JAEGWON KIM

THE ROLE OF PERCEPTION IN A PRIORI KNOWLEDGE: SOME REMARKS

(Received 20 February, 1981)

Empiricism about mathematical knowledge, and about what is commonly thought to be a priori knowledge such as knowledge of analytic truths, has been a distinctive and influential position, if not a dominant trend, in contemporary philosophy. This global empiricism goes back, at least, to John Stuart Mill, whose views on knowledge of mathematics and logic are best characterized as 'inductivism' or 'low empiricism'. As is well known, he regarded mathematical truths, at least the simple ones of arithmetic and geometry, as low-level inductive generalizations from perceived objects and events which directly instantiate them. Thus, the evidential relation that a mathematical truth, say, '3 + 2 = 5', bears to its experiential basis is the same, according to Mill, as the relation that 'All ravens are black' bears to the observation of black ravens. Comtemporary Logical Positivists, who proclaimed themselves as the legitimate heirs to the Empiricist tradition of Hume, Mill and Mach, were never quite clear on the issue of mathematical and logical knowledge: their official doctrine was of course that mathematical truths are 'tautological' or 'analytic', arising out of certain features of language, and that our knowledge of these truths, while a priori, does not constitute a refutation of empiricism. However, they never made clear exactly how these truths 'arise out of language' or whether our knowledge of them depends, evidentially or otherwise, on our *knowledge* of language.[1] As a result, it is not clear why the Positivists thought that the linguistic theory of *a priori*, if it is correct, would be a defense of global empiricism. In challenging the concept of analyticity and the existence of a priori knowledge, Quine has come closer to the spirit of global empiricism that the Positivists. His 'high empiricism' contrasts with Mill's inductivism in that it regards mathematics not as a low-level inductive science but as an integral component of our holistic epistemic scheme whose evidential connection to observation and low-level generalizations, like that of fundamental theoretical principles of physics, highly indirect. Knowledge of mathematics and logic, on this view,

Philosophical Studies **40** (1981) 339–354. 0031-8116/81/0403-0339$01.60

remains broadly empirical in that nothing in the total scheme, including mathematics and logic, is exempt from possible reconsideration and expulsion under the pressure of observation and experience.

I do not intend to discuss Quine's views, or other similar views, in this paper,[2] but concentrate on Mill's inductivism. There is a broad consensus among philosophers that Mill's theory is wrong – not just wrong but wrong in an obvious and simplicistic way. However, I do not believe that there has been a clear appreciation of the reasons why inductivism about mathematics is wrong, and I also think that there is an important element in Mill's approach that is right. A detailed examination of some aspects of Mill's inductivism, I think, can help in placing in proper perspective recent discussions of mathematical knowledge, aprioricity, and the causal approach to empirical knowledge.

I

Let us begin by considering what Mill would say about our knowledge of a simple arithmetical truth, say '3 + 2 = 5'.[3] There are three elements of his theory that are of interest to us:

(1) The statement '3 + 2 = 5' is not about numbers considered as abstract entities, for there are no such things. It is about three pebbles and two pebbles, three elephants and two elephants, and so on; that is, the statement is about objects accessible to our experience.

(2) The statement '3 + 2 = 5' is *inductively confirmed* by its favorable instances – by observing that three pebbles and two others amount to five pebbles, and so forth, in the same way the observation of black ravens inductively confirms the generalization 'All ravens are black'. That is to say, the arithmetical statement is an inductive generalization supported by observation of its favorable instances and refuted by observation of unfavorable instances.

(3) Aprioricism about '3 + 2 = 5' is gratuitous. Even those who accept this position must grant that the statement is also confirmed inductively by its favorable instances, and that inductivism provides a sufficient account of our knowledge of this truth.

> There is no need, therefore, to resort to aprioricism to explain mathematical knowledge and thereby complicate our overall epistemology.

For us the second and third claims are crucial. Notice first that the second claim by itself is not sufficient to give us a full-fledged inductivism: for it is conceivable that although an arithmetic truth receives confirmatory support from observation of favorable instances, this support is not enough to generate *knowledge,* and perhaps not enough to warrant our rational acceptance of it as true. A full-fledged inductivism must also show that we can *know* an arithmetical truth *solely* on the basis of this kind of low-level induction. Mill's third claim contends that this is possible: inductive evidence is sufficient to give us mathematical knowledge, and it is all we need in order to explain the possibility of mathematical knowledge. It is clear, however, that the second claim is more fundamental; it is pressuposed by the third, and without it the inductivism of the sort Mill defends cannot get started.

A popular, and widely accepted, criticism of Mill's second claim takes the following form:[4] if '3 + 2 = 5' is inductively confirmable, then it must also be inductively disconfirmable; but it is in fact not inductively disconfirmable, there being no *conceivable* observation of pebbles, elephants, microbes or what have you that would make it reasonable for us to doubt its truth. We would always explain away any apparent discrepancy between the arithmetical truth and observation by casting doubt on some aspects of the observation; we would never impugn the arithmetical truth. This objection, however, is far from conclusive; it is open to the inductivist to make the following reply: our inability to 'conceive' compelling counterexamples to '3 + 2 = 5' is not due to any real metaphysical or logical impossibility (these modalities are suspect notions in any case), but it is to be explained psychologically in terms of the strongly entrenched position it enjoys in our epistemic scheme. Everywhere we look, Mill says, we see simple arithmetical and logical truths confirmed and positively instantiated so that we become psychologically unable to countenance the possibility of their being false. Thus, hewing to the general Humean lines here, the inductivist suggests that the impossibility is in the eye of the beholder, not in the nature of reality beholden; familiarity breeds necessity, or at least an illusion of necessity.

Apart from this possible reply, the criticism also has the limitation that it leaves the epistemic role of observation of instances of arithmetical truths totally unaccounted for. It is certainly plausible to think that counting pebbles and fingers is epistemically relevant to checking or finding out the sum of 3 and 2. A child who has checked this sum with pebbles or his fingers has placed himself in a stronger epistemic position with respect to '3 + 2 = 5'. Mill's explanation in terms of a simple model of induction may be wrong, but the situation clearly calls for an explanation.

Another objection to Mill can be adapted from some observations Roderick Chisholm has made concerning our knowledge of the relations of exclusion and inclusion holding for properties such as colors.[5] In case of an enumerative induction confirming a generalization of the form 'All *A*'s are *B*', if what was thought to be a positive instance of the generalization, that is, something that is both *A* and *B*, later turns out not to have been one – that is, if what was thought to be a black raven later turns out to be a black grackle or just a brown shoe – then the confirmatory force of the observation is entirely voided. However, in case of the observational confirmation of '3 + 2 = 5' this is not the case. If it turns out that it was only in a dream that I counted three pebbles and two pebbles, or if it turns out that what I counted was not pebbles but seashells, that would make no difference to our enhanced epistemic position with respect to the arithmetical statement; it would not take away the confirmation that has accrued to this statement through what has turned out to be a case of mistaken perception. As Chisholm puts it for the case of color exclusions, "if we happen to find our perception was unveridical, this finding will have no bearing upon the result"[6].

I think this point is an important one, but as thus formulated there is a plausible initial reply to it. In the case of 'All ravens are black' and black ravens, if what was thought to be a black raven turns out to be a brown shoe, then there *never* was a confirming instance. In the case of '3 + 2 = 5' and the pebbles, however, if the pebbles turn out to have been seashells, this does not entirely void the confirming instance; for we now have, instead of three pebbels and two amounting to five pebbles, a new positive instance, of three seashells and two more amounting to five seashells. This explains why our confidence in '3 + 2 = 5' remains undisturbed. On the other hand, if what we thought we counted as three and two pebbles turn out to have been four and two (that is, if we miscounted), then this discovery would make a difference to our epistemic status with regard to the arithmetical statement; the discovery would void the increment in epistemic support accorded to the arithmetical statement by the mistaken counting.

While this reply meets the objection under discussion, there is a fundamental point implicitly shared by both the objection and the reply: what is important, and epistemically relevant, in counting pebbles or fingers to verify '3 + 2 = 5' is not the fact that pebbles or fingers are involved in the perceptual situation; it is rather the fact that the numerical properties three and two, and also five, are perceived to be instantiated in a certain relationship. The arithmetical statement concerns these numerical properties, and whether pebbles or fingers or seashells instantiate them is immaterial. We will recur to this point below, for now, let us continue our focus on the possible differences between the case of checking pebbles to verify '3 + 2 = 5' and the standard case of inductive confirmation.

In a standard case of induction favorable instances are typically *cumulative* in their confirmatory effect. This is not to say that the increase in some numerical measure of 'degree of confirmation' is correlated by some simple function with the number of positive instances; nor is it to deny that in special cases in which appropriate background information is available, a single-case induction may be entirely sufficient to warrant the inductive conclusion. The point is merely that generally speaking and everything being equal, the more favorable instances you have, the better confirmed is your generalization, that the more often your generalization has survived potential falsifiers, the stronger is the credibility that accrues to it. But this cumulative effect of favorable instances seems conspicuously absent when we consider 'inductive confirmation' of arithmetical statements or logical or analytic truths. Does the observation of unmarried bachelors inductively confirm 'All bachelors are unmarried'? It would be quite senseless to say: "Now that I have checked only seven bachelors – they all confirm the generalization – I had better check a few more just to make it really sure...". Similarly, in the case of '3 + 2 = 5', it would be senseless to reason: "Well, now that I have checked this out with my fingers, I should also check it out with my toes, and maybe some pebbles, too, to make it double sure. You could help out by checking it out with your toes and fingers, too,...". There may be a point in repeating the experiment by counting the fingers again, but this is to ensure that the counting was done correctly, that is, to make sure that there *was* a 'positive instance'; it is not to multiply positive instances.

This brings out another point: in a normal induction, we think it is not the mere number of positive instances but the variety in the kinds of instances that makes a significant impact on the credence we should attribute to a generalization. Thus, we should check not just the ravens in North America, but also sample them in South America and Asia; we should check various kinds of metals, not just a large sample of the same metal, in verifying the proportionality of thermal and electrical conductivity in metals . Now this requirement of variety seems totally out of place when we consider '3 + 2 = 5': if we counted our fingers right, and verified the relationship for these fingers, there is no need whatever to go to toes, nails, or flower pots. Unlike in the typical cases of inductive confirmation, neither the number of instances nor their variety seems to affect the epistemic relation of '3 + 2 = 5' to its 'positive instances'.

I think that these two points of difference are sufficient to indicate that the epistemic relationship between '3 + 2 = 5' or 'All bachelors are unmarried' and their instances is different in nature from the inductive relationship holding between inductive generalizations and their instances. In fact, I believe it is best not to think of the former relation as an *evidential* relation, a relation between evidence and hypothesis, or between justifying grounds and knowledge claim. It is better to think of the observation involved in counting pebbles as a *cue* that prompts our apprehension of the truth of '3 + 2 = 5' rather than as evidence for the truth of this proposition.

II

The upshot of our discussion thus far is that empirical observation does not function in case of simple arithmetical truths like ' 3 + 2 = 5' in the way it does in case of standard inductive generalizations. Whatever epistemic role we may ultimately assign to experience in relation to mathematical knowledge, it does not play the role of inductive evidence to generalizations. Mill was surely wrong is using the model of induction to explain the epistemic role of experience for mathematical knowledge. But there is one aspect of his theory that we should salvage. It is this: we *see* in our perceptual experience *three* pebbles and *two* pebbles, and *see* also *that* they make up five pebbles. That is to say, *we perceive in our experience of the world, perhaps also even within our minds, numerical or mathematical properties instantiated, and we also perceive certain numerical or mathematical relationships to obtain.*

If three pebbles and two more adding up to five pebbles seems too complicated a fact to 'perceive', we may begin with a simpler example: the three pebbles over here are more numerous than the two pebbles over there. It could hardly be contested that we see or perceive a fact of this sort as we see or perceive any physical fact. Perceptual discrimination of this kind is as readily made, and as common in our daily life, as perceptual discrimination of any other sort, such as that one object is longer, larger, or heavier than another, that one color is greener or more saturated than another, that the green round spot is on top of the red triangle, and countless others. Notice also that in these other cases, too, mathematical properties and relations may be involved, though they are geometric ones such as being longer than, being round and being triangular. These are among the many common-sense perceptual judgments we make every day, and often make correctly; without the ability to make them our ability to find our way about in the world, and to survive, would be severely limited. It is well known, from extensive psychological studies, that a normal human percipient can make accurate perceptual judgments of the number of dots in random patterns flashed on a screen for a short time (around one-fifth of a second) when the number is equal to or less than seven.[7] The number seven is important here since when more than seven dots are involved not only do subjects tend to make errors but they also tend to estimate the number rather than 'directly perceive' it, or as some experimenters put it, *subitize* it. Animals, too, can be conditioned to respond differentially to colors, shapes, and numbers of dots.

The points I want to establish here, something that I hope will come to everyone as completely obvious, is this: as objects of perceptual discrimination and judgment, there is nothing unusual, uncommon or mysterious about numerical properties and relations or, more generally, mathematical properties and relations. Seeing that something is round, that these are three green dots, that the dots over here are more numerous than those over there, that there are more dots on the screen now than just a moment ago, and so on are just as common, and practically and psychologically unproblematic, as seeing that these dots are green, the dot on the left is larger and greener than the one on the right, and so on. There probably are highly complex and abstract mathematical properties which have no significant relationships to the perceptual world, but it is not their being *mathematical* properties, whatever this may come to mean, that make them inaccessible to perception. Numerical properties do not differ in respect of perceptual accessibility from sundry physical properties such as colors, shapes, odors, warmth and cold. They are among those 'sensible qualities' the Empiricists used to talk about; as may be recalled, number was thought to be a 'primary quality' of objects.

Human perception is a causal process involving the features of the object or situation perceived and the states of our sense organs and nervous system. Just as the character of our perceptual experience of there being a green dot is causally determined in part by the state of affairs of there being a green dot, so our perceptual experience of there being three dots out there, or that there are more green dots than red ones, is causally determined by there being three green dots, or there being more green dots than red ones. Causal efficacy of these states of affairs involving numerical properties and relations are of course not limited to perceptual experience; a bomb can be rigged up to detonate just in case green dots outnumber red ones on the screen. Equally obviously, there being three green dots is a causal effect of certain antecedent events or actions. Like any other concrete states of affairs these states of affairs involving numerical properties are links in the pervasive causal network of this world. In this respect there is no difference between mathematical properties instantiated in physical situations on the one hand and the so-called physical properties on the other. If no mathematical properties are realized in the physical world, there would be not much reason to worry about mathematics. And some of these physically realizable mathematical properties are also perceptually accessible.

When Mill discusses the nature of mathematics and our knowledge of it, these facts about mathematical properties in relation to perception and the physical world are taken for granted. There is no worry about mathematical properties being in some sense suprasensory, supraphysical realities inaccessible to perceptual processes. Mathematical properties are abundantly exemplified everywhere we look, just as physical properties are, and there is no special mystery about our epistemic access to them. Often properties are said to be 'abstract universals', and thought not to exist in the concrete space-time world; even if one adopts this view, mathematical properties are not thereby made more abstract or more suprasensory than physical ones; it remains an open possibility that we have the same sort of access to the former as to the latter.

One reason I dwell on these rather obvious points is that they appear to have escaped the notice of some philosophers who for the past decade or so have concerned themselves with the alleged problem of reconciling the semantics of mathematical language, which posits mathematical entities such as numbers, with an empiricist account of mathematical knowledge. The standard current source of this problem is Paul Benacerraf's 1973 paper, 'Mathematical Truth'.[8] The following passage by W. D. Hart sets forth the problem with admirable succinctness:[9]

> But it is a metaphysical axiom that natural numbers are causally inert. So a causal condition on mathematical knowledge seems unsatisfiable. This is the shape of the recent problem of platonism and the causal theory of knowledge. But there lies behind it a much older problem. Consider: Empiricism is the only real theory of knowledge; all the rest are nonstarters. Despite the failure of nerve by most classical empiricists, empiricism is the doctrine that all knowledge is a posteriori (analyticity and so forth were confused ideas). All a posteriori knowledge is justified ultimately by experience. As H. P. Grice argues, experience necessarily requires causal interaction with the objects experienced. But mathematical objects are necessarily causally inert. So platonism seems incompatible with empiricism. Yet platonism is the only adequate theory of mathematical truth. So platonism and empiricism also seem separately undeniable.

Many issues are raised here, but I want to focus on the claim, 'the metaphysical axiom' as Hart calls it, to the effect that mathematical objects are 'causally inert'. This is supposed to mean that such things as numbers, sets, and geometric figures cannot enter into causal relations, either as causes or effects. As a result, they are not knowable through experience, for all experience involves causal interaction with the objects experienced.

The points I made earlier concerning our perceptual access to numerical and other mathematical properties can be appreciated when they are set against these claims and inferences presented by Hart. The principal claim that I want to advance in this connection in this: mathematical properties, including numbers, are no worse off than such sundry physical properties as color, mass, and volume, in respect of causal efficacy. In the perceptual context, which is of primary interest to us, numbers and numerical relationships present no greater mystery or puzzles than colors and shapes. If mathematical properties are causally inert because they are 'abstract', then physical properties, qua properties, are no less abstract and should be just as causally impotent. If mathematical knowledge cannot be explained on the causal model, then we should expect the model to be unsuited as a model of knowledge of physical properties as well. Conversely, if there can be knowledge of physical properties that is in accord with the requirements of empiricism or the causal theory, that is presumptive evidence that knowledge of mathematical objects meeting the same general requirements must also be possible. At any rate, the inference we must resist is 'Abstract; therefore, causally inert; therefore, unperceivable and unknowable'. Our case against this inference depends fundamentally on two points: first, this inference applies to physical properties with equal force, leading to absurdities; second, our perceptual access to numerical and other mathematical properties and relations is in fact very much like our perceptual access to physical properties and relations.

It may be objected that the present approach construes numbers as properties of sets or classes, and that these entities themselves are abstract entities incapable of entering into causal relations. The force of saying that something is 'abstract' or 'platonic' has never been made clear. One sense sometimes attached to 'abstract' is that of 'eternal': an abstract object in this sense neither comes into being nor perishes. Another closely related sense is that of not being in space and time. Abstract entities in this sense are atemporal and nonspatial; they lack location in space-time. A third sense is that of 'necessary'; abstract entities in this sense are said to 'exist necessarily'. It is by no means obvious that these three senses are equivalent; for example, one traditional concept of God makes him abstract in the first and third sense but not in the second.

It is only the second sense of 'abstract' which may exclude abstract entities from causal relations. Spatio-temporal contiguity is one of the conditions laid down by Hume as necessary ingredients in the causal relation. Although direct contiguity in space-time is not required for noncontiguous causation, causally related objects must be in some definite spatio-temporal relationship to each other. How else could one explain why an object, which is qualitatively indistinguishable from a second, does not causally interact with a third while the second does? Abstract objects in the sense of lacking in all spatio-temporal properties including locations seem logically incapable of entering into causal relations.

The line of argument we are considering here, namely that since sets are not spatio-temporal objects, they cannot be perceived, strikes me as a lame objection. If ascribing a numerical property to these dots on the paper, that they are five in number, is to have the consequence that we are attributing a property to some suprasensible object in the platonic realm, inaccessible to human perception, then something is wrong with the descriptive apparatus being used, and it is this apparatus, and the metaphysical assumptions that underlie it, that need to be changed, and not our belief that the number five is visibly instantiated by these dots right in front of us. More specifically, there is first of all the possibility of attributing numerical properties not to sets but to *physical aggregates* or *mereological sums*, a possibility that has recently been worked out by Kessler.[10] Second, the metaphysical assumption that sets or classes are abstract in the present sense is by no means obvious or compelling. That is, it is not clear why we should not say that the class of these dots, as well as the dots themselves, is right here on this piece of paper, that this class came into existence when little Johnny made the dots with his ballpoint, that it moves when the piece of paper is moved, and that it goes out of existence when the paper is burned to ashes in the fireplace. What precisely is the reason for consigning this class to the platonic realm accessible only to 'intellectual insight'? It might be said that what I have in mind here is the mereological sum which has the five dots as its parts but not the set of these five dots. But whether mereological sums are concrete (if all of their parts are) and whether sets of classes are abstract are questions that are to be answered not by looking at some preexisting matter of fact about sets or sums but by making conventional decisions. If we take mathematical theories of sets as definitive of the concept of a set or class, they have nothing to say about the abstractness or concreteness of sets, and nothing about their spatiality or temporality. That all sets are abstract, that they are all nonspatiotemporal, that they cannot admit of changes in their membership, and so on are philosophical doctrines more often taken for granted than explicitly and openly argued.[11] Perhaps some sets are abstract and some are concrete. Perhaps sets are concrete if their members are concrete and abstract if their members are abstract.[12]

So I stand by my claim that we perceive this collection of five dots and that class of three, and also perceive that the first is more numerous than the second. It is open to us to adopt the language of mereological sums if necessary; but I want to stress that there is no compelling reason to think that the set-theoretical language is ruled out because of the alleged abstractness of sets. Thus, there is no reason to think that we cannot say both that natural numbers are properties of sets, and that we have perceptual access to some sets and their numerical properties.

It is perhaps useful to point out that the claim is not that *all* mathematical properties, or even all natural numbers; are perceptually accessible. Just as not all physical properties are perceptually accessible – there are the so-called 'theoretical properties' – only some mathematical properties are accessible in our perceptual experience. What epistemological role these perceivable mathematical properties plays vis-à-vis those that are not perceivable is a question that still needs to be addressed, and we can expect it to share points of similarity with the much debated problem about the relation between 'theoretical terms' and 'observational terms' in philosophy of science. The possibility exists that neither of these questions is a particularly useful or illuminating way to study the epistemological issues in the respective areas; what needs to be noticed is simply that the situation is symmetric, in interesting respects, as between mathematical and physical knowledge.

III

We concluded earlier that Mill was wrong about the nature of the epistemic relation between our knowledge that $3 + 2 = 5$ and our seeing that three fingers and two more add up to five. Whatever it is, the relationship is not one of inductive confirmation, the relation that inductive evidence bears to the generalization it supports. What then is it? I am not ready to offer a theory here; I shall instead present some considerations and suggestions that may go some way toward adumbrating the shape of an account that better fits with the facts to be explained. Let us begin with a closer look at Chisholm's theory of 'intuitive induction'.

Although Chisholm does not discuss numerical examples, the kind of numerical examples we are concerned with will come under his 'intuitive induction'; at any rate it is instructive, I believe, to think of these cases of apprehending simple arithmetical truths on the model of intuitive induction. We are in agreement with Chisholm's negative conclusion: intuitive induction differs in kind from the usual varieties of induction in that in the former observed instances do not serve as evidence or justification for the conclusion; they serve merely as an 'occasion'[13] for our coming to know the conclusion. We said that the observation functioned as a 'cue' to trigger our apprehension of the conclusion, and this is putting the same point in a slightly different way. According to Chisholm,[14] intuitive induction has four distinguishable stages: (1) the perception of individual things (a red dot, a green dot; three fingers, two fingers, etc.); (2) abstraction of general features or properties from the perceived individual objects (color red, green; three, two, etc.); (3) an intuitive apprehension of certain relationships holding between these properties (that red 'excludes' green; that three and two makes five); (4) the further apprehension that the relationships apprehended hold necessarily.

Chisholm's description makes one thing clear; the reason why particular observed instances are of no evidential significance is that their work has been completed, and they drop out of the picture, once the subject has reached the second stage by making proper abstractions. Once proper general concepts have been acquired, their causal origins do not matter: for all we know, they may have been implanted in our minds by the Evil Genius or some sort of injection or radiation, although of course in these cases no intuitive induction would have taken place. Notice, further, that Chisholm does not claim that all relations holding for the abstracted concepts can be intuitively apprehended; some would have to be apprehended, if at all, through a long and complex process of inference. The claim is that there are some significant relations between the abstracted features which can be directly and intuitively apprehended, and that these can serve as 'axioms'[15] to generate further truths about these entities. I believe we can add that this procedure of inference and construction can generate not only new truths but also new *concepts*: thus, the familiar foundational scheme emerges for both truths and concepts. What such generative processes might be like in detail and whether truths and concepts thus generated will suffice to explain the whole of mathematical knowledge are the important questions that will need to be discussed.

Returning to Chisholm's intuitive induction, we must observe that there is at least one respect in which the four stages discerned by him do not accommodate our numerical examples. When we count our fingers to figure out what the sum of three and two is, the main point of the exercise is not to abstract out the numerical properties three, two, and five. In a typical case of this kind, we are already in possession of the requisite concepts: that is why we are able to count to three, two, and five. The point of the exercise is to ascertain whether a certain relationship holds among these numbers. It is perhaps possible, at least for some of us, intuitively to apprehend that the sum of three and two is five, without counting fingers or pebbles, once we have the three numbers. But this is not the point; the point is rather that in the case under consideration, the counting does seem to aid in an essential way the intuitive apprehension, at Chisholm's stage (3), of a relationship holding among these numbers.

There seems to be various ways that can be tried to accommodate the numerical examples of this kind within the Chisholmian scheme; one obvious way would be to revise Chisholm's stage (3) to make room for certain kinds of perceptual constructions like counting and the drawing of figures to check geometric relationships; another would be to leave Chisholm's scheme as it is for 'basic' concepts and 'axioms' and try to work out a role for perceptual constructions in the process of inference leading from these basic concepts and axioms to more complex concepts and derived truths. Whichever route is taken, the guiding idea is that the role of perception is that of a causal cue of a certain kind, not as justificatory evidence, for the a priori truths apprehended through its aid. It is what triggers the human cognitive mechanism into appropriate action, and given the particular sort of cognitive apparatus that humans are genetically endowed with, certain types of perceptual stimuli may in fact be causally necessary to generate a priori knowledge.

It is useful to compare these reflections with Plato's doctrine of recollection,[16] a theory that most philosophers would now consider a mere curiosity. Briefly, Plato's claim was that all our knowledge is prenatal, and that what we take to be a process of acquiring knowledge is really a process of recollecting the knowledge we previously had but have forgotten. In spite of his well-known disparagement of the senses as a source of knowledge, Plato appears to assign, in this doctrine, an indispensable role to sense-perception: the disembodied soul, unencumbered by the demands of the body, can contemplate the Forms directly, and apprehend their relationships intuitively and with certainty; however, as embodied beings, we must be prompted by appropriate perceptual cues in order to recollect. It seems, therefore, that the role of sense-perception in Plato's theory of recollection is akin to its role in Chisholm's theory of intuitive induction; in both, its role is not to provide evidence but to function as 'occasion' or 'trigger'.

In a recent article[17] Philip Kitcher has worked out a detailed analysis of a priori knowledge following the classical model of priori as that which is 'independent of experience'. Kitcher's analysis is based on the reliability approach rather than the traditional approach in which the concept of justification plays the central role, but what is of interest to us is the general picture of human knowledge and, in particular, of a priori knowledge that his analysis presents. Knowledge is thought of as the output of the cognitive mechanism which is genetically built into us and which develops and matures partly as a matter of biological process and partly as a result of the sensory input applied to it. A priori knowledge is that part of its output which is independent of the specific sensory input in the sense that the output is invariant with respect to the particular input applied as long as there has been a sufficient amount of it to generate the concepts needed for the formulation of the propositional output. The similarity between Kitcher's account of the role of sensory input in the generation of a priori knowledge and Chisholm's treatment of observation as an element in intuitive induction is evident. In both accounts the chief role assigned to sensory input is to generate the requisite concepts, and it is only in this way that a priori knowledge 'depends on experience'. Once the appropriate concepts are on hand, a priori knowledge will be produced by the cognitive mechanism regardless of the character and quantity of the sensory input that happens to be provided. The limitation I pointed out earlier in Chisholm's model in handling numerical examples seems to apply also to Kitcher's analysis: sensory input may have an epistemic role to play beyond that of providing raw material out of which the requisite concepts are developed. There are many significant differences between Chisholm's and Kitcher's accounts, as may be expected from the fact that whereas Chisholm's general approach is largely traditional Kitcher's is that of 'naturalized epistemology'. What this cursory comparative look at Plato's doctrine of recollection, Chisholm's account of intuitive induction, and Kitcher's analysis of a priori indicates is that each of these diverse approaches is compatible with the conception of the epistemic role of perception for a priori knowledge that I have tried to delineate in this paper. The substantive task of analysis and theory construction remains. My project here was to mark out a direction in which further work in this area may fruitfully proceed.[18]

The University of Michigan

NOTES

[1] And what sort of knowledge of language is involved, e.g. mere competence to speak a language or propositional knowledge about a language.
[2] For a good discussion of the Quinean approach to a priori see Adam Morton, A guide Through the Theory of Knowledge (Dickenson, Endocino, Calif., 1977), ch. 8.
[3] For Mill's views, see: A System of Logic, Book II, esp. chs. 5, 6, and 7.
[4] See, e.g., Carl G. Hempel, 'On the nature of mathematical truth', reprinted in P. Benacerraf and H. Putnam (eds.), Readings in the Philosophy of Mathematics (Prentice-Hall, Englewood Cliffs, N.J., 1964). For interesting recent defenses of Mill's general position, see Glenn Kessler, 'Frege, Mill, and the foundations of arithmetic', Journal of Philosophy 77 (1980), pp. 65–79; Philip Kitcher, 'Arithmetic for the Millian', Philosophical Studies 37 (1980), pp. 215–236.
[5] Theory of Knowledge, 2nd ed. (Prentice-Hall, Englewood Cliffs, N.J., 1977), ch. 3.
[6] Theory of Knowledge, p. 39.
[7] See, e.g., E. L. Kaufman et al., 'The discrimination of visual numbers', American Journal of Psychology 62 (1949), pp. 498–525.
[8] Journal of Philosophy 70 (1973), pp. 661–679.
[9] Review of Mark Steiner, Mathematical Knowledge, Journal of Philosophy 74 (1977), pp. 124–125.

[10] In the paper cited in Note 4.

[11] For a discussion of some of these issues, see Kit Fine, 'Modal set theory' (forthcoming in Noûs). The only argument that I can think of against giving spatio-temporal locations to sets would run as follows: "If the set of those dots is located on this piece of paper where the dots are located, there are many other sets that must be located at that very same place – e.g., the power set of that set, *its* power set, ... , *ad infinitum*. Furthermore, if sets are located in physical space they must be physical objects. We would then have the result that an infinite number of distinct physical objects is located on this spot." I don't see why some technical conventions about locating sets would not adequately solve this problem.

[12] In a recent article, 'Perception and mathematical intuition' (Philosophical Review 89 [1980], pp. 163–196), Penelope Maddy makes much the same claims that I make in this paragraph, although I had not had the opportunity to read this article when the present paper was written. See especially page 179 of her paper. There are other interesting points in her paper that are echoed in this paper, but the general epistemological thrust of her views is different from mine. See also in this connection the interesting paper by Michael Resnik, 'Mathematical knowledge and pattern cognition', Canadian Journal of Philosophy 5 (1975), pp. 25–39.

[13] Chisholm, Theory of Knowledge, p. 39.

[14] Op. cit., p. 38.

[15] On Chisholm's notion of 'axiom', op. cit., pp. 41ff.

[16] The primary sources are the *Meno* and the *Phaedo*.

[17] 'A priori knowledge', Philosophical Review 89 (1980), pp. 3–23.

[18] I am grateful to Michael Resnik and Gary Rosenkrantz for helpful comments and criticisms on an earlier draft of this paper.