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# THE ELASTIC CONSTANTS OF CORD-RUBBER LAMINATES

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# I. STATEMENT

In Ref. 1 it was shown that for those cases where all plies of a multi-ply laminate were loaded in such a way that their cords were in tension, or alternately, if all plies were loaded in such a way that all their cords were in compression, then exact expressions for the moduli of the resulting orthotropic structures could be found. These moduli were expressed in terms of the cord half-angle $\alpha$ and the four elastic constants of a single sheet of the material making up the laminate.

In Ref. 2 it was shown from some simple approximations that the four elastic constants of a single sheet could be reduced to one ratio of two of these elastic constants. Under these conditions, it should be possible to calculate and plot the over-all orthotropic elastic constants of the complete laminate as a function of the cord half-angle and this single ratio of elastic constants $\frac{G_{xy}}{E_x}$.

## II. SUMMARY

The elastic constants of a multi-plied orthotropic structure may be determined and plotted as a function of the cord half-angle $\alpha$ and one single numercial quantity representative of the degree of anisotropy of a single sheet of material used in the laminate.

These calculations show that the cord stiffness and end count ply an important part in the elastic modulus of extension $E_\xi$ at small cord angles. At larger cord angles these factors cease to be important and the over-all orthotropic modulus is determined primarily by the angle of the cords and the type of rubber used in the sheets. The cross modulus $F_{\xi\eta}$ may also be determined by similar techniques, as may the orthotropic shearing modulus $G_{\xi\eta}$. Both of these are dependent upon cord stiffness and end count to some extent over the entire range of cord angles, particularly the latter term which is extremely sensitive to this variable. Graphical values of these functions are presented and should be useful in estimating the effects of design changes on the stiffness of cord-rubber laminates.

## III. DETERMINATION OF ELASTIC CONSTANTS

Equations (15) and (17) of Ref. 1 give expressions for the modulus of elasticity $E_\xi$, the cross modulus $F_{\xi\eta}$ and the shearing modulus $G_{\xi\eta}$ associated with an orthotropic body made up of two plies with included cord half-angle $\alpha$. These moduli would hold equally well for multi-plied structures, such as 2, 4, or 6 plies, provided that the cords in all the plies are equally loaded. The equations relating these elastic constants to the $a_{ij}$ terms discussed in Refs. 1 and 2 are

$$\frac{1}{E_\xi} = \left(a_{11} - \frac{a_{13}^2}{a_{33}}\right)$$

$$\frac{1}{F_{\xi\eta}} = \left(a_{12} - \frac{a_{13}a_{23}}{a_{33}}\right)$$

$$\frac{1}{G_{\xi\eta}} = \left\{a_{31}\left[\left(\frac{a_{12}}{a_{11}}\right)\left(\frac{a_{11}a_{23} - a_{12}a_{13}}{a_{11}a_{22} - a_{12}^2}\right) - \frac{a_{31}}{a_{11}}\right] - (a_{32})\left(\frac{a_{11}a_{23} - a_{12}a_{13}}{a_{11}a_{22} - a_{12}^2}\right) + a_{33}\right\} \tag{1}$$

Multiplying each of these terms by the modulus $G_{xy}$ gives

$$\frac{G_{xy}}{E_\xi} = \left[(a_{11})(G_{xy}) - \left(\frac{a_{13}}{a_{33}}\right)(a_{13})(G_{xy})\right]$$

$$\frac{G_{xy}}{F_{\xi\eta}} = \left[(a_{12})(G_{xy}) - \left(\frac{a_{23}}{a_{33}}\right)(a_{13})(G_{xy})\right]$$

$$\frac{G_{xy}}{G_{\xi\eta}} = \left\{(a_{31})(G_{xy})\left[\left(\frac{a_{12}}{a_{11}}\right)\left(\frac{a_{11}a_{23} - a_{12}a_{13}}{a_{11}a_{22} - a_{12}^2}\right) - \frac{a_{31}}{a_{11}}\right] - (a_{32})(G_{xy})\left(\frac{a_{11}a_{23} - a_{12}a_{13}}{a_{11}a_{22} - a_{12}^2}\right) + (a_{33})(G_{xy})\right\} \tag{2}$$

It is seen from the form of Eqs. (2) that the $a_{ij}$ terms enter either as products with $G_{xy}$ or else as ratios with the other $a_{ij}$ terms. Thus, each of these elastic constants $E_\xi$, $F_{\xi\eta}$, and $G_{\xi\eta}$ can be reduced to a function of two variables, the cord half-angle $\alpha$ and the degree of anisotropy $\frac{G_{xy}}{E_x}$ , as previously shown in Ref. 2. These constants can be calculated and plotted as functions of the two variables. This has been done and the results are presented graphically in Figs. 1, 2, and 3.

In Ref. 2 it is shown why some uncertainty still exists concerning the exact value of the ratio $E_x/F_{xy}$. In previous reports, such as Refs. 2 and 3, where numerical calculations had been performed to present information, the calculations were invariably made in two sets, one using a value of one-half for $E_x/F_{xy}$ and the other using a value of one-third. In all previous calculations negligible differences were caused by the use of these two values of $E_x/F_{xy}$ and the common data resulting from the use of either was presented. This situation prevails even here with respect to the data presented in Figs. 1 and 3. However, the data of Fig. 2 concerning the quantity $F_{\xi\eta}/G_{xy}$ is the only example found where the use of these two different values gave slightly different results. Figure 4 shows the calculated values of $F_{\xi\eta}/G_{xy}$ vs. cord half-angle for $E_x/F_{xy} = 1/3$; the resulting values differ but little from those of Fig. 2, as can be readily observed.

It is seen that cord angles play a most dominant role in the variation of these elastic constants. In particular, the variation of extensional modulus $E_\xi$ with cord angle is extremely steep. This indicates that many applications of laminated structures might exist in other situations where the elastic characteristics must be controlled over wide ranges.

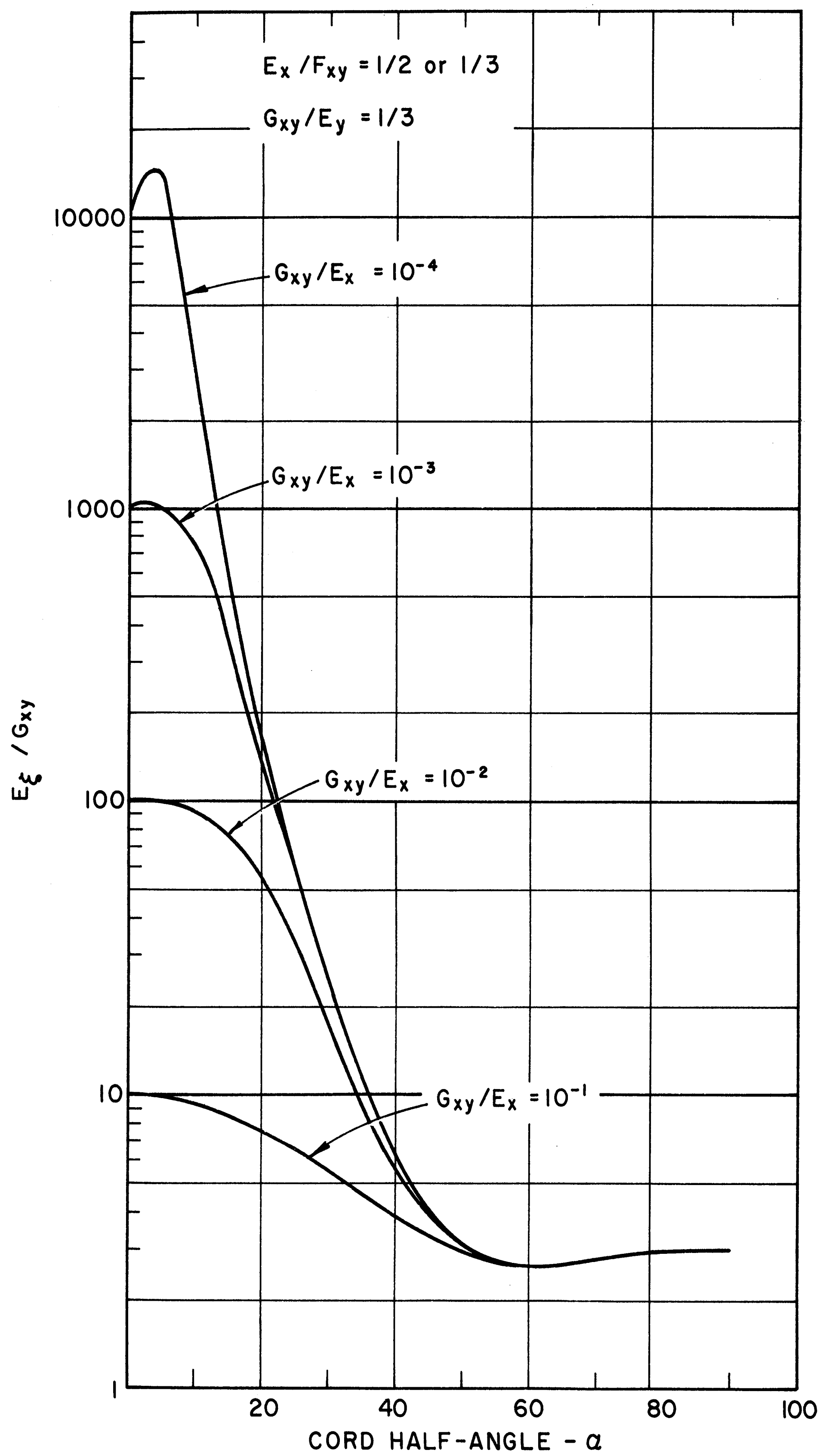


Fig. 1. $E_\xi/G_{xy}$ vs. cord half-angle $\alpha$ for various values of $G_{xy}/E_x$.

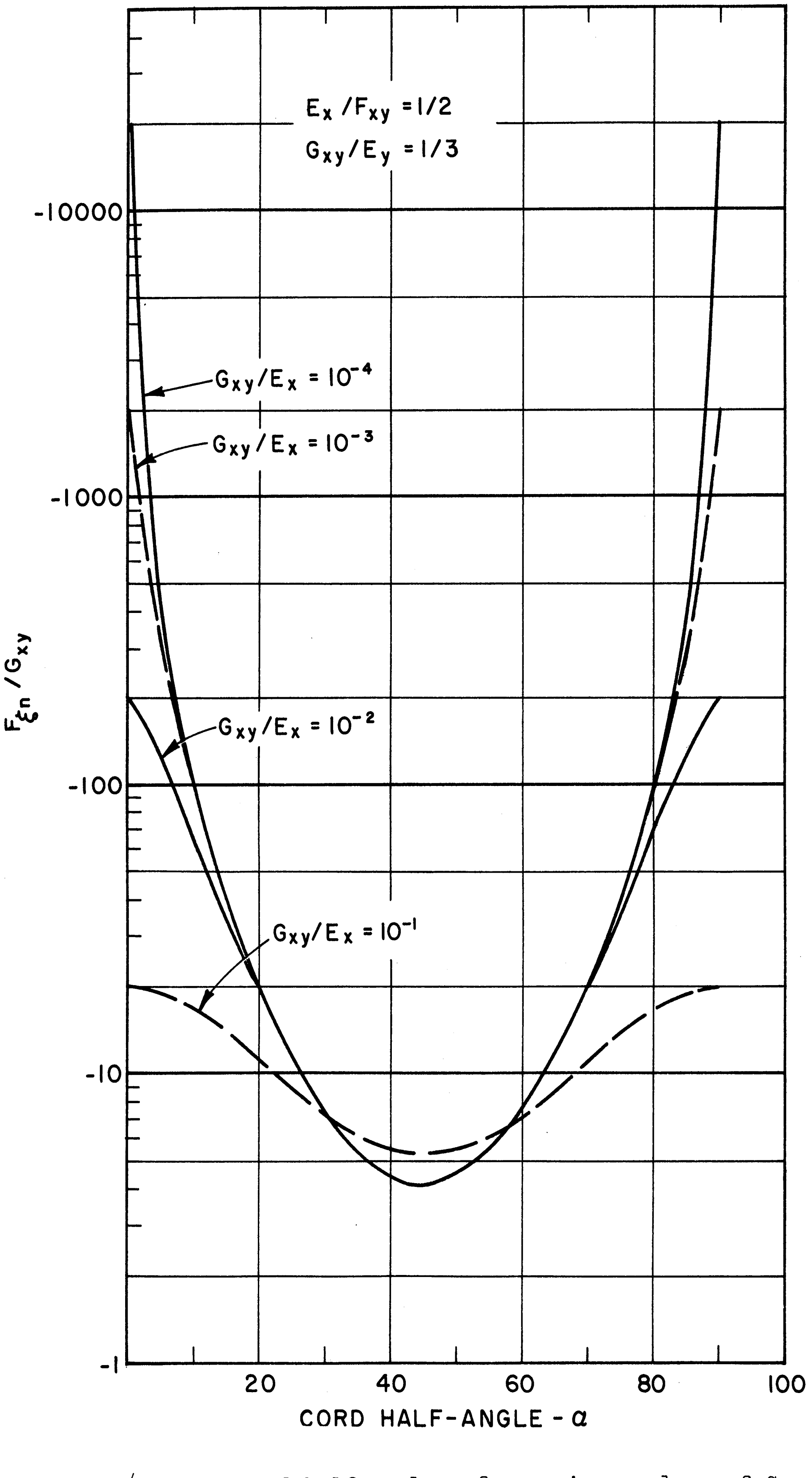


Fig. 2. $F_{\xi\eta}/G_{xy}$ vs. cord half-angle $\alpha$ for various values of $G_{xy}/E_x$.

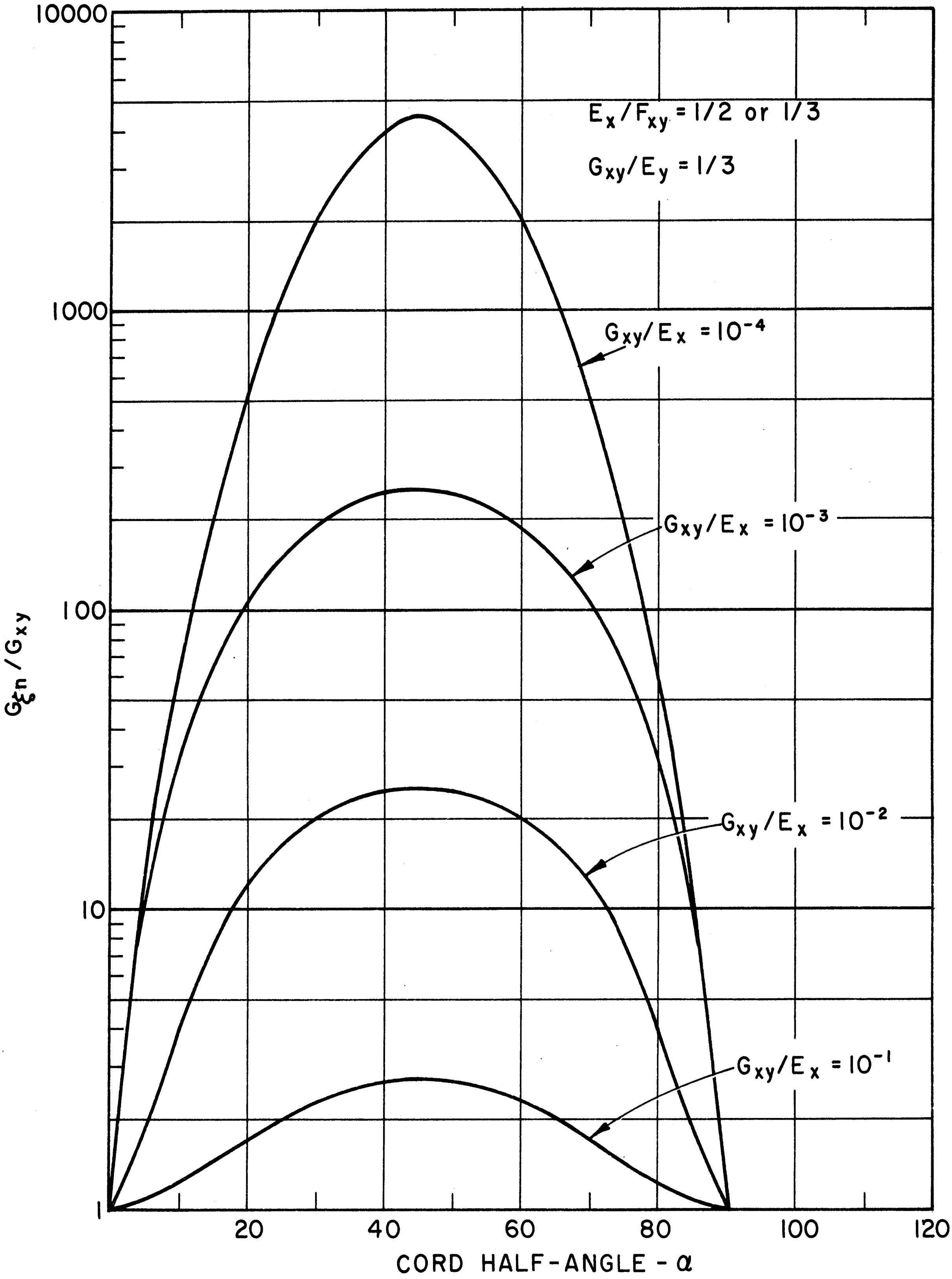


Fig. 3. $G_{\xi\eta}/G_{xy}$ vs. cord half-angle $\alpha$ for various values of $G_{xy}/E_x$.

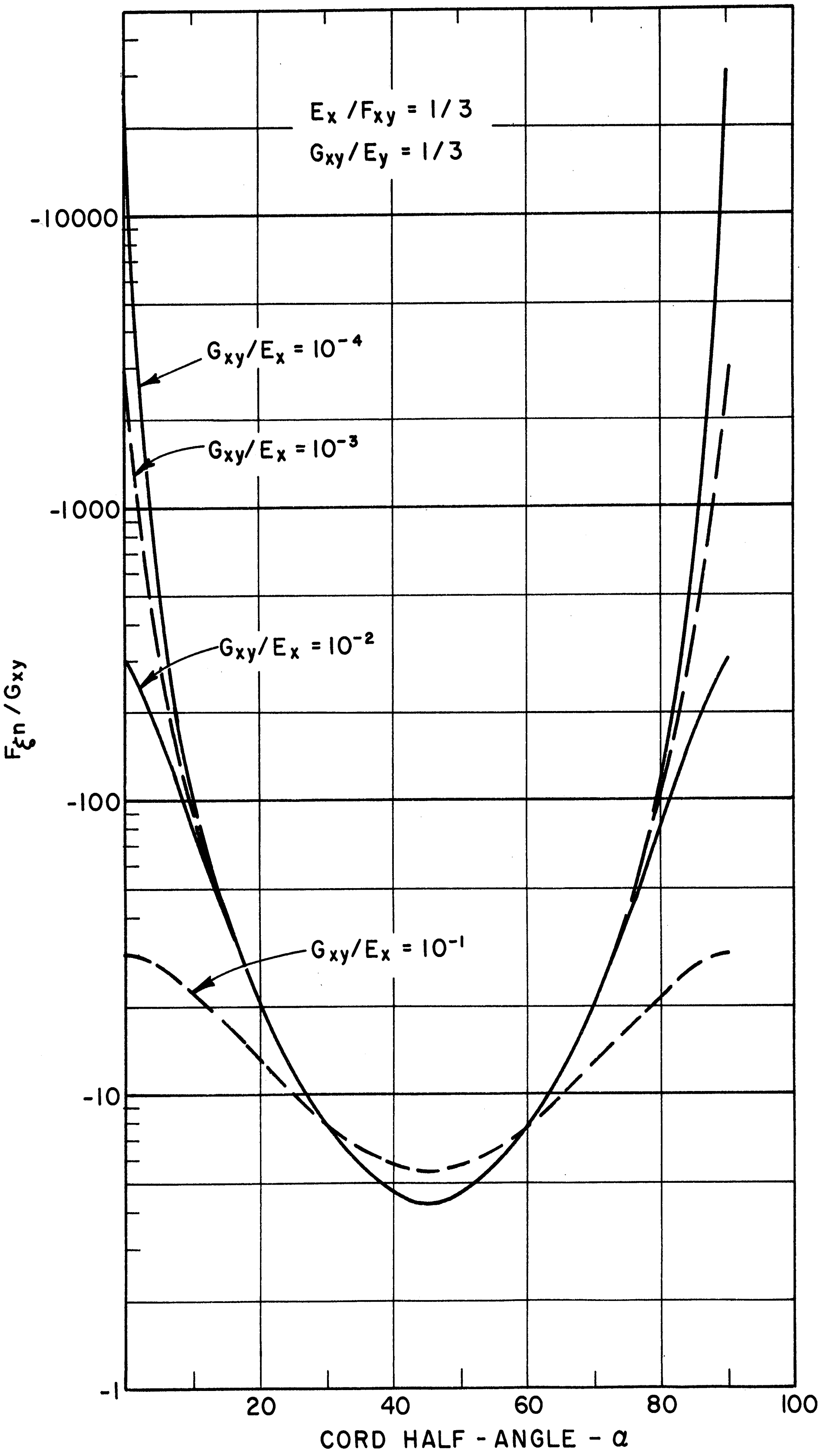


Fig. 4. $F_{\xi\eta}/G_{xy}$ vs. cord half-angle $\alpha$ for various values of $G_{xy}/E_x$, with $E_x/F_{xy} = 1/3$.

An interesting deviation from the relatively insensitive reaction of $E_\xi/G_{xy}$ and $F_{\xi\eta}/G_{xy}$ to changes in the ratio of $G_{xy}/E_x$ is provided by $G_{\xi\eta}/G_{xy}$, shown in Fig. 3. It may be seen that this quantity is highly dependent on both cord angle and the ratio $G_{xy}/E_x$, and further, that immense changes in shear stiffness are possible through design changes.

A knowledge of both $G_{xy}$ and $G_{xy}/E_x$ is required before Figs. 1-4 can be used quantitatively. Both of these elastic constants can be estimated by Eqs. (5) and (6) of Ref. 2.

## IV. ACKNOWLEDGMENTS

The calculations necessary for presenting this information were performed by Mr. Richard N. Dodge with assistance from Mr. D. H. Robbins, Mr. D. E. Zimmer, and Miss Gwendolynne Chang. Thanks are due to them for their care and patience in this work.

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