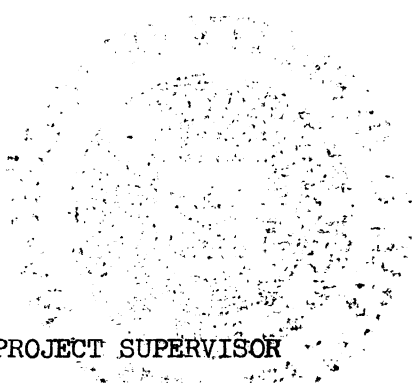


FINAL REPORT OF PROJECT M720-1
OCTOBER 15, 1946 THROUGH JUNE 30, 1954

BY

DR. A. BRUCE CLARKE



PROJECT SUPERVISOR

A. H. COPELAND, SR.

PROFESSOR OF MATHEMATICS

CONTRACT N6 ONR 232-1

PROJECT M720-1

JUNE, 1954

W.M.

UNR 1280

I. Purpose of Project

Operations research came into being as a full-fledged adjunct to the nation's armed forces under the pressures of World War II. Scientists of all kinds attempted to apply their specialized knowledge to problems of national defense. Among the many mathematicians struggled to put into mathematical form the extremely difficult problems of efficient utilization of forces and supplies in the field. Under the stress of the national emergency these mathematicians abandoned the time-honored isolation of their profession and concentrated on obtaining answers to practical problems. Where classical theories did not apply they proceeded by methods of approximation and improvisation. Of course, many mistakes were made, and progress at first was slow, as the workers became acquainted with the unfamiliar concepts and standards. However, by the war's end the operations research branches had proved their value many times and were being called upon to recommend decisions at the highest levels.

After the war, operations research departments were set up on a permanent basis by all service branches. Various problems were, however, recognized. First, it proved difficult to obtain competent personnel for these departments. The combination of adequate knowledge and research ability in mathematics, in particular, and willingness to concentrate full-time on the applied problems of operations research, was rare. Second, a large number of problems encountered in World War II which could only be solved approximately or empirically, indicated that the basic mathematical theory required for many applications was missing. For example, further basic researches in game theory and stochastic processes were required. Third, while it was realized that in the event of another national emergency scientists would gain turn their abilities to this field, it seemed de-

sirable to attempt to reduce the "breaking in" period of adjustment and study required.

In view of these problems, Project M720-1 was set up at the University of Michigan in the fall of 1946 by the Office of Naval Research. The result of this project was to establish an operations research group within the framework of the Department of Mathematics of the University. Membership was on a part-time basis by faculty and graduate students of the Department, and it was desired that the group should have as broad a base as possible. Although originally under the direct control of ONR, after one year's time overall supervision passed to the Operations Evaluation Group in Washington, although ONR continued its financial support.

The purpose of this group was three-fold. First, work was to be done on specific operations research problems initially proposed by ONR, and later on other problems recommended by OEG. Much of this work was to be carried out by graduate students under faculty supervision. Second, basic mathematical research in the fields of game theory, stochastic processes, and differential equations, was to be carried on both on an individual basis and by means of study groups. Third, it was desired to spread knowledge of the techniques of operations research and interest in the subject among the members of the department and graduate students to increase the supply of available personnel trained in the the field and thereby to strengthen the country.

Although originally scheduled for a twelve month period only, on an experimental basis, a series of extensions continued the project for over seven years through June, 1954. Although during this period various changes in emphasis and group composition took place, the three objectives listed above were never changed.

II. Contractual History

University of Michigan Engineering Research Institute Project M720-1 was formed under Contract N6 ONR-232, Task Order #1 issued by the Office of Naval Research, and dated October 15, 1946. This project was originally scheduled to run over the 12 month period from October 15, 1946 through October 14, 1947, with authorized expenditures of \$20,900.

By Amendment #2, dated May 20, 1947, the project was extended to September 30, 1948, and the authorized amount increased to \$49,620 by the addition of \$28,720 to the previous total.

Amendment #4, September 28, 1948, extended the project for another year through September 30, 1949, increasing the authorized amount by \$5,300 to a total of \$54,920.

Amendment #5, May 5, 1949, increased the authorized total by \$8,800 to \$63,720.

At the conclusion of this period, Amendment #6, dated September 6, 1949, extended the contract for a further year through September 30, 1950, by adding \$25,580 to the previously authorized amount bringing the total to \$89,300.

By Amendment #8, July 1, 1950, the contract was extended for another year through September 30, 1951, and the authorized amount was increased to \$119,300 by addition of \$30,000.

Amendment #9, April 6, 1951, extended the contract for two more years through September 30, 1953 and added \$60,000 to make the authorized total \$179,300.

During the summer and fall of 1952 a cut-back in the funds available for the Office of Naval Research necessitated the curtailment of the project. After some discussion it was decided that funds to the amount

of \$14,000 should be returned to ONR, the balance of \$165,300 being deemed sufficient to carry the project through its expiration date of September 30, 1953, [Amendment #11, January 11, 1953]. Various attempts were made by ONR to obtain funds to continue the project, however, the final decision was that it would terminate on the above date.

On September 11, 1953, an extension of this termination date to June 30, 1954, on a no-funds basis, was authorized to permit the writing of this report outlining the activities of the project during the seven year period from October 15, 1946 to September 30, 1953 [Amendment #13].

Amendments #3, 7, 10 and 12 refer to minor contractual adjustments.

III. Research Program of Project

The research done by Project M720-1 falls into several broad categories in most of which both basic theoretical work and specific applications were studied, together with a number of more-or-less isolated special problems.

On its formation in 1946, the project was initially assigned the following three problems:

(a) Non-linear Differential Equations: In particular, incompletely determined equations were studied, that is, equations having certain indeterminacy terms corresponding to the inaccuracies inherent in physical measurements. This problem was studied from the theoretical viewpoint of classifying permissible solutions functions, and also from the computational viewpoint. This work was done under the direction of Professor Wilfred Kaplan, assisted by Professors M. O. Reade, E. H. Rothe, and C. J. Coe. Most of the results in this direction are contained in project reports numbered R 5, R 26, and R 27, together with summaries in the various progress

reports, (See Appendix I).

(b) **Interception and Search Theory:** A number of classified problems in this field were studied under the direction of Professor A. H. Copeland, Sr. Howard Raiffa, and Professors C. C. Craig and R. M. Thrall. The results of this study are contained in reports R 6, R 13, R 14, R 15, R 16, R 17, R 22, R 24, memoranda M 2, M 6, M 7, M 8, M 11, M 12, M 13, M 14, M 16, M 17, M 18, and tables T 1, T 2, T 3 together with summaries in the various progress reports, (See Appendix I).

(c) **Prediction Theory and Continuous Stochastic Processes:** Considerable theoretical and computational research was done on the problem of predicting future position of aircraft by means of radar plots. In connection with this problem general investigations of continuous random processes were undertaken. This work was done under the direction of Max A. Woodbury and Professor A. H. Copeland, Sr., and was reported in reports R 14, R 18, R 19, R 20, and memoranda M 7, M 9, M 10, M 20, (See Appendix I).

While work on the above three problems proceeded, a number of further problems were posed by OEG. These fall mainly into two categories:

(d) **Game Theory:** A number of problems involved applications of game theory. Among these were questions on proper inventory supply aboard ships, aerial combat, and pursuit war games, optimum submergence tactics of submarines. Simultaneously a general theoretical study of mathematical game theory was undertaken. This general topic attracted the interest of many of the project members, and during the last three or four years of the project figured largest in the group's studies. This research was directed by Professors A. H. Copeland, Sr., R. M. Thrall, and Dr. W. Kincaid, and was reported in reports R 23, R 25, R 28, R 30, R 33, R 34, R 35, R 36, R 40, R 41, and memoranda IMR 23, (See Appendix I).

(e) Waiting Line Theory: Several problems given by OEG appeared to depend on knowledge of the probability distributions of various types of waiting lines. This problem was attacked theoretically and results tabulated. This research was under the direction of Dr. A. B. Clarke, and was published in reports R 32 and R 39, (See Appendix I).

A number of special problems which do not fall into the above categories were also studied. For instance, the conditions under which a submarine is physically capable of pursuing and attacking a given target were studied in R 37, and a special small sample probability problem was studied in R 38.

IV. Conclusion

In summing up the accomplishments of the project, it would appear natural to consider its success in the three objectives outlined in Section I: namely, study of specific operations research problems, basic pure mathematical research, and training of operations research personnel.

The best evidence that the group well fulfilled their obligations to the Navy in specific operations research problems is seen in the numerous time extensions and fund increases authorized by ONR. As was seen in Section II, the initial authorization for one year and \$20,900 was eventually extended to more than seven years and \$165,300. This would appear to be convincing evidence of the satisfaction of the contractor with the work done by the group.

As mentioned previously, the theoretical work of the project was directed mainly towards the study of non-linear differential equations, game theory, and stochastic processes. In this process at least four Ph. D. theses were written directly under the auspices of the project, while several

others grew naturally from the students' research on these problems. A number of papers were published in professional journals by members detailing some of their fundamental research. Several promising young mathematicians specializing in the above fields had their interest in their subject first aroused while they were members of this group.

Although no records were maintained on the performance of students after leaving the project, it is known that at least thirteen entered fields connected with operations research work. This would be a measure of the effectiveness of the group in carrying out the third of its general aims. There also resulted from these operations a much closer liaison between the theoretical mathematicians of the group and the armed forces, with a greater mutual understanding of the others' problems and desires.

Appendix I : Publications

REPORTS

- R 1 Bimonthly Progress Report on Contract No. N 6 onr 232-1: March 3, 1947
- R 2 Proposal: March 31, 1947
- R 3 Basic Mathematical Research on Differential Equations and Interception Problems for March-April, 1947: May 3, 1947
- R 4 Bimonthly Progress Report for May and June, 1947, Contract No. N6 onr 232-1 Basic Mathematical Research on Differential Equations and Interception Problems: July 3, 1947
- R 5 Report on the Analysis of Non-linear Differential Equations by Wilfred Kaplan: August, 1947
- R 6 On Interception Probabilities by Howard Raiffa: August, 1947
- R 7 Bimonthly Progress Report for July and August, 1947, Contract No. N6 onr 232-1 Basic Mathematical Research on Differential Equations and Interception Problems: September 3, 1947
- R 8 Bimonthly Progress Report for September and October, 1947, Contract No. N6 onr 232-1 Basic Mathematical Research on Differential Equations and Interception Problems: November 3, 1947
- R 9 Bimonthly Progress Report for November and December, 1947, Contract No. N6 onr 232-1 Basic Mathematical Research on Differential Equations and Interception Problems: January 3, 1948
- R 10 Bimonthly Progress Report for January and February, 1948, Contract No. N6 onr 232-1 Basic Mathematical Research on Differential Equations and Interception Problems: March 3, 1948
- R 11 Bimonthly Progress Report for March and April, 1948, Contract No. N6 onr 232-1 Basic Mathematical Research on Differential Equations and Interception Problems: May 3, 1948

- R 12 Bimonthly Progress Report for May and June, 1948, Contract No. N6 onr 232-1,
Basic Mathematical Research on Differential Equations and Interception
Problems: July 3, 1948
- R 13 Time Analysis of Collision Course by Howard Raiffa, assisted by John Hocking,
T. Slattery and Daniel Naymik: September, 1948 SECRET
- R 14 The Prediction Problem by A. H. Copeland, Sr. Assisted by A. H. Copeland, Jr.,
M. A. Woodbury, John Hocking: October 15, 1948 CONFIDENTIAL
- R 15 Penetration Analysis of Collision Courses by J. Hocking and T. Slattery.
Assisted by M. A. Woodbury, H. Raiffa, and D. Rippe: February, 1949 SECRET
- R 16 Collision Course Directed by Radar by C. L. Hammer and D. Naymik.
Assisted by M. Woodbury: February, 1949 CONFIDENTIAL
- R 17 Time Analysis of Collision Course II by T. Slattery: March, 1949 SECRET
- R 18 Optimum Methods of Linear Prediction by G. Feicht and M. Woodbury.
Assisted by C. Dolph: May, 1949
- R 19 Equivalent Formulation of the Optimum Method by A. H. Copeland, Jr.,
C. Hammer, and D. Rippe: May, 1949 CONFIDENTIAL
- R 20 Vectoring Errors by A. H. Copeland, Sr. Assisted by C. Hammer and
A. H. Copeland, Jr. : July, 1949
- R 21 Qualitative and Quantitative Analysis of the Solutions of Non-Linear
Differential Equations by W. Kaplan. Assisted by T. Slattery, June, 1949
- R 22 Approximate Formula for Variance of Miss by C. Hammer, D. Rippe and
A. H. Copeland, Jr.: July, 1949
- R 23 Analysis of a One-Person Game by W. M. Kincaid. Assisted by W. Scott,
J. Chover and A. H. Copeland: November, 1949
- R 24 Pursuit Courses Using Tangential Lead and Computer Leads by C. Hammer
and A. H. Copeland, Jr. : May, 1950 CONFIDENTIAL

- R 25 The Theory of Games Applied to the Attack Phase by Gerald Thompson:
January, 1950 RESTRICTED
- R 26 A Topological Study of the Level Curves of Harmonic Functions by
William Boothby: April 15, 1949
- R 27 A Set of Parabolic Regular Curve Families Filling the Plane and Certain
Related Riemann Surfaces by Helen Cullen: September 15, 1947
- R 28 An Algorithm for the Determination of All Solutions of a Two-Person
Zero Sum Game with a Finite Number of Strategies by H. Raiffa, G. Thompson,
and R. M. Thrall: September, 1950
- R 29 Progress Report: May, 1951 CONFIDENTIAL
- R 30 Arbitration Schemes for Generalized Two Person Games by H. Raiffa:
June, 1951
- R 31 Progress Report: December, 1951 CONFIDENTIAL
- R 32 On the Solution of the Telephone Problem by A. B. Clarke: March, 1952
- R 33 A Type of Inventory Problem by W. M. Kincaid. Assisted by D. Darling:
May, 1952
- R 34 On the Submergence Problem by B. Brainerd: July, 1952 RESTRICTED
- R 35 On Certain Game-Like Problems by J. Hocking and W. Feit: August, 1952
- R 36 Computational Technique for the Inventory Bomb Problem by K. Leisenring
and W. Brown: January, 1953
- R 37 The Submarine Approach Problem by E. Crisler and M. Stewart: December, 1952
RESTRICTED
- R 38 The Small Samples Problem by Betty Ullman: March, 1953
- R 39 The Time Dependent Waiting Line Problem by A. B. Clarke: March, 1953
- R 40 The Submarine Submergence Problem by D. Storvick and W. Feit: May, 1953
CONFIDENTIAL
- R 41 One Parameter Solution of a Game of Pursuit by J. Gil de Lamadrid:
August, 1953

MEMORANDA

- M 1 Harmonic Analysis and Linear Forecasting (Woodbury) in M720-1 R 7
- M 2 Past History Analysis with Respect to Vector Velocities that are not Constant (Raiffa) in M720-1 R 7
- M 3 Prediction in Stochastic Processes (Woodbury) in M720-1 R 8
- M 4 Minutes of Meeting, CNO-OEG, 3827 Navy Building, Washington, D. C.
December 16, 1947
- M 5 Minutes of Meeting, University of Michigan, Ann Arbor
January 27, 1948
- M 6 Phase II (Raiffa) SECRET
- M 7 A collision course directed by radar-- analytical solution (Copeland)
CONFIDENTIAL
- M 8 A collision course directed by radar--graphical solution (Copeland)
CONFIDENTIAL
- M 9 Three methods of choosing random numbers (Copeland) CONFIDENTIAL
- M 10 On the best linear prediction position for rectilinear motion CONFIDENTIAL
- M 11 Course along the major axis of an error ellipse (Copeland) CONFIDENTIAL
- M 12 Circular Pursuit Courses (Coe) CONFIDENTIAL
- M 13 Phase II--continuation of M 6 (Raiffa) SECRET
- M 14 Expected collision point as a function of radar range and "dead" time
(Raiffa and Hocking) SECRET
- M 16 Isochrones for circle plus collision course (Raiffa) SECRET
- M 17 Analytical Aids for graphical solution RESTRICTED
- M 18 Prediction Errors in the Error Ellipse Course CONFIDENTIAL
- IM 1 On Air Interception Conference; 6,7,8 April 1948, Room 3601, Navy Building,
Washington, D. C.

IM 2 Analysis of Collision Course (Raiffa)

IM 3 Commitments of project members (Woodbury)

IMR 20 Continuous Prediction-Unidimensional Course

IMR 23 Optimum Tactics Against Continuous Fire

TABLES

T 1 Analytical Aids for Graphical Solutions CONFIDENTIAL

T 2 Tables of Random Numbers and Equivalent Random Probits January 20, 1949

T 3 Variances Associated with Vectoring Errors August, 1949

Appendix II : Personnel

Project Supervisors

R. M. Thrall	September, 1946 through June, 1948
M. Woodbury	July, 1948 through June, 1949
A. H. Copeland, Sr.	July, 1949 through June, 1954
W. M. Kincaid (Assoc. Director)	July, 1949 through June, 1954

Faculty Members

A. B. Clarke	February, 1952 through June, 1954
C. J. Coe	October, 1947 through November, 1949
A. H. Copeland, Sr.	November, 1946 through June, 1954
C. C. Craig	November, 1946 through October, 1949
D. A. Darling	December, 1950 through September, 1953
C. L. Dolph	May 1949 through September 1949
F. Harary	June, 1949 through September 1950
W. Kaplan	November 1946 through September 1948
W. Kincaid	December, 1948 through June, 1954
K. Leisenring	March, 1952 through May, 1953
G. R. Livesay	July 1951 through July, 1952
M. O. Reade	June, 1947 through September, 1948
E. A. Rothe	February, 1948 through September, 1948
W. Scott	April, 1949 through August, 1949
R. M. Thrall	November, 1946 through June, 1954
C. Titus	August, 1953 through September, 1953
M. Woodbury	January, 1948 through June, 1949 (formerly Research Assistant)

Assistants in Research

Lynn Albers	January 1948 through December, 1948
W. F. Bauer	November, 1946 through September, 1949
W. S. Bicknell	June, 1947 through January, 1948
W. Boothby	October, 1947 through December, 1948
B. Brainerd	May, 1951 through May, 1953
Ilse Brauer	April, 1950 through June, 1952
Jagna Braunthal	April, 1949 through June, 1949
C. F. Briggs	June, 1949 through September, 1949
W. Brown	April, 1952 through October, 1952
J. Chover	February 1949 through February 1950
C. Clark	April 1949 through June 1949
A. Containo M	May 1949 through June 1949
S. D. Conte	July 1949 through September 1949
A. H. Copeland, Jr.	March 1948 through June, 1950
Caroline Crippen	June, 1951 through August 1951
Earl Crisler	May 1950 through September 1952
Marilyn Cross	October 1951 only
Myrle Cross	October 1951 only
Helen Cullen	October 1947 through December 1948
D. Dillon	April 1949 through June, 1949
P. Doyle	August, 1951 through February 1952
D. Dubois	June, 1948 through December, 1948
R. Else	June, 1948 through December, 1948
G. T. Feicht	September, 1948 through December, 1949
W. Feit	July, 1952 through June, 1953
W. Fox	March, 1951 through January 1952
E. Fritz	May, 1949 through June, 1949

Assistants in Research, continued

R. Getoor	March, 1952 through September, 1953
J. Gil de Lamadrid	September, 1952 through September, 1953
E. Gordon	May, 1949 through June, 1949
C. Hammer	June, 1948 through September, 1950
M. Hildebrandt	June, 1948 through January, 1950
J. Hocking	February, 1948 through June, 1951
W. Hoffman	July, 1952 through September, 1953
J. Jewett	March, 1951 through January, 1952
C. Kilby	October, 1951 through September, 1952
B. Lapidus	October, 1947 through December, 1947
J. Lawrence	June, 1947 only
J. P. Line	February 1951 through September, 1951
R. MacDowell	September, 1950 through December, 1950
D. F. Mela	May, 1947 through December, 1949
R. A. Meridith	September, 1948 through July, 1949
M. Naghdi	February 1948 through December, 1948
D. Naymik	October, 1947 through December, 1948
P. Overberg	June, 1949 through September, 1949
C. L. Perry, Jr.	June, 1947 through June, 1949
C. Quillen	February, 1949 through July, 1949
H. Raiffa	February, 1947 through June, 1951
D. Rippe	May, 1947 through January, 1951
Jane Rothe	September, 1949 through September, 1950
C. Rutherford	April, 1949 through June, 1949
E. St. Clair	October, 1947 through June, 1948
C. C. Sams	February, 1947 through June, 1947
T. G. Slattery	June, 1947 through June, 1949

Assistants in Research, continued

B. Soffe, Jr.	May, 1948 through June, 1948
M. Stewart	April, 1948 through December, 1952
J. Stoddard	July, 1952 only
D. Storvick	October, 1951 through September, 1953
R. Stratton	April, 1949 through June, 1949
G. Thompson	June, 1949 through June, 1951
R. Tofte	April, 1949 through June, 1949
Betty Ullman	September, 1950 through September, 1953
D. Wall	April, 1952 through September, 1952
J. Watson	May, 1949 through June, 1949
M. Woodbury	June, 1947 through December, 1947 (later, faculty)

Computers

G. Cargo	November, 1952 through June, 1953
G. F. Lunger	August, 1948 through June, 1951
J. Osborn	September, 1950 through June, 1951

Secretaries

Blanche Arner	May, 1950 through January, 1952
Lynda Copeland (Secretary-Computer)	September, 1948 through June, 1950
Marilyn Cross	July, 1953 through September, 1953
Kathryn Hanchon	September, 1952 through June, 1954
Mona Rosenthal	October, 1952 through June, 1953
Marion Vogelsong (Secretary-Computer)	November, 1946 through June, 1950
Florence Clarke (Secretary-Research Ass't)	February, 1952 through August, 1952

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