A PROBABILISTIC METHODOLOGY TO ASSESS THE RISK OF GROUNDWATER QUALITY DEGRADATION

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Abstract. An approach to assess the *risk* of groundwater quality degradation with regard to fixed standards, based on Disjunctive Kriging (*DK*) is presented. The *DK* allows one to evaluate the Conditional Probability (CP) of overriding a given threshold of concentration of a pollutant at a given time, and at a generic point in a considered groundwater system. The result of such investigation over the considered area can be plotted in form of maps of *spatial risk*. By repeating this analysis at different times, several *spatial risk* maps will be produced, one for each considered time. By means of non-parametric statistics, the temporal trend of the CPs can be evaluated at every point of the considered area. The *trend index*, assessed by means of a sort of classification of the trend values obtained as described above, can be superimposed on the most recent values of the *spatial risk* (i.e.: the most recent values of probability). Consequently a classification of the risk of groundwater quality degradation results with which to weigh both the spatial distribution and the temporal behaviour of the probability to exceed a given standard threshold. The methodology has been applied to values of nitrate concentration sampled in the monitoring well network of the Modena plain, northern Italy. This area is characterised by intensive agricultural exploitation and hog breeding along with industrial and civil developments. The influence of agriculture on groundwater results in a high nitrate pollution that limits its use for potable purposes.

Keywords: conditional probability, disjunctive kriging, groundwater management, groundwater quality, risk

1. Introduction

A fundamental step in groundwater resources management and environmental planning is the assessment of the risk expected after the occurrence of natural (Gundersen *et al.*, 1998) or anthropogenic events (Van der Eerden *et al.*, 1998).

According to Varnes (1984), *risk* is generally defined as the combination of *hazard* and *vulnerability*; the hazard represents the probability that a potentially detrimental event of given characteristics occurs in a given area, for a time period; the vulnerability is the degree of the intrinsic weakness of the system.

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The vulnerability of aquifers to pollution is rather easy to estimate because their transport characteristics do not change appreciably with time. However the hazard is difficult to quantify since it is caused by a number of processes, which are rarely time independent and involve non-homogeneous variables. The ability to define the risk of groundwater qualitative impairment directly relative to given standards can represent a valid alternative to counting exceedances of the standards.

A methodology that aims to assess a *risk* level related to an event considered dangerous to a groundwater quality system, should include the following phases:

(a) outlining of the critical event based on the probability of an effect (pollution);
(b) monitoring and spatial localisation of the effects produced by the potentially harmful event on the groundwater system (measurement of the concentration of the pollutant in the water);
(c) evaluation of the temporal recurrence of the event or of its effects;
(d) assessment of the levels of *risk* of quality degradation incorporating the results of the previous phases.

In this study, intensive agricultural practices, based on heavy use of chemicals, represent the critical event; the produced damage (exceedance of water quality standards) is pollution of groundwater. Agriculture in the study area has been practised over large areas and continuously year after year.

In the proposed approach the *risk of effective impairment* is defined as the result of a classification that combines the most recent level of risk (*index of spatial risk*) with information related to its temporal behaviour (*index of temporal risk)*. The *risk* of groundwater quality degradation due to a pollutant is defined as the Conditional Probability (CP) that the true value of a given pollutant is greater (or lower) than a given threshold value (standard). A number of recent research reports commend the use of non-linear geostatistics to estimate conditional probability.

Matheron (1976) firstly presented the basis of *DK* and the use of hermitian models. In the following years other important papers were published on both theory and application of *DK*. Yates *et al*. (1986a, b) reviewed the theory of gaussian *DK* and presented an application to mapping electrical conductivity and the probability of exceeding a given threshold. Yates and Yates (1988) demonstrated the usefulness of *DK* to management decision-making. Webster and Oliver (1989) presented different case-studies concerning *DK* application to different physical and chemical parameters. Carr and Mao (1993) compared *Disjunctive Kriging* (*DK*) and *Generalized Probability Kriging* showing that the two methods produce similar results if data are normally distributed. Soares *et al*. (1993) plotted spatial probability maps of SO_2 concentrations by using *Indicator Kriging*. Oks *et al*. (1993) have used *Non-Parametric Probability Kriging* to map lead contamination in terms of conditional probability that the concentration exceeds a given threshold. Rivoirard (1994) showed that *DK* could evaluate the probability that true values in a block are lower than a given threshold even if the data are characterised by a strongly skewed distribution. Consequently, if a significant number of observations are available, the *DK* allows one to evaluate, at a given time t, the CP that the concentration of a pollutant in groundwater exceeds a given threshold at a specific location of the study area.

Considering the threshold as the *maximum allowable concentration* (MAC) for drinkable use of the water resource these CP can be reviewed as the risk of degradation with regard to potable use. Starting from a number of sampled values of concentration, it is possible to plot risk maps related to the pollutant at the considered time t. By repeating this procedure n times, for each sampling period, several realisations of the temporal behaviour of the *risk* can be done. Non-parametric statistics (e.g. Spearman Rank Test), not dependent on the characteristics of the data distribution, allow one to show the temporal trend of the CP.

The relationship between the most recent *risk* (*spatial risk*) and its temporal behaviour (*temporal risk*) can also be depicted through subjective, but not arbitrary, judgements of experts. GIS techniques provide a technological framework to develop a crossed classification (Burrough, 1989) producing maps of *effective risk*. A two-dimensional table is a convenient tool to drive this super-imposition of the indices of spatial and temporal risk.

This approach was applied to the aquifer of Modena (Northern Italy). The qualitative standard considered was the *maximum allowable concentration* of nitrate in groundwater used for potable purposes. This area is exploited intensively for agriculture and animal breeding and these activities produce high loads of nitrates in groundwater (Goodchild, 1998; Fraters *et al.*, 1998). Water monitoring from 1990 to 1996 showed that, nitrate concentrations often exceed potable values in most of the area, particularly following the periods of fertiliser and manure application.

In this article, risk is referred to the limits fixed by Italian law (DPR 236/88, 1988).

2. Material and Methods

The proposed approach is based on the use of Disjunctive Kriging (*DK*) to estimate the conditional probability of an exceedance of a given threshold over a given area. The risk of groundwater quality impairment with regard to a fixed standard was defined as the conditional probability (risk) that a threshold of concentration of a generic pollutant exceeds a given value (standard). The methodology consisted of the following operative steps:

(a) Monitoring of groundwater quality by means of collection of water samples from wells and measurement of concentration values of pollutant (nitrate concentration in space and in time);

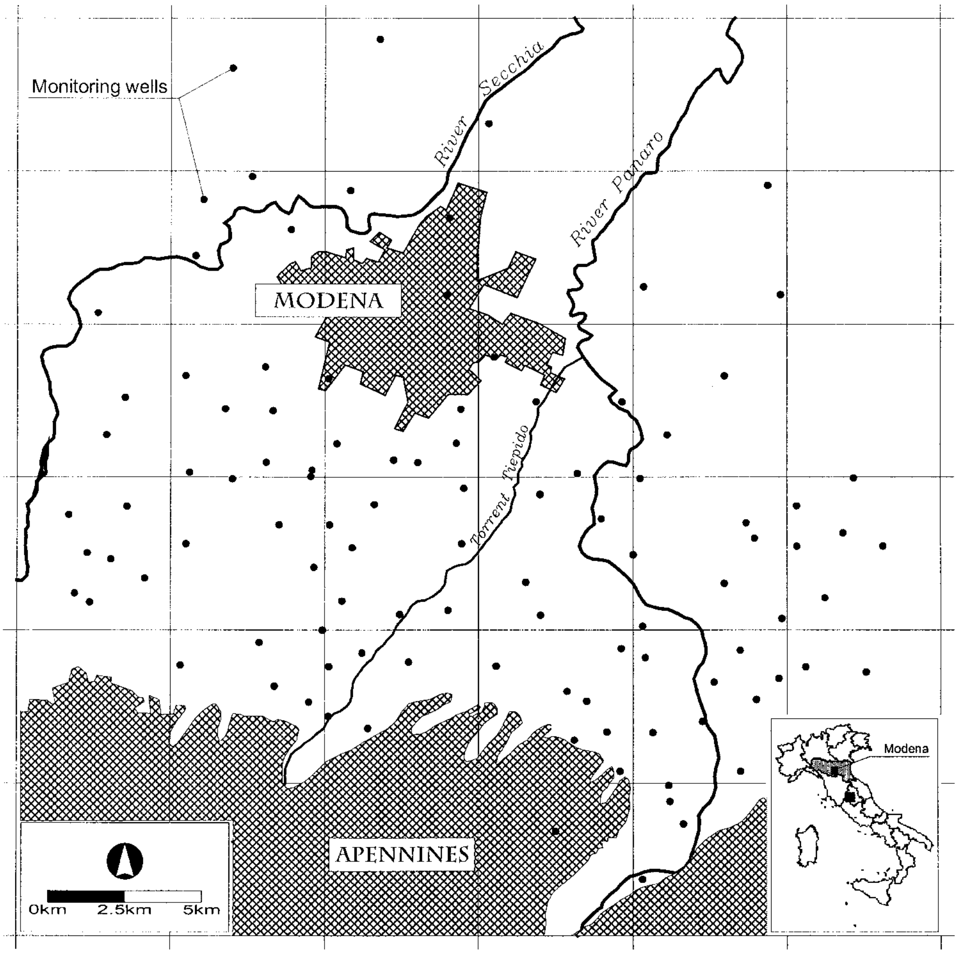

Figure 1. Study area.

(b) Estimation of the general statistics of the distribution of the concentration data (mean, variance, kurtosis, skewness, etc.);

(c) Evaluation of the spatial behaviour of the sampled pollutant by means of structural analysis (variography) and cross-validation (Journel and Huijbregts, 1978);

(d) Transformation of data (preliminary phase for *DK* application). The use of the *DK* estimator is based on the assumption of a known probability distribution of data, which is stationary throughout the study region; data are usually transformed to a normal distribution (Rao and Hsieh, 1991). Normality was obtained by means of the following operation $z(x_i) = \phi[y(x_i)]$, where $z(x_i)$ is the sampled value of the pollutant at the generic locations x_i and $y(x_i)$ is the correspondent gaussian transformation. The operator ϕ is a linear combination of an infinite number of *Hermite polynomials,* H_k, that can be expressed as

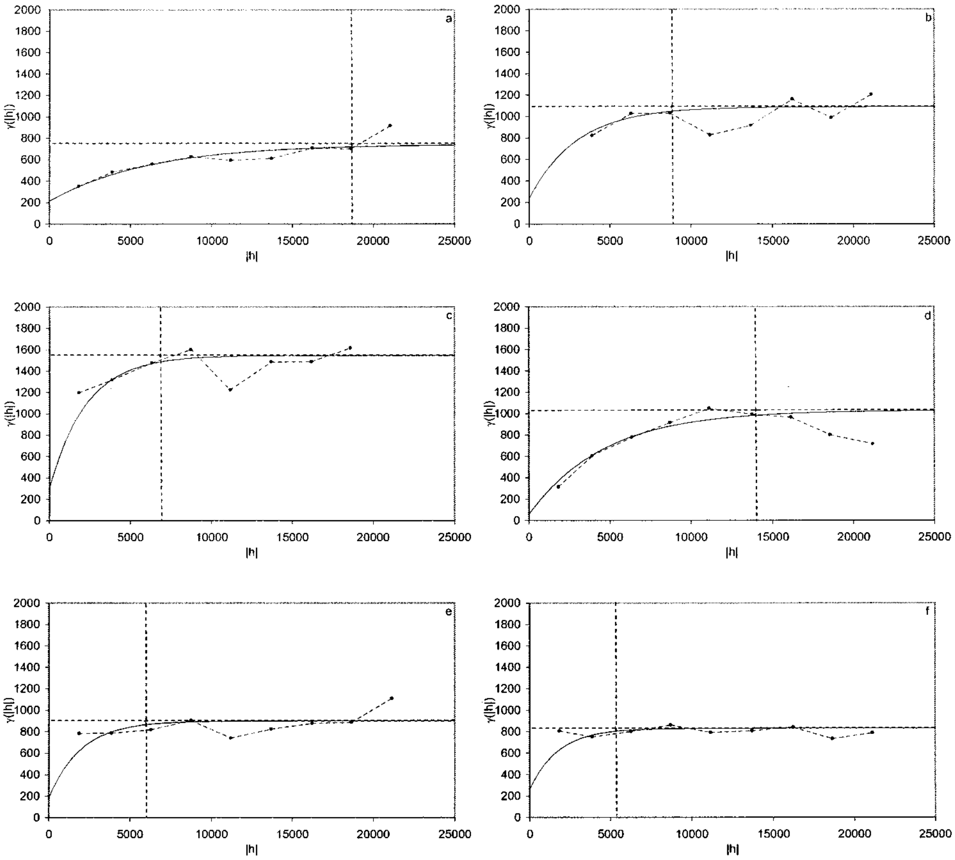

Figure 2a. Experimental (– – –) and theoretical (——) variograms of nitrate concentration from 1990 to 1992 (a, c, e: spring season; b, d, f: fall season).

follows (Yates *et al.*, 1986a):

$$\phi[y(x_i)] = \sum_{k=0}^{\infty} C_k H_k[y(x_i)] \, . \tag{1}$$

The number K of the Hermite polynomials for the approximation of the infinite expansion (Equation (1)), and the corresponding C_k coefficients are determined by means of *Hermite integration* (Abramovitz and Stegun, 1965). Then the conditional probability $P(x_0)$ that the true value in x_o exceeds a given threshold z_c can be obtained by means of the following expression

$$P(x_0) = 1 - G(y_c) - g(y_c) \sum_{k=0}^{K} H_{k-1}[y_c] H_k[y(x_0)]/k\,! \tag{2}$$

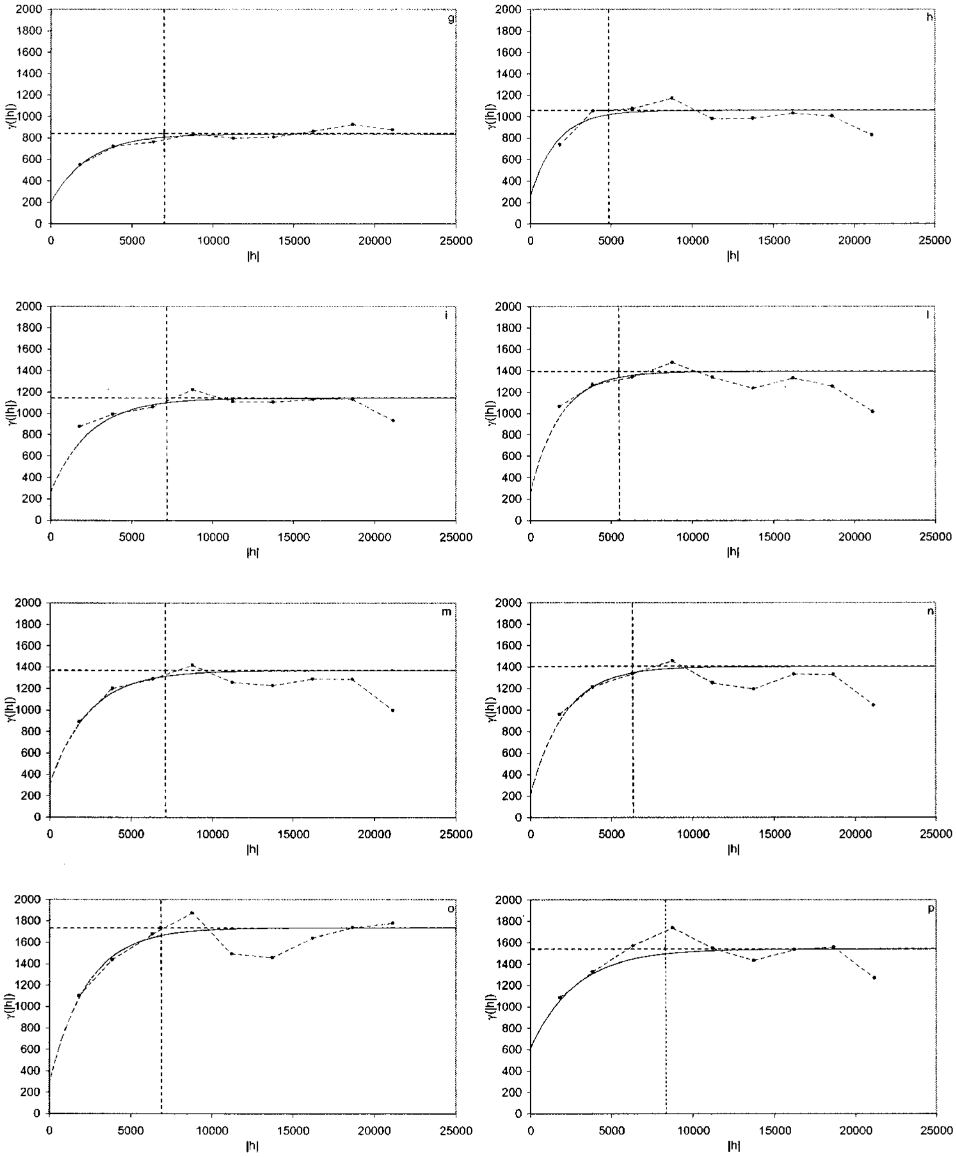

Figure 2b. Experimental (– – –) and theoretical (——) variograms of nitrate concentration from 1993 to 1996 (g, i, n, o: spring season; h, l, n, p: fall season).

where $G(y_c)$ is the standard gaussian cumulative; $g(y_c)$ is the standard normal probability density function; y_c is the normal transformation of z_c; K is the number of *Hermite* terms, and, for each $k = 0, \ldots, K$, $H_k[y(x_0)]$ is obtained by kriging separately the kth *Hermite* factor evaluated in each data sample (Rivoirard, 1994).

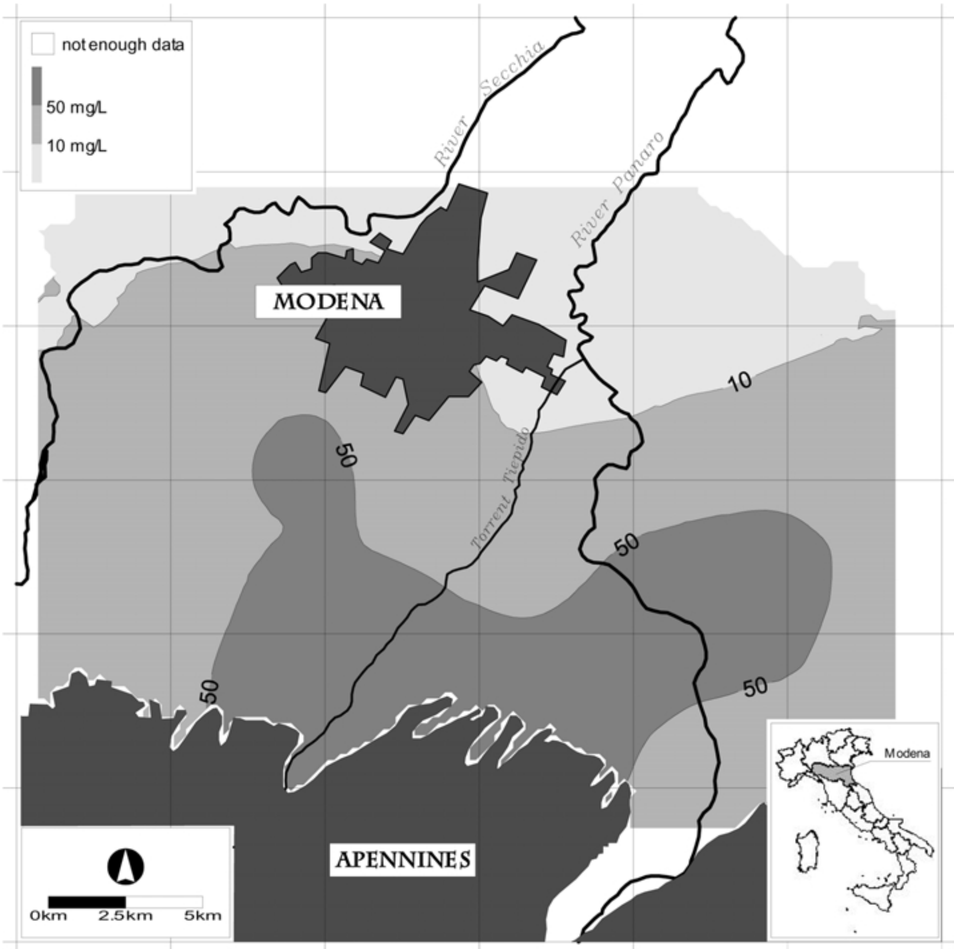

Figure 3. Example map of the estimated values of nitrate concentration by disjunctive kriging.

(e) Kriging estimation of concentration values over the whole study area and evaluation of the kriging estimation variances. Maps provide an accessible representation of the results produced in this phase and in most of the following.

(f) Evaluation of the probability conditioned (Yates *et al.*, 1986c) by the given threshold of concentration (usually the *maximum allowable concentration* or/and the *guide value* for drinkable use of water) at a given time, and all over the study area. The corresponding maps represent levels of *spatial risk*.

(g) Application of a non-parametric statistical test to evaluate temporal information (*temporal risk*). If the procedure described above is applied at different times, it can be conveniently used to consider the temporal information. Once the CP have been evaluated at a specific point of the study area at different times t_i, a non parametric test, i.e. Spearman test (Sneyers, 1975) can be used to

TABLE I
Categories of *effective risk*

Index of spatial risk	Index of trend (temporal risk)		
Classes of present CP	Positive trend	No trend	Negative trend
0.0–0.2	Low	Very low	Very low
0.2–0.4	Mean	Very low	Very low
0.4–0.6	High	Mean	Low
0.6–0.8	Very high	Very high	Mean
0.8–1.0	Very high	Very high	High

assess trends during the study time period. In fact, at a given time t, the spatial risk represents a layer (in time dimension) of information of the probability to exceed the threshold value. The Spearman test treats the different layers as numbers (probabilities) located in time, without any assumption on the distribution of these numbers. The value of the Spearman correlation coefficient (r) in every location of the study area was analysed by means of a two tails hypothesis test, using the table of the critical values of r. If the null hypothesis $H_0 : r = 0$ was rejected at a 5% significance level, a positive or negative trend was assigned at the given location when a positive, respectively negative, value of r characterised that location. If the null hypothesis was accepted, a no trend index was assigned at the location. Again the results can be presented in form of maps of *temporal risk*.

(h) Identification of a classification of the risk of groundwater quality impairment considers both its spatial distribution and temporal behaviour. The final aim of this methodology is to define a classification of the risk (*effective risk*) with respect to a given standard that include both temporal (index of *trend*) and spatial (index of *spatial risk*) information. Each index represents respectively, the temporal behaviour of the risk in terms of trends of CP and the present level of risk in terms of most recent CP evaluated over the considered area. By superimposing these two indexes, a range of five classifications of the *effective risk* (Table I) can be produced.

The final product of such methodology is a map of levels of the *effective* (spatial and temporal) *risk* of groundwater quality degradation relative to a given standard.

3. Case Study

In this article, the methodology has been applied to a case study of the concentration of nitrate in the aquifer of Modena (Figure 1). Probability maps have been

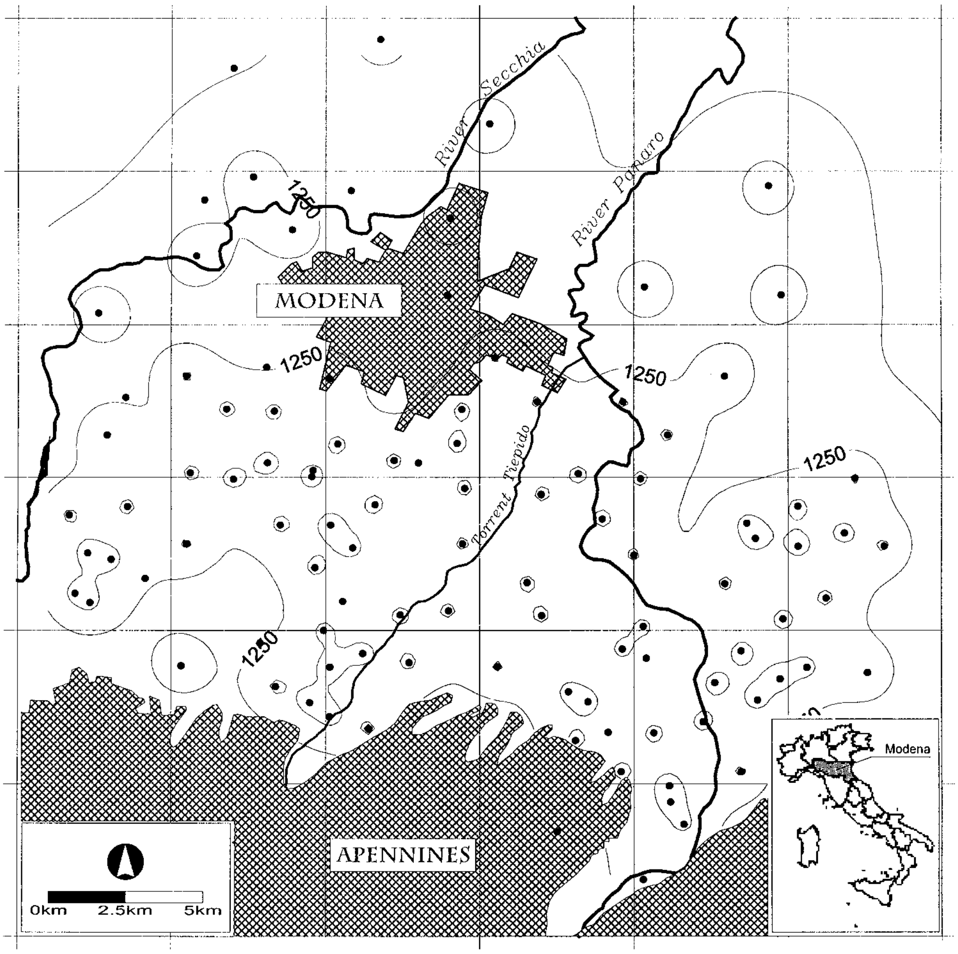

Figure 4. Example map of the disjunctive kriging estimation variances (mg $L^{-1})^2$ (same season of Figure 3).

plotted relative to two different values of the threshold. The first one has been fixed at 10 mg L^{-1} and the second equal to 50 mg L^{-1}, which are, respectively, the *guide value* (GV), and the *maximum allowable concentration* (MAC) established by the Italian law for potable use of the water.

The study area is large, about 24 000 ha, and it is located in Northern Italy. It is crossed by the rivers Secchia and Panaro, and it is bounded by the Apennines on the southern edge. From a geologic standpoint, the high and average plain, included between the Apennine on the South and the sedimentary basin of the river Po on the North, is basically characterised by a system of interconnected alluvial fans. The water table decreases (relative depths below land surface) from the southwestern area to the northeastern one, and it is normally higher in spring

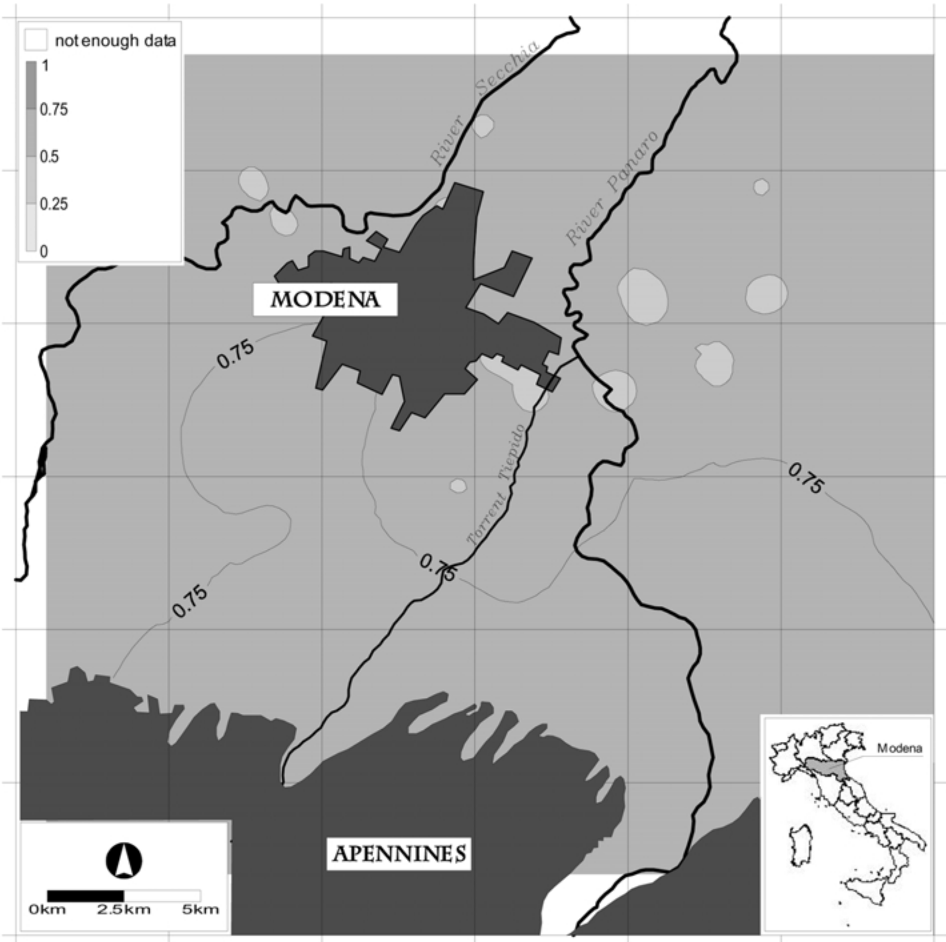

Figure 5. Example map of the *spatial risk* related to the threshold of 10 mg L^{-1} of nitrate concentration.

than in autumn. A change in hydrogeologic conditions of the aquifer from unconfined to confined or semi-confined, in the vicinity of Modena, was pointed out in a former study (Paltrinieri *et al*., 1990). Agricultural land use extends over 80% of the area and the principal crops are wheat, other cereals (maize), fodder, vegetables, vineyards and orchards. The major consequence of these practices has been the high concentrations of nitrate, which can be found in the aquifer.

Nitrate concentration values sampled in about 90 monitoring wells spread across the area were considered in this study. They were collected through 14 seasons, spring and autumn, from 1990 to 1996. Nevertheless, from 1990 to 1992 the monitoring network was not well organized and samples of nitrate concentration resulted often smaller than the following ones being the data set often incomplete, with several sampling points missing in each season. Furthermore statistical analysis

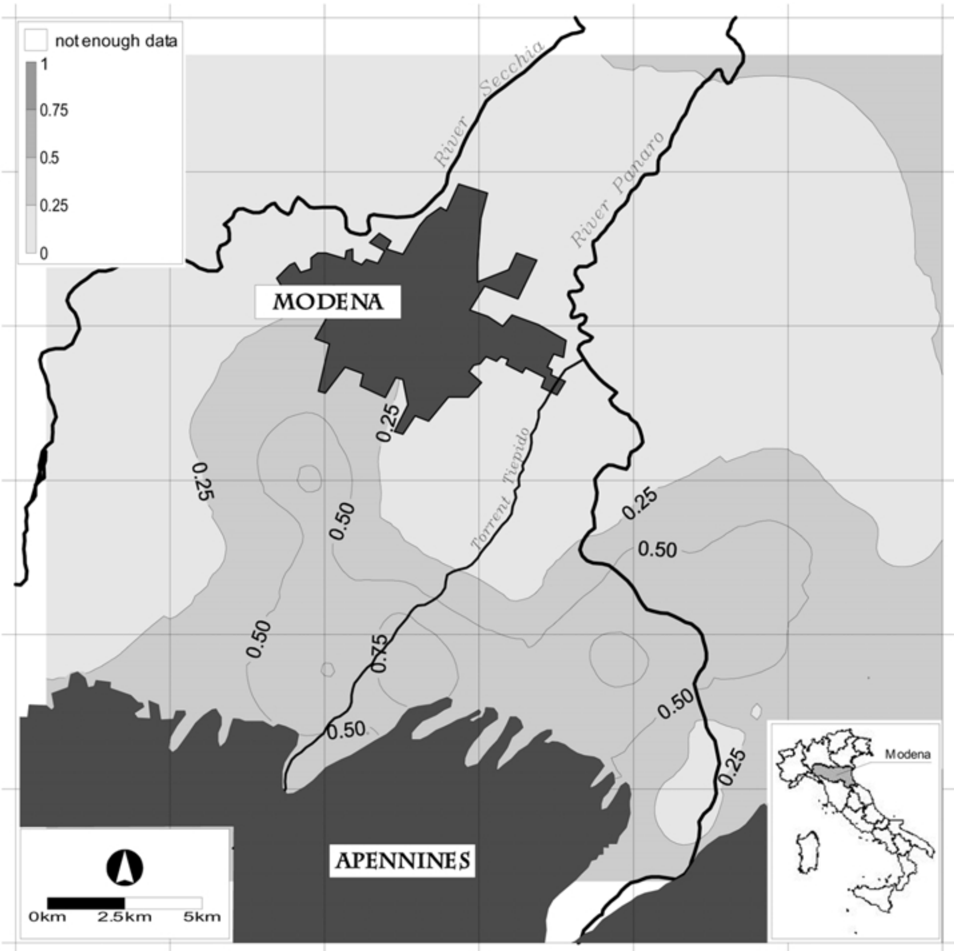

Figure 6. Example map of the *spatial risk* related to the threshold of 50 mg L^{-1} of nitrate concentration.

constantly outlined the presence of anomalous values (outliers), which were eliminated when the anomalies were not explained (error of measurement, error of transcription, etc.).

Anyway, in the whole observation period, the mean values of nitrate concentrations over the area did not seem to be particularly different from spring to autumn and ranged between 30 and 39 mg L^{-1}. The data distribution is almost asymmetric in the seasons considered and the skewness coefficients confirm this observation. Tests of the concentration distribution for normality by the Kolmogorov-Smirnov (Kendall and Stuart, 1966) method were rejected as not significant; the data were not normally distributed. The structural analysis (variography) was executed, but, because of the described lack of data from 1990 to 1992, the experimental vario-

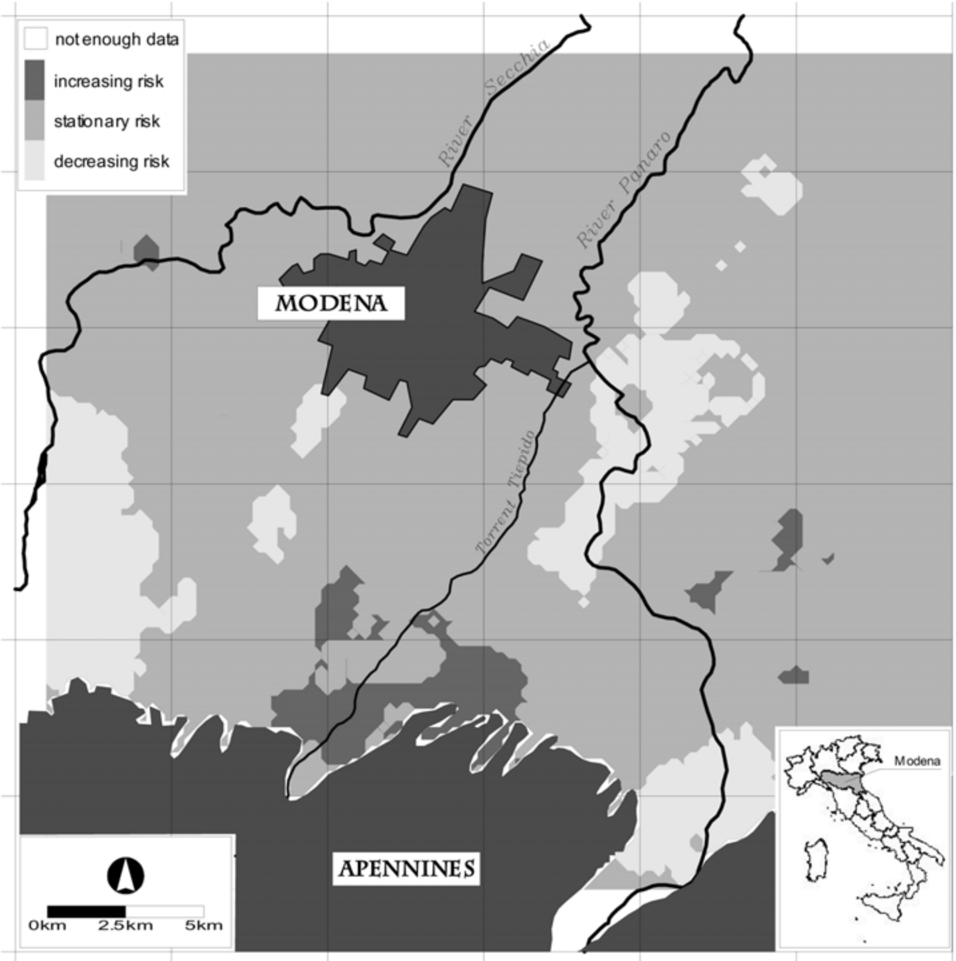

Figure 7. Example map of the *temporal risk* related to the threshold of 10 mg L^{-1} of nitrate concentration.

grams did not show a strong spatial dependence during these years; furthermore, the data pairs used for the first and second points of the variograms were often scarce compared with those used for the following points (less than 1/3). On the contrary, the experimental variograms, from 1993 to 1996, all appeared to reach stationarity.

Anyway, more importance and credibility to the spatial behaviour of nitrate concentration described by the experimental variograms, determined for the last four years, was given on the basis of the previous considerations also supported by the knowledge and experience of technicians, involved in studies on groundwater pollution in the study area. This is why the exponential model was forced to the

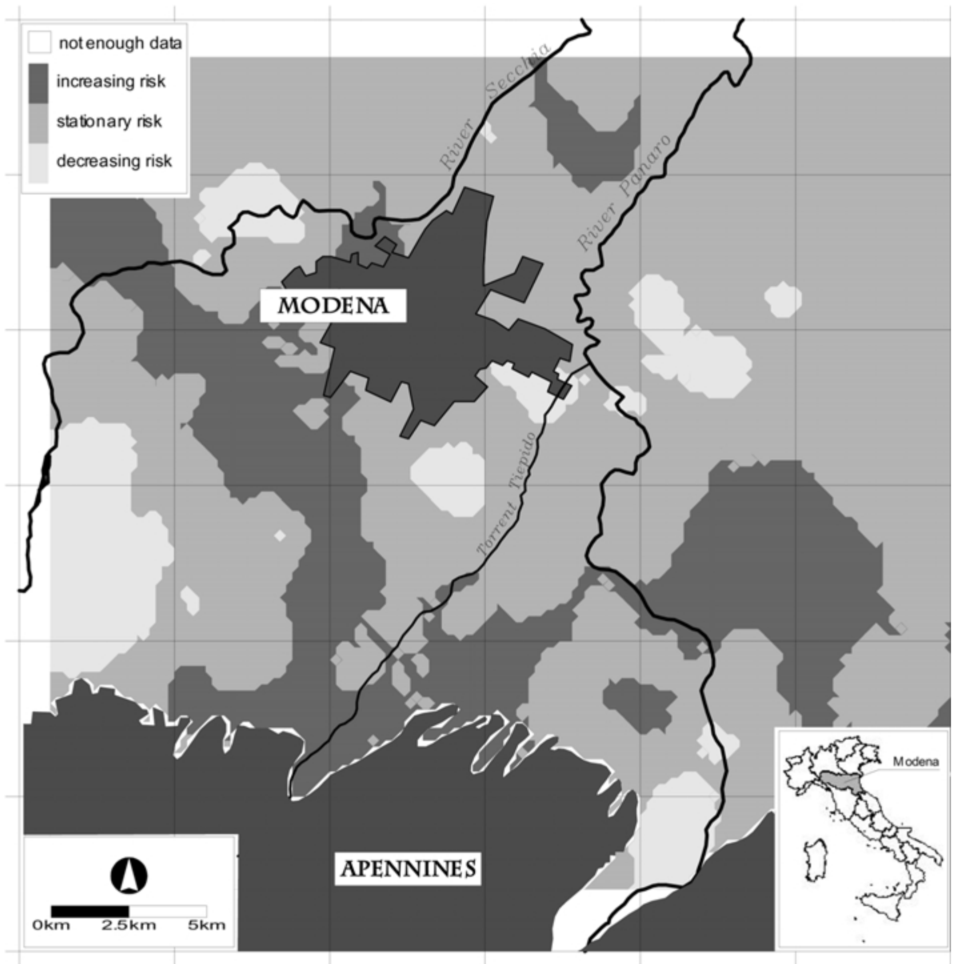

Figure 8. Example map of the *temporal risk* related to the threshold of 50 mg L^{-1} of nitrate concentration.

experimental variograms for years from 1990 to 1992 confident that this would have improved, and not altered, the study.

The exponential variogram model was chosen to reproduce the spatial behaviour of nitrate concentration near the origin since it showed the most consistent fit to the data. The nuggets were always less than the 30% of the total variance and all the theoretical variograms reach a constant value after a distance usually ranging between 5 and 10 km (Figure 2). These variograms were positively tested by means of the cross validation method, which confirmed the absence of systematic errors in the estimations and a substantial consistency of the variance.

The *DK* was applied in order to draw the maps of the *spatial risk* since this technique allows calculation of the conditional probability at any point in the study

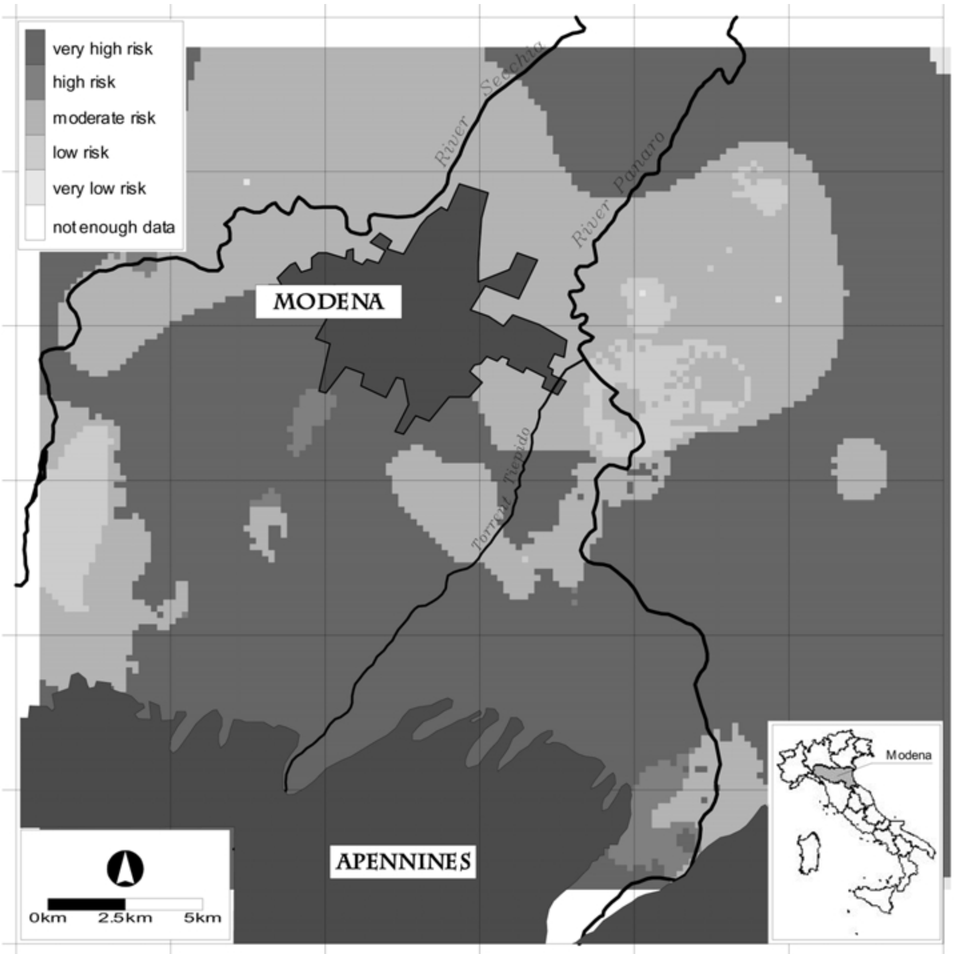

Figure 9. Example map of the *effective risk* related to the threshold of 10 mg L^{-1} of nitrate concentration.

area. The first step in a *DK* estimation process consisted in finding the right combination between the *Hermite* integration and the *Hermite* polynomial of order k.

In the study, the poorest *Hermite* approximation was found in the spring 1994; the difference between the *Hermite* approximation and the data variance was equal to –11%. In just three cases the approximation overestimated the data variance, by a maximum value of 6%. Changes in the averages (first *Hermite* coefficient) were negligible. *Hermite* underestimated 10 times out of 14 and the difference was rather low (maximum value 1.6 mg L^{-1}); in the other 4 cases *Hermite* approximation overestimated (maximum value 1.8 mg L^{-1}).

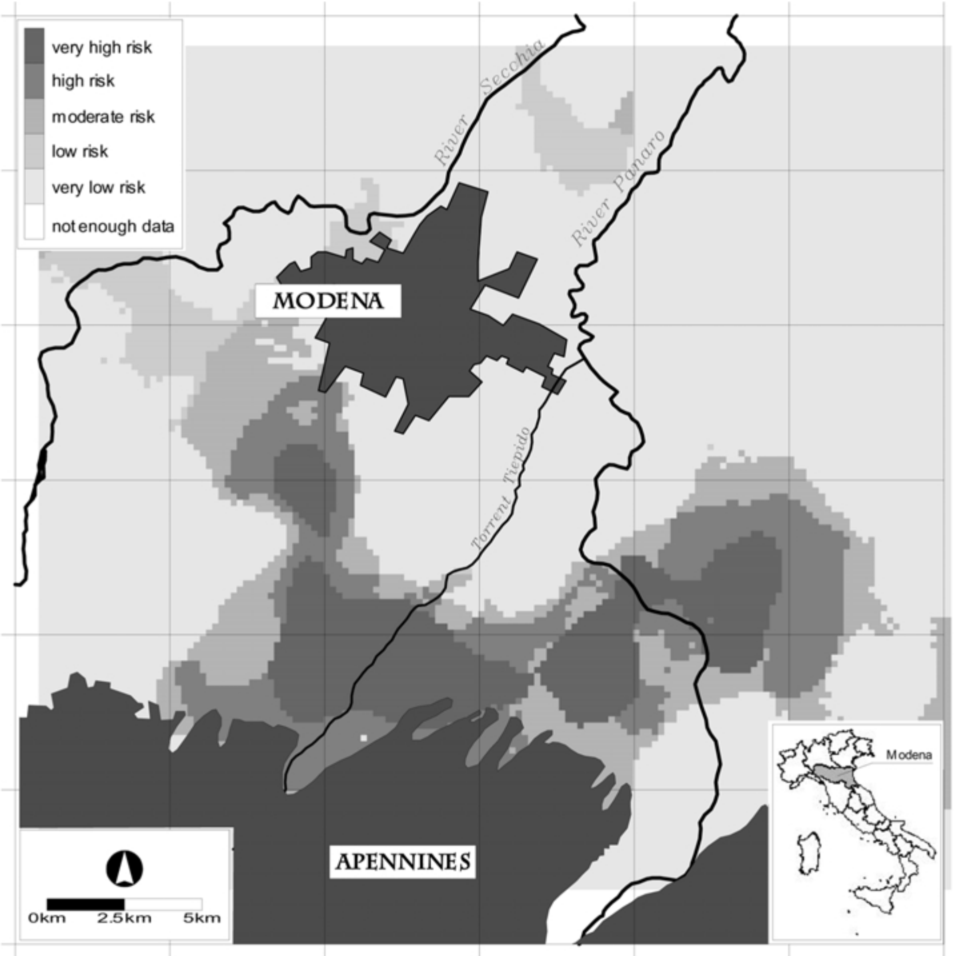

Figure 10. Example map of the *effective risk* related to the threshold of 50 mg L^{-1} of nitrate concentration.

The results of the first analysis were considered adequate. The second step was focused on the application of *DK* in order to estimate and draw maps of nitrate concentration (Figure 3) and the corresponding estimation variances (Figure 4). The third step involved computing the conditional probabilities to exceed the two given thresholds (10 and 50 mg L^{-1}) and plotting the maps of *spatial risk* (Figures 5 and 6).

The CP maps represent a simple measure of the risk at a particular time. The questions, at this point, were: 'since the considered event is time recurrent (not to say periodic), can we synthesise temporal information brought by the whole set of maps in a single form? And how can we do it?'.

A possible way to summarise the temporal evolution of the risk in a single map consisted in using a non-parametric statistical test applied to the temporal arrays of the probability values in each estimation cell. In this article the *Spearman* test was used to consider temporal information in terms of trend indices. Figures 7 and 8 show the maps of the levels of *temporal risk* for both the thresholds considered in this study.

The maps associated with the *trend indices* provided a measure of the temporal behaviour of risk but they were not able to take into account the most recent level of risk. A matrix including both sets of information (Table I) was developed defining classes of *effective risk* ranging from very high to low. Figures 9 and 10 show the maps of the *effective risk* related to the chosen thresholds. By comparing these two maps the following observation can be underlined: the map related to the *guide value* (*GV*) threshold (Figure 9) shows a very large area where *effective risk* has been classified *very high*. On the contrary, near the rivers and where the aquifer is mainly confined, it is considered *moderate* or *low*, probably because of freshwater recharge from the rivers. Figure 10, related to the *maximum allowable concentration* (*MAC*) threshold, shows that most of the area can be associated with a *very low* or *low* level of *effective risk*. However, a zone where it is *very high* is located between the towns of Spilamberto and Sassuolo, in the Southern part of the studied area where the effects of anthropogenic activities are very evident (Paltrinieri *et al.*, 1990).

4. Conclusion

Geostatistics allows one to estimate the probability that a known critical event produces harm to groundwater quality in excess of a given concentration threshold. This evaluation, developed in the form of maps, can be considered as a map of *spatial risk*, useful for water resources management efforts. Disjunctive kriging (*DK*) is a robust computational tool, which allows description of the spatial distribution of conditional probability. Possible alternatives to *DK* are geostatistical simulations (parametric and non parametric) and Indicator Kriging (*IK*).

The aim of the present article was to use *DK* as a methodology to assess the risk of groundwater quality impairment, defined as the probability that nitrate concentrations exceed a given threshold. This is an important result, since the *DK* results offer a basis for management decisions.

The methodology has been applied to the aquifer of the plain of Modena (Northern Italy) where nitrate concentrations had been sampled twice a year between 1990 and 1996. The groundwater monitoring network of about 90 wells was spread almost uniformly over the considered area. The number of samples ranged from 78 to 98 for most of the seasons but it was a little bit smaller during the first three years. The total number of data points (during the seven years) was 1271.

Using *DK*, several probability maps were plotted referred to two thresholds, respectively the *guide value* (*GV*) and *maximum allowable concentration* (*MAC*) for drinkable use of water. These maps provide a useful representation of the *spatial risk* for groundwater quality due to nitrate concentration, mainly produced by agricultural practices. Obviously, as the threshold increases the area of highest risk decreases. But groundwater in these acutely impaired areas cannot be used for human supply and actions should be taken to restore the aquifer quality.

Temporal information contained in the data set has been tested in order to evaluate the possibility of considering it to assess *temporal trends* in the risk. In general this may not be possible, since risk is related to an event and, consequently, when the event ceases the consequences produced by it could end. But, in the particular case considered in this article, the known critical event was time recurrent, since use of nitrate fertiliser or of manure disposal continue over the area, year after year. This means that the consequences produced by the considered event cannot become flushed out but, rather, they fluctuated from a worse situation to a better one. In situations where temporal information is available, it may be useful to develop a *trend index* able to describe the temporal behaviour of the *spatial risk* and a matrix of classification considering again the *trend index* associated with the *indexes of most recent spatial risk*. These measures can also be used to judge the performance of chemical and land use management practices intended to improve groundwater and surface water quality.

Acknowledgements

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References

Abramovitz, M. and Stegun, A.: 1965, *Handbook of Mathematical Functions*, Dover, New York.

Burrough, P. A.: 1989, *Principles of Geographical Information Systems for Land Resources Assessment*, Oxford University Press, Oxford.

Carr, J. and Mao, N.: 1993, 'Comparison of Disjunctive Kriging to Generalized Probability Kriging in Application to the Estimation of Simulated and Real Data', in A. Soares (ed.), *Geostatistics Troia 1992*, Kluwer Academic Publishers, Dordrecht, pp. 13–24.

DPR n.236/88: 1988, *Attuazione della Direttiva CEE n.80/778 Concernente la qualità delle acque destinate al consumo umano, ai sensi dell'art. 15 della Legge 16 Aprile 1987*, G.U. n. 183 suppl. del 30 giugno, Roma.

Fraters, D., Boumans, L. J. M., van Drecht, G., De Haan, T. and and De Hoop, W. D.: 1998, 'Nitrogen monitoring in groundwater in the sandy regions of The Netherlands', *Env. Poll.* **102**, 479–485.

Goodchild, R. G.: 1998, 'EU Policies for the reduction of nitrogen in water: The example of the Nitrates Directive', *Env. Poll.* **102**, 737–740.

Gundersen, P., Callesen, I. and De Vries, W.: 1998, 'Nitrate leaching in forest ecosystems is related to forest floor C/N ratios', *Env. Poll.* **102**, 403–407.

Journel, A. G. and Huijbregts, Ch. J.: 1978, *Mining Geostatistics*, Academic Press, London.

Kendall, M. G. and Stuart, A.: 1966, *The Advanced Theory of Statistics*, Hafner, New York.

Matheron, G.: 1976, 'A Simple Substitute for Conditional Expectation: The Disjunctive Kriging', in *NATO-ASI: Advanced Geostatistics in the Mining Industry*, pp. 221–236.

Oks, J. P., Leenaers, H. and Krzanowski, R. M.: 1993, 'Probability Kriging as a Decision Support Tool for Local Soil Pollution Problems', in A. Soares (ed.), *Geostatistics Troia 1992*, Kluwer Academic Publishers, Dordrecht, pp. 673–684.

Paltrinieri, N., Pellegrini, M. and Zavatti, A.: 1990, *Studi sulla vulnerabilità degli acquiferi*, Pitagora Pubbl., Bologna.

Rao, A. R. and Hsieh, C. H.: 1991, 'Estimation of variables at ungaged locations by empirical orthogonal functions', *J. Hydrol.* **123**, 51–67.

Rivoirard, J.: 1994, *Introduction to Disjunctive Kriging and Non-Linear Geostatistics*, Spatial Information Systems, Clarendon Press, Oxford.

Sneyers, R.: 1975, *Sur l'analyse statistique des sèries d'observations*, Organisation Météorologique Mondiale, Note technique n. 143, 192.

Soares, A., Tavora, J., Pinheiro, L., Freitas, C. and Almeida, J.: 1993, 'Predicting Probability Maps of Air Pollution Concentration: A Case Study on Barreiro/Seixal Industrial Area', in A. Soares (ed.), *Geostatistics Troia 1992*, Kluwer Academic Publishers, Dordrecht, pp. 625–636.

Van der Eerden, L. J. M., De Visser, P. H. B. and van Dijk, C. J.: 1998, 'Risk of damage to crops in the direct neighbourhood of ammonia sources', *Env. Poll.* **102**, 49–53.

Varnes, D. J.: 1984, *Commision on Landslides and Other Mass-Movements-IAEG. Landslide Hazard Zonation: A Review of Principles and Practices*, The UNESCO Press, Paris.

Yates, S. R., Warrick, A. W. and Myers, D. E.: 1986a, 'Disjunctive kriging I. Overview of estimation and conditional probability', *Water Resour. Res.* **22**, 615–622.

Yates, S. R., Warrick, A. W. and Myers, D. E.: 1986b, 'Disjunctive kriging II. Examples', *Water Resour. Res.* **22**, 623–630.

Yates, S. R., Warrick, A. W. and Myers, D. E.: 1986c, 'A disjunctive kriging program for two dimensions', *Comp. Geosci.* **12**(3), 281–313.

Yates, S. R. and Yates, M. V.: 1988, 'Disjunctive kriging as an approach to management decision-making', *Soil Sci. Soc. Am. J.* **52**, 1554–1558.

Webster, R. and Oliver, M. A.: 1989, 'Optimal interpolation on isarithmic mapping of soil properties: VI. Disjunctive kriging and mapping the conditional probability', *J. Soil Sci.* **40**, 497–512.