

PREFACE  
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CONNECTING RESEARCH, PRACTICE AND THEORY IN THE  
DEVELOPMENT AND STUDY OF MATHEMATICS EDUCATION<sup>1</sup>

The gap between theory and practice is not new. Neither is the divide between mathematics and mathematics education, nor between the professionals who work in these fields. Emerging, however, are new efforts to build connections, focused on problems of practice, in order to develop and study mathematics education. These endemic problems and new perspectives on their resolution are the subject of this special issue of *Educational Studies in Mathematics*.

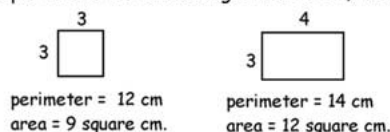
Consider the following well-known case: Liping Ma's (1999) study of Chinese and U.S. primary teachers of mathematics. With *Knowing and Teaching Mathematics*, Ma brought to the world's attention a critical problem of teaching quality. Based on structured interviews with primary teachers in China and the United States, Ma painted contrasting portraits of the ways in which these two groups know mathematics for their work as teachers. The interview tasks (Ball, 1988; Ma, 1999) were framed by the sorts of questions that come up in the course of teaching: How would you explain the procedure for subtracting two-digit numbers to second graders? How would you deal with students in your class who were having the following difficulty with the multiplication algorithm:

$$\begin{array}{r} 123 \\ \times 645 \\ \hline 615 \\ 492 \\ 738 \\ \hline 1845 \end{array}$$

Or, if a student in your class made this claim about perimeter and area, how would you respond?



As the perimeter of a rectangle increases, its area also increases:



Teachers' responses in Ma's study differed markedly. The U.S. teachers often fumbled, sometimes focusing only on the steps of the procedure, and sometimes distorting key mathematical issues. Chinese teachers more often explained the mathematical content in ways that were correct mathematically as well as focused on fundamental meanings. They had useful vocabulary with which to discuss and explain the underpinnings of the ideas. For example, where U.S. teachers used procedural terms such as 'borrowing' and 'crossing out' to discuss subtraction computation, the Chinese teachers talked conceptually about 'decomposing' numbers.

Ma's analysis of the interview data led her to propose that knowing mathematics for teaching depended on a "profound understanding of fundamental mathematics." She provided numerous detailed examples of what was involved in knowing mathematics in this way, as well as of the impoverished knowing characteristic of underdeveloped mathematical knowledge. Teachers with PUFM (profound understanding of mathematical knowledge) described the curricular development of a topic and were able to isolate key ideas and sequence their emphasis. They anticipated typical difficulties in learning the content and knew explanations or representations designed to counter these difficulties. And they made important connections within and across ideas.

Noteworthy is that this study's findings attracted broad international attention from researchers, mathematicians, and practitioners of mathematics teaching and teacher education. It centered on a critical problem of practice – widely perceived weaknesses in teachers' mathematical knowledge. However, no useful way existed to characterize these perceived weaknesses. Without an effective understanding of what was lacking, efforts to remedy it were often disappointing.

What stands out is Ma's approach to the problem. Her approach was multi-dimensional, deploying prior theory, research, and methodological techniques to examine teachers in two different cultures. The resources she deployed drew on mathematics as a discipline, research on teaching, teacher knowledge, and student learning; curriculum analyses; theories of teacher knowledge, and methods of studying such knowledge. A fruitful strategy, her approach allowed a new perspective on a problem of practice. Although many recognized that what distinguished knowledgeable practitioners was not how *much* mathematics they knew, but *what* they knew

and *how* they could articulate and use it in teaching, most policymakers saw little alternative but to increase mathematics requirements for teachers. Ma's results highlighted the importance of developing new approaches to develop teachers' usable mathematical knowledge, and reliable and valid tools to measure such knowledge. Although it was decidedly practice-centered, the study contributed to the trajectory of research in this area, advanced theory, and offered a new avenue for policy and measurement.

Over the past several years, researchers at the University of Michigan have been involved in developing a practice-based theory of mathematical knowledge for teaching, and in developing reliable and valid measures of such knowledge (Ball and Bass, 2003; Hill and Ball, in press; McCallum, 2003). One site for this work has been the detailed study of practice, with an eye to the mathematical issues that arise in the course of regular daily teaching as teachers ask questions, listen to students, use drawings, interpret and use curriculum materials, and talk about mathematics. Mathematics as a discipline has offered lenses for investigating the quotidian work of teaching, and for uncovering the hidden but significant mathematical demands of that work. Based on this research into practice, mathematicians, teachers, teacher educators, psychometricians, and mathematics education researchers have collaborated on the challenging task of designing credible questions to assess mathematical knowledge as it is used in teaching. Individuals and groups with different kinds of expertise, training, and experience have worked side by side, deeply engrossed in the work. A genuine respect for one another has grown because of the challenge of doing the work, and the varied expertise that has so clearly been needed to do it well. In this instance, the very differences that have so often created barriers to communication and joint work, and that have divided communities with a stake in mathematics education, have now been deployed successfully as valuable resources.

Not many stories exist of research that centers on practice and that draws on and contributes to the perspectives and expertise of multiple communities. The biologist and curriculum theorist, Joseph Schwab, identified two companion approaches necessary for bridging theory and practice. Because these methods are not reducible to general rules, he argued that they were 'arts' of scholarly practice and of practically oriented scholarship. He termed one 'the arts of the practical' and the second 'the arts of eclectic' and described them as the quintessential methods to bridging theory and practice that places problems of practice at the center (Schwab, 1974). Schwab believed that no theory could completely account for the particularities and dynamics of practice. The 'arts of the practical' organize practice in ways theory cannot, accounting for the crevices of the particu-

lars in specific situations. The 'arts of eclectic' permit a practice-focused approach to theory that makes it relevant to and useful for understanding and engaging in practice. With these 'arts,' scholars and practitioners use theory in the service of practice, and in so doing improve both practice and theory. Theoretically derived lenses permit views of practice that help to 'see' and frame its problems. In complement, the conversion of theories to perspectives, questions, and ways of acting permits the detail of practice to shape the meaning and use of general abstractions.

Improving the teaching and learning of mathematics, of particular students, in a wide range of educational settings, is a core problem of practice. It is *the* core problem, toward which work on all other important problems contributes. Ma, for example, probes the nature of teachers' mathematical knowledge because she believes that it bears on their effectiveness. Working on a problem effectively depends on knowledge and expertise of many different kinds. Diverse forms of expertise can help to identify critical questions, and essential resources with which to approach them. Some forms of expertise derive from practice, others from theory. The expertise of making things happen with actual children in real classrooms is one such form of practical expertise; the constellation of insight and skills born of experience with mathematics as a discipline is another. Systematic inquiry into learning, teaching, contexts, and mathematics complements such wisdom of practice, while theory offers tools to organize inquiry, which in turn changes and improves theory.

However, the history of work in mathematics education reveals patterns of *prioritizing theory over practice* or of *working on practice without reference to theory*, and a *divide among mathematicians, mathematics education researchers, and practitioners* of mathematics teaching at all levels. Allegiance to particular theories and particular professional camps has, at times, led to a pattern of letting theory and camp perspective determine the identification and solution of problems. In other instances, the outright rejection of theory, and of research, has permitted commonsense and opinion to guide educational practice in mathematics.

This special issue of *Educational Studies in Mathematics* is devoted to bringing theory and research together with practice, in large part by fostering productive relationships among scholars and practitioners around core problems of practice. The papers reflect the very theme of the special issue in that they are simultaneously accounts of the practices of boundary crossing and theory about the nature of such work. They offer useful and illuminating resources for those interested in these endemic problems of practice and research.

The papers had their genesis in a Working Group of mathematics education researchers, mathematicians, and practitioners at ICME-9 in Tokyo in 2000. Across four sessions, participants – over 130 in all – investigated how mathematics education research and theory is and might be connected (a) to other scholarly fields and (b) to practice. Focal questions were: How does mathematics education research intersect with theory and practice of scholarship in other fields? Where does interaction among disciplinary communities in the context of research in mathematics education happen effectively? What are some of the barriers to such interactions? What special roles do mathematicians and mathematics as a discipline have in mathematics education research, and what roles might they have? The group also probed ways in which new approaches to research attempt to cross the traditional divide between theory and basic research, on the one hand, and solving problems of teaching and learning, on the other. Participants discussed what was involved in deriving questions directly from issues of practice and in working closely with practitioners, as well as what is afforded and what pitfalls exist in designing theoretically focused experiments in practice. The Working Group explored how these sorts of efforts affect the definition of research problems in mathematics education and the development of methods. How do they affect the claims of research and what counts as ‘evidence’ for those claims?

Each paper in this special issue intertwines theory and practice, identifying and articulating problems of practice, and theorizing about the practice of this work. Some papers describe projects in which unusual partnerships are formed among people with different knowledge and expertise, with different interests around the construction of theory and the development of practice. Others examine the roles of practice and theory in the development of each. A prominent theme running through the papers is a scrutiny of the nature of knowledge, and a consideration of what is involved in knowing in and about practice. Another is a probing of the divide between mathematicians and mathematics educators, and between researchers and teachers, and thoughtful accounting of what it takes to cross those boundaries respectfully and productively.

The papers by Lyn English and by Maria Bartolini Bussi and Luciana Bazzini offer firsthand accounts of multi-tiered collaborative work in which practice is studied and developed. Each account offers detail about the ways in which mathematics and didactical or pedagogical theories are intertwined in the course of work on the design, enactment, and analysis of instruction. In English’s paper, we see the multi-tiered possibilities for both beginning and more experienced practitioners to learn from collaborative work on modeling; in the Bartolini Bussi and Bazzini paper, we see how

mathematics can be wielded to improve both designs for representation and learning and their analysis in practice. Barbara Jaworski offers a theoretical framework for the practices of such 'co-learning partnerships' that involve people with different kinds of expertise, and aim both at the development of theory and practice. Four dimensions structure Jaworski's framework: knowledge and learning, inquiry and reflection, insider and outsider, individual and community. Her article, offering theory for understanding the practices of such work, investigates the usefulness of the framework by trying it out on three different projects that have published accounts of their work. This article, too, then, blends theory with practice.

Ruhama Even and Baruch Schwarz provide a detailed case of the use of theory to understand practice, systematically engaging in analysis of a classroom episode from two distinctly different theoretical perspectives: cognitive theory on one hand, and sociocultural theory on the other. Their vivid analysis illuminates the implications of such choices. It also highlights the incompleteness of any one perspective, and raises important questions about the intertwining of theory to develop and understand teaching and learning. Resonant with Schwab's vision of the 'arts of eclectic,' their approach offers firsthand experience with the particularities of practice and the theory-dependent nature of 'data.' In practice, multiple interactions influence the course of teaching and learning; any single theory necessarily focuses on some such interactions and obscures others.

Using a different angle on practice, Jerry Goldin offers a perspective on the social and epistemological dimensions of boundary crossing. Again, examining problems of practice endemic to such efforts, he explores the cultural divides that impede communication and productive work between mathematicians and mathematics educators. Like Bartolini Bussi and Bazzini, Goldin sees these divides as serious problems of practice. At the heart of the issues, he argues, are fundamentally different views of knowledge and a lack of respectful regard for or inquiry into these epistemological differences. Pointing briefly to some promising exceptions to the persistent patterns of disrespect and lack of connection, Goldin argues concretely and forcefully for developing the practices of working between disciplinary mathematics and mathematics education.

Willi Dörfler proposes the creation of a new, legitimate field of inquiry, *mathematicology*, or the study of mathematics. A field devoted to inquiries into the practices of justification, representation, communication, and problem framing and solving would offer a territory that would, by necessity, require the expertise of different kinds of people with different kinds of training and experience. It would legitimate inquiry into problems

of practice and theory that are otherwise peripheral to existing fields, and would create a new space for collective work.

Each of these papers offers crucial resources for reconsidering the relations of theory and practice, of connections among communities of expertise and practice. Striking, however, is the ways in which the papers, and the special issue as a whole, mirror and represent its very aim. Where Liping Ma stopped, these authors have ventured. These are papers fundamentally about problems of practice. They are about a variety of problems of mathematics teaching and learning. But a reader who sees only this will miss what is most special about this special issue. These are authors simultaneously engaged in the practice of fostering productive connections between research and teaching, between mathematics as a discipline and the mathematics of teaching. These papers offer readers perspectives on *that* practice, of fostering productive connections, and *its* problems. The issue as a whole takes the Schwabian approach: With practice at the core, resources of theory are eclectically marshaled to illuminate problems of practice. Delving into the layered nature of this work will afford readers new vantage points on the enterprise of improving mathematics teaching and learning. Hopefully, the perspectives articulated by these authors can help to move our communities forward in tackling these fundamental challenges.

#### NOTE

1. The ideas in this preface have been developed and enriched through discussions and writing with Mark Hoover.

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