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Mach's Principle, Mass, and the Fine Structure Constant

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A modified form of Mach's principle is proposed, and its consequences are discussed.

1. INTRODUCTION

It was Mach[1] who suggested that the mass m of a particle is due to the gravitational force of the universe (Mach's principle). This leads, in particular, to the relation

$$mc^2 = GMm/R \tag{1}$$

from a dimensional consideration, where c and G are the velocity of light and the gravitational constant, respectively, and M and R are the mass and the radius of the universe. Equation (1) gives an estimate of the uniform density of the universe[1]:

$$\rho_M = 3c^2/4\pi GR^2 = 2.1 \times 10^{-29} \quad \text{g cm}^{-3} \tag{2}$$

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[1] From the Hubble constant $H^{-1} = 1.3 \times 10^{10}$ years $= 4.1 \times 10^{17}$ sec, one estimates that $R = c/H = 1.2 \times 10^{28}$ cm.

while the density inferred from the observation of the matter distribution of galaxies[2] is

$$\rho = 2 \times 10^{-31} \quad \text{g cm}^{-3} \tag{3a}$$

according to Wagoner *et al.*[3], and

$$\rho = 7 \times 10^{-31} \quad \text{g cm}^{-3} \tag{3b}$$

according to others.[4] The contribution to the density from the intergallactic matter is more uncertain, although possibilities of having the density higher than that given in (3a) or (3b) have been discussed.[5,6]

The relation (1) brings with it a conceptual difficulty, however. In fact, interactions other than gravitation, i.e., the strong, the electromagnetic, and the weak interactions, can contribute to the mass via self-energy, according to quantum field theory. It would be unreasonable to associate the entire mass of a particle with the gravitation effect.

In order to reconcile Mach's principle with this criticism, we propose to modify the principle (Section 2) and discuss its consequences (Section 3).

2. A MODIFIED FORM OF MACH'S PRINCIPLE

Based on the discussion of the preceding section, we add the self-energy $(\delta m)c^2$ due to elementary particle interactions on the right-hand side of Eq. (1),

$$mc^2 = (GMm/R) + (\delta m)c^2 \tag{1'}$$

Comparing Eq. (1′) with

$$m = m_0 + \delta m \tag{4}$$

where m and m_0 represent the physical (observed) and the bare mass, respectively, we conclude that

$$m_0 c^2 = GMm/R \tag{5}$$

i.e., *the bare mass of an elementary particle is caused by the gravitational force of the universe*. Note that, being a self-consistent equation, the r.h.s. of Eq. (5) contains the physical mass m, not the bare mass m_0. In the absence of interactions other than gravitation of the universe, Eq. (5) coincides with Eq. (1)

It then follows from Eq. (5) that

$$m_0/m = \rho/\rho_M = GM/Rc^2 \tag{6}$$

i.e., the ratio of the bare mass to the physical mass is the universal constant at a fixed time. Since $\delta m \neq 0$, we should have

$$\rho \neq \rho_M$$

in general. If we consider Eq. (3) as a typical estimate of ρ, we suspect that the physical mass is mostly due to self-energy.

3. DISCUSSION

We now discuss some consequences of Eq. (6).

3.1. The Mass of the Electron and the Fine Structure Constant

Using the expression for the self-energy of the electron due to the electromagnetic interaction,

$$\delta m_e = (3\alpha/2\pi) m_e \ln(\Lambda/m_e), \qquad \text{for} \quad \Lambda \gg m_e \tag{7}$$

we obtain

$$1 = (\rho/\rho_M) + (3\alpha/2\pi)\ln(\Lambda/m_e) \tag{8}$$

where Λ is the cutoff mass and $\alpha = e^2/4\pi\hbar c = 1/137$ is the fine structure constant. An immediate conclusion from Eq. (8) is that

$$\rho/\rho_M < 1 \tag{9}$$

which is in accord with Eqs. (2) and (3).

If we assume that $\rho/\rho_M \ll 1$, then we have an estimate of the cutoff parameter,[2]

$$\Lambda = e^{2\pi/3\alpha} m_e = 10^{124.5} m_e \tag{10}$$

Such a huge number for Λ is close to[3]

$$\Lambda = \hbar/r_G c = M\hbar c/G m_e^2 = 2.5 \times 10^{126} m_e \tag{11}$$

where r_G is the cutoff distance. It would correspond to the distance inside which the gravitational energy of two electrons exceeds the rest energy of the universe,

$$G m_e^2/r_G = Mc^2 \tag{12}$$

[2] In Eq. (8), we have used the expression for the self-energy in the lowest-order perturbation. If the higher-order calculation gives the term $[\alpha \ln(\Lambda/m_e)]^n$, Eq. (10) must be modified accordingly.

[3] The idea of the gravitational cutoff in quantum electrodynamics was speculated many years ago; see, e.g., Ref. (7).

We may call the distance $r_G = 1.5 \times 10^{-137}$ cm the *gravitational radius of the electron*. Note that we have assumed the validity of quantum electrodynamics (QED) to an extremely short distance. So far, no breakdown of QED has been found only to the distance $\sim 10^{-14}$ cm.

From a different point of view, Eq. (11) suggests an empirical for the fine structure constant[4]

$$\alpha = \frac{2\pi/3}{\ln(M\hbar c/Gm_e^2)} = \frac{2\pi/3}{\ln[(R\hbar c^3/G^2 m_e^3)(\rho/\rho_M)]}$$
$$= 1/139 \qquad \text{for} \quad \rho/\rho_M = 10^{-2} \tag{13}$$

In the case where $\rho/\rho_M \approx O(1)$, we may choose the cutoff parameter

$$\Lambda = M \tag{14}$$

which leads to

$$m_0/m = \rho/\rho_M = \tfrac{1}{3} \tag{15}$$

according to Eq. (8). If $1 - (\rho/\rho_M) \ll 1$, we should have a much smaller cutoff parameter.

3.2. The Lehmann Theorem and Nonelementarity of Spin-Zero Particles

The self-energy of a spin-zero hadron, such as a pseudoscalar meson, is negative provided that it is elementary in the sense that its field operator appears in the Lagrangian and satisfies the canonical commutation relations (Lehmann's theorem[(9)]). If, on the other hand, the electron is elementary, Eqs. (6) and (7) require that the self-energy of *any* elementary particle be positive. In order to avoid a contradiction, therefore, we have to conclude that any spin-zero hadrons are not elementary, but bound states of, say, quarks.[5]

3.3. Interdependence of Various Interactions

We cannot use a perturbational calculation, such as in Eq. (7), for the hadronic self-energy because of the large coupling strength involved. The cutoff or the form factor due to the strong interactions also should be con-

[4] An estimate for the density of the universe is given by solving Eq. (13) as $\rho/\rho_M = 3 \times 10^{-4}$. This number is a bit smaller than those of Eq. (3). However, we should remember that we are just making an estimate of the order of magnitude. A different form for the relationship between the fine structure constant and the gravitational constant has been discussed in Ref. (8).

[5] See, however, Ref. (10), which points out the ambiguity of the Lehmann theorem due to the $\lambda\varphi^4$ interaction.

sidered in this case. The universality of Eq. (6), however, implies that some correlation should exist between various interactions, although we cannot specify how they are related. It is interesting to point out that the analysis[11] of the Gell-Mann–Levy–Cabibbo angle of the weak interaction currents seems to suggest a possible correlation between the electromagnetic and the weak interactions.

3.4. Mass of the Quark

In a quark model, the bare masses of the quarks have been estimated in various ways,[12] giving invariably a range from approximately a few million electron-volts to a few hundred million electron-volts, while a lower bound for the physical quark mass is considered to be $\sim$5 GeV, if they exist at all.[13] This may be in accord with the estimate in footnote 4, which leads to the quark mass $\gtrsim$200 GeV.

3.5. Time Variation of the Physical Constants

In the model of an expanding universe, the radius R increases with time. Hence, $(1/G)m_0/m$ must be a decreasing function of time, provided the total mass of the universe is constant in time.

The case where the ratio m_0/m decreases in time may be excluded immediately,[6] since otherwise $m_0/m = \rho/\rho_M$ would have been greater than one in the past, contradicting Eq. (9). Hence, the gravitational constant G may be an increasing function of time.

The possibility of G decreasing in time has been discussed in the literature,[14,15] but the criticisms raised there are marginal for the assumed age of the universe, 1.3×10^{10} years. For increasing G, similar discussions can be made. According to Teller's argument,[15] it would correspond to a freezing temperature on the earth about 300 million years ago, which seems unlikely from various geological evidence. However, this argument cannot be taken as a conclusive one since there could be many other causes which influence the earth's temperature (heat due to radioactivity, condition of the atmosphere, temperature variation of the sun, change of the orbit or the axis of rotation of the earth, etc.).

For simplicity, let us assume that $m_0/m \ll 1$ and is time-independent. Then, according to Eqs. (8) and (11), the fine structure constant becomes time-dependent[8,16] (increasing in time): If $G \propto t^n$, we obtain

$$\alpha^{-1} = (3n/2\pi)\ln(1/t) + \text{const} \tag{16}$$

[6] A slowly decreasing m_0/m may be tolerable if the time corresponding to the limit $m_0/m = \rho/\rho_M \to 1$ could be considered as the beginning of the expansion of the universe.

which leads to

$$\frac{1}{\alpha}\frac{d\alpha}{dt}\bigg|_{t=H^{-1}} = \frac{3\alpha H}{2\pi} = (2.7 \times 10^{-13} \quad \text{years}^{-1}) \times n \tag{17}$$

This value should be compared with the Dyson inequality[16]

$$-4 \times 10^{-13} y^{-1} \leqslant (1/\alpha)\, d\alpha/dt \leqslant 3 \times 10^{-13} \quad \text{years}^{-1} \tag{18}$$

The bound[7] of Eq. (18) has been deduced from the data on the terrestrial occurrence of the nuclei ^{187}Re and ^{187}Os.

As for the astrophysical test of the time variation of the fine structure constant we note that, for $n = 1$,

$$\begin{aligned}\frac{\alpha(\text{2.5 billion years ago})}{\alpha(\text{present time})} &= \frac{137.0356}{137.0356 + (3/2\pi)\ln(13/10.4)} \\ &= 0.9992\end{aligned} \tag{19}$$

according to Eq. (16), while the corresponding value estimated from the fine structure of emission lines of five radio galaxies[17] is

$$\alpha(z = 0.2)/\alpha(\text{lab}) = 1.001 \pm 0.002 \tag{20}$$

where $z = \Delta\lambda/\lambda$ stands for the red-shift parameter. The values in Eqs. (19) and (20) are consistent with each other within the quoted error.

APPENDIX

1. Extensive investigations have been carried out by Dicke and others on Mach's principle and possible time variation of the gravitational constant, the fine structure constant, and the other constants in nature. Among others, Refs. 8 and 20 (and the references quoted therein) should be mentioned. In these articles, it has been also pointed out that the Eötvös experiment provides a stringent bound for the space variation of the fine structure constant.

[7] A recent experiment seems to give a more stringent bound[18]

$$(1/\alpha)\, d\alpha/dt \leqslant 2 \times 10^{-14} \quad \text{years}^{-1} \tag{18'}$$

Then, the time variation of Eq. (16) for $|n| \sim 1$ may be excluded. Note, however, that the terrestrial test is not without ambiguity because of a possible time variation of the other (strong or weak) coupling constants. For an experimental bound on the time variation of the gravitational constant, see Ref. 19.

2. In the text, we have assumed that the mass of the universe M is independent of time. One may define M to be the effective mass of the universe contained inside the mass horizon, since there might be no communication with matter lying beyond. If this is the case, M may be a decreasing function of time in an expanding universe. The formula (16), however, should be still valid if the time dependence $M\hbar c/Gm_e^2 \approx t^{-n}$ is assumed.

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REFERENCES

1. E. Mach, *The Science of Mechanics* (The Open Court Publishing Co., Chicago, 1919); R. H. Dicke, in *Gravitational and Relativity* (W. A. Benjamin, New York, 1964), Chapter 7.
2. R. J. Gould, *Ann. Rev. Astron. Astrophys.* **6**, 195 (1968); I. D. Novikov and Ya. B. Zeldovic, *Ann. Rev. Astron. Astrophys.* **5**, 627 (1967).
3. R. Wagoner, W. Fowler, and F. Hoyle, *Astrophys. J.* **148**, 3 (1967).
4. J. H. Oort, Solvay Conference 1958; S. Van den Bergh, *Z. Astrophys.* **53**, 219 (1961); T. Kiang, *Monthly Notices Roy. Astron. Soc.* **122**, 263 (1961).
5. A. Sandage, *Astrophys. J.* **133**, 355 (1961); see also *Phys. Today* **23**, 34 (1970).
6. J. H. Oort, in *Galaxies and the Universe* (Columbia University Press, New York, 1968), p. 1.
7. L. D. Landau, in *Niels Bohr and the Development of Physics*, W. Pauli, ed. (Pergamon Press, London, 1955), p. 52.
8. P. J. Peebles and R. H. Dicke, *Phys. Rev.* **127**, 629 (1962); **128**, 2006 (1962); P. J. Peebles, *Ann. Phys. (N.Y.)* **20**, 240 (1962).
9. H. Lehmann, *Nuovo Cimento* **11**, 432 (1954).
10. J. W. Moffat, *Nucl. Phys.* **13**, 150 (1959).
11. R. Gatto, G. Sartori, and M. Tonin, *Phys. Letters* **28B**, 128 (1968); N. Cabibbo and L. Miani, *Phys. Letters* **28B**, 131 (1968).
12. Y. Nambu and G. Jona-Lasinio, *Phys. Rev.* **122**, 345 (1961); **124**, 246 (1961); Z. Maki and I. Umemura, *Progr. Theoret. Phys. (Kyoto)* **38**, 1392 (1967); H. Koyama, *Progr. Theoret. Phys. (Kyoto)* **38**, 1369 (1967); Y. Tomozawa, *Phys. Rev.* **186**, 1504 (1969); S. Okubo, *Phys. Rev.* **188**, 2293 (1969).
13. Yu. M. Antipov *et al.*, *Phys. Letters* **29B**, 245 (1969); L. W. Jones, in *Symmetries and Quark Models*, R. Chand, ed. (Gordon and Beach, New York, 1970), p. 225.
14. P. A. M. Dirac, *Nature* **139**, 323 (1937); *Proc. Roy. Soc. (London)* **A165**, 198 (1938).
15. E. Teller, *Phys. Rev.* **73**, 801 (1948); P. Pochoda and M. Schwarzschild, *Astrophys. J.* **139**, 587 (1964); G. Gamow, *Proc. Natl. Acad. Sci. (U.S.)* **57**, 187 (1967); R. H. Dicke, in *Gravitation and Relativity* (W. A. Benjamin, New York, 1964), Chapter 8.

16. G. Gamow, *Phys. Rev. Letters* **19**, 759 (1967); F. J. Dyson, *Phys. Rev. Letters* **19**, 1291 (1967); A. Peres, *Phys. Rev. Letters* **19**, 1293 (1967).
17. J. N. Bahcall and M. Schmidt, *Phys. Rev. Letters* **19**, 1294 (1967).
18. F. J. Dyson, private communication.
19. I. I. Shapiro *et al.*, *Phys. Rev. Letters* **26**, 27 (1971).
20. D. W. Sciama, *Monthly Notices Roy. Astron. Soc.* **113**, 34 (1953); R. H. Dicke, *Rev. Mod. Phys.* **29**, 355 (1957); *Nature* **183**, 170 (1959); *Science* **129**, 62 (1959); *Am. Scientist* **47**, 25 (1959); in *Science in Space*, L. V. Berkner and H. Odishaw, eds., (McGraw-Hill, New York, 1961), p. 96; in *Entratto da Rendiconti della Scuola Internazionale di Fisica "E. Fermi" XX Corso*, C. Moeller, ed. (Academic Press, New York, 1963), p. 1; *Theoretical Significance of Experimental Relativity* (Gordon and Breach, New York, 1964).