Enhanced Nonlinear Dynamics for Accurate Identification of Stiffness Loss in a Thermo-Shielding Panel*

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Abstract. The dynamics of a panel forced by transverse loads and undergoing limit cycle oscillations and chaos is investigated. The nonlinear von Karman plate theory is used to obtain a model for healthy and damaged panels. Damage is modeled by a loss of stiffness in a portion of the plate. The presence of low levels of damage is identified by using an external nonlinear excitation and analyzing the attractor of the resulting dynamics in state space. Most of the current studies of such problems are based on linear theories and linear structures. In contrast, the results presented are obtained by using and enhancing nonlinear and chaotic dynamics, and have the advantage of an increased accuracy in detecting damage and monitoring structural health.

Key words: chaotic dynamics, damage detection, nonlinear fluid-structural systems

1. Introduction

The identification of stiffness loss in a plate is needed in several applications of structural health monitoring. In such applications, the use of non-destructive evaluation is exploited for detecting structural changes which indicate damage [24, 77]. In general, the ability to quantify and measure structural integrity is an important advantage as it increases safety while reducing operation costs and lengthening the lifetime of aging assets.

Identifying loss of stiffness is one of the most common means to detect damage. In turn, detecting the loss of stiffness is accomplished by monitoring a *feature* of the system. Vibration responses (frequency response functions, mode shapes and natural frequencies), electromagnetic properties, ultrasound, are examples of such features. The most common monitoring techniques are focused on detecting the presence of damage without any concern for its precise location, level or extent. In such approaches, the feature for a healthy and a damaged system are compared, and the identified differences usually are not quantified, but they are used as a red/green indicator. More advanced techniques use location-dependent features and focus on the identification of the location and/or the extent of damage. Such techniques require distributed sensors, more comprehensive information about the system, and a quantifiable feature whose measure may be correlated with the extent and location of damage. Nevertheless, such techniques are the most advantageous because they allow for the estimation of the remaining life of the system, and are the basis for the elimination of time-scheduled inspections, and a transition from time-based to condition-based maintenance.

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Monitoring structural integrity is particularly important in aerospace applications [14, 29, 56] where detecting the onset and propagation/evolution of damage during normal operation of the monitored system is highly desired. Among the most important technical difficulties faced by current methods are detecting incipient damage (of small level) and taking adequate measurements during normal operating conditions. These issues have limited the available techniques to mostly offline detection of large damage.

Most current techniques for damage detection are vibration-based, i.e. they are focused on determining changes in the frequencies and modes of vibration [14, 24, 36, 49, 60]. Other similar techniques use wavelet analyses [3–5, 62], Ritz vectors [11, 77], stochastic approaches [57, 64], subspace updating [1, 26, 32, 50, 51, 72, 78, 79], or evolutionary algorithms [80]. All these methods have been developed for monitoring linear structures. However, only a few methods are currently available for detecting damage in nonlinear structures. Most of the nonlinear methods are based on system identification [2, 25, 34, 38–40, 61], while a few use neural networks [6, 37, 76], or Lyapunov exponents [12, 52, 67, 73]. These nonlinear methods have important limitations, e.g., some have difficulty tackling high-dimensional systems, others do not predict the damage location or level, or do not detect simultaneous damages.

However, our results discussed in the present paper show that the use of nonlinearities holds a great potential. We have observed that the changes in the shape of attractors of the dynamics are large for nonlinear systems and may be used for damage detection. For example, results have been obtained for a pinned–pinned panel with structural nonlinearity depicted in Figures 1 and 2. These results, shown in Figures 3 and 4, demonstrate that the nonlinear response of the panel excited harmonically changes dramatically when the panel stiffness changes. A mere 4% change in stiffness leads to qualitative (large) changes in the attractor shape as shown in Figure 3. Moreover, the attractors change with both the location and extent of damage. For example, a small loss of stiffness (10%) along a small extent (5% of panel length) can clearly be distinguished in Figure 4 from larger damage levels (50% loss) and larger damage extents (20% length). In contrast, linear methods cannot detect the damage, as shown in Figure 4.

The main advantages of *nonlinear* vibration-based damage detection are: (i) nonlinearities are a pervasive presence in many systems affected by damage, (ii) linear techniques are not accurate when applied to nonlinear systems because often the presence or level of damage cannot be distinguished (by linear methods) from the nonlinear healthy system dynamics, (iii) nonlinear-based methods can provide greater sensitivity and performance.
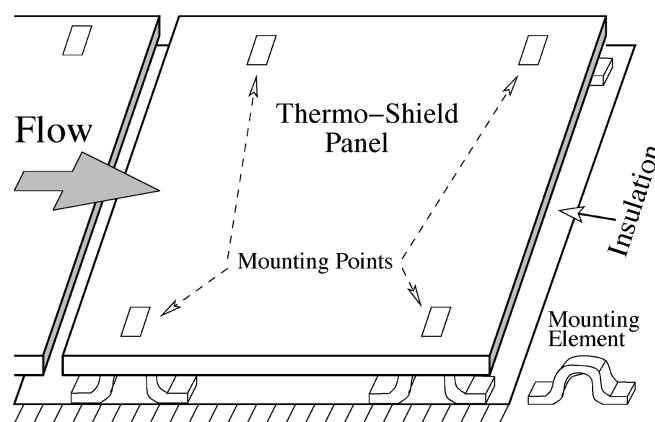

Figure 1. Thermo-shielding panel subjected to internal thermal loads and interacting with an external (supersonic) flow (i.e., an aeroelastic system).
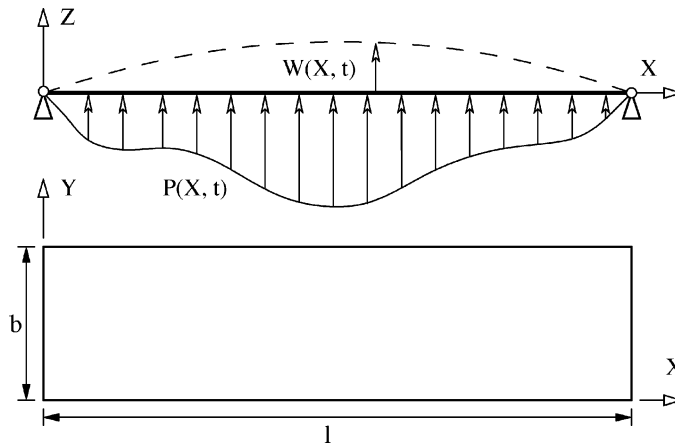
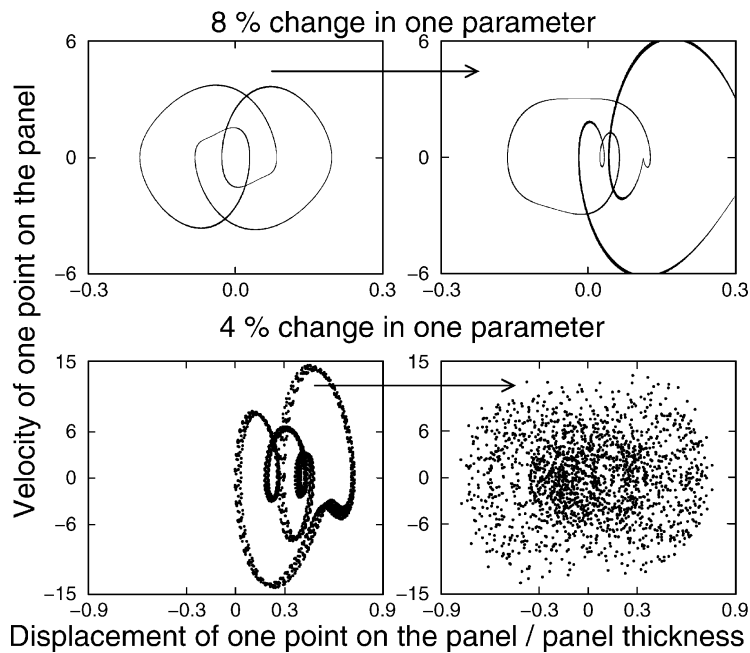*Figure 2.* Two-dimensional panel subject to transverse loading.

Figure 3. Nonlinear systems are sensitive to parametric variations, and small parameter variations lead to dramatic changes in attractor shapes.

Nonlinear phenomena are important not only in large, but also in small, amplitude vibrations when the strength of nonlinearity is high. An example of such a nonlinearity is a crack in a plate or a loose joint which may lead to changes in linear characteristics such as mode shapes [55], stiffness and damping, and can also induce nonlinear dynamics by introducing Coulomb friction and allowing the opening and closing of a gap (piece-wise linear dynamics with possible impacts [30, 69, 70]). Linear methods have difficulties because they are independent of the amplitude of vibration, and thus, the stiffening or softening of a healthy system due to nonlinearities may be erroneously assumed to be caused by damage. As an example, the co-existence of multiple attractors (for the same undamaged structure under the same excitation and) for the dynamics of a point on a panel with structural nonlinearity excited by unsteady
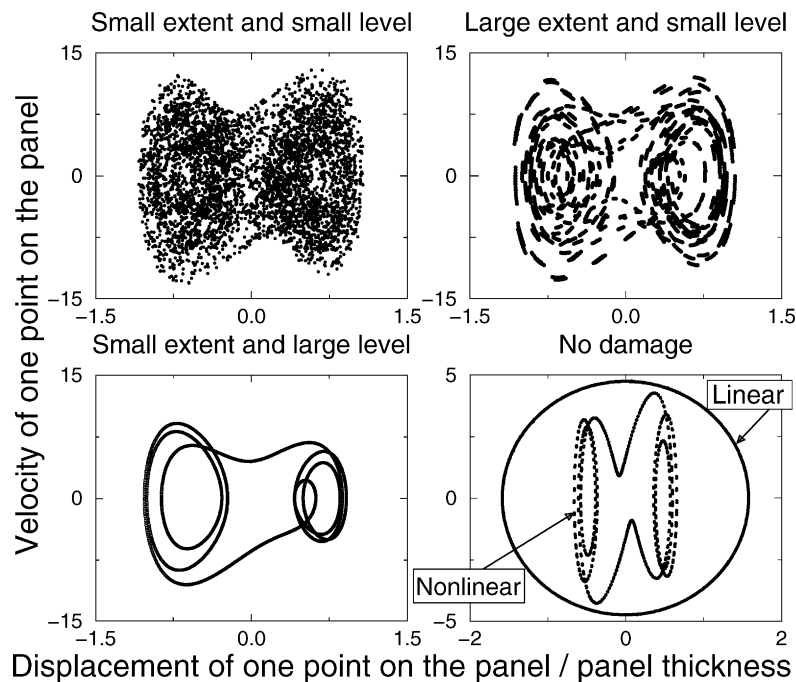
Small extent and small level   Large extent and small level

Small extent and large level   No damage

Figure 4. The attractors of nonlinear systems are sensitive to damage presence, extent, and location, while linear models fail to model the dynamics.

flow [22] may be erroneously interpreted by linear methods as being due to damage. Additionally, nonlinear systems are more sensitive to parametric changes than linear systems (especially in the neighborhood of bifurcations [27, 63]). Nonlinear and chaotic vibrations have been observed in many physical contexts including airplane wings [15–17, 31, 59], panels forced by flow-induced loads [8–10, 21, 22, 44, 53, 54], rotors [28], pipes [46–48], heat exchangers [13, 33, 47, 66], micro-systems [41, 68] such as micro-plates, and thermo-shielding panels [22].

2. Modeling

As an example of the methodology, consider damage detection in a thermo-shielding panel mounted on a rigid substructure and subjected to unsteady aerodynamic forces as depicted in Figure 1. A simplified model for the panel is shown in Figure 2. The deformation of the panel occurs primarily in the transverse direction (Z axis), and that deformation is of the order of the panel thickness. Although this deformation is small, structural nonlinearity is present (and it is strong) due to the coupling between stretching and bending of the panel. Energy dissipation of the aeroelastic system is also considered in the model by accounting for internal and external damping. Finally, the damaged regions of the panel are modeled as exhibiting a local reduction in the bending stiffness of the panel.

2.1. STRUCTURAL MODEL

The panel shown in Figure 2 is modeled as a one dimensional, homogeneous, isotropic, and elastic thin plate with pinned–pinned end points. The thickness h is considered much smaller than the length l. Also, the width b is considered much larger than the length l. These assumptions hold for many

cases of practical interest (such as a thermo-shielding panel). Nevertheless, the proposed approach for the accurate identification of loss of stiffness may be applied equally well to other cases, where these assumptions do not hold. For a linear elastic isotropic material, the stress–strain relation may be expressed as:

$$\sigma_{xx} = \frac{E}{l - v^2} \epsilon_{xx}, \tag{1}$$

where E is Young's modulus and v is Poisson's ratio. To model the structural nonlinearity caused by the in-plane stretching of the panel due to bending, the von Karman's strain–displacement relation is used as follows:

$$\epsilon_{xx} = \frac{\partial U}{\partial X} + \frac{1}{2}\left(\frac{\partial W}{\partial X}\right)^2 - Z\frac{\partial^2 W}{\partial X^2}, \tag{2}$$

where U is the displacement of the panel along the X-axis, and W is the displacement of the panel along the Z-axis. Thus, the moment acting on the cross section of the panel (of unit width) may be expressed as:

$$M = \int_{-h/2}^{h/2} \sigma_{xx} Z dZ = -D\frac{\partial^2 W}{\partial X^2} \text{ (clockwise)}, \tag{3}$$

where D is a coefficient characterizing the bending stiffness of the panel, $D = Eh^3/12(1 - v^2)$.

Damping in this system has two sources: (a) external aerodynamic damping, which is due to the interaction between the panel and the flow, and (b) internal damping, which is due to the viscosity of material of the panel. The aerodynamic damping causes energy loss as a non-conservative force F_e proportional to velocity of the panel. For certain flow velocities, the aerodynamic damping may be negative, which leads to flutter. A more elaborate aerodynamic model has been used to account for the interaction between the panel and the flow [22, 23]. For simplicity, in the present analysis we consider that the flow is stationary, and the aerodynamic damping is simply proportional to the local velocity of the panel. Also, the external damping is approximated as linear viscous damping of the form:

$$F_e = C_e \frac{\partial W}{\partial t},$$

where C_e is an external damping parameter. The material damping is considered to be a viscous damping stress proportional to the strain rate, as follows:

$$\sigma_{xx}^d = \zeta \frac{\partial \epsilon_{xx}}{\partial t},$$

where ζ is a material damping constant. Similar to Equation (3), the moment M^d corresponding to the damping stress and acting clockwise may be expressed as:

$$M^d = \int_{-h/2}^{h/2} \sigma_{xx}^d Z dZ = -C_i \frac{\partial^3 W}{\partial X^2 \partial t}, \tag{4}$$

where C_i is defined as the internal damping parameter, and may be expressed as $C_i = \zeta h^3/12$. Combining Equations (3) and (4), the total moment $\bar{M}$ acting clockwise on the cross section of the unit width panel is obtained as:

$$\bar{M} = M + M^d = -D\frac{\partial^2 W}{\partial X^2} - C_i \frac{\partial^3 W}{\partial X^2 \partial t}. \tag{5}$$
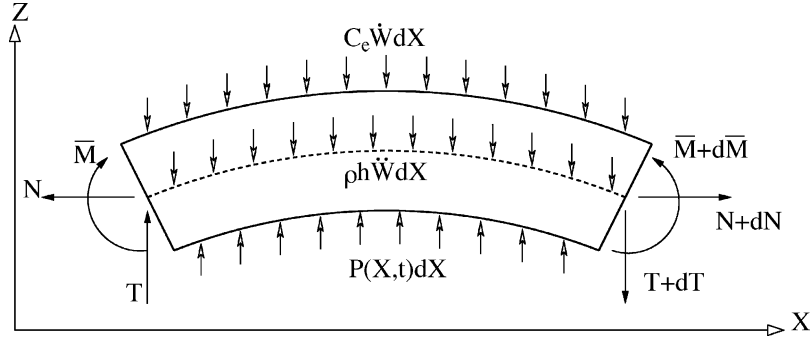
Figure 5. Forces acting on an infinitesimal element of the panel including distributed transverse, axial, and bending forces.

Figure 5 shows the equilibrium of forces acting on an infinitesimal element of the panel (of unit width). This equilibrium of forces may be expressed as $\sum F_Z = 0$, and may be used to obtain:

$$\frac{dT}{dX} = -C_e \dot{W} - \rho h \ddot{W} + P(X, t). \tag{6}$$

Also, the equilibrium of moments $\left(\sum M_Y = 0\right)$ may be used to obtain:

$$N \frac{dW}{dX} - \frac{d\bar{M}}{dX} + T = 0. \tag{7}$$

Differentiating Equation (7) with respect to X and substituting into Equations (5) and (6), the governing equation for the panel dynamics may be expressed as:

$$DW'''' + \rho h \ddot{W} + C_i \dot{W}'''' + C_e \dot{W} - NW'' - P(X, t) = 0, \tag{8}$$

where W' and $\dot{W}$ represent the spatial and time derivatives of W, while ρ is the mass density, and $P(X, t)$ is the transverse distributed loading. N is the in-plane tension load, which is assumed to be uniform along the entire length of the panel. N may be expressed as:

$$N = Eh \left[\eta_0 + \frac{1}{l} \int_0^l \frac{1}{2} W'^2(\xi) d\xi \right], \tag{9}$$

where the first term (η_0) is the initial axial strain, and the second term is the stretching due to bending. Substituting Equation (9) into Equation (8), one obtains:

$$DW'''' + \rho h \ddot{W} + C_i \dot{W}'''' + C_e \dot{W} - Eh\eta_0 W'' - \frac{Eh}{2l} \left[\int_0^l W'^2(\xi) d\xi \right] W'' - P(X, t) = 0. \tag{10}$$

The boundary conditions corresponding to the pinned–pinned panel are:

$$W(X = 0, t) = 0, \quad \bar{M}(X = 0, t) = 0, \quad W(X = l, t) = 0, \quad \bar{M}(X = l, t) = 0. \tag{11}$$

Next, Equation (10) is nondimensionalized by introducing the nondimensional variables x, w, and τ defined as $x = X/l$, $w = W/h$, and $\tau = t/\sqrt{\rho h l^4 / D}$. One obtains

$$w'''' + \ddot{w} + C_I \dot{w}'''' + C_E \dot{w} - R_x w'' - S \left[\int_0^1 w'^2(\xi) d\xi \right] w'' - F(x, \tau) = 0, \tag{12}$$

where $w' = \partial w/\partial x$, and $\dot{w} = \partial w/\partial \tau$. The nondimensional coefficients in Equation (12) may be summarized as follows: internal damping $C_I = C_i/\sqrt{\rho h l^4 D}$, external damping $C_E = C_e l^2/\sqrt{\rho h D}$, in-plane pre-load $R_x = E h \eta_0 l^2/D$, coefficient characterizing the stretching induced by bending $S = E h^3/(2D) = 6(1 - \nu^2)$, and external excitation force $F(x, \tau) = l^4/(hD)P(x, \tau)$. The nondimensional boundary conditions may be expressed as:

$$w(x = 0, \tau) = 0, \quad w''(x = 0, \tau) + C_I \dot{w}''(x = 0, \tau) = 0,$$
$$w(x = 1, \tau) = 0, \quad w''(x = 1, \tau) + C_I \dot{w}''(x = 1, \tau) = 0.$$

2.2. Damage Model

In many damage scenarios, yielding occurs in a small region of the panel, or a crack is initiated and propagates in the material. Such types of damage lead to a reduction of the local bending stiffness coefficient $\bar{D}$. The bending stiffness and Young's modulus for a damaged panel are denoted by $\bar{D}$ and $\bar{E}$. Similar to Equation (10), the governing equation for the damaged region of the panel may be expressed as:

$$\bar{D} W'''' + \rho h \ddot{W} + C_i \dot{W}'''' + C_e \dot{W} - \bar{E} h \eta_0 W'' - \frac{\bar{E} h}{2l}\left[\int_0^l W'^2(\xi)d\xi\right]W'' - P(X, t) = 0. \tag{13}$$

The same nondimensional variables x, w, and τ defined for the undamaged panel are used to nondimensionalize Equation (13). One obtains:

$$S_r \cdot w'''' + \ddot{w} + C_I \dot{w}'''' + C_E \dot{w} - S_r \cdot R_x w'' - S_r \cdot S\left[\int_0^1 w'^2(\xi)d\xi\right]w'' - F(x, \tau) = 0, \tag{14}$$

where S_r is referred to as a stiffness reduction factor, $S_r = \bar{D}/D$, and characterizes the damage level in the panel. No damage is present when S_r equals unity, and small values of S_r indicate large damage.

3. Nonlinear Feedback Excitation

Most current techniques for damage detection are based on observing the dynamics under ambient loads, or harmonic excitation. The main advantage of using a simple harmonic external excitation is that the excitation is easy to generate and measure. However, such an excitation is not best suited for interrogating a nonlinear system for the purpose of identifying small changes in its parameters. Furthermore, current vibration-based methods are designed for linear systems and minimize as much as possible the influence of nonlinearities in the dynamics during measurements. The aim of the method proposed herein is to exploit and enhance (where necessary) nonlinearities. The main advantage of using such an approach is a much enhanced sensitivity. To enhance nonlinearity, we propose an active approach where the system is actively interrogated by applying a nonlinear *feedback* excitation. As opposed to the usual external excitation, the nonlinear feedback excitation only requires the active/online measurement of the dynamics and a feedback loop. The specific form of the nonlinear feedback is dependent on each particular application. The excitation may include quadratic, cubic or other nonlinearities.

As opposed to the usual response to a harmonic excitation, the dynamics of the monitored system is chaotic during the interrogation phase. For linear or nonlinear systems undergoing limit cycle oscillations, the attractors are curves in the state space whose shape may be used as an indicator of damage [30]. However, the sensitivity of the shape of these curves is limited [23] because their shape is not very complex. Increasing the geometric complexity of the attractor of the dynamics is shown here to be a venue to increase sensitivity. Specifically, the nonlinear feedback excitation may be used to induce chaotic dynamics, which (usually) has attractors that are expanded spatially [18–20]. The increased sensitivity to parametric variation is demonstrated in Figure 7 where bifurcation diagrams are shown for varying damage level S_r, varying damage extent (i.e. the fraction of panel where loss of stiffness occurs – denoted by E_r), and varying damage location (along the panel). A clear distinction in the dynamics may be observed for various levels, extents and locations of damage. The changes revealed in the bifurcation diagrams are also reflected in the shape of the attractor of the dynamics.

The active interrogation of the system leads to changes in its dynamics so that the attractor (in state space or in embedded coordinates) is a complex geometric shape. This shape is used as a feature in a damage detection approach, i.e. given a (measured) attractor, damage is detected by comparing the shape of this attractor with its undamaged shape. This shape is complex, and its characterization is designed to provide the damage level as well as location. This approach is different from other attractor-based methods such as chaotic excitation. Chaotic excitation has been used for damage detection with success [35, 58, 67, 71, 75], but the attractor variance, and Lyapunov exponents which were used as features could not predict the damage location, and their sensitivity was moderate. The proposed approach is radically different. For example, when a chaotic excitation is applied to a linear system, superposition still holds and each frequency component of the chaotic signal is filtered through the transfer function of the structure as any other type of external (non-feedback) excitation would be. However, in the nonlinear feedback approach, superposition does not hold. The excitation and the system are merged in a new system whose dynamics is very sensitive to changes in its parameters.

In order to make the dynamics of the panel sensitive to the level, the extent, and the location of the damage in the panel, a harmonic excitation with a nonlinear feedback force component is applied at the middle point of the panel. Thus, the transverse distributed loading $F(x, \tau)$ in Equation (12) can be expressed as $F(\tau)\delta(x - 1/2)$, where $F(\tau)$ is response function of the input force, and δ is the Dirac delta function.

The structural nonlinear term in Equation (12), $S[\int_0^1 w'^2(\xi)d\xi]w''$, plays an important role in the complex dynamics of the panel. However, in cases such as a panel constrained by pinned-roller boundary conditions, the nonlinear effect is weak. When the structural nonlinearity and the in-plane pre-load are small, the panel dynamics subjected to a harmonic excitation force leads to harmonic oscillation. To enhance the sensitivity of these vibrations to parametric variations, a nonlinear feedback force is used in combination with the harmonic force. As a result, the input force may be expressed as:

$$F(\tau) = F_0 \sin \omega\tau + \lambda w\left(\frac{1}{2}\right)^2 w''\left(\frac{1}{2}\right) + \gamma w''\left(\frac{1}{2}\right), \tag{15}$$

where $w(1/2)$ and $w''(1/2)$ are the displacement and the curvature at the middle point of the panel, and λ and γ are control parameters. The form of the nonlinear feedback force is drawn from the form of two terms in Equation (12), $S[\int_0^1 w'^2(\xi)d\xi]w''$ and $R_x w''$, corresponding to structural nonlinearity and axial pre-load. They are designed to increase the nonlinearity and enhance the instability in the linear panel.
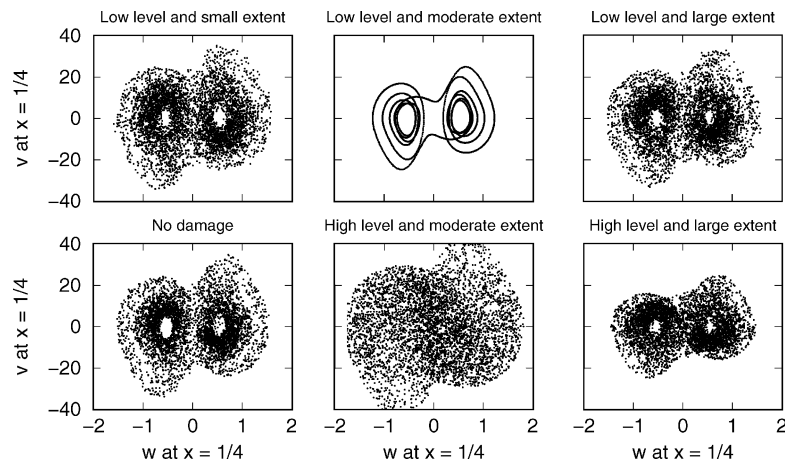
Figure 6. Attractors sampled in time for a healthy panel and three damaged panels with low or high levels of damage and various extents of damage ($v = \dot{w}$).

4. Results

To investigate the changes in the shape of attractors for various parametric values, we consider two approaches. First, we investigate the local distribution of points in an attractor. Second, we quantify the location of bifurcations and the onset of certain types of dynamic behavior by means of bifurcation diagrams.

Figure 6 shows the attractors of the dynamics sampled in time and obtained for a healthy panel and damaged panels with low and high levels of damage and various damage extents. The low level of damage is a 10% loss of stiffness (i.e. $S_r = 0.9$), while the high level of damage is a 50% loss of stiffness (i.e. $S_r = 0.5$). Three values are considered for the extent of the damage: small extent (1.25% of panel length), moderate extent (5% of panel length), and large extent (14% of panel length). The first key result demonstrated in Figure 6 is that the attractor of the dynamics changes significantly when the extent of damage changes. Figure 6 also shows the attractors obtained for panels with a high level of damage and various damage extents. The attractor undergoes geometric changes when parametric variations occur, such as is the case in the presence of damage. Also, Figure 7 shows that the geometric changes in the attractor shape occur for all parameters of interest, i.e. the level, location, and extent of damage.

Histograms may be used to characterize the shape and distribution of points in an attractor. The histograms shown in Figure 8 demonstrate that changes in attractors may be quantified for a (constant and low) level of damage and a varying extent of damage. The change in the attractor shape is dramatic when the damage varies from small to large extent. The moderate extent of damage affects the dynamics greatly, and leads to a collapse of the attractor onto a limit cycle, as shown in Figure 6. More subtle changes occur for the case of high damage. Similar to the case of low damage, the attractor changes shape greatly when the extent of damage is varied while the level of damage is constant and high, as shown in Figure 9. Moreover, a comparison of the histograms shown in Figures 8 and 9 reveals that the changes in the histograms are distinct for various damage levels when compared to various damage extents. Thus, both the level and the extent of damage may be detected.

An alternate approach to detecting the level and extent of damage is to detect the onset of bifurcation for varying parameters of the nonlinear feedback excitation. Bifurcation diagrams are known to reflect
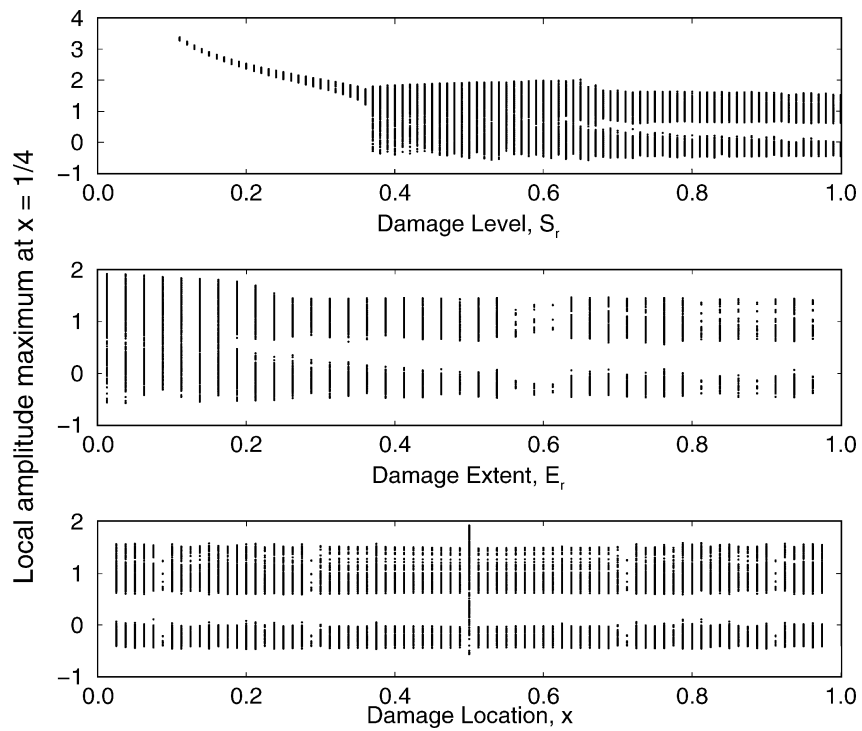
Figure 7. Bifurcation diagram for varying damage level, extent and location, showing the dramatic changes in the dynamic attractor for varying damage characteristics.
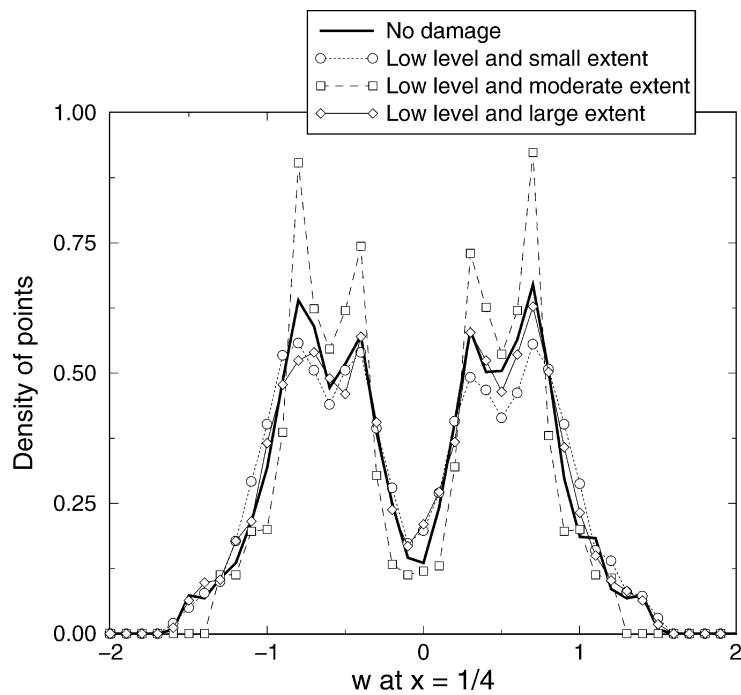

Figure 8. Histogram of the point distribution in an attractor for the dynamics of a system with low damage and various extents.
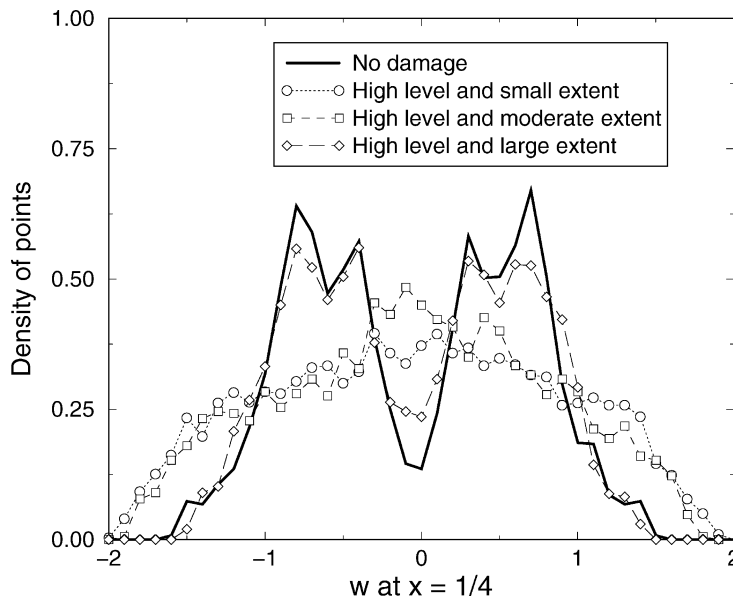
Figure 9. Histogram of the point distribution in an attractor for the dynamics of a system with high damage and various extents.
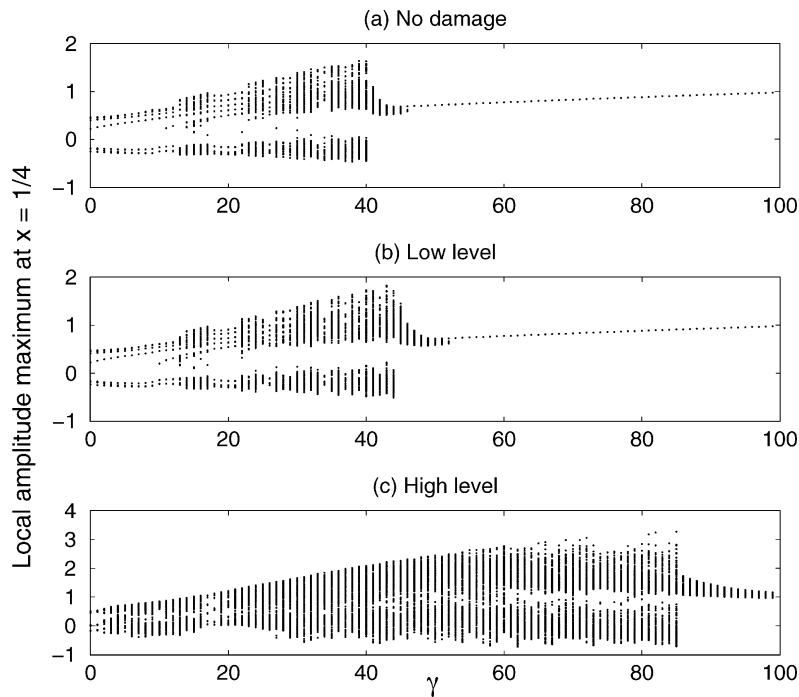

Figure 10. Bifurcation diagram for varying linear factor γ in the nonlinear feedback excitation, showing the dramatic changes in the dynamic attractor for varying nonlinear feedback characteristics.

accurately qualitative and quantitative changes which may occur in the dynamics of nonlinear systems [7, 27, 42, 43, 45, 63, 65, 74]. In the context of damage detection, we construct bifurcation diagrams by varying the control parameters λ and γ. Such diagrams are possible to measure because they require only the modification of the controller and not of the structure. In most physical systems, it is usually

difficult to change structural parameters (for testing purposes). In the present approach, such changes are not necessary because the variation in the global dynamics is accomplished by the variation of the nonlinear feedback characteristics. To demonstrate this approach, Figure 10 shows the local amplitude maxima at the quarter point location ($x = 1/4$) for various values of γ and for three separate cases: no damage, low level, and high level of damage. One may clearly identify dramatic changes in the bifurcation diagram for the three structures. For example, the onset of limit cycle oscillations occurs at clearly distinct values of γ (of 45, 50 and over 100).

5. Conclusions

A novel approach to actively interrogate a linear or nonlinear structure for the purpose of detecting parametric changes has been presented. The parametric changes identified are indicative of the presence, location and level of damage in the system. The main advantage of the proposed method is its high sensitivity to parametric variations, which makes it particularly well suited for detecting incipient damage.

Numerical examples have been presented for a panel with pinned–pinned end points forced by transverse loads and undergoing limit cycle oscillations and chaos. The nonlinear von Karman plate theory has been used to obtain a model for the panel, while damage has been modeled by a loss of stiffness in a portion of the plate. The sensitivity of this nonlinear system to parametric changes (enhanced by nonlinear feedback excitation) has been shown to be an effective tool in detecting structural changes and identifying stiffness loss in a thermo-shielding panel.

Most of the current studies of damage detection are based on linear theories and linear structures. In contrast, the results presented here are obtained using nonlinear and chaotic dynamics, and have the advantage of an increased accuracy in detecting damage and monitoring structural health. The sensitivity obtained by enhancing nonlinear dynamics and exploiting the features of chaotic dynamics is much higher than the sensitivity of linear analyses.

Acknowledgements

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