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UNIVERSITY OF MICHIGAN
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PROGRESS REPORT
THE MATHEMATICS OF MEASUREMENT BY PARTIAL ORDERING

By

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and

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Project M965

OFFICE OF NAVAL RESEARCH, U. S. NAVY DEPARTMENT
CONTRACT NO. Nonr 374(00), NR 041-011

June 1, 1952

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UMR0957

PROGRESS REPORT FOR

CONTRACT Nonr:37400

THE MATHEMATICS OF MEASUREMENT BY PARTIAL ORDERING

A. REPORT OF C. H. COOMBS

- 1) A General Theory of Psychological Scaling: this monograph is in press and I have spent considerable time recently in editorial tasks such as proof reading.
- 2) On the Method of Single Stimuli: this is a further development of the above scaling theory in the context of the Method of Single Stimuli. Conventional methods of analyzing multiple alternative items by Guttman scalogram analysis can be shown to infere certain conditions over and above unidimentionality. A new method is developed which avoids these extra conditions.
- 3) Measurement of a Social Utility: within the context of the general theory of scaling above a social utility is developed under the condition of a single latent attribute, which gives each individual an equal vote but weights individual utilities according to the strength of the preference. This requires the assumption of the existence of a common unit of measurement but it does not need to be computed or estimated to provide a simply ordered social utility.

I have given several talks on this at the Cowles Commission Seminar and Staff Meeting.

B. REPORT OF R. M. THRALL

- 4) Utility theory. I have been engaged in investigation of certain aspects of the axiomatic foundations of utility theory. This is in part a continuation of work which I began at RAND in the summer of 1951.

The starting point is the set of axioms used by Von Neumann and Morgenstern, and the method is to study what remains when these axioms are weakened in various ways. Of course, each weakening of the axiom results in a theory which has broader scope than the original one. The first part of this work has been described in several RAND research memoranda:

Hausner, Melvin

An Embedding of a Mixture Space in a Vector Space
RM-697, 4 October 1951.

Hausner, M., and J. G. Wendel

An Embedding of a Utility Space in an Ordered Vector Space
RM-698, 4 October 1951.

Hausner, M., and J. G. Wendel

Ordered Vector Spaces
RM-716, 5 November 1951.

Thrall, R. M., and N. C. Dalkey

A Generalization of Numerical Utilities - I
RM-724, 16 November 1951.

Currently I am considering generalization of the axiom of substitutibility of indifferent events in probability mixtures. I have discussed this work at a Staff Meeting of the Cowles Commission.

- 5) Social organizations. Professor R. C. Angell of the University of Michigan, Department of Sociology, proposed the following problem. Consider two organizations A and B (e.g., Rotary and Masons) in a community. We wish to know which organization is "higher" in the community. Suppose that organization A is divided into three subgroups A_1, A_2, A_3 of equal size where A_1 is made up of the most active members (officers, committee members, etc.) of A; A_2 is the middle strata; and A_3 the bottom strata of A. Let B_1, B_2, B_3 be a similar stratification of B into equal subclasses. Let $\sum_{ij} = \text{Order } A_i \cap B_j / \text{Order } A \cap B$ and let $R = \|\sum_{ij}\|$. The matrix R gives information about the relative social "height" of the two organizations; one could use the sign of the function $f(R) = \sum_{i,j} (i-j) \sum_{ij}$ to measure which is higher. The order thus obtained would not be transitive but might serve as a basis for some more refined order relation.

The subdivision into three equal parts can readily be generalized to the case of n equal parts. The further generalization to the case of n unequal parts creates a new problem which I call the bias problem. For example, if $A = B$ and $A_1 \supset B_1, A_1 \cup A_2 \supset B_1 \cup B_2$ then we get $\sum (i-j) > ij < 0$ which would give A higher than itself. Apparently one should first adjust the matrix R to a new matrix R^* in such a way as to compensate for the unequal subdivisions and then use $f(R^*)$ for the order criterion. I have devised two methods for doing this, each of which has the desirable property $f(R^*) = 0$ for $A = B$, but neither of which is entirely satisfactory in that they do not yield $R^* = \frac{1}{n} I_n$ when $A = B$. Details of this study will be continued in a working paper to be written soon.

C. GENERAL REPORT

- 6) Experimental program. The work on utility theory and other studies in game theory indicate the need for experimental work to test the extent of validity of various axioms. The Ford Foundation has made a grant to support a seminar in the coming summer to design experiments in this field. This seminar and the ensuing experimental program should be of great help in the theoretical studies covered by this ONR contract. Conversely, the theoretical studies will help give direction to the summer seminar, and we have been much occupied this spring semester in various preliminaries to the summer program.
- 7) Mr. Walter Feit has continued his study of the problem of mapping various partly ordered sets onto simply ordered sets, and of related problems.
- 8) We are continuing our study on a general model for measurement theory and now have a rough draft of the first part.
- 9) Distribution list. Enclosed is a distribution list for technical reports submitted for your approval.
- 10) The paper on the lattice theoretic analogue of the midpoint problem now available in reprint form. A counterexample to the conjective mentioned has been found and this has led to a renewed attack on the question as to which orders of the midpoints of n ordered points are possible. The lattice problem gives an upper bound

which is more known to be too large for $n > 5$.

- 11) Interdepartmental seminar. Summaries of recent meeting enclosed.

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