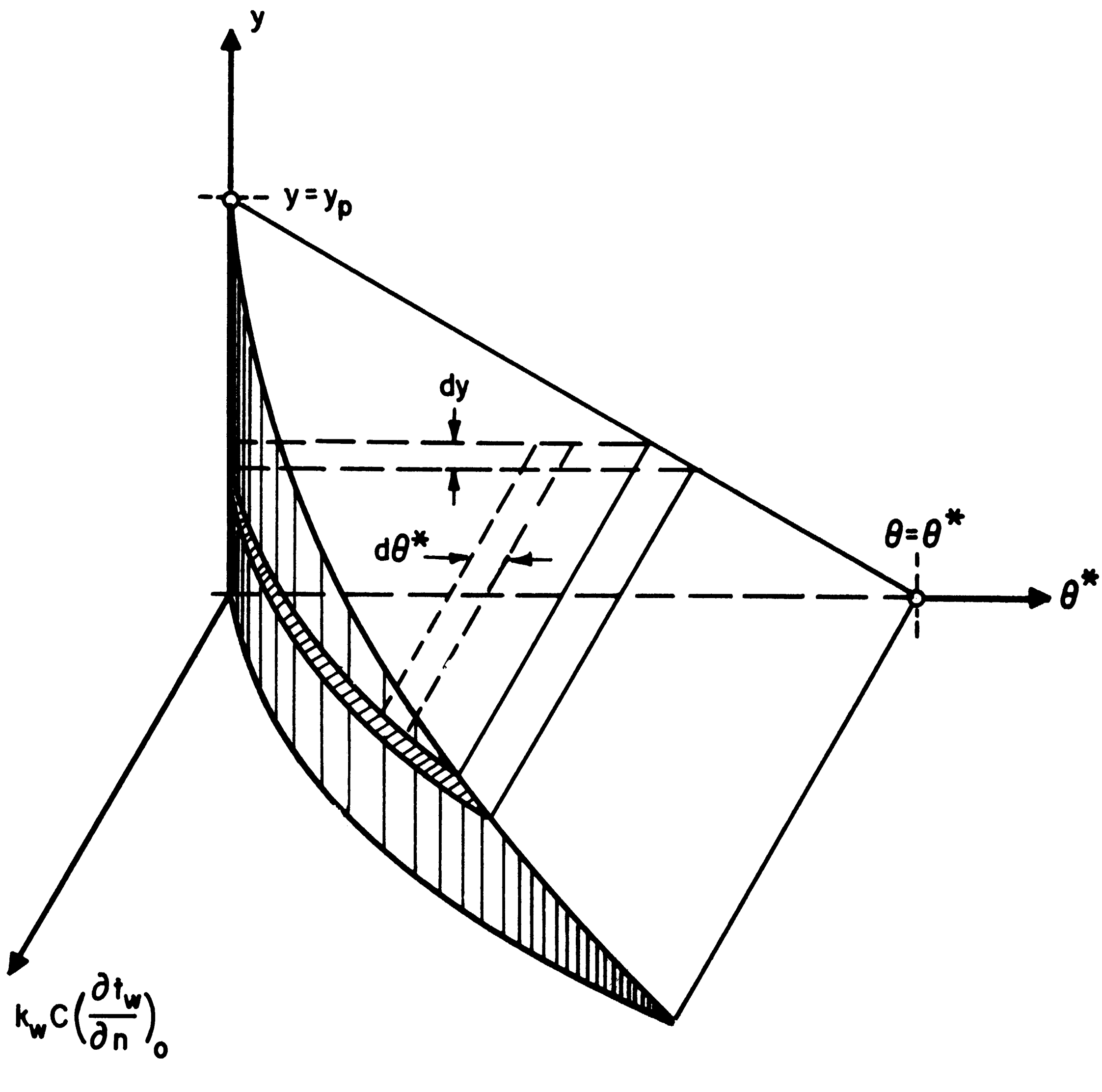


Fig. 5 - Graphical representation of Eq. (29)

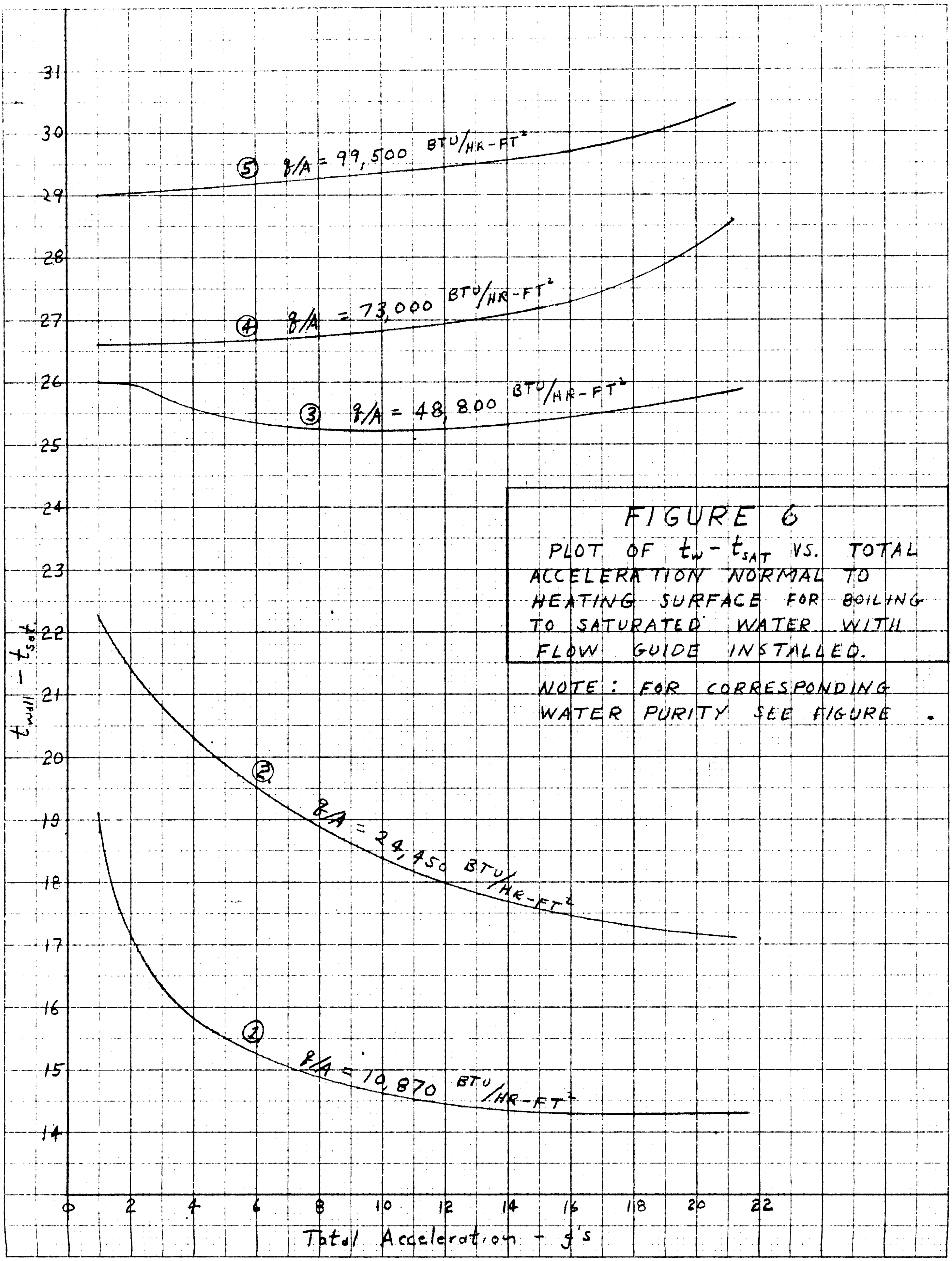


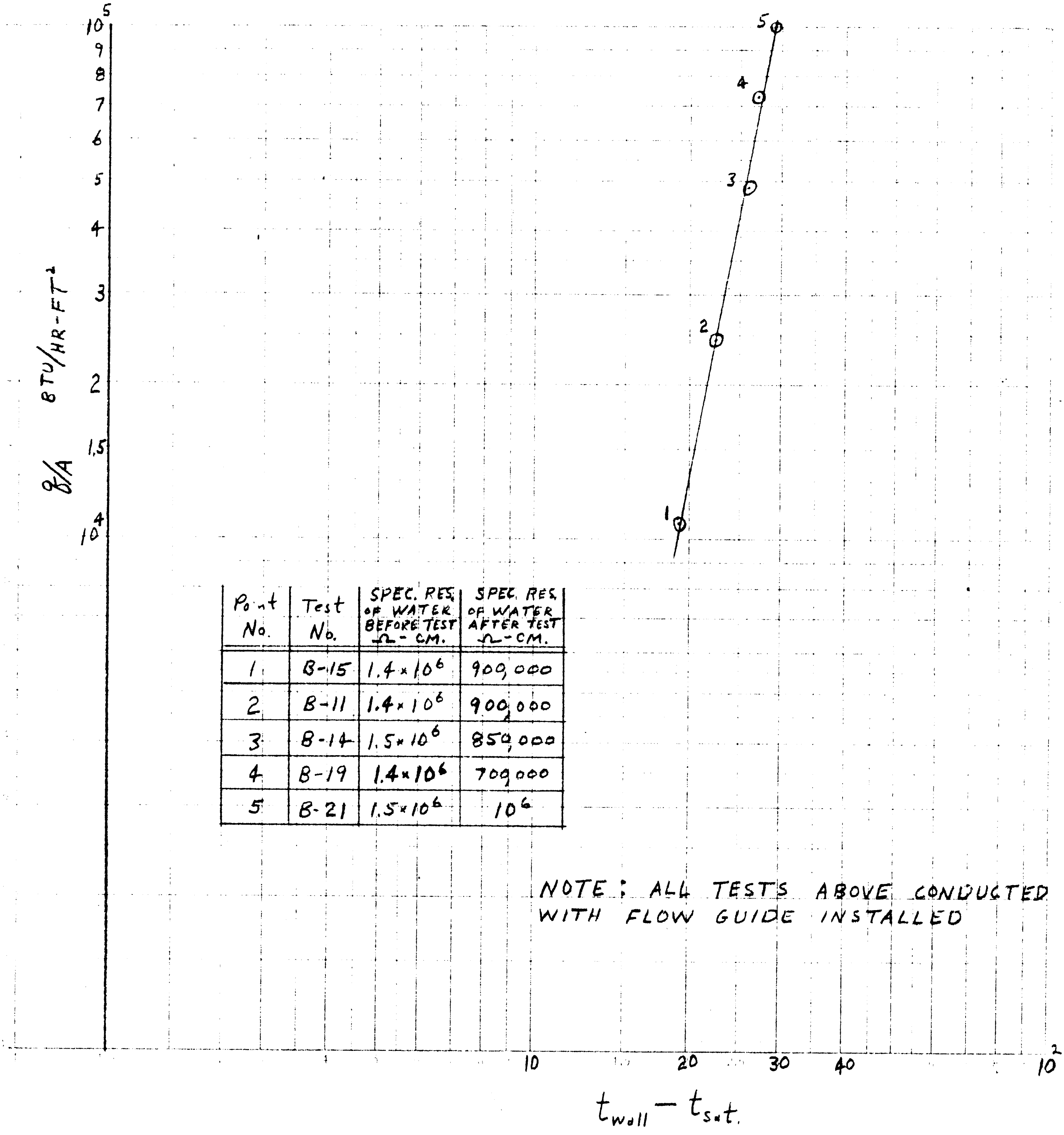
FIGURE 6

PLOT OF $t_w - t_{SAT}$ VS. TOTAL ACCELERATION NORMAL TO HEATING SURFACE FOR BOILING TO SATURATED WATER WITH FLOW GUIDE INSTALLED.

NOTE: FOR CORRESPONDING WATER PURITY SEE FIGURE .

FIGURE 7

HEAT FLUX VS $t_{WALL} - t_{SAT.}$ FOR SATURATED BOILING OF DISTILLED WATER WITH SYSTEM SUBJECTED TO STANDARD GRAVITATIONAL ACCELERATION.



Point No.	Test No.	SPEC. RES. OF WATER BEFORE TEST Ω-CM.	SPEC. RES. OF WATER AFTER TEST Ω-CM.
1	B-15	1.4×10^6	900,000
2	B-11	1.4×10^6	900,000
3	B-14	1.5×10^6	850,000
4	B-19	1.4×10^6	709,000
5	B-21	1.5×10^6	10^6