

## QUANTITY AND QUANTIFICATION\*

In [2], Richard E. Grandy proposes that the distinction between *mass* and *sortal* terms be understood as a difference in levels of predicates. Mass terms – like ‘gold’ and ‘water’ – are first-level predicates; sortal terms – like ‘ring’ and ‘statue’ – are second-level predicates. A natural language with terms of both sorts is to be regarded as a kind of second-order language, with quantifiers appearing at (at least) two levels.

Mass terms apply to *stuff*; sortal terms apply to *things*. First-level variables range over stuff – or *quantities* of stuff; second-level variables (first-level predicate variables) range over things.

Grandy proposes to analyze things as *relations* between quantities of stuff and points in time, subject to the condition that each such relation associates at most one quantity of stuff with each temporal point. The idea is that things are best thought of as functions – not always defined – from times to quantities of stuff. Thus a gold ring is identified with the function whose values are the quantities of gold that constitute the ring from molding to melting. Moreover, the spatial location of a thing is determined by the spatial location of the stuff that constitutes it; so the spatial location of the ring at any moment is the spatial location of its gold.

In this way, a difference emerges between the sentences ‘There is gold on the table’ and ‘There is a ring on the table’. The first has the simple form

$$(1) \quad \exists x (G(x) \wedge O(x, \text{now})),$$

where  $G$  and  $O$  are first-level predicates for ‘gold’ and ‘on the table’, and  $x$  is a first-level variable ranging over quantities of stuff. The second sentence, however, is cast something like

$$(2) \quad \exists R(\mathfrak{R}(R) \wedge \exists x(R(x, \text{now}) \wedge O(x, \text{now}))),$$

where  $\mathfrak{R}$  is a second-level predicate for ‘ring’ and  $R$  is a first-level predicate variable ranging over thing(-relation)s, appearing once in subject position and once in predicate position.

By rendering mass and sortal terms distinctly in this way, Grandy is able

to resolve some familiar puzzles. How can we affirm ‘The ring is the gold which was on the table’ and ‘The ring is new’ without implying that the gold which was on the table is also new? The answer, essentially, is that the expression ‘the ring’ is ambiguous. The sentence ‘The ring is the gold which was on the table’ is of the form

$$(3) \quad \iota x \exists R (\mathfrak{R}(R) \wedge R(x, \text{now})) = \iota x (G(x) \wedge O(x, \text{then})).$$

Here ‘the ring’ means the quantity of stuff that currently constitutes a ring – as in the description  $\iota x \exists R (\mathfrak{R}(R) \wedge R(x, \text{now}))$ . The sentence ‘The ring is new’, on the other hand, comes out as

$$(4) \quad \mathcal{N}(\iota R \exists x (\mathfrak{R}(R) \wedge R(x, \text{now}))),$$

where  $\mathcal{N}$  is a second-level predicate for ‘new’. In this case, ‘the ring’ refers to the thing itself, and not to the stuff of which it is constituted – as in the description  $\iota R \exists x (\mathfrak{R}(R) \wedge R(x, \text{now}))$ . Clearly, there is no inference from (3) and (4) to

$$(5) \quad N(\iota x (G(x) \wedge O(x, \text{then})))$$

– ‘The gold which was on the table is new’ – in which ‘new’ appears as a first-level predicate,  $N$ .

I agree with Grandy that both mass and sortal terms are best treated as predicates, but I do not share his conviction that they are predicates of different levels. Not that I believe them to be on the same level; it is just that I see little to prompt the alternative opinion. Given Grandy’s views on the nature of the entities involved, the sentence ‘There is a ring on the table’ could as well be cast as

$$(6) \quad \exists r (R(r) \wedge \exists x (\langle x, \text{now} \rangle \in r \wedge O(x, \text{now}))),$$

where  $R$  here is a first-level predicate – unquantified – for ‘ring’ and  $r$  is a first-level variable (perhaps with its range restricted to things).

The point is that nothing in the metaphysics of Grandy’s proposal requires second-order predication and quantification, as he remarks himself. Nevertheless, he argues that higher-order logic is already essential to the representation of sortal predicates like ‘bit of gold’, in which ‘bit of’ appears to apply syntactically to a mass term to produce a sortal. However, while one might agree about the utility – even the necessity – of representing ‘bit of gold’ as  $\mathcal{B}(G)$ , where  $\mathcal{B}$  is a predicate-forming operator correspond-

ing to ‘bit of’, Grandy lacks an independent argument to show that  $\mathcal{B}(G)$  is second-level rather than first-level. (It only confuses the matter – and perhaps begs the question – to call  $\mathcal{B}$  a “constant relation expression”.) Grandy does note what seems to be a “systematic difference” in the grammars of the two sorts of term. But I fail to see how this entails a difference of level.

Indeed, there may be a reason for preferring a uniformly first-order approach. There are predicates, like ‘new’, that apply equally to both stuff and things. Granted, the newness of the ring may have nothing to do with the newness of the gold that constitutes it – the gold may well be old. But ‘new’ is anyhow *applicable* to both the ring and the gold. Thus on Grandy’s analysis not only do expressions like ‘the ring’ turn out to be ambiguous, so also do certain predicates. These must be represented – as in (4) and (5) – by predicates of different levels, and while there may be no logical objection to the appearance of such duplicates, it will nevertheless be incumbent upon the semantic theory for such a language to account for the connection in meaning between them.

Let me conclude my remarks on the linguistic aspect of Grandy’s proposals with the observation that – whichever syntactic analysis be chosen – a great deal of theory must be attributed to any language containing both mass and sortal terms. For, as already noted, corresponding to every thing-relation covered by a predicate there needs to be a statement to the effect that the relation is many-one. And Grandy indicates further axioms – for example, that to say that a thing is a ring is to say that it is sometimes instantiated. It is not clear, however, where such theoretical constraints are to appear. As axioms in the object language? As part of the metatheory? More needs to be understood of Grandy’s conception of semantics for natural languages.

Now I turn to the metaphysical aspect of Grandy’s paper.

Responding to the objection that his analysis obliterates the concrete-abstract distinction – the objection that things (anyway, physical objects) are concrete whereas relations are abstract – Grandy asserts that what sort of thing something is should be accounted for in terms of the best semantic theory for a language containing words for things of that sort: “Whatever that semantics [for English] associates with ‘table’ is what ‘table’ refers to in English.” This reply is meager enough, of course, in the absence of any substantive account of how a semantic theory for a natural language might

interpret sentences with mass and sortal terms in the way Grandy advocates. But, regardless of how well his theory of stuff and things meshes with this-or-that semantic theory for this-or-that natural language, it is clear that Grandy *is* offering a theory – what may be called a formalization of certain notions implicit in discourse containing mass and sortal terms.

It is sometimes said that sortal terms differ from mass terms in that sortal terms “divide their reference” whereas mass terms do not. According to Grandy, mass terms do, after all, divide their reference; they divide it among quantities of stuff. What is a quantity of stuff – of gold, for instance? Grandy declares the notion to be a technical one, akin to but not the same as that discussed by Cartwright in [1]. For Grandy, “it suffices that a quantity be spatio-temporally coherent and consist uniformly of the same type of stuff.” He says further that “[a] quantity of gold ... contains many other quantities of gold.”

This last remark causes some perplexity. If a quantity of gold is indeed divisible into further quantities of gold, then although sentence (1) – ‘There is gold on the table’ – is intelligible enough, I do not know how to construe a sentence like

$$(7) \quad \exists!x(G(x) \wedge O(x, \text{then}))$$

– ‘There was exactly one quantity of gold on the table’. The point is not that (7) is false or that it is necessarily false. Rather, the idea that quantities are divisible in this way simply calls into question the very meaningfulness of quantification and identity in connection with mass terms and quantities of stuff.

The question raised here is important for Grandy. For he regards it as an attractive feature of his analysis that good sense can be made of sentences like (3) – ‘The ring is the gold which was on the table’. But (3) seems likely always to be false, or meaningless, inasmuch as it implies (7).

The sensible thing might seem to be to give up the idea of divisibility. Still, sentences like (7) – and hence (3) – are puzzling. Any good-sized quantity of gold will contain other quantities of gold. But how many? Does it make sense to ask? Questions like this require resolution before quantity and quantification can be used successfully to explain the behavior of mass and sortal terms.

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## BIBLIOGRAPHY

- [1] Cartwright, Helen Morris, 'Quantities', *The Philosophical Review* 79 (1970), 25–42.  
[2] Grandy, Richard E., 'Stuff and Things', this issue, pp. 479–485.

## NOTE

\* The present paper revises one I read as a reply to an earlier version of Richard E. Grandy's 'Stuff and Things' at the seventieth annual meeting of the American Philosophical Association, Eastern Division, in Atlanta, 27 December 1973. I thank Duane T. Williams for helpful criticism of this and my previous efforts.