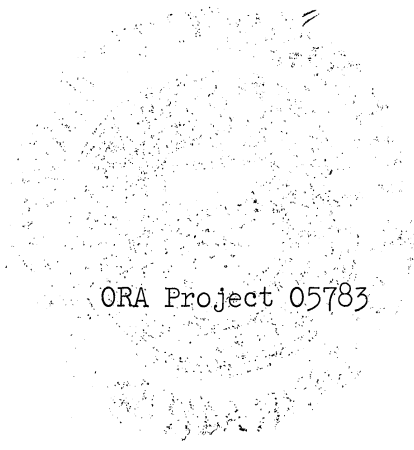# THE UNIVERSITY OF MICHIGAN
COLLEGE OF ENGINEERING
Department of Nuclear Engineering

Technical Report

THE EFFECT OF THE PORT VOID IN THE PREDICTION

OF THERMAL NEUTRON BEAM PORT CURRENT

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ACKNOWLEDGMENTS

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NOMENCLATURE

c	Number of neutrons produced per collision
d	Linear extrapolation distance
h	Duct length
j	Neutron current
k	Multiplication constant
ℓ	Neutron mean free path
r	Radial position
$\bar{r}$	Position vector
s	Neutron source
t	Time
v	Neutron velocity
z	Axial position
z_0	Extrapolated endpoint
B^2	Buckling
D	Neutron diffusion coefficient
E	Neutron energy
L	Neutron diffusion length
R	Duct radius
S	Surface area
T	Neutron temperature
$\mathcal{J}$	Neutron streaming flux
α	Largest eigenvalue of diffusion equation

κ Inverse of diffusion length (1/L)

λ Radioactive decay constant

μ Cosine of angle between neutron direction and z axis

ν Average number of neutrons born per fission

ξ Average logarithmic energy decrement

σ Microscopic neutron cross section

τ Fermi age

ϕ Neutron flux

χ^i Fraction of neutrons born in energy group i

Σ Macroscopic neutron cross section

$\bar{\Omega}$ Unit vector denoting direction of travel

Subscripts

     a   Absorption

     act   Activation

     as   Asymptotic

     eff   Effective

     epi   Epithermal

     f   Fission

     g   Geometric

     m   Material

     o   Scalar

     s   Scattering

     t   Total

NOMENCLATURE (Concluded)

th   Thermal

tr   Transport

R   Removal

T   Transverse

TC   Thermal cutoff

Superscripts

f   Fast

q   Source

th   Thermal

M   Milne

N   Normalized

ABSTRACT

A method is presented for the prediction of the exit thermal neutron current from a reactor beam port. The results of duct experiments at the Ford Nuclear Reactor afford confidence in the validity of the analytical technique. The motivation for the study is provided by a prevalent desire to enhance thermal neutron beam intensity for spectrometer investigations.

The analysis employs elementary P-1 theory, which is inspired by an examination of the infinite half-space limit. A return current boundary condition at the source plane of the port is developed by summing current contributions from the lateral surfaces. An idealization of the scalar flux distribution along the lateral surfaces of the port permits a diffusion theory solution for the source plane angular flux as a function of the port radius and the unperturbed flux distribution in the medium.

Activation measurements of the scalar flux distributions at the source planes of 1-1/2-, 3-, and 5-in. diameter aluminum ducts were performed in light water and graphite. Measurements of the exit current from the ducts provided with an internal collimator to eliminate wall leakage contributions were carried out in light water. The largest duct perturbation was observed for the 5-in. duct in water. In this case the source plane scalar flux was depressed by 50 percent from the unperturbed flux, and the angular flux in the direction of the duct exit was 61 percent of the unperturbed value. Comparison of the data with analytical results indicates that the exit current is predicted very well. The source plane scalar flux is predicted with somewhat less accuracy.

A background survey for the enhancement of the thermal neutron beam intensity at the Ford Nuclear Reactor is presented in an appendix. Variations in reflector materials and core-reflector configurations were investigated. The results suggest the employment of a D_2O-reflected core in a slab configuration.

I. INTRODUCTION

A. Statement of the Problem and Motivation for the Study

This dissertation develops a method for predicting the exit thermal neutron current from a reactor beam port. The necessary removal of a portion of the diffusing medium to accommodate a beam port exerts a significant influence on the behavior of the thermal neutrons in the vicinity of the port. Inasmuch as these neutrons comprise a source for the beam, the beam port perturbs the intensity and spectrum of the exit beam. Experimental results are presented which test the analytical method under realistic reactor conditions.

The motivation for this study arises from a demand for beam intensity enhancement. An estimate of the influence of the beam port on the exit current intensity is necessary for any thorough enhancement study, although the problem has a great deal of intrinsic interest as well. The increasing importance of neutron spectrometer experiments has revealed the need for the examination of problems associated with neutron beam extraction.[1]

Roughly, the neutron flux, $\phi(h)$, at the exit of a beam port of length h and cross sectional area A is given by:

$$\phi(h) = \frac{\phi(0) A}{4\pi h^2} \quad , \qquad (1.1)$$

where $\phi(0)$ is the neutron flux at the source plane of the port. The

cross-sectional area of the beam is generally limited by the resolution requirements of the experiment. Thus the intense neutron densities available in the reactor may be attenuated by orders of magnitude at the beam port exit.

B. Related Published Studies

Studies for the enhancement of thermal neutron beam intensity have emphasized the reduction of fast neutron and gamma ray background. The split core concept,[2] resulting in the design of the National Bureau of Standards Reactor,[3] involves the removal of fuel from the midplane region of a D_2O moderated and reflected core. The tangential beam port concept, applied to the Brookhaven High Flux Beam Reactor[4] (HFBR), involves placement of the beam ports tangential to the core. Furthermore, the D_2O moderated and reflected HFBR is undermoderated so as to produce exceptionally high thermal flux peaking in the reflector.

The effects of duct introduction into a reactor system have been examined primarily from the standpoint of the influence on the multiplication constant of the reactor. Behrens[5] revised the magnitude of L^2, the diffusion area of a medium containing gas-coolant passages, to account for the additional streaming leakage. Reynolds et al.[6] investigated the reactivity effects of large voids in the reflector of the Pool Critical Assembly at Oak Ridge National Laboratory. In a theoretical examination of the effect on reactivity of a reflector duct Baraff et al.[7]

treated the problem by ordinary perturbation theory. These studies do
not yield information concerning the perturbed scalar flux in the vicin-
ity of the void or the current within the void.

Shielding considerations have prompted the examination of the trans-
port of neutrons down long ducts.[8] The influence of the duct walls on
the attenuation of neutrons by ducts was studied by Simon and Clifford[9]
and they concluded that the direct streaming contribution predominates
for $h/R \gg 1$. Piercey[10] attempted to separate the contributions to the
streaming flux at various distances along the duct axis in measurements
at the LIDO reactor. In none of these investigations, however, was con-
sideration given to the duct influence on the streaming source.

C. Outline of This Study

In Chapter II a simple analytical method is developed to predict
the thermal neutron current at the exit of an evacuated duct from a
knowledge of the unperturbed scalar flux distribution in the medium.
We consider a long radial* duct introduced into the reactor reflector,
presenting a small reactivity perturbation to the multiplying system.
The infinite half-space problem is reviewed from the standpoint of the
anisotropic limit of the finite void. This permits comparison between
exact solutions and the P-1 approximation, and motivates a P-1 approach
to the present problem. A pseudo boundary condition on the scalar flux

*A radial duct refers to a duct whose axis lies along a radius of the re-
actor core.

is developed from the consideration of a finite return current to the duct
mouth. Utilization of this boundary condition permits a diffusion theory
solution for the perturbed scalar flux distribution at the duct mouth.
The diffusion theory solution is used in conjunction with the P-1 angular
distribution to predict the exit current. The return current to the
mouth is evaluated by summing contributions from the lateral surfaces of
the duct. Finally, some consideration is given to the scalar flux dis-
tribution along the lateral surfaces of the duct; although the realistic
problem is not solved, a reasonable boundary condition is derived from
idealizations of the physical situation.

A number of activation experiments were conducted outside of the
core of the Ford Nuclear Reactor. These are described in Chapter III.
Initial comments concern the reactor, the positioning of ducts, the
normalization procedure, and the foil detection techniques. Measurements
of the unperturbed scalar flux distributions in the water reflector and
reactor core are presented. Preliminary 1-1/2-in. duct experiments in
water and a small graphite insert are described. The scalar flux data
at the mouth and the collimated exit current for 1-1/2-, 3-, and 5-in.
ducts in water are displayed. Data are presented for the attenuation
of the thermal flux within the 1-1/2- and 5-in.-ducts. A measurement of
the neutron temperature perturbation at the mouth of the 5-in. duct is
described. The final section describes the scalar flux measurements at
the mouth of the ducts in the graphite reflector. All unperturbed scalar
flux measurements are compared with group-diffusion calculations. Scalar

flux data at the duct mouths and exit currents are compared with predictions based upon the analysis presented in Chapter II.

The discussion in Chapter IV considers the assumptions inherent in the analysis. The limitations of diffusion theory for the prediction of the unperturbed scalar flux in the reflector are pointed out, and the seriousness of these limitations from the point of view of the present analysis is considered. An attempt to predict the realistic scalar flux distributions along the duct walls is discussed. Duct reactivity evaluations obtained from two-dimensional group-diffusion calculations based upon initial estimates of the wall gradients are compared with experiment. In addition, items from previous chapters which require further consideration or clarification are discussed in Chapter IV.

A survey directed toward enhancement of the thermal neutron beam intensity at the Ford Nuclear Reactor is presented in Appendix A. The investigation considers variations in reflector materials and core-reflector configurations for radial beam ports. The results are relegated to an appendix because the work was in the nature of a background study preliminary to the main body of this dissertation. In this survey, consequently, the beam port influence on the exit current is neglected. This can be considered a conservative omission from the standpoint of the reflector materials study, since the longer diffusion lengths possessed by the most desirable reflectors give rise to lower beam port current depressions. The method described in Chapter II, however, can be used as a calculational refinement for the configurations of particular interest and a specified beam port diameter.

II. ANALYSIS

A. Orientation

The configuration of interest is a long cylindrical void inserted

into a diffusing medium. This is sketched in Figure 2.1. Neutrons
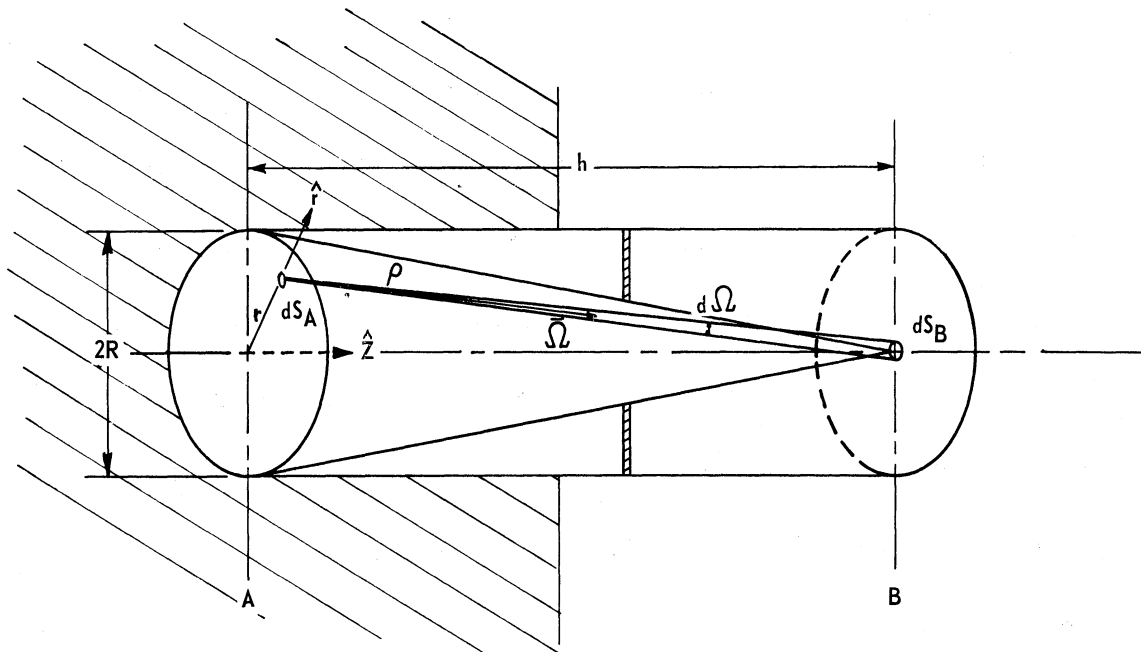
Figure 2.1. Void configuration.

streaming from all points on the interface between the diffusing medium

and the void form a beam at B far from interface A.* Assume that the beam

is collimated, so that a detector placed on the axis at B optically views

only the front interface at A. We are interested in the number of neutrons

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*Henceforth interface A will often be referred to as the mouth of the
duct and B as the duct exit.

per unit time passing through a unit area normal to the axis at B. This quantity will be called $j_B^+(E)$, the partial current at B. $j_B^+(E)$ is equivalent to the integral over the surface, S_A, of the angular current with energy, E, leaving $\hat{z}\,dS_A$ directed into solid angle $d\Omega$ about $\bar{\Omega}$ intercepting dS_B:

$$j_B^+(E) = \int_{S_A} dS_A \; \hat{z}\cdot\bar{\Omega} \; \phi_A(\bar{r},\bar{\Omega},E) \; \frac{d\Omega}{dS_B} \;. \qquad (2.1)$$

Then for any fragment of the energy spectrum:

$$j_B^+(E_0 \to E_1) = \int_{E_0}^{E_1} dE \int_{S_A} dS_A \; \hat{z}\cdot\bar{\Omega} \; \phi_A(\bar{r},\bar{\Omega},E) \; \frac{d\Omega}{dS_B} \;, \qquad (2.2)$$

or the thermal current can be expressed as:

$$j_{B_{th}}^+ = \int_0^{E_{TC}} dE \int_{S_A} dS_A \; \hat{z}\cdot\bar{\Omega} \; \phi_A(\bar{r},\bar{\Omega},E) \; \frac{d\Omega}{dS_B} = \int_{S_A} dS_A \; \hat{z}\cdot\bar{\Omega} \; \phi_A^{th}(\bar{r},\bar{\Omega}) \; \frac{d\Omega}{dS_B} \;.$$

The problem then reduces to that of solving for the angular current at interface A. The formulation is initiated by writing the steady state Boltzmann equation for the angular flux, $\phi(\bar{r},\bar{\Omega},E)$, in space, angle, and energy:

$$\bar{\Omega}\cdot\bar{\nabla} \; \phi(\bar{r},\bar{\Omega},E) + \Sigma(\bar{r},E)\phi(\bar{r},\bar{\Omega},E) = s(\bar{r},\bar{\Omega},E)$$

$$+ \int d\Omega' \int dE' \; \phi(\bar{r},\bar{\Omega}',E') \Sigma_S(\bar{r},\bar{\Omega}'\to\bar{\Omega},E'\to E) \;. \qquad (2.4)$$

The conventional symbols used in reactor physics are employed, and are defined in the Nomenclature. Hypothetically a solution for the geometry under consideration can be obtained after the source and scattering kernel are specified.

In light of the complexity of the geometry, the pursuit of an exact solution for $\phi(\bar{r},\bar{\Omega},E)$ is nearly futile. A numerical approach, utilizing a high speed digital computer is feasible, but besides lacking in generality, would be prohibitively time-consuming. Rather, we seek a technique which yields the angular flux to a sufficiently high degree of approximation to provide realistic predictions for $j_{B_{th}}^{+}$, and which is simple enough to be useful for practical computations. To this end, P-1 theory is examined.

B. Infinite Half-Space Limit

In order to assess the validity of a P-1, or diffusion theory, approach consider the case which brings about the highest degree of anisotropy at interface A, that of a full vacuum boundary. Allowing R, the void radius, to go to infinity and considering monoenergetic neutron diffusion in an infinite half-space of fixed composition, exact solutions can be obtained for the steady state transport equation (assuming isotropic scattering):

$$\mu \frac{\partial \phi}{\partial z} + \frac{1}{\ell} \phi(z,\mu) = s(z,\mu) + \frac{c}{2\ell} \int_{-1}^{1} d\mu' \, \phi(z,\mu'), \qquad (2.5)$$

and comparisons may be drawn with P-1 theory results. Two cases are
particularly well known. These are the classical Milne problem, s = 0,
and the case of constant production, s = constant.

We shall examine the angular flux in the case of constant production
for a unit isotropic source per unit volume. The coordinate frame is
sketched in Figure 2.2. Davison[11] expresses the angular flux at the
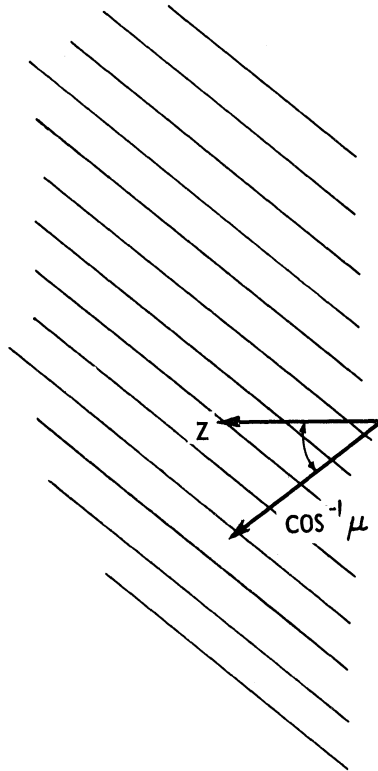
Figure 2.2. Infinite half-space geometry.

boundary in the constant production problem in terms of the Milne solution:

$$\phi^q(o,\mu) = \frac{L+\ell\mu}{c|j^M(o)|} \phi^M(o,\mu), \text{ for } \mu < 0 \quad . \tag{2.6}$$

For c = 1:*

$$\left| j^M(0) \right| = \frac{1}{\sqrt{3}} \phi_0(0) \; , \qquad (2.7)$$

and thus:

$$\left[\phi^q(0,\mu) \right]_{c=1} = \frac{\sqrt{3}}{c}(L+\ell\mu)\left[\phi^M(0,\mu) \right]^N, \text{ for } \mu < 0. \qquad (2.8)$$

where $\left[\phi^M(0,\mu) \right]^N$ is the tabulated[12] Milne angular distribution normalized to unit scalar flux at the boundary.

The P-1 equations for isotropic scattering and a constant isotropic source are:

$$\frac{d}{dz} \phi_1(z) + \Sigma_a \phi_0(z) = s_0$$

$$\frac{1}{3} \frac{d}{dz} \phi_0(z) + \Sigma_t \phi_1(z) = 0 \; . \qquad (2.9)$$

The angular flux is given by:

$$\phi_{P-1}(z,\mu) = \sum_{n=0}^{1} \frac{2n+1}{2} \phi_n(z) P_n(\mu) \; . \qquad (2.10)$$

The solution for the scalar flux is expressed as:

$$\phi^q_{0_{P-1}}(z) = Ce^{-\kappa z} + \frac{s_0}{\Sigma_a} \; , \qquad (2.11)$$

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*Our concern with reflector materials indeed directs attention to values of c close to 1. For light water, c = .994.

where:

$$\kappa^2 \equiv 3\Sigma_t\Sigma_a = \frac{1}{L^2} \quad . \tag{2.12}$$

Applying the Marshak boundary condition:

$$\int_0^1 \mu\,\phi(0,\mu)d\mu = 0 \quad , \tag{2.13}$$

and setting the source equal to unity, one obtains for the P-1 angular

distribution at the boundary:

$$\phi_{P-1}^q(0,\mu) = \frac{2}{\kappa}\frac{\phi_{P-1}^N(0,\mu)}{1 + \frac{2}{3}\frac{\kappa}{\Sigma_t}} \quad . \tag{2.14}$$

$\phi_{P-1}^N(0,\mu)$ is the familiar form of the normalized P-1 angular distribution,

and is given by:

$$\phi_{P-1}^N(0,\mu) = \frac{1}{2} - \frac{3}{4}\mu \quad . \tag{2.15}$$

The normalized P-1 and Milne angular distributions are compared in

Figure 2.3.

A comparison of the scalar fluxes is obtained by integrating Eqs.

(2.8) and (2.14) over all angles, yielding:

$$\left[\frac{\phi_{o\,P-1}^q(0)}{\phi_o^q(0)}\right]_{c=1} = 1.155 \quad . \tag{2.16}$$

The quantity of particular interest for the beam port problem is the
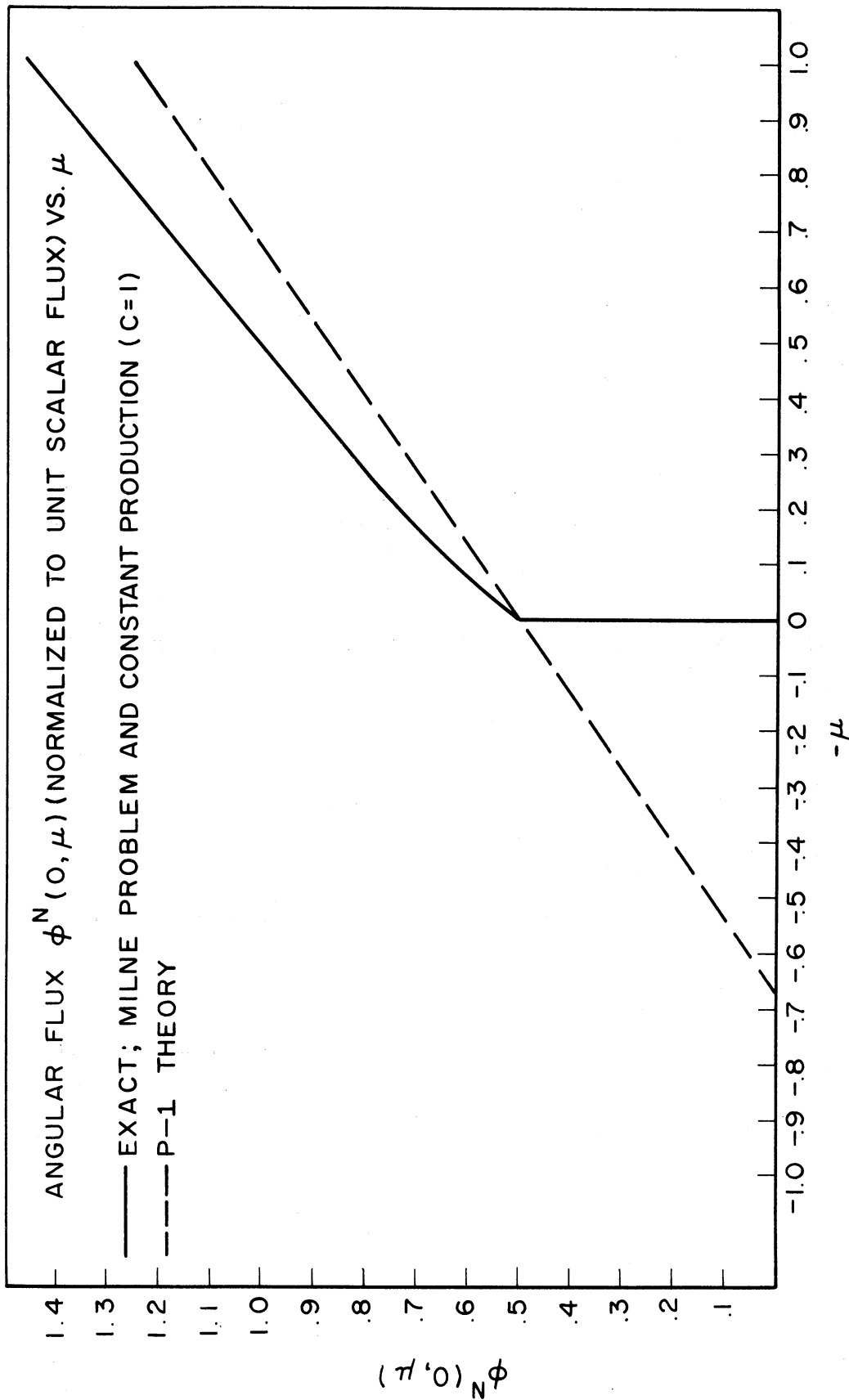
Figure 2.3. Normalized angular flux for c = 1; exact and P-1 approximation.

angular flux close to $\mu = -1$. Setting $\mu = -1$ in Eqs. (2.8) and (2.14) we

obtain:

$$\left[\frac{\phi_{P-1}^{q}(0,\mu)}{\phi^{q}(0,\mu)}\right]_{\substack{c=1 \\ \mu=-1}} = .993 \quad . \qquad (2.17)$$

The same result is obtained for the Milne problem ($s = 0$) if both the P-1

solution and the exact solution are normalized to the exact value of the

scalar flux at the boundary. Thus the P-1 approach gives rise to an

angular flux for very small angles which differs by less than 1 percent

from the exact solution. It should be pointed out that the P-1 solution

is free of transport corrections.

C. Diffusion Theory Boundary Condition

Since a P-1 approach provides an excellent approximation for the

angular flux in the limiting case of the infinite half-space, we shall

examine the finite void using diffusion theory. The current returning to

the mouth of the void originates from neutrons leaving the lateral sur-

faces. This forms a basis for a boundary condition at A which is phys-

ically realistic and consistent with the general formulation of diffusion

theory.

In addition to the assumption of low capture in the medium, $c \cong 1$,

and small angles to the detector, $(R/h)^2 \ll 1$, the following two assumptions

are made:

1. The perturbed scalar flux is flat over the entire duct mouth. This permits us to neglect the radial dependence at the mouth.

2. The fast flux is unperturbed by the void. As a corollary to this the assumption is made that the overall reactor fission source perturbation is negligible.

Specification of the partial current at A coupled with the continuity requirements at the boundary between the multiplying medium and the reflector completely specifies the diffusion theory solution for the scalar flux in the medium. It is convenient, however, to transform the current condition into one on the scalar flux, ϕ_0. Proceeding analogously to the familiar treatment of the limiting full vacuum interface[13] and referring to Figure 2.4:

$$j_A^- = \frac{\phi_0^A}{4} + \frac{\ell_{tr}}{6} \left. \frac{d\phi_0}{dz} \right|_A . \qquad (2.18)$$

Assuming a linear decay of the scalar flux past A:

$$\phi_0(z) = z \left. \frac{d\phi_0}{dz} \right|_A + \phi_0^A , \qquad (2.19)$$

we solve for the distance, d, past the interface, at which $\phi_0(d) = 4 \, j_A^-$:

$$4 \, j_A^- = d \left. \frac{d\phi_0}{dz} \right|_A + \phi_0^A . \qquad (2.20)$$
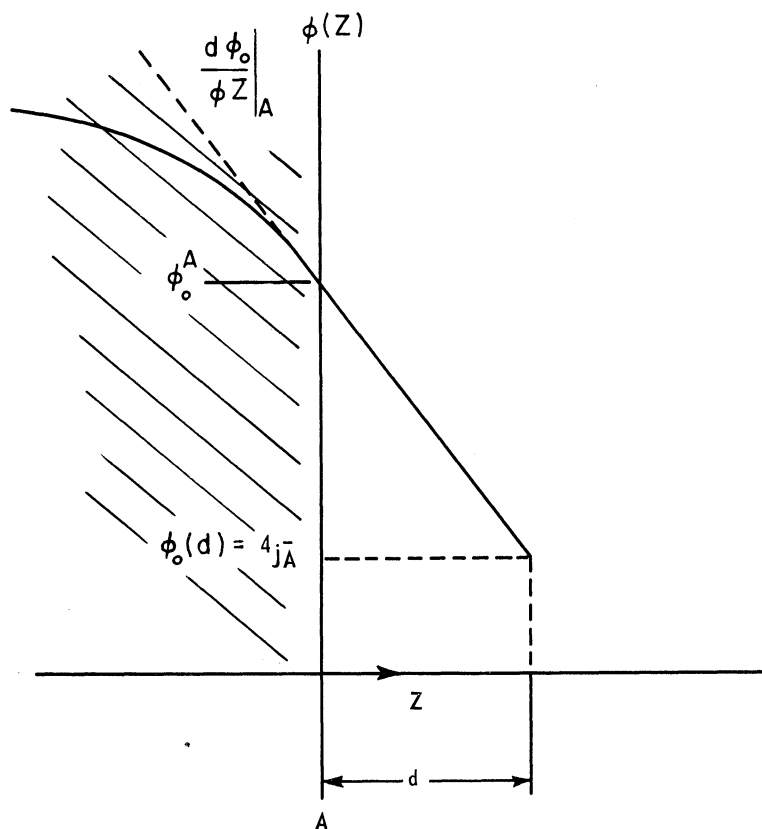
Figure 2.4. Linear extrapolation at vacuum interface.

Combining Eqs. (2.18) and (2.20) we obtain:

$$\phi_o(\tfrac{2}{3} \ell_{tr}) = 4 \, j_A^- , \qquad\qquad (2.21)$$

the well-known diffusion theory extrapolation condition, applied here to a vacuum boundary possessing a finite return curent.

It is shown in Appendix D that the application of this pseudo boundary condition on the scalar flux rather than the actual one involving the partial current introduces negligible error for $c \sim 1$. Furthermore, the employment of a linear extrapolation distance, d, rather than the extra-polated endpoint, z_o, is valid for values of c close to one:[11]

$$\frac{d}{z_0} \cong c^{-1/2} \text{ for } c \cong 1 \ . \qquad\qquad (2.22)$$

D. Angular Flux Distribution

The P-1 approximation to the angular flux is needed to calculate duct currents. It is instructive to derive it directly from the integral form of the transport equation. In the constant cross section approximation the angular flux is given by (Eq.(4-16) of Reference 11):

$$\phi(\overline{r},\overline{\Omega}) = \int_0^\infty \left[\left(\frac{c}{4\pi \ell}\right)\phi_0(\overline{r}-\eta\overline{\Omega}) + s(\overline{r}-\eta\overline{\Omega})\right] e^{-\eta/\ell} d\eta. \qquad (2.23)$$

Figure 2.5 is a vector diagram pertinent to Eq. (2.23). The source term in the equation is defined per unit solid angle.
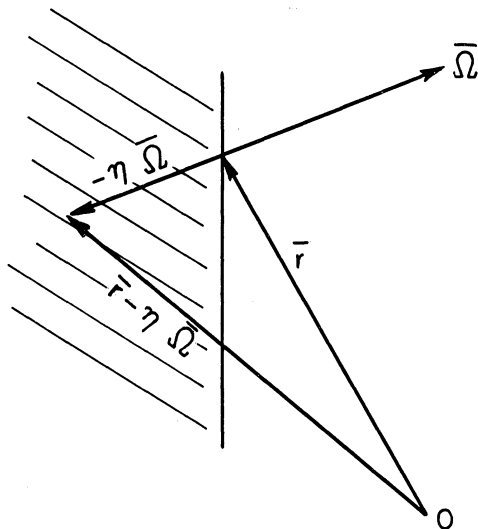
Figure 2.5. Vector diagram at vacuum interface.

Expanding any function, $f(\overline{r}-\eta\overline{\Omega})$, about the point $\overline{r}$ in a Taylor series

gives:

$$f(\overline{r}-\eta\overline{\Omega}) = f(\overline{r}) + (-\eta\overline{\Omega}\cdot\overline{\nabla})f(\overline{r}) + \frac{1}{2!}(-\eta\overline{\Omega}\cdot\overline{\nabla})^2 f(\overline{r}) + \ldots \quad . \quad (2.24)$$

Accordingly expanding the scalar flux and source in the integral equation, performing the integration over η, and retaining two terms yields:

$$\phi(\overline{r},\overline{\Omega}) \simeq \frac{c}{4\pi}[\phi_0(\overline{r}) - \ell\overline{\Omega}\cdot\overline{\nabla}\phi_0(\overline{r})] + \ell[s(\overline{r}) - \ell\overline{\Omega}\cdot\overline{\nabla}s(\overline{r})] \quad . \quad (2.25)$$

E. Derivation of j_B^+

Using the angular distribution presented in Eq. (2.25) and diffusion theory solutions for the source and scalar flux distributions at the mouth, we are in a position to derive an expression for j_B^+, the partial current at the exit of a long collimated duct. Restricting ourselves to the geometry illustrated in Figure 2.1, and assuming polar symmetry:

$$\overline{\nabla} = \hat{r}\frac{\partial}{\partial r} + \hat{z}\frac{\partial}{\partial z} \quad , \qquad (2.26)$$

$$\overline{\Omega} = \hat{r}\left(-\frac{r}{\rho}\right) + \hat{z}\left(\frac{h}{\rho}\right) \quad , \qquad (2.27)$$

and:

$$d\Omega = \frac{dS_B}{\rho^2}\left(\frac{h}{\rho}\right) \quad . \qquad (2.28)$$

The angular flux at A in the direction of a detector on the axis at B

is expressed as:

$$\phi_A(\bar{r},\bar{\Omega}) \cong \frac{c}{4\pi}\left\{\phi_0(\bar{r}) - \ell\left[\frac{h}{\rho}\frac{\partial\phi_0}{\partial z}\Big|_{\bar{r}} - \frac{r}{\rho}\frac{\partial\phi_0}{\partial r}\Big|_{\bar{r}}\right]\right\}_A$$

$$+ \ell\left\{s(\bar{r}) - \ell\left[\frac{h}{\rho}\frac{\partial s}{\partial z}\Big|_{\bar{r}} - \frac{r}{\rho}\frac{\partial s}{\partial r}\Big|_{\bar{r}}\right]\right\}_A \circ \tag{2.29}$$

Assume that ϕ_0, s, and their derivatives are constant over the emitting

surface, so that:

$$\phi_A(r,\bar{\Omega}) \cong \frac{c}{4\pi}\left[\phi_0 - \ell\frac{h}{\rho}\frac{d\phi_0}{dz}\right]_A + \ell\left[s - \ell\frac{h}{\rho}\frac{ds}{dz}\right]_A \circ \tag{2.30}$$

Substituting this expression for the angular flux into Eq. (2.3) and

integrating over the surface at A results in:

$$j_B^+ \cong \frac{c}{4}\left\{\left(\frac{R^2}{h^2+R^2}\right)\phi_0 - \frac{2}{3}\ell\left[1 - \left(1 + \frac{R^2}{h^2}\right)^{-3/2}\right]\frac{d\phi_0}{dz}\right\}_A$$

$$+ \pi\ell\left\{\left(\frac{R^2}{h^2+R^2}\right)s - \frac{2}{3}\ell\left[1 - \left(1 + \frac{R^2}{h^2}\right)^{-3/2}\right]\frac{ds}{dz}\right\}_A \circ \tag{2.31}$$

For a long duct, $R^2/h^2 \ll 1$, so that the expression reduces to:

$$j_B^+ \cong \frac{R^2}{h^2}\left\{\frac{c}{4}\left[\phi_0 - \ell\frac{d\phi_0}{dz}\right] + \pi\ell\left[s - \ell\frac{ds}{dz}\right]\right\}_A \circ \tag{2.32}$$

The slowing-down source can be computed from the fast flux obtained in a

few-group analysis. Assuming fast flux isotropy:

$$s = \frac{\Sigma_R^f \phi_0^f}{4\pi} \ .$$

F. Derivation of $\bar{j}_A$

Equation (2.21) specifies the diffusion theory boundary condition to

be used in obtaining the distribution of the perturbed scalar flux in the

vicinity of the duct mouth. It is necessary to sum contributions from

all lateral surfaces of the duct in order to derive an expression for $\bar{j}_A$

in terms of the scalar flux distribution along the walls of the duct. The

geometry is illustrated in Figure 2.6.
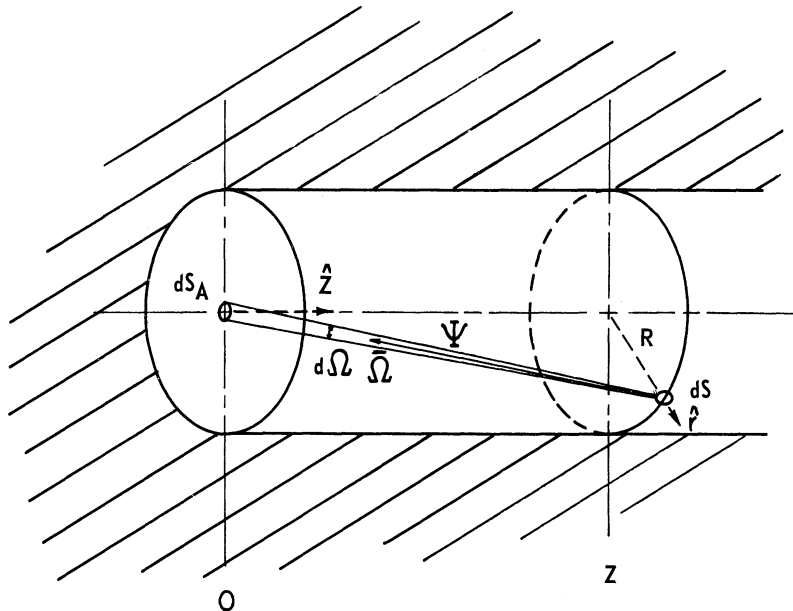
Figure 2.6. Configuration for calculation of $\bar{j}_A$.

Equation (2.25) is employed once again for the angular flux along

the lateral surfaces of the duct. Assuming that the slowing-down source

contribution is negligible, and that $c \approx 1$:

$$\phi(z,R,\bar{\Omega}) \cong \frac{1}{4\pi}\left[\phi_0(z,R) - \ell\, \bar{\Omega}\cdot\bar{\nabla}\, \phi_0\Big|_{\substack{z \\ R}}\right] . \qquad (2.34)$$

Since the mouth is treated in one dimension, it will suffice to solve

for the return current at the center. Referring to Figure 2.6:

$$\bar{\Omega} = -\hat{r}\,\frac{R}{\psi} - \hat{z}\,\frac{z}{\psi} , \qquad (2.35)$$

$$d\Omega = \frac{dS_A}{\psi^2}\,\frac{z}{\psi} , \qquad (2.36)$$

$$\psi^2 = R^2 + z^2 , \qquad (2.37)$$

and:

$$dS = 2\pi\, R dz . \qquad (2.38)$$

The return current at the mouth is expressed as:

$$\bar{j_A} = \int\limits_{S} dS(-\hat{r}\cdot\bar{\Omega})\phi(z,R,\bar{\Omega})\,\frac{d\Omega}{dS_A} . \qquad (2.39)$$

Substituting the angular flux:

$$\phi(z,R,\bar{\Omega}) \cong \frac{1}{4\pi}\left\{\phi_0(z,R) + \ell\left[\frac{R}{\psi}\,\frac{\partial\phi_0}{\partial r}\Big|_{\substack{R \\ z}} + \frac{z}{\psi}\,\frac{\partial\phi_0}{\partial z}\Big|_{\substack{R \\ z}}\right]\right\} , \qquad (2.40)$$

into Eq. (2.39), and supposing that the void extends to infinity, we

obtain for the return current at the mouth:

$$j_A^- = \frac{R^2}{2}\left\{\int_0^\infty dz \frac{z\,\phi_O(R,z)}{(R^2+z^2)^2} + \ell\int_0^\infty dz \frac{z^2 \frac{\partial\phi_O}{\partial z}\Big|_z^R}{(R^2+z^2)^{5/2}} + \ell R\int_0^\infty dz \frac{z \frac{\partial\phi_O}{\partial r}\Big|_z^R}{(R^2+z^2)^{5/2}}\right\}.$$

(2.41)

In order to assess the validity of this expression, let us examine

the limits as R→0 and R→∞. It is first convenient to transform variables

such that:

$$j_A^- = \frac{1}{2}\left\{\int_0^\infty dz \frac{z\phi_O(R,Rz)}{(1+z^2)^2} + \ell\int_0^\infty dz \frac{z^2 \frac{\partial\phi_O}{\partial z}\Big|_{Rz}^R}{(1+z^2)^{5/2}} + \ell\int_0^\infty dz \frac{z \frac{\partial\phi_O}{\partial r}\Big|_{Rz}^R}{(1+z^2)^{5/2}}\right\}.$$

(2.42)

When R becomes small, the scalar flux and its derivatives are slowly vary-

ing with respect to the remainder of the integrand, and can be taken out-

side the integral sign. The integrations yield:

$$\lim_{R\to 0} j_A^- = \frac{\phi_O(0,0)}{4} + \frac{\ell}{6}\,\hat{z}\cdot\bar{\nabla}\,\phi_O\Big|_0^0 ,$$

(2.43)

which is the partial current at A in the absence of the void.

In the limit of large R the rest of the integrands are slowly vary-

ing with respect to ϕ_O and its derivatives, and accordingly:*

---

*We assume here that the scalar flux is monotonically decreasing with z;

the unperturbed flux is of the form: $\phi(Rz) = \sum_i C_i e^{-a_i Rz}$.

$$\lim_{R \to \infty} \bar{j_A} = 0 \; , \qquad\qquad (2.44)$$

the vacuum interface condition. Equation (2.41), then, has the desired limits.

G. Scalar Flux Distribution Along the Lateral Surfaces

The final task, and evidently the most formidable, is the evaluation of the scalar flux distribution along the lateral surfaces of the void. We expect the average perturbation of the angular flux along the duct walls contributing most heavily to the return current to be less than that at mouth. This is because of the considerably more favorable view factors along the walls. For small void diameters it is reasonable to assume that the angular flux along the walls is unperturbed.

A sound approach to the problem , however, consists of a current balance over all surfaces. Such a procedure has been carried out for a void of finite length along the axis of a cylindrical reactor.[14] Considering for the moment a long cylindrical void, the radial component of the gradient at a point (R, z_1) on the surface is obtained by equating the outgoing (in the positive radial direction) partial current to the contributions from all surfaces of the void. The angular distribution is given by Eq. (2.34). Figure 2.7 illustrates the geometry. Carrying out
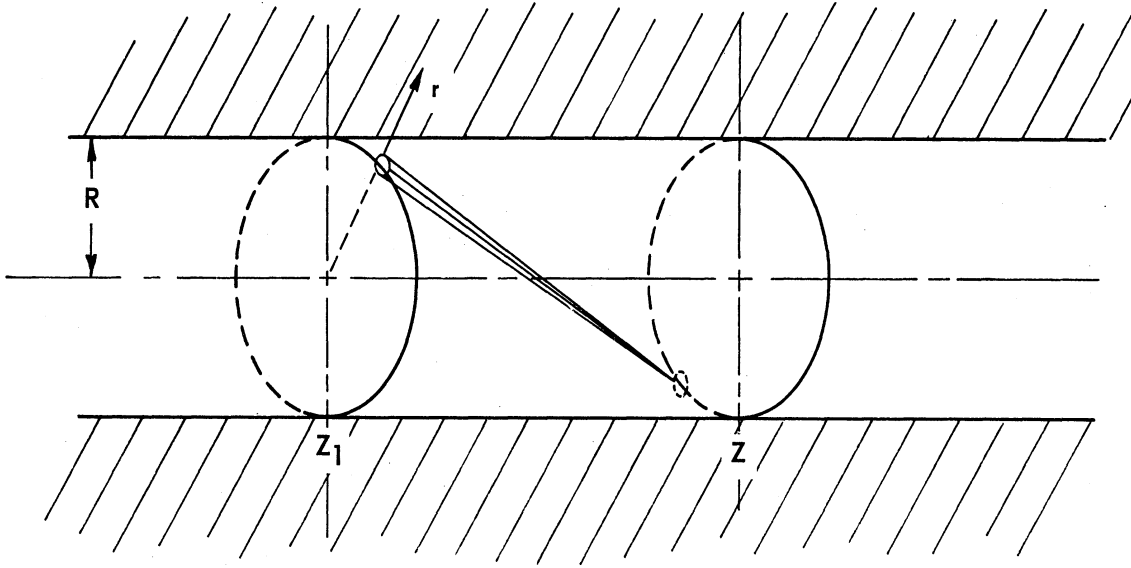
Figure 2.7. Current balance geometry.

the angular integrations and solving for the radial component of the

gradient, one obtains:

$$- \ell \, \frac{\partial \phi_0 / \partial r |_{z_1}^{R}}{\phi_0(R, z_1)} \;=\; - \frac{3}{2} + \frac{3}{2\pi R} \Bigg\{ \int dz \, \frac{\phi_0(R, z)}{\phi_0(R, z_1)} \, f_1(p) \;+\; \ell \int dz \, \frac{\partial \phi_0 / \partial r |_{z}^{R}}{\phi_0(R, z_1)} \, f_2(p)$$

$$- \ell \int dz \, \frac{\partial \phi_0 / \partial z |_{z}^{R}}{\phi_0(R, z_1)} \, f_3(p) \Bigg\} \;, \tag{2.45}$$

where:

$$f_1(p) \;=\; \frac{\pi}{4} \left[2 - (2 + p^2)(1 - p^2)^{1/2} \right] \;, \tag{2.46}$$

$$f_2(p) \;=\; \frac{1}{3p} \left[(2(1 - p^4) + 3(2 - p^2)) E(p^2) - (1 - p^2)(8 + p^2) K(p^2) \right] \;, \tag{2.47}$$

$$f_3(p) \;=\; \frac{1}{3} \left[\pm (1 - p^2)^{1/2} \right] \left[(2 + p^2) K(p^2) - 2(1 + p^2) E(p^2) \right] \;, \tag{2.48}$$

$$p^2 \equiv \frac{1}{1 + \frac{(z_1-z)^2}{4R^2}} \quad ; \quad \pm(1-p^2)^{1/2} \equiv \frac{z_1-z}{\left[(z_1-z)^2 + 4R^2\right]^{1/2}} \quad ,(2.49)$$

and K and E are the complete elliptic integrals of the first and second kinds.[15]

The integral equation is not easily soluble. Zimmerman assumed a separable solution, in which case Eq. (2.45) reduces to:

$$\ell \frac{\partial\phi_0/\partial r\big|_{z_1}^{R}}{\phi_0(R,z_1)} = \frac{1 - \frac{1}{\pi R}\int dz \frac{\phi_0(z)}{\phi_0(z_1)} f_1(p) + \frac{\ell}{\pi R}\int dz \frac{\partial\phi_0/\partial z}{\phi_0(z_1)} f_3(p)}{\frac{2}{3}\left\{1 + \frac{3}{2\pi R}\int dz \frac{\phi_0(z)}{\phi_0(z_1)} f_2(p)\right\}} \quad .$$

$$(2.50)$$

The assertion was made that although the separable approach lacks in rigor, it possesses the correct limits (for the void along the axis of a cylindrical reactor).

The trivial case of the infinite cylinder with a constant flux along the walls, although not directly applicable to our problem, is instructive and does permit a separable solution. For this case $\phi_0(z)/\phi_0(z_1)$ = 1, and $\partial\phi_0/\partial z$ = 0, and the integrations are performed from $-\infty$ to ∞. The numerator in Eq. (2.50) vanishes, and we obtain:

$$\ell \frac{\partial\phi_0/\partial r\big|_{z_1}^{R}}{\phi_0(R,z_1)} = 0 \quad , \qquad (2.51)$$

for all z_1.

In the spirit of the Zimmerman approach, a linear decaying flux of

the form $\phi_0(z) = a+bz$ also leads to a zero gradient condition for the infinitely long cylinder. This can be shown by examining the point $z_1 = 0$. For this case $\phi_0(z)/\phi_0(z_1) = 1+cz$ and $\frac{\partial\phi_0/\partial z}{\phi_0(z_1)} = c$, and the numerator of the right-hand side of Eq. (2.50) becomes:

$$1 - \frac{1}{\pi R} \int_{-\infty}^{\infty} dz(1+cz)f_1(p) + \frac{\ell}{\pi R} \int_{-\infty}^{\infty} dz\, c\, f_3(p) \quad . \tag{2.52}$$

The terms multiplied by the constant are odd functions, and their interals are zero. The remaining terms are identical to the constant flux case, giving:

$$\ell\, \frac{\partial\phi_0/\partial r\big|_0^R}{\phi(R,0)} = 0 \quad . \tag{2.53}$$

This example, however, is somewhat of a mathematical artifice, in that the result is not the same for $z_1 \neq 0$; and so the assumption of separability is not strictly valid (moreover, the linear expression permits negative fluxes).

The actual void geometry consists of a semi-infinite cylinder bounded by an interface at $z = 0$ (See Figure 2.8). The current balance at any point $z_1 > 0$ now contains additional terms from the contribution of the mouth. The isotropic and z component of the gradient contributions are derived in Appendix E and are of the form:

$$\frac{3}{2}\frac{\phi_0(0)}{\phi_0(z_1)}\left[\frac{\frac{1}{2} + \frac{z_1^2}{4R^2}}{\left(1 + \frac{z_1^2}{4R^2}\right)^{1/2}} - \frac{z_1}{2R}\right] - \frac{6}{\pi}\,\ell\, z_1^2 \frac{\partial\phi_0/\partial z\big|_0}{\phi_0(z_1)} \int_0^R r\,dr\, I_2(r,R,z_1) \quad . \tag{2.54}$$
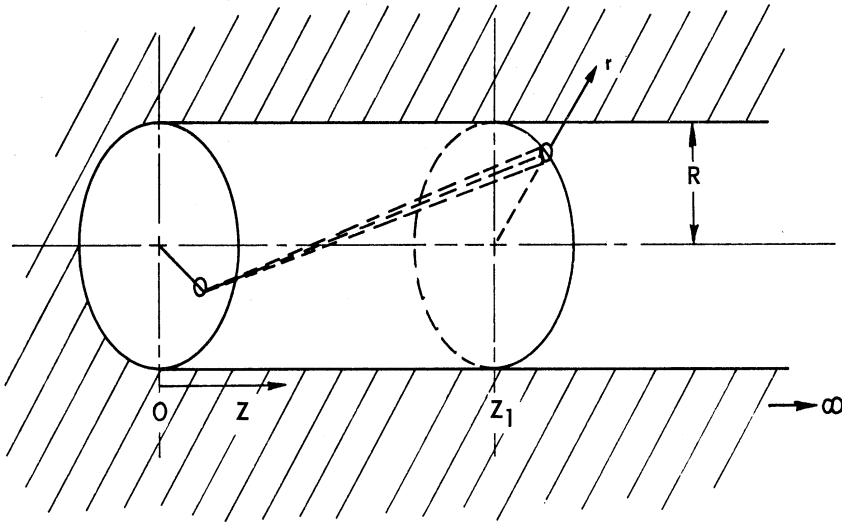
Figure 2.8. Current balance geometry including front interface.

These additional terms must be added to the right-hand side of Eq. (2.45).

Postulating as before a constant flux everywhere along the boundaries

such that $\partial\phi_0/\partial z|_0 = 0$ and $\dfrac{\phi_0(0)}{\phi_0(z_1)} = 1$, and assuming separability, one

finds that the contribution from the mouth is equivalent to that from

the negative half of the infinite cylinder studied previously. This

leads once again to the condition:

$$\left. \frac{\partial\phi_0/\partial r}{\phi_0} \right|_{\substack{R \\ z_1}} = 0, \quad \text{for all } z_1 > 0 \quad . \tag{2.55}$$

We have not solved the physically interesting problem, however. In

fact the argument for the $R \to 0$ limit of Eq. (2.41) demonstrates that a

constant flux along the lateral surfaces does not give rise to a duct

perturbation at the mouth at all. The realistic unperturbed scalar flux

is likely to be a decaying exponential or sum of exponentials, in which

case a z dependent radial component of the gradient is expected along the duct walls. Positive near the mouth, the radial component of the gradient becomes negative at an axial distance from the mouth such that the streaming contributions exceed the unperturbed flux in the medium.

In light of the complexity of the actual physical situation, the pursuit of an accurate representation of the scalar flux distribution at the duct walls will be avoided. Instead we assume that the zero gradient boundary condition is a reasonable approximation. This is equivalent to direct utilization of the unperturbed scalar flux if the unperturbed flux distribution is radially flat in the vicinity of the duct. The validity of this zero order perturbation treatment will be assessed by its success in predicting the results of experiment.

III. EXPERIMENT

A. The Ford Nuclear Reactor

The experiments were conducted on the south face of the Ford Nuclear

Reactor. The FNR is a swimming pool facility similar in design to the

Bulk Shielding Reactor at Oak Ridge National Laboratory.[16] The fuel is

of the MTR type. Regular elements contain 140 grams of U^{235} in 18 plate

3 in. x 3 in. subassemblies. Partial elements, which house the poison

rods, contain 71 grams of U^{235} in 9 plate subassemblies. A more complete

description of the facility is contained in the literature.[17]

The core configuration adopted for the experiments is illustrated

in Figure 3.1. Occasional minor variations in the fuel loading were ne-

cessitated by criticality considerations, but changes were confined to

the north extremities of the core. The specific elements on the south

face were left unchanged during the period of experiments.

B. Insertion and Positioning of Ducts

A duct, the duct support, and a foil holder are sketched in Figure

3.2. Variable sized collars mounted on the aluminum plate of the support

accommodate the ducts for positioning. The weighted plug bolted to the

bottom of the plate fits into a south matrix hole of the grid plate which

is furnished with an offset dowel pin to guarantee angular placement.

BEAM TUBE

PNEUMATIC TUBE

A,B,C SHIM RODS
CR   CONTROL ROD
G   GRAPHITE
FC   FISSION CHAMBER
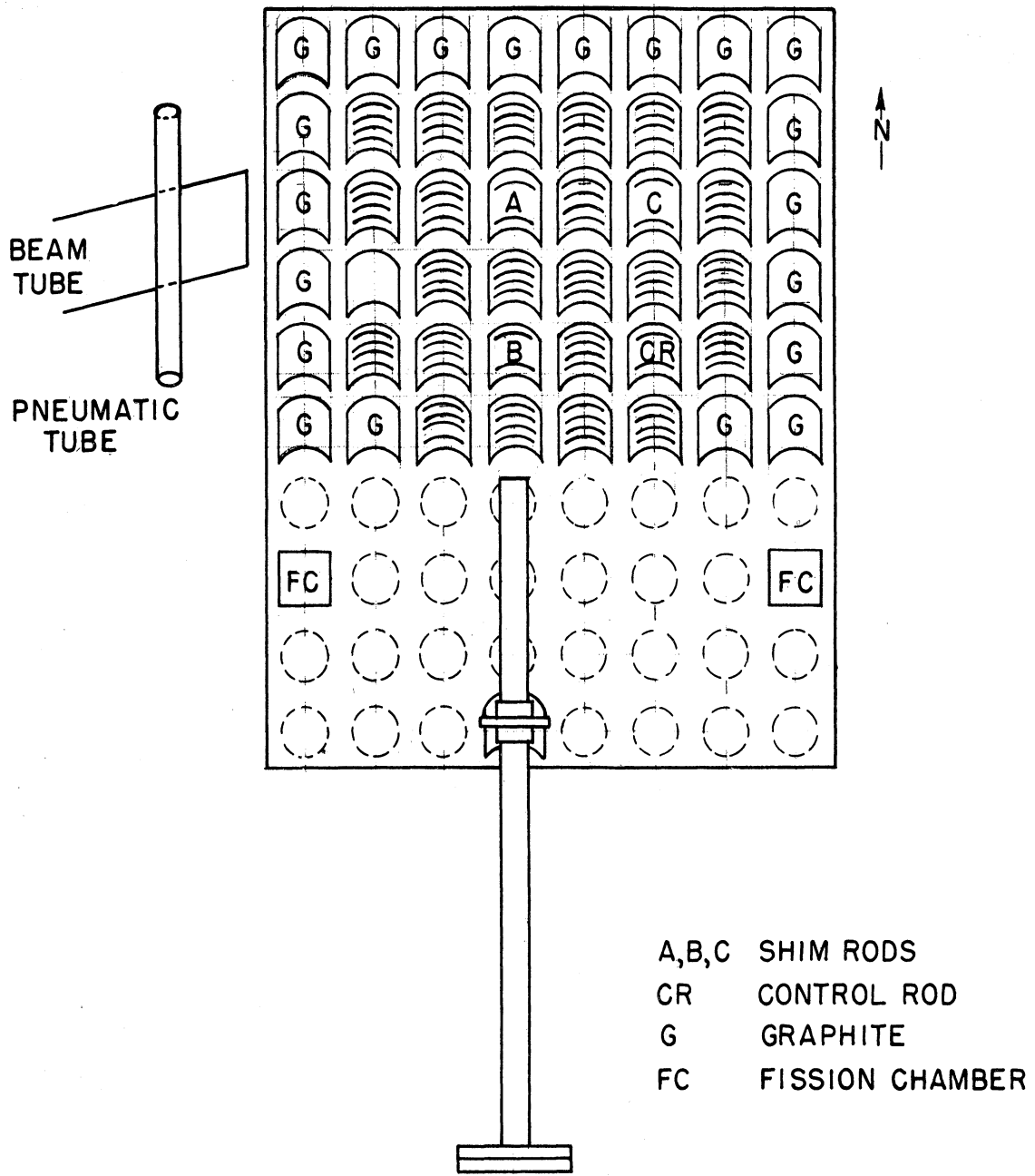
Figure 3.1. Ford Nuclear Reactor core configuration.

Streaming Duct

Foil Holder

4'

Duct Support with Normalizer
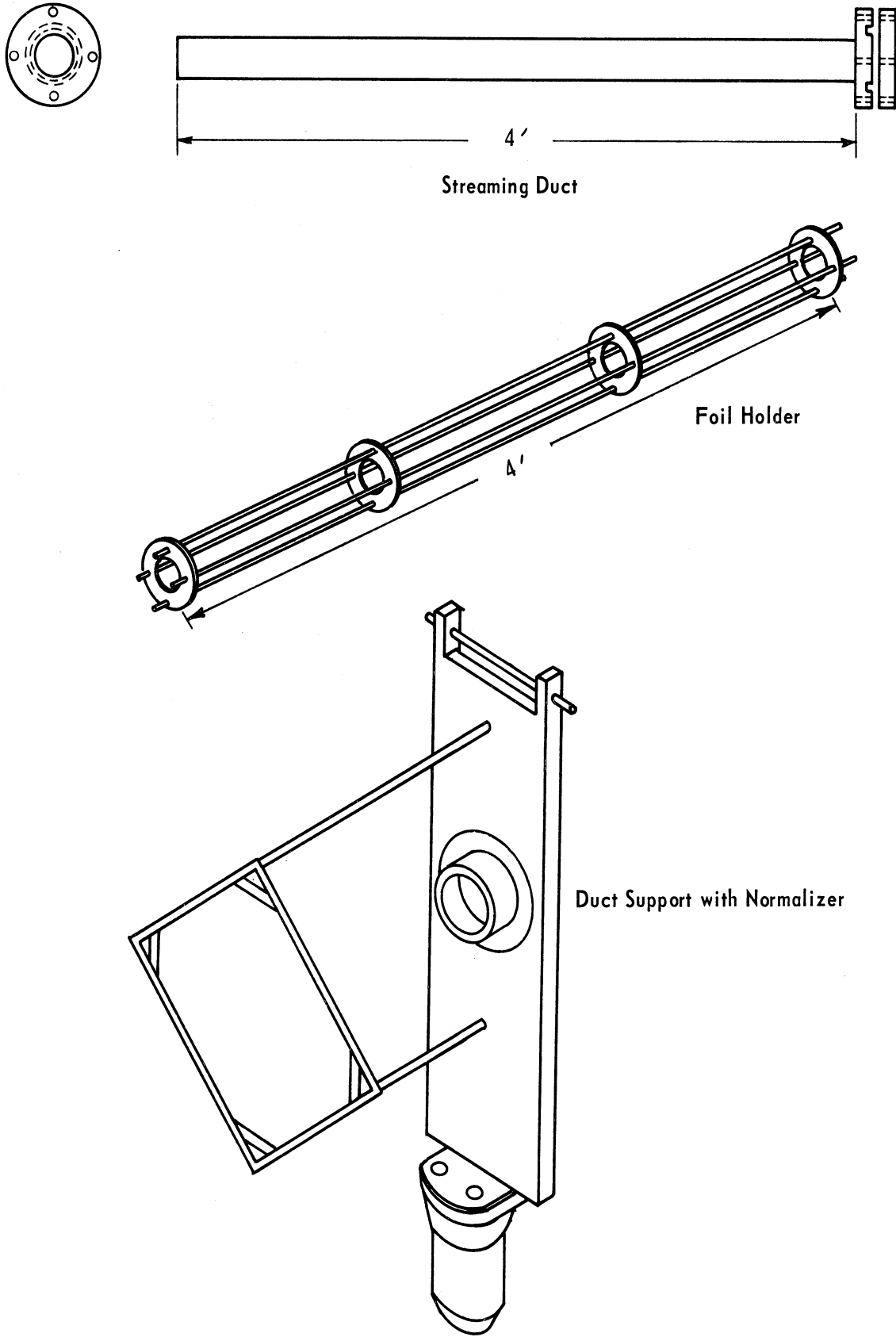
Figure 3.2. Streaming duct, foil holder,
and duct support with normalizer.

Figure 3.3 is a photograph of the 1-1/2-in. duct encased within the 4-in.

graphite insert positioned adjacent to the core.
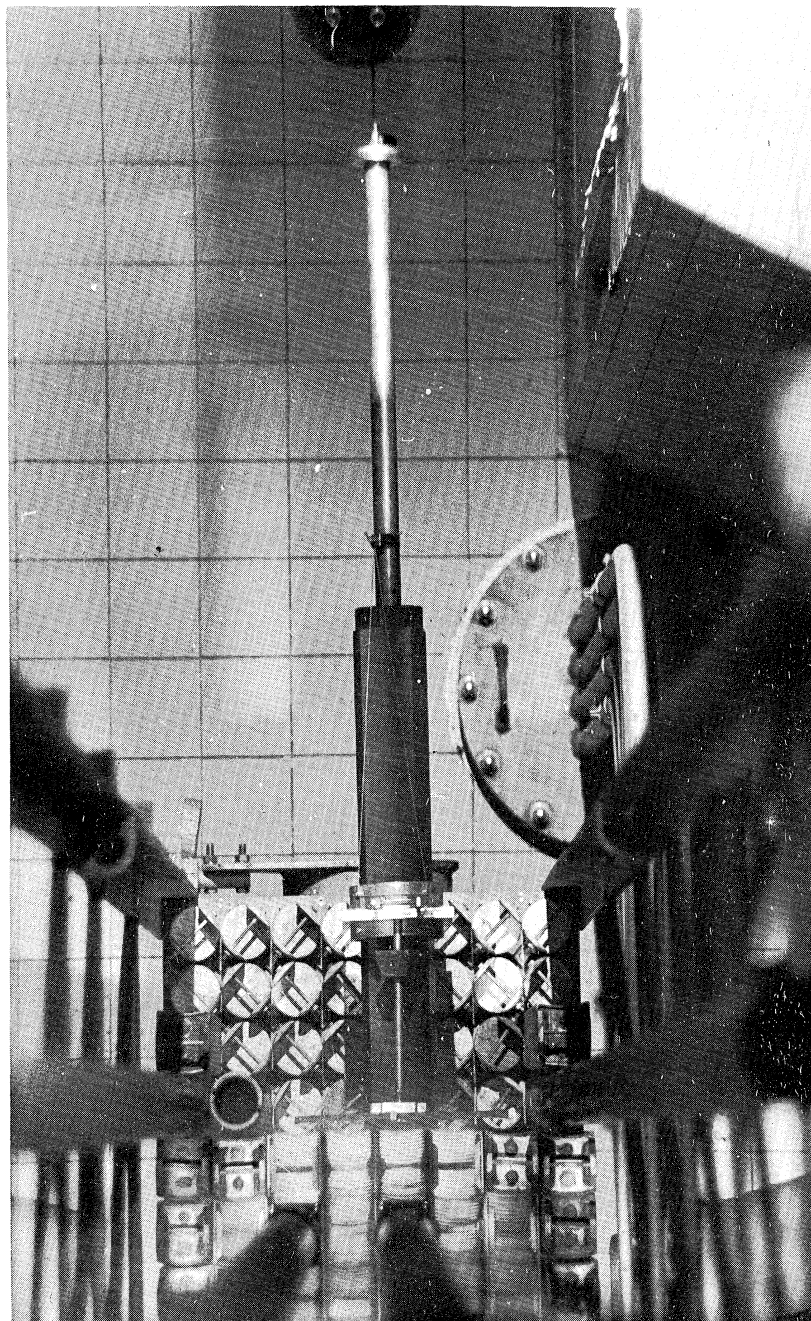
Figure 3.3. Photograph of 1-1/2-in. duct encased in
4-in. graphite insert.

Table III.1 is a list of the dimensions of the three aluminum ducts

employed in the water runs. Also tabulated are the reactivity worths of

the ducts positioned 4 in. below the centerline of the core in the col-

umn pictured in Figure 3.1. The ducts were evacuated to eliminate atten-

uation of neutrons by air and water vapor.

TABLE III.1

DUCT SPECIFICATIONS

O.D., in.	Wall Thick- ness, in.	Front Window Thick- ness, in.	Length, ft	Reactivity δk/k Axial Distance from Core		
				.76 cm	2.16 cm	6.16 cm
1-1/2	.0625	.020	4	...	.007%	...
3	.083	.040	4	...	.03%	...
5	.187	.062	4	.14%	.10%	<.01%

Runs were made at various low power levels in the range of 500 watts

to 70 kilowatts. The duration of each run was chosen so that duct in-

sertion and withdrawal occupied a time interval of less than 1-1/2 per-

cent of the total irradiation. Spacing of runs was limited by the induced

activity of the metal, which would have decayed in less than one-half

hour had the material been pure aluminum. The presence of alloying con-

stituents in the aluminum extended the cooling time considerably for high power activations.

C. Normalization

Because the experiments were performed at variable reactor power on different days, it was necessary to employ a precise and reliable procedure to normalize the runs to one another. First attempts at normalization using the pneumatic tubes on the west face of the core proved unsatisfactory. Although the activations in the pneumatic system agreed within ± 2 percent with a fixed monitor viewing the west face at the exit of a beam port, the proportionality between the flux intensity on the west face and at the duct position was destroyed by rod manipulations, xenon buildup, and core movements.

Normalization was finally accomplished with the aid of the pinwheel frame shown in Figure 3.2. Foils were mounted at the four corners of the diamond which were each 6-1/2 in. from the duct axis and 1/2 in. from the reactor core. That the duct itself exerted no influence on the normalizing foils was verified by comparing the normalization of consecutive 5 in. void and no-void runs with the fission chambers positioned as in Figure 3.1. The influence of poison rods was minimized by holding B shim rod 100 percent withdrawn and the control rod greater than 40 percent withdrawn during an irradiation. Furthermore, the duct axis was positioned 4 in. below the core midplane. That the spectrum was not changing

significantly for the purpose of normalization was confirmed by irradiat-

ing two cadmium-covered foils simultaneously with the bare normalizing

foils. Typical normalization data are displayed in Table III.2. Com-

parison of the root-mean-square deviation between the four positions set

a normalization error upper limit of approximately ± 3 percent. Repeated

runs were reproducible within these limits.

D. Detectors

Metallic foil activators were used for all of the measurements.

Gold, copper, manganese, and lutetium were each utilized to some extent.

Table III.3 is a list of the pertinent foil characteristics.

Gold was used for most of the measurements. It could be obtained

chemically pure and possesses a high activation cross section together

with a long half-life. In addition, the large 4.9 ev resonance cross

section permits epithermal activations at a well-defined energy. Bare

and cadmium-covered 1/16 in. x 1/16 in. x .001 in. foils were employed at

the duct mouth with no significant perturbation induced in larger foils

irradiated at the exit of the duct. The small foils at the mouth could

be counted in a well counter soon after irradiation at the flux levels

necessary to activate the foils at the exit. Weighing was accomplished

on an electrostatic balance.

Epithermal fluxes were measured by completely surrounding the foils

with 20 mils of cadmium. Incomplete cadmium enclosure was shown to lead

35

TABLE III.2

NORMALIZATION DATA
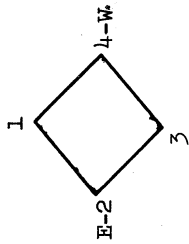
Run No.	Date	Duct Size, in.	Control Rod Position, %	Nominal Reactor Power Level*, kw	Mon. 1	Mon. 2 Bare	Mon. 2 Thermal	Mon. 3	Mon. 4 Bare	Mon. 4 Thermal	Avg.	Rms Deviation $\left\{\frac{1}{4}\sum_{i=1}^{4}(m_i-\bar{m})^2\right\}^{1/2}$, %
32	9/4	1-1/2	40	5	1.0	1.0	1.0	1.0	1.0	1.0	1.0	---
28	9/4	H_2O	42	1	.200	.204	.201	.210	.207	.211	.205	2.5
31	9/4	H_2O	40	0.5	.099	.0995	.097	.099	.099	.102	.99	1.5
39	9/12	1-1/2	58	70	10.75	10.9	11.05	11.2	10.6	10.9	11.0	1.5
40	9/12	1-1/2	40	70	8.35	8.25	8.13	8.32	8.40	8.15	8.12	2.0
44	9/14	5	80	35	4.15	3.96	3.81	3.90	4.09	4.19	4.01	3.5
61	12/4	5	70	10	1.75	1.69	1.64	1.82	1.71	1.71	1.73	3.5

*As recorded by the reactor instrumentation.

TABLE III.3

FOIL CHARACTERISTICS

Detector	Abundance, %	Thickness, in.	Princ. Activity Counted	Ref. 18 σ_{act}(.025 ev) b.	Ref. 19 $\int_{epi} \sigma_{act}(E) \frac{dE}{E}$ b.	Purity
Au	100	.001	.41 Mev γ	98	1558	Excellent
Mn	100	.002	.845 Mev γ	13.3	11.8	Poor Cu and Ni Impurities
Cu^{63}	69 Nat. Abund.	.005	.51 Mev Annih. γ from β^+	4.3	4.4	Excellent
Lu^{176}	2.6 Nat. Abund. in 2.5 Lu-Al Alloy	.010	.208 Mev γ	360 b. at .14 ev Reson.	---	Excellent

to large errors. Cadmium cutoff energies are derived by Dayton and
Pettus.[20] A 20 mil covering of cadmium around any of the foils listed
in Table III.3 affords a cutoff energy for an equivalent perfect filter
of .448 ev in an isotropic flux and .342 ev in a monodirectional flux.
The region between these two energies is estimated to contribute less
than 1/2 percent to the total activation in an assumed Maxwellian
spectrum fitted with a 1/E tail at .18 ev.

The foils were washed in acetone, weighed, and covered with a thin
layer of paper and tape. It was determined that the covering material
introduced no perturbation in the foil activation by carrying out several
trial irradiations with various thicknesses of paper and tape. Bare and
cadmium-covered foils were placed at least one inch apart at the mouth
of the ducts. The perturbation induced by the cadmium on the bare foil
at the mouth and on the one at the exit of the duct was estimated to be
well within the normalization error.[21]

E. Counting Equipment

Except for occasional utilization of the multichannel analyzer,
counting was done in a well counter equipped with a 3 in. x 3 in. NaI (Tl)
scintillation crystal backed with a Dumont No. 6292 photomultiplier tube.
A Radiation Instrument Development Laboratory Model 4951 scaler provided
the high voltage to the photomultiplier. The discriminator was set well
above the noise level and the high voltage was maintained in the center

of the counting plateau at 1275 volts. Reproducible positioning of foils
in the well was accomplished with the aid of a test tube device incorporat-
ing a centering rod.

Stability of the system over long intervals of time was important.
Figure 3.4 illustrates the measured decay of a gold foil over a 7-day
period. Subsequent count rates were kept below the maximum in the figure
to eliminate dead time corrections and to avoid jamming of the mechanical
register. An absolute activity determination on the multichannel analyzer
set an efficiency of 52 percent for the .41 Mev gold gamma rays.

F. Unperturbed Flux Distributions for the Water Reflector

The first experiment was done to verify that the current at the
exit of the evacuated 1-1/2 in. duct was far in excess of the unperturbed
flux in the vicinity of the duct exit. If this were the case, activations
measured at the duct exit could be attributed solely to streaming con-
tributions from the region close to the mouth of the duct. Thermal* and
epicadmium gold reaction rates were measured in the water bordering the
south face of the reactor core out to four feet. The measurements were
performed at the midplane of the core one row east of the position dis-
played in Figure 3.1. The relative reaction rates are plotted in Figure
3.5.

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*The thermal reaction rate is taken to mean the difference between bare
and cadmium-covered measurements.
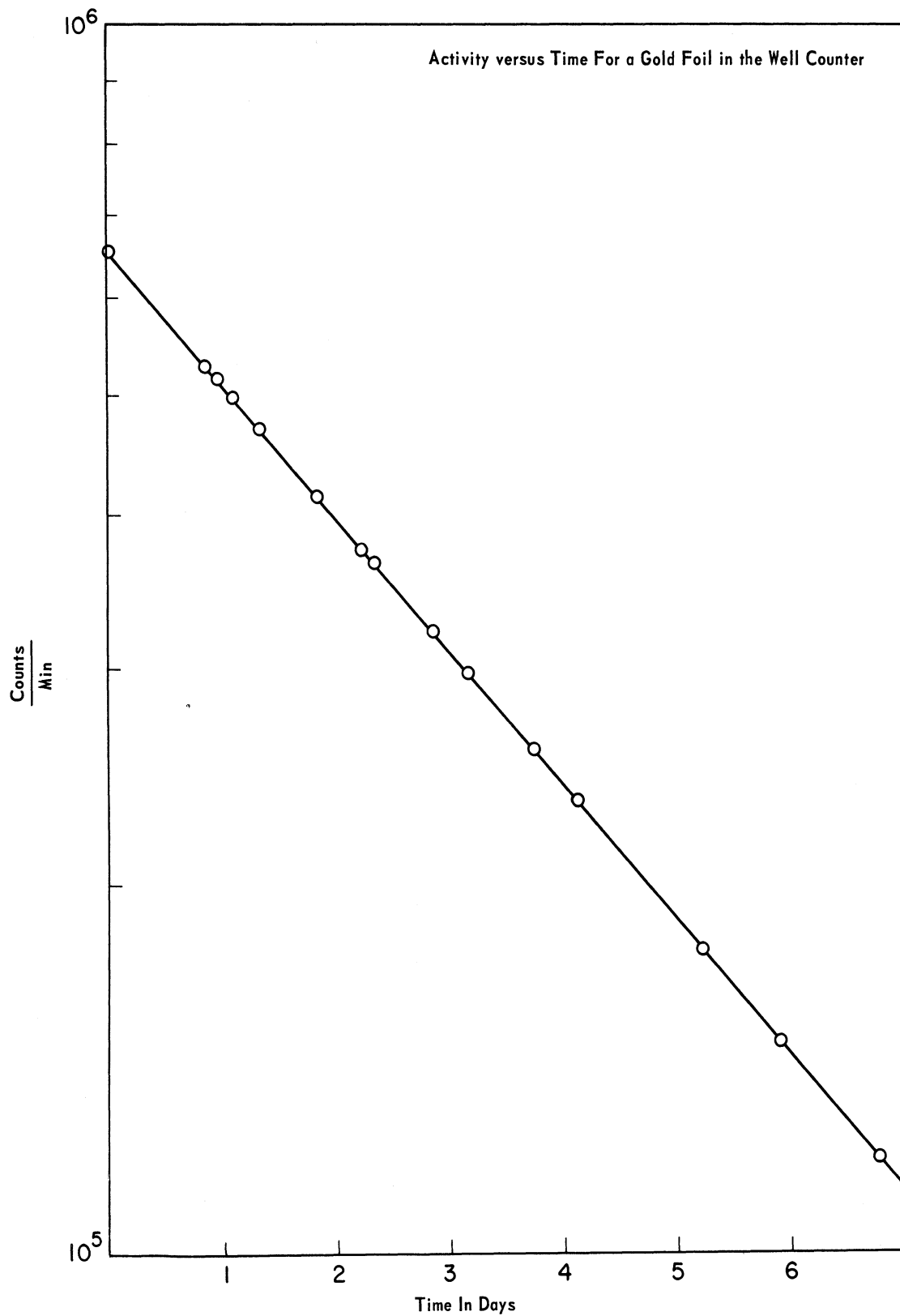
Figure 3.4. Decay of a gold foil counted in the well counter.
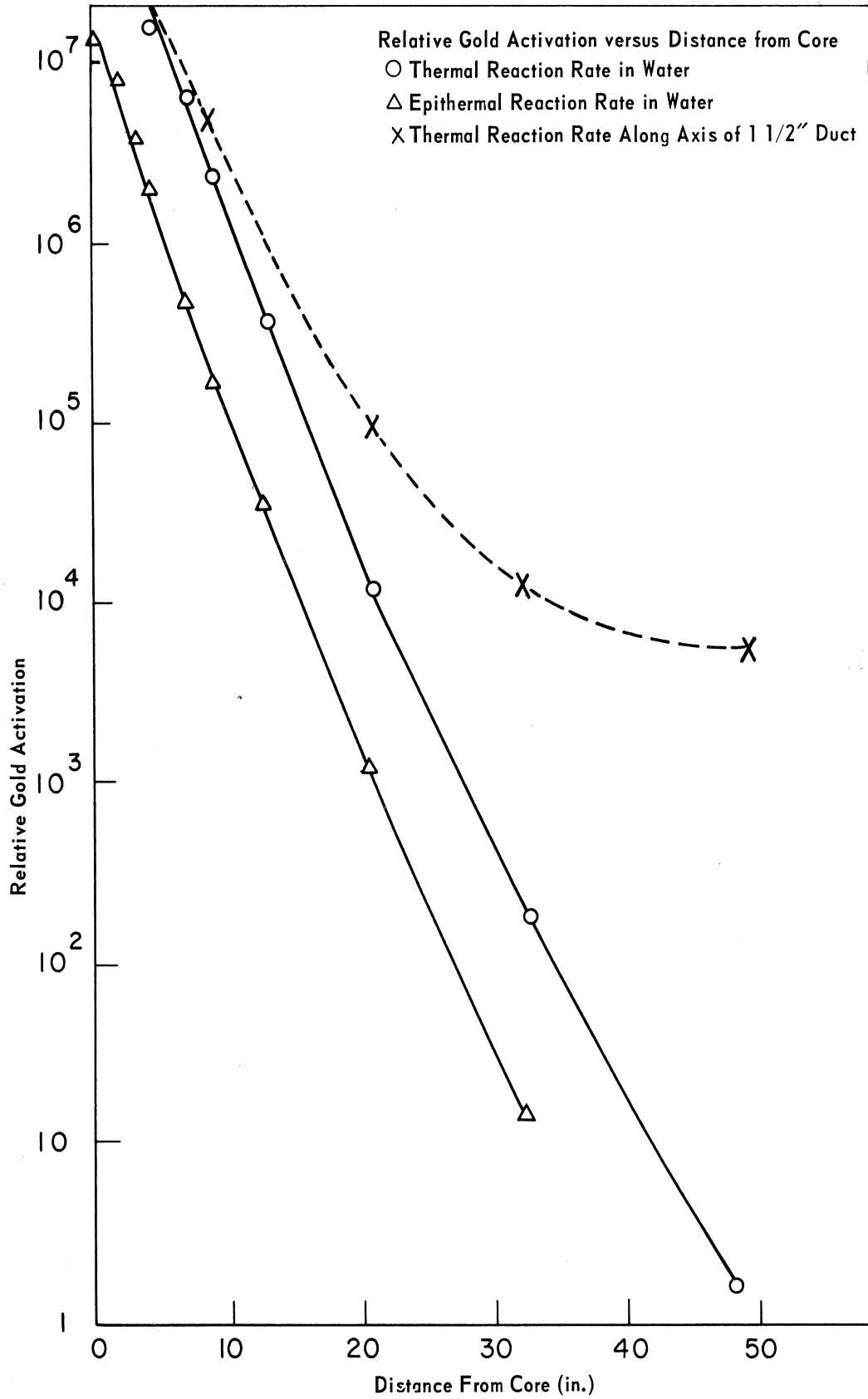
Figure 3.5. Thermal and epicadmium flux out to four feet in water reflector and along axis of 1-1/2-in. duct.

Thermal reaction rate data for foils placed within the 1-1/2 in. duct positioned 5/16 in. from the reactor core are shown in the same figure. The bare and cadmium-covered measurements were made separately to prevent severe mutual shielding. The results demonstrate that the streaming current in the duct exceeds the unperturbed flux in the medium by greater than two orders of magnitude 34 in. from the duct mouth.

Of peripheral interest is the flux distribution in the core. Foil insertions were made between the plates of the fuel using 10 mil aluminum venetian blind slats for positioning. The disadvantage factor in the fuel plates of the FNR is nearly unity,[22] and for this reason measurements in the channels between plates are representative of the homogenized medium. Since the gold cadmium ratio is low within the core, copper foils were used for this measurement. The data are shown in Figure 3.6. The flags indicate the uncertainty in position and normalization. The results for groups three and four of a four-group, one-dimensional machine calculation are normalized to the data.

The flux peak in the two off-center fuel elements can be explained by the close proximity to the water channels serving the poison rods, which were nearly 100 percent withdrawn during the course of the experiment. Since access to the partial element was limited, the peaking effect in the vicinity of a water channel is demonstrated in Figure 3.7 by an east-west flux distribution in the element bordering the east side of the control rod element (see Figure 3.1).
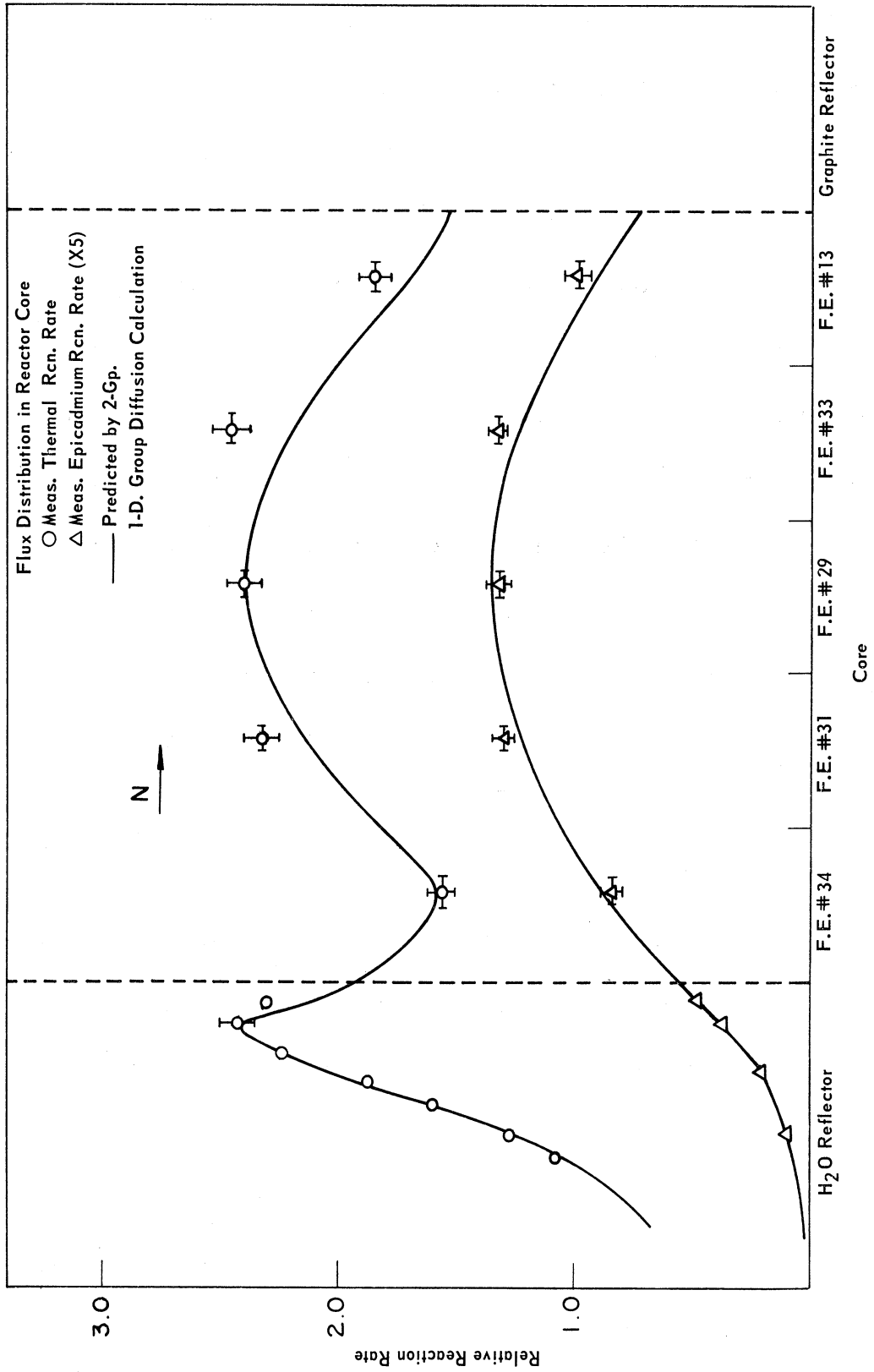
Figure 3.6. Thermal and epicadmium flux in the core and the water reflector.
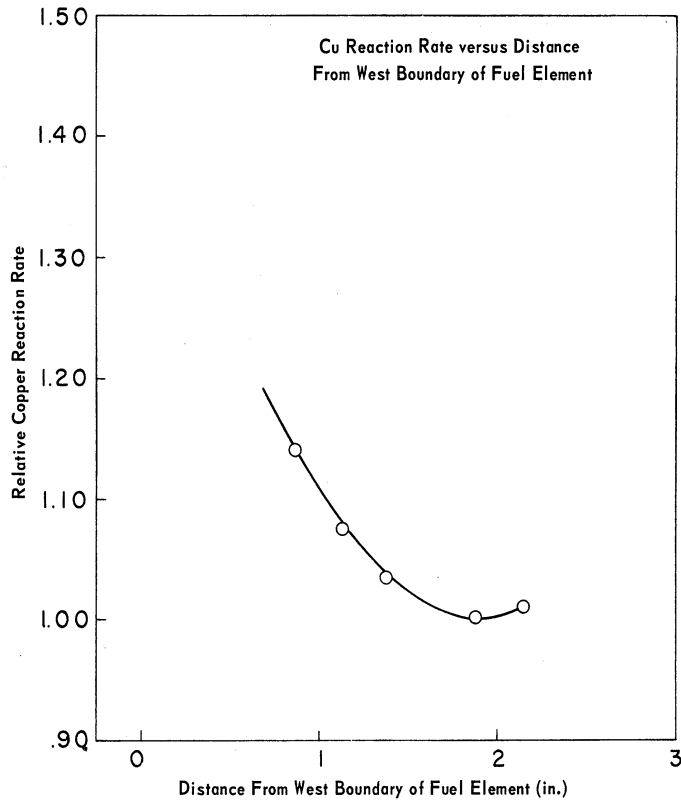
Figure 3.7. East-west traverse of thermal flux in the fuel element bordering on the east of the control rod element.

Most of the streaming measurements in water were performed with the duct axis positioned 4 in. below the core midplane in the column illustrated in Figure 3.1. This position was chosen to minimize the perturbing influence of the control rod. The unperturbed duct axis flux distribution at this position is plotted in Figure 3.8 out to 22 cm from the core. The data are normalized at 2.16 cm to the results of a two-group, two-dimensional group-diffusion code.

The data in Figure 3.8, as well as most of the subsequent results, are plotted on the basis of detector reaction rates. This presents the
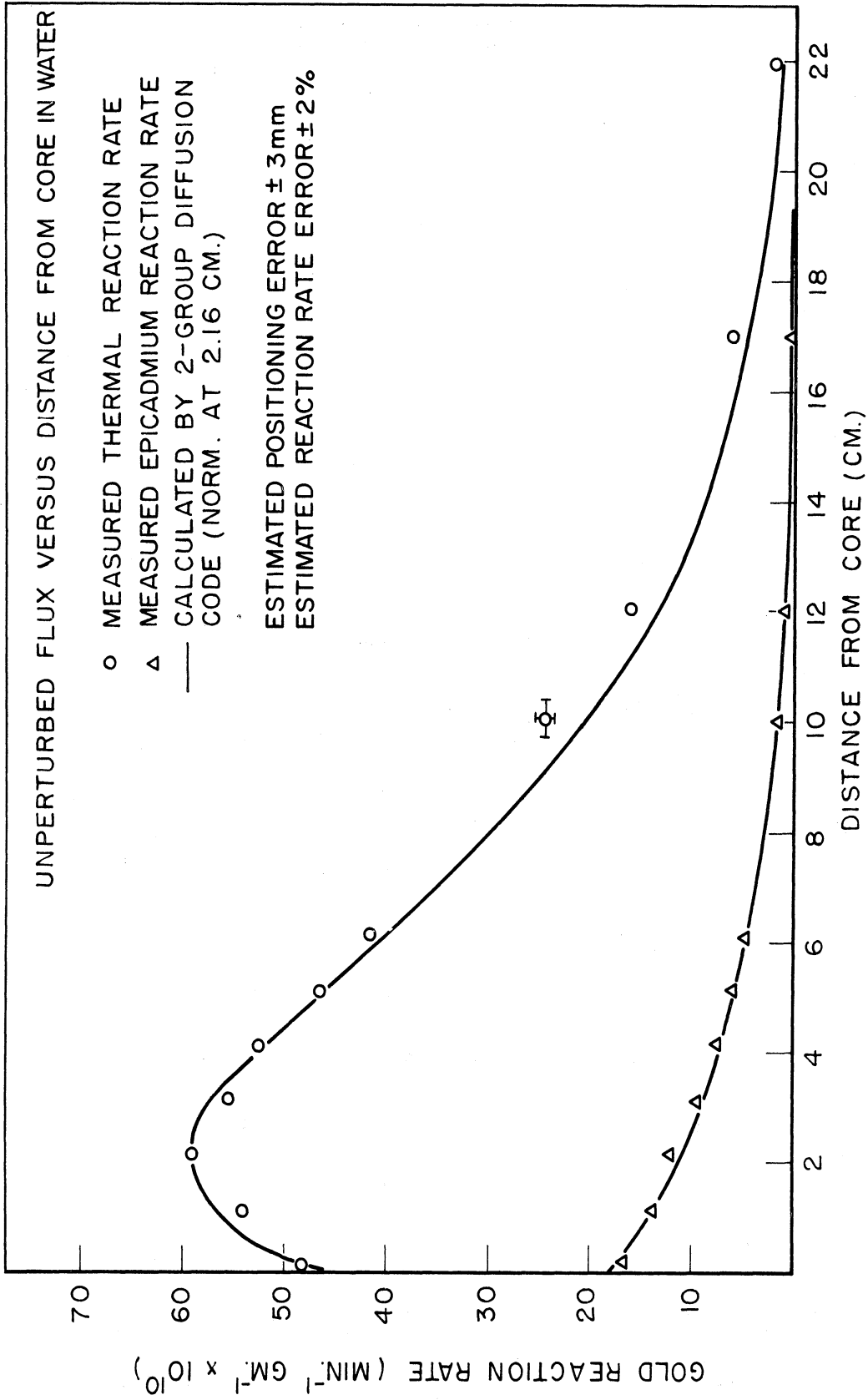
Figure 3.8. Unperturbed thermal and epicadmium flux in water reflector.

data in its raw form, yet it is a convenient form for interpretation in the present study. The normalization technique discussed earlier permits direct comparison of the results of measurements employing identical detectors.

Corrections for flux perturbations induced by the foils must be made before the ratios of epithermal to thermal fluxes can be evaluated. From the results of Dalton and Osborn[23] we estimate a low correction to the thermal flux measured by small 1 mil gold foils in water:

$$\frac{\phi^{th}}{\phi^{th}}_{meas.} = 1.04 \ .$$ (3.1)

The self-shielding induced by the 4.9 ev gold resonance brings about a considerably larger correction to the epicadmium reaction rate:[24]

$$\frac{\left\{\int_{epi} \sigma_a(E) \frac{dE}{E}\right\}}{\left\{\int_{epi} \sigma_a(E) \frac{dE}{E}\right\}_{meas.}} = 2.2 \ ,$$ (3.2)

for 1 mil gold foils.

Applying these corrections, a comparison can be drawn with the resulting ratios of epithermal to thermal flux predicted by the four-group code. Assuming a 1/E epicadmium spectrum and adopting the epithermal absorption integral in Table III.3 we get:

$$\frac{\phi^3}{\phi^4} = \frac{\int_{.625\ ev}^{5.53\ Kev} \phi(E)dE}{\int_{0}^{.625\ ev} \phi(E)dE} = .205, \qquad (3.3)$$

at 2.16 cm in the water reflector (Figure 3.8). The four-group code predicts that ϕ^3/ϕ^4 = .195, a 5 percent deviation from the measured ratio.

G. Preliminary Duct Experiments

With the evacuated 1-1/2-in. duct positioned at various distances

from the core, bare and epicadmium reaction rates were measured at the

duct exit. The data are presented in Figure 3.9. The thermal data are

compared to predictions for the angular flux, obtained by using the un-

perturbed flux distribution in Eq. 2.41 to compute the boundary condition,

a group-diffusion calculation to obtain the perturbed flux distribution

at the duct mouth, and Eq. 2.30 for the angular flux. It is noteworthy

that the peak occurs at a duct position closer to the core than the po-

sition of the unperturbed flux peak in the medium.

The same measurement was performed in a 4 in. x 4 in. x 28 in. graph-

ite insert bored with a 1-1/2-in. axial hole to accommodate the duct.

The insert was positioned flush against the reactor core as shown in

Figure 3.3, and the duct was moved in steps of 1 in. by introducing graph-

ite plugs into the hole in front of the duct. The thermal flux data in

the solid insert as well as the streaming data at the duct exit are plotted

in Figure 3.10. The dotted lines represent the previous water data for com-

parison.
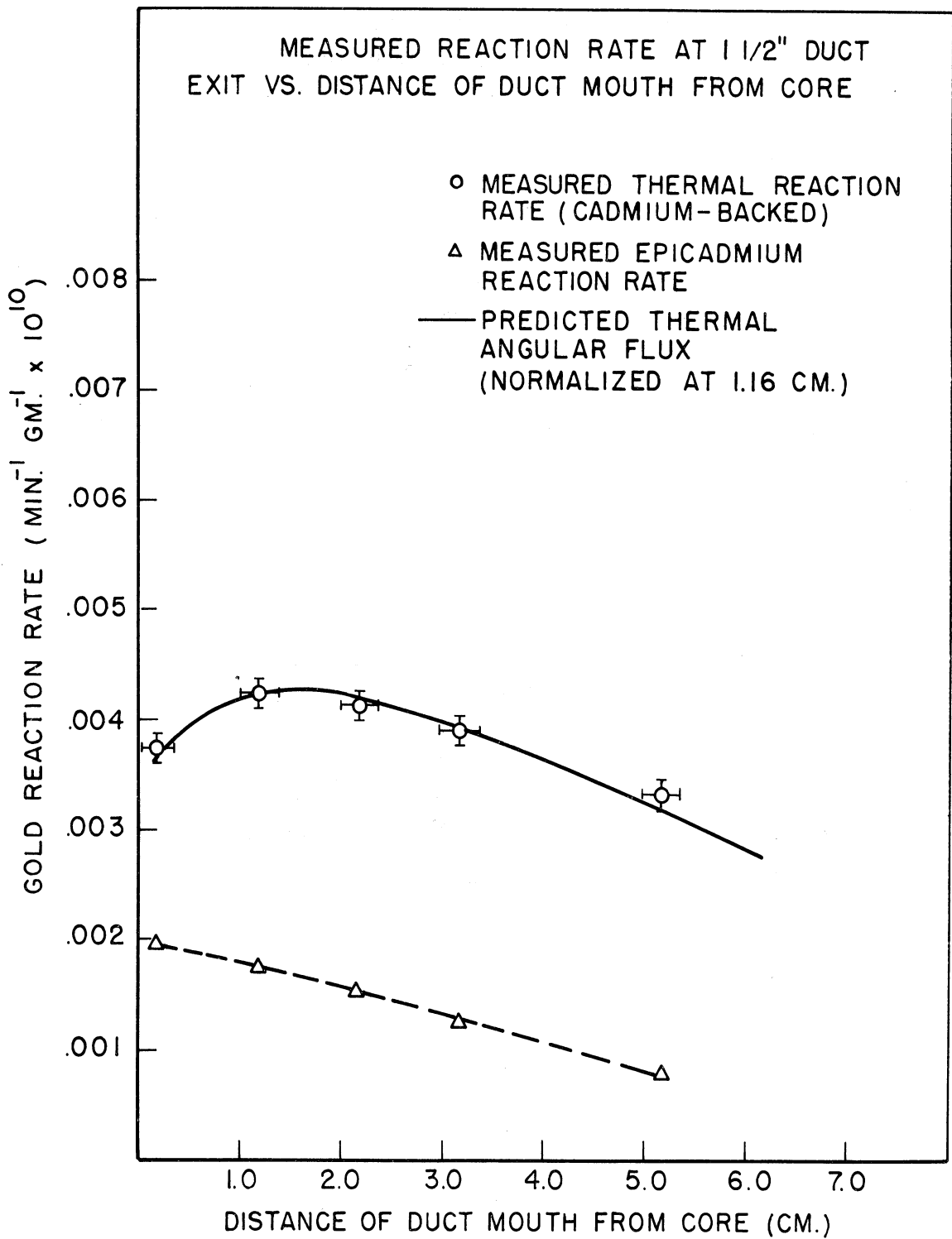
Figure 3.9. Reaction rate at exit of 1-1/2-in. duct in water.
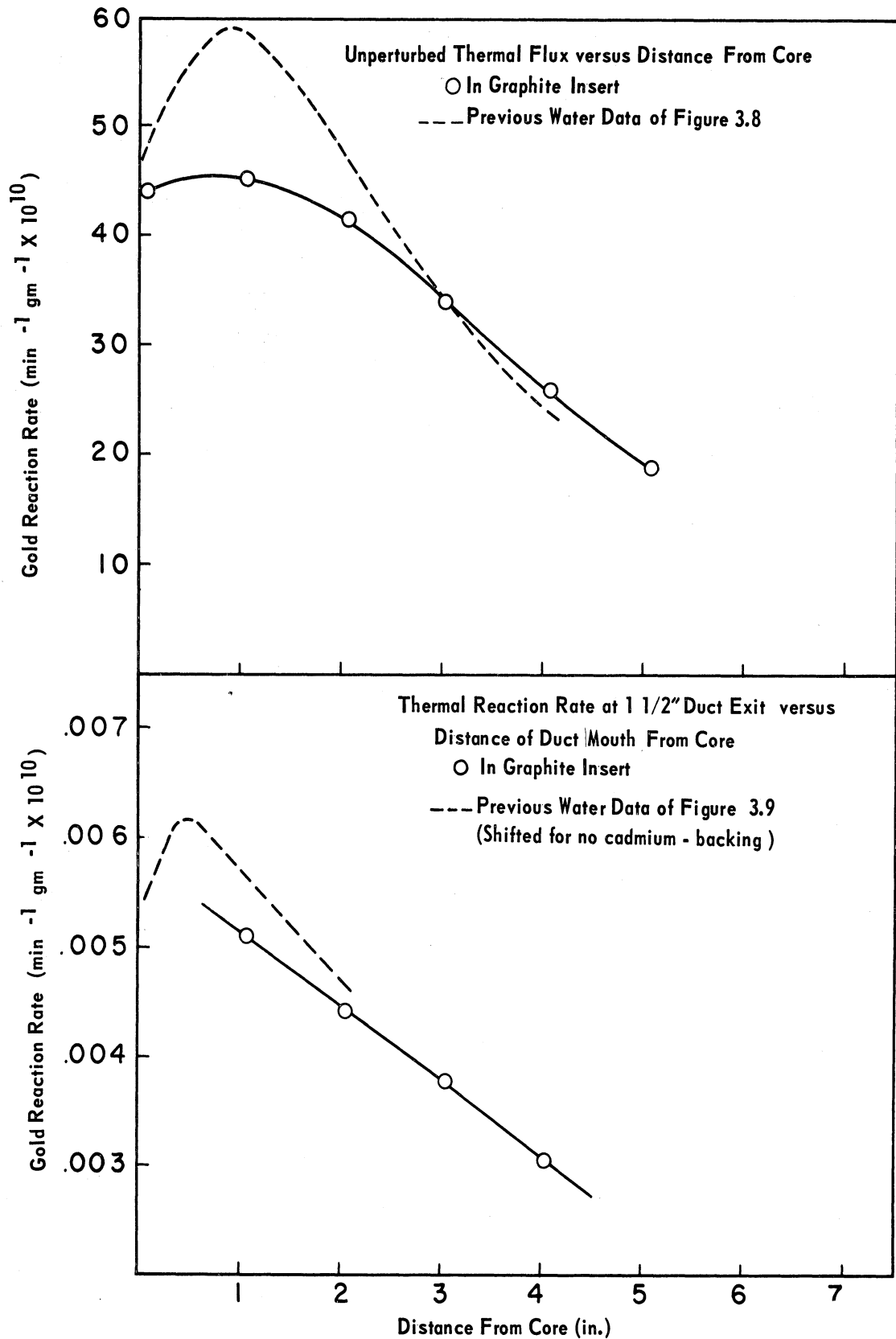
Figure 3.10. Thermal reaction rate in 4-in. graphite insert and at the exit of 1-1/2-in. duct encased in 4-in. graphite insert.

To compare the data with predictions for the actual magnitude of $j^+_{B_{th}}$ at the duct exit it was necessary to measure the leakage contributions from the duct walls and also the effect of activation from back-scattering at the exit. Measurements with the duct positioned at 2.16 cm in water resulted in the compilation of Table III.4. A 23 percent back-

TABLE III.4

PREDICTION OF ACTIVATION AT EXIT OF 1-1/2 INCH DUCT IN WATER

Exit Measurement

Bare reaction rate	$.00691 \times 10^{10}$	$gm^{-1}\text{-}min^{-1}$
Cd.-backed reaction rate	$.00531 \times 10^{10}$	"
Cd.-covered reaction rate	$.00142 \times 10^{10}$	"
Thermal monodirectional reaction rate	$.00389 \times 10^{10}$	"
Reaction rate from walls of duct	$.00082 \times 10^{10}$	"
Net thermal monodirectional reaction rate from mouth of duct	$.00307 \times 10^{10}$	"
Net thermal monodirectional reaction rate from mouth of duct $j^+_{B_{th}}$ (corr. for cntg. 1/4 in. foil)	$.00322 \times 10^{10}$	"

Mouth Prediction

Direct streaming from .6875 in. of radius of duct	$.00286 \times 10^{10}$	"
Transmission through wall from last .0625 in. of radius	$.00035 \times 10^{10}$	"
Net direct streaming prediction from mouth $j^+_{B_{th}}$	$.00321 \times 10^{10}$	"

scattering contribution was revealed by backing the bare foils with cadmium. A wall contribution of 21 percent was measured by covering the mouth of the duct with cadmium. A 5 percent correction originated from

Figure 3.13. Collimator design.

neutron detector at the exit to view .905 sq. in. of the mouth regardless of the duct diameter.

The high epithermal content of the spectrum at the collimator exit necessitated employment of a detector possessing a relatively low resonance integral. Manganese combined this attribute with a high activation rate.* The bare and cadmium-covered foils were irradiated simultaneously by offsetting the covered foils by 3/8 in. from the centerline. A copper impurity in the manganese required that the foils be counted in the single channel well counter within one manganese half-life.

The results of the collimator measurements are plotted in Figure 3.14.

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*The reactivity perturbations induced by the introduction of the larger ducts caused power fluctuations in the reactor system. Since the measuring foils and normalizing foils possessed different decay constants, the transient was not automatically corrected for by the normalizing foils. It was necessary to evaluate the normalization by the analysis presented in Appendix F.

Figure 3.11. Perturbed radial flux distributions at duct mouths.
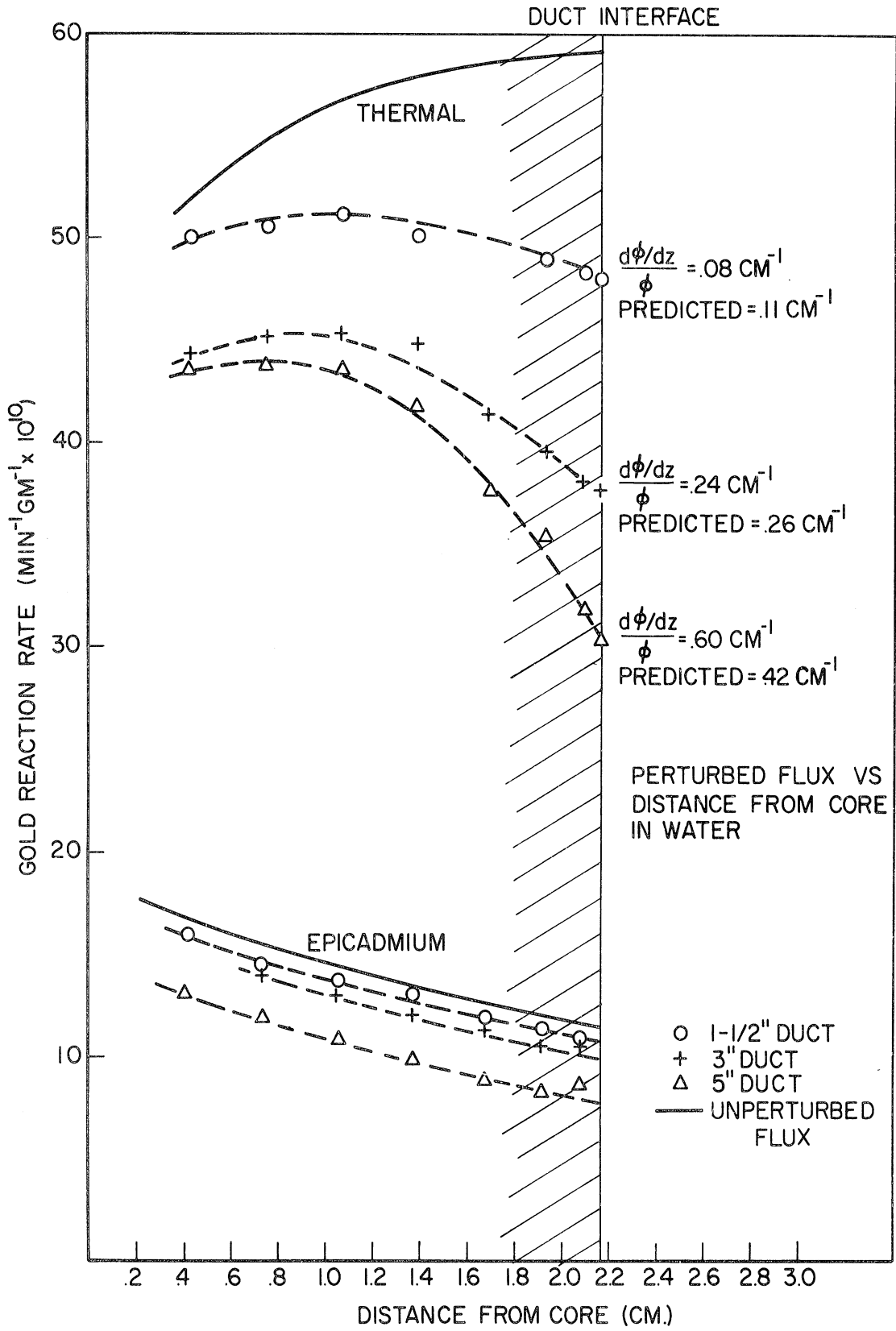
Figure 3.12. Perturbed axial flux distributions at duct mouths.

Figure 3.13. Collimator design.

neutron detector at the exit to view .905 sq. in. of the mouth regard-

less of the duct diameter.

The high epithermal content of the spectrum at the collimator exit

necessitated employment of a detector possessing a relatively low resonance

integral. Manganese combined this attribute with a high activation rate.*

The bare and cadmium-covered foils were irradiated simultaneously by off-

setting the covered foils by 3/8 in. from the centerline. A copper im-

purity in the manganese required that the foils be counted in the single

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ing foils and normalizing foils possessed different decay constants,
the transient was not automatically corrected for by the normalizing
foils. It was necessary to evaluate the normalization by the analysis
presented in Appendix F.

Figure 3.14. j^+_{Bth} at collimator exit.

The solid line presents the prediction for $j_{B_{th}}^+$. The data and the pre-dictions for $j_{B_{th}}^+$ are normalized to a value of unity for no void.

3. Thermal Flux Attenuation Within the Empty Ducts

The behavior of the flux within the empty duct is related to the main topic of this study, and a series of experiments was performed to examine the attenuation of the forward component of the thermal flux. Bare and cadmium-covered measurements were performed separately. The detectors consisted of 1/16 in. square 1 mil gold foils. The bare foils were backed with cadmium, which posed the formidable problem of mutual shielding. In order to avoid the laboriousness of separate runs for each foil position, the five foils were positioned to avoid shadowing by spacing them 60° apart. Investigations within the 5 in. duct demonstrated that the radial flux was reasonably flat near the exit of the 5 in. duct (Figure 3.15).

The thermal reaction rates within the 1-1/2 in. and 5 in. ducts as a function of axial distance from the mouth are plotted in Figure 3.16. The measured slopes at 44 in. are recorded in the figure.

I. Thermal Spectrum

A question of some concern was the influence of the void on the thermal spectrum, and consequently the spectral effect on the apparent perturbations measured. Results published by Walton et al.[25] indicate no
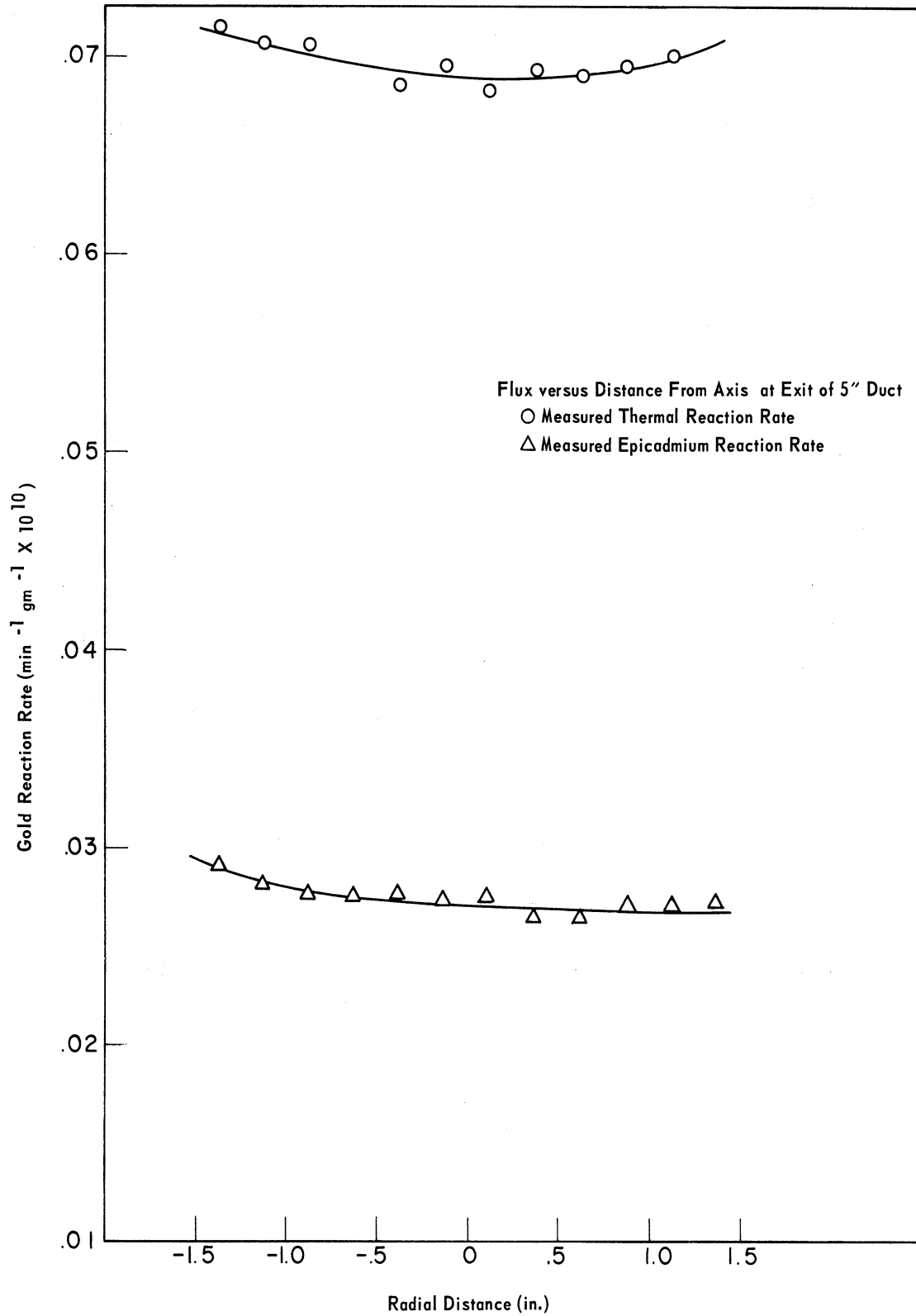
Figure 3.15. Radial variation of flux at 5-in. duct exit.
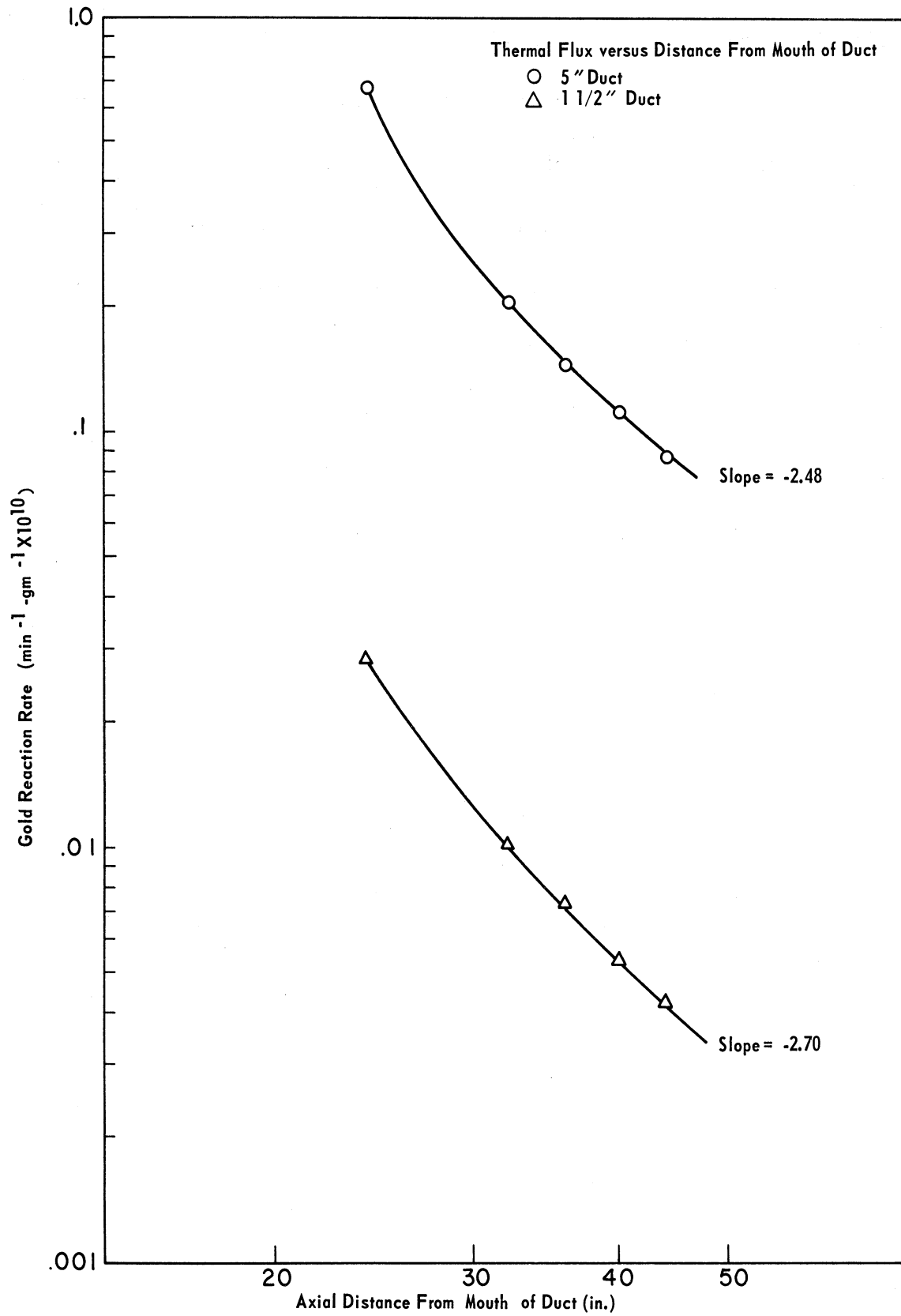
Figure 3.16. Thermal flux attenuation within 1-1/2- and 5-in. ducts.

hole effect on the spectrum in polyethylene with voids up to 2 in. in diameter. The effect of the 5 in. duct on the thermal neutron temperature at the mouth was measured using the .14 ev resonance in the Lu^{176} absorption cross section as a spectral index. Bare and cadmium-covered 10 mil, 2-1/2 percent Lu-Al foils were irradiated simultaneously with manganese* at the mouth of the voided 5 in. duct and at the same positions with the duct filled with water. The activities were measured on a multichannel analyzer corrected for drift with a Co^{59} standard. The resultant activities are listed in Table III.5.

TABLE III.5

NEUTRON TEMPERATURE MEASUREMENT

H_2O Medium

Bare Lu^{176}	21.82×10^5 $gm^{-1}-min^{-1}$
Cd.-covered Lu^{176}	...
Bare Mn	11250×10^5 "
Cd.-covered Mn	356×10^5 "
$A^{th}(Lu^{176})/A^{th}(Mn)$	.00200

5 in. Void

Bare Lu^{176}	10.99×10^5 $gm^{-1}-min^{-1}$
Cd.-covered Lu^{176}	...
Bare Mn	5561×10^5 "
Cd.-covered Mn	276×10^5 "
$A^{th}(Lu^{176})/A^{th}(Mn)$	.00208

---

*Manganese was used as the 1/v detector because Lu^{175} possesses a large resonance integral, giving a cadmium ratio of only 1.59 for the voided duct.

The ratio of the reaction rate of Lu^{176} to that of $1/v$ detector is

calculated using the form of the resonance given by Schmid and Stinson:[26]

$$\sigma(E)\sqrt{E} = \frac{\text{Const.}}{1 + 1108(E-.142)^2} \quad . \qquad (3.4)$$

The calculated ratio, normalized to unity at $0°C$, is plotted up to $100°C$

in Figure 3.17. In this range of temperatures, the observed ratios sug-

gest a temperature rise at the mouth of the voided duct of $10° \pm 5°C$. A

$10°C$ temperature rise at $300°K$ would account for a 1.7 percent depression

in the average activation cross section of a $1/v$ detector.

J. Measurements in Complete Graphite Reflector

Graphite was chosen as a second medium in which to examine duct

effects. The selection of graphite was based upon diffusing properties

widely different* from those of light water, as well as accessibility at

the FNR. The south face of the core illustrated in Figure 3.1 was covered

with three rows of 3 in. x 3 in. graphite reflector elements, except for

two columns at the center, which were left open to accommodate the solid

graphite (reactor grade) insert sketched in Figure 3.18. The axis of the

5 in. hole bored through the graphite was along the centerline of the

reactor core. Graphite plugs were designed to accommodate 1-1/2 and 3 in.

aluminum ducts.

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*The thermal diffusion coefficient for light water is .155 cm and for pure
graphite .915 cm.
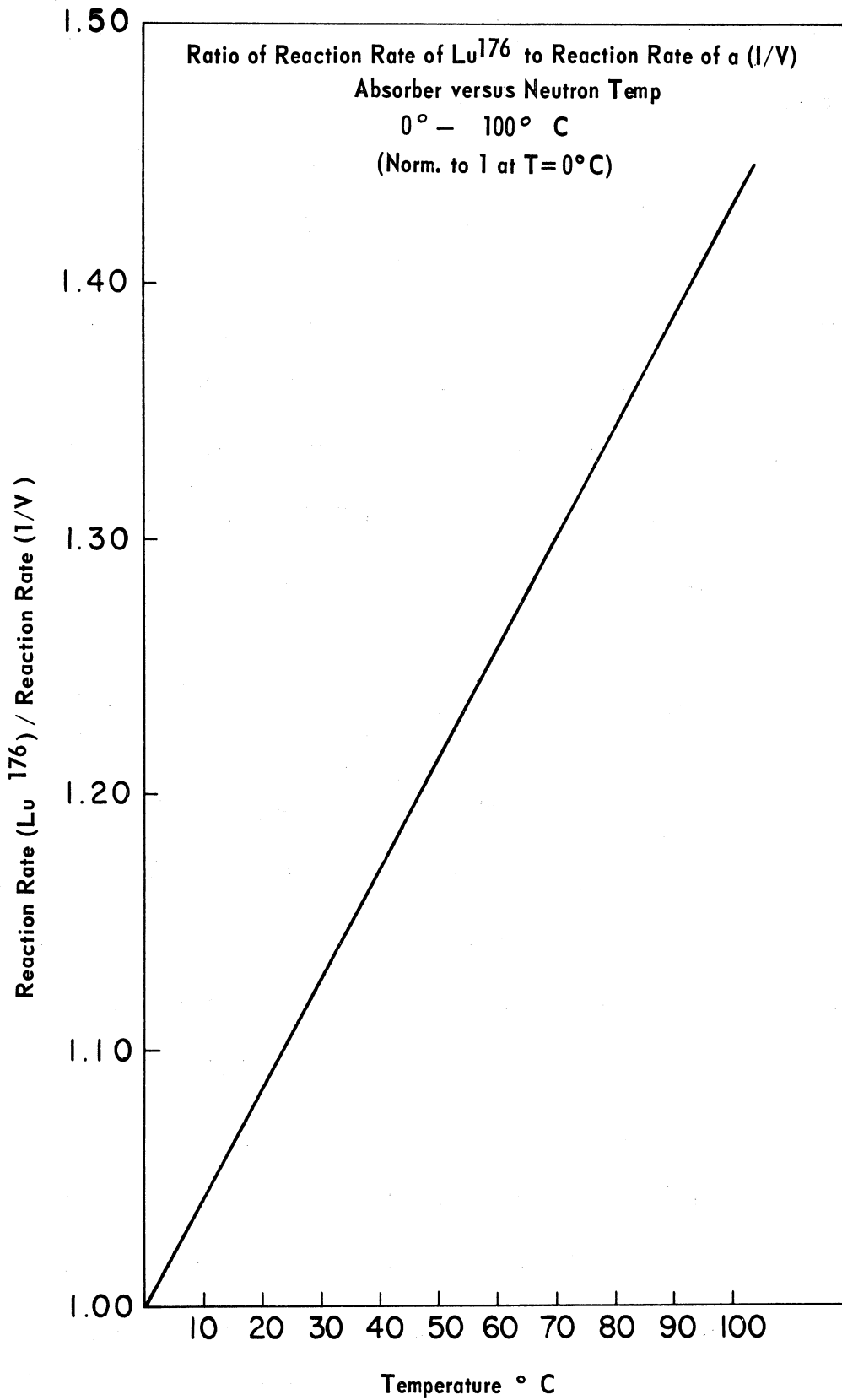
Figure 3.17. Ratio of reaction rate of Lu^{176} to reaction rate of 1/v absorber.
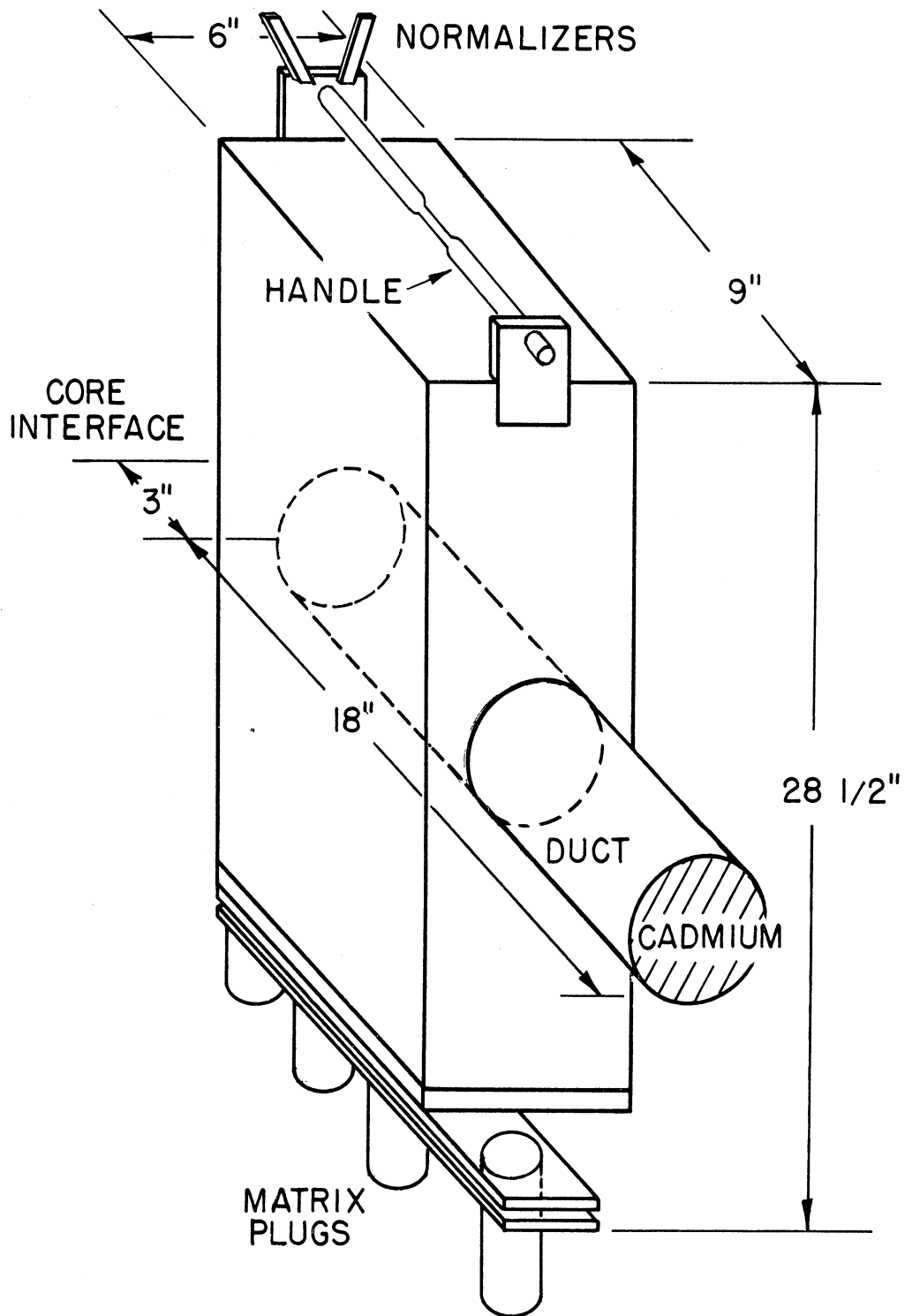
Figure 3.18. Graphite insert.

Measurements were made with the insert positioned against the south face of the reactor core. The side clearance was such as to allow no more than a .050 in. water gap between the insert and the columns of reflector elements. Time limitations allowed only scalar fluxes at the duct mouth to be measured. For this reason, the duct lengths were limited to 1-1/2 ft., which is effectively a void of infinite length from the standpoint of the perturbation at the mouth of the largest duct. The ducts were evacuated and positioned 3 in. from the core-graphite interface.

It was anticipated that water absorption might be a serious problem. To avoid the consequences of water penetration the graphite insert was coated with a thin layer of an Epoxy resin. It was verified that this was an effective sealer by soaking coated and uncoated samples of graphite in water and weighing shortly after removal.

Since the insert represented a large amount of reactivity, the reactor was brought critical with the insert in place on the south face of the core. The normalization was accomplished with four foils extending into the water from lucite brackets. Comparing the normalization results with counts from the fission chambers revealed that normalization was not influenced by the presence of the ducts.

Inasmuch as the thermal neutron mean free path is long in graphite, simultaneous bare and cadmium-covered measurements were avoided. A cadmium ratio of 25 in the graphite permitted the use of bare foil measure-

ments exclusively. If the epicadmium flux had been perturbed by the

5 in. duct to the same extent as the thermal flux, such a procedure could

lead to an error of no more than 1-1/2 percent.

Figure 3.19 presents the results of unperturbed flux measurements

in the solid graphite insert. The results are normalized at 7.62 cm to

the distributions obtained by a two-group, one-dimensional diffusion

calculation. A radial traverse is shown in Figure 3.20.

The flux measurements at the duct mouth were made at two positions

3/8 in. from the centerline, and agreed within 1 percent. The data are

plotted in Figure 3.21. The solid line is the predicted scalar flux at

the mouth obtained by using the measured unperturbed flux distribution

to compute the boundary condition. Figure 3.22 displays the axial dis-

tribution at the mouth of the 5 in. duct and compares the data with the

diffusion theory result.

GRAPHITE | H₂O

UNPERTURBED FLUX
VERSUS DISTANCE
FROM CORE IN
GRAPHITE

o MEASURED THERMAL
  REACTION RATE

△ MEASURED
  EPICADMIUM
  REACTION RATE (XIO)

—CALCULATED BY
  2-GP. DIFFUSION
  CODE (NORM. AT
  7.62 CM.)

ESTIMATED PO-
SITIONING ERROR
±2mm

ESTIMATED REAC-
TION RATE ERROR
±2%

COPPER REACTION RATE (MIN.$^{-1}$ CM^{-1} x 10^6)

DISTANCE FROM CORE (CM.)
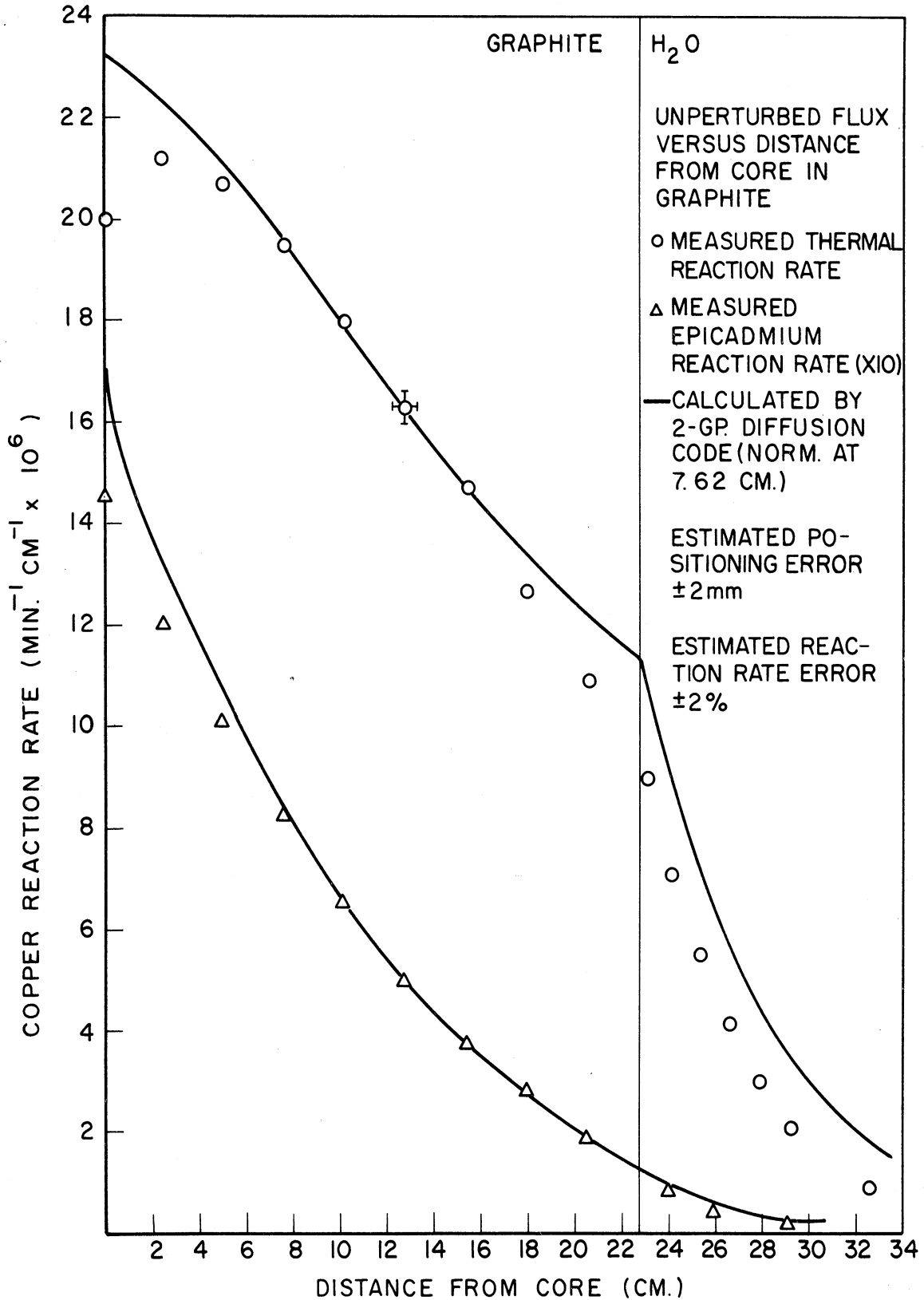
Figure 3.19. Unperturbed thermal and epicadmium flux in graphite insert.
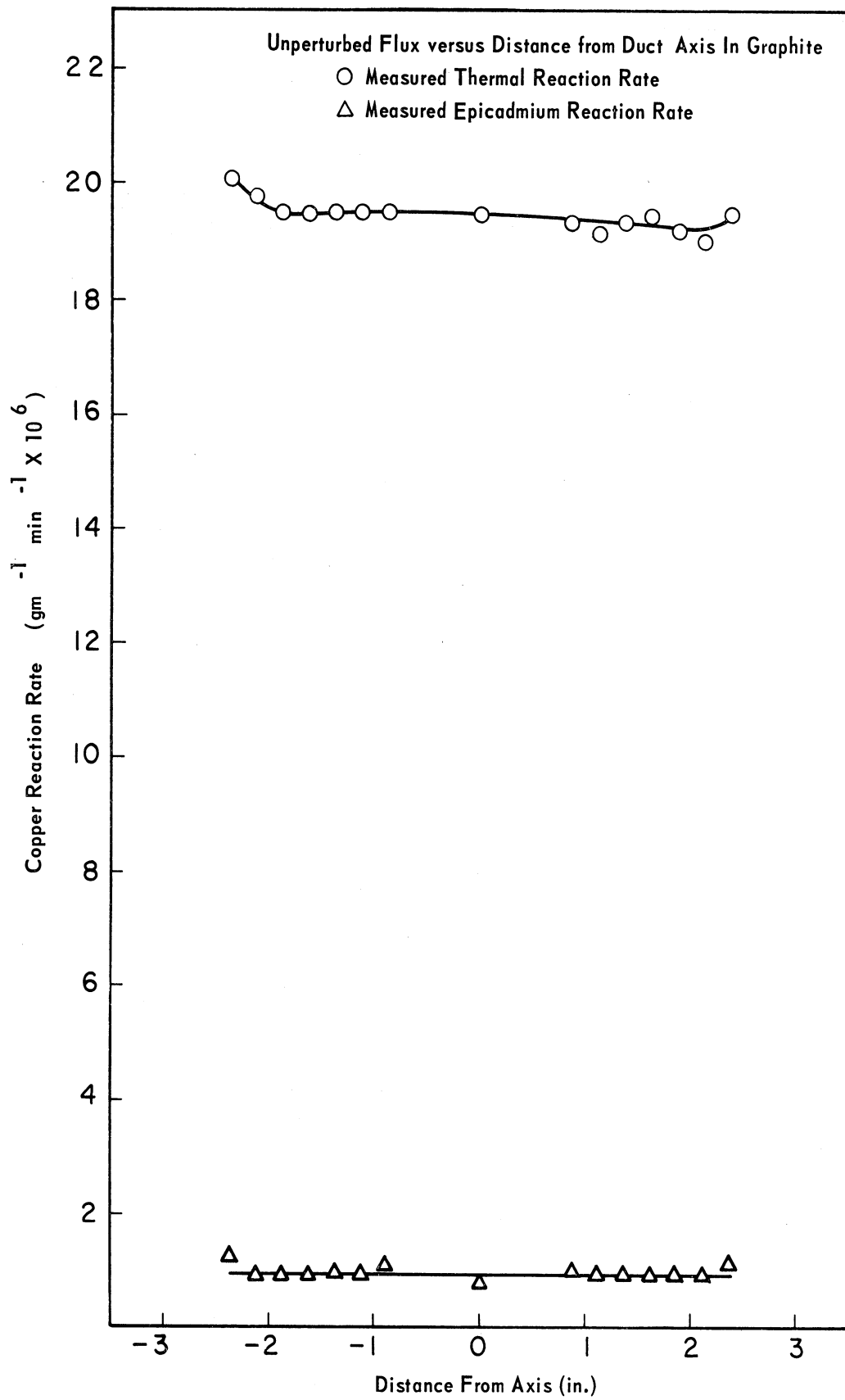
Figure 3.20. Radial flux distribution in graphite insert.
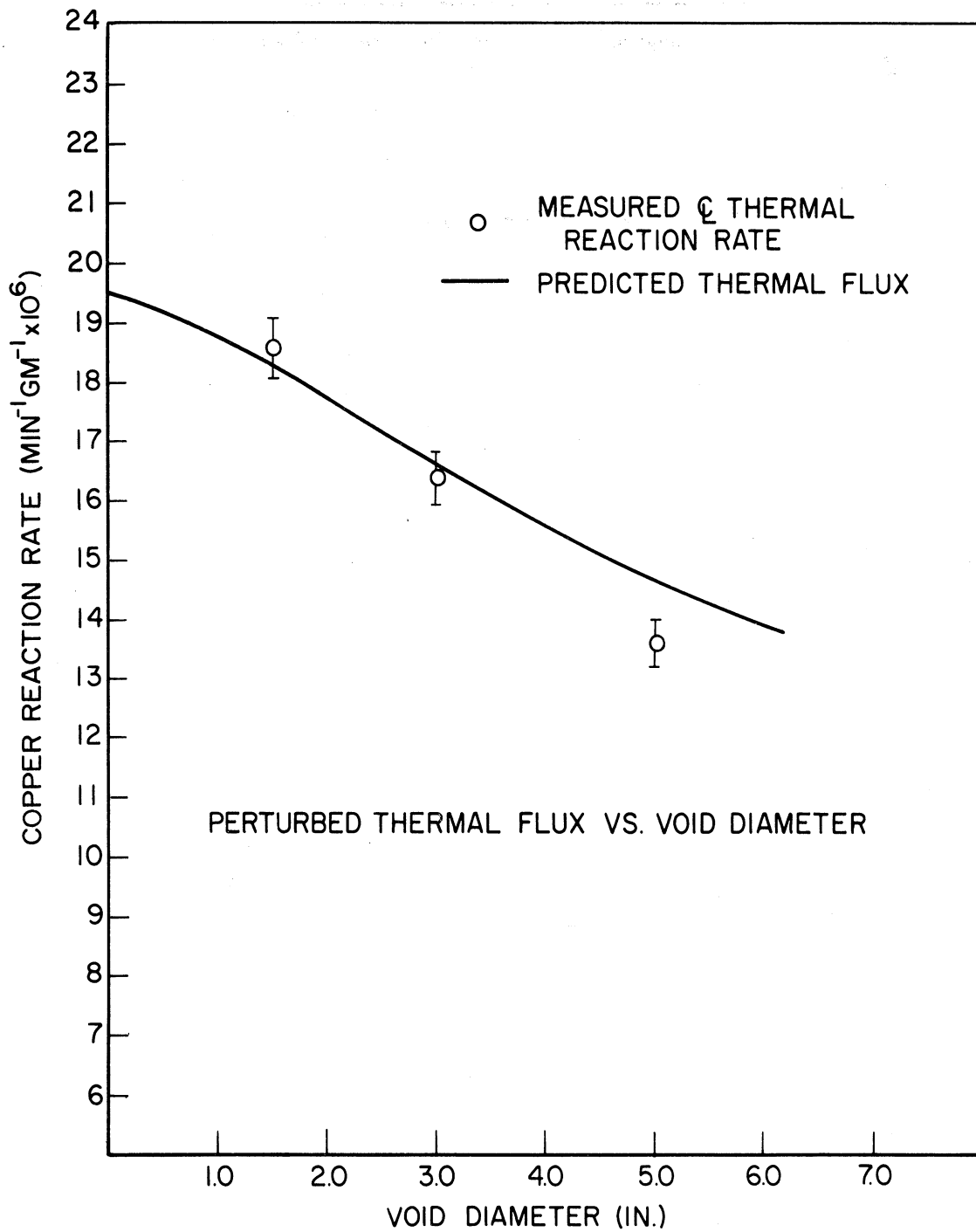
Figure 3.21. Perturbed thermal flux at center of duct mouths in graphite.
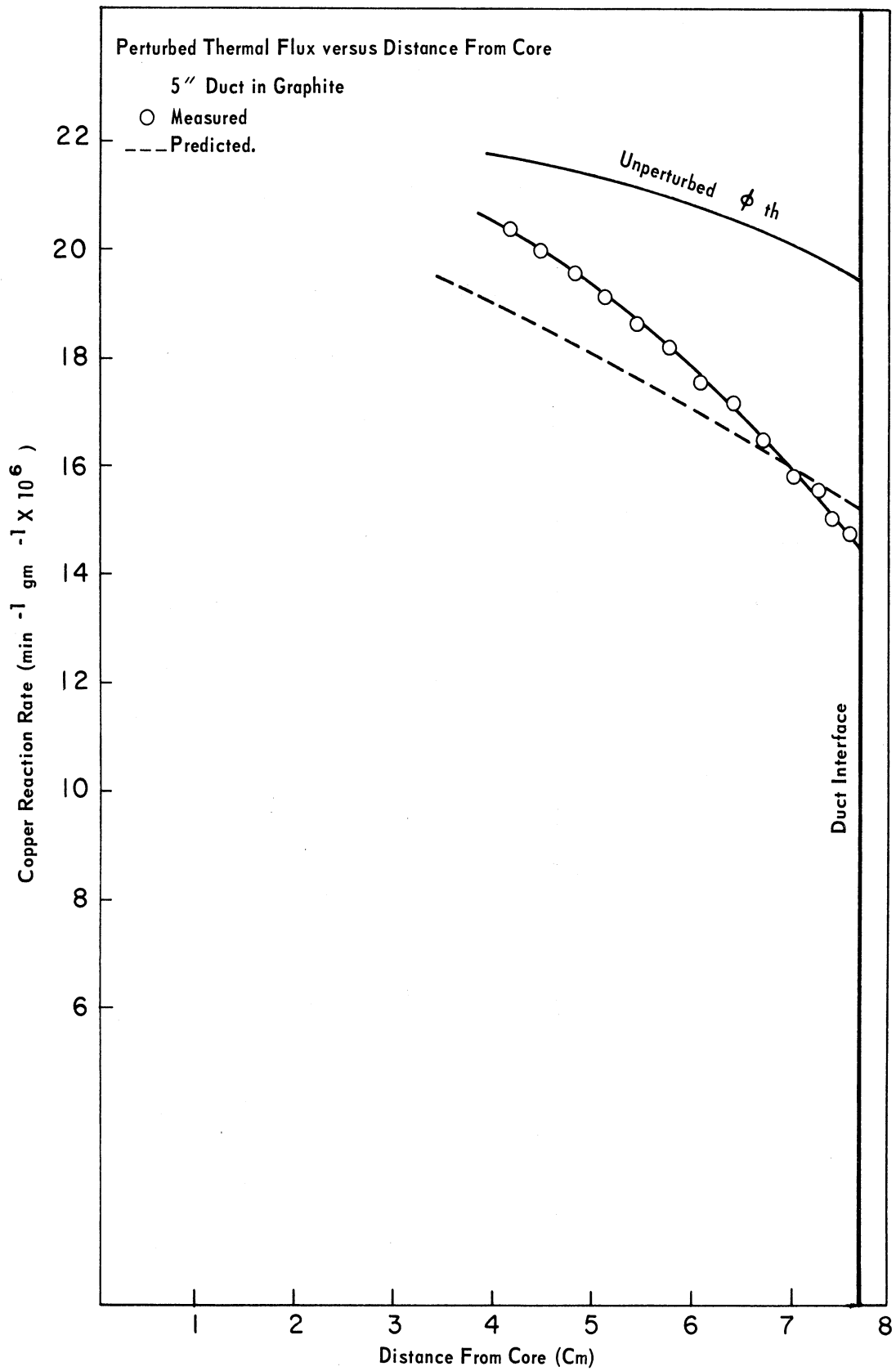
Figure 3.22. Axial distribution of thermal flux at mouth of 5-in. duct in graphite.

IV. DISCUSSION

A. Unperturbed Thermal Flux Distributions

The measured unperturbed thermal flux distributions, rather than

the computed ones, were used to obtain the predictions presented in

Chapter III. Utilization of the computed unperturbed thermal flux dis-

tributions, however, leads to no more than 3 percent disparity in the

prediction of the duct mouth thermal flux at the positions studied.

This is significant in that it suggests that duct perturbations can be

predicted from diffusion theory flux distributions without the need for

experiment. It should be pointed out, however, that the unperturbed

flux shape in the immediate vicinity of the duct mouth must be accu-

rately described by theory.

It is generally recognized that few-group diffusion theory has

limitations when used to predict accurate flux distributions in re-

flectors. The cross section averaging procedure[27] and the high degree

of fast flux anisotropy[28] give rise to deviations between theory and

experiment. Theory and experiment are compared in Figures 3.8 and 3.19.

The percentage deviation increases with distance from the core, although

for the graphite reflector the situation is complicated by the existence

of interfaces between highly dissimilar media. Utilization of more than

two groups in the few-group scheme does not improve the agreement. No

attempt has been made to include spectral effects close to the core-

reflector interface, nor has the curvature of the fuel element, which introduces up to 1/4 in. of water in front of the graphite insert, been considered. Infinite medium, Maxwellian-averaged cross sections were used in the reflector region. Presumably accurate predictions for the flux distribution far into the reflector region can be attained provided that appropriate transport and multigroup techniques are utilized. A thorough examination of this problem is beyond the scope of this study.

B. 1-1/2-In. Duct Experiments

The agreement between the experimental data shown in Figure 3.9 and the prediction based upon the perturbed angular flux at the duct mouth is good. It should be recalled, however, that unlike the calculations the measurements include the leakage contribution from the walls of the duct. This implies that the wall leakage contribution in the region examined is either a small or a constant percentage of the mouth contribution. Measurements with the duct at 2.16 cm indicate a 21 percent contribution from the walls. This number compares well with the results of similar measurements reported by Piercey.[10] Summing contributions from the lateral surfaces based upon the unperturbed thermal flux in the medium, a wall leakage contribution of 15 percent is predicted. The disparity of 6 percent could be accounted for by an albedo* contribution

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*The albedo contribution is taken to mean neutrons of all energies which, having entered the duct through the mouth, make at least one scattering collision in the wall before they diffuse back into the duct and are detected as thermal neutrons.

of the fast neutrons passed by the cadmium.

Subtraction of the wall leakage contribution permits the prediction of $j_{B_{th}}^{+}$. The excellent agreement presented in Table III.4 suggests that any thermal neutron albedo contribution from the mouth at the exit of the 1-1/2 in. duct lies within the experimental error.

The 1-1/2 in. duct was provided with the internal collimator sketched in Figure 3.13 to completely eliminate duct wall and albedo considerations. A measurement was performed with identical copper foils at the duct mouth and the collimator exit. This permitted a convenient comparison between the experimental value and the prediction for the mouth contribution to the exit thermal activation. The difference between the two was within the uncertainty in the collimator dimensions. Unfortunately no data comparable to the uncollimated data of Figure 3.9 were obtained.

C. Thermal Flux Attenuation
Within the Empty Ducts

The "duct streaming" measurements within the empty 1-1/2 in. and 5 in. ducts were motivated more out of academic interest than from any direct bearing on the main theme of this study. It was of interest to determine whether the thermal flux far from the mouth of the duct obeyed the $1/r^2$ geometrical attenuation predicted by Simon and Clifford.[9] The cadmium-backed foils were allowed to view all surfaces of the duct directly in front of them in order to include the albedo contribution. For the 1-1/2 in. duct h/R = 60 at the farthest measured distance from the

mouth, and for the 5 in. duct $h/R = 18$.

The experiment did not correspond to the configuration studied by Simon and Clifford in that they treated a point isotropic source at the duct mouth. The distributed source available experimentally at the duct mouth does not alter the results of their analysis. The presence of direct streaming sources in the duct walls, however, is estimated to give rise to a slightly more rapid attenuation than $1/r^2$.

If we assume an attenuation of the form $1/r^n$, the measurements (Figure 3.16) yield exponents of $n \simeq 2.7$ for the 1-1/2 in. duct and $n \simeq 2.5$ for the 5 in. duct. The large value of the exponent for the 1-1/2 in. duct is surprising, since the thermal albedo contribution from the mouth was estimated from other measurements (see Section IV. B) to be insignificant. In spite of the precautions taken, however, mutual shadowing between foils might have taken place.

D. Radial Flux Distributions at the Duct Mouth

We have assumed in the analysis that the flux is flat over the entire duct mouth. Examination of the distributions in Figure 3.11 reveals that this is not entirely correct, but that most of the recovery in the flux occurs close to the walls of the duct. The skewing of the perturbed thermal flux distributions is caused by the asymmetry of the unperturbed radial distribution. In any case, the analytical technique treats only the center of the duct mouth. The degree of accuracy achieved

in applying the analysis to the entire source plane, then, depends upon

the extent of collimation utilized in a practical situation. The average

perturbed scalar flux over the entire duct mouth is approximately 10 per-

cent higher than the flux minimum at the center.

E. Fast Flux Perturbation

In assuming a negligible fast flux perturbation at the duct mouth

it was anticipated that even significant changes would affect the thermal

flux in only a minor way. It is conceivable that the epithermal region

of the spectrum (for example, .625 ev to 5 Kev) could be treated by the

analytical method applied to the thermal flux, but no attempt was made

to predict duct perturbations in the epithermal region. Epicadmium gold

activations, which primarily measure the 4.9 ev flux, indicate a 26 per-

cent scalar flux depression at the mouth of the 5 in. duct. Epicadmium

manganese activations, constituting absorption in a 340 ev resonance

plus a significant portion of the 1/v portion of the spectrum, indicate

a 22 percent scalar flux depression. It is reasonable to assume that the

duct perturbation to the higher energy, more nearly monodirectional* com-

ponents of the spectrum is smaller. The integrated effect from the point

of view of slowing down sources into the thermal region is difficult to

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*Figure 4.1 displays angular distributions for the very fast flux (.82 to
 10 Mev) derived from an S_8 calculation.
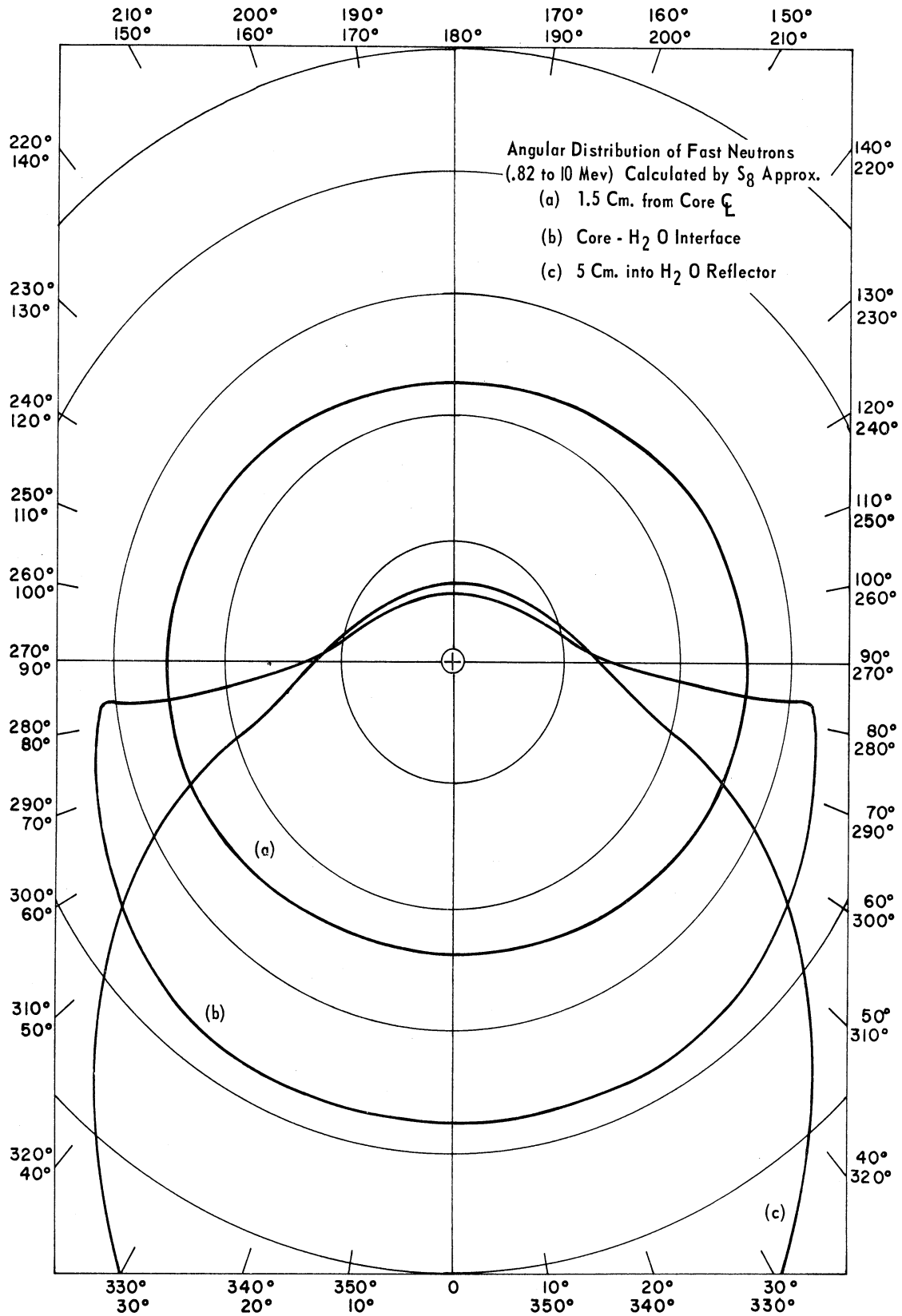
Figure 4.1. Angular distribution of fast neutrons.

evaluate. Figure 4.2 presents the calculated effect of assuming various group 1 (fast) flux depressions at the mouth on the group 2 (thermal) flux for the 5 in. duct. This is computed on the basis of a constant thermal return current to the duct mouth. The results suggest that a 25 percent depression in the integrated fast flux at the duct mouth only leads to a 3 percent depression in the thermal flux.
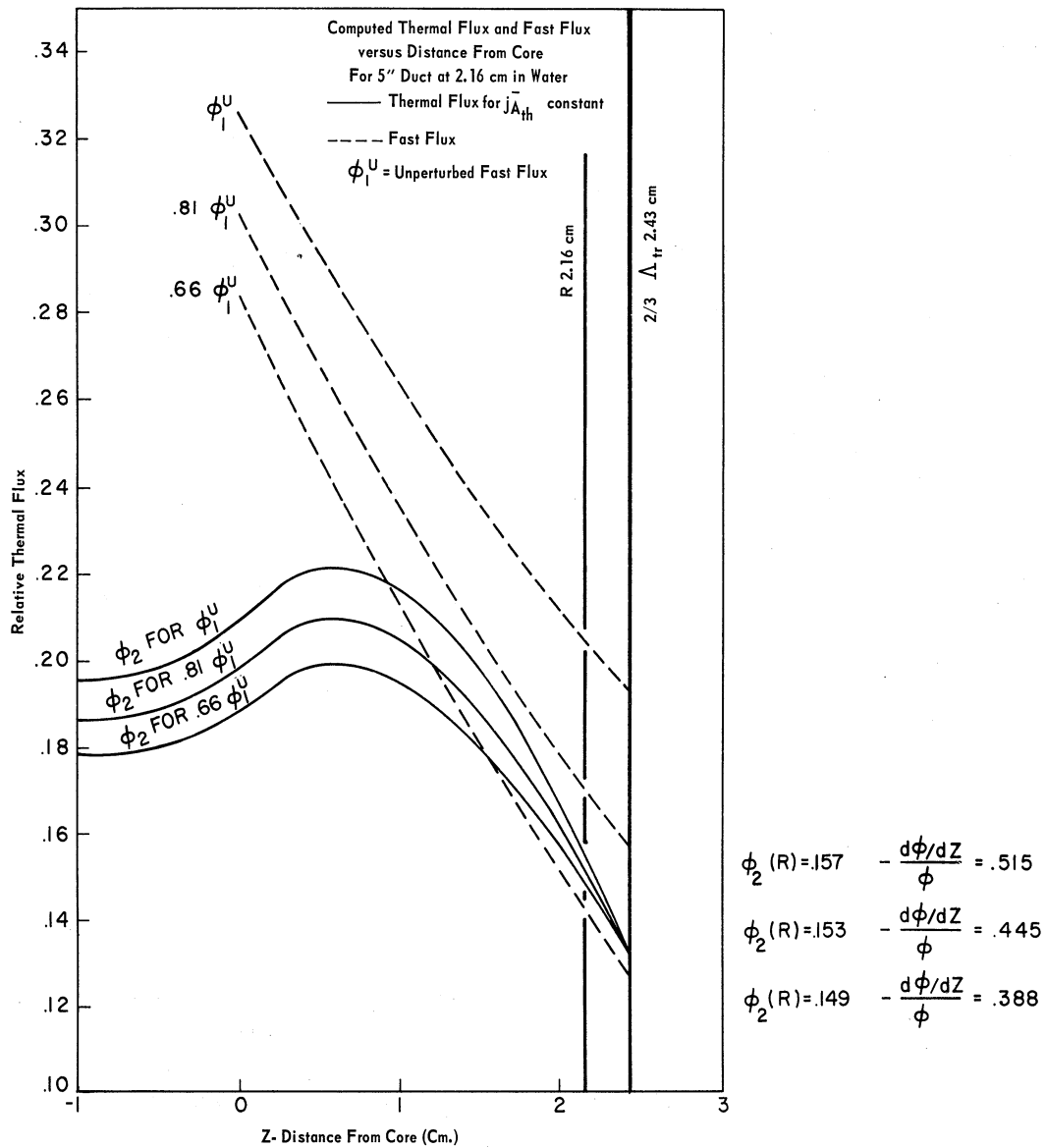
Figure 4.2. Effect on predicted thermal flux of fast flux depression.

The analysis is inapplicable to ducts which appreciably perturb the overall fission source in the multiplying medium. The assumption of small fission source perturbations should, of course, be realized for small ratios of the void area to the core face area, or for voids inserted sufficiently far from the reactor core (i.e., for small reactivity effects). The cross-sectional area of the 5 in. duct in the experimental configuration is of the order of 5 percent of the total area of the reactor face normal to the duct axis. Epicadmium flux measurements along the walls of the 5 in. duct indicate no significant depression in the fast flux. Ducts which appreciably perturb the reactor fission source may be amenable to an iteration analysis involving one and two-dimensional group-diffusion codes.

F. Thermal Spectrum

The $10° \pm 5°C$ hardening of the thermal spectrum measured at the mouth of the 5 in. duct is insignificant in the present study (the effect was estimated to lower the average activation cross section of a $1/v$ detector by less than 2 percent). The fact that a measurable effect was observed, however, suggests that spectral hardening induced by the void may be important in experimental determinations of the neutron spectrum at the exit of a beam port.

Measurements at the Ford Nuclear Reactor using a crystal spectrometer[29] at the exit of the 6 in. beam port sketched in Figure 3.1 determined

a neutron temperature of $40° \pm 5°C$. The temperature of the pool water is

$28°C$ at a reactor power level of one megawatt. It is reasonable to as-

sume that the 3 in. of graphite and 1-1/2 in. of water between the mouth

of the port and the core are sufficient to bring the core neutrons into

equilibrium with the diffusing medium. Using the Brown prescription[30]

for the hardening effect in water:

$$T_{eff} = T_0 \left[1 + .876 \frac{\Sigma_a(T_0)}{\Sigma_{s\infty}} \right] , \qquad (4.1)$$

we expect a neutron temperature in the unperturbed medium of less than

$32°C$. The difference of $8° \pm 5°C$ between the estimated unperturbed

neutron temperature and the measured value is presumably due to spectral

hardening induced by the port void.

G.   Scalar Flux at the Duct Mouth

The thermal flux measurements at the center of the 5 in. duct mouth

lie approximately 7 percent below the predicted values in both media, as

seen in Figures 3.11 and 3.21. Assuming for the moment that the unperturbed

flux is a correct representation for the distribution at the duct walls,

attenuation by the aluminum walls of the duct, which is difficult to

evaluate, in addition to the fast flux depression and hardening of the

thermal spectrum, which have been discussed earlier, all act to depress

slightly the measured thermal flux at the mouth. The current measurement

at the exit of the collimated duct in water, however, shown in Figure 3.14, agrees well with the predicted value. The arguments presented in Section II.B indicate that in the limit of the infinite half-space $(\bar{j}_{A_{th}} = 0)$ P-1 theory affords an adequate prediction for the angular current at the boundary (for $\mu = -1$), but the P-1 scalar flux prediction is 15 percent higher at the boundary than the exact quantity. Since the experimental data conform to this trend (in water $\bar{j}_{A_{th}}$ is less than 50 percent of the unperturbed return current to the 5 in. duct mouth), it is reasonable to ascribe the discrepancy in the scalar flux to measurable deviations from diffusion theory. Further evidence for deviations from diffusion theory is afforded by the comparison of the measured and predicted gradients at the mouth. In water the measured gradient is 40 percent higher than the predicted value from diffusion theory, and in graphite the measure-ment is 70 percent higher. Examination of the Milne problem reveals that the exact solution for the gradient possesses a logarithmic infinity at the mouth.

A measurement of $j_{B_{th}}^{+}$ was not performed in the complete graphite re-flector. However, we infer from the results of the perturbed scalar flux measurements at the duct mouths that the analysis applied to the water medium should be equally valid in graphite. Figure 4.3 is a plot of the prediction for $j_{B_{th}}^{+}$ in graphite. It should be pointed out that since the constant cross section approximation used to obtain Eq. (2.25) is not strictly valid here, this equation is not rigorously applicable. The re-sults of a numerical integration of Eq. (2.23), however, agree to within
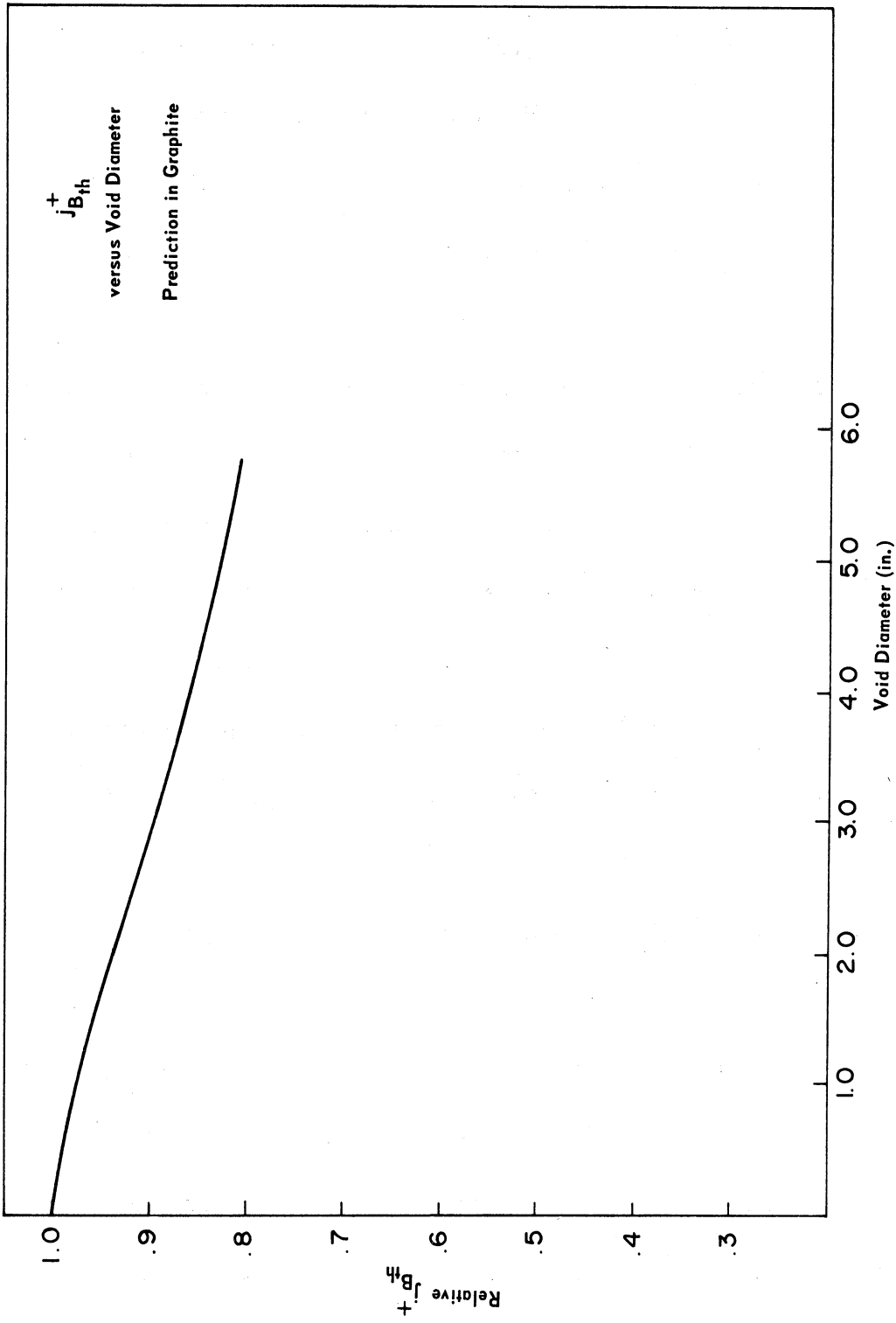
Figure 4.3. Predicted $j_{B_{th}}^+$ for graphite.

one percent with predictions based upon the constant cross section approximation.

H. Scalar Flux Distribution Along the Duct Walls

An idealization of the scalar flux distribution along the duct walls has been incorporated into the framework of the analysis. Predictions within this framework have compared favorably with the experimental results. Nevertheless, it would have been more satisfying to use the realistic flux distribution in Eq. (2.41) to compute the return current. Up to this point, however, an examination of the behavior of the realistic scalar flux distribution along the duct walls has been avoided.

Information concerning the realistic duct wall flux distribution was accessible experimentally. The configuration which gave rise to the largest flux perturbation at the mouth, the 5 in. duct in water, was examined. Foil strips fastened to thin lucite brackets were mounted perpendicular to the duct wall at 1, 3, 5, and 8 cm from the mouth. The relative thermal reaction rates for strips mounted on the east and west sides of the duct are plotted versus radial distance from the duct wall in Figure 4.4. The ratio of the radial component of the gradient at the wall to the scalar flux, $\dfrac{\partial \phi / \partial r}{\phi}\Big|_{z}^{R}$, is recorded by each curve.*

Figure 4.5 is a plot of the perturbed scalar flux distribution along

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*The fact that the gradients are higher on the east side of the duct originates from the asymmetry of the unperturbed radial distribution.
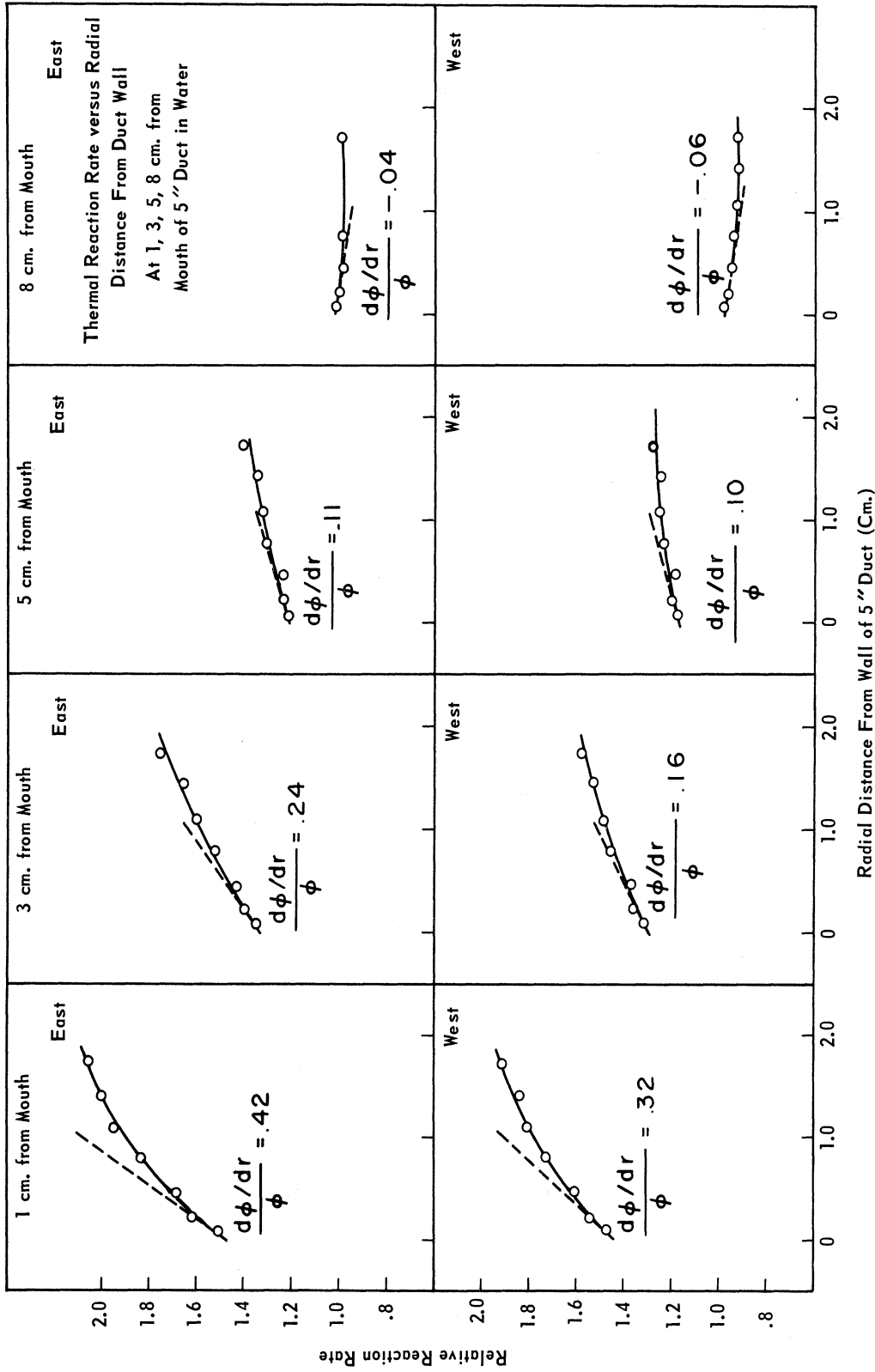
Figure 4.4. Radial thermal flux behavior at four positions along 5-in. duct wall.

the 5 in. duct wall compared with the unperturbed scalar flux distribution

in the medium out to 8 cm from the mouth. As anticipated, the scalar

flux is initially depressed close to the mouth, but recovers rapidly and

eventually exceeds the unperturbed flux where the radial component of the

gradient has become negative. Unfortunately, no data were obtained beyond

8 cm. However, assuming for the moment that the perturbed distribution

is identical to the unperturbed distribution beyond 7 cm, utilization of

the perturbed distribution in the calculation of $\overline{j_{A_{th}}}$ predicts a value for

the duct mouth scalar flux which is 10 percent lower than that obtained

from the unperturbed distribution. Since we have concluded earlier that

the unperturbed distribution predicts the correct P-1 value of the duct

mouth scalar flux, the enhancement of the perturbed distribution beyond

7 cm must compensate for the depression close to the mouth.

An attempt to predict the realistic behavior of the thermal flux

distribution along the duct walls was made by the following method. An

initial estimate of the quantity $\left.\frac{\partial\phi/\partial r}{\phi}\right|_z^R$ was obtained for several dis-

crete axial positions along the duct wall using the unperturbed axial

flux distribution in Eq. (2.50) (revised to include the two terms given in

expression (2.54) to account for the mouth contribution), the separated

form of the current balance relationship. A computer program written to

evaluate the initial estimate of $\left.\frac{\partial\phi/\partial r}{\phi}\right|_z^R$ is presented in Appendix G.

The initial estimate of the relative radial component of the gradient at

the mouth, $\left.\frac{\partial\phi/\partial r}{\phi}\right|_0^R$, was obtained in the usual way. The relative gradients
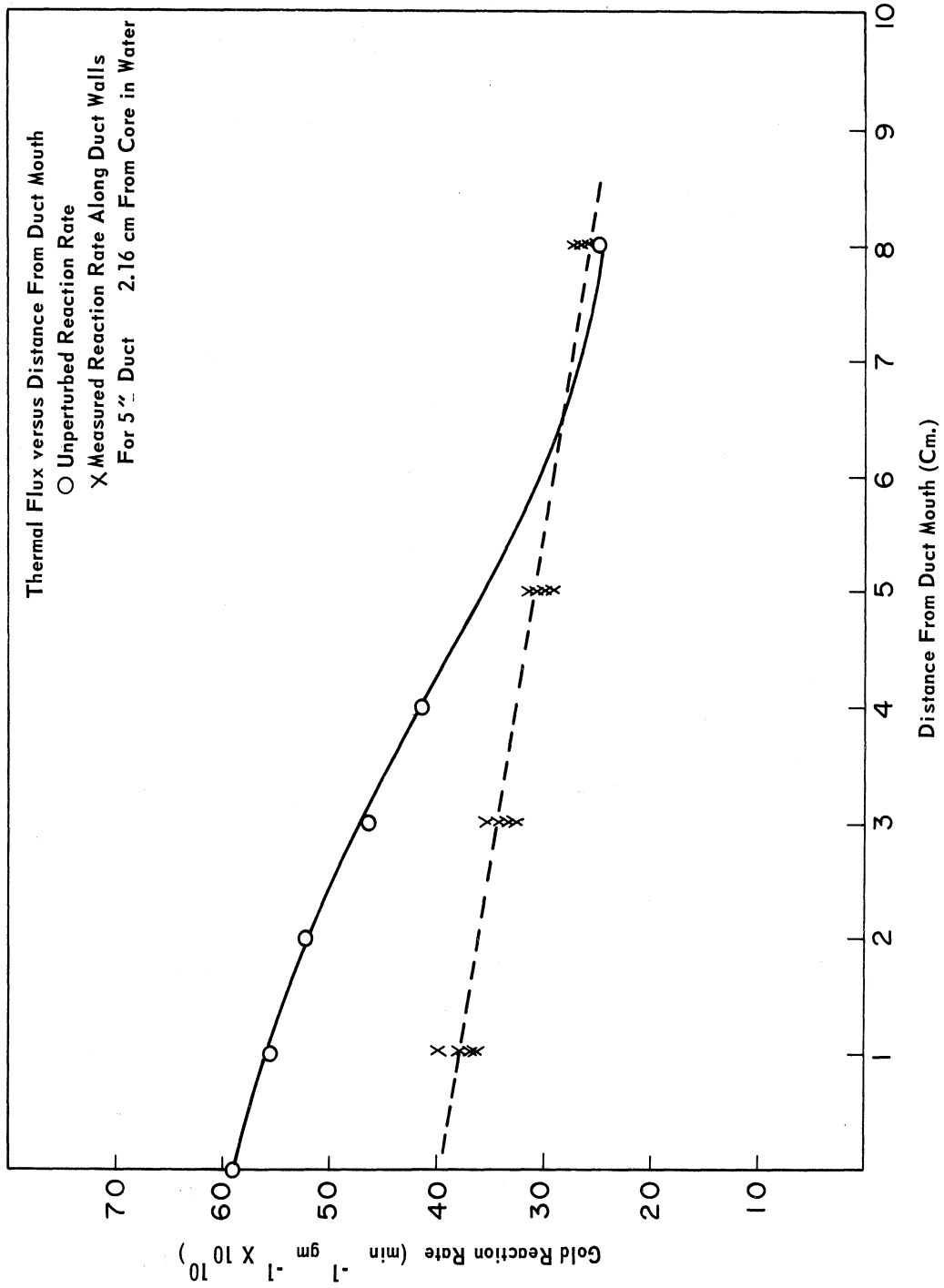
Figure 4.5. Axial thermal flux distribution along walls of 5-in. duct.

were used for a z varying boundary condition along the water-duct inter-
faces in a two-dimensional group-diffusion code.* The cylindrical
geometry approximation to the actual configuration is sketched in Figure
4.6. The calculations were performed with two energy groups, and the
fast flux was assumed unperturbed.

It was intended that the resulting perturbed flux distribution be
used in the unseparated current balance relationship, Eq. (2.45) (re-
vised to include the terms in expression (2.54)), to obtain improved es-
timates for $\frac{\partial\phi/\partial r}{\phi}\Big|_{z}^{R}$. The procedure forms a basis for an iteration
technique which was expected to converge rapidly. The initial estimates
for $\frac{\partial\phi/\partial r}{\phi}\Big|_{z}^{R}$, however, did not permit convergence of the group-diffusion
code for the 3 in. or 5 in. ducts. Even when the computation was forced
into convergence by diminishing the absolute magnitudes of $\frac{\partial\phi/\partial r}{\phi}\Big|_{z}^{R}$ far
from the mouth, the computed flux distributions were unrealistic, tending
to increase with distance from the core. The initial estimates and al-
tered values (to achieve code convergence) of $\frac{\partial\phi/\partial r}{\phi}\Big|_{z}^{R}$ for the 1-1/2, 3 and
5-in. ducts in water are recorded in Table IV.1. Comparisons with the
measured gradients at the walls of the 5 in. duct recorded in Figure 4.4

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*The TWENTY GRAND code, described in Appendix B, provides for a log-
arithmic-derivative or "rod-region" boundary condition, specified by:

$$D^i \frac{d\phi^i}{d\vec{n}} = -C^i \phi^i$$

for energy group i. C^i is supplied as input data for any region spe-
cified as a "rod-region."
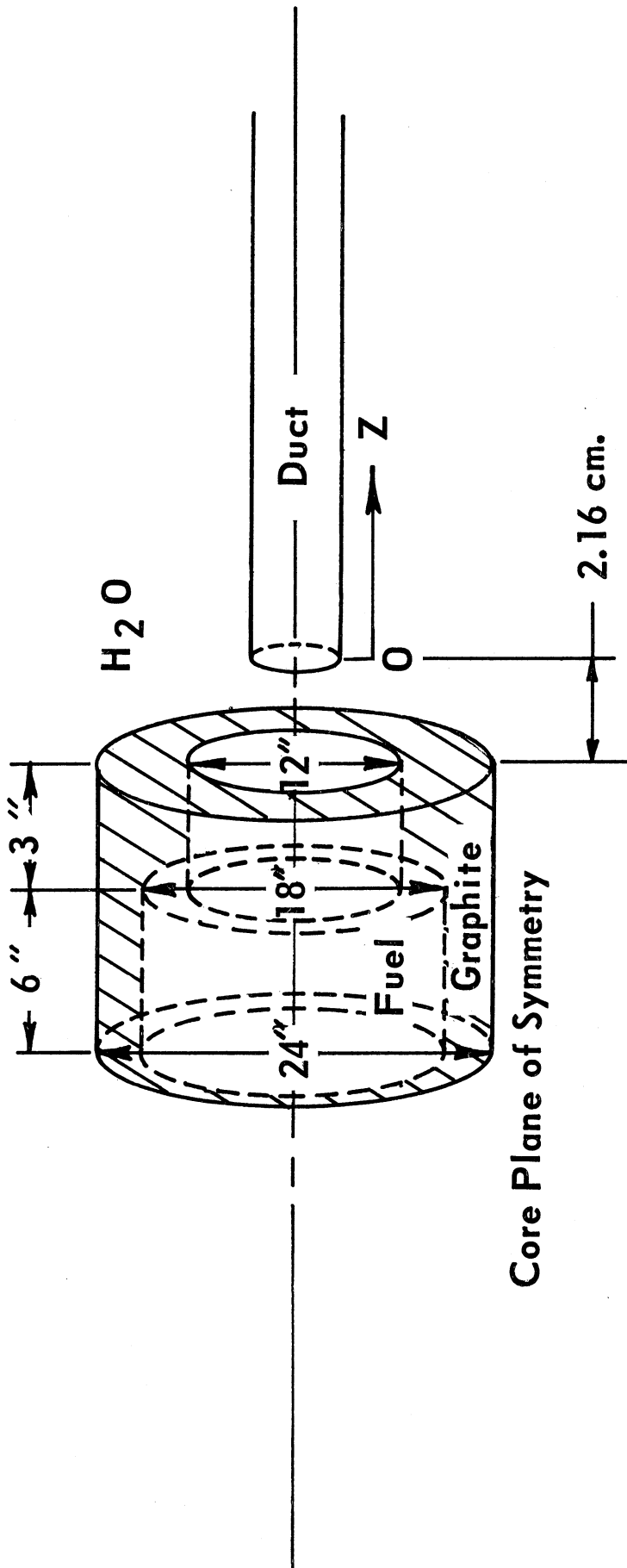
Figure 4.6. Cylindrical geometry approximation to core-duct configuration.

TABLE IV.1

FIRST ESTIMATE OF THE RADIAL COMPONENT OF THE GRADIENTS FOR BOUNDARY
CONDITION IN TWO-DIMENSIONAL CALCULATION

z Distance from Duct Mouth (cm)	1-1/2 In. Duct		3 In. Duct		5 In. Duct	
	$\dfrac{\partial\phi/\partial r}{\phi}\Big\|_{z}^{R}$ cm^{-1}	Altered $\dfrac{\partial\phi/\partial r}{\phi}\Big\|_{z}^{R}$ cm^{-1}	$\dfrac{\partial\phi/\partial r}{\phi}\Big\|_{z}^{R}$ cm^{-1}	Altered $\dfrac{\partial\phi/\partial r}{\phi}\Big\|_{z}^{R}$ cm^{-1}	$\dfrac{\partial\phi/\partial r}{\phi}\Big\|_{z}^{R}$ cm^{-1}	Altered $\dfrac{\partial\phi/\partial r}{\phi}\Big\|_{z}^{R}$ cm^{-1}
0	.120	Unaltered	.240	.240	.400	.400
1	.150		.340	.340	.547	.547
2	.104		.265	.265	.450	.450
3	.051		.164	.164	.310	.310
4	.000		.057	.057	.149	.149
5	-.047		-.050	-.050	- .020	-.020
6	-.090		-.154	-.154	- .193	-.193
7	-.130		-.254	-.254	- .365	-.365
8	-.165		-.350	-.350	- .536	-.450
9	-.198		-.442	-.396	- .704	-.450
10			-.531	-.396	- .870	-.450
11					-1.033	-.450
12					-1.195	-.450
13					-1.355	-.450
14					-1.516	-.450
15					-1.678	-.450
16					-1.841	-.450
17					-2.006	-.450
18					-2.175	-.450

indicate that the estimated gradients close to the duct mouth are not unreasonable. The apparent deadlock is attributed to the large negative estimates of the gradients far from the duct mouth. Either these values or the treatment of the negative gradients by the code (as a negative absorption in the duct region) is unrealistic.

I. Reactivity Predictions

A far more encouraging prediction emerges from an examination of the eigenvalues computed by the two-dimensional code. The duct reactivity calculated by using the altered estimates of the duct wall gradients listed in Table IV.1 compare favorably with experiment. This is not surprising, since the fundamental eigenvalue of the diffusion equation is relatively insensitive to the flux distribution. However, inasmuch as the altered gradient estimates for the 5 in. duct provided a highly unrealistic flux distribution, an additional calculation was performed that utilized only the positive gradients ($\frac{\partial \phi / \partial r}{\phi}\Big|_z^R$ was set equal to zero past 4 cm). Considering the degree of approximation to the realistic geometry, the result of this calculation was also in reasonable agreement with experiment. The reactivity results are presented in Table IV.2.

TABLE IV.2

DUCT REACTIVITY—MEASURED AND PREDICTED
DUCTS AT 2.16 CM FROM CORE IN WATER

Duct Diameter (in.)	$\delta k/k$ Measured, %	$\delta k/k$ Predicted using altered $\frac{\partial\phi/\partial r}{\phi}\Big\|_{z}^{R}$, %	$\delta k/k$ Predicted using positive $\frac{\partial\phi/\partial r}{\phi}\Big\|_{z}^{R}$ (Zero past 4 cm), %
1-1/2	-.007	-.007	—
3	-.03	-.04	—
5	-.10	-.11	-.16

V. CONCLUSIONS

The main results of the analysis and measurements discussed in Chapters II, III, and IV may be summarized as follows:

1. P-1 theory can be used to predict adequately the thermal neutron current at the exit of a collimated beam port. The source plane scalar flux can be predicted using P-1 theory with somewhat less accuracy. The method has been applied by computing the return current at the duct source plane in terms of the measured distribution of the unperturbed scalar flux in the reflector. The return current is used in a few-group diffusion calculation in the form of a pseudo boundary condition on the scalar flux. Predicted exit thermal neutron currents for ducts up to 5 in. in diameter in the water reflector at the Ford Nuclear Reactor agree within ± 3 percent with activation measurements. Predicted source plane scalar fluxes in water and graphite reflectors agree within 7 percent with the measurements.

2. The method can be applied satisfactorily if the calculated distribution of the unperturbed scalar flux in the reflector as obtained from few-group diffusion theory is used to compute the return current. The unperturbed scalar flux distributions in the reflector deviate significantly from the few-group diffusion theory results close to boundaries and far from the reactor core. Predictions for the beam port exit currents obtained from the calculated distributions, however, deviate by only 3 percent from those obtained from the measured distributions.

3. An iteration technique was proposed in an attempt to predict the realistic distributions of the scalar flux along the lateral surfaces of the ducts. Unfortunately, the initial estimates of the radial component of the gradients (obtained by separation of variables) failed to produce convergence of the two-dimensional group-diffusion code. Further work is needed to explore this difficulty.

4. The results of a survey to optimize thermal neutron leakage flux from the Ford Nuclear Reactor clearly indicate that:

   a. The optimum geometry is a slab reactor with the beam port external to the core face normal to the smallest core dimension.

   b. An increase in power density by improved reflection is more desirable than an increase in integrated power because of a reduction in the ratio of fast to thermal flux.

   c. D_2O is the most effective reflector material for this purpose.

APPENDIX A

A SURVEY FOR THE ENHANCEMENT OF THERMAL
NEUTRON LEAKAGE FLUX

1. Introduction

This survey is directed toward an optimization of the thermal

neutron flux intensity and the neutron spectrum in a reflector region

of the Ford Nuclear Reactor (FNR) for beam port applications. The

FNR was briefly described in Chapter III. Although the results of the

investigation pertain to a swimming pool reactor facility of the FNR

type, they should apply at least qualitatively to a more general type

of thermal reactor.

For purposes of efficiency, as well as compliance with economic

and physical limitations at the existent reactor facility, the following

constraints were established:

(a) The core composition was held constant. For this the FNR

     regular fuel element was adopted (partial elements and poison

     rods were disregarded).

(b) The integrated reactor power was held constant.

(c) The vertical core dimension was fixed at 24 in., the height

     of the "meat" portion of the FNR fuel element.

(d) The beam port was aligned with a radius of the core outside

     of the fuel region.

(e) The beam port influence on the exit current was neglected in

this phase of the study.

In keeping with these constraints, the survey considers variations in

reflector materials and core-reflector configurations.

The computer codes utilized for the numerical computations are

described in Appendix B. Compositions and group constants are tab-

ulated in Appendix C. Four-group diffusion theory was adopted. The

four-group scheme, with breakpoints listed in Appendix C, has been

highly successful in treating small, enriched, light-water reactors.[31]

Moreover, a four-group study affords some useful information concerning

the neutron spectrum.

2. Three-Dimensional Simulation

Three-dimensional group-diffusion calculations would be prohib-

itively time-consuming for an inquiry of this nature. Two-dimensional

calculations were performed whenever possible, but they were limited in

usefulness, since criticality search routines were not included. There-

fore, most of the computations were performed using one-dimensional

simulation of the three-dimensional geometry. This was accomplished by

holding two of the dimensions, y and z, constant and searching for the

critical size and flux distribution in a variable third dimension, x.

Leakage in the y and z directions was incorporated as a transverse buckl-

ing.

The transverse buckling was obtained in the following manner. One-dimensional flux distributions in the y and z directions were acquired for each of the four groups for constant y and z dimensions (leakage in in the x direction was accounted for by allowing the code to search for the critical transverse buckling). Then for each group the bucklings transverse to the x direction were derived from the following relationships:

$$<(B_y^2)>^i \; = \; \frac{- \int\limits_{core} \nabla^2 \phi^i(y) dy}{\int\limits_{core} \phi^i(y) dy} \quad , \tag{A.1}$$

and:

$$<(B_z^2)>^i \; = \; \frac{- \int\limits_{core} \nabla^2 \phi^i(z) dz}{\int\limits_{core} \phi^i(z) dz} \quad . \tag{A.2}$$

Transverse buckling components which were used in the acquisition of the subsequent results are listed in Table A.I.* The total buckling transverse to the x-direction for the ith group is given by:

$$<(B_t^2)_{T_x}>^i \; = \; <(B_y^2)>^i + <(B_z^2)>^i \quad . \tag{A.3}$$

---

*Transverse bucklings obtained for the core were used in all regions. Refinements to some of the subsequent results could be achieved by using region-dependent as well as group-dependent bucklings.

TABLE A.I

TRANSVERSE BUCKLINGS

(cm^{-2})

	Cold-Clean Core				Equil. Xe, 8.4% BU Core			
	$(B_1)_T^2$	$(B_2)_T^2$	$(B_3)_T^2$	$(B_4)_T^2$	$(B_1)_T^2$	$(B_2)_T^2$	$(B_3)_T^2$	$(B_4)_T^2$
Vertical	.00296	.00253	.00153	-.00472	.00298	.00253	.00156	-.00448
3 Rows Horiz.	.00933	.00493	.00499	-.00026	---	---	---	---
Total	.01229	.00746	.00652	-.00498	---	---	---	---
4 Rows Horiz.	.00635	.00337	.00341	-.000202	.00651	.00346	.00348	-.000057
Total	.00931	.00590	.00494	-.00492	.00949	.00599	.00504	-.00454
5 Rows Horiz.	.00464	.00248	.00252	-.00016	.00475	.00254	.00255	-.000042
Total	.00760	.00501	.00405	-.00488	.00773	.00507	.00411	-.00452
7 Rows Horiz.	.00283	.00152	.00155	-.000117	.00289	.00155	.00156	-.000035
Total	.00579	.00405	.00308	-.00484	.00587	.00408	.00312	-.00451
8 Rows Horiz.	---	---	---	---	.00235	.00126	.00127	-.000032
Total	---	---	---	---	.00533	.00379	.00283	-.00451

3. Streaming Flux

The scalar flux distributions, which together with the fundamental eigenvalue constitute the principal output of the group-diffusion codes, are not the quantities of foremost interest from the point of view of the present study. Rather, we are interested in the angular flux at the source plane in the direction of the beam port exit. The angular flux is given by Eq. (2.23) in the constant cross section approximation. If the constant cross section restriction is removed:

$$\phi(\overline{r},\overline{\Omega}) = \frac{1}{4\pi} \int_0^\infty d\eta \left[\Sigma_s(\overline{r}-\eta\overline{\Omega})\phi_0(\overline{r}-\eta\overline{\Omega}) + s(\overline{r}-\eta\overline{\Omega}) \right] e^{-\int_0^\eta \Sigma_t(\overline{r}-\eta'\overline{\Omega})d\eta'}.$$

(A.4)

$s(\overline{r}-\eta\overline{\Omega})$ is defined here as the source integrated over all angles.

For long ports, $(h/R) \gg 1$, it is the forward flux, close to $\mu = -1$, which is detected at the exit. In the limit of a collimated port of infinite length, Eq. (A.4) can be approximated by the one-dimensional integration:

$$\phi(x,\mu=-1) \cong \frac{1}{4\pi} \int_0^\infty dx' \left[\Sigma_s(x-x')\phi_0(x-x') + s(x-x') \right] e^{-\int_0^{x'} \Sigma_t(x-x'')dx''}.$$

(A.5)

Then for any position x in the reflector and energy group i, we define the streaming flux:

$$\mathcal{J}^i(x) = \int_0^\infty dx' [\Sigma_s^i(x-x')\phi_o^i(x-x') + s^i(x-x')]e^{-\int_0^{x'} \Sigma_t^i(x-x'')dx''}$$

(A.6)

The streaming flux is emphasized in the subsequent presentation of results. Since beam port perturbations were neglected in this survey, the unperturbed scalar flux and source distributions were used to compute this quantity. Slowing down sources were neglected. Subroutine STREAM, written to compute $\mathcal{J}^i(x)$ and the partial current, $j_i^+(x)$, in conjunction with the FOG code, is listed together with the revised main program of FOG in Section A.7.

4. Configurational Effects

BARE REACTOR EXAMINATION

In addition to allowing a simple analysis, the bare reactor exhibits fast neutron leakage behavior which is similar to that of the reflected core. Examination of the behavior of fast neutron leakage should provide considerable insight into the influence of the reactor configuration on the thermal neutron flux intensity in a reflected system, inasmuch as the removal of fast neutrons constitutes a thermal neutron source in the reflector.

The flux in a bare homogeneous reactor satisfies the Helmholtz equation:

$$\nabla^2\phi + B_g^2\phi = 0 . \tag{A.7}$$

The total leakage is given by:

$$- \int D(\overline{r})\nabla^2\phi(\overline{r})d^3r = B_g^2 \int D(\overline{r})\phi(\overline{r})d^3r , \tag{A.8}$$

and the integrated power may be expressed as:

$$Power = const. \int \Sigma_f(\overline{r})\phi(\overline{r})d^3r . \tag{A.9}$$

For a fixed integrated power and spatially independent properties:

$$. \quad Total\ Leakage \sim B_g^2 . \tag{A.10}$$

A critical bare reactor, however, must satisfy the condition:

$$k_{eff} = 1 = \frac{k_\infty e^{-B_m^2 \tau}}{1 + L^2 B_m^2} . \tag{A.11}$$

The material buckling, B_m^2, is a function only of the reactor material

composition, and for a critical system it must be equal to the smallest

eigenvalue, the geometrical buckling, of Eq. (A.7). Thus:

$$Total\ Leakage \sim B_g^2 = B_m^2 , \tag{A.12}$$

which is a constant for a critical bare reactor of homogeneous composi-

tion operating at a fixed power level, regardless of the core geometry.

The average leakage per unit area, however, is a function of the

total surface area of the core. The minimum critical volume and surface area for bare reactors of three geometries[13] are listed in Table A.II.

TABLE A.II

CRITICAL VOLUME, AREA, AND RELATIVE LEAKAGE
FOR BARE REACTORS OF THREE GEOMETRIES

Geometry	Minimum Critical Volume	Critical Surface Area	Relative Average Leakage/Area
Sphere	$130/B^3$	$125/B^2$	1.0
Finite Cylinder	$148/B^3$	$155/B^2$	.806
Rectangular Parallelepiped	$161/B^3$	$179/B^2$	.698

The fourth column of the table does not express the complete story, however. The leakage is strongly peaked at the center of a face of the parallelepiped, whereas it is uniform over the entire surface of the sphere. Using Gauss's theorem, the total leakage from the reactor can be expressed as:

$$- D \int_{\text{vol.}} d^3r \; \nabla^2 \phi(\overline{r}) \; = \; - D \int_{\text{surf.}} dS \; \hat{n} \cdot \overline{\nabla} \phi \Big|_{\overline{r} = \overline{R}} \; . \qquad (A.13)$$

The flux distribution in a bare cube of linear dimension c is given by (assume the reactor is large enough so that $c \simeq c+d$):

$$\phi(x,y,z) = A \cos \frac{\pi x}{c} \cos \frac{\pi y}{c} \cos \frac{\pi z}{c} \quad . \qquad (A.14)$$

The average leakage per unit area is given by:

$$-\frac{D}{c^2} \int_{-c/2}^{c/2} dy \int_{-c/2}^{c/2} dz \frac{\partial}{\partial x} \phi(x,y,z) \Big|_{x=c/2} = \frac{4DA}{\pi c} \quad , \qquad (A.15)$$

whereas the peak leakage per unit area occurs at the center of the face, and is equal to:

$$- D \, \hat{n} \cdot \overline{\nabla} \, \phi(\overline{r}) \Big|_{\overline{r}=\overline{R}} = - D \frac{\partial}{\partial x} \phi(x,y,z) \Big|_{\substack{y=0 \\ z=0 \\ x=c/2}} = \frac{DA\pi}{c} \quad . \ (A.16)$$

Thus for a cube, $\dfrac{\text{Peak Leakage/area}}{\text{Average Leakage/area}} = 2.46$, and

$\dfrac{\text{Peak Leakage/area for a cube}}{\text{Average leakage/area for a sphere}} = 1.72$. This result indicates that

the bare parallelepiped possesses an advantage over the more efficient

reactor geometry from the point of view of maximum leakage.

Now let us estimate the effect of varying two of the dimensions of

a rectangular parallelepiped, holding the third dimension constant. We

designate the fixed dimension by c, and the variable dimensions by a

and b. For a bare critical parallelepiped operating at constant power,

the geometrical buckling is given by:

$$B_g^2 = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{c}\right)^2 = \text{constant} = 3\left(\frac{\pi}{c}\right)^2 \quad , \qquad (A.17)$$

which is the buckling for a cube. The total leakage from one face of

the reactor is expressed by:

$$- D \int_{-c/2}^{c/2} dz \int_{-b/2}^{b/2} dy \frac{\partial}{\partial x} \phi(x,y,z) \Big|_{x = a/2} = \frac{4DA}{\pi} \left(\frac{bc}{a} \right) \quad . \quad (A.18)$$

Using Eq. (A.17), we obtain for the leakage out of a face normal to

dimension a:

$$\text{Leakage} = \frac{4DAc}{\pi} \frac{1}{\left(\frac{2a^2}{c^2} - 1 \right)^{1/2}} \quad , \quad \frac{a}{c} > .707 \quad . \quad (A.19)$$

Dividing by the area of the face, the average leakage/area is given by:

$$\frac{\text{Average Leakage}}{\text{Area}} = \frac{4DA}{\pi c} \frac{1}{(a/c)} \quad . \quad (A.20)$$

Table A.III is a list of the total leakage and average leakage per unit

area (normalized to unity for a cube) from a face of the parallelepiped

normal to dimension a.

TABLE A.III

RELATIVE LEAKAGE FROM A FACE FOR BARE
PARALLELEPIPEDS OF VARIOUS SHAPES

a/c	Total Leakage	Leakage/Area
.8	1.89	1.25
.9	1.27	1.11
1.0	1.00	1.00
1.1	.84	.909
1.2	.73	.834

Beam extraction from a face which is normal to the slender core dimension is suggested.

THE WATER REFLECTED PARALLELEPIPED

One group of neutrons is insufficient to study the reflected parallelepiped, thus it is convenient to solve the four-group equations on the computer. The configurations examined are shown in Figure A.1. Reactor cores with 3, 4, 5, and 7 transverse elements were considered. The critical x dimensions recorded in Figure A.1 were obtained from dimension searchs using the three-dimensional simulation technique discussed earlier. The fuel (cold-clean composition) was surrounded by light water except for symmetric rows of graphite normal to the y-axis. Figures A.2-A.5 display the x-axis scalar flux and streaming flux distributions. Three of the four computed groups are presented in these and subsequent plots.

Table A.IV is a list of pertinent information from Figures A.2-A.5.
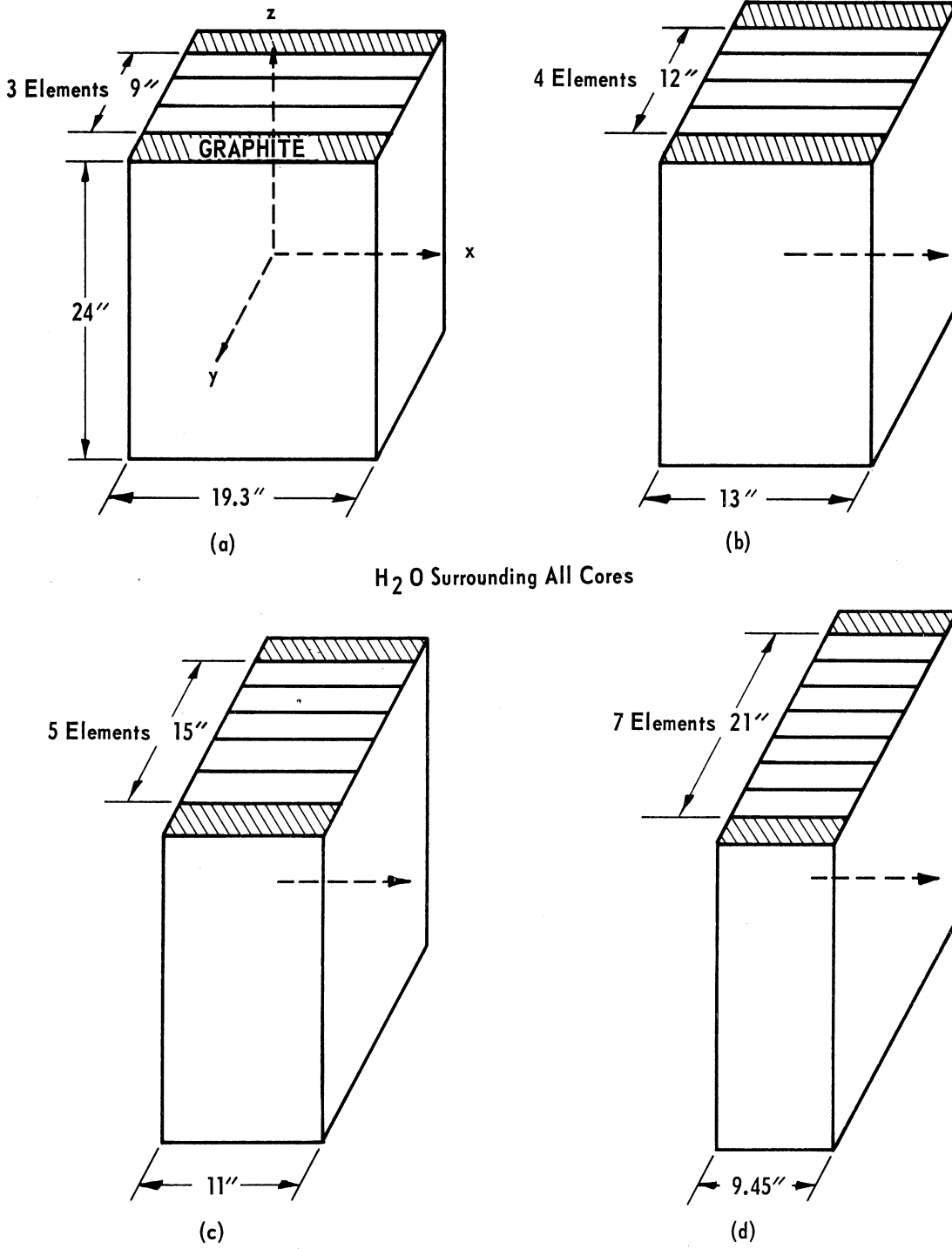
Figure A.1. Core geometries for H_2O reflected configuration studies (cold-clean fuel).
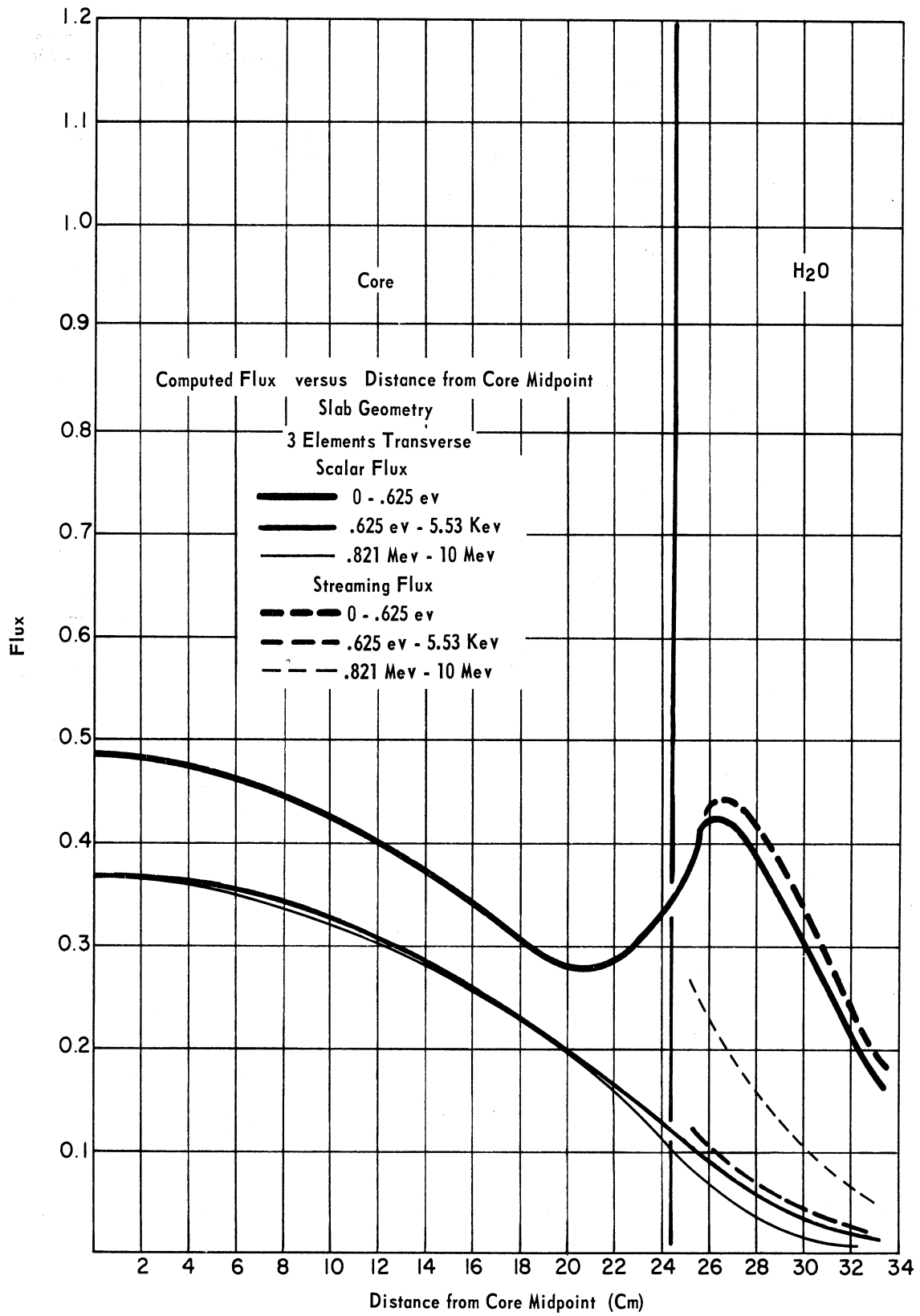
Figure A.2. Computed flux distributions; H_2O reflected core; 3 elements transverse.
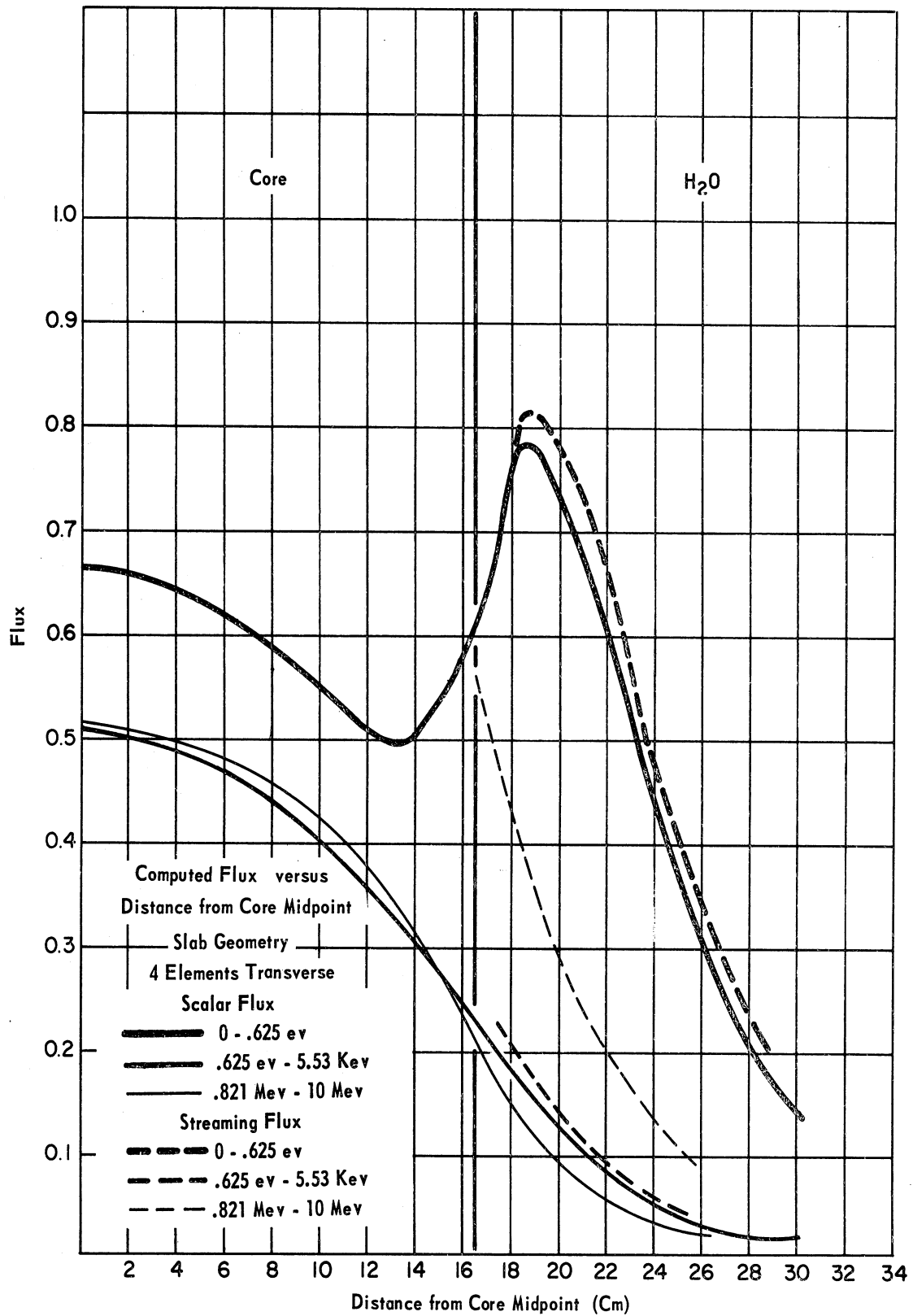
Figure A.3. Computed flux distributions; H_2O reflected core; 4 elements transverse.
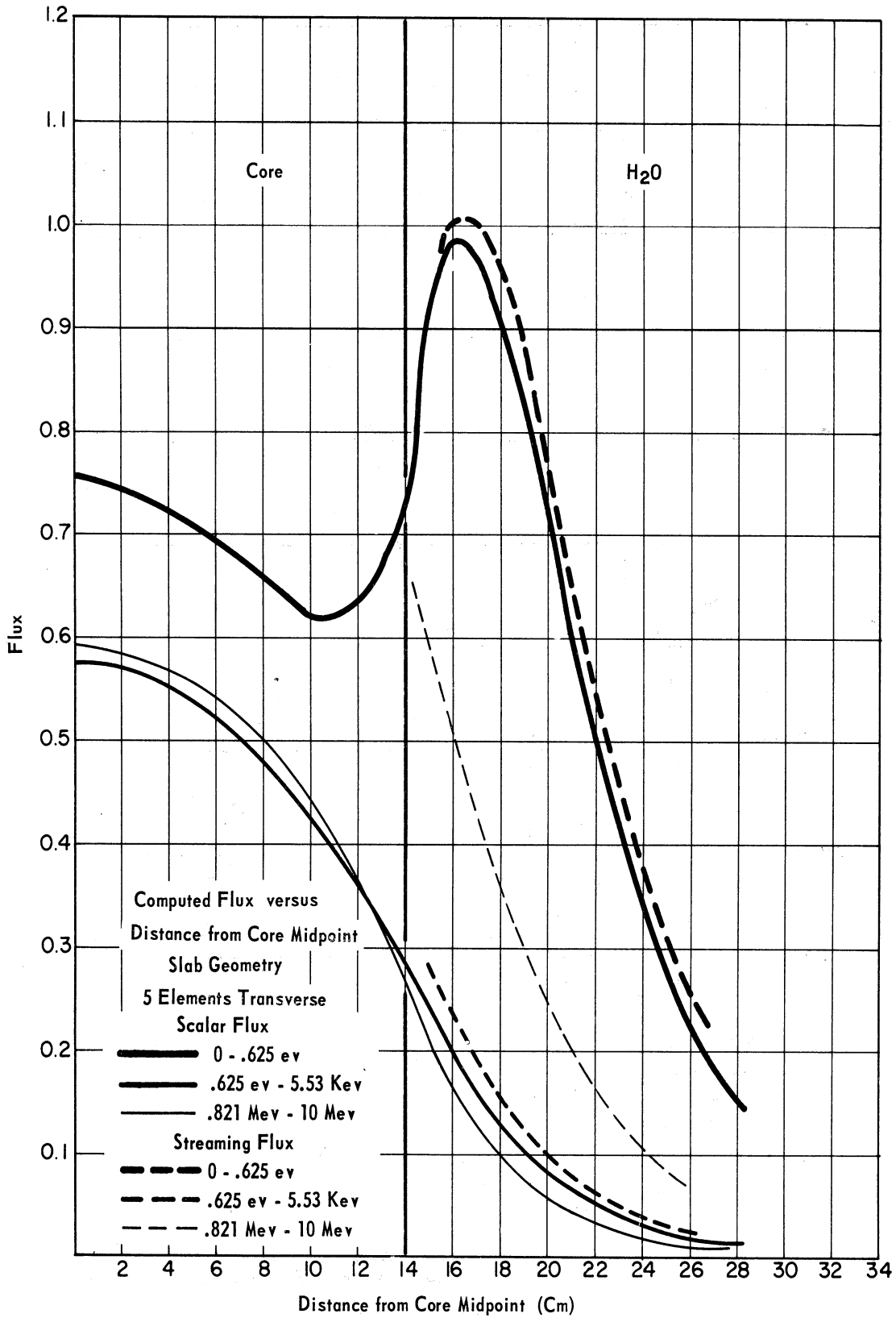
Figure A.4. Computed flux distributions; H_2O reflected core; 5 elements transverse.
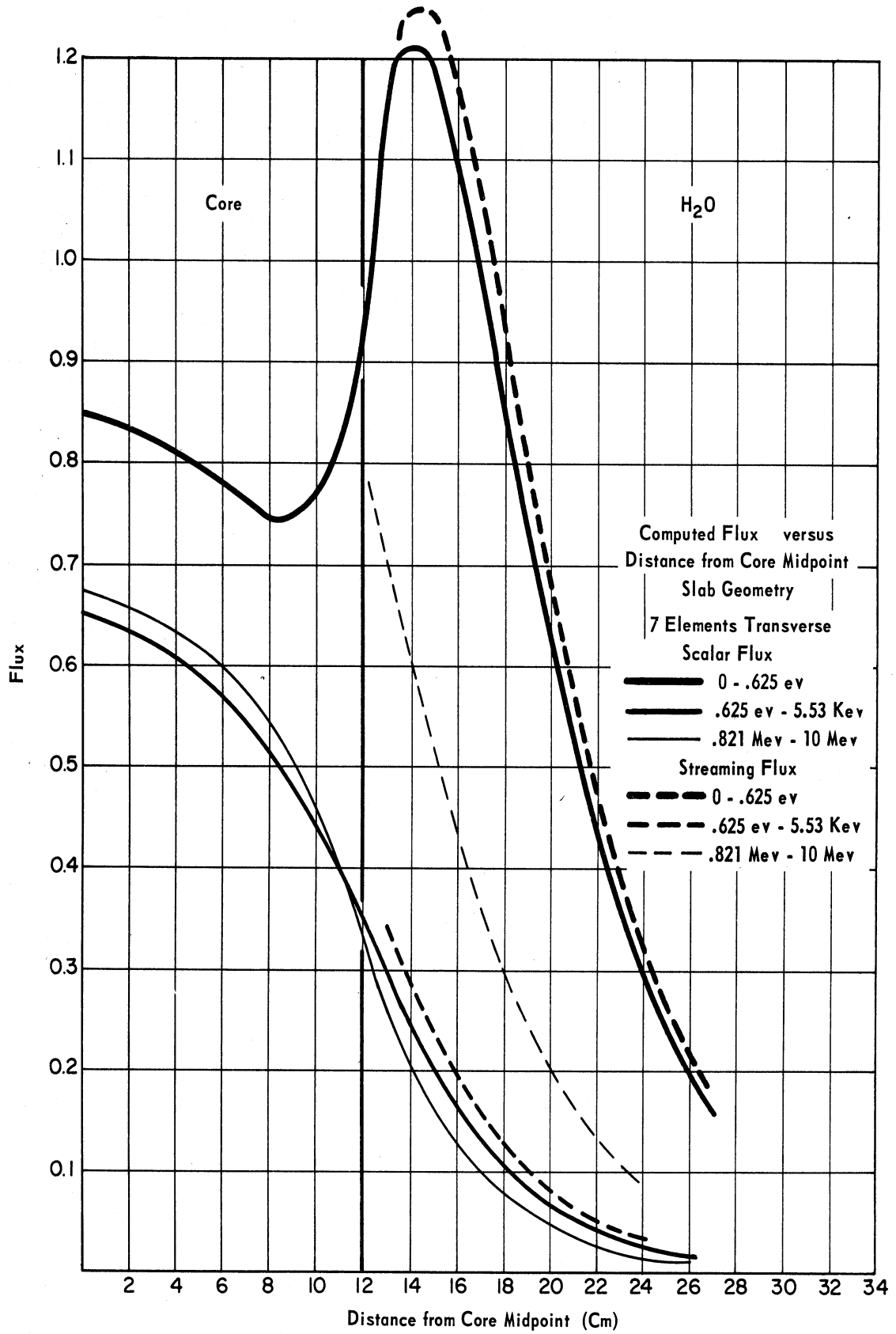
Figure A.5. Computed flux distributions; H_2O reflected core; 7 elements transverse.

TABLE A.IV

COLD-CLEAN H_2O REFLECTED PARALLELEPIPED CORES OF

VARIOUS SHAPES

	S^4_{max}	$(S^4-S^3)_{max}$	$(S^4-S^1)_{max}$	(S^3/S^4_{max})	(S^1/S^4_{max})	$1/V$ in.$^{-3}$
3 Elem. Trans. Cold-Clean Core	.448	.358	.256	.202	.467	.00024
4 Elem. Trans. Cold-Clean Core	.811	.647	.478	.202	.434	.00027
5 Elem. Trans. Cold-Clean Core	1.06	.794	.599	.180	.388	.00025
7 Elem. Trans. Cold-Clean Core	1.25	.990	.755	.208	.432	.00021
7 Elem. Trans. Eq. Xe- 8.4% BU Core	.965	.770	.596	.215	.422	.00016

The quantities chosen here and in the subsequent studies to be of major

interest are: (i) the maximum value of the thermal streaming flux, $\mathcal{J}^4_{max}$;

(ii) the maximum difference between the thermal and epithermal and the

thermal and fast streaming fluxes, $(\mathcal{J}^4 - \mathcal{J}^3)_{max}$ and $(\mathcal{J}^4 - \mathcal{J}^1)_{max}$;

(iii) the ratio of epithermal to thermal and fast to thermal streaming

fluxes at the location of the maximum thermal streaming flux, $(\mathcal{J}^3/\mathcal{J}^4_{max})$

and $(\mathcal{J}^1/\mathcal{J}^4_{max})$; (iv) the inverse volume of the core, $1/V$, a measure of

the average power density of the reactor. An additional listing in Table A.IV

compares the cold-clean, seven-element transverse core with a core containing equilibrium Xe and 8.4 percent fuel burnup.

Table A.IV strongly suggests beam extraction adjacent to a face which is normal to a slender dimension of the core. Quantitative justification is presented for deforming the core to the optimum configuration (maximum power density), and going past this point if possible.

LOCALIZED CORE ALTERATIONS

It was of interest to examine the effects of an uneven fuel distribution. Figure A.6 presents the results of a calculation comparing the leakage flux of a core with clean fuel in the center and 8.4 percent fuel burnup on the outside with that of a core endowed with the converse loading. In the latter core the clean and burned-up fuels were loaded in approximately even proportions, while in the former core the clean fuel was moved to the central zone while enough burned-up fuel was loaded on the circumference to maintain criticality. The calculation was accomplished in cylindrical geometry, and the burned-up composition is that listed in Table C.I (minus the xenon). The slightly reduced size of the configuration with the clean fuel in the center gives rise to higher thermal streaming flux in the reflector; however, the increase is only 5 percent.

Localized fuel additions in the vicinity of a beam port would merely act to depress the thermal flux while increasing the fast flux. The effect of small water gaps at the center of the core was explored, but
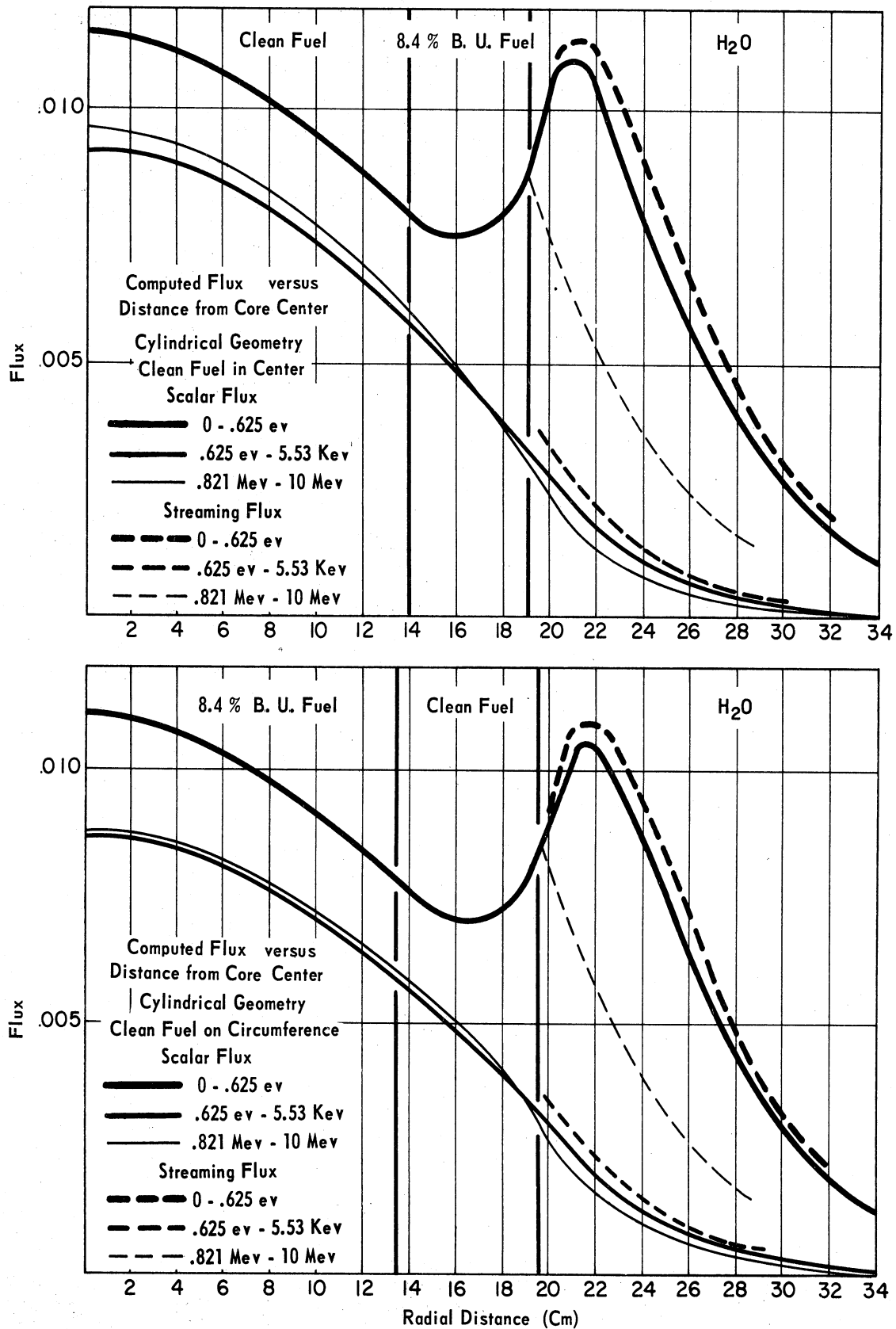
Figure A.6. Computed flux distributions; comparison of depleted fuel in center and on circumference of core.

yielded no net improvement. (A 3 in. gap, while giving rise to a 250

percent local flux peak, would increase the fuel loading by 12 percent,

and decrease the reflector thermal flux by 14 percent.)

5. The Effect of Reflector Materials

NEARLY INFINITE REFLECTORS IN CYLINDRICAL GEOMETRY

The initial examination of reflector materials was carried out in

cylindrical geometry for a cold-clean core composition. The core was

surrounded by a thickness of at least one migration length of each mate-

rial to achieve the effect of a nearly infinite reflector. Group in-

dependent transverse bucklings were used for simplicity.* The results

of the computations are plotted in Figures A.7-A.12. Results for H_2O,

graphite, BeO, D_2O, and combinations of graphite and H_2O are presented.

BeO results are presented because the calculations indicated that this

material is at least as effective as pure Be.

Table A.V, composed of significant information from Figures A.7-A.12,

points to the near linear dependence of the thermal streaming flux on

the average reactor power density (tabulated as the inverse core volume).

BeO, clearly the best reflector, increases the thermal streaming flux by

a factor of 3.2 while raising the average power density by a factor of

---

*B_T^2 = .00165 cm^{-2} was adopted here for the core and B_T^2 = .00125 cm^{-2} for
the reflector. These numbers were obtained from epicadmium flux trav-
erses in the FNR.[17]
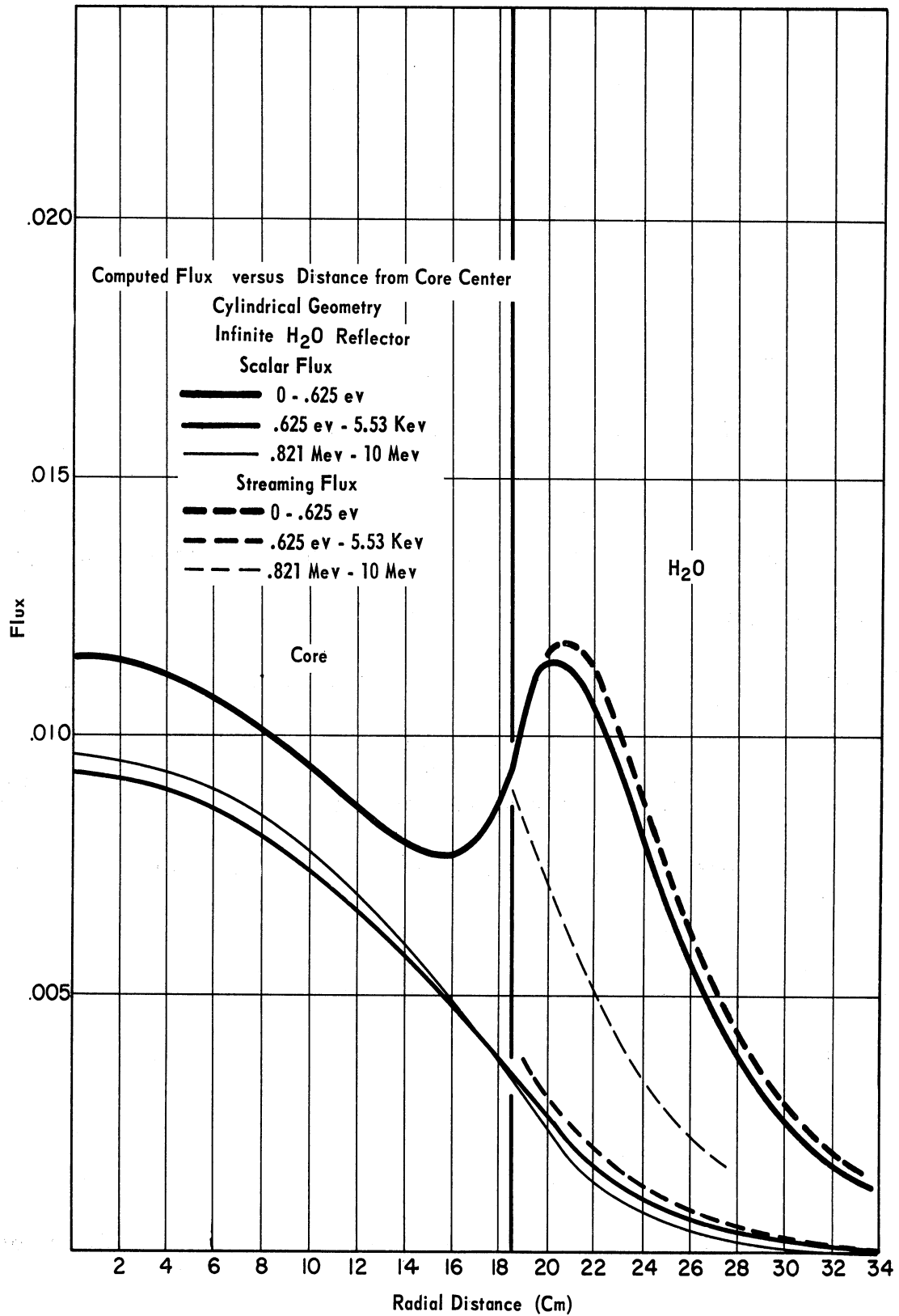
Figure A.7. Computed flux distributions; infinite H_2O reflector; cylindrical geometry.
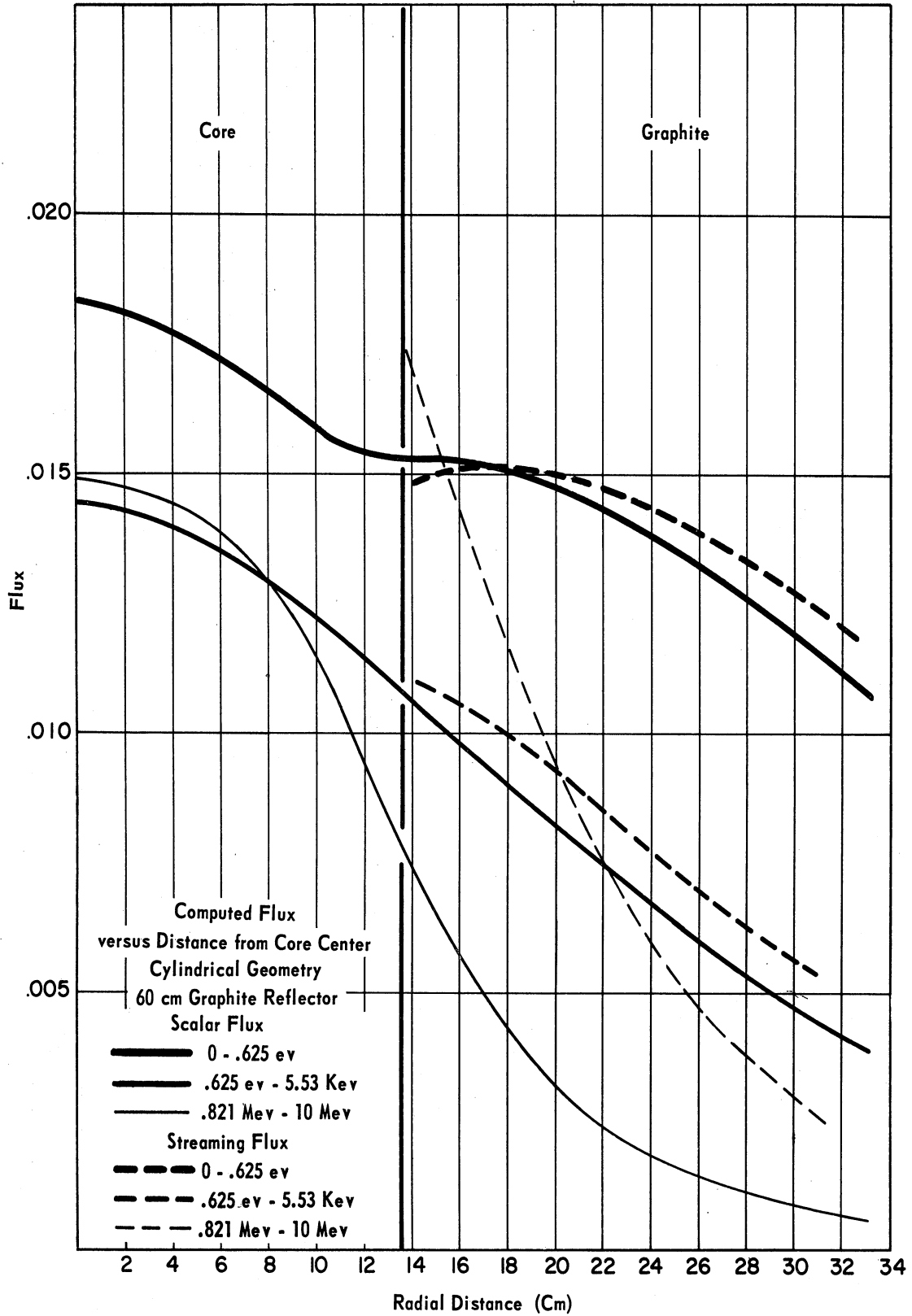
Figure A.8. Computed flux distributions; infinite graphite reflector; cylindrical geometry.
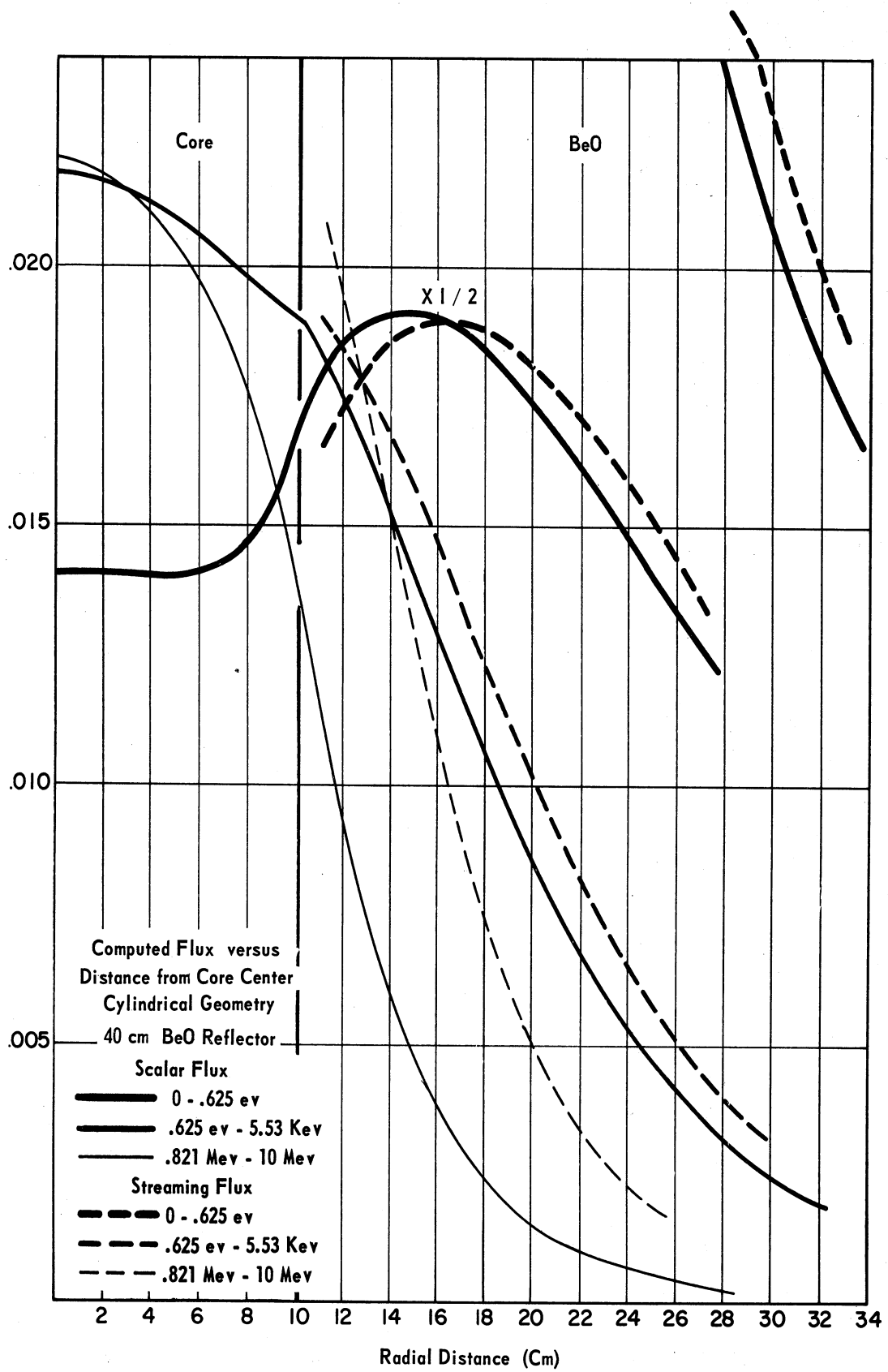
Figure A.9. Computed flux distributions; infinite BeO reflector; cylindrical geometry.
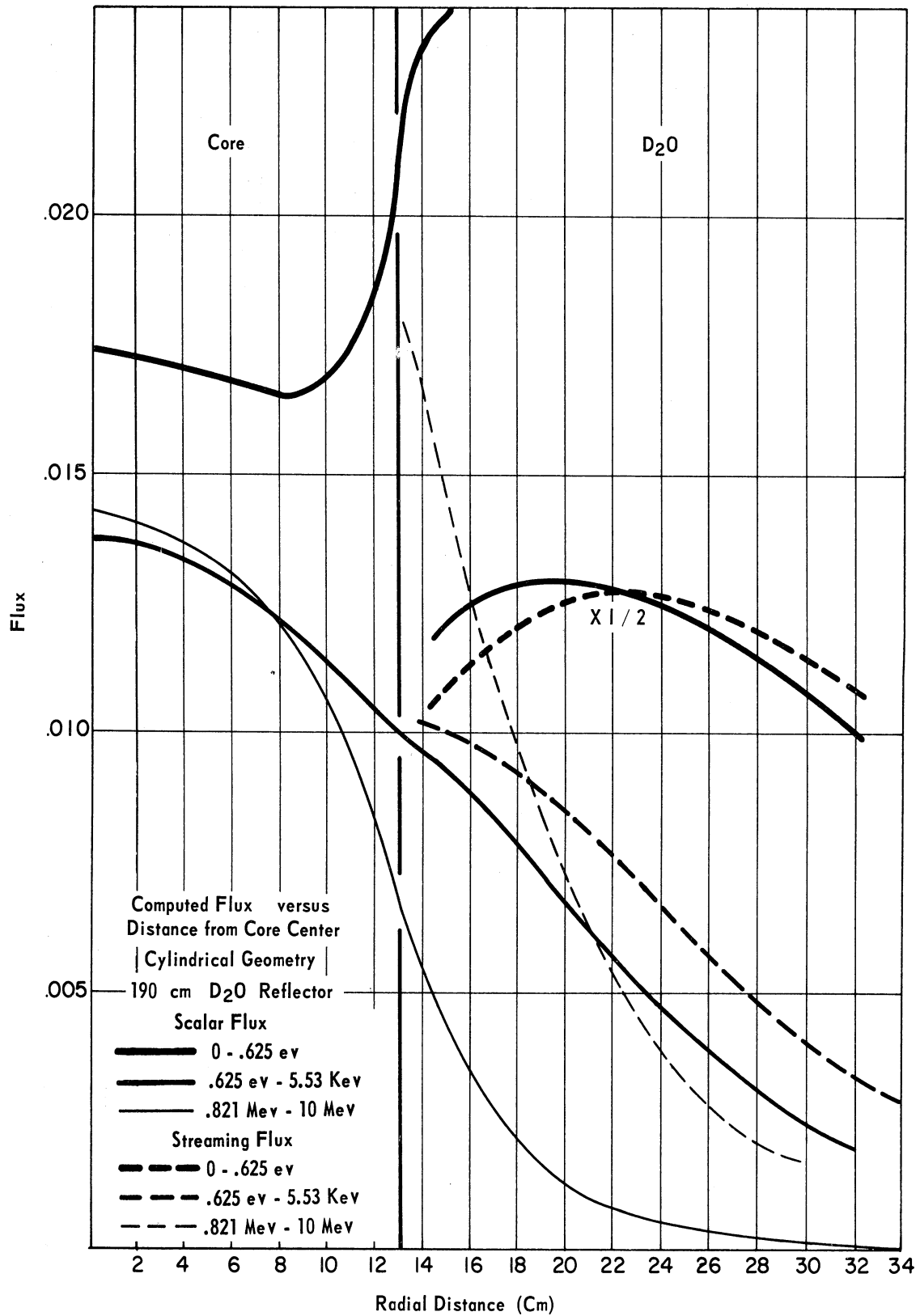
Figure A.10. Computed flux distributions; infinite D₂O reflector; cylindrical geometry.
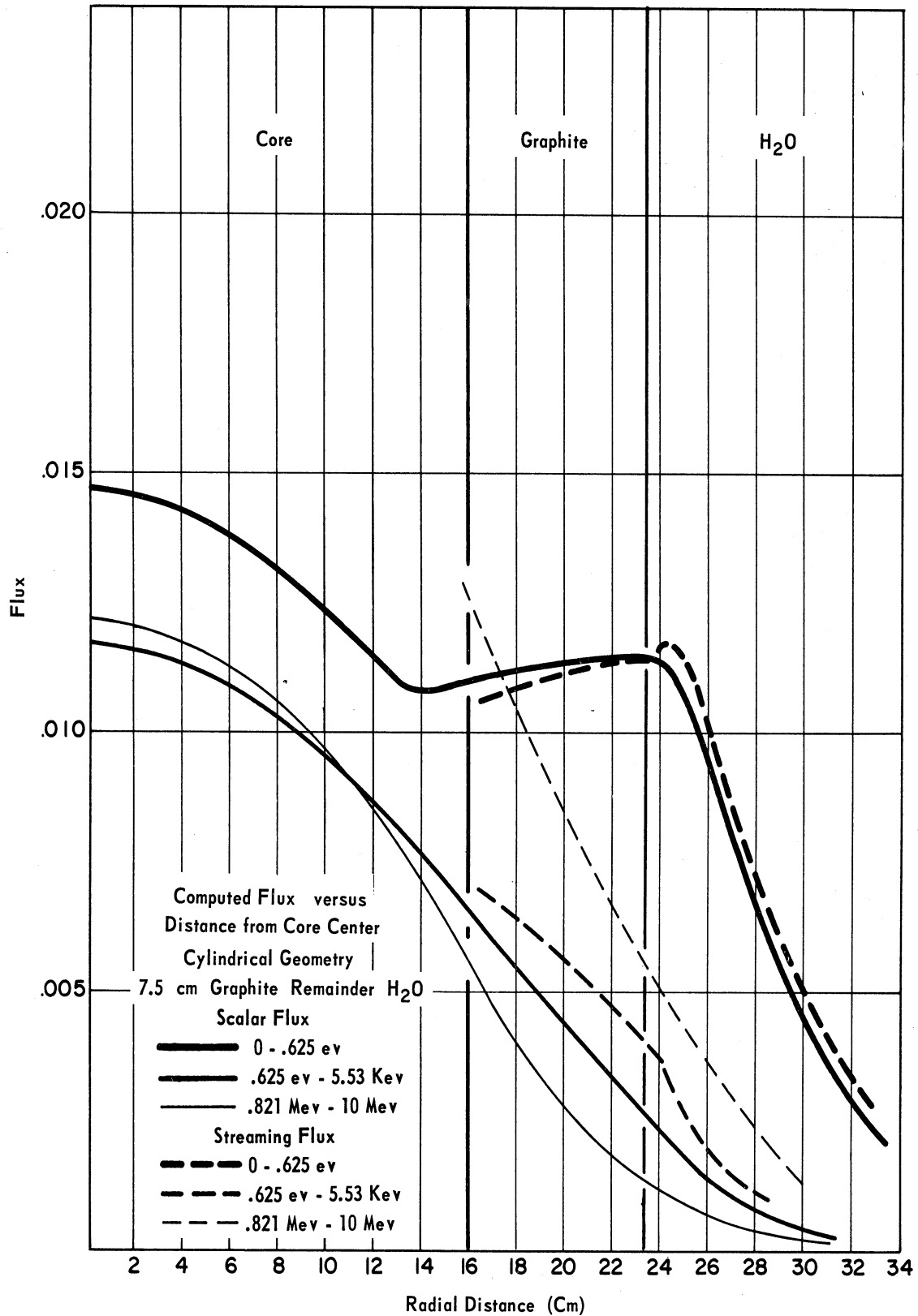
Figure A.11. Computed flux distributions; 7.5 cm graphite-remainder H₂O; cylindrical geometry.
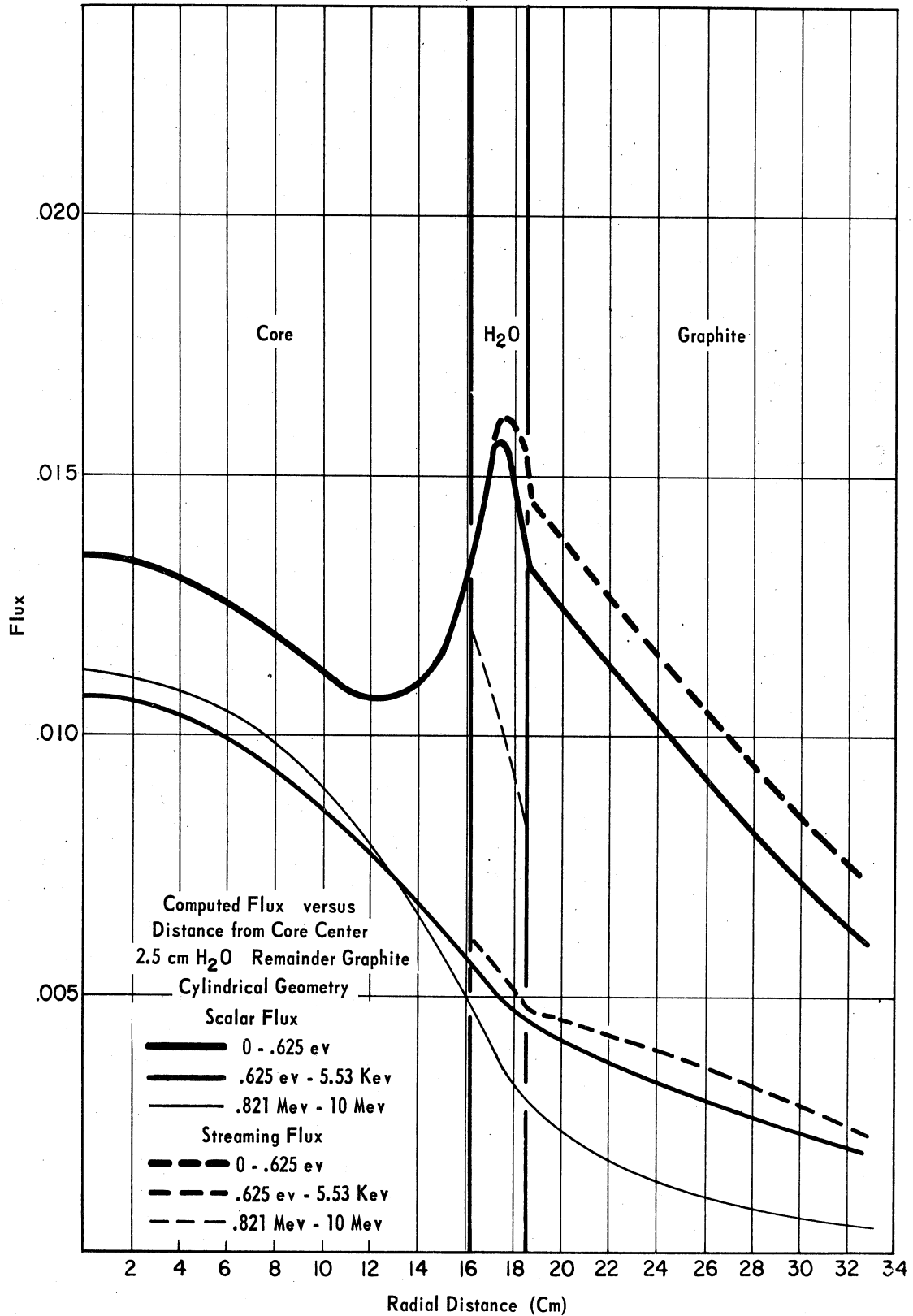
Figure A.12. Computed flux distributions; 2.5 cm H_2O-remainder graphite; cylindrical geometry.

TABLE A.V

INFINITE REFLECTOR—CYLINDRICAL GEOMETRY

VARIOUS REFLECTOR MATERIALS

	S^4_{max}	$(S^4-S^3)_{max}$	$(S^4-S^1)_{max}$	(S^3/S^4_{max})	(S^1/S^4_{max})	$1/V$ in.$^{-3}$
H$_2$O Refl.	.0119	.0095	.0061	.218	.546	.00025
Graph. Refl.	.0151	.0071	.0098	.642	.690	.00046
BeO Refl.	.0379	.0258	.0309	.368	.258	.00083
D$_2$O Refl.	.0255	.0191	.0224	.279	.183	.00050
7.5 cm Graph. Remainder H$_2$O	.0118	.0088	.0070	.305	.432	.00034
2.5 cm H$_2$O Remainder Graph.	.0161	.0108	.0068	.329	.609	.00033

3.3 (compared with the H$_2$O reflected core). The behavior of both BeO and D$_2$O in extending the thermal streaming flux peak farther from the core than in H$_2$O is a reason for the improved spectral ratio, $\left(\phi^1/\phi^4_{max}\right)$. This fact presents a strong motivation for increasing the average reactor power density by improved reflection rather than integrated power boost. Infinite reflection by D$_2$O increases the average power density by a factor of 2 over the H$_2$O reflected core, whereas $(\phi^4-\phi^1)_{max}$ goes up by a factor of 3.7.

Infinite graphite reflection shows up poorly in this set of calculations. Graphite used in conjunction with water demonstrates some im-

provement. The effect of 1 in. of water located between the core and

the graphite is conspicuous. This configuration gives rise to significant

gains in $\mathcal{J}^4_{max}$ and $(\mathcal{J}^4 - \mathcal{J}^3)_{max}$.

REFLECTOR INSERTS

The effect of reflector inserts of graphite, BeO, and D_2O was ex-

amined for the equilibrium Xe, 8.4 percent fuel burnup composition. The

geometry is sketched in Figure A.13. The fundamental core dimensions

shown in Figure A.13a were established from a one-dimensional simula-

tion study. Calculations were performed in XY geometry, and the vertical

transverse bucklings recorded in Table A.I were used to account for leak-

age in the third dimension. The insert widths were chosen to enable the

inserts to accommodate a six inch port. The reactivity contributions,*

$\delta k/k$, are recorded in Table A.VI along with the other significant quan-

tities. Figures A.15-A.18 present the scalar flux and streaming flux

distributions along the x-axes of the inserts. Figure A.19 and the

bottom row of Table A.VI present the results of a calculation of the

core displayed in Figure A.14, possessing full 6 in. layers of BeO.

The insert study permits a comparison of the effects of H_2O, graphite,

D_2O, and BeO independent of power density influences (a somewhat arti-

ficial situation). Light water excels from the standpoint of suppression

---

*The core dimensions were held constant, therefore all of the inserts,
possessing better reflector properties than H_2O, constituted positive
reactivity perturbations. Enhancement of the thermal flux arising from
an increase in power density, then, does not appear here.
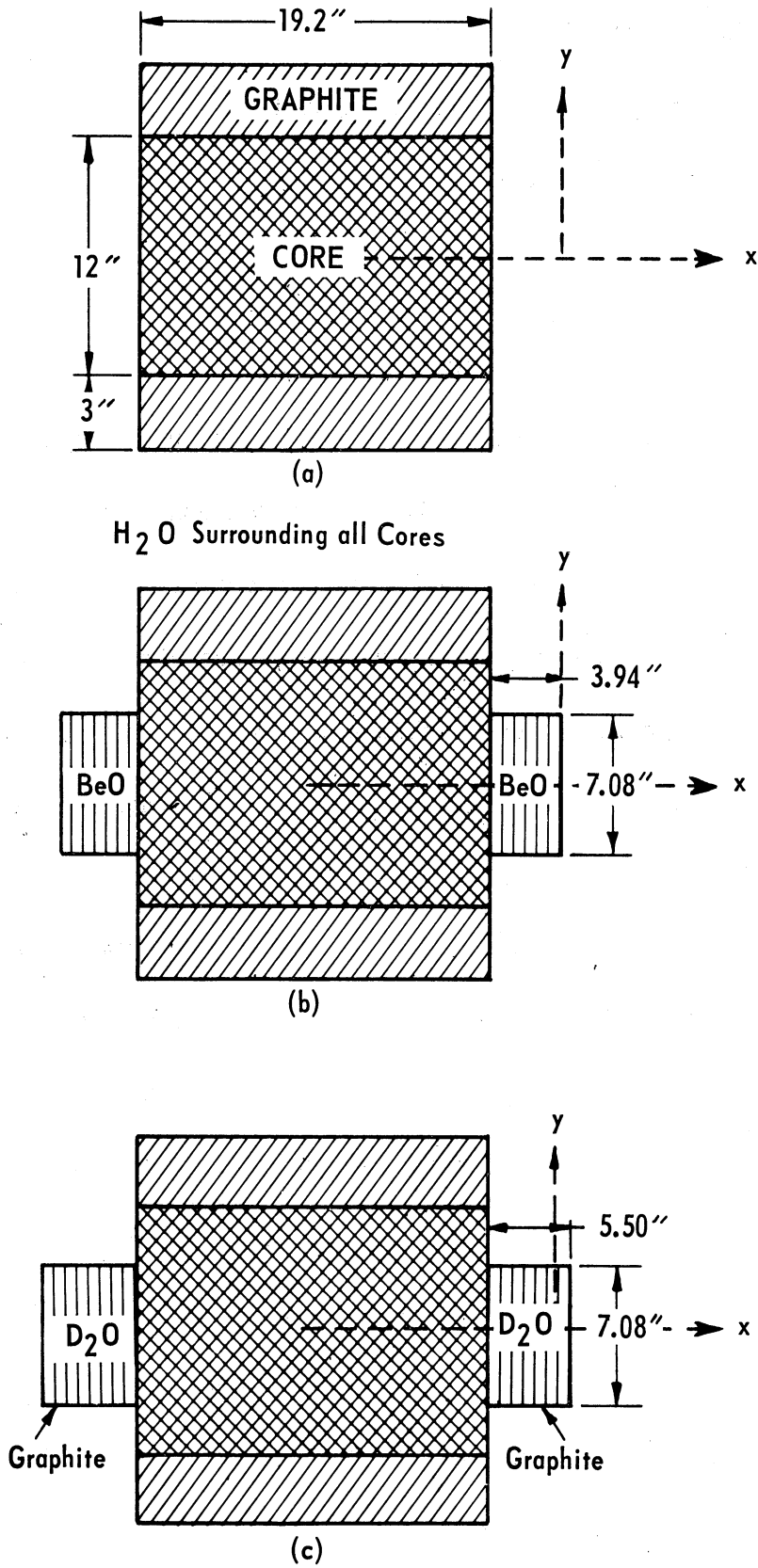
Figure A.13. Core geometries for material insert studies.
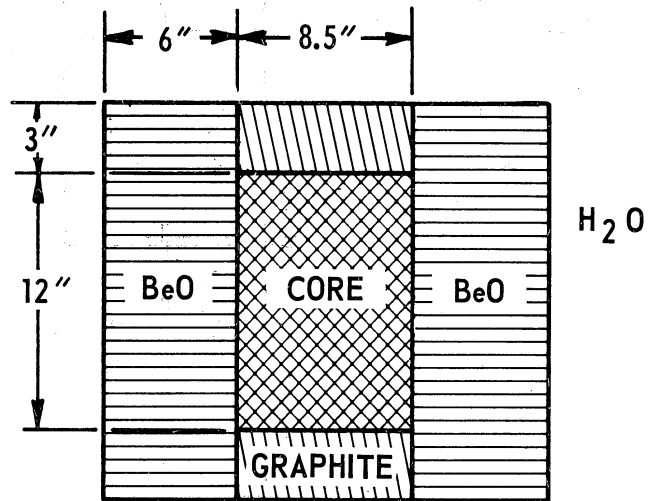
Figure A.14. Core geometry for 6 in. BeO reflector.

TABLE A.VI

TWO-DIMENSIONAL CALCULATIONS; FIXED SIZED CORE—INSERTS OF VARIOUS MATERIALS

	S^4_{max}	$(S^4-S^3)_{max}$	$(S^4-S^1)_{max}$	(S^3/S^4_{max})	(S^1/S^4_{max})	$\delta k/k$	$1/V$ in. $^{-3}$
Full H$_2$O	.0316	.0234	.0173	.266	.506	---	.00018
Graph. Insert	.0288	.0225	.0211	.271	.347	+1.92%	.00018
BeO Insert	.0288	.0205	.0212	.389	.375	+2.3 %	.00018
D$_2$O Insert	.0361	.0279	.0289	.266	.266	+1.86%	.00018
6 in. BeO	.0980	.0649	.0759	.330	.212	---	.00041
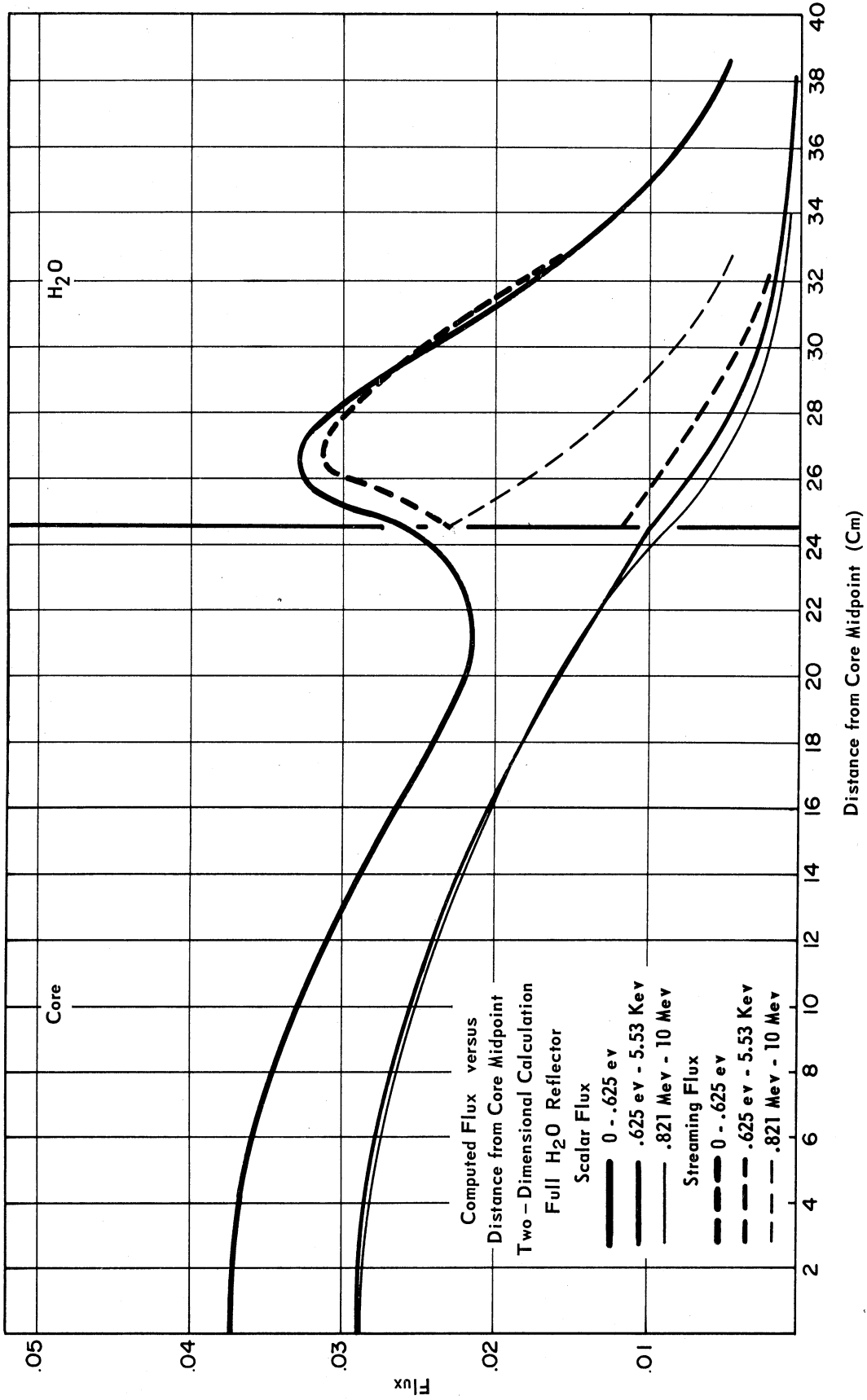
Figure A.15. Two-dimensional computed flux distributions; H_2O reflected.
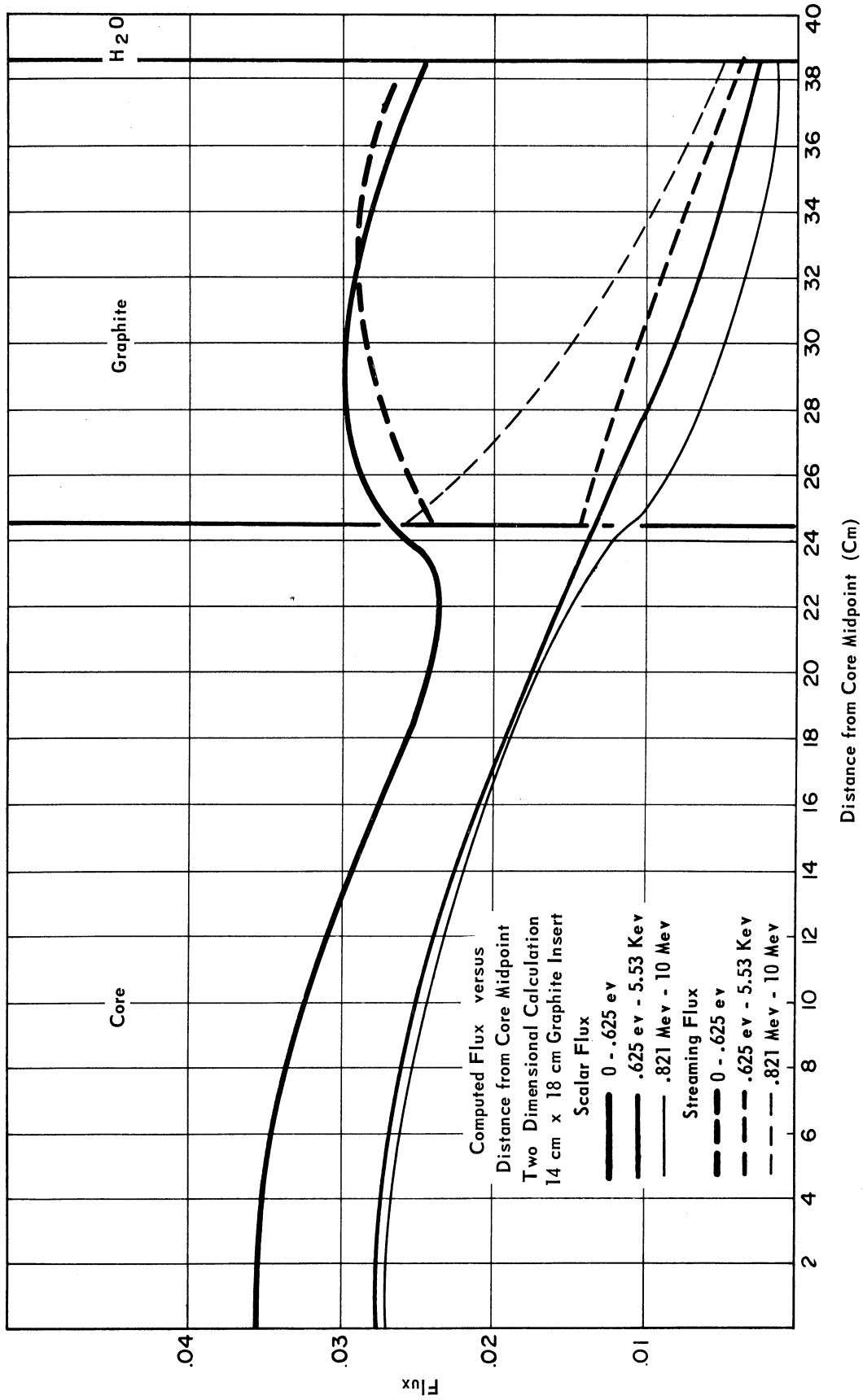
Figure A.16. Two-dimensional computed flux distributions; graphite insert.
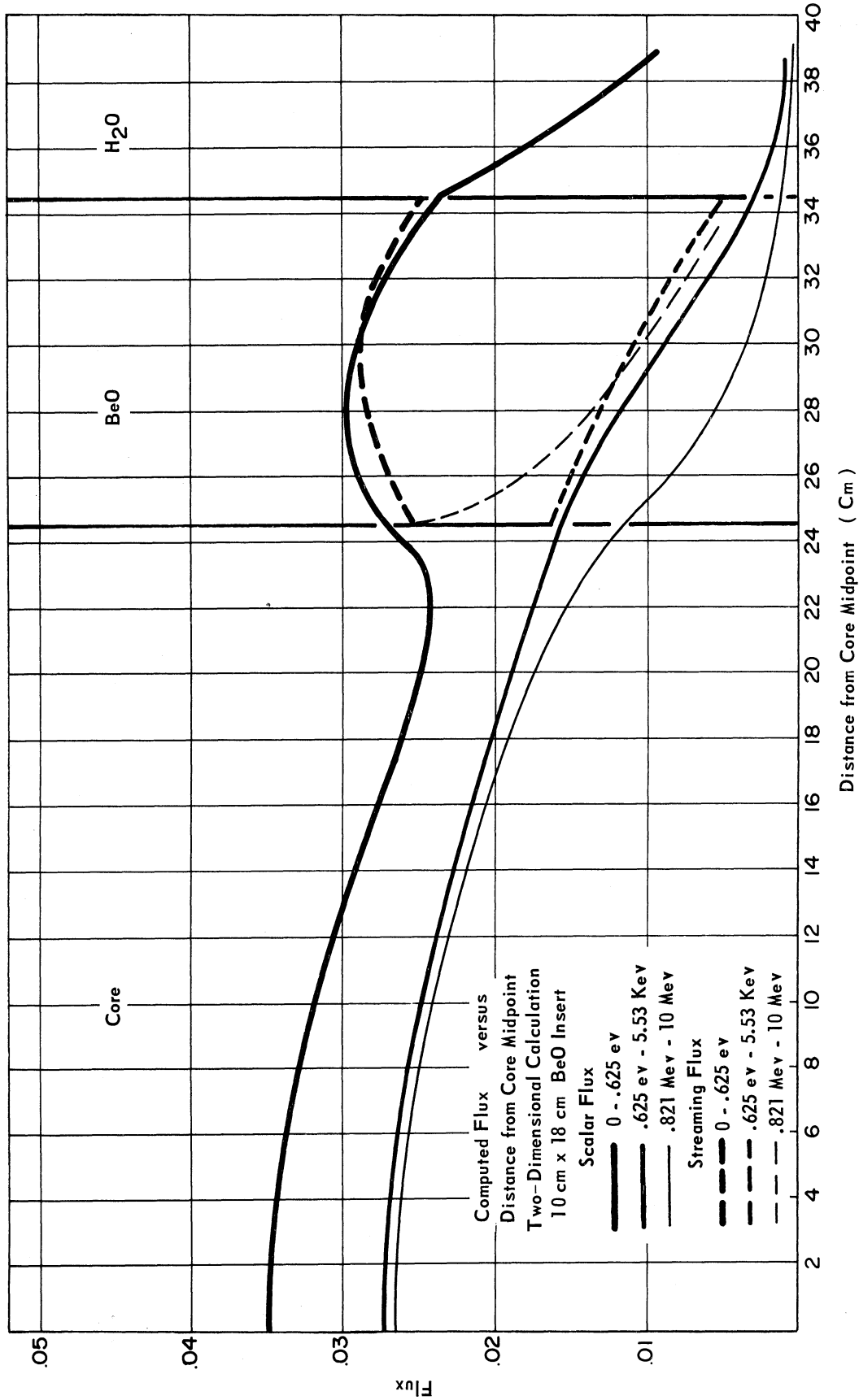
Figure A.17. Two-dimensional computed flux distributions; BeO insert.
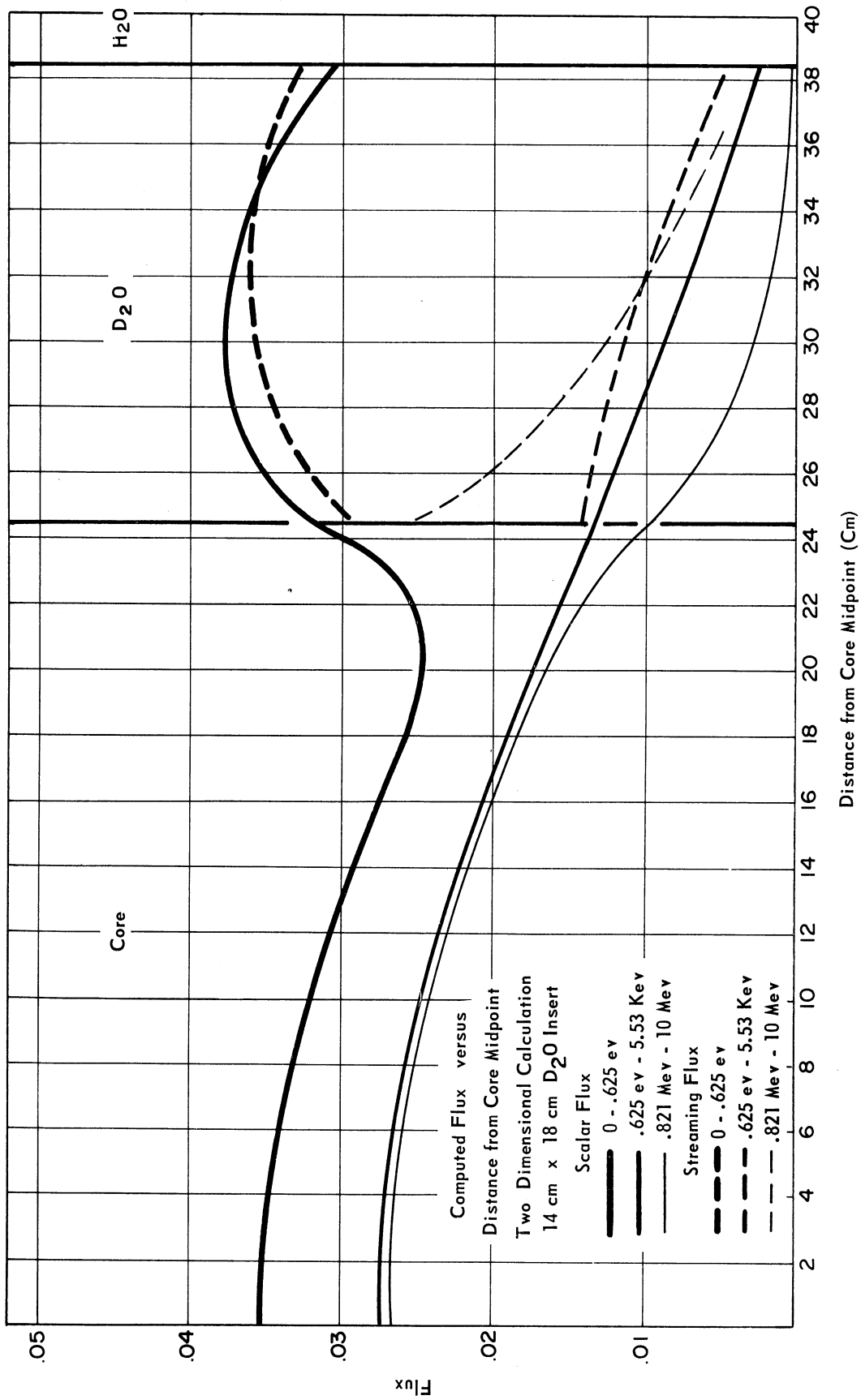
Figure A.18. Two-dimensional computed flux distributions; D_2O insert.
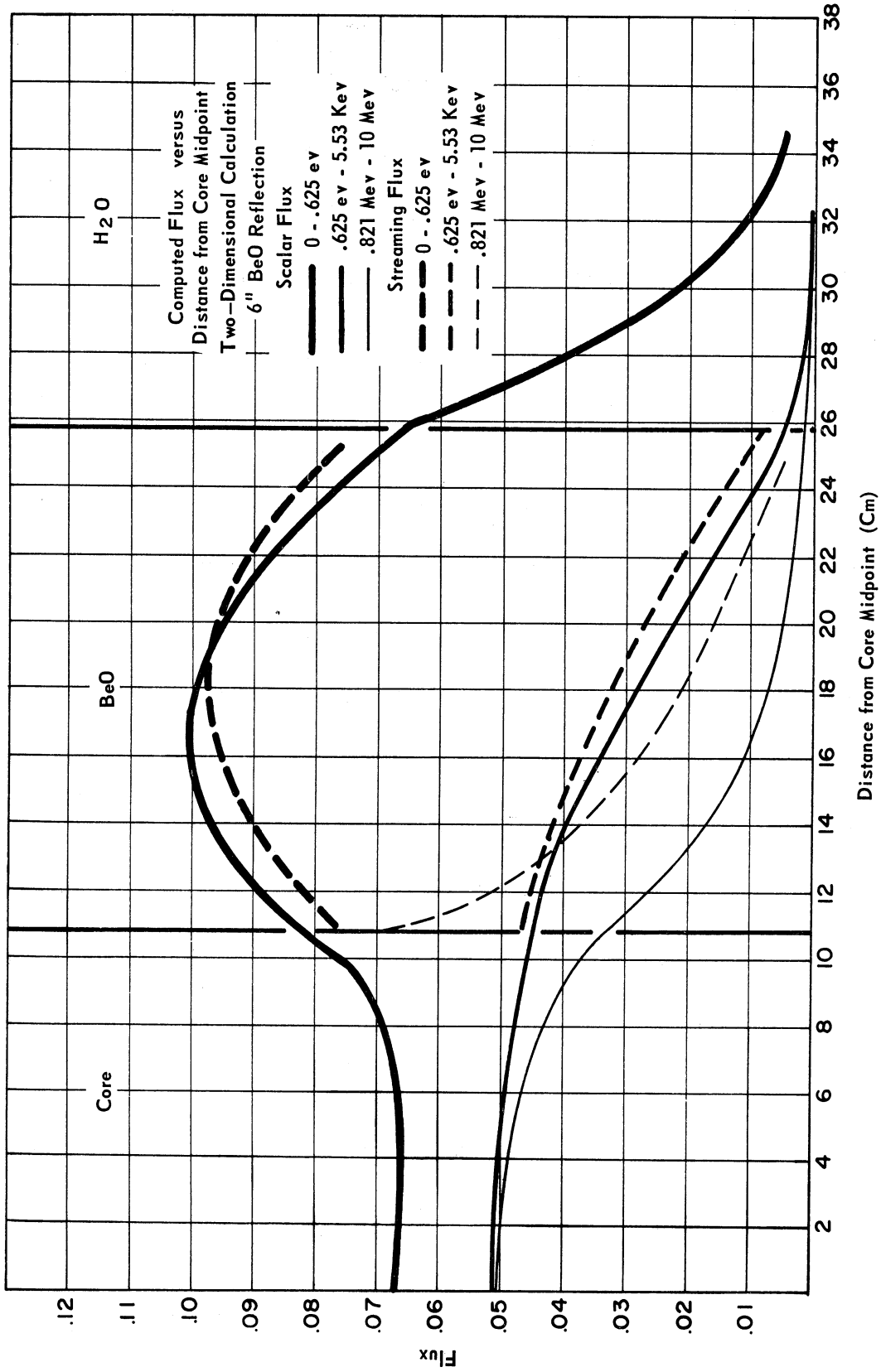
Figure A.19. Two-dimensional computed flux distributions; 6 in. BeO reflector.

of the epithermal content of the spectrum, exemplified by comparison

of the quantities $(\mathscr{J}^3/\mathscr{J}^4{}_{max})$ and $(\mathscr{J}^4 - \mathscr{J}^3)_{max}$ for all four materials.

Every reflector insert diminishes the ratio, $(S^1/S^4{}_{max})$. As before, we

attribute this effect to the longer diffusion lengths possessed by the

insert materials, extending the high thermal flux region farther into

the reflector. The D_2O insert results are particularly conspicuous. In

addition to a 65 percent increase in the quantity $(\mathscr{J}^4 - \mathscr{J}^1)_{max}$, the

D_2O insert brings about a 14 percent enhancement in the thermal flux.

REFLECTOR LAYERS IN PARALLELEPIPED GEOMETRY

The final study constitutes a comparison of the streaming flux for

3 in. and 6 in. layers of reflector materials completely covering two

faces of a rectangular parallelepiped reactor core. Figure A.20 is an

illustration of the geometry. The fundamental water reflected core is

sketched in Figure A.20a. Equilibrium Xe, 8.4 percent fuel burnup was

chosen for the core composition. Once again, H_2O, graphite, BeO, and

D_2O were examined. The configurations were simulated with one-dimen-

sional computations. The results are tabulated in Table A.VII. To-

gether with the quantities previously tabulated, an additional dimension,

d, is listed, which is the distance from the core to the reflector loca-

tion appropriate to the quantity alongside it. Figures A.22 and A.23

present the quantities $(\mathscr{J}^4 - \mathscr{J}^3)_{max}$ and $(\mathscr{J}^4 - \mathscr{J}^1)_{max}$ plotted versus

the dimension a.* Dimension a is an inverse measure of the relative

average power density.
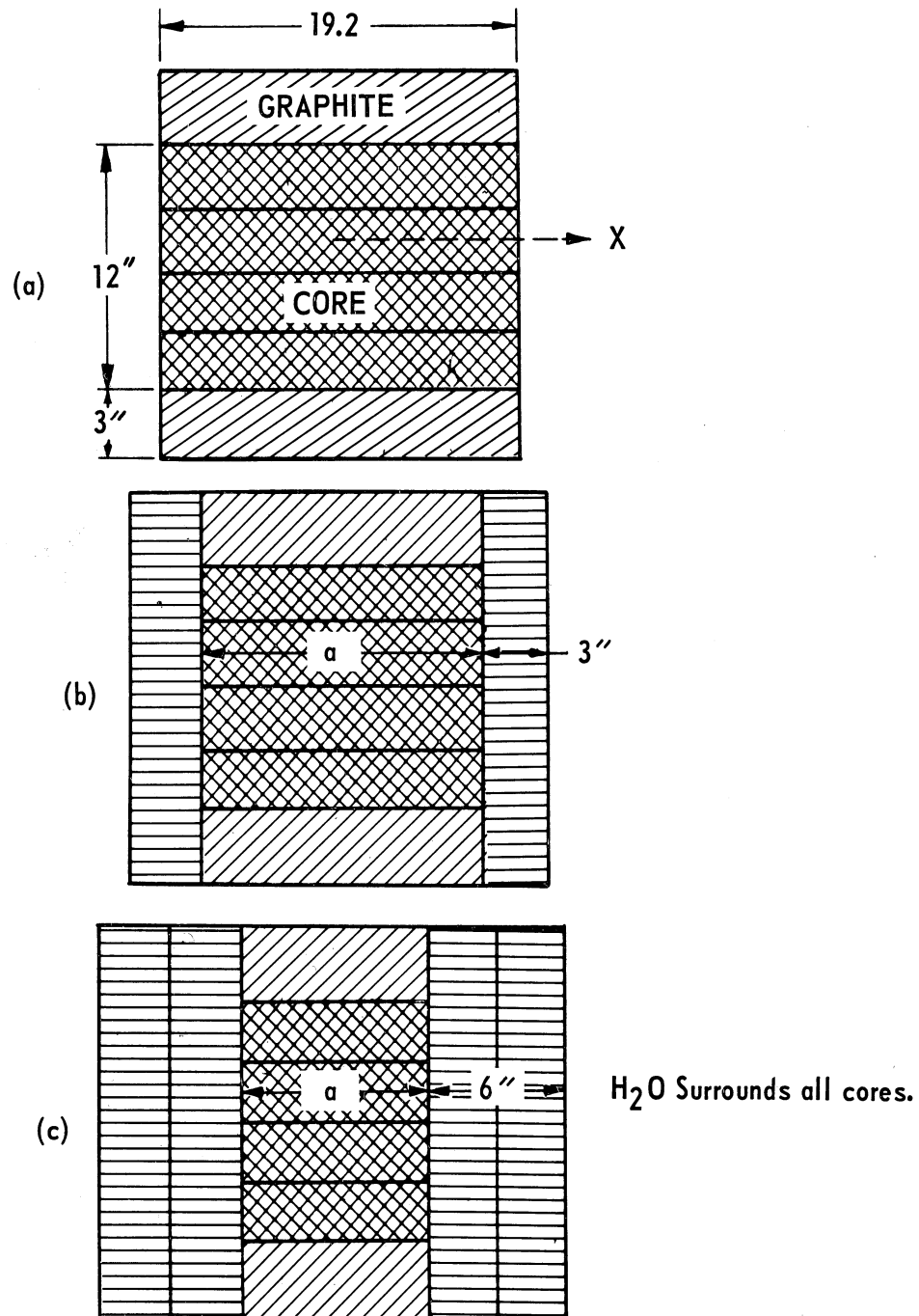
Figure A.20. Core geometries for 3 in. and 6 in. reflectors
of various materials.

*Additional points for 4-1/2 in. of the reflector materials were com-
puted to obtain the shapes of the curves.

The consequence of shrinking dimension a by extension of the transverse dimension, illustrated in Figure A.21, is compared in Table A.VII and Figures A.22 and A.23 with the results of reflection by 3 in. and 6 in. layers of reflector materials. Although this calculation was presented in Section A.4 for the cold-clean core composition, it is of interest to compare this configurational effect with the effect of reflection for the fuel composition presently under consideration.
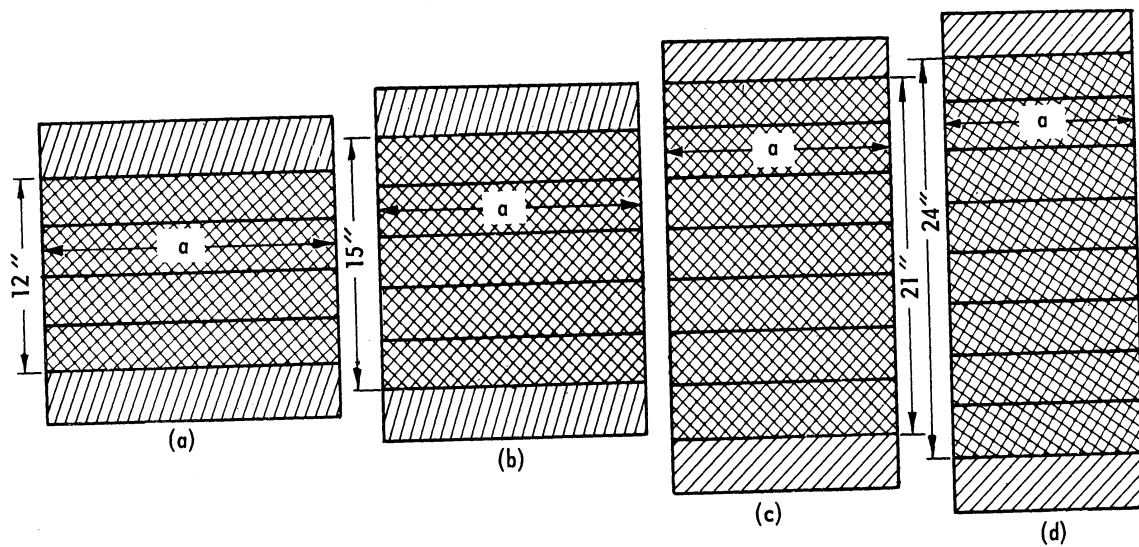
Figure A.21. Core geometries for H_2O reflected configuration studies (equilibrium Xe-8.4 percent burnup fuel).

The results appearing in Table A. VII are too numerous to discuss in detail. The geometry considered in this portion of the study has practical application for ports located 180° apart. Graphite makes a far better showing here than in the cylindrical geometry study. The maximum thermal streaming flux, (S^4_{max}), in this configuration increases more rapidly than the average power density with improved reflection. The results for reflection by 6 in. of D_2O are striking.* Figures A.22

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*Due to the calculational simplification of region-independent bucklings (thermal, in particular), the results for the 6 in. reflectors may be highly unrealistic.

TABLE A.VII

THREE AND SIX INCH REFLECTORS OF VARIOUS MATERIALS

	S^4_{max}	d in.	(S^3/S^4_{max})	(S^1/S^4_{max})	$(S^4-S^3)_{max}$	d in.	$(S^4-S^1)_{max}$	d in.	a in.	$1/V$ in.$^{-3}$
3 in. Graph. Refl. 4 Elem. Trans.	.614	3	.254	.324	.474	3	.408	3	15.5	.000223
6 in. Graph. Refl. 4 Elem. Trans.	1.28	4.3	.186	.174	1.08	5.6	1.09	5.6	10.9	.000317
3 in. BeO Refl. 4 Elem. Trans.	.773	2.1	.333	.307	.582	3.0	.600	3.0	12.9	.000268
6 in. BeO Refl. 4 Elem. Trans.	1.65	3.1	.250	.140	1.29	3.9	1.56	3.9	8.5	.000406
3 in. D_2O Refl. 4 Elem. Trans.	.853	3	.254	.232	.636	3	.656	3	14.4	.00024
6 in. D_2O Refl. 4 Elem. Trans.	3.45	4.3	.144	.074	2.99	4.9	3.22	4.9	5.45	.000635
H_2O Refl. 4 Elem. Trans.	.485	.9	.216	.464	.385	1.1	.282	1.3	19.2	.00018
H_2O Refl. 5 Elem. Trans.	.714	.9	.216	.444	.569	1.1	.428	1.3	15.0	.000185
H_2O Refl. 7 Elem. Trans.	.965	.9	.216	.422	.770	1.1	.596	1.3	12.2	.000163
H_2O Refl. 8 Elem. Trans.	1.039	.9	.216	.416	.830	1.1	.648	1.3	11.6	.00015
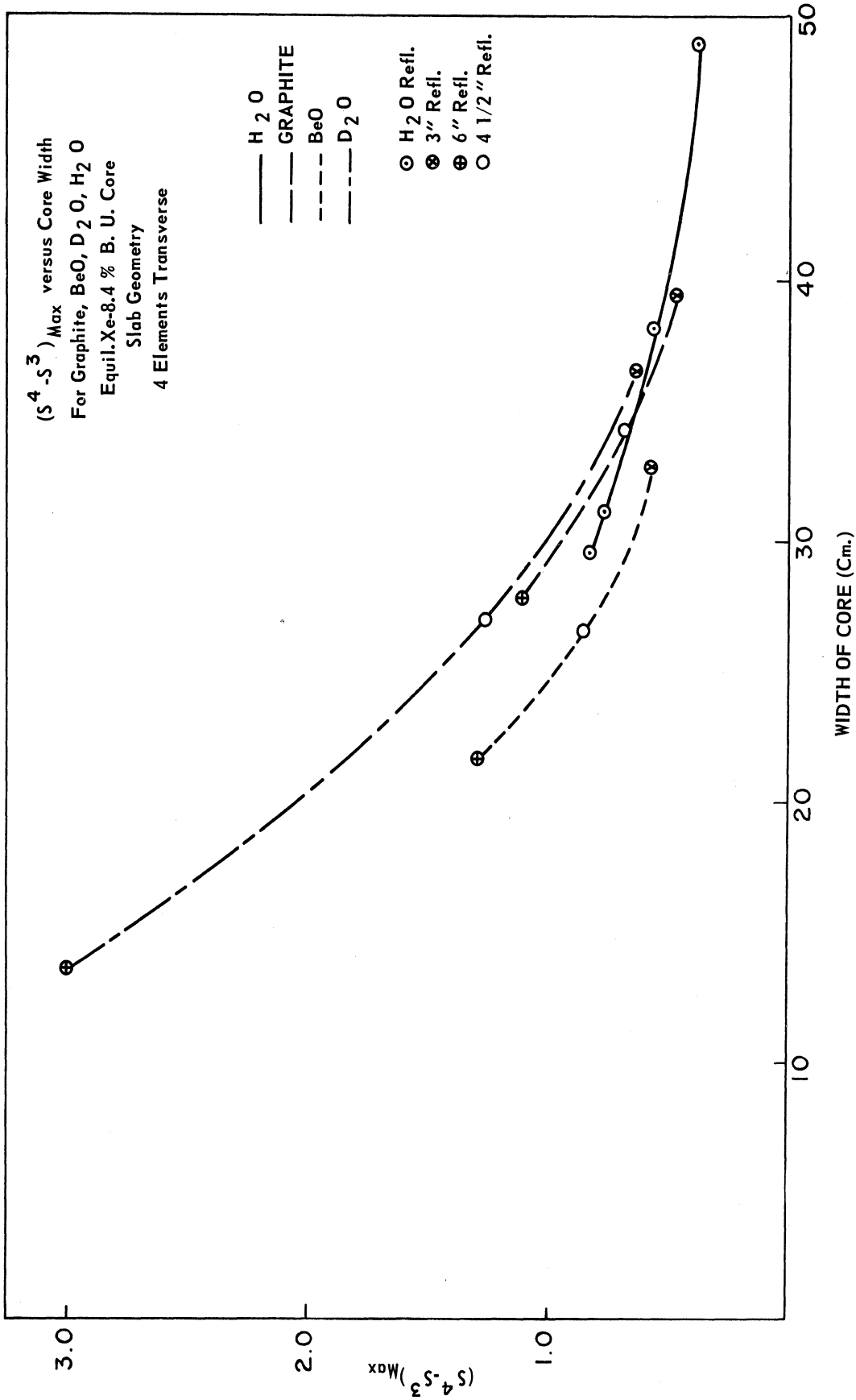
Figure A.22. $(S^4-S^3)_{max}$ for 3- and 6-in. reflection.
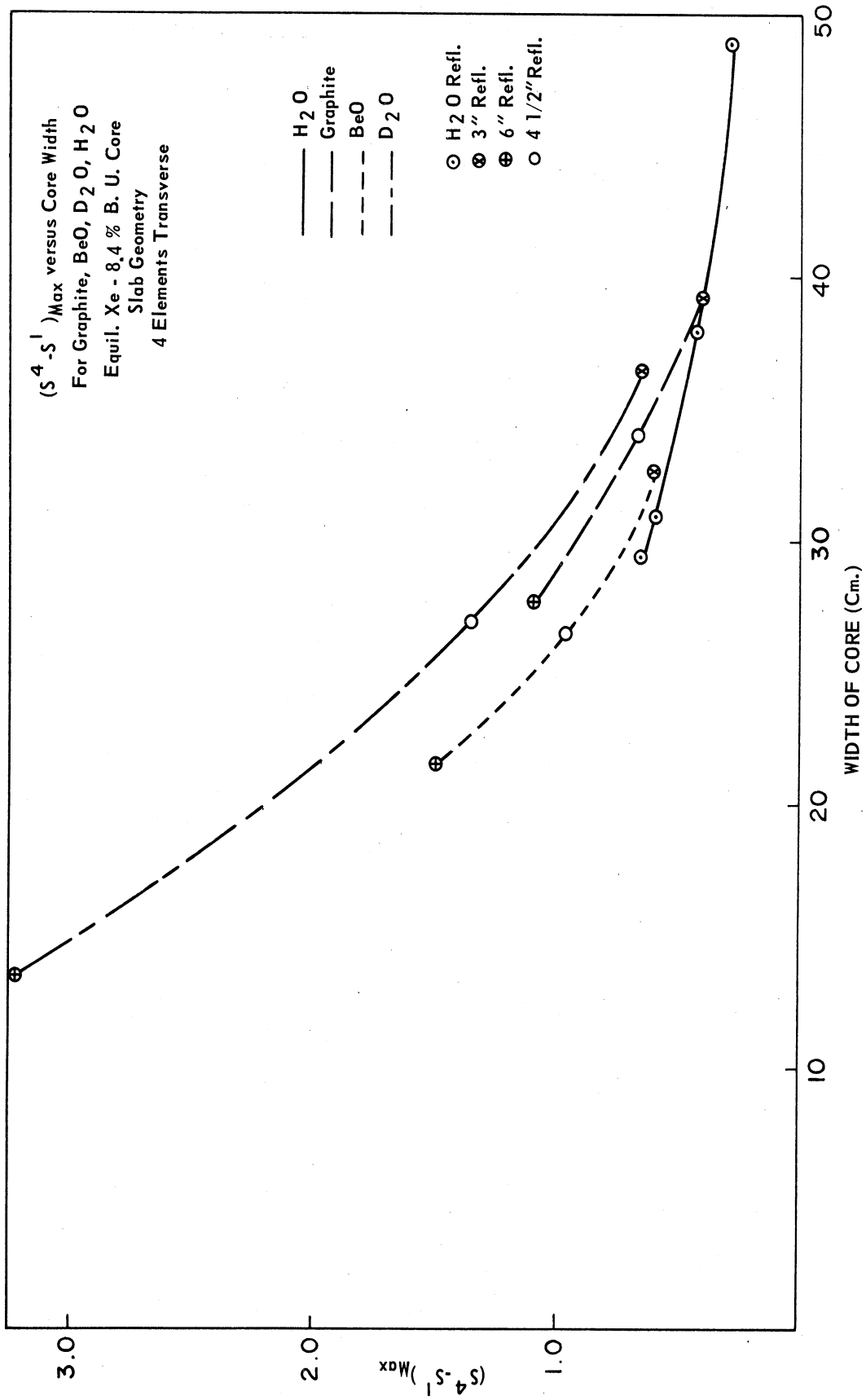
Figure A.23. $(S^4 - S^1)_{max}$ for 3- and 6-in. reflection.

and A.23 compare the reflectors on the basis of constant average power density (excluding the curves for H_2O reflection obtained by altering the ratio of the core dimensions). On this basis, D_2O excels, followed in order by graphite and BeO.

6. Final Remarks

Although the group constants used in the calculations were obtained in a consistent manner, a question may be raised regarding the validity of the cross sections. The source of the cross section library is cited in Appendix C. Much of the analysis reported in the literature[31] is based upon this set of cross sections or slight variations of it. The reasonable agreement between the predicted and measured distributions of the thermal and epicadmium fluxes presented in Figures 3.6, 3.8, and 3.19 tends to corroborate the validity of the group constants for H_2O, graphite, and the core.

In order to examine the validity of the D_2O group constants, a measurement was made with a limited quantity of D_2O, and the experimental results were compared with a group-diffusion calculation. A flux traverse was performed in a cylindrical D_2O insert 5 in. long and 5-1/2 in. in diameter placed against the center of the south face of the FNR.[32] The two-dimensional analysis utilized 4 groups in cylindrical geometry. A comparison of the measured and calculated thermal and epithermal flux distributions along the axis of the insert is presented in Figure A.24.
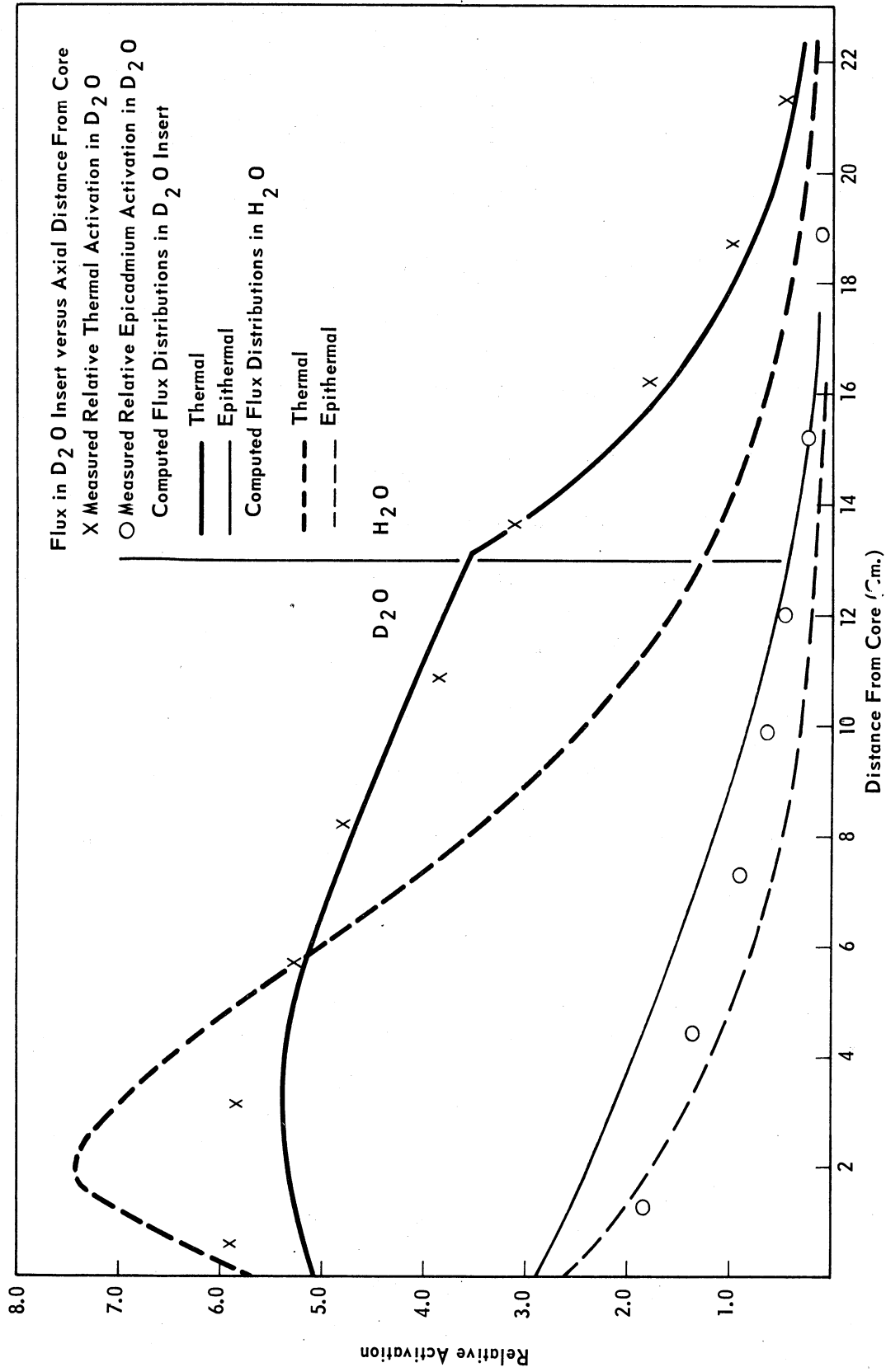
Figure A.24. Measured and computed flux distributions along axis of cylindrical D_2O insert.

Both the measured and calculated distributions were normalized to distributions obtained along the same axis in the complete light water medium (also plotted in Figure A.24).

Many of the results reported here were based upon the technique described in Section A.2, simulating three-dimensional geometry using one-dimensional computations. An examination of the validity of this method is afforded by comparing the results of actual two-dimensional calculations with the results of one-dimensional simulations. Since four-group, two-dimensional computer studies are extremely time-consuming, few comparisons were made. The first comparison was carried out for the core illustrated in Figure A.14, possessing symmetric 6 in. layers of BeO. The two-dimensional calculation resulted in an eigenvalue (See Eq. B.1) .6 percent lower than that obtained in the one-dimensional simulation. The water reflected core of Figure A.13a was examined next, the two-dimensional calculation yielding an eigenvalue 3 percent lower than the one-dimensional simulation. Finally, the deviation for the cold-clean, water reflected core shown in Figure A.1b was 2 percent. None of the discrepancies are serious, considering the disagreement between the results of group-diffusion calculations and precise criticality determinations for clean, homogeneous assemblies reported by Feiner.[33] It is curious, however, that the smaller cores yield better agreement. It was anticipated that the deviation between the one-dimensional simulation results and the two-dimensional results would be inversely proportional to the core size, owing to corner effects.

Apparently a reflector composition influence exists, as well.

A question may be raised concerning the accuracy of the calculated fast flux magnitudes in the reflector, ϕ^1 and ϕ^2. P-1 theory tends to overestimate the attenuation of the forward-peaked, high energy components of the spectrum. Experimental fast flux determinations are scarce. Since the reflector regions of greatest interest lie within 1 to 3 fast neutron mean free paths from the core, P-1 errors in the fast flux are presumed to be small.

Most of the core alterations examined in this appendix involved increases in the reactor power density. Practical considerations call for a remark concerning core heat removal capabilities. A cursory examination of this problem was carried out with an existent computer program,[34] originally written to evaluate the FNR two megawatt performance. The results of the calculation, assuming a constant coolant mass flow rate through the core and an integrated reactor power of one megawatt, indicate that in going from 24 to 4 regular fuel elements a maximum fuel cladding temperature rise of only 7°F is anticipated.

7. Subroutine STREAM

Subroutine STREAM, used in conjunction with the FOG group-diffusion code (control program 3), computes the partial current, defined by:

$$j_i^+(x) = \frac{\phi_0(x)}{4} - \frac{\ell_{tr}}{6} \frac{d\phi_0(x)}{dx} \quad , \tag{A.21}$$

and the streaming flux, defined by Eq. (A.6), at each mesh point in a designated region. The streaming flux integral is performed over the designated region plus the region before it. Thus for reasonable accuracy the width, w, of the region immediately before the designated region must be defined sufficiently large so that $\Sigma_t^i\, w \gg 1$ for each energy group. The Fortran program listing of the revised main program of FOG and subroutine STREAM are recorded here.

In order to call the subroutine L63 of the input data must be set different from zero. Then a master card and one card for each energy group is expected at the end of the FOG input deck. The card arrangement is as follows:

Master Card (Format 4I12)

| N(1) | = 0 | Bypass Subroutine |
| | = 1 | Utilize Subroutine |

| N(2) | = 0 | Do not compute j^+ |
| | = 1 | Compute j^+ |

| N(3) | = 0 | Do not compute ℓ |
| | = 1 | Compute ℓ |

| N(4) | = I | I is the FOG region designated for the computation. |

Cross Section Card* (Format 2F12.0)

| F(1) | = Σ_s^i | Scattering cross section in region I |
| F(2) | = Σ_s^i | Scattering cross section in region I-1 |

_____

*One card is required for each energy group.

```
C FOG CODE MAIN PROGRAM - WRITTEN AT NORTH AMERICAN AVIATION
C REVISED AT UNIVERSITY OF MICHIGAN , 1962


$COMPILE FORTRAN,PRINT OBJECT,PUNCH OBJECT,DUMP,EXECUTE        MAIN3000
C     CONTROL PROGRAM NO.3. THIS PROGRAM CONTAINS THE CRITICALITY SEARCHFOG23080
C     ES AND ADJOINT FLUX CALCULATIONS.                        FOG23090
C                                                              FOG23100
      DIMENSION  LM(40),LB(40),NPO(3),LBS(40),MR(40),L(250),A(5000),PHI(FOG23110
     1239,4),APHI(239,4),SOU(239),SOP(40),SOUP(239),BSA(40,4),SOPP(40)   FOG23120
     2,R(239),RI(239),NN(5),C(5),D(239),DP(40),G(239),GP(40),G1(239),G1PFOG23130
     3(40),BETA(239),DELTA(239),DELR(40),T(4),DIF(40,4),TRANS(40,     FOG23140
     44),  SIGT(40,4),CIH(4),OMEGA(4),OMEG1(4),FLP(239),BUCK(40),BUCK2(4FOG23150
     50,4),VUSIG(40,4),SIGPT(40),FL(239),SIGA(40,4),AC1(40),AC2(40),AC3(FOG23160
     640,4),AC4(40,4),AC5(40,4),BUCK3(4),AC6(40,4),AC7(40,4), AC8(40,4),FOG23170
     7AC9(40),AC10(40),CHI(40,4) ,BL(4),B2(4),B3(4),B4(4),B5(4),RW(40)  FOG23180
      DIMENSION NFU(40),SOUS(239),SOPS(40),SIP(4)               FOG23190
      COMMON L,A,APHI,D,G,G1,AC3,AC4,AC5,AC6,AC7,AC8,DP,NFU,C,NN  FOG23200
      EQUIVALENCE (L(249),N),(L(248),LOB)                       FOG23210
      EQUIVALENCE (L(1),NOG),(L(2),N1),(L(3),N2),(L(4),N3),(L(5),N4),(L(FOG23220
     16),N5),(L(7),M),(L(8),LM),(L(48),N6),(L(49),N7),(L(50),N8),(L(51),FOG23230
     2N9),(L(52),N10),(L(53),N11),(L(54),N12),(L(55),N13),(L(56),N14),(LFOG23240
     3(57),N15),(L(58),N16),(L(59),N17),(L(60),NPO),(L(63),N18),(L(64),NFOG23250
     419),(L(65),N20),(L(66),N21),(L(67),LBS),(L(107),N22),(L(108),N23),FOG23260
     5(L(109),MA),(L(110),MR),(L(150),MICT),(L(151),MICT2),(L(152),LB)  FOG23270
      EQUIVALENCE (A(4956),LB1),(A(4957),LB2),(A(4958),LBT),(A(4959),LB3FOG23280
     1),(A(4960),SOPS),(A(3340),SIP),(A(3344),LB5),(A(3345),GAM1)     FOG23290
      EQUIVALENCE (A(1),ESP1),(A(2),ESP2),(A(3),EIGEN2),(A(4),ESP3),(A(5FOG23300
     1),THET),(A(6),FMUL),(A(8),RW),(A(48),C1),(A(49),C2),(A(50),BUCK,BUFOG23310
     2CK1,BUCK3),(A(368),C3),(A(369),C4),(A(395),CIH),(A(400),BUCK2),    FOG23320
     3(A(560),TRANS,SOP),(A(720),AC1),(A(760),AC2),(A(800),VUSIG),(A(960FOG23330
     4),DELR),(A(1000),SIGA),(A(1160),AC9),(A(1200),DIF),(A(1360),G1P),(FOG23340
     5A(1560),AC10),(A(1400),CHI),(A(1600),OMEGA),(A(1604),OMEG1),(A(160FOG23350
     68),T),(A(1612),SIGPT),(A(1652),PG1),(A(1653),PG2),(A(1654),PG3),(AFOG23360
     7(1655),PG4),(A(1656),N100),(A(1657),EIGEN),(A(1658),EIGEN1)      FOG23370
      EQUIVALENCE (A(1659),P),(A(1660),GP),(A(1700),SOU),(A(1939),A1),( FOG23380
     1A(1940),SOPP),(A(1980),LT1),(A(1981),LT2),(A(1982),B1),(A(1983),BLFOG23390
     2),(A(1987),B2),(A(1991),B3),(A(1995),B4),(A(1999),W),(A(2085),B5),FOG23400
     3(A(2009),W1),(A(2004),W2),(A(2014),PSI1),(A(2019),PSI2),(A(2024),DFOG23410
     4ET),(A(2025),PIM),(A(2026),ERR),(A(2027),SUM),(A(2028),GAM),(A(202FOG23420
     59),NM1),(A(2034),NM2),(A(2038),MM1),(A(2039),NAA),(A(2044),NSKP), FOG23430
     6(A(2049),ICT),(A(2054),ICT1),(A( 2055),ICT2),(A(2056),ICT3),(A(205FOG23440
     77),DELRT),(A(2058),BUCKT),(A(2059),C10),(A(2064),SGPT)            FOG23450
      EQUIVALENCE (A(2600),SOUP),(A(2850),R),(A(3100),RI),(A(3350),FL),(FOG23460
     1A(3600),FLP),(A(3840),SIGT),(A(4000),PHI),(APHI,SOUS,BSA),(G,BETA)FOG23470
     2,(G1,DELTA),(A(2068),CON),(A(2069),CON1),(A(2074),FAC)           FOG23480
      CALL ZERO
      MICT=35                                                   FOG23490
      MICT2=10                                                  FOG23500
    5 READ INPUT TAPE 7,6                                       FOG23510
    6 FORMAT (72H                                               FOG23520
     1                                                          FOG23530
      WRITE OUTPUT TAPE 6,6                                     FOG23540
   10 CALL DATIN                                                FOG23550
   20 CALL OMCAL                                                FOG23560
      IF (N100-1) 30,5,5                                        FOG23570
   30 NSKP=0                                                    FOG23580
   40 CALL DAPR                                                 FOG23590
      IF (N22-1) 50,50,110                                      FOG23600
   50 CALL CONS                                                 FOG23610
   60 CALL FLUX                                                 FOG23620
      IF (N20) 70,90,65                                         FOG23630
   65 IF (N20-3) 70,70,80                                       FOG23640
   70 CALL CRIT1                                                FOG23650
      GO TO 90                                                  FOG23660
   80 CALL CRIT2                                                FOG23670
   90 CALL ANPR                                                 FOG23680
  100 IF (N22) 120,120,110                                      FOG23690
  110 CALL AFLUX                                                FOG23700
  120 IF (N18) 140,140,130                                      FOG23710
  130 CALL CHECK                                                FOG23720
      CALL REPR                                                 FOG23730
      CALL STREAM                                               FOG23685
  140 IF (L(250)) 5,5,150                                       FOG23740
  150 CONTINUE                                                  FOG23750
      END                                                       FOG23760
```

```
C SUBROUTINE STREAM WRITTEN AT U OF M TO BE USED IN CONJUNCTION WITH FOG


$ COMPILE FORTRAN,PRINT OBJECT,PUNCH OBJECT                    SUBSTREAM
      SUBROUTINE STREAM                                                    1
      DIMENSION  LM(40),LB(40),NPO(3),LBS(40),MR(40),L(250),A(5000),PHI(   1
     1239,4),APHI(239,4),SOU(239),SOP(40),SOUP(239),BSA(40,4),SOPP(40)     2
     2,RI(239),RI(239),NN(5),C(5),D(239),DP(40),G(239),GP(40),G1(239),G1P  3
     3(40),BETA(239),DELTA(239),DELR(40),T(4),DIF(40,4),TRANS(40,         4
     44),  SIGT(40,4),CIH(4),OMEGA(4),OMEG1(4),FLP(239),BUCK(40),BUCK2(4   5
     50,4),VUSIG(40,4),SIGPT(40),FL(239),SIGA(40,4),AC1(40),AC2(40),AC3(   6
     640,4),AC4(40,4),AC5(40,4),BUCK3(4),AC6(40,4),AC7(40,4), AC8(40,4),   7
     7AC9(40),AC10(40),CHI(40,4) ,BL(4),B2(4),B3(4),B4(4),B5(4),RW(40)     8
      DIMENSION  CURR(239),NNN(4),AA(2),SUMM(239,4)
      DIMENSION  NFU(40),SOUS(239),SOPS(40),SIP(4)                        10
      COMMON L,A,APHI,D,G,G1,AC3,AC4,AC5,AC6,AC7,AC8,DP,NFU,C,NN          11
      EQUIVALENCE (L(249),N),(L(248),LOB)                                 12
      EQUIVALENCE (L(1),NOG),(L(2),N1),(L(3),N2),(L(4),N3),(L(5),N4),(L(  13
     16),N5),(L(7),M),(L(8),LM),(L(48),N6),(L(49),N7),(L(50),N8),(L(51),  14
     2N9),(L(52),N10),(L(53),N11),(L(54),N12),(L(55),N13),(L(56),N14),(L  15
     3(57),N15),(L(58),N16),(L(59),N17),(L(60),NPO),(L(63),N18),(L(64),N  16
     419),(L(65),N20),(L(66),N21),(L(67),LBS),(L(107),N22),(L(108),N23),  17
     5(L(109),MA),(L(110),MR),(L(150),MICT),(L(151),MICT2),(L(152),LB)    18
      EQUIVALENCE (A(4956),LB1),(A(4957),LB2),(A(4958),LBT),(A(4959),LB3  19
     1),(A(4960),SOPS),(A(3340),SIP),(A(3344),LB5),(A(3345),GAM1)         20
      EQUIVALENCE (A(1),ESP1),(A(2),ESP2),(A(3),EIGEN2),(A(4),ESP3),(A(5  21
     1),THET),(A(6),FMUL),(A(8),RW),(A(48),C1),(A(49),C2),(A(50),BUCK,BU  22
     2CK1,BUCK3),(A(368),C3),(A(369),C4),(A(395),CIH),(A(400),BUCK2),     23
     3(A(560),TRANS,SOP),(A(720),AC1),(A(760),AC2),(A(800),VUSIG),(A(960  24
     4),DELR),(A(1000),SIGA),(A(1160),AC9),(A(1200),DIF),(A(1360),G1P),(  25
     5A(1560),AC10),(A(1400),CHI),(A(1600),OMEGA),(A(1604),OMEG1),(A(160  26
     68),T),(A(1612),SIGPT),(A(1652),PG1),(A(1653),PG2),(A(1654),PG3),(A  27
     7(1655),PG4),(A(1656),N100),(A(1657),EIGEN),(A(1658),EIGEN1)         28
      EQUIVALENCE (A(1659),P),(A(1660),GP),(A(1700),SOU),(A(1939),A1),(   29
     1A(1940),SOPP),(A(1980),LT1),(A(1981),LT2),(A(1982),B1),(A(1983),BL  30
     2),(A(1987),B2),(A(1991),B3),(A(1995),B4),(A(1999),W),(A(2085),B5),  31
     3(A(2009),W1),(A(2004),W2),(A(2014),PSI1),(A(2019),PSI2),(A(2024),D  32
     4ET),(A(2025),PIM),(A(2026),ERR),(A(2027),SUM),(A(2028),GAM),(A(202  33
     59),NM1),(A(2034),NM2),(A(2038),MM1),(A(2039),NAA),(A(2044),NSKP),   34
     6(A(2049),ICT),(A(2054),ICT1),(A( 2055),ICT2),(A(2056),ICT3),(A(205  35
     77),DELRT),(A(2058),BUCKT),(A(2059),C10),(A(2064),SGPT)             36
      EQUIVALENCE (A(2600),SOUP),(A(2850),R),(A(3100),RI),(A(3350),FL),(  37
     1A(3600),FLP),(A(3840),SIGT),(A(4000),PHI),(APHI,SOUS,BSA),(G,BETA)  38
     2,(G1,DELTA),(A(2068),CON),(A(2069),CON1),(A(2074),FAC)             39
      READ INPUT TAPE 7,59,(NNN(I),I=1,4)                                101
   59 FORMAT (4I12)                                                      201
      N24=NNN(1)                                                         301
      N25=NNN(2)                                                         401
      N26=NNN(3)                                                         501
      N27=NNN(4)                                                         601
      IF (N24) 19,19,1                                                     2
    1 IF(N25) 9,9,30
   30 DO 31 J=1,NOG
      NF=1                                                                 4
      LOL=1                                                               5
      DO 5 I=1,M                                                           6
      NF=NF+LM(I)                                                          7
      DO 2 K=LOL,NF                                                        8
      IF (I-M) 6,7,7                                                       9
    6 CURR(K) = (PHI(K,J)/4.0)-((DIF(I,J)/2.0)*(PHI(K+1,J)-PHI(K,J
     1))/DELR(I))                                                        11
      GO TO 2                                                            12
    7 CURR(K) = (PHI(K,J)/4.0) - ((DIF(I,J)/2.0) * (PHI(K,J)-PHI(K-1,J
     1))/DELR(I))                                                        14
    2 CONTINUE                                                           15
      IF (I-1) 3,3,4                                                     16
    3 LOL=LOL + LM(I) + 1                                                17
      GO TO 5                                                            18
    4 LOL = LOL+LM(I)                                                    19
    5 CONTINUE                                                           20
      WRITE OUTPUT TAPE 6,60,J
   60 FORMAT(35H    RADIUS     PLUS CURRENT   GROUPI3)                   23
      WRITE OUTPUT TAPE 6,61,(R(L),CURR(L)  ,L=1,NF)                     24
   61 FORMAT (2E16.8)                                                    25
   31 CONTINUE
```

```
   9 IF (N26) 19,19,10                                                      26
  10 I=N27
     WRITE OUTPUT TAPE 6,62,I
  62 FORMAT (55H1                          STREAMING FLUX  IN REGION
    113)
     DO 18 J=1,NOG
     READ INPUT TAPE 7,64,(AA(K),K=1,2)
  64 FORMAT(2F12.0)                                                        226
     SIGMS1=AA(1)                                                          326
     SIGMS2=AA(2)                                                          426
     WRITE OUTPUT TAPE 6,65,SIGMS1,SIGMS2,J
  65 FORMAT(23H SIGMS IN REGION I-1 ISE16.8,                               626
    121H SIGMS IN REGION I ISE16.8,8H   GROUPI3)
     LOTT = 0                                                               28
     I2=I-2                                                                128
     IF(I2)28,28,27
  27 DO 20 N=1,I2                                                           29
     LOTTT = LM(N)                                                          30
     LOTT = LOTTT + LOTT                                                    31
  20 CONTINUE                                                               32
  28 LOT = LOTT + 1                                                         33
     I3=LM(I)                                                              133
     DO 17 K=1,I3                                                           34
     SUMM(K,J)= 0.0
     I4=LOT+LM(I-1)                                                        135
     DO 13 L=LOT,I4                                                         36
     FL2=K                                                                 236
     FL3=LM(I-1)+LOT-L                                                     336
     IF(((SIGA(I-1,J)+SIGMS1)*DELR(I-1)*FL3+(SIGA(I,J)+SIGMS2)*            436
    1DELR(I)*FL2)-50.0)23,13,13                                            536
  23 IF(L-I4)25,11,11                                                      636
  25 IF (L-LOT) 11,11,12                                                    37
  11 STR=(((SIGMS1*PHI(L,J)+SOUP(L)*CHI(I-1,J))/2.0)*DELR(I-1))*EXP(
    1-((SIGA(I-1,J) + SIGMS1)         * DELR(I-1) * FL3                     39
    2) - ((SIGA(I,J)+SIGMS2        ) * DELR(I) * FL2))                      40
     GO TO 21                                                               41
  12 STR =  ((SIGMS1*PHI(L,J)+SOUP(L)*CHI(I-1,J))*DELR(I-1))*EXP(-((
    1SIGA(I-1,J) + SIGMS1 )*DELR(I-1)*FL3)-((SIGA(I,J)+SIGMS2
    2         )*DELR(I)*FL2))                                               44
  21 SUMM(K,J) = SUMM(K,J) + STR
  13 CONTINUE                                                               46
     N=LOT+LM(I-1)                                                          47
     N1=N+K                                                                147
     DO 16 L=N,N1                                                           48
     FL4=N+K-L                                                             148
     IF(((SIGA(I,J)+SIGMS2)*DELR(I)*FL4)-50.0)24,16,16                     248
  24 IF(L-N)15,15,26                                                       348
  26 IF (L-N-K) 14,15,15                                                    49
  14 STR=((SIGMS2*PHI(L,J)+SOUP(L)*CHI(I,J))*DELR(I))*EXP(-(SIGA(I,J)+S
    1IGMS2       )*DELR(I)*FL4)                                             51
     GO TO 22                                                               52
  15 STR = ((SIGMS2*PHI(L,J)+SOUP(L)*CHI(I,J))/2.0) * DELR(I)
    1*EXP(-(SIGA(I,J)+SIGMS2)*DELR(I)*FL4)                                 153
  22 SUMM(K,J) = SUMM(K,J) + STR
  16 CONTINUE                                                               55
  17 CONTINUE                                                               56
  18 CONTINUE                                                               61
     IF (NOG-3) 35,35,32
  32 WRITE OUTPUT TAPE 6,33
  33 FORMAT(69H0 PT. IN REGION    GROUP 1        GROUP 2        GROUP 3
    1      GROUP 4)
     WRITE OUTPUT TAPE 6,34,(K,SUMM(K,1),SUMM(K,2),SUMM(K,3),SUMM(K,4),
    1K=1,I3)
  34 FORMAT(I7,E22.8,3E14.8)
     GO TO 19
  35 IF (NOG-2) 39,39,36
  36 WRITE OUTPUT TAPE 6,37
  37 FORMAT(55H0 PT. IN REGION    GROUP 1        GROUP 2        GROUP 3)
     WRITE OUTPUT TAPE 6,38,(K,SUMM(K,1),SUMM(K,2),SUMM(K,3),K=1,I3)
  38 FORMAT (I7,E22.8,2E14.8)
     GO TO 19
  39 WRITE OUTPUT TAPE 6,40
  40 FORMAT(41H0 PT. IN REGION    GROUP 1        GROUP 2)
     WRITE OUTPUT TAPE 6,41,(K,SUMM(K,1),SUMM(K,2),K=1,I3)
  41 FORMAT(I7,E22.8,E14.8)
  19 RETURN                                                                 62
     END                                                                    63
```

APPENDIX B

DESCRIPTION OF THE CODES

Computer calculations were performed on the IBM-709, and later on

the IBM-7090. Some revision of the Fortran language codes were neces-

sary for compatibility with The University of Michigan executive sys-

tem.[35] Program revisions involved tape number changes, function name

revisions, memory zeroing, and occasional function deletions. In addi-

tion, since the executive system monitor occupies several thousand lo-

cations of the 32 K memory, it was often essential to shorten or break

down programs written for the scope of the entire memory.

Group-diffusion codes solve the following set of coupled equations:

$$-D^i \nabla^2 \phi^i + [D^i B_T^2{}^i + \Sigma_a^i]\phi^i + \sum_{\ell \neq i} \Sigma_R^{i \to \ell} \phi^i$$

$$= \chi^i \sum_{\ell} \frac{(\nu \Sigma_f)^\ell \phi^\ell}{\alpha} + \sum_{\ell \neq i} \Sigma_R^{\ell \to i} \phi^\ell \quad . \qquad (B.1)$$

Finite-difference methods along with a variety of convergence accelera-

tors are employed in obtaining solutions. Flexibility is generally built

into a code in the form of an assortment of boundary conditions and spe-

cific geometrical configurations.

FOG[36] is a one-dimensional few-group diffusion code providing for

as many as four energy groups and 239 space points. The code permits

a choice of three geometries, nine sets of boundary conditions, and five

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types of criticality searches. Downscattering between adjacent groups only is allowed.

TWENTY GRAND[37] is a two-dimensional group-diffusion code based upon the Equipoise convergence technique. Three thousand mesh points are accommodated, and the code was revised to handle as many as four energy groups (the original form accommodated six energy groups). Both XY and RZ geometries are available and neutron transfer between any of the groups is permitted. Zero flux, zero derivative, and logarithmic-derivative boundary conditions are utilizable.

Thermal neutron group constants were computed with TEMPEST[38], the Fortran version of the SOFOCATE[39] code. Cross sections, supplied in the form of a library deck, can be averaged over a thermal spectrum determined from (i) the Wigner-Wilkins equation for light moderators, (ii) the Wilkins equation for heavy moderators, or (iii) a Maxwellian distribution.

FORM,[40] the Fortran version of MUFT-IV,[41] a Fourier transform slowing-down code, was used to obtain fast group constants. Fast spectra are generated from the spatially Fourier-transformed slowing-down distribution by the P-1 or B-1 approximation, with or without the Selengut-Goertzel approximation. Cross section data are supplied on tape in the form of a 54-group cross section library from .625 ev to 10 Mev. The memory size limitations of The University of Michigan system required alteration of the main program, a listing of which appears at the end of this appendix.

```
C FORM (FORTRAN MUFT) PROGRAM - WRITTEN AT NORTH AMERICAN AVIATION
C REVISED UNIVERSITY OF MICHIGAN , 1962


$COMPILE FORTRAN, PRINT OBJECT, PUNCH OBJECT                    MAIN1000
      DIMENSION FILE3(7776),PROB(325,18),D3(644), D1(599),D2(1641),    0
     1DUMMY(5215)
      COMMON FILE3,D1,PROB,D2,NSD,IP,A,NED,D3,NADD,DUMMY,NFIRST
      CALL ZERO
      REWIND 3                                                      80
      NFIRST=1                                                      90
      CALL DELSET                                                   100
      CALL SEQPGM
      END

C                          SUBROUTINE DELSET

$ASSEMBLE,PUNCH OBJECT                                          ZERO0000
      ENTRY ZERO
ZERO  LXA *+4,1
      8STZ 77777,1
      TIX *-1,1,1
      TRA 1,4
      OCT 52661
      END



$COMPILE FORTRAN, PRINT OBJECT, PUNCH OBJECT                    MAIN2000
      DIMENSION FILE3(7776),PROB(325,18),D3(644), D1(599),D2(1641),    0
     1DUMMY(5215)
      COMMON FILE3,D1,PROB,D2,NSD,IP,A,NED,D3,NADD,DUMMY,NFIRST,
     1NTIME,NTEST,IP,NSD,NS,NI,NU,L,NALPHA,NIN,NRA
    1 CALL INPUT                                                  110
      CALL INP1                                                   120
      IF(NFIRST-1)2,2,4                                           130
    2 NTEST=0                                                     140
      NTIME=1                                                     150
      CALL READLB(IP,NSD,NTIME,NTEST)                             160
      IF(NTEST)12,3,12                                            170
   12 CALL SELPGM(1)
    3 IF(NADD)13,13,11                                            180
   11 CALL ADD                                                    190
   13 WRITE TAPE 3,(FILE3(I),I=1,7776)                            200
      CALL SELPGM(4)
    4 READ TAPE 3,(FILE3(I),I=1,7776)                             220
      CALL SEQPGM
      END

C                          SUBROUTINE INPUT
C                          SUBROUTINE INP1
C                          SUBROUTINE READLB
C                          SUBROUTINE ADD


$COMPILE FORTRAN, PRINT OBJECT, PUNCH OBJECT                    MAIN3000
      DIMENSION FILE3(7776),PROB(325,18),D3(644), D1(599),D2(1641),    0
```

```
     1DUMMY(5215)
      COMMON FILE3,D1,PROB,D2,NSD,IP,A,NED,D3,NADD,DUMMY,NFIRST,
     1NTIME,NTEST
      CALL GRUCON(NFIRST)
      CALL SEQPGM
      END

C                              SUBROUTINE GRUCON


$COMPILE FORTRAN, PRINT OBJECT, PUNCH OBJECT                     MAIN4000
      DIMENSION FILE3(7776),PROB(325,18),D3(644), D1(599),D2(1641),     0
     1DUMMY(5215)
      COMMON FILE3,D1,PROB,D2,NSD,IP,A,NED,D3,NADD,DUMMY,NFIRST,
     1NTIME,NTEST,IP,NSD,NS,NI,NU,L,NALPHA,NIN,NRA
      IF(NFIRST-1)6,6,7                                            240
    6 NTIME=2                                                     250
      CALL READLB(IP,NSD,NTIME,NTEST)                             260
      IF(NADD/10)15,15,14                                         270
   14 CALL ADD                                                    280
   15 WRITE TAPE 3,PROB                                           290
    7 CALL SEQPGM
      END

C                              SUBROUTINE READLB
C                              SUBROUTINE ADD


$COMPILE FORTRAN, PRINT OBJECT, PUNCH OBJECT                     MAIN5000
      DIMENSION FILE3(7776),PROB(325,18),D3(644), D1(599),D2(1641),     0
     1DUMMY(5215)
      COMMON FILE3,D1,PROB,D2,NSD,IP,A,NED,D3,NADD,DUMMY,NFIRST
      CALL SLODON(NFIRST)                                          300
      REWIND 3                                                     310
      CALL EDIT12                                                  320
      IF(NED-2)10,10,8                                             330
    8 CALL EDIT3                                                   340
      IF(NED-3)10,10,9                                             350
    9 CALL EDIT4                                                   360
   10 NFIRST=NFIRST+1                                              370
      CALL SELPGM(3)
   12 CALL SELPGM(1)                                          06000390
      END                                                         400

C                    SUBROUTINE SLODON
C                    SUBROUTINE EDIT12
C                    SUBROUTINE EDIT3
C                    SUBROUTINE EDIT4
```

APPENDIX C

COMPOSITIONS AND GROUP CONSTANTS

Table C.I is a listing of all compositions considered for this

study. Two classes of core compositions were examined. The first is

the cold-clean initial fuel loading of 140 gm/element. The second is

appropriate to a core with equilibrium xenon and 8.4% fuel burnup, rep-

resentative of the FNR operating at one megawatt during the fall of 1962.

Core compositions are presented for a homogenized regular fuel element

with the water between subassemblies averaged into the composition.

The homogenization of the subassembly is valid since the disadvantage

factor of the fuel is .988.[22] The existence of partial elements and

poison rods was neglected.

Thermal group constants were evaluated by averaging cross section

data over spectra computed by the TEMPEST code from 0-.625 ev. The

TEMPEST library of cross sections, most recently revised in February,

1961, was used throughout for consistency. A temperature of 300°K and

a material buckling of .0130 cm^{-2} were used as input data. Core con-

stants were evaluated using the Wigner-Wilkins light moderator approxi-

mation. Figure C.1 shows the Wigner-Wilkins spectrum computed for the

cold-clean core composition compared to a Maxwellian spectrum at 300°K.

Both spectra were normalized in the following manner:

$$\int_0^{.625 \text{ ev}} \phi(E)dE = 1.0 \ . \tag{C.1}$$

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Reflector constants were averaged over a Maxwellian spectrum at $300°K$ (the buckling was taken to be zero). Table C.II lists the thermal group constants for the compositions in Table C.I.

Fast group constants, evaluated by the FORM code, are entered in Table C.III. The FORM library tape, updated to May, 1962, was obtained directly from the North American Aviation Corporation, and was used as the source of all cross section data. The four-group scheme had the following breakpoints: .625 ev-5.53 Kev; 5.53 Kev-.821 Mev; .821 Mev-10 Mev. Constants were evaluated for a full U^{235} fission source spectrum using the B1 approximation and the self-consistent age approximation. A buckling of .0130 cm^{-2} was used to obtain core constants and .00001 cm^{-2} for the reflector. Resonance self-shielding factors (L-factors) of unity were used for both U^{235} and U^{238}.
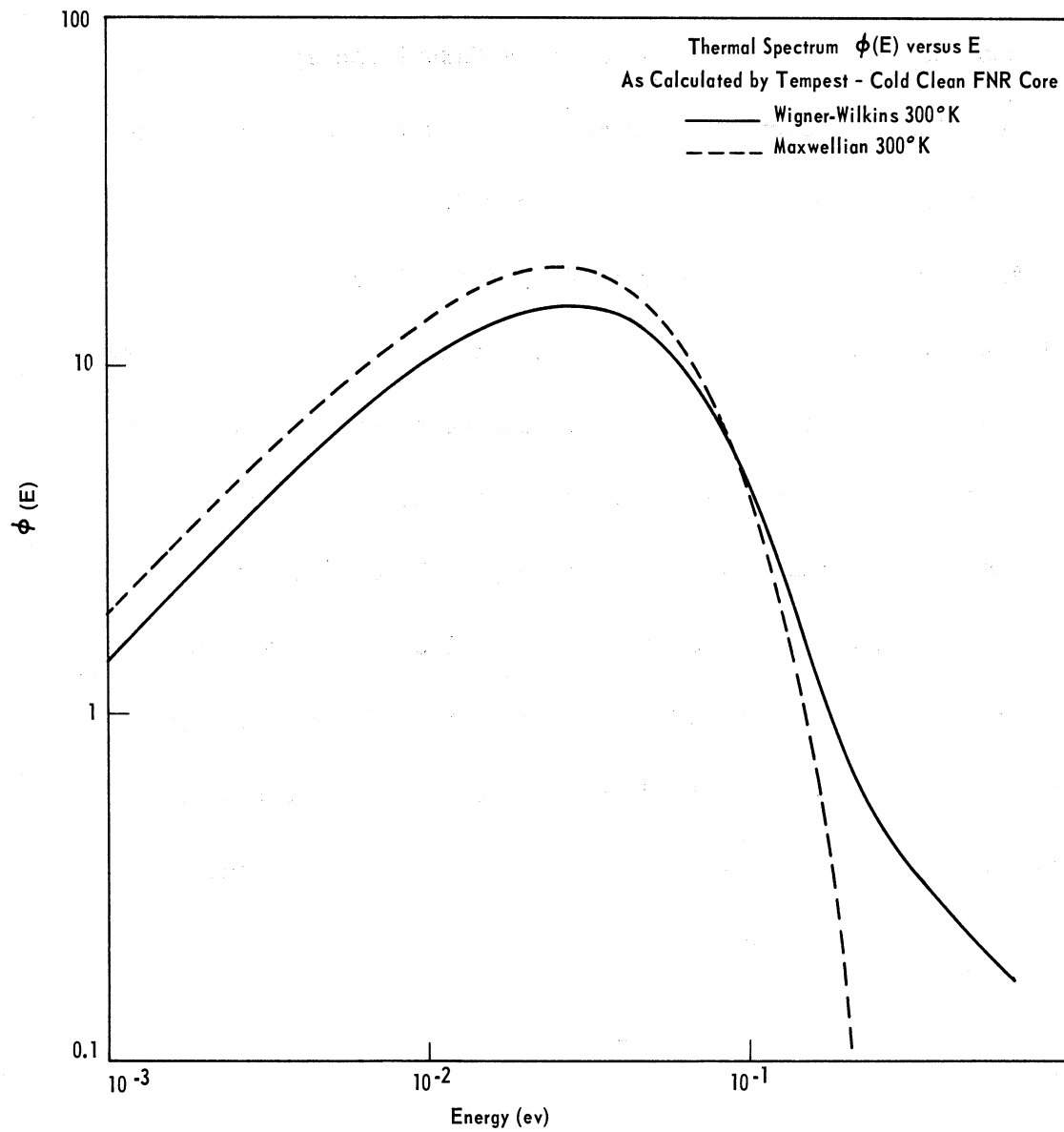
Figure C.1. Wigner-Wilkins and Maxwellian thermal spectra for cold-clean FNR core at 300°K.

TABLE C.I

COMPOSITIONS

	Cold-Clean Core		Equil. Xe 8.4% BU Core		Homogenized Graph.		Reflector				
	Volume Fraction	$\frac{Nuc.}{cm^3}$ x 10^{24}	Volume Fraction	$\frac{Nuc.}{cm^3}$ x 10^{24}	Volume Fraction	$\frac{Nuc.}{cm^3}$ x 10^{24}	Graph.	H_2O	D_2O	BeO	Be
								$\frac{Nuclei}{cm^3}$ x 10^{24}			
H_2O	.5845	---	.5845	---	.0694	---	---	---	---	---	---
H	---	.0392	---	.0392	---	.00464	---	.0670	---	---	---
O	---	.0196	---	.0196	---	.00232	---	.0335	.0331	.0728	---
Al	.4133	.0249	.4133	.0249	.0774	.00466	---	---	---	---	---
U^{235}	.00197	9.46×10^{-5}	.00181	8.67×10^{-5}	---	---	---	---	---	---	---
U^{238}	2.26×10^{-4}	1.08×10^{-5}	2.26×10^{-4}	1.08×10^{-5}	---	---	---	---	---	---	---
B^{10}*	---	---	$.20 \times 10^{-5}$	$.27 \times 10^{-6}$	---	---	---	---	---	---	---
Xe^{135}	---	---	---	$.85 \times 10^{-9}$	---	---	---	---	---	---	---
C	---	---	---	---	.854	.0685	.0803	---	---	---	---
D_2O	---	---	---	---	---	---	---	---	.0331	---	---
D	---	---	---	---	---	---	---	---	.0662	---	---
Be	---	---	---	---	---	---	---	---	---	.0725	.1236

*Equivalents to account for non-Xe^{135} fission product poisoning.

TABLE C.II

THERMAL GROUP CONSTANTS

	Cold-Clean Core	Equil. Xe 8.4% BU Core	Homogenized Graph.	Graph.	Reflector			
					H_2O	D_2O*	Be	BeO
$<\Sigma_a>$ cm^{-1}	.0629	.0622	.00256	.000263	.0195	.000029	.00108	.000651
$<D>$ cm	.2993	.2989	.693	.915	.1546	.831	.4155	.4354
$<\nu\Sigma_f>$ cm^{-1}	.0996	.0915	0.0	0.0	0.0	0.0	0.0	0.0
$<\Sigma_{tr}>$ cm^{-1}	1.377	1.378	.490	.364	2.485	---	.802	.7655
$<\Sigma_s>$ cm^{-1}	1.313	1.313	---	.384	3.06	.375	.741	.720

*Not available in TEMPEST library; obtained by averaging BNL-325 cross section data over a Maxwellian spectrum.

TABLE C.III

FAST GROUP CONSTANTS

	Cold-Clean Core	Equil. Xe* BU Core	Reflector					
			Homogenized Graph.	Graph.	H_2O	D_2O	Be	BeO
$< \Sigma_a >_1$ cm^{-1}	.000898	.000885	.000056	.000003	.00138	.00144	.00453	.00449
$< \Sigma_a >_2$	.000285	.000267	.000018	0.0	.000013	0.0	.000002	.000001
$< \Sigma_a >_3$	.00485	.00454	.000124	0.0	.000954	0.0	.000054	.000031
$< \Sigma_a >$	.00190	.00173	.000073	0.0	.000787	.000148	.000729	.000549
$< \Sigma_R >_1$ cm^{-1}	.07429	.07445	.0321	.0275	.1087	.0835	.0419	.0462
$< \Sigma_R >_2$	.08509	.08336	.0199	.0113	.1494	.0372	.0275	.0230
$< \Sigma_R >_3$	.08252	.0804	.0161	.00657	.1520	.0196	.0169	.0134
$< \Sigma_R >$	.0254	.0236	.00734	.00372	.0495	.0115	.00885	.00746
$< \nu\Sigma_f >_1$ cm^{-1}	.000306	.000281	0.0	0.0	0.0	0.0	0.0	0.0
$< \nu\Sigma_f >_2$	.000380	.000346	0.0	0.0	0.0	0.0	0.0	0.0
$< \nu\Sigma_f >_3$	.00593	.00539	0.0	0.0	0.0	0.0	0.0	0.0
$< \nu\Sigma_f >$	.00207	.00180	0.0	0.0	0.0	0.0	0.0	0.0
$< D >_1$ cm	2.017	1.920	2.369	2.410	2.048	2.175	1.908	1.442
$< D >_2$	1.156	1.144	1.085	1.049	1.057	1.230	.581	.549
$< D >_3$	.905	.907	.950	.934	.595	1.242	.486	.502
$< D >$	1.365	1.341	1.245	1.122	1.245	1.334	.7364	.6287
$< \Sigma_s >_1$ cm^{-1}	.201	.201	---	.163	.261	.178	.209	.285
$< \Sigma_s >_2$	.597	.597	---	.342	.761	.248	.636	.641
$< \Sigma_s >_3$	.910	.910	---	.377	1.385	.240	.736	.695

Subscripts: 1—.821 Mev to 10 Mev
2—5.53 Kev to .821 Mev
3—.621 ev to 5.53 Kev
Non-subscripted constant denotes .621 ev to 10 Mev.
*Xe^{135} cross sections were not available on the library tape, thus it was neglected here.

COMPARISON OF CURRENT BOUNDARY CONDITION AND PSEUDO

BOUNDARY CONDITION AT VACUUM INTERFACE
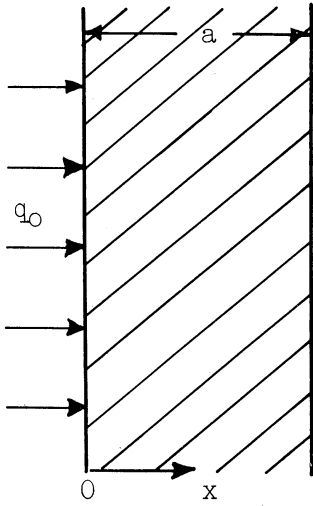
We desire to compare solutions to the two-group diffusion equations utilizing (i) the realistic boundary condition on the partial current and (ii) the pseudo boundary condition on the scalar flux. We examine a system which is similar to the one studied in the text but deviates sufficiently to allow an analytic solution. Consider a slab of width a, with fast neutrons entering the medium at $x = 0$. We write the two-group diffusion equations:

$$D_1 \frac{d^2 \phi_1}{dx^2} - \Sigma_{R_1} \phi_1 = 0 \qquad (D.1)$$

$$D_2 \frac{d^2 \phi_2}{\phi x^2} - \Sigma_{a_2} \phi_2 + \Sigma_{R_1} \phi_1 = 0 \quad . \qquad (D.2)$$

The solution for the flux in group 1 is:

$$\phi_1(x) = A e^{-\kappa_1 x} + B e^{\kappa_1 x} , \qquad (D.3)$$

and for group 2:

$$\phi_2(x) = C e^{-\kappa_2 x} + D e^{\kappa_2 x} + S_1 \phi_1(x) ; \qquad (D.4)$$

where:

$$\kappa_1^2 = \frac{\Sigma_{R_1}}{D_1} \quad , \tag{D.5}$$

$$\kappa_2^2 = \frac{\Sigma_{a_2}}{D_2} \quad , \tag{D.6}$$

and:

$$S_1 = \frac{\Sigma_{R_1}}{D_2} \left(\frac{1}{\kappa_2^2 - \kappa_1^2} \right) \quad . \tag{D.7}$$

Adopting the approach of Chapter II, we assume that the fast flux decays as it would in the infinite medium unperturbed by the void at a, allowing us to set B = 0. Then the thermal flux is given by:

$$\phi_2(x) = Ce^{-\kappa_2 x} + De^{\kappa_2 x} + S_1 Ae^{-\kappa_1 x} \quad , \tag{D.8}$$

where the constant A is determined by the source strength, q_o. Another constant is eliminated by a boundary condition on the thermal flux at x = 0. For the sake of argument assume that:

$$j_2^+(o) = \frac{\phi_2(o)}{4} - \frac{\ell_{tr_2}}{6} \frac{d\phi_2}{dx}\bigg|_o = 0 \quad , \tag{D.9}$$

which specifies the thermal flux within one arbitrary constant, C, giving:

$$\phi_2(x) = C \left\{ e^{-\kappa_2 x} - \frac{\left(1 + \frac{2}{3} \ell_{tr_2} \kappa_2 \right)}{\left(1 - \frac{2}{3} \ell_{tr_2} \kappa_2 \right)} e^{\kappa_2 x} \right\}$$

$$+ S_1 A \left\{ e^{-\kappa_1 x} - \frac{\left(1 + \frac{2}{3} \ell_{tr_2} \kappa_1 \right)}{\left(1 - \frac{2}{3} \ell_{tr_2} \kappa_2 \right)} e^{\kappa_2 x} \right\} \quad . \tag{D.10}$$

Applying the realistic boundary condition at $x = a$:

$$j_2^-(a) = \frac{\phi(a)}{4} - \frac{\ell_{tr_2}}{6} \left. \frac{d\phi}{dx}\right|_a = j_A^- \quad , \qquad (D.11)$$

the constant becomes:

$$c_i = \frac{4j_A^- - S_1A \left\{ \left(1 - \frac{2}{3}\ell_{tr_2}\kappa_1\right) e^{-\kappa_1 a} - \frac{\left(1 + \frac{2}{3}\ell_{tr_2}\kappa_1\right)}{(1 - 2/3\,\ell_{tr_2}\kappa_2)} \left(1 + \frac{2}{3}\ell_{tr_2}\kappa_2\right) e^{\kappa_2 a} \right\}}{\left(1 - \frac{2}{3}\ell_{tr_2}\kappa_2\right) e^{-\kappa_2 a} - \frac{\left(1 + \frac{2}{3}\ell_{tr_2}\kappa_2\right)}{\left(1 - \frac{2}{3}\ell_{tr_2}\kappa_2\right)} \left(1 + \frac{2}{3}\ell_{tr_2}\kappa_2\right) e^{\kappa_2 a}} \quad . \qquad (D.12)$$

Applying the pseudo boundary condition:

$$\phi(a+d) = 4j_A^- \quad , \qquad (D.13)$$

the constant becomes:

$$c_{ii} = \frac{4j_A^- - S_1A \left\{ e^{-\kappa_1(a+d)} - \frac{\left(1 + \frac{2}{3}\ell_{tr_2}\kappa_1\right)}{(1 - 2/3\,\ell_{tr_2}\kappa_2)} e^{\kappa_2(a+d)} \right\}}{e^{-\kappa_2(a+d)} - \frac{\left(1 + \frac{2}{3}\ell_{tr_2}\kappa_2\right)}{\left(1 - \frac{2}{3}\ell_{tr_2}\kappa_2\right)} e^{\kappa_2(a+d)}} \quad . \qquad (D.14)$$

Inasmuch as we have used the uncorrected diffusion theory extrapolation distance in the analysis:

$$e^{\pm\kappa(a+d)} = e^{\pm\kappa\left(a + \frac{2}{3}\ell_{tr}\right)} = e^{\pm\kappa a} e^{\pm\frac{2}{3}\ell_{tr}\kappa} \quad . \qquad (D.15)$$

Then for $\ell_{tr}\kappa \ll 1$:

$$e^{\pm\kappa(a+d)} \simeq e^{\pm\kappa a}\left(1 \pm \frac{2}{3}\ell_{tr}\kappa\right) \quad . \qquad (D.16)$$

It follows that:*

$$c_i \cong c_{ii} \quad . \qquad\qquad (D.17)$$

---

*For light water, $2/3\ \ell_{tr_2}\kappa_2 = .095$ and $2/3\ \ell_{tr_2}\kappa_1 = .052.$

APPENDIX E

DUCT MOUTH CONTRIBUTION TO THE RADIAL CURRENT AT THE WALL

Consider the contribution to the partial current in the radial direction at (R, z_1) from the mouth of a cylindrical void. The number of neutrons per second passing through dS_1 emanating from the mouth is given by:

$$j^+(R,z_1)dS_1 = \int_{S_A} dS_A \hat{z} \cdot \bar{\Omega} \phi(r,0,\bar{\Omega})d\Omega \quad . \tag{E.1}$$

The angular flux at the mouth, neglecting source contributions, is expressed by:

$$\phi(r,0,\bar{\Omega}) \simeq \frac{1}{4\pi}\left[\phi_0(r,0) - \ell\bar{\Omega} \cdot \bar{\nabla}\phi_0\Big|_{\substack{r\\0}}\right] \quad . \tag{E.2}$$

Assuming polar symmetry:

$$\bar{\nabla}\phi_0\Big|_{\substack{r\\0}} = \hat{r}\frac{\partial\phi_0}{\partial r}\Big|_{\substack{r\\0}} + \hat{z}\frac{\partial\phi_0}{\partial z}\Big|_{\substack{r\\0}} \quad . \tag{E.3}$$

Referring to Figure E.1:

$$dS_A = r\,dr\,d\beta \quad , \tag{E.4}$$

$$dS_1 = 2R\,d\Theta\,dz \quad , \tag{E.5}$$

and:

$$\bar{\Omega} = -\frac{h}{\psi}\hat{r} - \frac{p}{\psi}\hat{y} + \frac{z_1}{\psi}\hat{z} \quad . \tag{E.6}$$
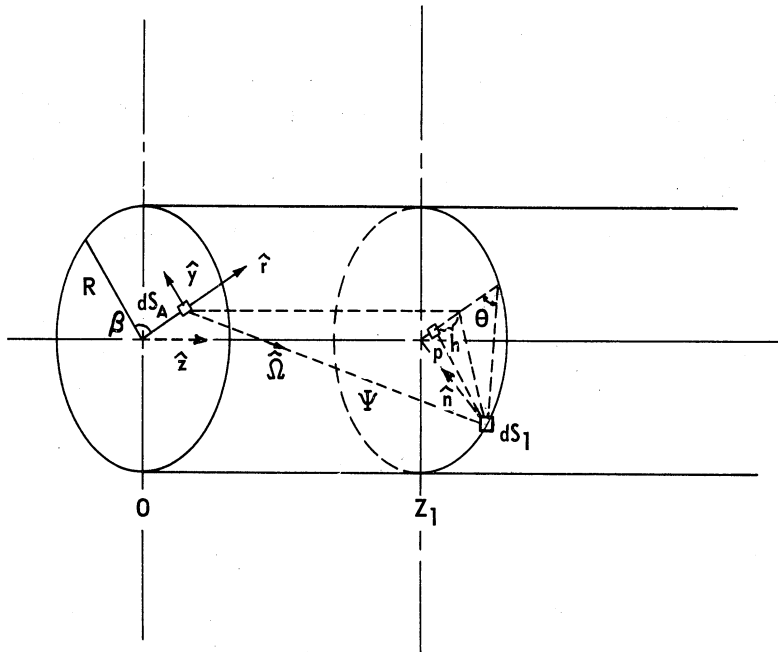
Figure E.1. Duct configuration for calculation of mouth contribution to wall radial current.

From geometrical considerations:

$$\hat{n} = \hat{r} \cos 2\theta + \hat{y} \sin 2\theta \quad , \tag{E.7}$$

$$p = R \sin 2\theta \quad , \tag{E.8}$$

and:

$$h = r + R \cos 2\theta \quad . \tag{E.9}$$

Accordingly:

$$d\Omega = \frac{dS_1}{\psi^2} (-\hat{n} \cdot \bar{\Omega}) = \frac{dS_1}{\psi^3} (R + r \cos 2\theta), \tag{E.10}$$

and;

$$\psi^2 = z_1^2 + p^2 + h^2 = z_1^2 + R^2 + 2rR \cos 2\Theta + r^2 \quad . \quad \text{(E.11)}$$

Using Eqs. (E.3) and (E.6), the angular flux takes on the form:

$$\phi(r,0,\overline{\Omega}) = \frac{1}{4\pi}\left\{\phi_0(r,0) - \ell\left[\frac{z_1}{\psi}\frac{\partial\phi_0}{\partial z}\bigg|_{\substack{r\\0}} - \frac{h}{\psi}\frac{\partial\phi_0}{\partial r}\bigg|_{\substack{r\\0}}\right]\right\} , \quad \text{(E.12)}$$

and the current is obtained by substituting the above expressions into Eq. (E.1):

$$j^+(R,z_1) = \frac{1}{2\pi R dz}\int_0^\pi d\Theta \int_0^{2\pi} d\beta \int_0^R r dr \frac{1}{4\pi}\left\{\phi_0(r,0)\right.$$

$$\left. - \ell\left[\frac{z_1}{\psi}\frac{\partial\phi_0}{\partial z}\bigg|_{\substack{r\\0}} - \frac{h}{\psi}\frac{\partial\phi_0}{\partial r}\bigg|_{\substack{r\\0}}\right]\right\}\frac{z_1}{\psi}\left(\frac{2R dz}{\psi^3}\right)(R + r \cos 2\Theta) \quad . \quad \text{(E.13)}$$

Assume that $\dfrac{\partial\phi_0}{\partial r}\bigg|_{\substack{r\\0}}$ is negligible in comparison with $\phi_0(r,0)$ and $\dfrac{\partial\phi_0}{\partial z}\bigg|_{\substack{r\\0}}$, which are taken to be constant over the entire mouth. Then Eq. (E.13) reduces to:

$$j^+(R,z_1) = \frac{z_1}{\pi}\left\{\phi_0(0)\int_0^R r dr \int_0^{\pi/2} d\Theta \frac{(R + r \cos 2\Theta)}{[z_1^2 + R^2 + r^2 + 2rR \cos 2\Theta]^2}\right.$$

$$\left. - \ell z_1\frac{\partial\phi_0}{\partial z}\bigg|_{\substack{r\\0}}\int_0^R r dr \int_0^{\pi/2} d\Theta \frac{(R + r \cos 2\Theta)}{[z_1^2 + R^2 + r^2 + 2rR \cos 2\Theta]^{5/2}}\right\} \quad . \quad \text{(E.14)}$$

Or equivalently:

$$j^+(R,z_1) = \frac{z_1}{\pi}\left\{\phi_0(0)\int_0^R r dr I_1(r,R,z_1) - \ell z_1\frac{\partial\phi_0}{\partial z}\bigg|_0\int_0^R r dr I_2(r,R,z_1)\right\} \quad .$$

$$\text{(E.15)}$$

The integration to obtain I_1 is easily performed, giving:

$$I_1(r,R,z_1) = \frac{\pi}{2} R \frac{(z_1^2+R^2-r^2)}{[(z_1^2+R^2+r^2)^2-4r^2R^2]^{3/2}} .$$
(E.16)

The second integration is considerably more difficult, but finally reduces to:

$$I_2(r,R,z_1) = \frac{q^3}{96(rR)^{5/2}(1-q^2)^2} \left\{ \left[(R+r)q^4-(R+3r)q^2+2r \right] K(q^2) \right.$$
$$\left. - \left[2(R+r)q^4-2(2R+r)q^2+2r \right] E(q^2) \right\},$$
(E.17)

where:

$$q^2 = \frac{4rR}{z_1^2+(r+R)^2} ,$$
(E.18)

and K and E are the complete elliptic integrals of the first and second kinds.[15]

The first integral with respect to r, giving the isotropic contribution to the current at (R,z_1), can be done analytically, yielding:

$$\int_0^R rdrI_1(r,R,z_1) = \frac{\pi}{4z_1} \left\{ \frac{\frac{1}{2} + \frac{z_1^2}{4R^2}}{\left(1 + \frac{z_1^2}{4R^2}\right)^{1/2}} - \frac{z_1}{2R} \right\} .$$
(E.19)

The second integral, representing the contribution of the z component of the gradient to the current, must be done numerically. Combining terms, we write the final form for the contribution to the partial current at (R,z_1) from the mouth:

$$j^+(R,z_1) = \frac{1}{4} \phi_0(0) \left\{ \frac{\frac{1}{2} + \frac{z_1^2}{4R^2}}{\left(1 + \frac{z_1^2}{4R^2}\right)^{1/2}} - \frac{z_1}{2R} \right\} - \ell \frac{z_1^2}{\pi} \frac{\partial \phi_0}{\partial z}\bigg|_0 \int_0^R rdrI_2(r,R,z_1) .$$
(E.20)

APPENDIX F

NORMALIZATION FOR TIME VARYING FLUX

Consider the normalization of an experiment conducted in a time varying flux, $\phi(t)$. For purposes of discussion assume that the appropriate integrations over energy have been performed.

The time rate of change of the concentration of a species i is expressed as:

$$\frac{dc^i}{dt} = R^i(t) - \lambda c^i \quad , \qquad (F.1)$$

where $R(t)$ is the time dependent reaction rate:

$$R^i(t) = \Sigma^i \phi(t) \quad . \qquad (F.2)$$

The activity at time t, the termination of an irradiation, is obtained by solving Eq. (F.1), and is given by:

$$A^i(t) = \lambda^i \int_0^t dt' R^i(t') e^{\lambda^i(t'-t)} = \lambda^i \Sigma^i \int_0^t dt' \phi(t') e^{\lambda^i(t'-t)} \quad . \qquad (F.3)$$

We define the time averaged value of the flux:

$$< \phi > = \frac{1}{t} \int_0^t \phi(t') dt' \quad . \qquad (F.4)$$

Setting the time dependent flux equal to the product of a steady state flux magnitude and a function of time:

$$\phi(t) = \Phi f(t) \quad , \qquad (F.5)$$

Eq. (F.3) becomes:

$$A^i(t) = \lambda^i \Sigma^i \Phi \int_0^t dt' f(t') e^{\lambda^i(t'-t)} \quad . \qquad (F.6)$$

We desire to obtain the average value of the flux measured by a detector of species a normalized to the average flux measured at another position and at the same time by a detector of species b. Then:

$$< \phi >^a_N \equiv \frac{< \phi >^a}{< \phi >^b} = \frac{\frac{1}{t} \Phi^a \int_0^t f(t') dt'}{\frac{1}{t} \Phi^b \int_0^t f(t') dt'} = \frac{\Phi^a}{\Phi^b} \quad , \qquad (F.7)$$

and substituting from Eq. (F.6):

$$< \phi >^a_N = \frac{A^a(t) \lambda^a \Sigma^a \int_0^t dt' f(t') e^{\lambda^a(t'-t)}}{A^b(t) \lambda^b \Sigma^b \int_0^t dt' f(t') e^{\lambda^b(t'-t)}} \quad . \qquad (F.8)$$

Since we are interested only in relative normalized quantities:

$$< \phi >^a_N \sim \frac{A^a(t)}{A^b(t)} e^{-(\lambda^a-\lambda^b)t} \frac{\int_0^t dt' f(t') e^{\lambda^a t'}}{\int_0^t dt' f(t') e^{\lambda^b t'}} \quad . \qquad (F.9)$$

For the trivial case of a and b identical Eq. (F.9) reduces to:

$$< \phi >^a_N \sim \frac{A^a(t)}{A^b(t)} \quad . \qquad (F.10)$$

APPENDIX G

COMPUTER PROGRAM FOR EVALUATION OF $\frac{\partial\phi/\partial r}{\phi}\Big|^R_z$

The Fortran program for the computation of $\frac{\partial\phi/\partial r}{\phi}\Big|^R_z$ along the duct

wall is listed here. The calculation assumes separation of variables

in the current balance relationship and evaluates Eq. (2.50) (revised to

include the terms in Expression (2.54) to account for the duct mouth

contribution) at discrete points along the duct wall for an unperturbed

axial flux distribution of a designated shape. Exponential, cosine,

linear, and flat flux shapes are permissible. The input data are of the

following form (Format I2, 8F8.4):

$N(1)$	$= 1$	Linear flux shape
	$= 2$	Flat flux shape
	$= 3$	Cosine flux shape
	$= 4$	Exponential flux shape

$F(1)$	
$F(2)$	Parameters for flux shape
$F(3)$	
$F(4)$	

$F(5)$	$= z_0$	Position of duct mouth (cm)
$F(6)$	$= R$	Duct Radius
$F(7)$	$= \ell$	Neutron mean free path in medium
$F(8)$	$= \delta$	Interval of computation

```
C WALL INTERFACE SEPARABLE CALCULATION - WRITTEN U OF M , 1962


$COMPILE FORTRAN,PRINT OBJECT,PUNCH OBJECT,EXECUTE,DUMP          BOUND
C COMPUTATION OF SEPARABLE BOUNDARY CONDITION FOR TUBE IN REFLECTOR          1
      DIMENSION TEG(3), SUM(3), GET(3),TEG1(3)                               2
C DEFINE FUNCTIONS USED IN PROGRAM                                           3
      PHI1F(ZZ) = AA+A*ZZ                                                    4
      PHI3F(ZZ) = AA*COS(A*ZZ+B)                                            5
      PHI4F(ZZ) = AA*EXP(A*ZZ) + BB*EXP(B*ZZ)                              6
      DPHI3F(ZZ) = -AA*A*SIN(A*ZZ+B)                                       7
      DPHI4F(ZZ) = AA*A*EXP(A*ZZ) + BB*B*EXP(B*ZZ)                        8
      PAINF(ZZ) = 1.0/SQRT(1.0+(Z2-ZZ)*(Z2-ZZ)/(4.0*R*R))                9
      QUEERF(Y) = SQRT(4.0*R*Y/((Z2-Z0)*(Z2-Z0)+(R+Y)*(R+Y)))            10
C READ AND PRINT INPUT DATA                                                 11
C NOTE, INDIC FOR FLUX, 1'LINEAR, 2'FLAT, 3'COS, 4'EXP                     12
    1 READ INPUT TAPE 7,2,INDIC,AA,A,BB,B,Z0,R,AMBDA,DECRE                 13
    2 FORMAT(I2,8F8.4)                                                     14
      WRITE OUTPUT TAPE 6,3                                                 15
    3 FORMAT(66H1COMPUTATION OF SEPARABLE BOUNDARY CONDITION FOR TUBE IN  114
   1 REFLECTOR)                                                           115
      WRITE OUTPUT TAPE 6,4,AA,A,BB,B                                      16
    4 FORMAT(26H0PARAMETERS FOR FLUX SHAPE/4H0AA=F8.4,4H  A=F8.4,4H BB=F  17
   18.4,4H  B=F8.4)                                                       18
      WRITE OUTPUT TAPE 6,5,Z0                                             19
    5 FORMAT(23H0POSITION OF Z0 IN CM.=F8.4)                              20
      WRITE OUTPUT TAPE 6,6,R                                              21
    6 FORMAT(23H0RADIUS OF TUBE IN CM.=F8.4)                              22
      WRITE OUTPUT TAPE 6,7, AMBDA, DECRE                                  23
    7 FORMAT(32H0TRANSPORT M.F.P. LAMBDA IN CM.=F8.4/                     24
   127H0INCREMENT FOR INTEGRATION=F8.4)                                   25
C SET Z2,COMPUTE PHIZ2, PHIZ0, DPHIZ0''PRINT FLUX SHAPE                    26
      Z2=Z0+DECRE                                                          27
    8 IF(INDIC-3) 9,10,11                                                  28
    9 IF(INDIC-1) 12,12,13                                                 29
   10 PHIZ2=PHI3F(Z2)                                                      30
      PHIZ0=PHI3F(Z0)                                                      31
      DPHIZ0=DPHI3F(Z0)                                                    32
      IF(Z2-(Z0+DECRE)) 14,14,24                                          33
   11 PHIZ2=PHI4F(Z2)                                                      34
      PHIZ0=PHI4F(Z0)                                                      35
      DPHIZ0=DPHI4F(Z0)                                                    36
      IF(Z2-(Z0+DECRE)) 15,15,24                                          37
   12 PHIZ2=PHI1F(Z2)                                                      38
      PHIZ0=PHI1F(Z0)                                                      39
      DPHIZ0=A                                                             40
      IF (Z2-(Z0+DECRE)) 16,16,24                                         41
   13 PHIZ2=1.0                                                            42
      PHIZ0=1.0                                                            43
      DPHIZ0=0.0                                                           44
      IF(Z2-(Z0+DECRE)) 17,17,24                                          45
   14 WRITE OUTPUT TAPE 6,18                                               46
      GO TO 22                                                             47
   15 WRITE OUTPUT TAPE 6,19                                               48
      GO TO 22                                                             49
   16 WRITE OUTPUT TAPE 6,20                                               50
      GO TO 22                                                             51
   17 WRITE OUTPUT TAPE 6,21                                               52
   18 FORMAT(24H0FLUX IS COSINE IN SHAPE)                                 53
   19 FORMAT(29H0FLUX IS EXPONENTIAL IN SHAPE)                            54
   20 FORMAT(24H0FLUX IS LINEAR IN SHAPE)                                 55
   21 FORMAT(22H0FLUX IS FLAT IN SHAPE)                                   56
   22 WRITE OUTPUT TAPE 6,23                                               57
   23 FORMAT(72H0  Z2      DENOM      F1      F2      F3      F4   L.D    58
   1PHI/PHI DPHI/PHI)                                                     59
C BEGIN COMPUTATION OF DENOM,F1,F2                                         60
   24 Z1=Z0                                                                61
      J=3                                                                  62
      DO 25 I=1,3                                                          63
      TEG(I)=0.0                                                           64
   25 CONTINUE                                                             65
      GO TO 27                                                             66
   26 Z1=Z1+DECRE                                                          67
   27 DO 28 I=1,3                                                          68
      GET(I)=TEG(I)                                                        69
   28 CONTINUE                                                             70
```

```
   29 IF(INDIC-3) 30,31,32                                                    71
   30 IF(INDIC-1) 33,33,34                                                    72
   31 PHIZ1=PHI3F(Z1)                                                         73
      DPHIZ1=DPHI3F(Z1)                                                       74
      GO TO 35                                                                75
   32 PHIZ1=PHI4F(Z1)                                                         76
      DPHIZ1=DPHI4F(Z1)                                                       77
      GO TO 35                                                                78
   33 PHIZ1=PHI1F(Z1)                                                         79
      DPHIZ1=A                                                                80
      GO TO 35                                                                81
   34 PHIZ1=1.0                                                               82
      J=2                                                                     83
   35 X=PAINF(Z1)                                                             84
      CALL IEF1(1.5707,X,E,F,G)                                               85
      IF(G-1.0) 102,102,100                                                   86
  100 WRITE OUTPUT TAPE 6,101,Z1                                              87
  101 FORMAT(47H          ELLIPTIC INT. OUTSIDE OF RANGE ,Z1=F8.4)            88
      E=1.0                                                                  188
      F=10.0                                                                 288
  102 TEG(1) = PHIZ1*((2.0*(1.0-X*X*X*X)+3.0*(2.0-X*X))*E-(1.0-X*X)           89
     1*(8.0+X*X)*F)/(PHIZ2*X)                                                 90
      TEG(2) = PHIZ1*(2.0-(2.0+X*X)*SQRT(1.0-X*X))/PHIZ2                      91
      IF(INDIC-2) 36,37,36                                                    92
   36 TEG(3) = (DPHIZ1*(Z2-Z1)*((2.0+X*X)*F-2.0*(1.0+X*X)*E))/                93
     1(PHIZ2*SQRT((Z2-Z1)*(Z2-Z1)+4.0*R*R))                                   94
   37 IF(Z1-Z0) 38,38,40                                                      95
   38 DO 39 I=1,3                                                             96
      TEG1(I) = TEG(I)                                                        97
      SUM(I) = 0.50*TEG(I)                                                    98
   39 CONTINUE                                                               100
      GO TO 42                                                                99
   40 DO 41 I=1,3                                                            101
      SUM(I) = SUM(I) + TEG(I)                                               102
   41 CONTINUE                                                               103
   42 DO 44 I=1,J                                                            104
      IF(ABSF(TEG(I))-ABSF(GET(I))) 43,26,26                                 105
   43 IF (ABSF(TEG(I))-0.0005*ABSF(TEG1(I))) 44,26,26                        106
   44 CONTINUE                                                               107
      DENOM = 0.6667*(1.0+DECRE*SUM(1)/(6.2832*R))                          108
      F1 = (1.0-DECRE*SUM(2)/(4.0*R))/DENOM                                 109
      F2 = DECRE*SUM(3)/(9.4248*R*DENOM)                                    110
C COMPUTE F3 AND F4                                                         111
      F3 = (PHIZ0/(2.0*R*PHIZ2*DENOM))*(((Z2-Z0)*(Z2-Z0)+2.0*R*R)/          112
     1SQRT(4.0*R*R+(Z2-Z0)*(Z2-Z0))-(Z2-Z0))                                113
      DEL = R/20.0                                                          114
      SUM = 0.0                                                             115
      Y=0.0                                                                 116
      TEG=0.0                                                               117
      IF (INDIC-2) 47,45,47                                                 118
   47 Y=Y+DEL                                                               119
      Q=QUEERF(Y)                                                           120
      CALL IEF1(1.5707,Q,E,F,G)                                             121
      IF (G-1.0) 105,105,103                                                122
  103 WRITE OUTPUT TAPE 6,104,Y                                             123
  104 FORMAT(48H          ELLIPTIC INT. OUTSIDE OF RANGE ,RAD=F8.4)         124
  105 TEG = (Q*Q*Q/(96.0*SQRT(Y*Y*Y*Y*Y*R*R*R*R*R)*(1.0-Q*Q)*(1.0-Q*Q)))   125
     1*Y*(((R+Y)*Q*Q*Q*Q-(R+3.0*Y)*Q*Q+2.0*Y)*F-(2.0*(R+Y)*Q*Q*Q*Q-2.0*
     2(2.0*-+Y)*Q*Q+2.0*Y)*E)                                               127
      IF ((R-Y)-0.0001) 45,45,46                                            128
   46 SUM = SUM + TEG                                                       129
      GO TO 47                                                              130
   45 SUM = SUM+0.50*TEG                                                    131
      F4 = (4.0*(Z2-Z0)*(Z2-Z0)*DPHIZ0*DEL*SUM)/(3.1416*PHIZ2*DENOM)       132
C COMPUTE FINAL VALUE AND PRINT                                            133
      BOUND = F1 + AMBDA*F2 - F3 + AMBDA*F4                                 134
      GRAD = BOUND/AMBDA                                                    135
      WRITE OUTPUT TAPE 6,48,Z2,DENOM,F1,F2,F3,F4,BOUND,GRAD               136
   48 FORMAT (8F9.4)                                                        137
      IF (INDIC-2) 50,49,50                                                 138
   49 IF ((Z2-Z0)-12.0) 51,52,52                                            139
   50 IF (PHIZ2-0.25*PHIZ0) 52,51,51                                        140
   51 Z2 = Z2 + DECRE                                                       141
      GO TO 8                                                               142
   52 GO TO 1                                                               143
      END                                                                   144
$DATA
```

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