DETECTION OF SIGNALS OF UNCERTAIN FREQUENCY

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ABSTRACT

Alternative models for extension of frequency sensitivity in human observers are discussed. One decision procedure for a multiple filter model is considered in some detail as a general model for decision situations in which each available response specifies a subset of the signal alternatives. Two experiments were conducted in an attempt to choose between a sweeping-filter model and a multiple-filter model. Detection in a two-alternative forced-choice experiment in which the signal could be one of two possible signals was measured as a test of the two models. The data, in connection with other available studies, are taken to show the need for a more complex model for auditory discrimination.
DETECTION OF SIGNALS OF UNCERTAIN FREQUENCY

1. INTRODUCTION

Some effort has been expended recently on the problem of constructing behavioral models which characterize ways in which the hearing mechanism may be extended in frequency sensitivity.

This work has a basis in Fletcher's critical-band concept,\(^1\) which attributed a narrow range of frequency sensitivity to observers in psychophysical detection experiments. Within this framework the next question to ask was: granted these results, how may the hearing mechanism be characterized when it must be sensitive to many different frequencies?

Some attempts have been made to answer this question in the terms of signal detectability theory. Two models seem at present to be the strongest candidates for consideration. Starting from the critical-band notion one approach has been to view the hearing mechanism as sensitive at any one moment to only a narrow range of frequencies, but able to sweep this band of sensitivity rapidly over the auditory spectrum. This mechanism is presumed to be central in nature, although for the purposes of this sort of analysis we need make no commitment to hypothetical physiological mechanisms.
An alternative model stemming originally from Helmholtz views the hearing mechanism as made up of many narrow-band filters which cover the auditory spectrum. A selective process is envisioned in which signals are detected on the basis of the outputs of selected ones of the filters, depending on the relevant frequencies.

The available experimental evidence is somewhat confusing if one attempts to choose between the two models. Two studies have been reported measuring detection of complex signals made up of mixtures of two sine waves. Marilli\textsuperscript{2} reports results consistent with a sweeping single-filter model and Green\textsuperscript{3}, using essentially the same methodology, reports results predicted from the multiple-filter model. The differential predictions were made from the detectabilities of the individual component signals when presented separately.

Tanner, Swets, and Green\textsuperscript{4} presented a somewhat different approach. On any one temporal forced-choice trial one of two known frequencies was presented and observers were to state which interval had contained a signal. The observers were not required to state which signal had been presented; they were merely to identify the interval. The data are interpreted as supporting a sweeping filter model. Veniar\textsuperscript{5,6} has reported two studies which enlarged the range of possible signals. Her equivocal results are interpreted as supporting neither model.

The present experiments are a simple extension of the Tanner, Swets, and Green study. Many more pairs of possible frequencies are considered, and an attempt was made in an extensive preliminary study to arrive at a stable prediction base enabling exact statement of how detectable the signals used were, using a traditional single-signal detection experiment. If the signals used in the uncertain signal
detection experiment are made as nearly equally detectable as possible, within the restriction that four observers are run together, precise predictions of performance can be made assuming each of the models.

2. THE EXPERIMENTS

2.1 Procedure

The experiments were conducted at the Psychophysical Laboratory, Electronic Defense Group, The University of Michigan Research Institute. In all experiments, a two-alternative forced-choice method was used. The apparatus and procedures have been described elsewhere.\(^7\) Since the time of this report an improved random source has been installed. It is an electronic spinning-disk device with random running time controlled by pulses from a radiation source, which are counted by a Geiger counter.\(^8\) A modification in procedure reported by Green\(^3\) was used which consisted of dropping the continuous background noise 10 db before each trial block and presenting the signal or signals being used. This was an attempt to remind the observers of the frequency characteristics of the signals.

Nine sine-wave frequencies were used: 500, 800, 900, 980, 1000, 1020, 1100, 1200, and 1500 cps. In a preliminary experiment, each frequency except 980 and 1020 cps was presented for at least 300 trials at each of four signal levels, and a plot of \(d'\) vs \((2E/N_o)^{1/2}\) gave estimates of signal voltage necessary to lead to equal detection at 90 percent correct for each observer at each frequency. Since the four observers were run simultaneously, a mean estimated voltage was computed for actual presentation in the main experiment.
In the main experiment a two-alternative temporal forced-choice procedure was also used, but for any experimental run each trial could have one of two frequencies presented. One of the two possible signals was always 1000 cps, and the other was one of the nine experimental signals. For each trial both the signal to be presented and the interval in which it was presented were chosen independently and randomly. The signal duration was 0.05 second. The observers' task on each trial was to specify only which one of the two intervals had contained a signal; they were not required to specify which signal. The signals were equal in detectability, and the observers always knew which signals were possible, since both signals were presented with the noise reduced before each trial. The same frequency pair was used for at least three runs of 100 trials before moving to a new pair.

After collecting data for each frequency paired with the 1000 cps signal, the noise level was increased to a level such that observers would have 75 percent correct as predicted from the single-tone preliminary experiment, and the uncertain signal procedure was repeated using the same signal voltages.

Two sets of observers were used. One group started with a noise of 0.016 volt RMS measured across their PDR-8 earphones. This was increased to 0.022 volt for the second condition. The second group of observers worked under considerably higher noise levels; 0.105 and 0.140 volt, respectively. All presentations were binaural.
3. PREDICTIONS

When the signal to be detected will be at either of two frequencies the observer must listen for both frequencies in both time intervals. The following simplified models are essentially the same as those presented by Veniar as alternatives to predict behavior in this sort of task.

3.1 Narrow-Band Scanning Model

This model was proposed by Tanner, Swets, and Green. It considers the auditory mechanism as being sensitive over a narrow band of frequencies at any one time. This band is viewed as being able to move rapidly over the audio spectrum. In the terms of this model the uncertain-frequency task would require the observer to sweep between both possible frequencies within each observation interval. It is clear that with greatly separated frequencies more time during the observation interval would be spent in sweeping than in listening for the two possible signals. With more than a certain amount of frequency separation a better strategy would be for the observer to listen for one of the two possible signals and to ignore the other. Clearly, when the ignored frequency was presented the observer would be expected to perform at no better than chance, or 50 percent correct, and when the signal listened for was presented the observer should do as well as in the preliminary experiment. When each signal is randomly selected for presentation 50 percent of the time and the two signals are equally detectable the predicted percent correct would be

\[ P(C) = .5 \left( p(C) \right) + .5 \times .5, \]
where $P_p(c)$ is the predicted percent correct for either signal, based on the preliminary experiment.

3.2 Multiple-Band Filter Model

Green\(^3\) has presented this model in some detail for the detection of multiple-component signals. The model is of a set of contiguous narrow-band filters covering the audible spectrum, any number of which can be activated as needed. If two signals are equally likely to occur in an interval, a decision as to which interval contained a signal is based on the outputs of the filters at the two frequencies. The calculation of predictions under the multiple-filter model are based on the presentation by Tanner and Birdsall.\(^9\) They define a set of two normal probability-density distributions on a decision axis, conditional on whether signal plus noise or noise alone is presented. The decision axis is shown to be a transformation of likelihood ratio. The output of each filter in the present discussion is considered to be the same as that for Tanner and Birdsall's Ideal Observer. The two-alternative forced-choice decision situation would provide two measures on the decision axis in each interval, one for each possible frequency. The optimal decision procedure would be to compute the \textit{a posteriori} probabilities of the occurrence of each possible signal in each interval, sum the probabilities in each interval, and select the largest sum.

Computation of this prediction turns out to be very difficult, so for the purposes of this paper two less efficient decision procedures are considered.

One less efficient decision procedure would be to add the two decision-axis values obtained in each time interval rather than the probabilities associated with them, and select the largest sum as
the basis for a decision. Although less efficient, this procedure is a great deal easier computationally and within this simplified conceptual scheme sets a lower bound on performance under the multiple-filter mode 1. In each interval values drawn from two normal variates are added; in the interval with noise alone the two have the same mean, and in the interval with signal plus noise the sum is of one value from a noise alone distribution and another from a signal plus noise distribution. Since all the distributions are assumed to have equal variance the two distributions of sums will have the same difference between means, but twice the variance. Then the obtained \( d' \), defined as the distance between the means of the new noise alone and signal plus noise distributions divided by their common variance, will be

\[
d'_{\text{obt}} = \frac{d'}{\sqrt{2}},
\]

where \( d' \) without subscript refers to the detectability of the two signals when presented alone as determined in the preliminary experiment. This value can be converted to a predicted percent correct as a function of \( d' \) by means of the formula

\[
P(c) = \int_{-\infty}^{\infty} F(x)g(x)dx,
\]

where \( x \) is the value of the larger of the two sums, \( F(x) = \int_{-\infty}^{x} f(x)dx \), \( f(x) \), and \( g(x) \) are the probability-density functions for noise alone and signal plus noise, respectively, and \( g(x) = f(x-d') \).

A decision procedure yielding slightly higher predicted performance is to consider the four obtained measures on the decision axis separately, with the interval containing the largest measure chosen to determine the response.
This procedure is worked out in some detail because it could apply to any task in which one of a number of specified alternatives is presented but each available response covers a set of the alternatives. This is the situation here: one of two signals could appear in one of two time intervals, giving four alternatives, but only the interval is specified by the response. At the output of each filter there are two distributions on the decision axis. This is an extension of the four-alternative forced-choice analysis given by Tanner and Swets.\textsuperscript{10} They show that the probability of being correct is given by the expression
\[ P_4(c) = \int_{-\infty}^{\infty} [F(x)]^3 g(x) dx, \]
where \( F(x) = \int_{-\infty}^{x} f(x) dx \), \( f(x) \) is the probability-density function on the decision axis for noise alone, and \( g(x) \) is the probability-density function for signal plus noise.

In the present case a response may also be scored as correct for the wrong reason, because a value from a noise alone distribution is the largest of the four measures but is in the correct interval. The probability of being correct under these conditions can be shown to be
\[ P(\bar{c}) = \int_{-\infty}^{\infty} [F(x)]^2 G(x)f(x) dx = \frac{1}{3} [1-P_4(c)], \]
where the terms are defined as before, and \( G(x) = \int_{-\infty}^{x} g(x) dx \).

Since the two events are mutually exclusive, and the actual score received in an experiment is the sum of the occurrences of both events, the above two probabilities may be added to get an overall predicted percent correct, \( P_{4,2}(c) \):
\[ P_{4,2}(c) = P_4(c) + \frac{1}{3} (1 - P_4(c)) \]

\[ = \frac{2}{3} P_4(c) + \frac{1}{3}. \]

The double subscript refers to the number of alternatives available and the number of response categories into which these are mapped.

3.3 Null Prediction

This is the prediction that in the uncertain-signal situation observers will do as well as when the signals were presented alone. This prediction has some theoretical content and will be discussed further after the data are presented.

The four theoretical predictions discussed above are graphed in Fig. 1 showing predicted percent correct as a function of the d' of the two alternative signals when presented alone. The curves represent the null prediction, the two decision procedures under the multiple-filter model, and the sweeping-filter model.

4. RESULTS AND DISCUSSION

4.1 First Experiment

Two experiments were run using different crews of observers. Data from the first experiment are presented in Figs. 2 and 3. On the abscissa are plotted the pairs of frequencies used. The connected lines are, in decreasing order, the predicted lower-bound on performance under the null prediction, the 2-filter model using the least efficient decision procedure (curve 3 of Fig. 1), and the sweeping-filter model. The circled points are the obtained data with at least 400 observations per point. The curves should be looked at from the center out; the
FIG. 1  PREDICTED PERCENT CORRECT AS A FUNCTION OF $d'$
FOR ALTERNATIVE MODELS
1000-1000 control condition should coincide with the null prediction, and, according to any of the models discussed except the null, performance should fall off on either side of center, finally reaching some asymptote. The two frequencies were not equally detectable for all conditions and all observers, due to averaging values over the observers in the preliminary experiment, so the predictions are not all exactly equal. In each case a mean percent correct between the predictions for both frequencies of a pair was plotted as the prediction for the pair. This gives slightly lower predictions for the sweeping-filter model than would be the case assuming that the observer would always listen for the more detectable of the two frequencies when he cannot sweep between them.

For each crew of observers the same signal voltages were held constant throughout. First, all the data shown on Fig. 2 were collected using a signal voltage which had led to about 90 percent detection in the preliminary experiment. After this, the noise background level was raised to a level at which about 75 percent correct was predicted, and the experiment was repeated. These data are shown in Fig. 3. This repetition was run because, in the light of the data, it was felt that performance with widely separated frequencies might be depressed if the detectability of the two possible signals was lowered by increasing the noise. As is clear from the data, only one observer shows a drop to the sweeping-filter prediction with increased noise. However, this observer's performance when the possible signals were alike in frequency was also considerably below the null prediction.

In no case, in this experiment, was the sweeping-filter prediction supported. In almost all cases performance was above the
FIG. 2  PREDICTED AND OBTAINED PERCENT CORRECT FOR OBSERVERS ON THE FIRST EXPERIMENT WITH NOISE = .016 V
FIG. 3 PREDICTED AND OBTAINED PERCENT CORRECT FOR OBSERVERS ON THE FIRST EXPERIMENT WITH NOISE = .022 V
lower-bound two-filter prediction, even when the two possible frequencies were separated by as much as 500 cycles. In all cases performance fell off as frequency separation between the signals was made larger; but the decrease was not so great as predicted by the sweeping-filter model.

The data from Observer 1 suggest another possibility, that in the uncertain signal situation the bandwidth of the auditory mechanism may be increased to near 500 cycles and held there, even when the possible signals were close together. This interpretation could be placed on any of the models to account for data of this sort. It might be argued that too many intermediate filters were being called into play or alternatively, that the sweep mechanism was covering too great a frequency range. Under either explanation greater noise power enters the decision than in the case of the single-signal preliminary experiment. At the noise level used in the first experiment the data point to a rejection of a sweeping-filter model for frequency sensitivity extension and a tentative acceptance of the multiple-filter model using a decision procedure somewhat better than the lower-bound procedure shown on the plots.

4.2 Second Experiment

At the conclusion of the first experiment it was felt that the rather low background noise-level which had been used might have been responsible for the results and that with a higher noise level the narrow-band properties of the hearing mechanism might be shown more clearly. With a new crew of observers the whole experimental procedure was repeated using a background noise about 12 db higher than before. Preliminary single-tone detection studies were run, signal
voltage levels were computed to give roughly 90 percent correct
detection, and the uncertain signal experiment was repeated. Figure
4 shows these data and the predictions under the models considered for
the three observers.

As in the first experiment the noise level was then increased
until the prediction under the null was about 75 percent. These data
are shown in Fig. 5.

Under these conditions, Observer B showed performance matching
the sweeping-filter model prediction. With as little as 100 cps difference
between the two tones the data suggest he could listen for only one of
the two signals. The other two observers yield the same sort of data
as before, never falling below the 2-filter model prediction.

5. DISCUSSION

In the light of the data now available it would seem that we
need a more comprehensive model than those considered in this paper
provide. Green\textsuperscript{3} and Marill\textsuperscript{2} both studying detection of complex tones,
present contradictory data. Green supports a multiple-filter model and
Marill supports a single-filter model. A procedural difference may be
a cause for this difference. Green used binaural and Marill used
monaural presentation. Whether or not this factor may be shown to
account for the obtained discrepancy, the models in present form are
not capable of generating precise predictions of the results.

A more immediate problem is posed by data reported by Veniar\textsuperscript{5,6}
Her interpretation of the results is that no model will predict all the
data. Tanner, Swets, and Green\textsuperscript{4} report an unknown signal experiment like
the one reported here. Using a four-alternative forced-choice procedure
FIG. 4 PREDICTED AND OBTAINED PERCENT CORRECT FOR OBSERVERS ON THE SECOND EXPERIMENT WITH NOISE = .105V
FIG. 5 PREDICTED AND OBTAINED PERCENT CORRECT FOR OBSERVERS ON THE SECOND EXPERIMENT WITH NOISE = .140 V
they obtained results supporting the sweeping-filter model with only 300 cps difference between the possible signals. An immediate conclusion is that the greatly oversimplified models thus far considered are not sufficient to handle those data as they have been reported. Rather than retreat into stating that observers might act in different ways under different circumstances there are enough data now available to show the need for a more comprehensive, more complex model.

In an attempt to retain the simplified models, Veniar argues that observers who never act as "narrow-band" observers, even in single-tone control experiments could produce results with little performance decrement under unknown signal conditions. Observers with greater decrement would then presumably have been originally listening with a more narrow band. However, such narrow-band observers would be expected to be the most efficient under the control condition, but this is not the case in the present study; the null predictions for Observer B in Fig. 5 are not higher than those for the other observers. This example has pointed out a further problem characteristic of extensions of the simplified models. It should be pointed out that the sweeping-filter model could be made ad-hoc to predict any data thus far reported if the sweep rate is considered to be variable and able to move rapidly enough over the auditory spectrum.

An attempt to develop a comprehensive model will not be made here. It has been made clear that such a model is needed, but the necessary analytic work has not yet been done. Such a model will probably have to take into account individual differences, not only in parameters such as capacity but also in the ways complex auditory discrimination problems are approached. The models thus far considered have assumed "internal filters" of fixed bandwidth, ordinarily with
square pass bands. These assumptions are undoubtedly unrealistic, and the available data require more realistic basic assumptions before we can expect to move much further with the present approach to understanding the performance of the hearing mechanism. There is enough consistency, even in the contradictory data, to invite the theoretical extension.

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FOOTNOTES


