

# Neutrinos in a Vacuum Dominated Cosmology

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We explore the dynamics of neutrinos in a vacuum dominated cosmology. First we show that such a geometry will induce a phase change in the eigenstates of a massive neutrino and we calculate the phase change. We also calculate the delay in the neutrino flight times in this geometry. Applying our results to the presently observed background vacuum energy density, we find that for neutrino sources further than 1.5 *Gpc* away both effects become non-trivial, being of the order of the standard relativistic corrections. Such sources are within the observable Hubble Deep Field. The results which are theoretically interesting are also potentially useful, in the future, as detection techniques improve. For example such effects on neutrinos from distant sources like supernovae could be used, in an independent method alternative to standard candles, to constrain the dark energy density and the deceleration parameter. The discussion is extended to investigate Caianiello's inertial or maximal acceleration (MA) effects of such a vacuum dominated spacetime on neutrino oscillations. Assuming that the MA phenomenon exists, we find that its form as generated by the presently observed vacuum energy density would still have little or no measurable effect on neutrino phase evolution, for neutrinos in the energy range of a few *eV*.

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KEY WORDS: Background vacuum energy; neutrino oscillations.

## 1. INTRODUCTION

In the last few years there have been two very interesting developments in the field of physics. On the one hand, recent observations [1] strongly suggest that the Hubble expansion does depart from that for a purely matter dominated universe. The leading explanation is that the universe is dominated by a mysterious low energy density vacuum  $\rho_\Lambda \sim (1.6 \times 10^{-3} \text{ eV})^4$  whose dynamical effects are similar to those of a cosmological constant  $\Lambda$ . On the other hand, recent experiments at

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several research centers, such as the Super-Kamiokande Collaboration [2], have provided compelling evidence that the neutrino may have a non-zero mass. Such evidence, coupled with the fact that they can traverse very large distances unimpeded, makes neutrinos good long range probes or good information carriers between any points cosmological distances apart. In this paper we study the evolutionary consequences on the propagation of massive neutrinos in the geometry of a vacuum dominated cosmology. The results are used to discuss the effects of the observed [1] background dark energy on neutrino dynamics.

The paper is arranged as follows. In section 2 we discuss neutrino oscillations in a vacuum dominated cosmology. Section 3 deals with neutrino flight time delay in this geometry. In section 4 we look for any neutrino phase changes that may result from inertial effects. Section 5 concludes the paper.

## 2. VACUUM INDUCED NEUTRINO OSCILLATIONS

Neutrinos are produced and also detected through weak interactions. At their production each particle emerges as flavor eigenstate  $|v_\alpha\rangle$ . It is now widely believed [3] that each such state is a coherent eigenstate of a linear superposition of mass eigenstates  $|v_i\rangle$  so that  $|v_\alpha\rangle = \sum_i U_{\alpha i}^* |v_i\rangle$ , where  $U$  is unitary. These mass eigenstates propagate in spacetime as a plane waves  $|v_i(x, t)\rangle = \exp(-i\Phi_i)|v_i\rangle$ , where  $\Phi_i$  is the phase of the  $i^{\text{th}}$  mass eigenstate. Because the various mass eigenstates may have different energy and momenta, they will propagate differently in space with the result that their changing phases may interfere. This implies that a neutrino initially produced at a spacetime point  $P(t_P, x_P)$  with a flavor,  $v_\alpha$ , in an eigenstate  $|v_\alpha\rangle$  may have evolved to a different flavor  $v_\beta$ , in an eigenstate  $|v_\beta\rangle$  by the time it is detected at a different spacetime point  $Q(t_Q, x_Q)$ . Such is the basis for neutrino oscillations. The evolutionary effects on the relative phases of such mass eigenstates can be driven by, among other things, the local geometry  $g_{\mu\nu}$  of the spacetime. Since the motion of a given mass state,  $|v_i\rangle$ , in a given geometry  $g_{\mu\nu}$  is geodesic, the phase  $\Phi_i$  can be written in a covariant form

$$\Phi_i = \frac{1}{\hbar} \int p_\mu dx^\mu = \frac{1}{\hbar} \int g_{\mu\nu} p^\mu dx^\nu, \quad (1)$$

where  $p_\mu$  is the conjugate momentum to  $x^\mu$ .

Recently several authors [4] have considered geometry effects on the phases of neutrinos propagating in the gravitational field of a massive body. The current consensus is that in such a geometry the resulting phase changes are negligibly small and also only grow as  $\ln r$ , where  $r$  is the distance traversed by the neutrino. In the present discussion we consider the motion of neutrinos in a vacuum dominated cosmology. In seeking for gravitationally induced phase changes arising purely from the vacuum we assume that the latter does induce, on spacetime, a de Sitter-type metric with an energy density  $\rho_\Lambda \sim (1.6 \times 10^{-3} eV)^4$ . In comoving

coordinates the line element for such geometry is

$$ds^2 = -(1 - \chi^2 r^2) dt^2 + (1 - \chi^2 r^2)^{-1} dr^2 + r^2 d\Omega^2, \tag{2}$$

where  $\chi$  is related to the cosmological constant  $\Lambda$  by  $\Lambda = 3\chi^2$ . As this spacetime is isotropic we can, with no loss of generality, restrict the motion to some fixed plane, say  $\theta = \frac{\pi}{2}$ . Thus, the phase  $\Phi_i$  acquired by  $|v_i\rangle$  propagating from point  $P(t_P, x_P)$  to point  $Q(t_Q, x_Q)$ , in such spacetime is

$$\Phi_i = \frac{1}{\hbar} \int_P^Q (p_t dt + p_r dr + p_\varphi d\varphi). \tag{3}$$

Further, since this spacetime admits two Killing vectors  $\partial_t$  and  $\partial_\varphi$ , the associated conjugate momenta  $p_t$  and  $p_\varphi$  are conserved quantities, so that

$$p_t = -m_i(1 - \chi^2 r^2) \frac{dt}{ds} = -E = \text{const.} \tag{4}$$

and

$$p_\varphi = m_i r^2 \frac{d\varphi}{ds} = \text{const.} \tag{5}$$

Such quantities represent, respectively, the energy and the angular momentum of the particle as seen by an observer in the region  $r \rightarrow 0$ . The conjugate momenta components can be linked by the mass-shell relation,  $g^{\mu\nu} p_\mu p_\nu - m_i^2 = 0$ . Thus for the geometry under consideration, we have

$$-(1 - \chi^2 r^2)^{-1} (p_t)^2 + (1 - \chi^2 r^2) (p_r)^2 + \frac{1}{r^2} (p_\varphi)^2 + m_i^2 = 0, \tag{6}$$

where

$$p_r = m_i(1 - \chi^2 r^2)^{-1} \frac{dr}{ds}. \tag{7}$$

To keep the discussion simple we shall focus on the radial motion. In this direction  $d\varphi = 0$  and the phase in (3) becomes

$$\Phi_{i(PQ)} = \frac{1}{\hbar} \int_{r_P}^{r_Q} \left( p_r - E \frac{dt}{dr} \right) dr, \tag{8}$$

where from (4) and (7) we have

$$\frac{dt}{dr} = -(1 - \chi^2 r^2)^{-2} \frac{E}{p_r}. \tag{9}$$

Using (6) with  $d\varphi = 0$  one can express the radial component of momentum  $p_r$  in terms of the constant energy  $E$ . Equation (8) then takes the form

$$\Phi_{i(PQ)} = \frac{1}{\hbar} \int_{r_P}^{r_Q} \left( (1 - \chi^2 r^2)^{-1} \sqrt{E^2 - m_i^2(1 - \chi^2 r^2)} - E \frac{dt}{dr} \right) dr. \tag{10}$$

As the neutrino propagates, the phases of the  $i^{\text{th}}$  and  $j^{\text{th}}$  mass states, say, will evolve differently so that at the detector  $Q(t_Q, x_Q)$  a phase difference  $\Delta\Phi = \Phi_{i(PQ)} - \Phi_{j(PQ)} = \Delta\Phi_r + \Delta\Phi_t$  is observable as an interference pattern. Here and, henceforth,  $\Delta\Phi_r$  and  $\Delta\Phi_t$  refer, respectively, to the spatial-momentum and the temporal-energy contributions to the phase difference. There are usually two different approaches used in calculating  $\Delta\Phi$ , which yield similar results.

In the one case one supposes that the neutrino mass eigenstates are relativistic in their entire flight from source to detector [3]. This allows a discussion of the motion in terms of geometric optics. In this approximation such states evolve as plane waves propagating on a null surface,  $ds = 0$ . The particle energy  $E$  can then be expanded in terms of  $E_{null}$ , the associated energy of the massless fields at the origin ( $r = 0$ ) (and which is constant along the null trajectory) and  $\Delta\Phi$  calculated by evaluating the integral in (10) along this null trajectory. While the method yields the correct results it does so at the expense of overshadowing (and seemingly countering) the physics in the central argument that the neutrino is massive. The method also runs into problems if some neutrinos may turn out to have significant mass. Nevertheless, its simplicity is appealing.

On the other hand, one can take the view that neutrinos are indeed massive, and classically evolve the various eigenstates along their geodesics with the energy  $E$  and the conjugate momentum  $p_\phi$  as constants of motion. This method, when applied to the case of a neutrino as a massive particle with localized energy, appears counter-intuitive for the following reason. If the various mass eigenstates with the same energy  $E$  and different radial momenta  $p_r$  start at the same initial spacetime point  $P(t_P, x_P)$ , it is difficult to see how they could end up at the same final spacetime point  $Q(t_Q, x_Q)$  so they can interfere. The only way such particles could interfere at  $Q(t_Q, x_Q)$  is if they started at different times  $t_P$  and  $t'_P$  so that there is an initial time difference  $\Delta t$  between their points of origin. However this runs one into a further counter-intuitive problem since in the first place the initial neutrino flavor  $\nu_\alpha$  was supposed to be a localized particle.

Nevertheless, each of the above approximations yields the correct results and we shall find it convenient in this treatment to utilize the latter approach with modifications. Our arguments are related to those of Battacharya et al. [4] where the authors consider a wave packet with a large flavor correlation length. In our treatment, we assume that at the initial spacetime point  $P(t_P, x_P)$  the mass states are produced with an energy width  $W$  related to an energy spread  $\Delta E$  about some average energy  $\langle E \rangle$ , which classically turns out to be the constant energy  $E$ . This spread then contributes a term  $\frac{\Delta E}{W}$  to the phase difference,  $\Delta\Phi$ . Consequently, the masses  $m_{i=1,2}$  will arrive at  $Q(t_Q, x_Q)$  simultaneously and hence interfere provided the temporal-energy contribution to the phase difference vanishes,

i.e.

$$\Delta\Phi_t = \Delta \frac{1}{\hbar} \int_p^Q p_r dt - \frac{\Delta E}{W} = 0. \tag{11}$$

With this, we then have that, for interference to take place, the only active contribution to the phase difference  $\Delta\Phi$  is  $\Delta\Phi_r$  given by

$$\begin{aligned} \Delta\Phi_r &= \Delta_{ij} \left[ \frac{1}{\hbar} \int_p^Q p_r dr \right] \\ &= \Delta_{12} \left[ \frac{1}{\hbar} \int_{r_p}^{r_Q} \left( (1 - \chi^2 r^2)^{-1} \sqrt{E^2 - m_{i=1,2}^2 (1 - \chi^2 r^2)} \right) dr \right], \end{aligned} \tag{12}$$

where  $\Delta_{12}$  takes the difference between the two integrals associated with the two different masses  $m_1$  and  $m_2$ . Noting that  $m_i^2 \ll E^2$  one finds, on reintroducing  $c$  into (12), that

$$\begin{aligned} \Delta\Phi_r &= \frac{\Delta m^2 c^3}{2\hbar E} (r_Q - r_P) + \frac{1}{8} \frac{\Delta m^4 c^7}{\hbar E^3} (r_Q - r_P) \\ &\quad - \frac{1}{24} \frac{\Delta m^4 c^7}{\hbar E^3} \chi^2 (r_Q^3 - r_P^3) + \dots, \end{aligned} \tag{13}$$

where  $\Delta m^2 = |m_1^2 - m_2^2|$  and  $\Delta m^4 = |m_1^4 - m_2^4|$ . The first two terms in (13) are [4] the usual flat space  $\Delta\Phi_0$  and the special relativistic correction  $\Delta\Phi_{rel}$ , respectively, for neutrino oscillations. The term in  $\chi^2$  is our result for the leading cosmological background contribution  $\Delta\Phi_\Lambda$  to the oscillation of neutrinos. Notice that this cosmological term grows as  $r^3$  while the first two terms only grow as  $r$ . Clearly for neutrino sources at cosmological distances  $r \sim \frac{1}{\chi}$  the cosmological term can be important. In particular, for the current estimates of the cosmological constant at  $\Lambda = 3\chi^2 \approx 10^{-56} cm^{-2}$ , the phase effects due to the geometry, on neutrinos from a source like a supernova, some 1.5Gpc away would be of the order of the special relativistic correction term. Since  $\Delta\Phi_\Lambda$  is opposite, in sign, to  $\Delta\Phi_{rel}$  then at such distances the relativistic corrections are significantly suppressed so that  $\Delta\Phi_r \rightarrow \Delta\Phi_0$ . One can compare this result to that due to the gravitational effect of a massive body. Bhattachrya et al. [4] have shown that in this latter geometry gravitational contributions to neutrino oscillations are virtually negligible and grow only as  $\ln \frac{r_Q}{r_P}$ , where  $r_Q < r_P$ . As one can infer from our result, the effect due to geometry under consideration evolves differently, becoming non-trivial at large distances.

### 3. NEUTRINO FLIGHT DELAY TIMES

One other possible effect due to the local geometry on the dynamics of a massive neutrino is the flight time. Here we are interested in the time delay between

the flights of neutrinos and photons as induced by the geometry of a vacuum dominated spacetime. The neutrino flight time  $\Delta t_\nu$  can be estimated from the preceding discussion. From (9) we have that  $\Delta t_\nu = -\int (1 - \chi^2 r^2)^{-2} \frac{E}{pr} dr$ . Applying (4) and (6) gives

$$\Delta t_\nu = - \int_{r_P}^{r_Q} (1 - \chi^2 r^2)^{-1} \frac{E}{\sqrt{E^2 - m_\nu^2 (1 - \chi^2 r^2)}} dr. \tag{14}$$

Since  $1 > \chi^2 r^2$  and since for the neutrino we always have  $E^2 \gg m^2$  we can expand each of the two terms of the integrand. This gives, to order  $\frac{m^4}{E^4}$

$$\Delta t_\nu = - \left( 1 + \frac{m^2}{2E^2} + \frac{3m^4}{8E^4} \right) (r_Q - r_P) - \frac{1}{3} \left( 1 - \frac{3m^4}{8E^4} \right) \chi^2 (r_Q^3 - r_P^3) + \dots \tag{15}$$

The first term is the flight time in a Minkowski space time. It contains the classical and higher order relativistic contributions. The second term  $\sim r^3$  results from the modifications of the spacetime geometry by the cosmological constant.

On the other hand, the flight time for a photon leaving the same spacetime point  $P(t_P, x_P)$  as the neutrino to the same detector  $Q(t_Q, x_Q)$  can be obtained by integrating  $(\frac{dt}{dr})_{null}$  along the null trajectory. We have from (9) that  $(\frac{dt}{dr})_{null} = -[(1 - \chi^2 r^2)^{-2} \frac{E}{pr}]_{pr=gu} E$ . Then the photon flight time  $\Delta t_\gamma$  is given by

$$\Delta t_\gamma = \int \left( \frac{dt}{dr} \right)_{null} dr = - \int (1 - \chi^2 r^2)^{-1} dr. \tag{16}$$

which evaluates to  $\Delta t_\gamma = -\frac{1}{2\chi} \ln[\frac{1+\chi r}{1-\chi r}]_{r_P}^{r_Q}$ . This result can be expanded for  $r < \frac{1}{\chi}$  to give

$$\Delta t_\gamma = -(r_Q - r_P) - \frac{1}{3} \chi^2 (r_Q^3 - r_P^3) - \frac{1}{5} \chi^4 (r_Q^5 - r_P^5) + \dots \tag{17}$$

The delay  $\Delta t_{\nu\gamma}$  in neutrino arrival time with respect to photon arrival time is then given by  $\Delta t_{\nu\gamma} = \Delta t_\nu - \Delta t_\gamma$ . From equations (15) and (17) we find (on re-introducing  $c$ 's) that

$$\Delta t_{\nu\gamma} = - \left( \frac{m^2 c^3}{2E^2} + \frac{3m^4 c^7}{8E^4} \right) (r_Q - r_P) + \left( \frac{m^4 c^7}{8E^4} \right) \chi^2 (r_Q^3 - r_P^3) + \dots \tag{18}$$

Our result departs from the usual result in flat space by the term

$$\delta t_{\nu\gamma} = \left( \frac{m^4 c^7}{8E^4} \right) \chi^2 (r_Q^3 - r_P^3) \tag{19}$$

resulting from the vacuum effects. We note that the only adjustable parameter in this result is the distance from the source to the detector. As in the phase change  $\Delta \Phi_r$

result in (13) we find that for the current estimates of the cosmological constant at  $\Lambda = 3\chi^2 \approx 5 \times 10^{-56} \text{ cm}^{-2}$ , the time delay effects due to the geometry, on massive neutrinos from a source like a supernova some 1.5 Gpc away would be of the order of the relativistic correction term. The result, then, is that for neutrino sources at distances  $r \gtrsim 1.5 \text{ Gpc}$  the relativistic corrections are, again, significantly suppressed by the (opposite sign) corrections  $\delta t_{\nu}$  due to the vacuum induced geometry.

#### 4. VACUUM INDUCED INERTIAL EFFECTS AND NEUTRINO PHASES

As a final consideration on gravitationally induced phase changes in neutrino eigenstates we take a look at the possible effects originating from a phenomenon that has lately been common in the literature, namely that of maximal acceleration (MA) of particles. We should mention that, to our knowledge, such a phenomenon has not yet been observed. In order to set the problem we find it useful to lay out the background. The geometry experienced by a particle of mass  $m$  accelerated in a background spacetime  $g_{\mu\nu}$  was first discussed by Caianiello [5]. According to Caianiello such a geometry is defined [5] on an eight-dimensional manifold  $\mathbf{M}_8$  by a metric  $d\tilde{s}^2 = g_{AB}dX^A dX^B$ , where  $(A = 0, 1, 2 \dots 7)$  and  $X^A = (x^\mu, \frac{c^2}{\mathcal{A}_m} \frac{d\tilde{x}^\mu}{ds})$ . Here  $g_{AB} = g_{\mu\nu} \otimes g_{\mu\nu}$ ,  $ds$  is the usual four-dimensional element given by  $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$ , with  $\mu = 0, 1 \dots 3$  and  $\mathcal{A}_m$  is called the maximal acceleration of the particle mass  $m$ , given by  $\mathcal{A}_m = \frac{2mc^3}{\hbar}$ . An effective four-dimensional spacetime that takes consideration of the maximal acceleration of the particle [6] can be defined as an imbedding in  $\mathbf{M}_8$ . The metric  $\tilde{g}_{\mu\nu}$  induced on such a hypersurface imbedded in  $\mathbf{M}_8$  gives rise to a line element

$$d\tilde{s}^2 = \sigma^2(x) g_{\alpha\beta} dx^\alpha dx^\beta, \tag{20}$$

where the conformal factor  $\sigma^2(x)$  is given by  $\sigma^2(x) = 1 + g_{\mu\nu} \frac{c^4}{\mathcal{A}_m^2} \ddot{x}^\mu \ddot{x}^\nu$ . The appearance of the quantity  $\mathcal{A}_m = \frac{2mc^3}{\hbar}$ , where  $m$  is the rest mass of the particle, implies that the geodesics are mass dependent, in violation of the equivalence principle. Moreover, the accelerations  $\frac{d^2x^\mu}{ds^2} = \ddot{x}^\mu$  which are related to the Newtonian force are not covariant quantities and, with respect to symmetries, the conformal factor  $\sigma^2$  is neither invariant nor can it be removed by general coordinate transformations [7]. The metric in (20) does not therefore satisfy the standard requirements of general relativity. The original derivations [5, 6] which aimed at relating quantum mechanics to gravity use special relativity in flat Minkowski ( $g_{\mu\nu} \rightarrow \eta_{\mu\nu}$ ) spacetime. One then assumes that the technique should yield reasonably accurate results, at least in a locally flat environment as generated by weak gravitational fields.

On a cosmological scale, such a weak field environment can be provided by the observed [1] background vacuum energy density in the form of a cosmological constant  $\Lambda = 3\chi^2$ , and whose metric is given by (2). We shall presently derive, based on (20), expressions for MA in a vacuum dominated cosmology and use such results to seek for any associated effects on the evolution of neutrino phases.

The symmetry of this spacetime leads to  $\sigma^2 = \sigma^2(r, \theta)$ . Accordingly a relativistic particle, like a neutrino mass  $m$ , moving in such a cosmological environment would experience a geometry

$$d\bar{s}^2 = \sigma^2(r, \theta)[-(1 - \chi^2 r^2)dt^2 + (1 - \chi^2 r^2)^{-1}dr^2 + r^2 d\Omega^2]. \tag{21}$$

One can restrict the motion to some fixed plane, say  $\theta = \frac{\pi}{2}$ . We then have that,

$$\begin{aligned} \sigma^2 = \sigma^2(r) = 1 + \frac{c^4}{\mathcal{A}_m^2} & \left[ (1 - \chi^2 r^2) \left( \frac{d^2 t}{ds^2} \right)^2 \right. \\ & \left. - (1 - \chi^2 r^2)^{-1} \left( \frac{d^2 r}{ds^2} \right)^2 - r^2 \left( \frac{d^2 \varphi}{ds^2} \right)^2 \right]. \end{aligned} \tag{22}$$

The quantities  $\ddot{x}^\mu = \frac{d^2 x^\mu}{ds^2}$  can be written down in terms of the total energy  $E$  and the angular momentum  $L$ . Using the mass-shell relation (6) and the energy conservation equations (4) and (5) we find that

$$\begin{aligned} \ddot{t} &= \left( \frac{E}{m} \right)^2 \frac{4\chi^4 r^2}{(1 - \chi^2 r^2)^4} \left[ \left( \frac{E}{m} \right)^2 - (1 - \chi^2 r^2) \left( 1 + \frac{L^2}{mr^2} \right) \right], \\ \ddot{r} &= \left[ \chi^2 r + \frac{L^2}{mr^3} \right]^2, \\ \ddot{\varphi} &= \frac{L^2}{mr^6} \left[ \left( \frac{E}{m} \right)^2 - (1 - \chi^2 r^2) \left( 1 + \frac{L^2}{mr^2} \right) \right]. \end{aligned} \tag{23}$$

Equations (22) and (23) give the conformal factor in a de Sitter spacetime as

$$\sigma^2(r) = 1 + \frac{c^4}{\mathcal{A}_m^2} \left[ \begin{aligned} & \left( \frac{E}{m} \right)^2 \frac{4\chi^4 r^2}{(1 - \chi^2 r^2)^3} \left[ \left( \frac{E}{m} \right)^2 - (1 - \chi^2 r^2) \left( 1 + \frac{L^2}{mr^2} \right) \right] \\ & - (1 - \chi^2 r^2)^{-1} \left[ \chi^2 r + \frac{L^2}{mr^3} \right]^2 \\ & - \frac{L^2}{mr^4} \left[ \left( \frac{E}{m} \right)^2 - (1 - \chi^2 r^2) \left( 1 + \frac{L^2}{mr^2} \right) \right]. \end{aligned} \right]. \tag{24}$$

As has been our approach we shall consider here, for the neutrino mass eigenstate  $|\nu_i\rangle$ , only the radial motion ( $d\varphi = 0$ ). Then the effective conformal

factor becomes

$$\sigma^2(r) = 1 + \frac{c^4}{\mathcal{A}_m^2} \left[ \frac{\chi^4 r^2}{(1 - \chi^2 r^2)} \right] \left\{ \left( \frac{E}{m} \right)^2 \frac{4}{(1 - \chi^2 r^2)^2} \times \left[ \left( \frac{E}{m} \right)^2 - (1 - \chi^2 r^2) \right] - 1 \right\}. \tag{25}$$

Following (1), the phase  $\tilde{\Phi}_i$  induced on a neutrino mass eigenstate  $|v_i\rangle$  propagating in this geometry is now given by  $\tilde{\Phi}_i = \frac{1}{\hbar} \int p_\mu dx^\mu$ , where  $\tilde{p}_\mu = m \tilde{g}_{\mu\nu} \frac{dx^\nu}{ds}$  is the four-momentum and  $\tilde{g}_{\mu\nu} = \sigma^2(r) g_{\mu\nu}$ . The mass-shell condition (3) is modified to  $\tilde{g}^{\mu\nu} p_\mu p_\nu - m_i^2 = 0$  and yields  $\tilde{p}_r = (1 - \chi^2 r^2)^{-1} \sqrt{E^2 - m_i^2 \sigma^2(r) (1 - \chi^2 r^2)}$ . As a result, one can now address the conditions for interference to take place at the detector  $\mathcal{Q}$ . Applying the same arguments leading to (12) one finds that the only active contribution to the phase difference  $\Delta\tilde{\Phi}$  is  $\Delta\tilde{\Phi}_r$  given by

$$\begin{aligned} \Delta\tilde{\Phi}_r &= \tilde{\Delta}_{ij} \left[ \int_P^{\mathcal{Q}} \tilde{p}_r dr \right] \\ &= \tilde{\Delta}_{12} \left[ \int_{r_P}^{r_Q} ((1 - \chi^2 r^2)^{-1} \sqrt{E^2 - m_{i=1,2}^2 \sigma^2(r) (1 - \chi^2 r^2)}) dr \right], \end{aligned} \tag{26}$$

where  $\sigma^2$  is given by (25) and where  $\Delta_{12}$  takes the difference between the two integrals associated with the two different masses  $m_1$  and  $m_2$ . Equation (26) can be evaluated to give  $\Delta\tilde{\Phi}_r = \Delta\Phi_r + \Delta\Phi_{\sigma(r)}$ , where  $\Delta\Phi_r$  is given by (13) and  $\Delta\Phi_{\sigma(r)}$  is the new contribution involving the MA term and is given (up to terms in  $r^3$ ) by,

$$\Delta\Phi_{\sigma(r)} = -\frac{\hbar}{6c^3} \left[ \frac{E^3}{c^4} \frac{\Delta m^4}{(m_1 m_2)^4} - E \frac{\Delta m^2}{2(m_1 m_2)^2} \right] \chi^4 (r_Q^3 - r_P^3). \tag{27}$$

Here we have, again, restored all  $c$ 's to facilitate numerical estimates. Clearly  $\Delta\Phi_{\sigma(r)}$  contributes differently to  $\Delta\tilde{\Phi}_r$  than the regular  $\Delta\Phi_r$  given in (13). One notices from equation (27) that the leading term in  $\Delta\Phi_{\sigma(r)}$  is second order in  $\chi^2$  and at the same time proportional to  $E^3$ . If we compare this leading MA term in (27) to the relativistic term  $\Delta\Phi_{rel} = \frac{1}{8} \frac{\Delta m^4 c^7}{\hbar E^3} t(r_Q - r_P)$  in (13) we see that even at cosmological distances  $r \sim \frac{1}{\chi}$  the ratio  $\frac{\Delta\Phi_{\sigma(r)}}{\Delta\Phi_{rel}}$  is controlled by the cosmological constant,

$$\frac{\Delta\Phi_{\sigma(r)}}{\Delta\Phi_{rel}} \approx \frac{4}{3} \left[ (\hbar c)^2 \frac{E^6}{(mc^2)^8} \right] \chi^2. \tag{28}$$

For supernova neutrinos, say, with energy  $E \sim 10 \text{ eV}$  and mass  $mc^2 \sim 0.1 \text{ eV}$  one can deduce that the above ratio is extremely small,  $\sim 10^2 \chi^2$ . Evidently,

$\frac{\Delta\Phi_{\sigma(r)}}{\Delta\Phi_{rel}}$  becomes even smaller for  $r \ll \frac{1}{\chi}$ , diminishing (see (27)) as  $\chi^4 r^2$ . Our estimates have not included the case for high energy neutrinos in the  $TeV$  energy range, such as those suspected from AGN sources (see for example Piriz and Wudka [4]). In this latter case (which we defer for a future report) the above ratio could, in theory, be significant for distant sources  $\sim \frac{1}{\chi}$ , although in practice the flux from such distant sources is likely to be vanishingly small. All in all, assuming the MA effect exists, its cosmological form still appears to have little or no measurable contribution to neutrino oscillations, for neutrinos in the energy range of  $\sim 10 eV$ . On the other hand it has recently been shown [8] that in a Schwarzschild geometry,  $\Delta\Phi_{\sigma(r)}$  can make significant contributions to  $\Delta\Phi$ .

## 5. CONCLUSION

In conclusion, we have investigated the gravitational effects of vacuum energy on the propagation of neutrinos. To isolate such effects, we assumed the vacuum defines on spacetime a de Sitter-type geometry (with a positive cosmological constant). It is found that such vacuum geometry induces a phase change  $\Delta\Phi_{\Lambda}$  in the neutrino eigenstates. This phase change grows as  $r^3$ , where  $r$  is the distance of the source from the detector. We have also calculated the neutrino delay time induced by such a geometry and found a similar cubic growth in the radial component of the motion. In particular, we find that for  $r \gtrsim \frac{1}{\chi}$  the phase change  $\Delta\Phi_{\Lambda}$  contribution to  $\Delta\Phi$  and the flight time delay  $\delta t_{v\gamma}$  contribution to  $\Delta t_{v\gamma}$  can both be of the order of their respective special relativistic contributions. Applying our results to background vacuum energy density associated with [1] the presently observed  $\Lambda \sim 5 \times 10^{-56} cm^{-2}$ , we find that for neutrino sources further than  $1.5 Gpc$  away both the above effects become non-trivial. Such sources are well within the Hubble Deep Field. The results which are theoretically interesting are also potentially useful, in the future, as detection techniques improve. For example such effects, on neutrinos from distant sources like supernovae, could be used in an independent method alternative to standard candles, to constrain the background dark energy density and the deceleration parameter. Undoubtedly, making use of such information depends on improved future techniques to record events from weak neutrino fluxes as those originating from such sources cosmological distances away.

Finally, the discussion was extended to investigate Caianiello's inertial or maximal acceleration (MA) effects of such a vacuum dominated spacetime on neutrino oscillations. Assuming that the MA phenomenon exists, we find that its form as generated by the presently observed  $\Lambda \sim 10^{-56} cm^{-2}$  would still have little or no measurable effect on neutrino phase evolution, for neutrinos in the range of a few  $eV$ , even at cosmological distances  $\sim \frac{1}{\chi}$ .

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Editor's note: The Preprint hep-ex/0206016 Five Years of Neutrino Physics with Super-Kamiokande by M. B. Smy, to appear in the proceedings of the conference: XXXVII Rencontres de Moriond: Electroweak Interactions and Unified Theories, represents a good review of the most recent results on neutrino physics.