THE UNIVERSITY OF MICHIGAN

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Tire and Suspension Systems Research Group

Technical Report No. 4

CORD LOADS IN CORD-RUBBER LAMINATES

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UMRI Project 02957

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October 1960

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I. STATEMENT

The calculation of cord loads in cord-rubber laminates must be preceded by a knowledge of the interply stresses acting in these laminates. This is because the total stress state acting on any particular lamina is made up of that portion of the total external stresses carried by this lamina plus the interply stresses acting on it. Thus only a partial answer to cord loads is given by considering only the effects of external stresses.

Reference 1 presented equations for the values of the interply stresses as functions of the elastic constants of the laminates and as functions of the cord half-angle. Having these expressions, it should be possible to combine them with the external stresses in such a way as to predict the cord loads for any external loading condition.

II. SUMMARY

By proper combination of the interply and external stresses, expressions for the stress in the direction of the cords may be obtained for those cases where the cords in all plies of a laminated structure are either in tension or in compression. This report will be restricted to those cases in which the stiffness of the cords is much greater than the stiffness of the rubber. Thus, if the stresses in the cord direction are known, it may be assumed that these stresses are equally distributed among the cords only, i.e., that the cords carry all the tension load or compression load in their direction.

Under this assumption, influence coefficients relating cord loads to applied external stress are presented. These indicate that the influence coefficients relating external normal stresses to cord loads are generally smaller than unity. However, the application of external shearing stresses can result in extremely high cord loads in those cases where the cord half-angles are either close to zero or close to 90°, since influence coefficients greater than 40 have been calculated.

III. CALCULATION OF CORD LOADS

As discussed in Ref. 1, the expressions for interply stresses as functions of the externally applied stresses may be written as

$$\sigma'_\eta = \frac{(a_{12}a_{13} - a_{11}a_{23})}{(a_{11}a_{22} - a_{12}^2)} \sigma_{\xi\eta}$$

$$\sigma'_\xi = - \left[\left(\frac{a_{12}}{a_{11}}\right)\frac{(a_{12}a_{13} - a_{11}a_{23})}{(a_{11}a_{22} - a_{12}^2)} + \frac{a_{13}}{a_{11}}\right] \sigma_{\xi\eta}$$

$$\sigma'_{\xi\eta} = - \frac{a_{13}}{a_{33}} \sigma_\xi - \frac{a_{23}}{a_{33}} \sigma_\eta \tag{1}$$

Assuming a two-ply laminate as before, it may be seen at once that the total stress state of one lamina is determined by the sum of the external stresses and the interply stresses just given. In constructing this sum, it will be assumed that both plies of the two-ply laminate have the same elastic properties, that is, they may both have cords in tension or they may both have cords in compression. The case of one ply being in tension and the other being in compression is specifically omitted from consideration here. Figure 1 shows one typical lamina of a laminated sheet with the stresses acting on it as assumed in this report. From the general transformation expressions used in Ref. 1, the expression for the stress in the cord direction may be written at once as

$$\sigma_x = (\sigma_\xi + \sigma'_\xi)\ell_{x\xi}^2 + 2(\sigma_{\xi\eta} + \sigma'_{\xi\eta})\ell_{x\xi}\ell_{x\eta} + (\sigma_\eta + \sigma'_\eta)\ell_{x\eta}^2$$

$$\sigma_y = (\sigma_\xi + \sigma'_\xi)\ell_{y\xi}^2 + 2(\sigma_{\xi\eta} + \sigma'_{\xi\eta})\ell_{y\xi}\ell_{y\eta} + (\sigma_\eta + \sigma'_\eta)\ell_{y\eta}^2 \tag{2}$$

where

$$\ell_{x\xi} = \cos\alpha \qquad \ell_{y\xi} = -\sin\alpha$$

$$\ell_{x\eta} = \sin\alpha \qquad \ell_{y\eta} = \cos\alpha \qquad \text{(a)}$$

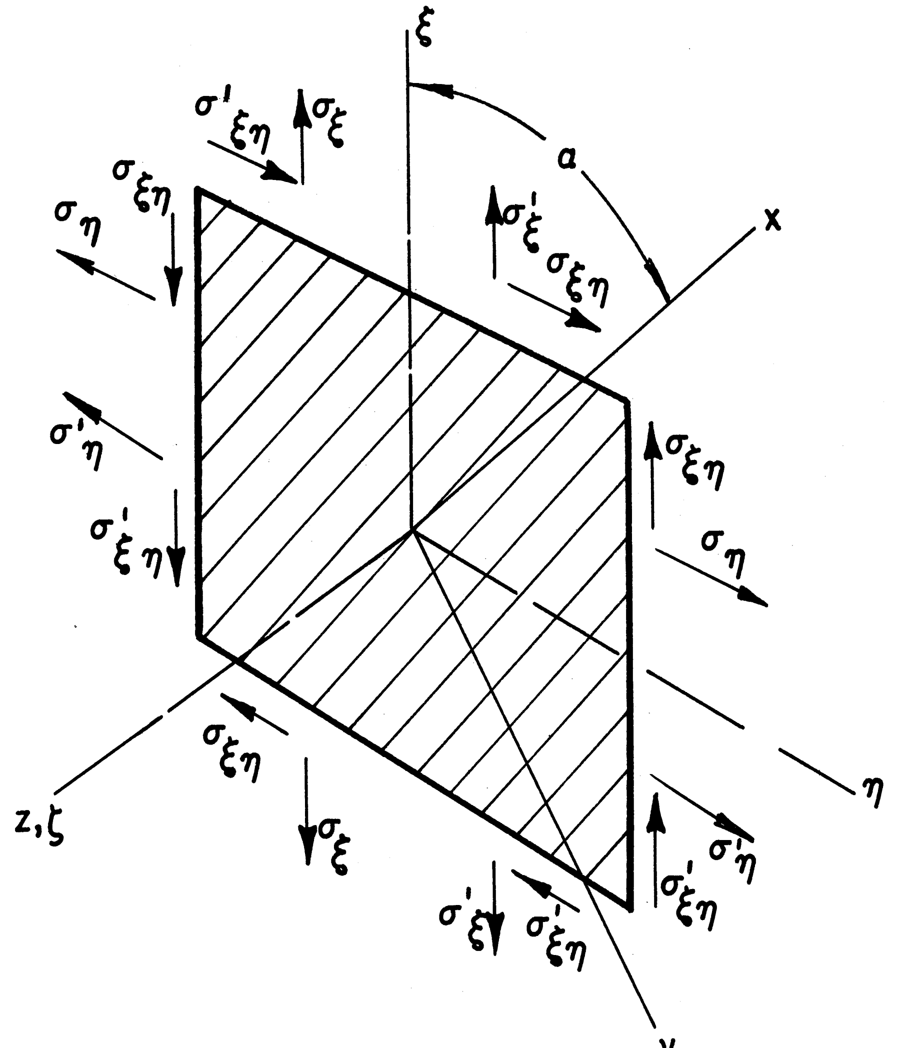

Fig. 1. Typical lamina with both interply and external stresses acting.

The cord stress cannot be determined by a direct examination of the stress σ_x in the x direction, since the cord may actually be forced into a state of compressive strain by means of stresses acting at right angles to it. Thus to determine the sign of stress in a cord it is necessary to examine the expression for the cord strain,

$$\epsilon_x \approx \frac{\sigma x}{E_x} - \frac{\sigma y}{F_{xy}} \tag{3}$$

Reference 2 shows that the modulus E_x may be approximated as

$$E_x \simeq \frac{(AE)_c(n)}{t} \tag{4}$$

where

n = end count
t = thickness of a single ply
A_c = cord cross-sectional area
E_c = cord modulus
$(AE)_c$ = cord spring constant per unit length of cord.

Equation (3) can now be written as

$$(\epsilon_x)(E_x) = \sigma_x - (\sigma_y)\frac{E_x}{F_{xy}} = \frac{(\epsilon_x)(AE)_c(n)}{(t)} \tag{5}$$

From this it is seen at once that the cord load may be obtained, since it is the product of the strain in the cord direction and the $(AE)_c$. This gives

$$P = (AE)_c(\epsilon_x) = \frac{t}{n}\left[\sigma_x - (\sigma_y)\left(\frac{E_x}{F_{xy}}\right)\right] \tag{6}$$

where P is the load in a single cord.

Equation (6) indicates the form of the expression giving the cord load. It is seen that, in the case of each individual laminate construction, it will be necessary to specify factors t and n, since these are variables of construction. The quantity in brackets may be calculated and plotted as a function of the other elastic variables of the system. This may be seen most easily by inserting the terms of Eqs. (2) into Eqs. (6), giving the final expression for cord load as

$$\begin{aligned}
P = (\epsilon_x)(AE)_c = \frac{t}{n}\Bigg[& \left\{\sigma_\xi - \left[\left(\frac{a_{12}}{a_{11}}\right)\left(\frac{a_{12}a_{13} - a_{11}a_{23}}{a_{11}a_{22} - a_{12}^2}\right) + \frac{a_{13}}{a_{11}}\right]\sigma_{\xi\eta}\right\}\cos^2\alpha \\
& + 2\left[\sigma_{\xi\eta} - \left(\frac{a_{13}}{a_{33}}\right)\sigma_\xi - \left(\frac{a_{23}}{a_{33}}\right)\sigma_\xi\right]\sin\alpha\cos\alpha \\
& + \left\{\sigma_\eta + \left(\frac{a_{12}a_{13} - a_{11}a_{23}}{a_{11}a_{22} - a_{12}^2}\right)\sigma_{\xi\eta}\right\}\sin^2\alpha \\
& - \frac{E_x}{F_{xy}}\left\{\sigma_\xi - \left[\left(\frac{a_{12}}{a_{11}}\right)\left(\frac{a_{12}a_{13} - a_{11}a_{23}}{a_{11}a_{22} - a_{12}^2}\right) + \frac{a_{13}}{a_{11}}\right]\sigma_{\xi\eta}\right\}\sin^2\alpha \\
& + 2\frac{E_x}{F_{xy}}\left[\sigma_{\xi\eta} - \left(\frac{a_{13}}{a_{33}}\right)\sigma_\xi - \left(\frac{a_{23}}{a_{33}}\right)\sigma_\eta\right]\sin\alpha\cos\alpha \\
& - \frac{E_x}{F_{xy}}\left\{\sigma_\eta + \left(\frac{a_{12}a_{13} - a_{11}a_{23}}{a_{11}a_{22} - a_{12}^2}\right)\sigma_{\xi\eta}\right\}\cos^2\alpha\Bigg]
\end{aligned} \qquad (7)$$

Equation (7) may be simplified considerably by rewriting it in terms of symbols in the form

$$P = \frac{t}{n}\left[(L)\sigma_\xi + (M)\sigma_\eta + (N)\sigma_{\xi\eta}\right] \qquad (8)$$

where L, M, and N are dimensionless influence coefficients, since the dimensions of the term on the left side, the cord load, are identical to the dimensions of the terms on the right, namely, the product of stress and thickness over end count. From Eq. (8) it may be seen that the factors L, M, and N are the only ones which need to be calculated and plotted. Their values are given by Eqs. (9):

$$L = \cos\alpha\left[\cos\alpha - 2\left(\frac{a_{13}}{a_{33}}\right)\sin\alpha\right] - \sin\alpha\left(\frac{E_x}{F_{xy}}\right)\left[\sin\alpha + 2\left(\frac{a_{13}}{a_{33}}\right)\cos\alpha\right]$$

$$M = \sin\alpha\left[\sin\alpha - 2\left(\frac{a_{23}}{a_{33}}\right)\cos\alpha\right] - \cos\alpha\left(\frac{E_x}{F_{xy}}\right)\left[\cos\alpha + 2\left(\frac{a_{23}}{a_{33}}\right)\sin\alpha\right]$$

$$N = -\left[\cos^2\alpha - \left(\frac{E_x}{F_{xy}}\right)\sin^2\alpha\right]\left[\left(\frac{a_{12}}{a_{11}}\right)\left(\frac{a_{12}a_{13} - a_{11}a_{23}}{a_{11}a_{22} - a_{12}^2}\right) + \left(\frac{a_{13}}{a_{11}}\right)\right] + 2\sin\alpha\cos\alpha\left[1 + \frac{E_x}{F_{xy}}\right] + \left[\sin^2\alpha - \left(\frac{E_x}{F_{xy}}\right)\cos^2\alpha\right]\left(\frac{a_{12}a_{13} - a_{11}a_{23}}{a_{11}a_{22} - a_{12}^2}\right) \qquad (9)$$

In regard to the numerical calculation of the terms L, M, and N, it may be seen that the only elastic constant which enters directly as a variable is $\frac{E_x}{F_{xy}}$. It may be recalled from the discussion of Ref. 2 that the ratio $\frac{E_x}{F_{xy}}$ was shown to be one-half. It was also shown there that ratios of the constants a_{ij}

depend only upon one variable, namely, the ratio $\frac{G_{xy}}{E_x}$, which has been found to span a range running from 10^{-4} to 10^{-1} for all observed cases for both cord tension and compression. It should be possible, then, to calculate the factors L, M, and N as functions of the variables $\frac{G_{xy}}{E_x}$ and the cord half-angle α. This has been done and the results are presented in Figs. 2, 3, and 4. Trial calculations showed that nearly identical curves resulted if $\frac{E_x}{F_{xy}}$ was chosen to be 1/3 instead of 1/2.

The cord loads may be determined from the information presented in these figures along with the use of Eq. (8), which requires a knowledge of the end count and the ply thickness. The numerical value of $\frac{G_{xy}}{E_x}$ may be found by the method described in Ref. 2.

Examination of the values of L, M, and N indicates that cord loads due to external normal stresses approach a maximum in the vicinity 30-35° of cord angle, that they approach zero between 55 and 60° of cord angle, and become negative for larger cord angles. Similar conclusions apply to M, which is the mirror image about the 45° line of L. In both of these curves, it is seen that a distinct possibility exists that cords may go into compression, as evidenced by the values of L in excess of 55°. These calculations indicate a cord half-angle somewhat larger than 55° as being that associated with zero cord tension due to the presence of external stresses, partly because the interply stresses, discussed in Ref. 2, become very small in this vicinity and contribute little to the cord loads here. If it were not for this, it might be expected that this zero load angle would be considerably different from that of 54.75°, derived from the usual condition of an inextensible cord imbedded in a rubber

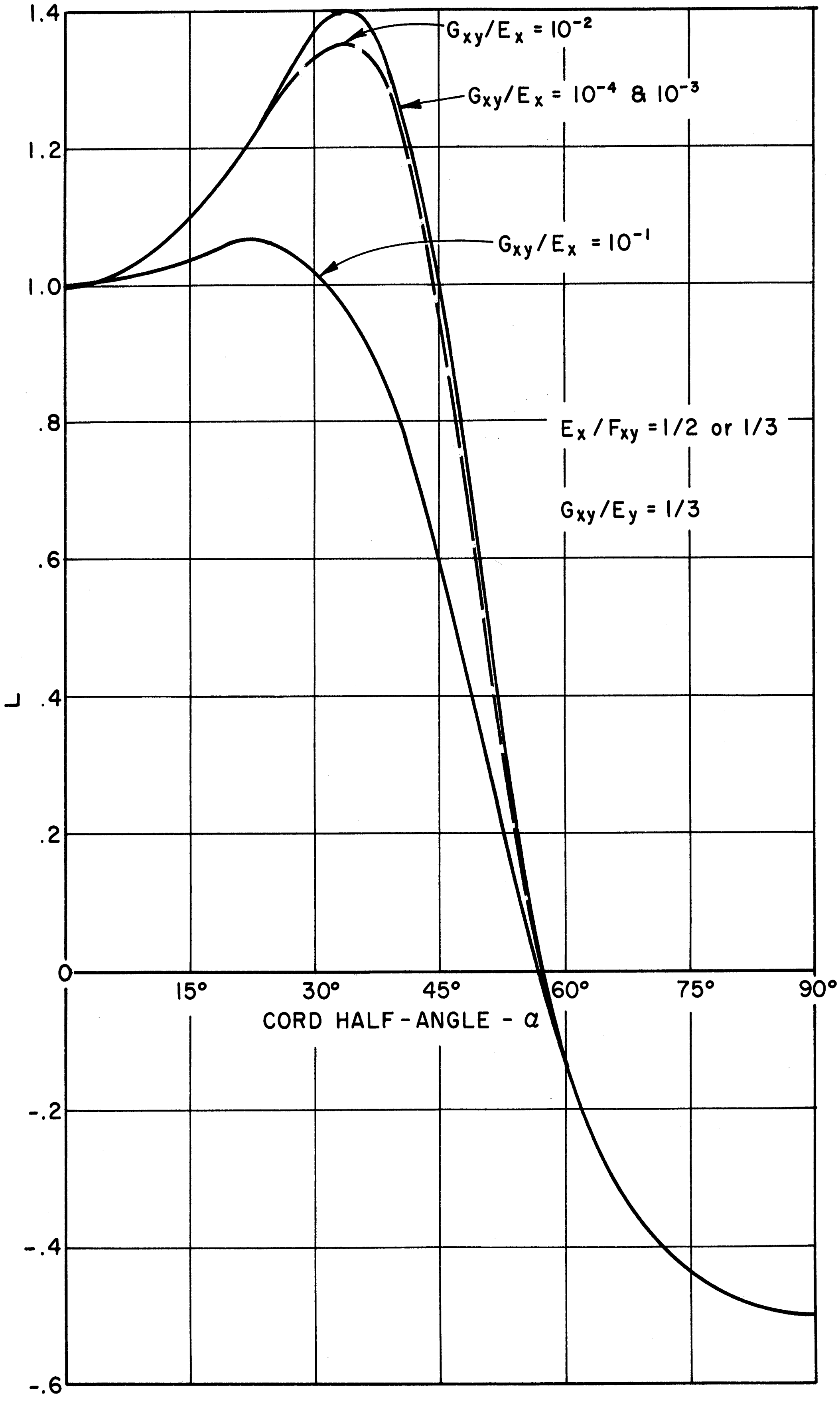

Fig. 2. L vs. half-angle α for a range of values of G_{xy}/E_x.

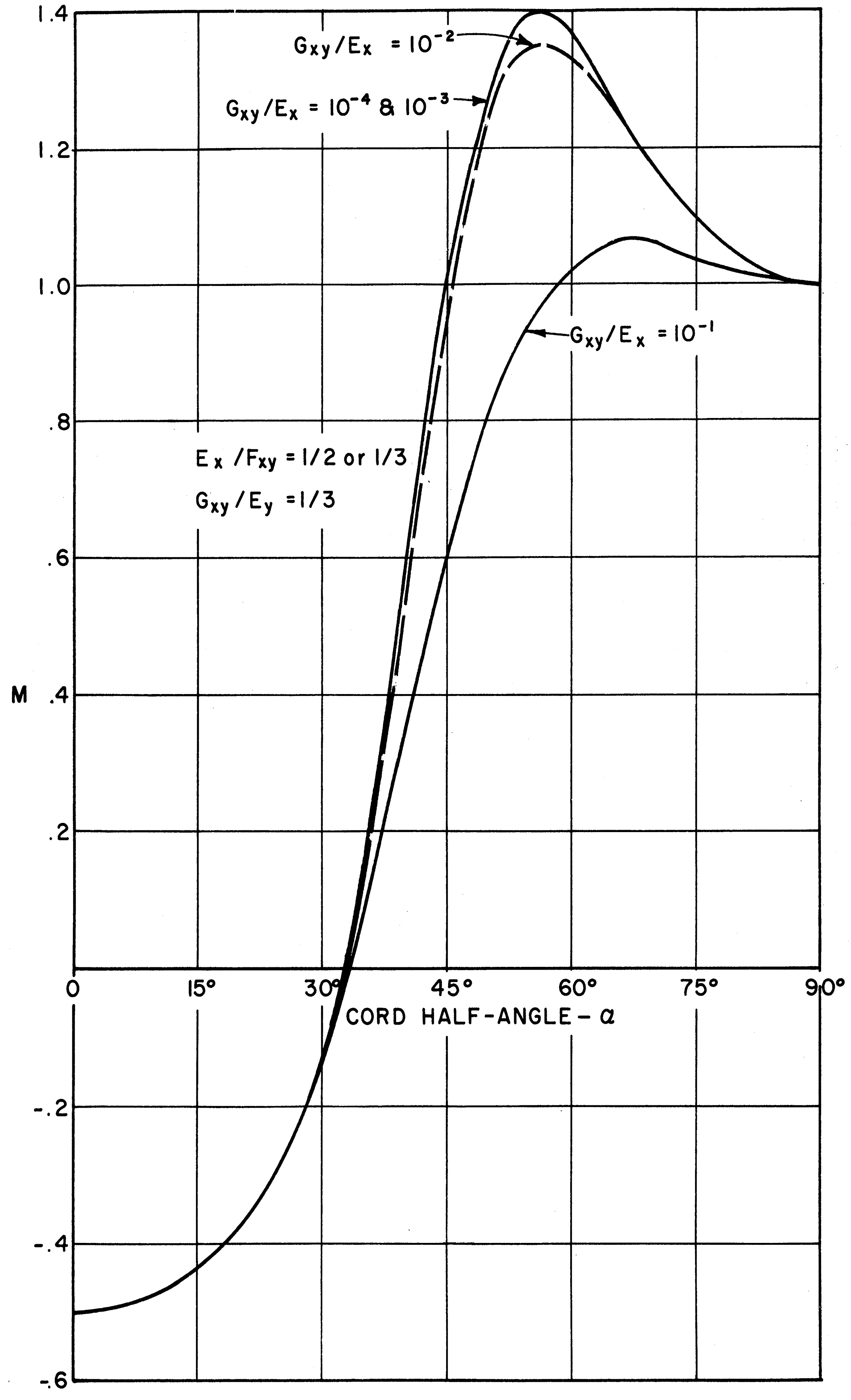

Fig. 3. M vs. half-angle α for a range of values of G_{xy}/E_x.

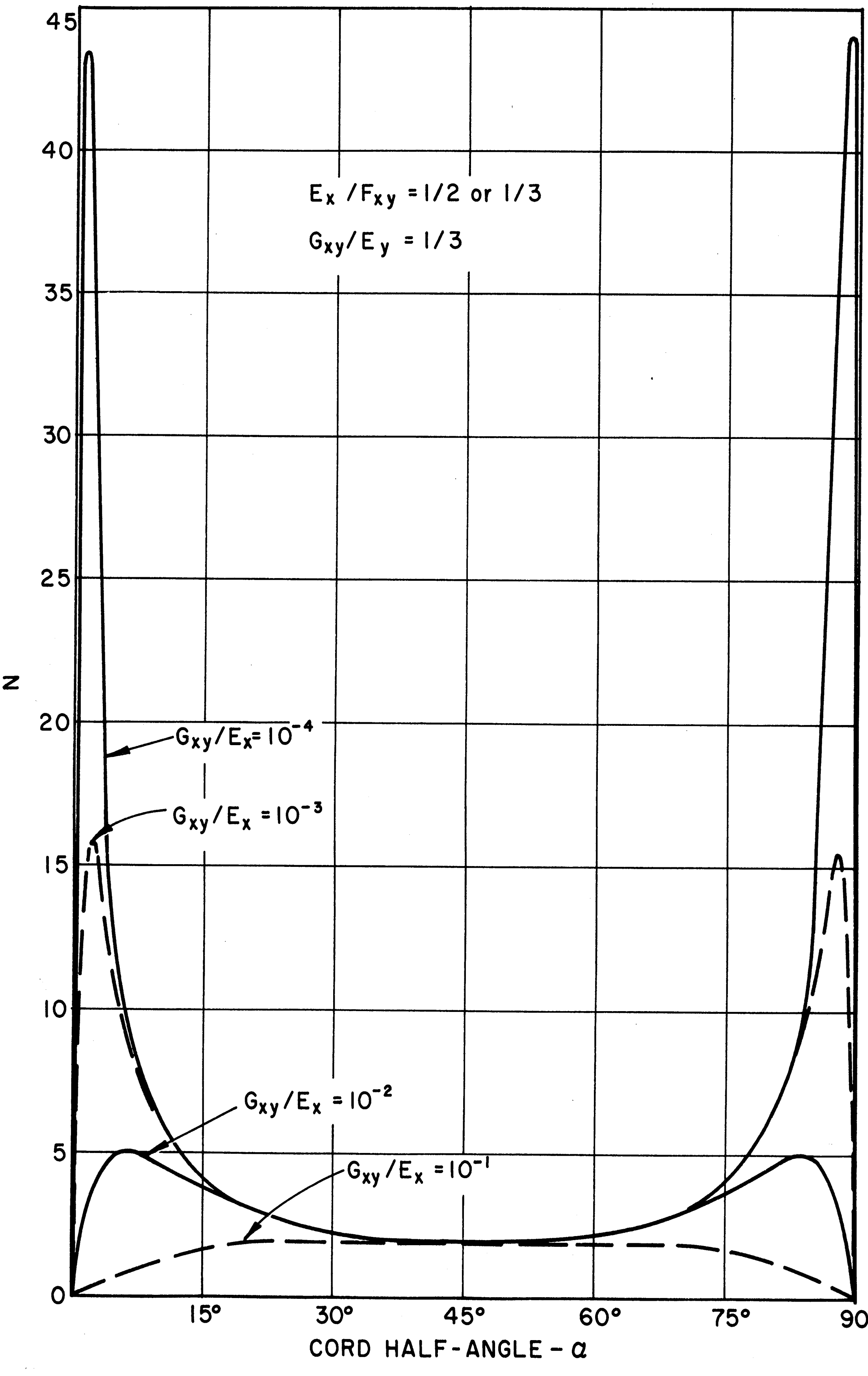

Fig. 4. N vs. half-angle α for a range of values of G_{xy}/E_x.

block whose Poisson's ratio is one-half.

Figure 4 shows that the presence of external shear stresses can result in extremely high cord loads under certain conditions. Generally, these conditions require that the cord half-angles be very small or else be very close to 90°, which is the equivalent, and also that the cords be quite stiff compared with the rubber matrix in which they are imbedded. This means that such structures as braided wire cords imbedded in rubber would be particularly susceptible to large cord loads if these laminates were used at small angles. It is quite surprising that cord loads of the magnitudes indicated can be generated, and it is hoped that some experimental evidence can be obtained on this point in the future.

IV. EXAMPLE

The example used in Ref. 2 will again be chosen here since it is a fairly simple geometry and can be used to illustrate all the principles involved. Consider a circular tube of 5-in. diameter such as shown in Fig. 5. We desire to determine the cord loads in each of the two plies of this tube due to the application of the same external stresses as previously used, these being

$$\sigma_\xi = 62.5 \text{ psi}$$

$$\sigma_\eta = 125.0 \text{ psi}$$

$$\sigma_{\xi\eta} = 6.38 \text{ psi}$$

It will be necessary here to assume that the end count and the ply thickness are known. The ply thickness, .050 in. was previously defined for purposes of calculating the stress magnitudes. The end count will be assumed to be 20 cords per inch.

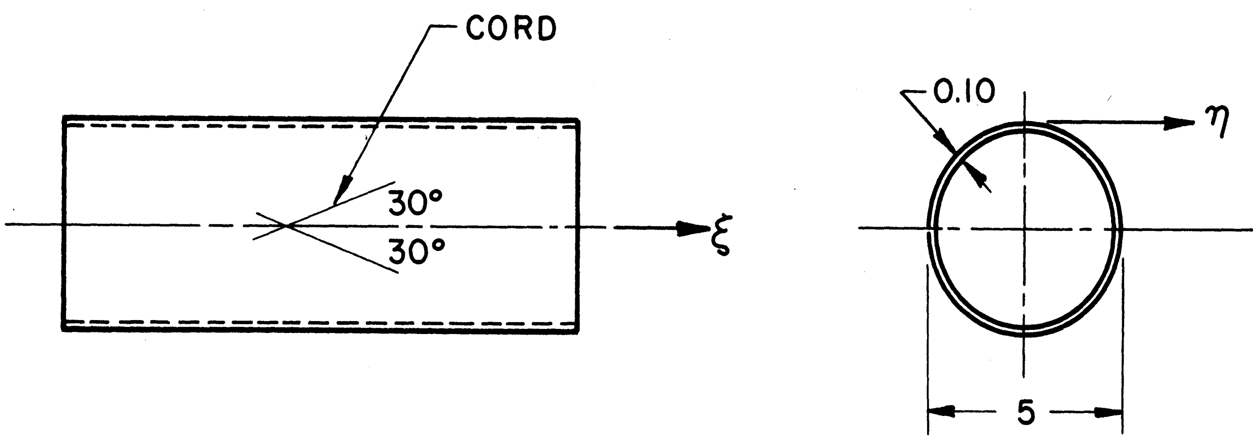

Fig. 5. Cylindrical tube of two-ply construction.

Utilizing the curves in Figs. 2, 3, and 4, along with the values of ply thickness and end count just given, the cord load in the tension ply may be obtained directly from the positive values of L, M, and N as given in Eq. (8). This gives, again assuming as in the previous example that $\frac{G_{xy}}{E_x} = 10^{-3}$,

$$P_{\substack{tens.\\ ply}} = \frac{.050}{20} \left[(1.375)(62.5) + (-0.125)(125.0) + (2.3)(6.38)\right]$$

$$= .212 \text{ lb per cord} \tag{10}$$

In regard to the compression ply, it is seen that the functions L, M, and N must be evaluated for negative angles α. Examination of Eqs. (9) indicates that both L and M are unchanged when evaluated for negative α in place of positive α. That means that the influence on cord load of normal stresses will be just the same in both plies of the tube in question. However, examination of Eqs. (9) further indicates that the term N does change sign when evaluated for negative angles in place of positive angles. In that case, the correct form of Eqs. (8) applicable to the other ply of this tube is:

$$P = \frac{.050}{20} \left[(1.375)(62.5) + (-0.125)(125.0) - (2.3)(6.38)\right]$$

$$= .139 \text{ lbs per cord} \tag{11}$$

From these calculations, it is seen that the cord loads are different in the two plies but that both sets of cords are still in a state of tension. A similar example could be presented utilizing all compression data but would not show anything novel, since the use of the curves of Figs. 2, 3, and 4 would be exactly the same.

V. ACKNOWLEDGMENTS

The calculations necessary for presenting this information were performed by Mr. Richard N. Dodge with assistance from Mr. D. H. Robbins, Mr. D. E. Zimmer, and Miss Gwendolynne Chang. Thanks are due to them for their care and patience in this lengthy task.

VI. REFERENCES

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