ENGINEERING RESEARCH INSTITUTE
THE UNIVERSITY OF MICHIGAN
ANN ARBOR

Technical Report

VACUUM-BOTTLE SUSPENSION SYSTEMS

S. K. Clark

ERI Project 2254

THE ARO EQUIPMENT CORPORATION
BRYAN, OHIO

April 1958

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ABSTRACT

A method is presented for designing suspension systems for liquid oxygen containers. These suspension systems have the properties of relatively high thermal conductive resistance while at the same time providing an overdamped spring support. The damping is obtained by proper use of dry friction devices and limits both the relative motion and imposed acceleration of the inner bottle to acceptably small values. The method is of advantage since it requires a smaller annular space than is presently used between the two bottles.

INTRODUCTION

The Aro Equipment Corporation requested a study of possible suspension systems in October, 1957, as part of its program to increase the resistance of its liquid oxygen containers to vibration and shock environments. This was to be a broad study since its immediate object was to review any possible designs which came to mind in an effort to find a satisfactory one.

DEVELOPMENT OF DESIGN

Any successful suspension system such as is required here must contain a spring element of some form or another. One practical means of limiting the resonant amplitudes of the sprung mass is the introduction of damping. The most common way of introducing damping into a system is the use of fluids, but the extremely low thermal resistance of fluids eliminated them from consideration here. Another reason to reject them is that any suspension system designed for this purpose must undergo the approximately 800°F temperature to which the bottle is subjected in outgassing. Another method, the use of an absorber, was not deemed practical here due to the flexibility of the mounting feet and outer shell; further, small design variations in these elements would require changes in the absorber design. In general, the absorber would be too sensitive to small design changes to be a very effective tool.

Ruling out fluid damping and the absorber, the only other methods of introducing damping are through hysteresis or dry friction. Hysteresis is an acceptable method and in general should work provided that enough energy can be removed from the system per cycle. However, nonmetallic materials (such as rubber, nylon, etc.) are the most efficient and these are not capable of standing the 800°F temperature mentioned earlier. For this reason, dry friction was chosen as the mechanism of damping, since it can be obtained easily between metallic surfaces which can withstand high temperatures.

A number of dry friction devices were investigated before a design geometry was selected which appeared to possess all the desired features listed below:

1. Maximum thermal resistance.

2. Good, controllable spring constant properties in the vertical direction and in any two arbitrary but mutually perpendicular horizontal directions.

3. Good, controllable dry friction properties in the directions listed in the previous item.

4. Small thickness, so that the evacuated annular space between bottles may be kept to a minimum.

5. Ease of manufacturing and assembly of the proposed design.

6. Ability to withstand outgassing temperatures of approximately 800°F.

The proposed design geometry uses the 5-liter spherical bottle as a numerical example, although the technique outlined here is applicable to bottles of different weight and size.

Reference to Fig. 1, taken from Den Hartog, indicates that successful operation of a dry friction system through the resonant range depends on obtaining enough dry friction so that F/P_0 is greater than about 0.9, in order for oscillations to be limited to finite values. F is the friction force and P_0 the exciting force. Figure 1 was constructed for the system shown in Fig. 2.

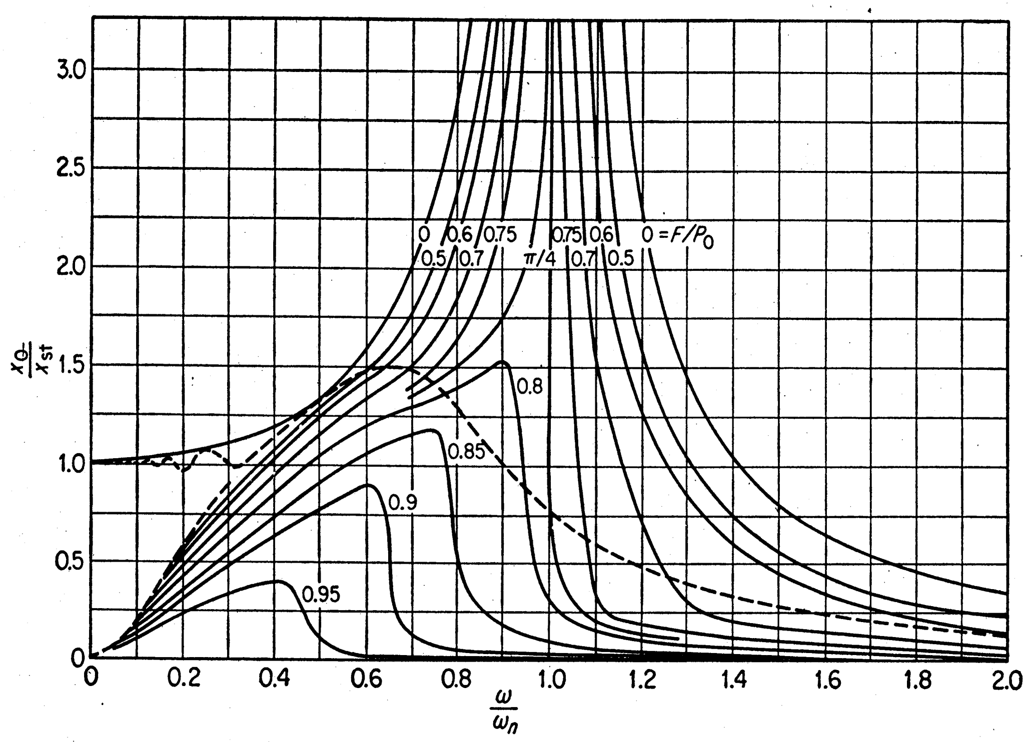

Fig. 1. Resonance diagram for a system with dry friction damping. By permission from Mechanical Vibrations, by J. P. Den Hartog. Copyright, 1956, McGraw-Hill Book Company, Inc.

From this, it is seen that the equation of motion becomes

$$m\ddot{x} + f(\dot{x}) + kx = P_o \sin \omega t \quad , \tag{1}$$

where $f(\dot{x})$ represents the damper force.

The system with which we are concerned is shown in Fig. 3. The equation of motion governing it is

$$m\ddot{x} + f(\dot{x}) + kx = my_o \omega^2 \sin \omega t \ . \tag{2}$$

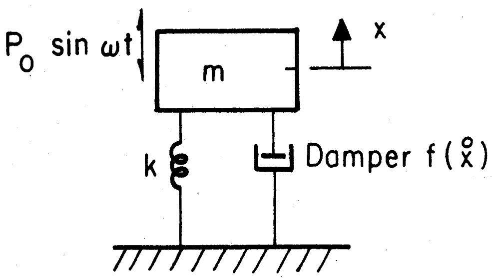

Fig. 2. Spring-mass system with dry friction damping.

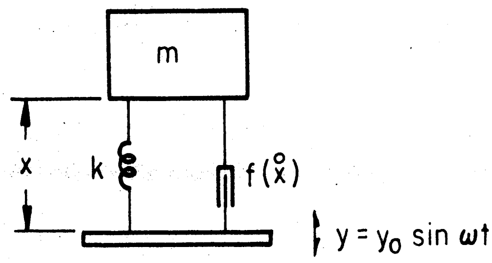

Fig. 3. Spring-mass system with dry friction damping and forced base motion.

The solution to Eq. (1) is plotted in Fig. 1, while Eq. (2) is identical to Eq. (1) upon substitution of P_o for $my_o\omega^2$, so Fig. 1 may be used to plot out the solution to Eq. (2). This is especially useful upon introducing the relation

$$y_o\omega^2 = \text{Maximum Base Acceleration} = a_B \ .$$

Thus, the ratio F/P_o now becomes, for the solution of Eq. (2), F/ma_B, and Fig. 1 indicates that the smallest value of this parameter which can be safely allowed in the vicinity of resonance is

$$\frac{F}{my_o\omega^2} = \frac{F}{ma_B} \approx 0.9 \ . \tag{3}$$

It is now desirable to preselect the natural frequency of the inner bottle on its suspension system. This selection is based on the general idea that the lower the natural frequency, the more isolation of inner from outer bottle will be obtained. High stresses in the suspension-system springs put a lower limit on the natural frequency of about 10 cps, and this figure will be used in all subsequent calculations. Any desired value of natural frequency which is higher could be chosen, but would result in less isolation.

At resonance an effective force P_o exists of magnitude[2]

$$P_o = (my_o\omega^2) = \frac{W}{g} \cdot (10x2\pi)^2 \text{ x } .040$$

$$= 0.4 \text{ W} = ma_B \quad . \tag{4}$$

Thus,

$$\frac{F}{P_o} = 0.9 = \frac{F}{0.4W} \text{ ,}$$

or

$$F \geq 0.36W \tag{5}$$

Equation (5) states that the magnitude of the dry friction force which must be generated to prevent motion in any given direction is .36 times the weight of the suspended inner bottle. This is one of the two basic design conditions, the other being the assumption that natural frequency will be held to 10 cps, as previously stated.

Design conditions as outlined above must be satisfied with the bottle in its full condition, since this is the most critical condition for both damping force and stress in the supporting springs. It will be shown later that the response of the system, under conditions other than full bottle loading, is more severe but not necessarily destructive to the system.

The principles just stated may be applied to a general spring-support system. Some work on this subject indicates that an optimum number of springs, namely, the number giving a minimum conductive path, must exist. In general, this minimum conductive path will be obtained with the minimum number of springs as a support system. This minimum number is four, for springs which are designed to take only compression. Another desirable feature of any design would be a distribution of spring constants so that the natural frequency of the inner bottle would be the same for any direction of motion. This can be nearly accomplished by a design in which three springs are placed at 45° below the circumferential equator of the spherical or near-spherical bottle, while a single spring at the topmost point acts as a hold-down device. Figure 4 shows the coordinate system used to describe the spring positions in Table I, where a spherical bottle of radius R_o is implied.

Calculation of the spring constant for vertical motion can be done by observing the geometry shown in Fig. 5. Assuming that one face of the spring may slip with respect to the inner bottle, then a spring deflection occurs which is described by

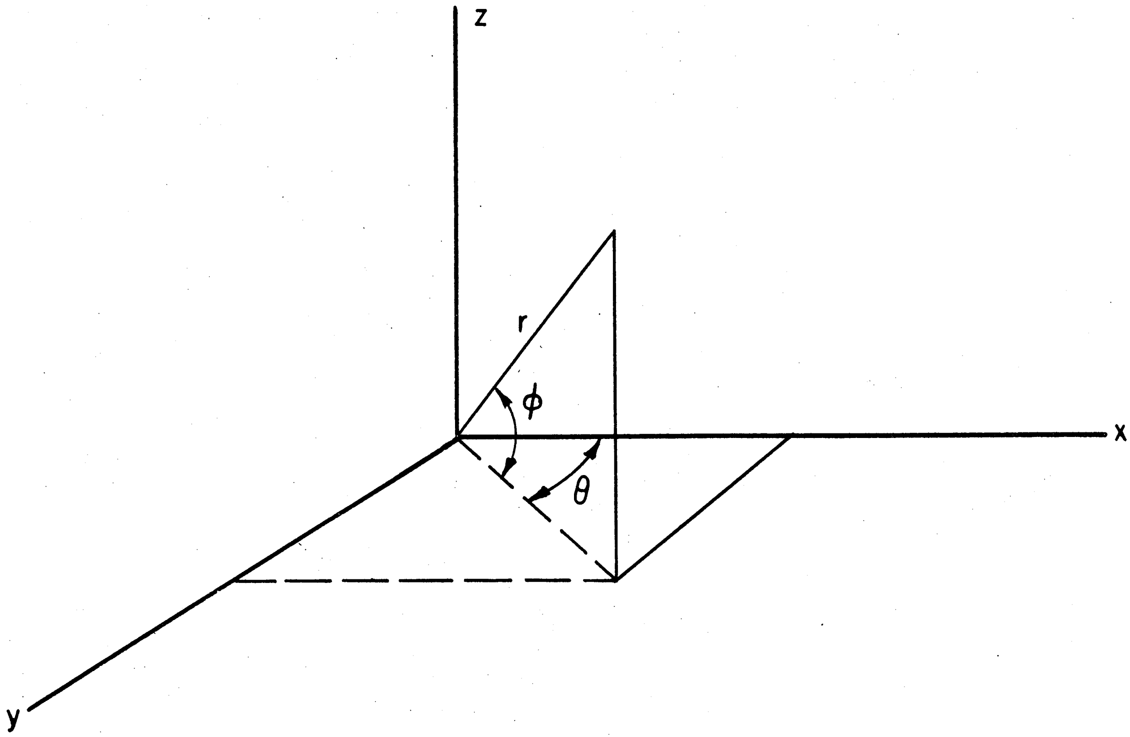

Fig. 4. Spherical coordinates for spring-position description.

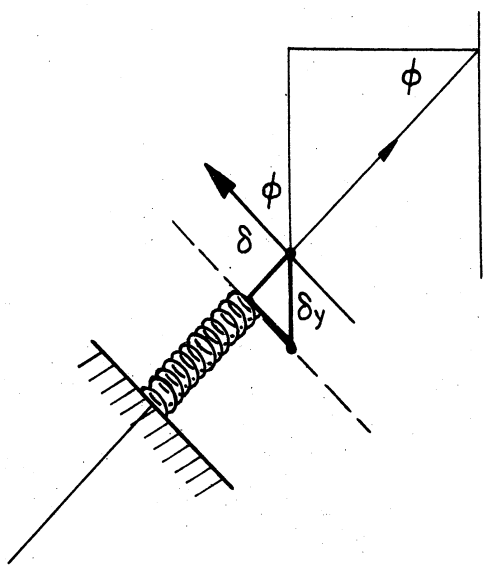

Fig. 5. Spring-deflection geometry.

$$\delta = \delta_y \cdot \sin \phi ,$$

where δ_y is the vertical bottle motion. Hence the restoring force is

$$F = (k \cdot \delta) \sin \phi = k\delta_y \sin^2 \phi . \qquad (6)$$

Two other identical springs exist so that the total restoring force is

$$F = 3k\delta_y \sin^2 \phi ,$$

or

$$k_{eq} = \frac{F}{\delta_y} = 3k \sin^2 \phi . \qquad (7)$$

For this design $\phi = 45°$ so that

$$k_{eq} = \frac{3}{2} k .$$

TABLE I

Spring No.	r	θ	ϕ
1	R_o	0	-45°
2	R_o	120°	-45°
3	R_o	240°	-45°
4	R_o	-	90°

The friction force will be directed perpendicular to the spring force, and will be in magnitude the product of spring force and friction coefficient so that

$$F_f = F_s \cdot f .$$

The vertically acting component of friction force is the one which is effective in absorbing energy, and is

$$F_f \cdot \cos \phi = F_s \cdot f \cdot \cos \phi .$$

Since 3 springs exist, the total vertical component of damping force is

$$3 F_f \cos \phi = 3F_s \cdot f \cdot \cos \phi . \qquad (8)$$

The average value of the spring force F_s may be computed since the total vertical components of both spring and friction forces must at least equal the weight, not counting any preload which might be present in the system. Figure 6 shows this geometry, where it must be assumed that the maximum possible

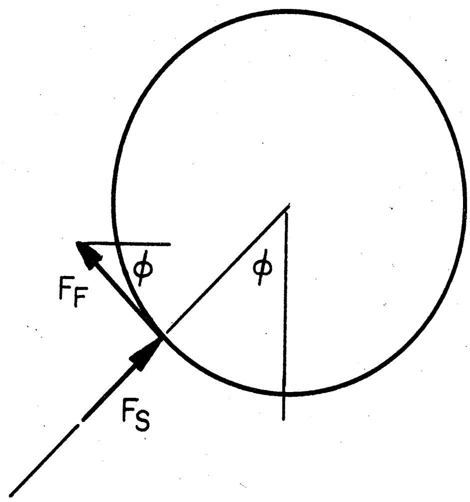

Fig. 6. Spring and friction force directions.

friction force is developed at the spring-bottle joint. The upward component of force is

$$F_s \cos\phi + F_F \sin\phi = F_s \cos\phi + fF_s \sin\phi .$$

For three springs, equilibrium of these forces with the bottle weight gives

$$3(\cos\phi + f \sin\phi)\, F_s = W,$$

or

$$F_s = \frac{W}{3(\cos\phi + f \sin\phi)} .$$

The vertical component of the total damper force is now, from Eq. (8),

$$F_{F_y} = 3f \cos\phi \cdot \frac{W}{3(\cos\phi + f \sin\phi)} = \frac{f \cos\phi}{\cos\phi + f \sin\phi} \cdot W . \qquad (9)$$

Equation (9) allows the angle ϕ to be chosen so that Eq. (5) can be satisfied.

For a typical copper-flashed inner-bottle surface, friction coefficients have been measured by experiment for stainless-steel-spring wire in contact with the copper. Details of those measurements are presented in the Appendix, but their main result is that an effective friction coefficient $f = 0.276$ was found. Using this, it is seen that an angle $\phi = 45°$ for placement of the springs results in Eq. (9) becoming

$$F_{F_y} = \frac{0.276 \frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2} + 0.276 \frac{\sqrt{2}}{2}} W = .216\, W .$$

Since a vertical damper force of 0.36W is required, then some preload will in general be necessary, or else a different angle ϕ must be picked. It is believed that the technique of adding preload is more satisfactory in the end, since it is necessary to hold the inner bottle down against upward motion. Thus, if a vertical preload equal to the weight W were put into the system by means of spring 4 of Table I, the resulting damping force component in the vertical direction would be

$$F_{F_y} = 2(.216W) = .432W ,$$

which is above the lower limit defined in Eq. (5).

The complete design geometry of the suspension system may now be illustrated in Fig. 7 as a function of weight, because it is possible to determine [from Eq. (7), the known spring positions, and the assumed natural frequency of 10 cps] the spring constant k of each individual spring in terms of the weight W, since

$$(10 \times 2\pi)^2 = \frac{\left(\frac{3}{2}k\right) g}{W} .$$

Solving for k, one obtains

$$K = 6.80\ W .$$

This is independent of preload.

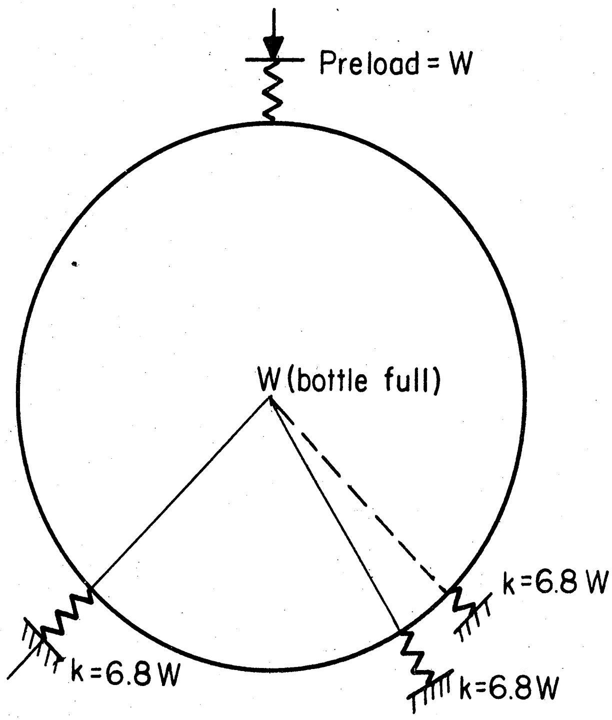

Fig. 7. Design geometry.

The spring constant of the top, or preloading, spring should be held to as low a value as possible. Spring stresses will limit this, but in general the aim here is to provide a preloading device which will exert a minimum influence on the spring constant of the remainder of the suspension system.

The exact shape of the springs is a matter in which the designer has some freedom, since the same spring constant can be obtained with a large number of different design geometries. The writer has made a short study of this and wishes to recommend "wave washers" or their wire equivalents as having a particularly desirable set of properties. Appreciable spring constants can be obtained with very small thicknesses, which means that the annular space between inner and outer bottles can be reduced. Also, the conductive paths, as measured by the length over area ratio of metal between inner and outer bottle, are usually quite long, and are of the same order of magnitude as that in the present "spoke-type" suspension. Finally, springs of this type contact the inner and outer bottles at points which have extremely small cross-sectional areas, and so provide additional thermal resistance.

With the construction shown in Fig. 7, it can be shown that the spring constant in any horizontal direction is identical to that in the vertical direction, so that all natural frequencies in translation are equal to the natural frequency previously calculated for vertical motion.

Frequencies of rotational and rocking motion were calculated and found to be close to those for translation, so that no unexpectedly high natural frequencies should exist.

Calculations were made from Fig. 1 of the displacement response of a mass supported by a suspension system designed along these principles. This response is plotted in Fig. 8, based on motion of the system in a vibration environment of Spec. Mil. E5272A.

These calculations were checked closely by a synthetic experimental system shown in Fig. 9, in which acceleration could be measured. Results from this system are plotted in Fig. 10, and are valuable in indicating the acceleration levels which will be encountered by the inner bottle. This information cannot be obtained by calculation due to the nonharmonic nature of the motion. In general the system behaved as expected in that relative motion became noticeable only at higher values of forcing frequency.

It should be stated here that so far only generalizations based on a system composed of a rigid mass, linear spring, and perfect dry friction damper have been made. Testing of these ideas with actual liquid gas containers and possible change or supplement of them is certainly in order, since the actual system is quite elastic, contains some fluid damping properties, and operates with a friction damper which is almost certainly not of constant friction coefficient as assumed in the analysis.

These principles will now be applied to a numerical example, the design of a suspension system for a 5-liter spherical bottle.

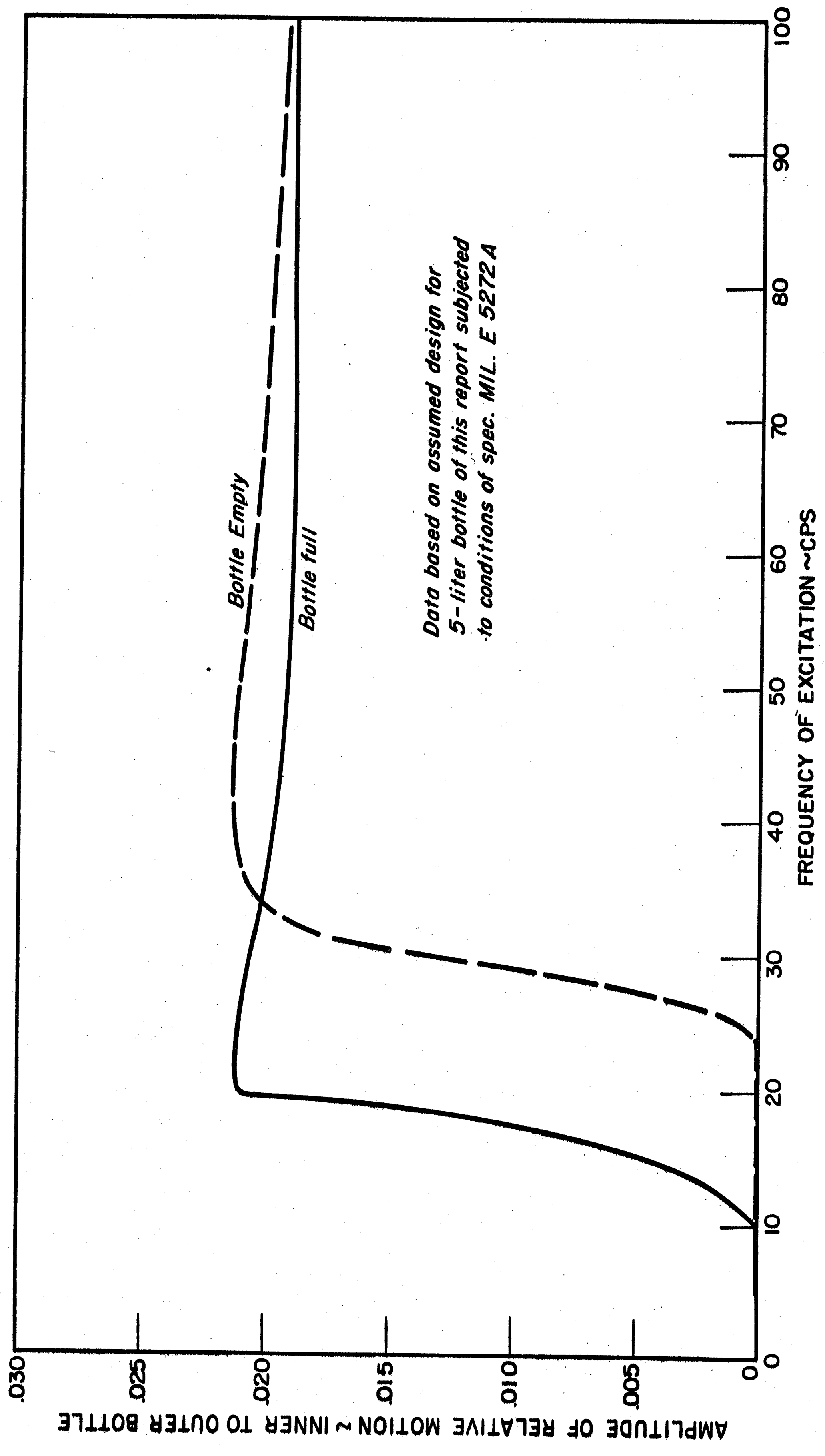

Fig. 8. Amplitude of relative motion of inner to outer bottle vs. frequency of excitation.

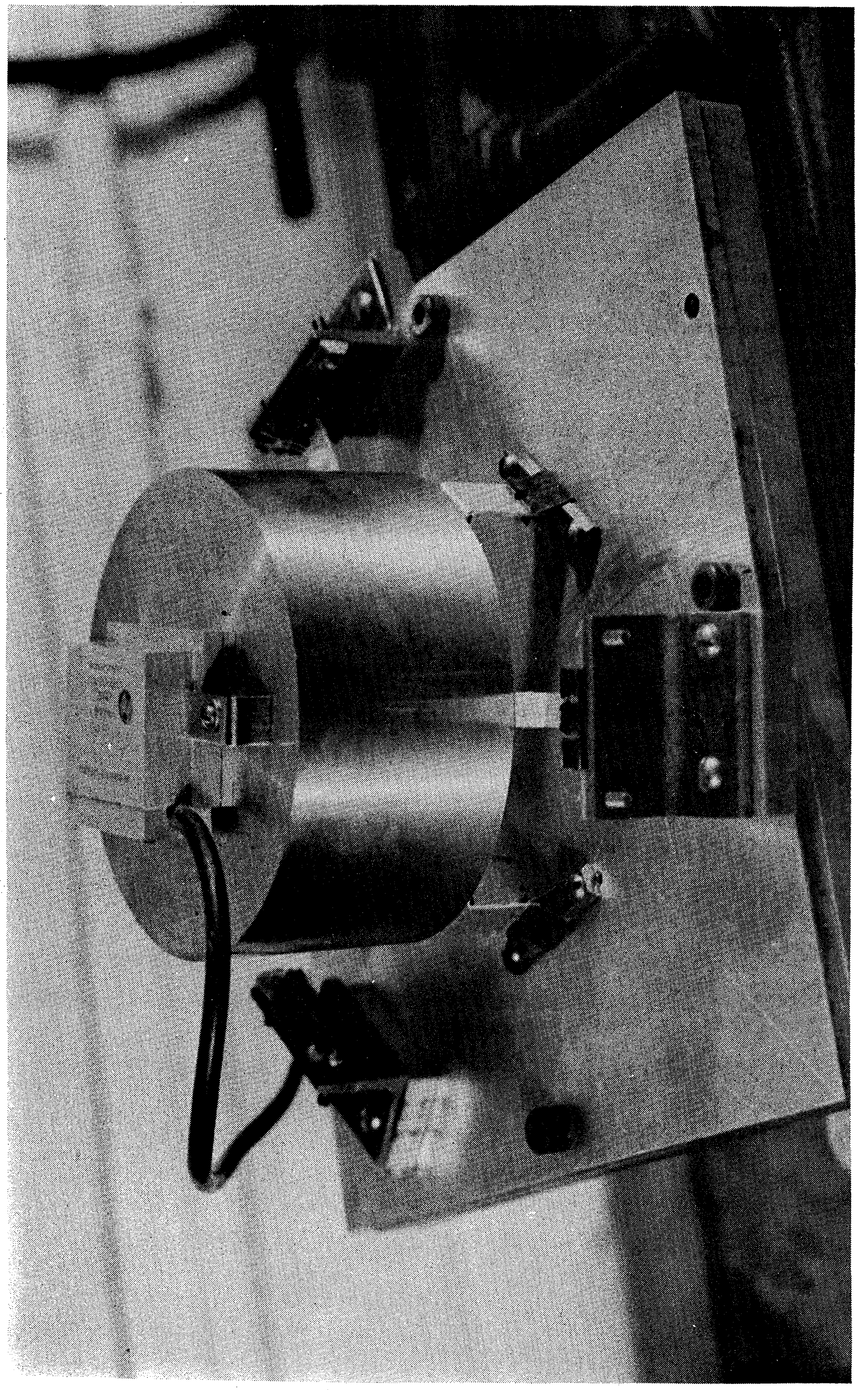

Fig. 9. Synthetic dry friction system for measurement of acceleration response.

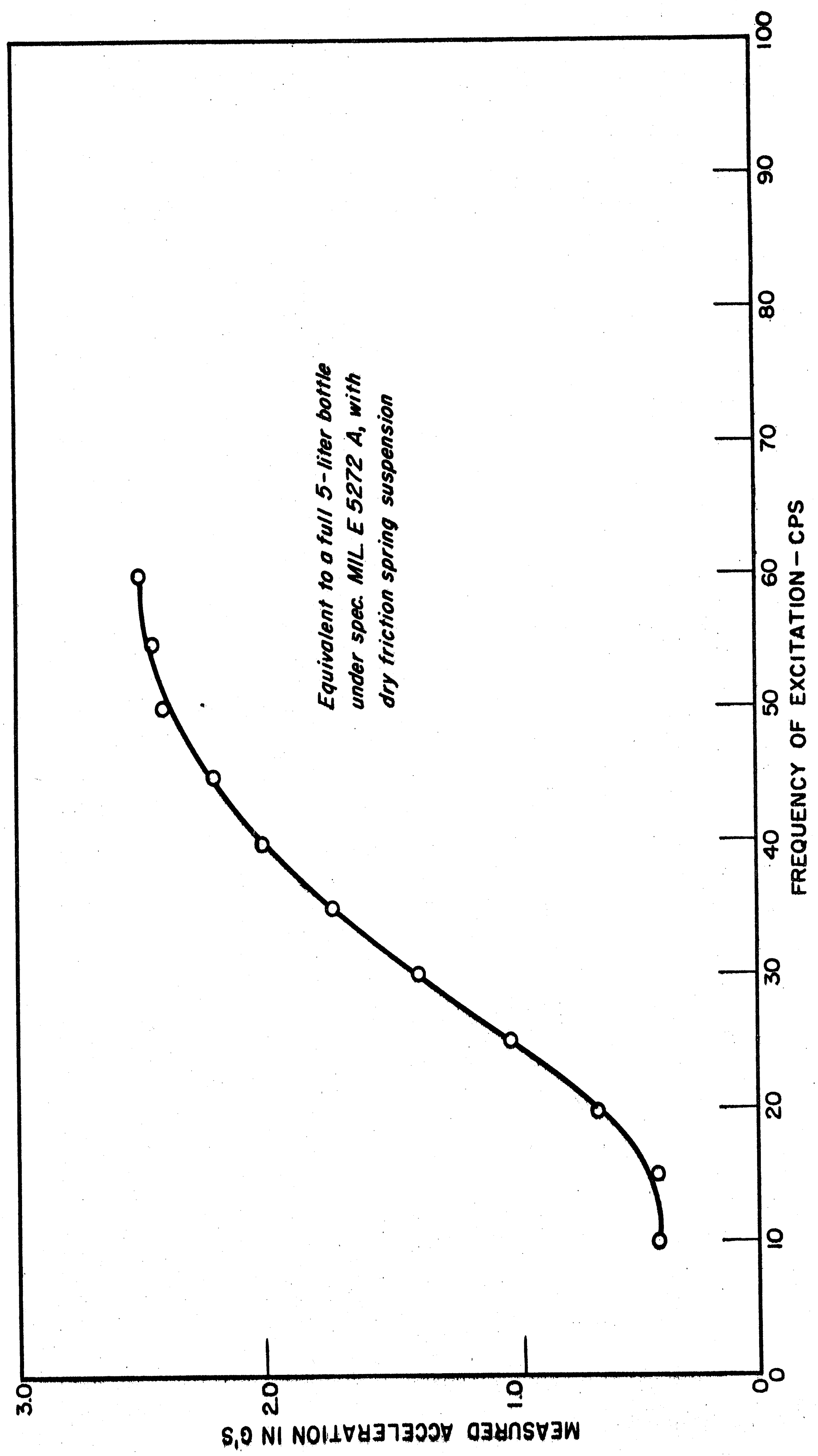

Fig. 10. Measured acceleration in G's vs. exciting frequency.

NUMERICAL EXAMPLE

For the 5-liter spherical bottle, weights were calculated as follows:

Liquid Oxygen	12.3 lb
Inner Bottle and Probe	4.5 lb
Total	16.8 lb

Keeping the natural frequency at 10 cps, the spring constant may be calculated as

$$K = 6.80\ W = 6.8 \times 16.8 = 114 \text{ lb/in.}$$

for each individual spring, assuming them to lie at an angle $\phi = 45°$. The preload must be set equal to about 17 lb, as previously discussed.

A large number of calculations have been run on various sizes of wave-washer springs for this type of service. The generalization which may be drawn from these calculations is that the three-wave wire washer is probably as good a geometry as it is possible to find. The Associated Spring Corp. Handbook[3] gives the following expressions for wave washers:

$$K = \frac{Ed^4N^4}{1.94\ D^3},$$

$$\sigma = \frac{3\pi\ PD}{4d^3N^2},$$

where

K = spring constant, lb/in.
E = modulus of elasticity, lb/sq in.
d = wire diam, in.
N = number of waves
D = mean diam of washer, in.
σ = stress, bending, psi
P = load, lb.

Using these, a spring is calculated having the following properties:

K = 114 lb/in.
D = 3 in.
d = .040 in.
E = 28×10^6 (stainless steel)
N = 3
σ = 104,000 psi, under a 33.6 lb load.

For the upper spring, a two-armed wire form is chosen as shown in Fig. 11.

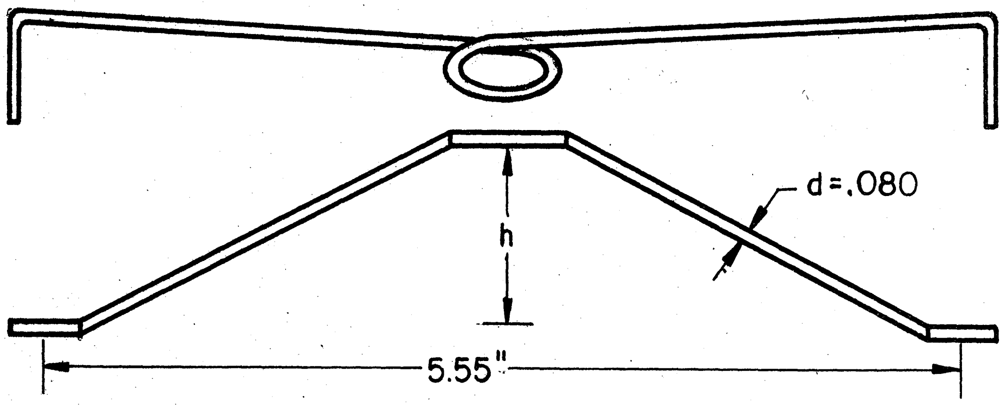

Fig. 11. Top-spring geometry.

Treating each half as a cantilever beam, one obtains the following design:

K = 50 lb/in.
d = .080
E = 28×10^6 (stainless steel)
σ = 100,000 psi for a load of 16-lb total
h - to suit the design.

The thermal resistance of this system may be approximated by computing the quantity

$$R = \frac{L}{A}$$

and summing for the entire system, giving a measure of the conductive thermal resistance. For this suspension

$$R \approx 71 \quad ,$$

while for the suspension it replaces,

$$R \approx 61 \quad ,$$

so that the loss of liquified gas by heat leakage should be better here than in the original suspension.

REFERENCES

1. Den Hartog, J. P., Mechanical Vibrations, 4th ed., McGraw-Hill Book Co., New York, 1956.

2. Mil. E5272A, Procedure 1, p. 10.

3. Handbook of Mechanical Spring Design, published and distributed by Associated Spring Corporation, Bristol, Connecticut.

APPENDIX

The coefficient of friction between stainless-steel wire and the copper-flashed inner surface of a liquid oxygen bottle was measured by rotating a weighted stainless-steel spring with respect to the copper-plated surface. The couple necessary to cause this rotation was measured. Six trials gave a mean value of coefficient of friction of 0.276, yielding a probable error of the mean of .005, and no readings were rejected by Chauvenet's criterion.