# ENGINEERING RESEARCH INSTITUTE UNIVERSITY OF MICHIGAN ANN ARBOR

#### Final Report

#### MATHEMATICAL RESEARCH

October 15, 1946, through June 30, 1954

A. BRUCE CLARKE

A. H. COPELAND, SR. Professor of Mathematics

Project M720-1

U. S. NAVY, OFFICE OF NAVAL RESEARCH CONTRACT N6-onr-232, TASK ORDER NO. 1

December, 1954

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# FINAL REPORT MATHEMATICAL RESEARCH

October 15, 1946, through June 30, 1954

#### I. PURPOSE OF THE PROJECT

Operations research came into being as a full-fledged adjunct to the nation's armed forces under the pressures of World War II. Scientists from many fields attempted to apply their specialized knowledge to problems of national defense. Among these was a number of mathematicians who struggled to place the extremely difficult problems of most efficient utilization of men and supplies in the field into mathematical form. Under the stress of the national emergency these mathematicians abandoned the time-honored isolation of their profession and concentrated on obtaining numerical answers to practical problems. Where classical theories did not apply they proceeded by methods of approximation and improvisation. Of course many mistakes were made, and progress at first was slow since the workers had to become acquainted with the unfamiliar concepts and standards. By the war's end, however, the operations research branches had proved their value and were being called on to recommend decisions at the highest levels.

After the war, operations research departments were established on a permanent basis by all service branches. However, various problems were recognized, among them the following: First, it proved difficult to obtain competent personnel for these departments. The combination of adequate knowledge and research ability in mathematics, in particular, and willingness to concentrate full time on the applied problems of operations research (without the stimulus of war) was rare. Second, the large number of problems encountered in World War II, which could only be solved approximately or empirically, indicated that the basic mathematical theory necessary for many applications was missing. For example, further basic research in game theory and stochastic processes was required. Third, while it was realized that in the event of another national emergency scientists would again turn their abilities to this field, it seemed desirable to attempt to reduce the "breaking-in" period of adjustment and study required.

In view of these problems, Project M720-1 was established at the University of Michigan in the fall of 1946 by the Office of Naval Research (ONR). The purpose of this project was to establish an operations research group within the framework of the Department of Mathematics of the University. Membership was to be on a part-time basis by faculty and graduate students of the department and it was desired that the group should have as varied a representation as possible. Although originally under the direct control of ONR, after one year's time overall supervision passed to the Operations Evaluation Group (OEG) in Washington, although ONR continued its financial support.

The program of this group was threefold. First, work was to be done on specific operations research problems proposed initially by ONR; later other problems were proposed by OEG. Much of this work was to be carried out by graduate students under faculty supervision. Second, basic mathematical research in the fields of game theory, stochastic processes, and differential equations was to be carried on, both on an individual basis and by means of study groups. Third, it was desired to spread both knowledge of the techniques of operations research and interest in the subject among the members of the department and graduate students in order to increase the supply of available personnel trained in the field, thereby strengthening the country.

Although originally scheduled for a twelve-month period only, on an experimental basis, a series of extensions made possible the continuation of the project for over seven years, through June, 1954. Although during this period various changes in emphasis and group composition took place, the three objectives listed above were never changed.

#### II. CONTRACTUAL HISTORY

University of Michigan, Engineering Research Institute Project M720-1 was formed under Contract N6 onr 232, Task Order No. 1, issued by the Office of Naval Research and dated October 15, 1946. This project was originally scheduled to run over the twelve-month period from October 15, 1946, through October 14, 1947, with authorized expenditures of \$20,900.

By Amendment No. 2, dated May 20, 1947, the project was extended to September 30, 1948, and the authorized amount increased to \$49,620 by the addition of \$28,720 to the previous total.

Amendment No. 4, dated September 28, 1948, extended the project for another year through September 30, 1949, increasing the authorized amount by \$5,300 to a total of \$54,920.

Amendment No. 5, dated May 5, 1949, increased the authorized total by \$8,800 to \$63,720.

At the conclusion of this period, Amendment No. 6, dated September 6, 1949, extended the contract for a further year through September 30, 1950, and added \$25,580 to the previously authorized amount, bringing the total to \$89,300.

By Amendment No. 8, dated July 1, 1950, the contract was extended for another year through September 30, 1951, and the authorized amount was increased to \$119,300 by the addition of \$30,000.

Amendment No. 9, dated April 6, 1951, extended the contract for two more years, through September 30, 1953, and added \$60,000 to make the authorized total \$179,300.

During the summer and fall of 1952, a reduction in the funds available for ONR necessitated the curtailment of the project. After some discussion it was decided that funds to the amount of \$14,000 should be returned to ONR, the balance of \$165,300 being deemed sufficient to carry the project through its expiration date of September 30, 1953 (Amendment No. 11, dated January 11, 1953). Various attempts were made by ONR to obtain funds to continue the project; however, the final decision was that it would terminate on the above date.

On September 11, 1953, an extension of this termination date to June 30, 1954, on a no-funds basis, was authorized to permit the writing of this report outlining the activities of the project during the seven-year period from October 15, 1946, to September 30, 1953 (Amendment No. 13). Amendment Nos. 3, 7, 10, and 12 refer to minor contractual adjustments.

#### III. RESEARCH PROGRAM OF PROJECT

The research done by Project M720-1 falls into several broad categories in which both basic theoretical work and specific applications were studied, together with a number of more-or-less isolated, special problems.

On its initiation in 1946, the project was assigned the following three problems:

(1) Nonlinear Differential Equations. In particular, incompletely determined equations were studied, that is, equations having certain indeterminacy terms corresponding to the inaccuracies inherent in physical measurements. This problem was studied from the theoretical viewpoint of classifying

permissible solution functions and also from the computational viewpoint. This work was done under the direction of Professor W. Kaplan, assisted by Professors M. O. Reade, E. H. Rothe, and C. J. Coe. Most of the results in this direction are contained in project reports R5, R26, and R27, together with summaries in various other progress reports (see Appendix I).

- (2) Interception and Search Theory. A number of classified problems in this field was studied under the direction of Professors A. H. Copeland, Sr., C. C. Craig, and R. M. Thrall, and Howard Raiffa. The results of this study are described in reports R6, R13, R14, R15, R16, R17, R22, and R24; memoranda M2, M6, M7, M8, M11, M12, M13, M14, M16, M17, and M18; and tables T1, T2, and T3, together with summaries in the various progress reports (see Appendix I).
- (3) Prediction Theory and Continuous Stochastic Processes. Considerable theoretical and computational research was done on the problem of predicting future position of aircraft by means of radar plots. In connection with this problem, general investigations of continuous random processes were undertaken. This work was done under the direction of M. A. Woodbury and Professor A. H. Copeland, Sr., and was described in reports R14, R18, R19, and R20; and memoranda M7, M9, M10, and M20 (see Appendix I).

While work on the above three problems proceeded, a number of further problems was posed by OEG. These problems fall mainly into two categories, as follows:

- (1) Game Theory. A number of problems involved applications of game theory. Among these were questions on proper inventory supply aboard ships, in aerial combat and pursuit, war games, and optimum submergence tactics of submarines. Simultaneously, a general theoretical study of mathematical game theory was undertaken. This general topic attracted the interest of many of the project members, and during the last three or four years of the project figured largest in the group's studies. This research was directed by Professors A. H. Copeland, Sr. and R. M. Thrall, and Dr. W. Kincaid, a summary of the results being included in reports R23, R25, R28, R30, R33, R34, R35, R36, R40, and R41; and memorandum IMR23 (see Appendix I).
- (2) Waiting-Line Theory. Several problems given by OEG appeared to depend on knowledge of the probability distributions of various types of waiting lines. This problem was attacked theoretically and results were tabulated. This research was under the direction of Dr. A. B. Clarke and described in reports R32 and R39 (see Appendix I).

A number of special problems which do not fall into the above categories was also studied. For instance, the conditions under which a submarine is physically capable of pursuing and attacking a given target were

studied in R37, and a special, small-sample probability problem was studied in R38.

#### IV. CONCLUSION

It would appear natural to consider among the accomplishments of the project its success in the three objectives outlined in Section I, namely, the study of specific operations research problems, basically pure mathematical research, and training of operations research personnel.

The best evidence that the group well fulfilled its obligations to the Navy in specific operations research problems is seen in the numerous time extensions and fund increases authorized by ONR. As was seen in Section II, the initial authorization for one year and \$20,900 was eventually extended to more than seven years and \$165,300. This would appear to be convincing evidence of the satisfaction of the contractor with the work done by the group.

As mentioned previously, the theoretical work of the project was directed mainly toward the study of nonlinear differential equations, game theory, and stochastic processes. In this process, four Ph.D. theses were written directly under the auspices of the project, while several others grew naturally from the students' research on these problems. A number of papers was published in professional journals by members detailing some of the fundamental research carried on by the project (see Appendix II). If this list were to include all papers based in part on such research or motivated by problems discussed in project-study groups, it would necessarily be much longer. The classified nature of a great deal of the group's work was a major factor in limiting the amount of outside publication.

Although no records were maintained on the performance of students after leaving the project, it is known that at least thirteen entered fields connected with operations research work. This would be a measure of the effectiveness of the group in carrying out the third of its general aims, i.e., to spread knowledge of the techniques of operations research and to increase the supply of available trained personnel. There also resulted from these operations a much closer liaison between the theoretical mathematicians of the group and the armed forces, with a greater mutual understanding of the other's problems and desires.

#### APPENDIX I

#### REPORTS, MEMORANDA, AND TABLES

#### A. Reports

- Rl Bimonthly Progress Report on Contract No. N6 onr 232, March 3, 1947.
- R2 Proposal, March 31, 1947.
- R3 Basic Mathematical Research on Differential Equations and Interception Problems for March-April, 1947, May 3, 1947.
- R4 Bimonthly Progress Report for May and June, 1947, Contract No. N6 onr 232, Basic Mathematical Research on Differential Equations and Interception Problems, July 3, 1947.
- R5 Report on the Analysis of Nonlinear Differential Equations, W. Kaplan, August, 1947.
- R6 On Interception Probabilities, H. Raiffa, August, 1947.
- R7 Bimonthly Progress Report for July and August, 1947, Contract No. Nó onr 232, Basic Mathematical Research on Differential Equations and Interception Problems, September 3, 1947.
- R8 Bimonthly Progress Report for September and October, 1947, Contract No. N6 onr 232, Basic Mathematical Research on Differential Equations and Interception Problems, November 3, 1947.
- R9 Bimonthly Progress Report for November and December, 1947, Contract No. N6 onr 232, Basic Mathematical Research on Differential Equations and Interception Problems, January 3, 1948.
- R10 Bimonthly Progress Report for January and February, 1948, Contract No. No onr 232, Basic Mathematical Research on Differential Equations and Interception Problems, March 3, 1948.
- Rll Bimonthly Progress Report for March and April, 1948, Contract No. N6 onr 232, Basic Mathematical Research on Differential Equations and Interception Problems, May 3, 1948.
- R12 Bimonthly Progress Report for May and June, 1948, Contract No. N6 onr 232, Basic Mathematical Research on Differential Equations and Interception Problems, July 3, 1948.

- R13 Time Analysis of Collision Course, H. Raiffa, assisted by J. Hocking, T. Slattery, and D. Naymik, September, 1948. SECRET
- R14 The Prediction Problem, A. H. Copeland, Sr., assisted by A. H. Copeland, Jr., M. A. Woodbury, and J. Hocking, October 15, 1948. CONFIDENTIAL
- R15 Penetration Analysis of Collision Courses, J. Hocking and T. Slattery, assisted by M. A. Woodbury, H. Raiffa, and D. Rippe, February, 1949. SECRET
- R16 Collision Course Directed by Radar, C. L. Hammer and D. Naymik, assisted by M. A. Woodbury, February, 1949. CONFIDENTIAL
- R17 Time Analysis of Collision Course II, T. Slattery, March, 1949. SECRET
- R18 Optimum Methods of Linear Prediction, G. Feicht and M. A. Woodbury, assisted by C. Dolph, May, 1949.
- R19 Equivilent Formulation of the Optimum Method, A. H. Copeland, Jr., C. Hammer, and D. Rippe, May, 1949. CONFIDENTIAL
- R20 Vectoring Errors, A. H. Copeland, Sr., assisted by C. Hammer and A. H. Copeland, Jr., July, 1949.
- R21 Qualitative and Quantitative Analysis of the Solutions of Nonlinear Differential Equations, W. Kaplan, assisted by T. Slattery, June, 1949.
- R22 Approximate Formula for Variance of Miss, C. Hammer, D. Rîppe, and A. H. Copeland, Jr., July, 1949.
- R23 Analysis of a One-Person Game, W. Kincaid, assisted by W. Scott, J. Chover, and A. H. Copeland, Sr., November, 1949.
- Pursuit Courses Using Tangential Lead and Computer Leads, C. Hammer and A. H. Copeland, Jr., May, 1950. CONFIDENTIAL
- R25 The Theory of Games Applied to the Attack Phase, G. Thompson, January, 1950. RESTRICTED
- R26 A Topological Study of the Level Curves of Harmonic Functions, W. Boothby, April 15, 1949.
- R27 A Set of Parabolic Regular Curve Families Filling the Plane and Certain Related Riemann Surfaces, H. Cullen, September 15, 1947.

- R28 An Algorithm for the Determination of All Solutions of a Two-Person Zero Sum Game with a Finite Number of Strategies, H. Raiffa, G. Thompson, and R. M. Thrall, September, 1950.
- R29 Progress Report, May, 1951. CONFIDENTIAL
- R30 Arbitration Schemes for Generalized Two-Person Games, H. Raiffa, June, 1951.
- R31 Progress Report, December, 1951. CONFIDENTIAL
- R32 On the Solution of the Telephone Problem, A. B. Clarke, March, 1952.
- R33 A Type of Inventory Problem, W. Kincaid, assisted by D. Darling, May, 1952.
- R34 On the Submergence Problem, B. Brainerd, July, 1952. RESTRICTED
- R35 On Certain Game-Like Problems, J. Hocking and W. Feit, August, 1952.
- R36 Computational Technique for the Inventory Bomb Problem, K. Leisenring and W. Brown, January, 1953.
- The Submarine Approach Problem, E. Crisler and M. Stewart, December, 1952. RESTRICTED
- R38 The Small Samples Problem, B. Ullman, March, 1953.
- R39 The Time-Dependent Waiting-Line Problem, A. B. Clarke, March, 1953.
- R40 The Submarine Submergence Problem, D. Storvick and W. Feit, May, 1953. CONFIDENTIAL
- R41 One Parameter Solution of a Game of Pursuit, J. Gil de Lamadrid, August, 1953.

#### B. Memoranda

- MI Harmonic Analysis and Linear Forecasting, M. A. Woodbury, M720-1 R7.
- M2 Past History Analysis with Respect to Vector Velocities That Are Not Constant, H. Raiffa, M720-1 R7.
- M3 Prediction in Stochastic Processes, M. A. Woodbury, M720-1 R8.

- M4 Minutes of Meeting, CNO-OEG, 3827 Navy Building, Washington, D. C., December 16, 1947.
- Minutes of Meeting, University of Michigan, Ann Arbor, January 27, 1948.
- M6 Phase II, H. Raiffa. SECRET

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- M7 A Collision Course Directed by Radar--Analytical Solution, A. H. Copeland, Sr. CONFIDENTIAL
- M8 A Collision Course Directed by Radar--Graphical Solution, A. H. Copeland. Sr. CONFIDENTIAL
- M9 Three Methods of Choosing Random Numbers, A. H. Copeland, Sr. CONFIDENTIAL
- M10 On the Best Linear Prediction Position for Rectilinear Motion. CONFIDENTIAL
- Mll Course along the Major Axis of an Error Ellipse, A. H. Copeland, Sr. CONFIDENTIAL
- M12 Circular Pursuit Courses, C. J. Coe. CONFIDENTIAL
- M13 Phase II--Continuation of M6, H. Raiffa. SECRET
- M14 Expected Collision Point As a Function of Radar Range and "Dead" Time, H. Raiffa and J. Hocking. SECRET
- M16 Isochrones for Circle Plus Collision Course, H. Raiffa. SECRET
- M17 Analytical Aids for Graphical Solution. RESTRICTED
- M18 Prediction Errors in the Error Ellipse Course. CONFIDENTIAL
- IML On Air Interception Conference; 6, 7, 8, April, 1948, Room 3601, Navy Building, Washington, D. C.
- IM2 Analysis of Collision Course, H. Raiffa.
- IM3 Committments of Project Members, A. H. Woodbury.
- IMR20 Continuous Prediction-Unidimensional Course.
- IMR23 Optimum Tactics against Continuous Fire.

#### C. TABLES

- Tl Analytical Aids for Graphical Solutions. CONFIDENTIAL
- T2 Tables of Random Numbers and Equivilent Random Probits, January 20, 1949.
- T3 Variances Associated with Vectoring Errors, August, 1949.

#### APPENDIX II

#### **PUBLICATIONS**

This list of outside publications is of necessity incomplete. This is due to the difficulty of deciding to what degree the work of various authors was affected by time spent in project-study groups.

- 1. Boothby, William M., "The Topology of Regular Curve Families with Multiple Saddle Points", Amer J Math, 73, 405-438 (1951).
- 2. Boothby, William M., "The Topology of the Level Curves of Harmonic Functions with Critical Points", Amer J Math, 73, 512-538 (1951).
- 3. Clarke, A. Bruce, "On Time-Dependent Waiting-Line Processes (Abstract)", Ann Math Statistics, 24, 491-492 (1953).
- 4. Clarke, A. Bruce, "The Theory of Delay Times and Waiting Lines", Conference on Modern Statistical Methods for Business and Industry, Proceedings, Carnegie Institute of Technology, Pittsburgh, Pennsylvania, April 30-May 1, 1953.
- 5. Darling, D. A. and Kincaid, W. M., "An Inventory Problem (Abstract)", J Operations Research Soc Amer, 1, 80 (1953).
- 6. Dolph, C. L. and Woodbury, M. A., "Representation Theory and Prediction of Stochastic Processes", <u>EMF</u>, <u>11</u>, Willow Run Research Center, University of Michigan, Ann Arbor, Michigan (March, 1950).
- 7. Dolph, C. L. and Woodbury, M. A., "On the Relation between Green's Functions and Covariances of Certain Stochastic Processes, and Its Application to Unbiased Linear Prediction", <u>Trans Amer Math Soc</u>, 72, 519-550 (1952).
- 8. Kaplan, Wilfred, "Numerical Methods in the Solution of Problems of Nonlinear Elasticity", <u>Proc Symposia Appl Math</u>, <u>1</u>, American Mathematical Society, New York, New York, 194-196 (1949).
- 9. Kaplan, Wilfred, "Dynamical Systems with Indeterminacy", Amer J Math, 72, 573-594 (1950).

- 10. Kaplan, Wilfred, "Some Methods for Analysis of the Flow in Phase Space",

  Proc of the Symposium on Nonlinear Circuit Analysis, Polytechnic Institute of Brooklyn, Brooklyn, New York, 99-106 (1953).
- 11. Motzkin, T. S., Raiffa, H., Thompson, G. L., and Thrall, R. M., "The Double Description Method", Annals of Math Studies, 2, No. 28, 51-73, Princeton University Press, Princeton, New Jersey (1953).
- 12. Raiffa, Howard, "Arbitration Schemes for Generalized Two-Person Games", Annals of Math Studies, 2, No. 28, Princeton University Press, Princeton, New Jersey, (1953).

#### APPENDIX III

#### PERSONNEL

#### A. Project Supervisors

R. M. Thrall

September, 1946, through June, 1948

M. A. Woodbury

July, 1948, through June, 1949

A. H. Copeland, Sr.

July, 1949, through June, 1954

W. Kincaid (Associate Director) July, 1949, through June, 1954

#### B. Faculty Members

A. B. Clarke

C. J. Coe

A. H. Copeland, Sr.

C. C. Craig

D. A. Darling

C. L. Dolph

F. Harary

r a marary

W. Kaplan

W. Kincaid

K. Leisenring

G. R. Livesay

M. O. Reade E. H. Rothe

W. Scott

R. M. Thrall

C. Titus

M. A. Woodbury

February, 1952, through June, 1954 October, 1947, through November, 1949 November, 1946, through June, 1954 November, 1946, through October, 1949 December, 1950 through September, 1953 May, 1949, through September, 1949 June, 1949, through September, 1950 November, 1946, through September, 1948 December, 1948, through June, 1954 March, 1952, through May, 1953 July, 1951, through July, 1952 June, 1947, through September, 1948 February, 1948, through September, 1948 April, 1949, through August, 1949 November, 1946, through June, 1954 August, 1953, through September, 1953

#### C. Assistants in Research

L. Albers

W. F. Bauer

W. S. Bicknell

W. Boothby

B. Brainerd

January, 1948, through December, 1948 November, 1946, through September, 1949 June, 1947, through January, 1948 October, 1947, through December, 1948 May, 1951, through May, 1953

January, 1948, through June, 1949

(formerly Research Assistant)

#### C. Assistants in Research (continued)

- I. Brauer
- J. Braunthal
- C. F. Briggs
- W. Brown
- J. Chover
- C. Clark
- F. Clarke
- A. Containo
- S. D. Conte
- A. H. Copeland, Jr.
- C. Crippen
- E. Crisler
- Marilyn Cross
- Myrle Cross
- H. Cullen
- D. Dillon
- P. Doyle
- D. Dubois
- R. Else
- G. T. Feicht
- W. Feit
- W. Fox
- E. Fritz
- R. Getoor
- J. Gil de Lamadrid
- E. Gordon
- C. Hammer
- J. Hocking
- W. Hoffman
- J. Jewett
- C. Kilby
- B. Lapidus
- J. Lawrence
- J. P. Line
- R. MacDowell
- D. F. Mela
- R. A. Meridith
- M. Naghdi
- D. Naymik
- P. Overberg
- C. L. Perry, Jr.
- C. Quillen
- H. Raiffa
- D. Rippe

- April, 1950, through June, 1952
- April, 1949, through June, 1949
- June, 1949, through September, 1949
- April, 1952, through October, 1952
- February, 1949, through February, 1950
- April, 1949, through June, 1949
- February, 1952, through August, 1952
- May, 1949, through June, 1949
- July, 1949, through September, 1949
- March, 1948, through June, 1950
- June, 1951, through August, 1951
- May, 1950, through September, 1952
- October, 1951, only
- October, 1951, only
- October, 1947, through December, 1948
- April, 1949, through June, 1949
- August, 1951, through February, 1952
- June, 1948, through December, 1948
- June, 1948, through December, 1948
- September, 1948, through December, 1949
- July, 1952, through June, 1953 March, 1951, through January, 1952
- May, 1949, through June, 1949
- March, 1952, through September, 1953
- September, 1952, through September, 1953
- May, 1949, through June, 1949
- June, 1948, through September, 1950
- February, 1948, through June, 1951
- July, 1952, through September, 1953
- March, 1951, through January, 1952
- October, 1951, through September, 1952
- October, 1947, through December, 1947
- June, 1947, only
- February, 1951, through September, 1951
- September, 1950, through December, 1950
- May, 1947, through December, 1949
- September, 1948, through July, 1949
- February, 1948, through December, 1948
- October, 1947, through December, 1948
- June, 1949, through September, 1949
- June, 1947, through June, 1949
- February, 1949, through July, 1949
- February, 1947, through June, 1951
- May, 1947, through January, 1951

#### C. Assistants in Research (continued)

J. Rothe

C. Rutherford

E. St. Clair

C. C. Sams

T. Slattery

B. Soffe, Jr.

M. Stewart

J. Stoddard

D. Storvick

R. Stratton

G. Thompson

R. Tofte

B. Ullman

D. Wall

J. Watson

M. A. Woodbury

September, 1949, through September, 1950

April, 1949, through June, 1949

October, 1947, through June, 1948

February, 1947, through June, 1947

June, 1947, through June, 1949

May, 1948, through June, 1948

April, 1948, through December, 1952

July, 1952, only

October, 1951, through September, 1953

April, 1949, through June, 1949

June, 1949, through June, 1951

April, 1949, through June, 1949

September, 1950, through September, 1953

April, 1952, through September, 1952

May, 1949, through June, 1949

June, 1947, through December, 1947

(later Faculty Member)

#### D. Computers

G. Cargo

L. Copeland

G. F. Lunger

J. Osborn

M. Vogelsong

November, 1952, through June, 1953

September, 1948, through June, 1950

August, 1948, through June, 1951 September, 1950, through June, 1951

November, 1946, through June, 1950

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Professor G. P. Wadsworth, Room 2-285 Massachusetts Institute of Technology	
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