

DISCUSSION ON THE PAPER "A NOTE ON THE CRACK-PLANE STRESS FIELD METHOD FOR ANALYSING SIFs AND ITS APPLICATION TO A CONCENTRIC PENNY-SHAPED CRACK IN A CIRCULAR CYLINDER OPENED UP BY CONSTANT PRESSURE" by Wang Qizhi*

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Recently my attention has been drawn to the above-mentioned article in which Dr. Qizhi employed the so-called *section method* for deriving an approximate analytical expression for the SIF for a penny-shaped crack in an infinitely long cylinder subjected to uniform axial load. The method is widely used in problems of structural mechanics. Probably Parton and Morozov first employed this method in fracture mechanics. However, subsequent development of the method is largely due to Brodachev and Kuli who demonstrated the effectiveness of the method in many elastostatic crack problems. In particular, these authors employed this method for deriving approximate analytical expressions for the SIFs for two arbitrarily located cracks, a penny-shaped crack, an external circular crack in isotropic as well as anisotropic bodies for all three modes. The basic results in this direction are summarised in their paper [1].

In brief, Brodachev and Kuli proceeded as follows. It is well-known that for a penny-shaped crack of radius a whose faces are subject to some axial stresses $p(r)$, the normal stress in the crack plane outside the crack is

$$\sigma_z(r,0) = \frac{2}{\pi\sqrt{r^2-a^2}} \int_0^a \frac{t\sqrt{a^2-t^2}p(t)dt}{r^2-t^2}; \quad a < r < \infty \quad (1)$$

Stress intensity factor for a mode I penny-shaped crack is determined as

$$K_I = \lim_{r \rightarrow a+0} [2(r-a)]^{1/2} \sigma_z(r,0) \quad (2)$$

Putting (1) into (2), we obtain the following expression for the SIF:

$$K_I^\infty = \frac{2}{\pi\sqrt{a}} \int_0^a \frac{tp(t)dt}{\sqrt{a^2-t^2}} \quad (3)$$

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Borodachev and Kuli next represented the expression (1) as the sum of a singular and a regular part:

$$\sigma_z(r,0) = K_I \sqrt{\frac{a}{r^2 - a^2}} - \frac{2}{\pi} \sqrt{r^2 - a^2} \int_0^a \frac{tp(t)dt}{(r^2 - t^2)\sqrt{a^2 - t^2}}; \quad a < r < \infty \quad (4)$$

which can be easily verified by simply putting (3) into (4) and comparing the resulting expression with (1). The above analysis is valid for an infinite space.

It is now assumed that an expression like (4) will also hold for an infinitely long cylinder of radius b containing a penny-shaped crack, whose faces are subjected to the axial stresses $p(r)$, where now K_I should be replaced by K_p , the SIF to be determined, that is, we assume that

$$\sigma_z(r,0) = K_I \sqrt{\frac{a}{r^2 - a^2}} - \frac{2}{\pi} \sqrt{r^2 - a^2} \int_0^a \frac{tp(t)dt}{(r^2 - t^2)\sqrt{a^2 - t^2}}; \quad a < r < b \quad (5)$$

We next sever the cylinder by the plane section $z = 0$ and using equilibrium conditions and evaluating a number of integrals, it can be shown that K_I is given by

$$K_I = 2\pi \int_0^a r_0 p(r_0) W(r_0) dr_0 \quad (6)$$

where the weight function $W(r_0)$ is expressed as

$$W(r_0) = \frac{1}{\pi^2 \sqrt{a(b^2 - a^2)}} \left\{ \arctan \sqrt{\frac{a^2 - r_0^2}{b^2 - a^2}} + \sqrt{\frac{b^2 - a^2}{a^2 - r_0^2}} \right\} \quad (7)$$

Note that the structure of formula (6) clearly shows that $W(r_0)$ is the SIF for the penny-shaped crack in the cylinder, corresponding to the case where the crack faces are subjected to a unit concentrated axial load uniformly distributed along a circular ring of radius r_0 ($r_0 < a$). In fact, the expression $W(r_0)$ may be regarded as the axisymmetric fundamental SIF for a penny-shaped crack, embedded in an infinitely long isotropic superposition principle (6) to generate expressions for the SIF corresponding to more general axisymmetric cases of loading.

Let us now consider some particular loading cases:

Example 1. If the faces of a penny-shaped crack are subjected to a pair of concentrated loads, then

$$p(r) = \frac{P}{2\pi r} \delta(r) \quad (8)$$

where $\delta(r)$ is the Dirac's delta function. Putting (8) into (6) and considering the well-known property of the delta function, we obtain

$$K_I = \alpha \frac{P}{\pi^2 a \sqrt{a}} \quad (9)$$

where

$$\alpha = 1 + \frac{\Lambda \arcsin \Lambda}{\sqrt{1 - \Lambda^2}}; \quad \Lambda = \frac{a}{b} \quad (10)$$

Example 2. In the case of uniform axial load acting on the crack faces, we have

$$p(r) = \sigma H(a - r) \quad (11)$$

where $H(\dots)$ is the Heaviside step function. Putting (11) into (6) and evaluating the resulting integrals, we obtain

$$K_I = \beta \frac{2\sigma\sqrt{a}}{\pi} \quad (12)$$

where

$$\beta = \frac{1}{2} \left\{ 1 + \frac{\arcsin \Lambda}{\Lambda \sqrt{1 - \Lambda^2}} \right\} \quad (13)$$

Analogous results can be obtained for any axisymmetric loading. The above results were first obtained by Borodachev and Kuli [1,2].

Dr. Qizhi states that his paper involves more detailed formulation of the problem. In fact, his results correspond to only a particular case of the general problem. If one is to solve the problem for different cases of loading using Dr. Qizhi's methodology, the same manipulations will have to be repeated every time. I agree with Dr. Qizhi to the extent that it is not always possible to access Russian literature, but he did have full cognizance of the source of the work [1] which is referred to and to a considerable extent reproduced in [2] (Ref. [9] of Dr. Qizhi's paper). He might have done better to have gone through this material before committing to solve a particular case of a problem already solved.

REFERENCES

[1] N.M. Borodachev and M.P. Kuli, *Problemi Prochnosti (Problems of Strength)* 2 (1982) 23-27, in Russian.

[2] V.Z. Parton and V.M. Morozov, *Mechanics of Elastic-Plastic Fracture*, Hemisphere Publishers (1989) translation of 2nd revised Russian edition.

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