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TRANSIENT THERMAL STRESSES BY AN ANALOGY

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Abstract

This paper utilizes the physical analogy which exists between the conduction of heat in solids and the diffusion of vapor in solids to study transient thermal stresses by a simple technique. Controlled amounts of vapors are allowed to diffuse into photo-elastic plastics in such a manner as to duplicate, on a slower time scale, the diffusion of heat into solid elastic materials. The diffusing vapors create a volumetric expansion of the plastic analogous to thermal strain. The resulting stress patterns may be readily observed by photo-elastic techniques, and may be transformed into thermal stresses existing in the real heat conduction problem by suitable constant multipliers. The solution of a strip initially at uniform temperature whose edges are suddenly subjected to some different temperature is used to calibrate one photo-elastic material and to verify the analogy.

INTRODUCTION

The conduction of heat in homogeneous isotropic solids is governed by the differential equation

$$\frac{\partial u}{\partial t} = K \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \qquad (1)$$

along with suitable boundary and initial conditions. The same equations and conditions govern the diffusion of gases and vapors, under certain restrictions, in organic solids[(1)*]. In general these restrictions are somewhat more severe than those associated with the flow of heat, and so this analogy is specifically restricted to those situations whereby proper control of the concentration of the diffusing medium Eq. (1) may be satisfied.

Transient solutions of Eq. (1) are well known for many problems, and are of particular value in those cases where thermal diffusivity may be considered constant over some range of temperature so that solutions of Eq. (1) represent real temperature variations. Stresses that are created by the transient temperature distributions are obtained from the equations of lasticity, usually under the assumption of constant modulus of elasticity, Poisson's ratio and thermal coefficient of expansion.

Since both heat and vapor diffusion follow Eq. (1), and since it is well known that the absorption of water vapor by certain

(1)* Superscripts refer to numbered references in the bibliography.

organic solids causes swelling to occur, then small water vapor concentration differences should cause swelling which, inside a restricted range, is linearly proportional to the quantity of water vapor absorbed, just as thermal expansion is proportional to the temperature. Since the equations of elasticity apply in both cases, then stresses should be set up due to vapor diffusion of the type governed by Eq. (1), which are exact analogs of those stresses arising from unsteady heat conduction. In general both the stress magnitude and the time scale of the phenomena are different in the analog, but fortunately the order of magnitude of the diffusion constants as compared with thermal diffusivity is such that the analog gives the very early time stress solution which is usually that portion of greatest interest. Solutions for thermal stresses in closed form or series solutions of rapid convergence for small times are generally difficult to obtain in even simple geometric shapes subjected to temperatures. Experimenal procedures are limited generally to the measurement of strains with finite size gages applied judiciously to models. However, because of the degree of effort required for mathematical or experimental solutions, and the cost of electronic calculator solutions, it was felt by the authors that a simple method yielding a point-by-point analysis of thermal stresses was desirable. Such a method has been developed based upon the analogy presented above.

General

It is a well known fact among photo-elastic investigators that their materials are subject to a so-called "time edge effect".

This effect is generally very undesirable as it tends to stress the surfaces or edges of carefully prepared models according to the atmosphere in which the models are stored, thus making static testing a difficult procedure unless careful controls are maintained. New materials not subject to this effect are constantly being sought. The "time edge effect" is primarily caused by maintaining the model in an atmosphere with a partial pressure of water which is not in equilibrium with the water content of the model. The resultant diffusion of water into or out of the model causes sharp initial gradients near the surface and the resultant volumetric changes in turn cause high compressive or tensile stresses. It has been noted in the author's laboratories that the resin CR-39 is in water equilibrium with an atmosphere at about 35% relative humidity at normal room temperature. Above that humidity the model adsorbs water and below it desorbs water. The process is apparently fully reversible. The time edge effect is, however, not solely associated with water. All photo-elastic materials contain natural organic vapors which usually will leave the surface of a specimen due to surface volatilization. This process is perhaps some ten times slower than the water process but is significant in controlled experiments. The simple expedient of saturating the atmosphere surrounding the specimen with the appropriate vapor by the use of finely divided powder of the specimen material and of many times the total surface area of the specimen has been observed to control this process.

An analogy between the controlled "time edge effect" and the transient thermal stress problem has been stated above. A simple experiment to check this analogy has been performed. The case chosen was the stress in the central region of a long thin strip whose faces were insulated, whose initial temperature was considered zero as a reference and whose edges were subjected to a step function change of temperature at time zero. The solution of the temperature problem is given in reference (2). The solution of the transient stress problem is observed (Ref. 3) to be a one-dimensional stress system in which the longitudinal stress is proportional to the temperature minus the average temperature. Figure 1 shows the variation of the σ_x stress across the half width of the specimen plotted as the internal to surface stress ratio for various values of the dimensionless parameter Kt/ℓ^2 where K is the diffusivity in $(\text{inches})^2/\text{seconds}$, t is the time in seconds, and ℓ is the half width of the model in inches. The curves represent theoretical values and the points represent experimentally measured values of model stress as produced by constant temperature water adsorption, normalized to the same scale. This experiment was carried out with the model at an equilibrium temperature of 98°C, which represents about the highest practical working temperature for the material used. Similar experiments have been conducted at room temperature with equally successful verification.

Another experiment concerning the problem of a disk initially at temperature zero, whose edges are suddenly heated, was conducted and the correlation of the photo-elastic measurement of maximum shear stress on the model, subjected to water diffusion, with the calculated transient thermal stresses was equally good.

A saw cut was carefully made in the model of Figure 1 at the conclusion of the test and it was observed that no disturbance of the stress pattern was produced. This is considered as strong evidence that shear stress solutions of three-dimensional problems will be possible by this method, although of course not with CR-39, due to its inherent optical retardation in one dimension. Tests of the casting resin "Castolite" were made which illustrated that this material might be about one-half as sensitive to water vapor diffusion as CR-39, but that it does operate as a three-dimensional material in the same manner.

Since the intention of this paper was not to investigate all plastics or all diffusion processes available, but only to point out the successful use of an analogy to the thermal stress problem, no tabulations of data will be given. A program of research is currently being carried out to determine the temperature-dependence of the diffusivities of several vapors in the more important photoelastic plastics. A report of these values along with the appropriate scale factors for converting photo-elastically-determined stresses into numerical values for thermal stresses in metals will be made the subject of another paper.

Conclusions

A procedure for determining transient thermal stresses in two or three dimensional problems has been proposed and verified. Considerable variation in experimental technique and materials is possible. Optimum selection of a photo-elastic material and diffusing vapor has not been attempted in this paper but will be reported shortly. Since the vapor diffusion is reversible, it appears that by simple mechanical ingenuity one could produce the solution for any reasonable time and position program of surface temperatures. The advantages of the method are:

1. Point shear stress analyses may be made.
2. Interior shear stresses may be measured by a slicing technique.
3. The stressing proceeds slow enough to allow isoclinics to be photographed.
4. Complete determination of the stresses by methods of oblique incidence would cause the equation of equilibrium to yield thermal gradients
5. Composite models may be studied.
6. No elaborate equipment is needed.

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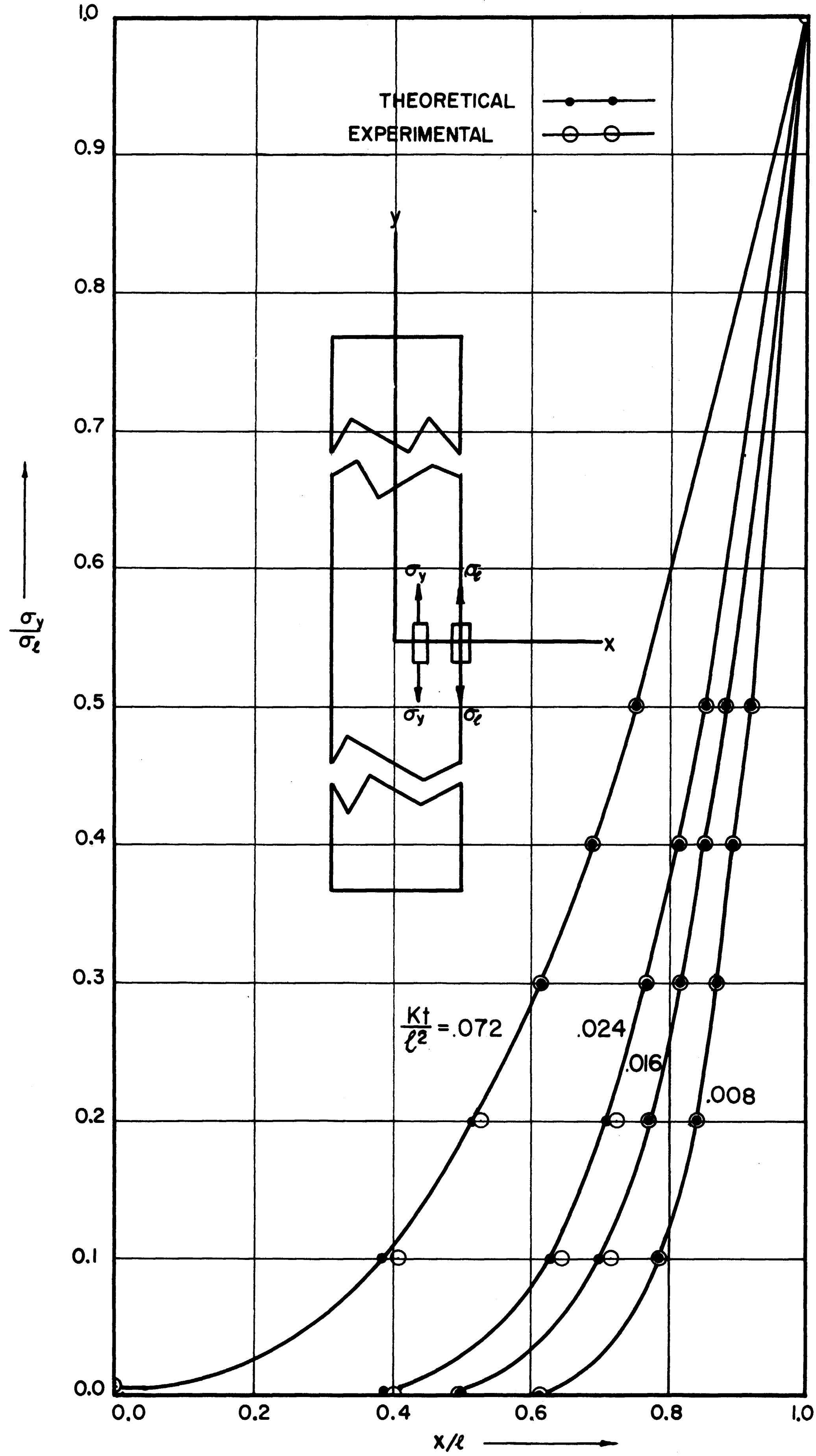

Figure 1. Comparison of experimentally measured thermal stress in a strip, with heated edges, as predicted by the diffusion analogy, with the results of the known theoretical solution.