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Report of Project MICHIGAN

OPTICAL DATA-PROCESSING AND FILTERING SYSTEMS

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PREFACE

Project MICHIGAN is a continuing research and development program for advancing the Army's long-range combat-surveillance and target acquisition capabilities. The program is carried out by a full-time Willow Run Laboratories staff of specialists in the fields of physics, engineering, mathematics, and psychology, by members of the teaching faculty, by graduate students, and by other research groups and laboratories of The University of Michigan.

The emphasis of the Project is upon basic and applied research in radar, infrared, acoustics, seismics, information processing and display, navigation and guidance for aerial platforms, and systems concepts. Particular attention is given to all-weather, long-range, high-resolution sensory and location techniques, and to evaluations of systems and equipments both through simulation and by means of laboratory and field tests.

Project MICHIGAN was established at The University of Michigan in 1953. It is sponsored by the U. S. Army Combat Surveillance Agency of the U. S. Army Signal Corps. The project constitutes a major portion of the diversified program of research conducted by Willow Run Laboratories in order to make available to government and industry the resources of The University of Michigan and to broaden the educational opportunities for students in the scientific and engineering disciplines.

Progress and results described in reports are continually reassessed by Project MICHIGAN. Comments and suggestions from readers are invited.

The work reported herein was conducted by the Radar Laboratory of Willow Run Laboratories, under the joint sponsorship of the U. S. Army Signal Corps and the U. S. Air Force, Wright Air Development Division.

Robert L. Hess
Technical Director
Project MICHIGAN

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Optical Data-Processing and Filtering Systems

ABSTRACT

Optical systems, which inherently possess two degrees of freedom rather than the single degree of freedom available in a single electronic channel, offer some advantages over their electronic counterparts for certain applications. Coherent optical systems have the added property that one may easily obtain many successive two-dimensional Fourier transforms of any given light amplitude distribution, or, by using astigmatic optics, one may obtain one-dimensional transforms. Therefore, most linear operations of an integral-transform nature are easily implemented. The optical implementation of integral transforms which are of importance to communication theory is discussed; the general problems of optical-filter synthesis and multichannel computation and data processing are introduced, followed by a discussion of potential applications. Astigmatic systems, which permit multichannel operation in lieu of two-dimensional processing, are treated as a special case of general two-dimensional processors. Complex input functions are discussed with reference to their role in coherent optical systems.

1
INTRODUCTION

Optical images inherently possess two degrees of freedom, as represented by the two independent variables which define a point on a surface. In this respect, optical systems differ basically from electronic systems, which possess only time as an independent variable.

Optical systems have the additional property that a Fourier transform relation exists between the light amplitude distributions at the front and back focal planes of a lens used in such a system. This property may be put to use in coherent optical systems. An optical arrangement which presents a space-domain function and successive Fourier transforms is easily implemented. As a result, integral-transform operations may often be carried out more conveniently in an optical system than in an equivalent electronic channel. Because of this Fourier transform relation, coherent optical systems behave, in many ways, analogously to electrical filters. The ease of synthesis of these optical filters has recently made them useful in some areas where only electrical filter networks were previously used.

In many problems arising in the field of communication engineering, a piece of electronic equipment is required to operate on an incoming signal so as to evalute an integral of either of the general forms.

$$I(x_o, y) = \int_{a(y)}^{b(y)} f(x, y)\, g(x-x_o, y)\, dx \tag{1}$$

where x_o, a, and b may be functions of time, or

$$I(x_o, y_o) = \int_c^d \int_{a(y)}^{b(y)} f(x, y)\, g(x-x_o, y-y_o)\, dx\, dy \tag{2}$$

where x_o, y_o, a, b, c, and d may be functions of time.

Processes such as those of cross-correlation, autocorrelation, convolution, spectral analysis, and antenna-pattern analysis are special cases of the integrals in Equations 1 and 2, as are also various linear integral transforms. The integral in Equation 2, which includes a second integration over the y-variable, allows the generation of two-dimensional transformations.

Electronic computation and evaluation systems for performing the above integrations exist, but they suffer from disadvantages inherent in systems possessing only one degree of freedom. In the electronic case, time is the only available independent variable. This is a severe restriction if either (1) the integral is to be evaluated for a large number of different values of the parameter y, or (2) an integration over y is also required. In such cases, scanning, time-sharing, or time-sequencing procedures must be employed.

In an optical system, however, two independent variables are available. Thus, the optical system can readily handle the two-dimensional operation without resort to scanning. Alternatively, for a one-dimensional process with a varying parameter, the second dimension can be used to provide a number of independent computing channels for various incremental values of the unused variable. In the optical system, the number of independent one-dimensional channels is limited only by the number of positions which can be resolved across the system aperture. The two-dimensional nature of an optical channel may therefore be exploited either to provide a true two-dimensional processor or to provide a multichannel filter bank. For certain types of operations, the second degree of freedom may permit a considerable equipment simplification; partly for this reason, interest in optical processing has been growing for the past several years.

The basic optical theory which permits a filter-theoretic description of an optical data-handling channel is by no means new. The Fourier transform relations upon which the spatial filtering is based are essentially the relations established by Huygens, Fresnel, and Kirchoff

(Reference 1), while the spatial filtering is essentially that proposed by Abbe in his theory of image formation in the microscope (Reference 1). In the past decade, optical systems have been extensively discussed in terms of communication theory (References 2-9). The present authors have made use of astigmatic optical systems to achieve multichannel operation, as described in Section 2. 4, and have synthesized complex filter functions of two variables. In addition, bipolar and complex signals have been recorded as transparencies with positive transmittance by use of a carrier frequency, and then optically converted back into bipolar or complex form by appropriate spatial filtering.

This report outlines some of the fundamental principles and techniques useful in understanding and designing coherent optical systems and indicates areas of potential applications.

2
THEORY of COHERENT OPTICAL SYSTEMS

2. 1. ELEMENTARY OPTICAL SYSTEMS

Consider a piece of film having a transmittance function T; in general, T = T(x, y), a function of the two variables which define the film plane P_1. If this transparency is illuminated with light of intensity I_o (Figure 1), the emergent intensity distribution is

$$I(x, y) = I_o T(x, y)$$

Suppose that a second transparency is overlaid on the first, or that the first transparency is imaged on the second (with unity magnification, for the sake of simplicity). The intensity distribution of the beam which is emergent from the second transparency is then

$$I(x, y) = I_o T_1(x, y) T_2(x, y)$$

where T_1 and T_2 are the transmittance functions of planes P_1 and P_2.

Integration over a plane could be accomplished by imaging the region of integration onto a detector whose resolution elemental size is greater than the image of the region. The detector might, for example, be a photocell or a small region of photographic film. Figure 2

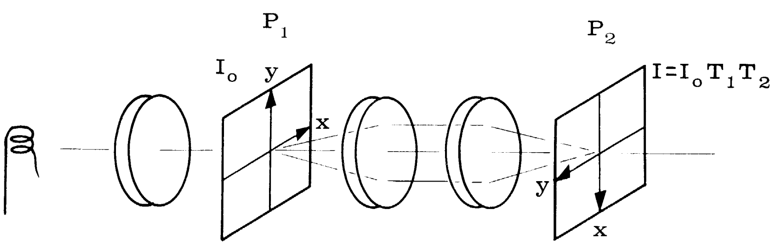

FIGURE 1. OPTICAL MULTIPLICATION

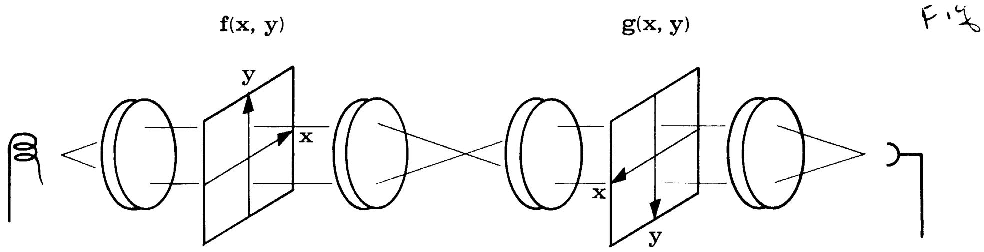

FIGURE 2. OPTICAL SYSTEM FOR TWO-DIMENSIONAL INTEGRATION

shows a simple optical system that evaluates the integral

$$I = \int_{a_1}^{b_1} \int_{a_2}^{b_2} f(x, y)g(x, y)\, dx\, dy \tag{3}$$

Alternatively, the integration is accomplished if the detector samples equally the light emerging from all parts of the plane of integration. The second technique is in general the more practical.

In the event integration in only one dimension is required, and the remaining dimension is to be used to achieve multichannel operation, a method must be found to limit the integration to only one dimension. Astigmatic optical systems, in which the focal properties are different for the two dimensions, can achieve this result. The integral evaluated is then of the form

$$I(y) = \int_{a(y)}^{b(y)} f(x, y)g(x, y)\, dx \tag{4}$$

The astigmatic optical system shown in Figure 3 differs from that of Figure 2 only through the presence of the cylindrical lens which counteracts, for the y-dimension only, the inherent tendency of lens L_2 to integrate the light emerging from the plane P_2.

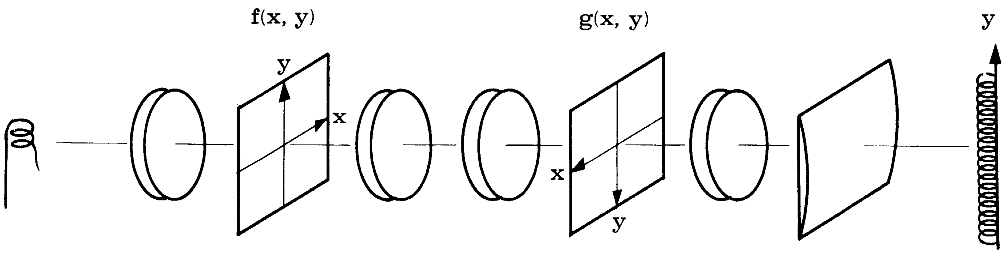

FIGURE 3. OPTICAL SYSTEM FOR ONE-DIMENSIONAL INTEGRATION

The more general case of evaluating

$$I(y, x_o) = \int_{a(y)}^{b(y)} f(x, y)g(x-x_o, y)\, dx \tag{5}$$

is easily handled by merely transporting the g-transparency across the aperture in the x-dimension. This is achieved by recording g(x) on a strip of film which is then translated across the optical aperture while all other elements remain stationary.

The optical systems discussed thus far impose the constraint that f and g must be everywhere positive. This arises because the transmission functions are merely energy ratios and are therefore always positive numbers.

$$0 \le T(x, y) \le 1 \text{ if } T = \frac{I(x, y)}{I_o}$$

In general, the f's and g's of interest are bipolar in nature. The representation of these f's and g's requires the use of a scaling factor k and a bias level B. Then $T(x, y) = B + kf(x, y)$, where B and k are chosen such that $0 \le T \le 1$ for all x, y, and yet the term kf must not be negligible compared to B[1]. If one tries to evaluate an integral of the form 1, the integral generated is of the form

$$I(y, t) = \int_{a(y,t)}^{b(y,t)} \left[B_1 + k_1 f(x, y)\right]\left[B_2 + k_2 g(x, y)\right] dx \tag{6}$$

Error terms arise because of the presence of the bias levels. Even though the B's and k's are known, the removal of these errors or their exact calculation depends on the nature of f and g. The use of a "coherent" optical system will often eliminate certain of these difficulties which arise in the elementary systems described above.

2.2. COHERENT OPTICAL SYSTEMS

A coherent optical system is one in which the relative phases of the light waves in various parts of the system are invariant with time. For coherence to be achieved, a point source of light is required. Any two points in an optical system utilizing a point source have a relative phase which is time invariant.

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[1] If kf << B, the kf term may become obscured by grain noise, stray light, and so forth. On the other hand, if kf is fairly large, clipping may occur on signal peaks. An optimization process which determines the best choice of B and k for a particular set of signal and noise statistics and tolerable distortions is therefore necessary in some cases.

The analysis of an optical system can in principle always proceed from the field equations. However, if only physical dimensions that are much greater than the light wavelength are considered, the analysis is simplified to the application of Huygens' principle. In this report, the signals take the form of transparencies, the detail of which is sufficiently coarse that Huygens' principle can be applied.

A monochromatic electromagnetic wave is described by giving its magnitude and phase as a function of the three space variables. For example,

$$E_x = A(x, y, z) \cos[\omega t + \phi(x, y, z)]$$

where A is an amplitude factor, ϕ is a phase factor, and ω is the radian frequency of the wave, represents one component of the electric field vector. Since polarization effects are not of interest, any field vector can be denoted by E. Since only a few values of Z (where the optic axis is taken in the Z direction) are of concern, the field at z_1 is denoted by E_1, rather than E(Z).

Coherent optical systems have three fundamental properties which allow these systems to be analyzed by Fourier methods. Before the analysis is begun, two conventions will be adopted which will simplify the later discussions.

The first convention is that a wave of the form

$$E_1 = A(x, y) \cos [\omega t + \phi(x, y)]$$

will be written as

$$\hat{E}_1 = A(x, y)e^{j\phi(x, y)}$$

(Whenever a letter appears with a "$\wedge$" it is a complex function.) The motivation for adopting this representation is that all the significant features of the optical system are time invariant and that ω, the temporal radian frequency, acts, in a sense, like a carrier frequency.

The second convention relates to the description of transparencies. Consider a thin transparency described by a transmission function $t^2(x, y)$ and by a thickness function $\frac{\alpha(x, y)}{2\pi(n - 1)}$ (in wavelengths). Note that $0 \leq t \leq 1$, and n is the index of refraction of the transparency. One can say that this transparency represents a signal function, S(x, y), given by

$$S(x, y) = t(x, y)e^{j\alpha(x, y)}$$

One derives the first property by noting that a light wave,

$$E_1 = A(x, y) \cos [\omega t + \phi(x, y)]$$

incident on a transparency with transmission t^2 and thickness $\frac{\alpha}{2\pi(n - 1)}$, yields an emergent wave

$$E_2 = A(x, y)t(x, y) \cos [\omega t + \phi(x, y) + \alpha(x, y)]$$

or,

$$\hat{E}_2 = \hat{S}\hat{E}_1 \quad (7)$$

The second basic property is related to the energy of the wave. The energy, ξ, of a light wave, E, is proportional to the time average of E^2. Thus the second property of coherent optical systems is

$$\xi = k\hat{E}\hat{E}^* \quad (8)$$

where * denotes conjugate. Since the only possible outputs of optical systems are in the form of energy-sensitive detectors (films, photoelectric cells, etc.), the complex representation gains added significance. In fact, the complex representation is sufficient for analyzing optical systems as "black boxes."

Consider the optical system shown in Figure 4. The following statements, which are demonstrated in Appendix A, summarize the third and most significant property of the complex representation. If the light waves at planes P_1, P_2, and P_3 are denoted by $\hat{E}_1(x_1, y_1)$, $\hat{E}_2(x_2, y_2)$, and $\hat{E}_3(x_3, y_3)$, respectively, then $\hat{E}_3$ and $\hat{E}_1$ form a Fourier pair to within a phase factor $e^{j\beta(x_3, y_3)}$, or

$$\hat{E}_3(x_3, y_3) = \mathcal{F}\left[\hat{E}_1(x_1, y_1)\right] \cdot e^{j\beta(x_3, y_3)} \quad (9a)$$

where $\mathcal{F}$ denotes the Fourier transform. Plane P_1 is anywhere between L_1 and L_2, and β is a function of Z_1. An exact Fourier transform exists between $\hat{E}_3$ and $\hat{E}_2$.

$$\beta(x_3, y_3) = 0 \text{ for } Z_1 = Z_2 \quad (9b)$$

an exact Fourier transform does not exist anywhere else (with respect to lens L_2).

$$\beta(x_3, y_3) \neq 0 \text{ for } Z_1 \neq Z_2 \quad (9c)$$

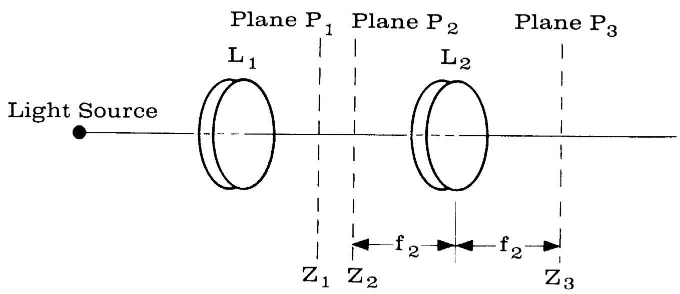


FIGURE 4. FOURIER TRANSFORMS IN AN OPTICAL SYSTEM

A comment is perhaps in order at this point, to avoid confusion in some of the optical diagrams to follow. In conventional Fourier transform theory, the transformation from the time domain (the analog of the spatial domain) to the frequency domain requires the kernel function $e^{-j\omega t}$, and the transformation from frequency to time employs the conjugate kernel $e^{j\omega t}$. A lens always introduces the kernel $\exp j\frac{2\pi}{\lambda f}(x_n x_{n+1} + y_n y_{n+1})$ in passing from plane P_n to plane P_{n+1}. Therefore, in an optical system one takes only successive transforms rather than a transform followed by its inverse. The effect of using a kernel of the wrong polarity, however, is simply to reverse the coordinate system of the transformed function. Therefore, by reversing the coordinate system of the appropriate planes of Figure 5, it is possible, in effect, to take inverse transforms, and make the optical system consistent with the conventions of Fourier transform theory.

The selection of the appropriate labeling may be considered with the aid of Figure 5. Arbitrarily call P_1 a spatial-domain plane, and consider the direction of propagation of the light wave to be from left to right. Lens L_1 introduces the kernel function $\exp j\frac{2\pi}{\lambda f}(x_2 x_1 + y_2 y_1)$. By defining

$$\omega_x = \frac{-2\pi}{\lambda f} x_2$$

and

$$\omega_y = \frac{-2\pi}{\lambda f} y_2$$

the kernel becomes $\exp -j(\omega_x x + \omega_y, y)$, which is in accord with convention.

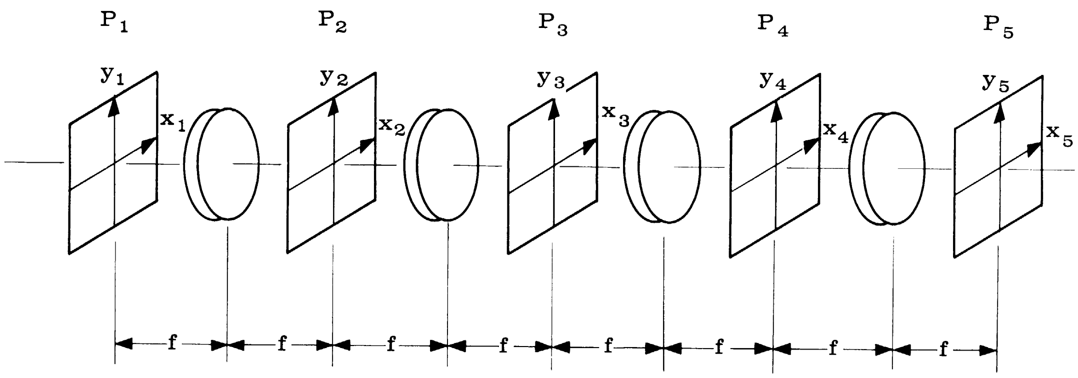


FIGURE 5. SUCCESSIVE OPTICAL TRANSFORMS

Recalling that $\mathfrak{F}\{\mathfrak{F}[f(x)]\} = f(-x)$, one then observes that x_1 is mapped into $-x_3$ as a result of the two successive transforms. In other words, the image at P_3 is reversed. If the coordinate system of P_3 is reversed,

$$\hat{E}(P_3) = \mathfrak{F}^{-1}\left[\hat{E}(P_2)\right] = \mathfrak{F}^{-1}\left\{\mathfrak{F}\left[\hat{E}(P_1)\right]\right\} = \hat{E}(P_1),$$

to within bandwidth limitation.

The resultant coordinate systems for successive planes is shown in Figure 6. By adopting this scheme of labeling, each frequency domain is represented by the Fourier transform of the spatial domain to its left, and each spatial domain is represented by the inverse Fourier transform of the frequency plane to its left. Note, however, that this scheme of coordinate assignment is valid only for the case of the light wave traveling from left to right.

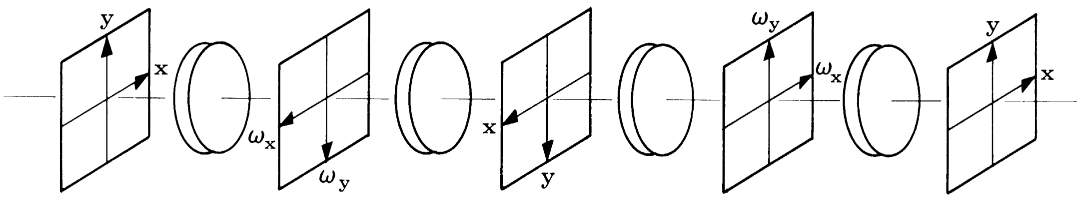


FIGURE 6. STANDARD COORDINATES

2.3. FILTER SYNTHESIS

The properties of coherent optical systems thus presented permit the synthesis of a wide range of optical filters. Such an optical filter consists of a transparency inserted at some appropriate position in the optical system.

In the optical system of Figure 7, a signal s(x, y) is inserted at plane P_1. At plane P_2, the spectrum $S(\omega_x, \omega_y)$ is displayed. Suppose a transparency $R(\omega_x, \omega_y)$ to be inserted also at plane P_2. Such a transparency modifies the spectral content of the signal, effecting the operation

$$\hat{V}(\omega_x, \omega_y) = \hat{S}(\omega_x, \omega_y)\,\hat{R}(\omega_x, \omega_y)$$

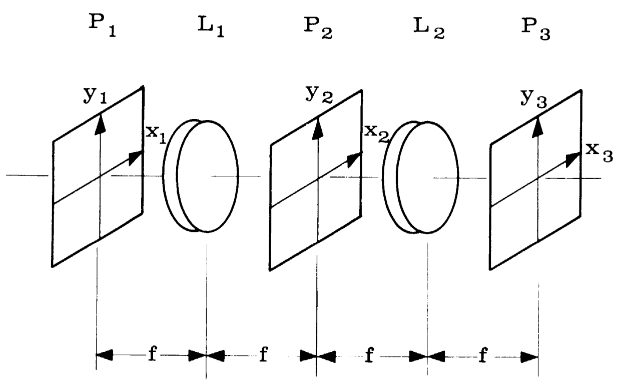


FIGURE 7. THE TWO-DIMENSIONAL FILTER

The transparency in general has a complex transmittance $\hat{R} = |R| e^{j\psi}$. The amplitude portion is obtained by varying the optical density and the phase portion is obtained by varying the thickness, which in turn varies the phase retardation.

At plane P_3, the signal is transformed back to the spatial domain, and is given by

$$\hat{v}(x, y) = \iint s(x-\alpha, y-\beta)\, \hat{r}(\alpha, \beta)\, d\alpha\, d\beta \tag{10}$$

where

$$\hat{v}(x, y) = \mathfrak{F}^{-1}\left[\hat{V}(\omega_x, \omega_y)\right]$$

and

$$\hat{r}(x, y) = \mathfrak{F}^{-1}\left[\hat{R}(\omega_x, \omega_y)\right]$$

This is a convolution integral.

The transparency $\hat{R}$ in its simplest form might be a slit or other aperture. Such apertures are low-pass or bandpass spatial filters. A stop becomes a band-rejection filter. The inclusion of a phase plate causes a phase shift of one portion of the spectrum with respect to the remainder. Complex filter functions are possible; because it appears that one has independent control over both phase and amplitude, a wide variety of filter functions can be synthesized.

As an alternative to inserting a transparency in the plane P_2 (the frequency domain), a transparency $\hat{r}(x, y)$, also with complex transmittance, can be introduced into the spatial domain at P_1. If provision is made for translating $\hat{s}(x, y)$ (in the x-y plane) relative to $\hat{r}(x, y)$, the signal at plane P_2 becomes $\hat{V}'(x', y') = \mathfrak{F}\left[\hat{s}(x-x', y-y')\hat{r}(x, y)\right]$. Here x' and y' measure the lateral displacement between s and r. At the position $\omega_x = \omega_y = 0$, the integral becomes

$$\hat{V}'(x', y') = \iint \hat{s}(x-x', y-y')\hat{r}(x, y)\, dx\, dy \tag{11}$$

which has the form of a cross-correlation. By reversing the coordinate system of $\hat{s}$ prior to recording, i.e.,

$$x - x' \longrightarrow x' - x$$
$$y - y' \longrightarrow y' - y$$

one obtains the convolution integral

$$V'(x', y') = \iint s(x'-x, y'-y) r(x, y)\, dx\, dy$$

This is identical in form with Equation 10. Therefore, two methods are available for synthesizing a required transfer function:

(1) The frequency-domain synthesis, in which a complex transmittance function (called a filter) is introduced into the frequency domain, plane P_2, and operates directly on the frequency spectrum; and

(2) The spatial-domain synthesis, in which a complex transmittance function (called a reference function) is introduced into the spatial domain, plane P_1, and operates directly upon the signal function.

The two techniques produce the same result, as indeed they should. The display is different, however. With the frequency-domain operation of Equation 10, an area display is produced in which the variables, x, y, are the coordinates of the plane P_3. With the spatial-domain operation, the output display is only a point (namely $\omega_x = \omega_y = 0$) and the coordinates x', y' are generated as functions of time by physically displacing $\hat{s}$ with respect to $\hat{r}$. The spatial-domain instrumentation requires a scanning mechanism; the filter technique does not.

It is possible and often advantageous to divide a required operation on $\hat{s}$ into two portions, one carried out in the spatial domain and the other in the frequency domain. The output at plane P_3 then becomes

$$\mathcal{F}^{-1}\left\{R_2(\omega_x, \omega_y)\,\mathcal{F}\left[\hat{s}(x, y)\hat{r}_1(x, y)\right]\right\} \qquad (12)$$

where $\hat{r}_1$ is a reference function inserted in plane P_1 (the spatial-domain plane), and R_2 is a filter function inserted in plane P_2 (the frequency-domain plane).

A class of filters that has received considerable attention in communication theory is the matched filter, which has a transfer that is the complex conjugate of the signal spectrum to which the filter is matched. Such filter maximizes the ratio of the signal squared to the root-mean-square noise when the noise is white gaussian. The signals of concern here are two-dimensional and can also be complex. However, the above criterion for the matched filter still remains valid.

For the time-domain signals of electronics, the matched-filter criterion

$$\hat{R}(\omega) = \hat{S}^*(\omega)$$

implies the time-domain relationship for the impulse response is

$$r(t) = s(-t)$$

For the optical case, since complex signals are possible, the corresponding relations are

$$\hat{R}(\omega_x, \omega_y) = \hat{S}^*(\omega_x, \omega_y)$$

and

$$\hat{r}(x, y) = \hat{s}^*(-x, -y)$$

The construction of the matched filter may be carried out by successive operations in the spatial and frequency domains, as suggested by Equation 12. At plane P_1 of Figure 7 a function $\hat{h}_1(x, y)$ is inserted. A filter $\hat{H}_2(\omega_x, \omega_y)$ is inserted at plane P_2 and the output is taken from P_3. The operation at P_1 produces

$$\hat{s}(x-x_1, y-y_1)\hat{h}_1(x, y)$$

and the filter $\hat{H}_2(\omega_x, \omega_y)$ produces, at plane P_3, the result

$$\iint \hat{s}(\alpha-x, \beta-y)\hat{h}_1(\alpha, \beta)\hat{h}_2(x_3 - \alpha, y_3 - \beta)\, d\alpha\, d\beta$$

where

$$\hat{h}_2(x, y) = \mathcal{F}^{-1}\left[\hat{H}_2(\omega_x, \omega_y)\right]$$

If the observation in plane P_3 is confined to the position $x_3 = y_3 = 0$, the above expression reduces to

$$\iint \hat{s}(\alpha-x_1, \beta-y_1)\hat{h}_1(\alpha, \beta)\hat{h}_2(-\alpha_1 - \beta)\, d\alpha\, d\beta$$

The matched-filter condition is

$$\hat{h}_1(\alpha, \beta)\hat{h}_2(-\alpha, -\beta) = \hat{s}^*(\alpha, \beta)$$

or

$$\hat{H}_2(\omega_x, \omega_y) = \mathcal{F}\left[\frac{\hat{s}^*(-\alpha, -\beta)}{\hat{h}_1(-\alpha, -\beta)}\right]$$

In most cases, the signal introduced into the optical system will be written as a real function, i. e. , written as a density variation on photographic film. However, it is often advantageous to use complex transparencies (phase control) for the synthesis of a required transfer function. For such a transparency $\hat{r}(x, y)$ there are three cases of interest:

(a) $\hat{r}(x, y)$ is a varying-density transparency with uniform phase retardation over the aperture.

(b) $\hat{r}(x, y)$ has uniform density over the aperture, but with phase retardation, $\phi(x, y)$.

(c) $r(x, y)$ has both varying density and varying phase retardation over the aperture.

The transparency in each case has, respectively, the form

(a) $\hat{r}(x, y) = A(x, y)$.

(b) $\hat{r}(x, y) = Ae^{j\phi(x, y)}$; A = constant.

(c) $\hat{r}(x, y) = A(x, y)e^{j\phi(x, y)}$.

In each case, $A(x, y)$ is a real number $0 \leq A(x, y) \leq 1$, and $\phi(x, y)$ is real.

Consider the complex plane of Figure 8. All values of r(x, y) may be represented by a set of points within or on the unit circle. The functions $\hat{r}(x, y)$ of case (a) lie on the real line OA. The functions r(x, y) of case (b) lie on some circle within the unit circle ABCDA, whereas the functions $\hat{r}(x, y)$ of case (c) may occupy any set of points within or on the unit circle.

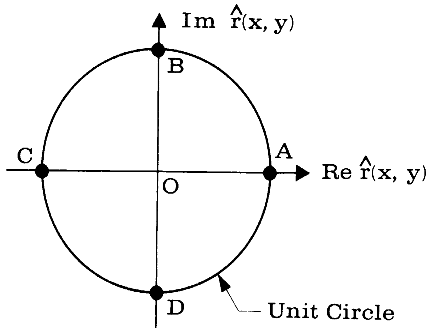


FIGURE 8. COMPLEX AMPLITUDE-TRANSMISSION PLANE

A special case of (b) is worth noting: when $\phi(x, y)$ assumes only two values, differing by π (as, for example, 0 and π). This is a binary code. In terms of Figure 8, the function is confined to two points lying on COA. Such a phase function is comparatively easy to generate since techniques for producing phase gratings are well developed. When this is combined with case (a), the entire line COA is utilized. Thus real functions of positive and negative values can be used, as in the case of the electronic channel. In this case, however, no bias level is required.

The ability to process complex functions finds practical application in certain problems arising in radar and communication systems. In the most general case, one has a carrier signal which may be both amplitude- and phase-modulated. The modulated signal then takes the form

$$f(t) = A(t) \cos \left[\omega_c t + \alpha(t)\right]$$

which may be represented by the complex function

$$f(t) = A(t)e^{j\alpha(t)}$$

In a coherent system, one can make use of the phase function as well as the amplitude function.

Generally, $\hat{f}(t)$ is to be subjected to some form of filter operation. For example, it $\hat{f}(t)$ represents a coded pulse, the operation of interest might be a cross-correlation against a reference signal $\hat{s}(t)$. The operation can be performed optically if the complex functions $\hat{s}$ and $\hat{f}$ are appropriately recorded.

Within the present state of the art it is difficult, however, to record simultaneously the modulus and argument of $\hat{f}$ in the form of a transmissivity and thickness variation. If the complex function $\hat{f}(t)$ is shifted in frequency by W (where W is greater than the highest significant frequency contributing to $Ae^{j\alpha}$), an alternative method becomes available. Only the real part of the frequency-translated $\hat{f}(t)$, i. e.,

$$\mathrm{Re}\left\{A(t)e^{j[\alpha(t) + Wt]}\right\}$$

need be recorded, and this is displayed solely as a transmission variation. The frequency shift is effected using a rotary phase shifter or equivalent device, while the real part is obtained through the use of a synchronous detector operating at the carrier frequency. Let r(t) be the resulting function. The spectral display of the optical system is such that the positive and negative frequency components of a signal are independently available for attenuation and/or phase shifting. If the negative frequencies of r(t) are then removed in an optical system, the resulting function is $A(t)e^{j[\alpha(t) + Wt]}$. (This is shown in Appendix B.)

Two observations deserve mention at this point. First, the presence of the radian frequency W should be taken into consideration in the optical filter. Second, the simplification in recording which is provided by the above technique exacts a price. One must double the bandwidth of the electronic channel at all points following the synchronous detector.

2.4. ASTIGMATIC OPTICAL SYSTEMS

The optical systems discussed thus far perform two-dimensional operations. This feature is useful if the signal to be processed is a function of two variables, for then the signal can be displayed as a two-dimensional function and processed simultaneously in both variables. An electronic system, having time as its only available independent variable, would require a scanning technique to perform the two-variable operation.

More often the signal is one-dimensional in nature and the additional variable is not required. In such a case, the second variable can be used to provide a multiplicity of independent channels so that many one-dimensional signals can be processed simultaneously. The signals to be processed are written as $\hat{s}_y(x)$ and are stacked on the transparency with respect to the y-variable. The resultant transparency is of the form $\hat{s}(x, y)$ as before, except that now y is a parameter which takes on as many values as there are independent channels to process. The limit on this number is, of course, the number of resolvable elements available across the y-dimension of the optical-system aperture.

The processing is to be done with respect to the x-variable only. The y-dimension channels must remain separated. The optical system of Figure 7 when modified as shown in Figure 9, performs in the required manner.

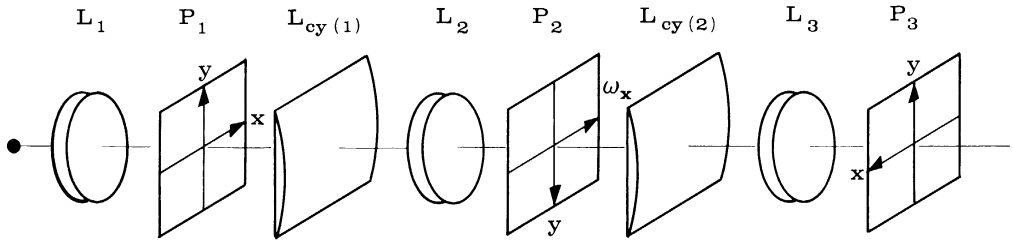


FIGURE 9. MULTICHANNEL OPTICAL SYSTEM

The plane P_1, as before, has the coordinate system (x, y). At the Fourier transform plane P_2, a display of (ω_x, y) is desired; i.e., one wishes to effect a Fourier transform with respect to x only, while preserving the y-dimension. A cylindrical lens, which has focal power in one dimension only, can effect a one-dimensional Fourier transformation. To display the y-dimension at P_2, plane P_1 is imaged at P_2, i.e., a double Fourier transformation with respect to the y-variable is instituted between P_1 and P_2.

A cylindrical lens $L_{cy(1)}$ in combination with a spherical lens L_2 is placed between planes P_1 and P_2. The cylindrical lens exerts focal power in the y-dimension only and by itself produces a Fourier transformation with respect to y. The lens L_2, by itself, introduces a two-dimensional Fourier transformation. The two lenses in combination produce a double transformation with respect to y and a single transformation with respect to x. Plane P_2, therefore, has ω_x and y as its coordinates. Ideally, of course, one would like to transfer the y-dimension of P_1 directly on plane P_2. This is possible only by imaging P_1 onto P_2, which, of course, implies a double Fourier transformation.

At the plane P_2 a filter element, $R(\omega_x, y)$, is inserted. This function is interpreted as a multichannel one-dimensional filter which processes each channel independently. A similar cylindrical-spherical lens combination between planes P_2 and P_3 results in the inverse transformation with respect to ω_x. Thus, the output plane P_3 with coordinates x, y displays the input function after modification by the filter.

A simplification is possible if the signals in all channels are processed identically. Separation of the channels at the plane P_2, or frequency plane, is no longer necessary. In this event, the cylindrical lenses are not required and the optical system reverts to that of Figure 6. The function displayed at the frequency plane P_2 is, as earlier, $S(\omega_x, \omega_y)$. The filter element, being independent of ω_y, takes the form $R(\omega_x)$. The function displayed at P_3 is modified with respect to the x-dimension frequencies only. The operation can be written as

$$\hat{v}(x, y) = \mathcal{F}^{-1}\left[\hat{S}(\omega_x, \omega_y)\hat{R}(\omega_x)\right] = \int \hat{s}(x-\alpha, y)\hat{r}(\alpha)\, d\alpha$$

As in the two-dimensional processor, the required transfer function can be synthesized in the spatial domain. The optical system of Figure 9 suffices, except that the output is taken from plane P_2 and the portion of the system beyond this plane is not required (Figure 10). The integral evaluated when a slit is placed along the line $\omega_x = 0$ is

$$\hat{V}'(x', y) = \int \hat{s}(x-x', y)\, \hat{r}(x, y)\, dx$$

where, as before, y is a parameter providing multichannel operation.

Instrumentations which require only one-dimensional processing need be coherent in one dimension only. Therefore, in the optical systems described in this section, the point source of illumination can be replaced by a line source oriented parallel to the y-dimension. This is of practical advantage because the available light flux can be increased by several orders of magnitude.

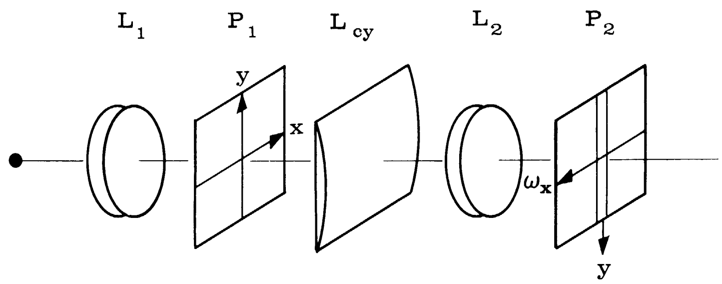


FIGURE 10. SPATIAL-DOMAIN FILTERING

3
POTENTIAL APPLICATIONS

The first part of this report has been devoted to an exposition of the theory of coherent optical systems in a framework convenient for the purpose of filter design or the design of data-handling systems. Emphasis has been placed on a filter-theoretic approach with no loss of generality, since all the integral-transform operations of interest in the field of communication theory can be described as filtering operations.

With the theory thus far presented, it is possible to discuss some potential applications of coherent optical systems. Innovations which are useful in particular problems would be incorporated into the simple optical systems already discussed. Amplitude and phase control is available over two-dimensional regions in both the space and spatial-frequency domains; this permits great flexibility of filter configurations.

However, many of the problems of interest at present are inherently one-dimensional but have a varying parameter. If an optical configuration is sought, one is naturally led to an astigmatic arrangement. It is useful to consider an alternative approach to astigmatic optical systems; they may be viewed as multichannel data-handling (or computing) systems. This alternative viewpoint is developed in Section 3. 1, and a discussion of possible applications appears in Section 3. 2. The treatment in Section 3. 1 will not involve any new theoretical concepts and may be omitted in a first reading; however, it is intended to give some additional insight into the application of astigmatic systems.

3. 1. A MULTICHANNEL OPTICAL COMPUTER

The astigmatic systems of Section 2. 4 can be instrumented as a multichannel optical computer with broad capabilities. The optical system can evaluate integrals of the form:

$$I(\omega_x, x_o, y) = \int_{a(y)}^{b(g)} f(x-x_o, y)g(x, y)e^{-j\omega_x x} dx \qquad (13)$$

The usefulness of an optical technique which evaluates integrals of this form will become evident when some potential applications are discussed in Section 3. 2. However, the ease with which optical computations of the above form can be made warrants a few remarks at this point.

(1) The functions f(x, y) and g(x, y) can be recorded readily. The photographing of intensity-modulated cathode-ray-tube presentations, or their equivalent, is a direct means for converting signal waveforms of very large bandwidths into static form. While care is required in the design of the camera and film-transport mechanisms when multichannel operation is incorporated, the process is essentially straightforward. Moreover, the staticizing of the signals means that signals with bandwidths of the order of 100 mcs can be recorded.

(2) The Fourier-transformation capability of the coherent optical system permits simple sorting of spectra and easy processing in the frequency domain.

(3) The number of independent channels which can be handled simultaneously is limited only by the resolution of the photographic film and the resolving power of the lenses. Generally, at least 20 channels per millimeter of film can be accommodated.

The multichannel capability is readily demonstrated by placing an opaque obstacle with sides parallel to the x-axis in the plane P_1 of Figure 9. This removes light from the corresponding interval in planes P_2 and P_3. Moving this obstacle in the y-direction in P_1 (while keeping its edges parallel to the x-axis) causes a corresponding image motion in planes P_2 and P_3. The edges of these corresponding image regions have a sharpness determined by the resolution of the optical system in the y-direction.

Equipment in laboratory use employing commercially available components is capable of providing about 20 channels per millimeter. Approximately 500 to 1000 channels may therefore be obtained using 35-mm-size optical components.

A further demonstration of the multichannel capability follows. Let a transparency A(x, y) be inserted into the plane P_1 of Figure 9. This transparency consists of a number of grating strips with varying spatial frequency, the various gratings being stacked in the y-dimension. Each grating is, in effect, a square wave written about a bias level (see Figure 11).

In the plane P_2, a spectral analysis of A(x, y) with respect to x will be found in each increment of y. The result of photographing the light distribution in P_2 is given in Figure 12. It will be noted that all channels have a recorded signal at the center. This corresponds, channel by channel, to the average level of illumination emergent from that channel in plane P_1. The images in each strip show several lines on each side of the central image; these lines correspond to the fundamental and harmonics comprising the square wave in each y increment.

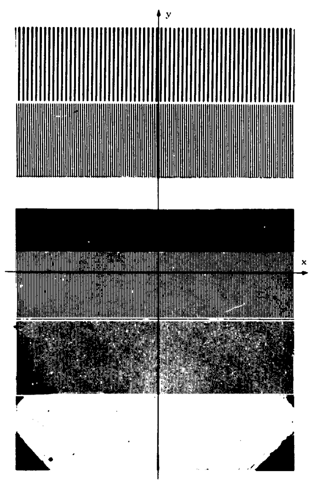


FIGURE 11. MULTICHANNEL DIFFRACTION GRATING

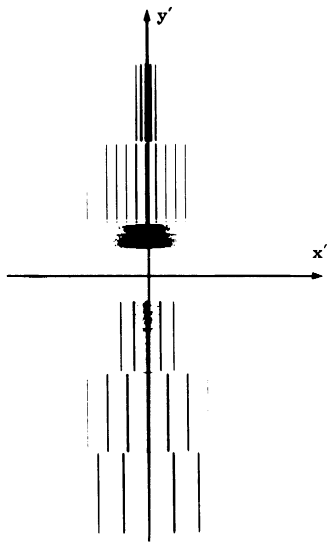
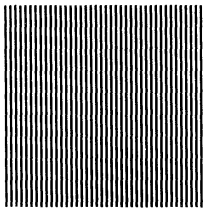
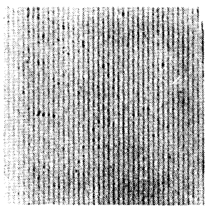


FIGURE 12. SPECTRAL ANALYSIS OF THE MULTICHANNEL DIFFRACTION GRATING

In some instances, for example, in taking a Fourier transform and making a frequency analysis, the output is taken from plane P_2. Alternatively, the spectrum at P_2 can be modified by means of stops, phase plates, or density filters, and the modified signal displayed at plane P_3. A photograph of the output at P_3 is shown in Figure 13 for one channel only. If an obstacle is placed on the optical axis in plane P_2, the central image (i. e. , the d-c or "bias" term) is removed and the contrast between the dark and light regions disappears, as shown in Figure 13(b). This interesting behavior can be understood when one recalls that the eye (like any photographic recorder) responds to light intensity, the square of the light amplitude. Before removal of the bias level, the total light amplitude (bias plus square wave) was everywhere positive. Removal of the central image (the d-c or average level) causes the amplitude of the light reaching P_3 to be bipolar; both positive and negative amplitudes appear. Since the eye and/or photographic recorders can only sense the square of this amplitude, positive and negative amplitudes cannot be distinguished from each other. Therefore, the contrast in P_3 vanishes when the d-c component is removed through filtering. Imperfect symmetry of the positive and negative half-cycles, and the loss of high-frequency components which accentuate the corners, tend to cause the residual effects. In particular, these are the dark lines between the half-cycles and the remaining slight contrast between positive and negative half-cycles.[2]



(a) Square-wave diffraction grating

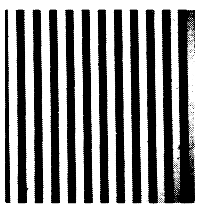


(b) Image of (a) after bias is removed

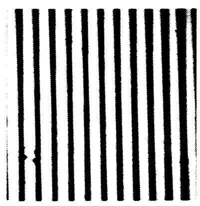
FIGURE 13. OUTPUT AT P_3 FOR ONE CHANNEL

[2]A useful operation closely related to the above discussion is performed in the phase-contrast microscope. Here contrast is improved by retarding the d-c component by a quarter wavelength of the illumination. A quarter-wave plate placed on the axis of the system in the transform plane P_2 effects the retardation.

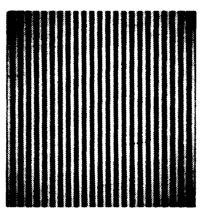
If the central image and first sideband on each side of the central image are allowed to pass plane P_2, another interesting feature may be demonstrated. If all higher-order images are removed at P_2, the image in plane P_3 resembles the pattern of P_1 except that the amplitude variation is now sinusoidal rather than square. The optical filter has passed the d-c and the fundamental frequency, but rejected all harmonics. Figure 14(a) shows the distribution in plane P_1, and Figure 14(b) shows the resulting image in P_2. If, instead, the central image and the second sideband on each side (but not the first) were accepted, the image again would be sinusoidal, with twice as many lines/unit length present in P_3 as in P_1 (Figure 14 c). This corresponds to selection of the average value and the second harmonic. Any frequencies present in the spectral decomposition of A(x, y) (which is usually not completely symmetric in practice), may be selected and passed by the filter to form a resultant image in P_3, which is the transform of the spectrum in P_2.



(a) Square-wave diffraction grating



(b) Image of (a) when filter transmits
d-c and fundamental only



(c) Image of (a) when filter transmits
d-c and second harmonic only

FIGURE 14. FILTERING OF SQUARE WAVE

In the process of evaluating an integral in the form of Equation 13, it is often useful to perform a filtering operation on f or g before taking the product fg. The optical configuration of Figure 9 is modified to that of Figure 15. Here, lens L_4 and cylindrical lens $L_{cy(3)}$ have been added. A transparency,

$$A_1(x, y) = B_f + f(x, y)$$

is placed in P_1, and a second transparency,

$$A_2(x, y)\ B_g + g(x, y)$$

is placed in P_3, with the coordinate axis properly chosen to correct for image-inversion effects. B_f and B_g are bias levels for the two transparencies.

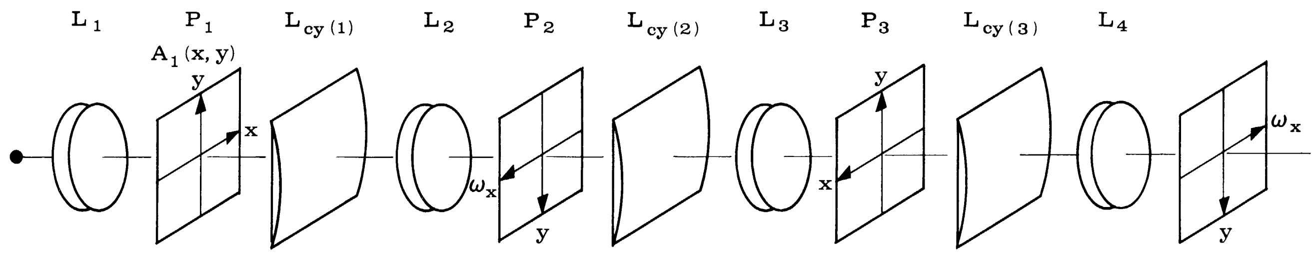


FIGURE 15. MODIFIED MULTICHANNEL OPTICAL SYSTEM

In the absence of filtering in plane P_2, an image of P_1 would be formed in P_3. This would illuminate $A_2(x, y)$ such that the amplitude emergent to the right of P_3 would be the product

$$A_3(x, y) = A_1(x, y)A_2(x, y) = \left[B_f + f(x, y)\right]\ \left[B_g + g(x, y)\right]$$

therefore,

$$A_3(x, y) = B_fB_g + B_gf(x, y) + B_fg(x, y) + f(x, y)g(x, y) \qquad (14)$$

The presence of the bias terms results in three extranous terms which in general interfere with the carrying out of the desired operation. In particular, if a cross-correlation is to be performed, the output is taken from plane P_4 at the position $\omega_x = 0$. Unfortunately, the term B_fB_g, being a constant, puts energy into P_4 at $\omega_x = 0$. This seriously reduces the contrast in the output. Some advantages of filtering in plane P_2 are now apparent. If an obstacle is placed in P_2 at $x_2 = 0$, the d-c component, B_f, of the function $A_1(x, y)$ may be removed. This makes the light amplitude incident on plane P_3 proportional to f(x, y) without the bias level. Phase information is present and the bipolar function f(x, y) has its positive and negative peaks 180^o out of phase with each other. Thus the first and third terms of the right-hand side of Equation 14 vanish.

A question now arises concerning the possible error in evaluating Equation 13 caused by the second term of the right-hand side of Equation 14. The term $B_g f(x, y)$ can only be troublesome if f(x, y) has a d-c component (i. e., if it contains spatial frequency zero in the x-direction). In this case, a component due to this d-c level will appear at $\omega_x = 0$ in plane P_4. The error caused by this term can, however, be removed from the system output by recording both f(x, y) and g(x, y) on a common carrier frequency. If the functions resulting from writing f(x, y) and g(x, y) as a modulation on a common carrier are designated as

$$A_1'(x, y) = B_f + f'(x, y)$$

and

$$A_2'(x, y) = B_g + g'(x, y)$$

then $B_g f'(x, y)$ will produce no output at $\omega_x = 0$, and it can be shown that

$$\left[\int f'(x-x_o, y)g'(x, y)e^{j\omega_x x}\,dx\right]_{\omega_x = 0} = \left[\int f(x-x_o, y)g(x, y)e^{j\omega_x x}\,dx\right]_{\omega_x = 0} \tag{15}$$

A proof of this equality is given in Appendix B.

3.2. APPLICATIONS TO COMMUNICATION THEORY

In preparing to discuss some possible applications of optical data-processing techniques, it is useful to list some well-known mathematical operations which can be expressed in the form of Equation 13. All the operations are taken as one-dimensional, but all except the Laplace transform permit a varying parameter. It is obvious that integration cannot be performed over an infinite interval, as some of the general expressions demand. However, in most practical cases f(x, y) and g(x, y) will vanish outside some interval. In Table I, L represents the achievable aperture limits of the optical system.

A few realistic problems will now be discussed in terms of optical data processing and presentation.

TABLE I. SOME AREAS OF APPLICATION

$I(\omega_x, x_o, y)$	$f(x-x_o, y)$	$g(x, y)$	$e^{j\omega_x x}$	a(y)	b(y)
Fourier Transform	f(x)	1	$e^{-j\omega_x x}$	-L	+L
Laplace Transform	$f(x - x_o)$	g(x)	$\omega_x = 0$	-L	+L
Cross-Correlation	$f(x - x_o)$	f(x)	$\omega_x = 0$	-L	+L
Auto-Correlation	$f(x_o - x)$	g(x)	$\omega_x = 0$	-L	+L
Convolution	f(x)	e^{-xy}	$e^{-j\omega_x x}$	0	+L

3.2.1. FOURIER TRANSFORMS. The Fourier transform provides the foundation for such areas as filter theory, spectral analysis, antenna-pattern analysis, and modulation theory, all of which are essential to the general field of electronics.

An optical spectrum analyzer is easily constructed to accommodate input signals in the form of a film record of a cathode-ray-tube display. The technique of optically displaying the spectrum may be particularly useful in applications where one may wish to alter the spectrum in a prescribed manner, as in speech transmission and coding.

The study of antenna patterns also presents some interesting possibilities. It is well known that the far-field power pattern of an antenna can be determined from the squared modulus of the Fourier transform of the current distribution at the antenna aperture. In cases where one is interested in the effects of various illumination errors on the power pattern, the Fourier transform properties of coherent optical systems permit visual observation of the resulting patterns as the optical aperture illumination is appropriately varied to correspond to antenna illumination errors.

One interesting mechanization is of particular interest to the filter designer. If a matched filter is to be designed for a signal of pulsed form with a constant repetition rate, the necessary filter is a comb filter. An electronic comb filter frequently is constructed using recirculating delay lines, a fairly complex mechanization. An optical comb filter, however, is merely a diffraction grating whose spacing, for a pulsed input signal, would be determined by the pulse-repetition frequency of the signal.

The following experiments demonstrate some additional capabilities of optical processors.

First optical modulation and demodulation will be demonstrated. A transparency having in one channel an amplitude function of the form

$$A_1(x) = B_1 + \sigma_1 \cos \omega_c x$$

is multiplied by a second transparency

$$A_2(x) = B_2 + \sigma_2 \frac{\cos \omega x}{|\cos \omega x|}$$

The first function, $A_1(x)$, is the carrier, together with a bias term S_1, again required so that positive and negative values of the carrier can be represented as density variations on photographic film. The second function, $A_2(x)$, is a square wave, along with a bias term S_2. For convenience in carrying out the experiment and in displaying the result, a relatively low carrier frequency has been chosen. The square-wave function is shown in Figure 16(a) and the carrier in Figure 16(b). The experimental results are shown in Figure 17.

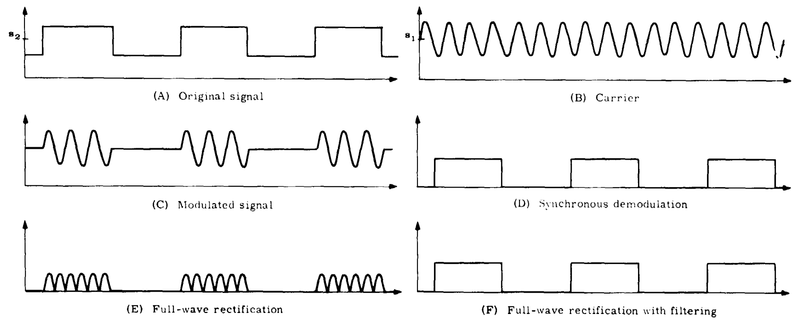


FIGURE 16. MODULATION AND DEMODULATION

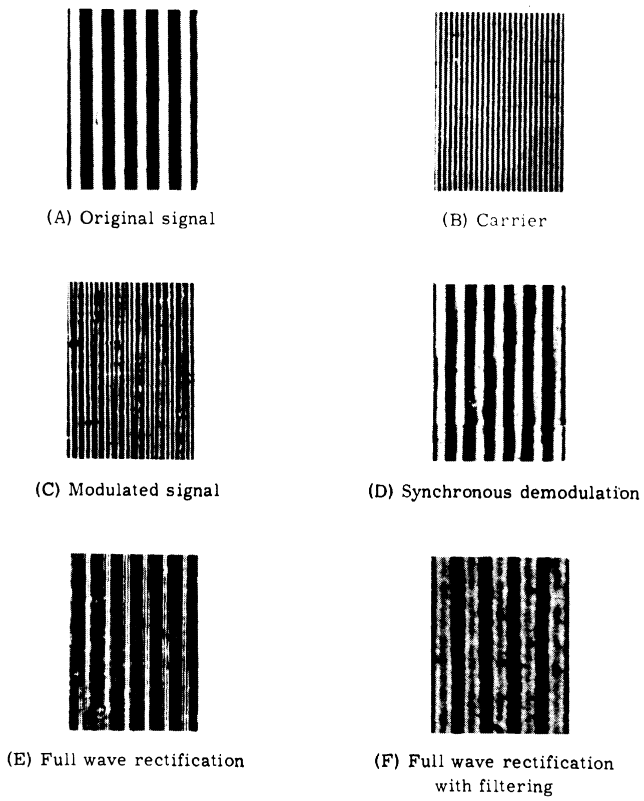


FIGURE 17. EXPERIMENTAL RESULTS

The modulation process consists of a multiplication and a filtering operation. The former is carried out in the spatial domain. After the multiplication, frequencies are generated which are not part of the modulation function; these are the cross-product terms resulting from the bias S_1. Appropriately positioned stops act as stop-band filters and remove these components.

The resulting function can be written as

$$B_1B_2 + B_2\sigma_1\left(1 + \frac{\cos \omega x}{|\cos \omega x|}\right) \cos \omega_c x$$

which is in the standard form of an amplitude-modulated carrier. Again, a bias S_1S_2 is required in order to be able to record the result. The modulated wave is shown in Figure 16(c).

To demodulate the wave, two methods are available, each having an electronic analog. The first is a linear operation, somewhat analogous to synchronous demodulation. The spectral display at the frequency plane produces two spectral images for each frequency, symmetrically positioned about zero frequency. These can be regarded as produced from the exp $(j\omega_c t)$ and exp $(-j\omega_c t)$ portions of the function cos $\omega_c t$, i.e., from the positive and negative frequencies. Suppose, now, that only half of the frequency spectrum is transmitted (either the left or right half-plane). Also, let the zero frequency be eliminated. The modulated wave becomes

$$B_2\sigma_1\left(1 + \frac{\sigma_2}{B_2} \frac{\cos \omega x}{|\cos \omega x|}\right) e^{j\omega_c x}$$

A recording device, being insensitive to phase, records

$$\sigma_1(B_2 + \sigma_2 \frac{\cos \omega x}{|\cos \omega x|}$$

as shown in Figure 16(d). This is the original function before modulation.

Alternatively, by removal of the bias term, the modulated wave becomes full-wave rectified when recorded, as in Figure 16(e). Additional filtering to remove the carrier-frequency terms produces the result in Figure 16(f). The analogy of this operation to the detection operation of electronics is quite apparent.

A second experiment demonstrates the use of independent phase and amplitude control over portions of the spectrum. The coherent optical system is used to obtain the derivative of a function. This is done in the frequency domain. Differentiation of a spatial-domain function corresponds to multiplication of the transform by ω. Therefore, the appropriate filter for differentiation consists of the superposition of: (1) a transparency that is opaque at the center or zero-frequency region and that becomes more transparent at higher frequencies; and (2) a half-wave phase plate needed to produce a reversal of sign for negative frequencies. Figure 18 shows a pulse and its first two derivatives. The second derivative is obtained by use of a transparency with transmittance proportional to ω^2.

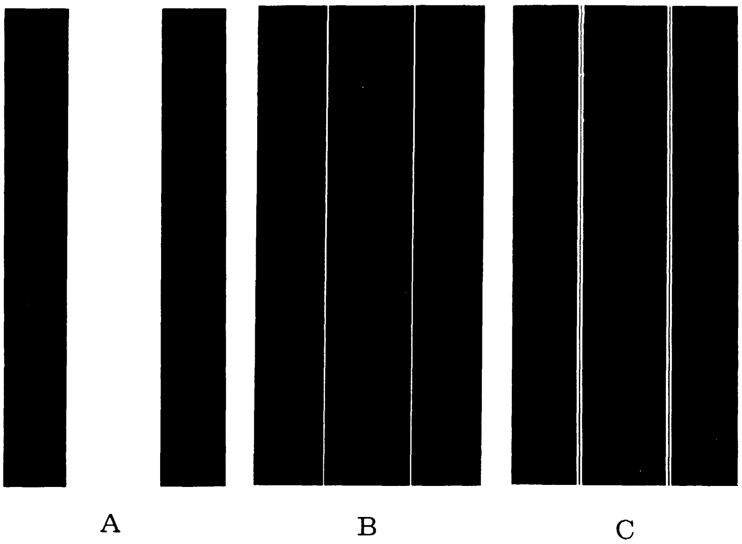


FIGURE 18. DIFFERENTIATION

3.2.2. LAPLACE TRANSFORMS. The Laplace transform is given by

$$F(s) = \int_0^\infty f(x)e^{-sx}\,dx$$

where s is a complex variable. Let $s = \alpha + j\beta$; then,

$$F(s) = F(\alpha, \beta) = \int_0^\infty f(x)e^{-\alpha x}e^{-j\beta x}\,dx$$

which has the form of a Fourier transform of the product of two functions (except for the lower limit on the integral). Here β plays the role of a spatial frequency while one of the two functions contains a parameter α. The optical configuration to be used now follows from the discussion of astigmatic systems. The planes P_1, P_2, P_3, and P_4 in Figure 15 are successive Fourier transform pairs with respect to the variable x. At plane P_1, therefore, one inserts an f(x), which is uniform over all values of y. At plane P_3, P_1 is imaged onto the function $e^{-\alpha x}$, where the y-dimension is used to provide the various values of α. Finally, the function $F(\alpha, \beta)$ is displayed at plane P_4, where α and β correspond to the y- and x-directions, respectively.

3.2.3. CROSS-CORRELATION. The principles of optical cross-correlation have already been described; however, the simplicity of the mechanization in certain cases is so extreme as to deserve special mention.

Suppose f(x) and g(x) are two real-valued, narrow-band functions which are to be cross-correlated. (The narrow-band condition, bandwidth < center frequency, can be satisfied by proper modulation). The following arrangement then yields the cross-correlation of f(x) and g(x):

(a) The multichannel optical system of Figure 9 is extended to have successive transform planes (in the x-dimension) $P_1, \ldots, P_4$.

(b) At P_1 the function $B_f + f(x)$ is placed in one channel. (For B_f sufficiently large, no thickness modulation is necessary and the function is purely a varying-transmission transparency.)

(c) Plane P_2 has a stop at zero frequency (on axis) which serves as a high-pass filter to reject B_f and accept f(x).

(d) At plane P_3, the resulting function f(x) is imaged onto $B_g + g(x)$, thus forming the product $f(x)\left[B_g + g(x+x_o)\right]$, where $x_o = x_o(\tau)$, a displacement, may be a function of time.

(e) Plane P_4 then contains the zero-frequency component of the integral of the above product on the system axis. Therefore, for each value $x_o(\tau)$, the output $h(\tau)$ is given by

$$h(\tau) = \int f(x) g\left[x + x_o(\tau)\right] dx$$

It is simple to reject $B_g f$ because f(x) is a narrowband function.

One possible immediate application of optical cross-correlation as described above is in the reduction of data on atmospheric turbulence, where phase records are correlated to measure scale of turbulence, effective wind velocities, etc.

The extension of the technique to autocorrelation and convolution is obvious from the above discussion.

3.2.4. DISPLAY OF THE AMBIGUITY FUNCTION. The ambiguity function is a useful tool in the analysis of radar and communication systems. Thus far there has been little success in the area of synthesis. Though one would like to specify the characteristics of the ambiguity function and then derive the signal function which yields these characteristics, the synthesis procedure is undeveloped and one generally resorts to a trial-and-error method utilizing computer techniques.

As an example of the principles of coherent optical systems, three methods of displaying the ambiguity function will be given.

The ambiguity function, $\hat{A}(t, \omega)$, associated with the function $\hat{f}(x)$ is given by

$$\hat{A}(\tau, \omega) = \int \hat{f}(x) \hat{f}^*(x + \tau) e^{-j\omega x} dx$$

Generally, one is concerned with the modulus squared of $A(t, \omega)$ rather than with $A(t, \omega)$ itself.

It has been shown that in an optical system there exist planes, $P_1, P_2 \ldots P_r$, that are successive transform planes. These planes may be used in either a two-dimensional sense or in a multichannel sense. In order to make use of these concepts, one makes the following observations.

(1) The ambiguity function, A, is the Fourier transform of the product of two functions, $\hat{f}(x)$ and $\hat{f}^*(x+\tau)$;

or

(2) The ambiguity function, A, is the cross-correlation of two functions, $\hat{f}^*(x)$ and $\hat{f}(x)e^{-j\omega x}$;

or

(3) If $\hat{F}(\omega)$ is the Fourier transform of $\hat{f}(x)$, then $A(t, \omega)$ is the Fourier transform of the product $F(\xi - \omega)F^*(\xi)$ (where * denotes conjugate).

Each of these observations leads to a different implementation.

(1) In the first case, the function f(x) is placed at plane P_1 (i. e., for each value of y, the same function, f(x), is displayed). The function $f(x+\tau)$ is placed at plane P_3, and the y-dimension is used to get various values of τ. The product of f(x) and $f(x+\tau)$ has now been completed by use of the multichannel capability. Finally, at plane P_4 the ambiguity function is displayed.

(2) The function $f(x)e^{-j\omega x}$ is placed at plane P_1 and the y-dimension is utilized for ω. The function $f(x+\tau)$ is passed through plane P_3, thereby utilizing real time for τ. The output is taken from a vertical slit placed on the optical axis at plane P_4. At any instant of time, $A(\omega, \tau_o)$ is displayed.

(3) The third implementation is similar to the first except that the spectra $F(\omega-\omega_o)$ and $\overline{F(\omega)}$ are placed at planes P_1 and P_3, respectively, and the y-dimension corresponds to ω.

4
CONCLUSIONS and COMMENTS

The treatment of optical systems, primarily coherent optical systems, which has been presented in this report shows that optical data-processing and filtering systems may often present advantages over their electronic counterparts. The chief advantages stem from the following properties.

(a) Optical systems are inherently two-dimensional.

(b) Coherent systems inherently generate successive Fourier transform pairs.

(c) Independent control over phase and amplitude of special components can be easily effected.

(d) Multiplication can be effected by a simple imaging process.

Many other interesting properties then follow from these four.

However, many practical difficulties may arise to offset the possible advantages of a data-handling system. These, in general, originate from the use of photographic film as a medium for generating the function transparencies. The noise-like effects of film grain, perturbations in emulsion thickness, and the effect of spurious scattering of light, all bear further investigation and in some cases present serious limitations. The question of complete independence between phase and amplitude control in optical filters is treated in a manner which attributes ray properties to the light wave and therefore bypasses the question of field phenomena when very small objects are allowed to intercept the wave; a scattering analysis is probably in order for this problem. Lens systems limit resolution and hence channel capacity and channel density. The delay time involved in developing photographic film may be intolerable for some applications. Investigations which will dictate the ultimate limitations and practicability of optical computing or data-handling systems are far from complete. However, the flexibility and inherent simplicity of optical channels appear to assure this technique a promising role in forthcoming filtering and data-handling problems.

Appendix A
TRANSFORM RELATIONS in COHERENT SYSTEMS

In Section 2.2 of this report the following three properties of coherent optical systems are given:

$$\hat{E}_3(x_3, y_3) = \mathfrak{F}\, \hat{E}_1(x, y_1)e^{j\beta(x_3, y_3)} \tag{9a}$$

$$\beta(x_3, y_3) = 0 \text{ for } Z_1 = Z_2 \tag{9b}$$

$$\beta(x_3, y_3) \neq 0 \text{ for } Z_1 \neq Z_2 \tag{9c}$$

These properties are demonstrated with the aid of Figure 4. Suppose lens L_1 to form a collimated beam of monochromatic light. Calculation of $\hat{E}_3$ at x_3, y_3, given $\hat{E}_1$, requires finding the optical-path length from x_1, y_1 to x_3, y_3. $\hat{E}_3$ is then the integral, over plane P_1, of $\hat{E}_1$ properly delayed in phase according to the distance r:

$$\hat{E}_3 = \frac{1}{j\lambda}\iint \frac{1 + \cos\theta}{2d}\hat{E}_1(x_1, y_1)e^{-j\frac{2\pi r}{\lambda}}dx_1\, dy_1$$

where λ = wavelength of light

d = amplitude attenuation factor resulting from distance between planes P_1 and P_3

$\frac{1 + \cos\theta}{2}$ = the obliquity factor

r = the distance between the points (x_1, y_1) and (x_3, y_3)

In the systems to be considered, $\frac{1}{j\lambda}$ is dropped because absolute phase and amplitude are of no consequence, the d in the denominator is dropped because the distance attenuation is negligible, and the obliquity factor is dropped because θ is always sufficiently small that cos $\theta \approx 1$. The above expression becomes

$$\hat{E}_3 = \int \hat{E}_1 \exp\left[-j\frac{2\pi}{\lambda} r(x_1, y_1, x_3, y_3)\right]dx_1\, dy_1$$

To calculate $r(x_1, y_1, x_3, y_3)$, consider the configuration of Diagram 1.

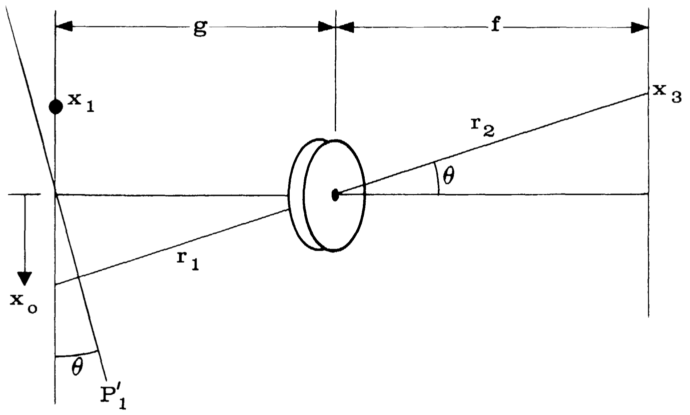


DIAGRAM 1

A plane wave, P_1, making an angle θ with the normal (as shown) is brought to a focus at x_3, where

$$x_3 = f \sin \theta$$

This implies that the optical distance between x_3 and any point on plane P_1' is a constant, c. This constant is

$$c = r_1 + r_2 = \sqrt{g^2 - x_o^2 \cos^2 \theta} + \sqrt{f^2 + x_3^2} = g + f + \left(1 - \frac{g}{f}\right)\frac{x_3^2}{2f}$$

for small θ, and using $\frac{x_o}{g} = \frac{x_3}{f}$. The distance from plane P_1 to x_3 is obtained by adding the term

$$-x_1 \sin \theta = \frac{x_1 x_3}{f}$$

the total distance from x_1 to x_3 then becomes

$$r(x_1, x_3) = g + f + \left(1 - \frac{g}{f}\right)\frac{x_3^2}{2f} - \frac{x_1, x_3}{f} = A_o + B_o x_3^2 - \frac{x_1}{f} x_3$$

This approach can be carried out in two dimensions to yield

$$r(x_1, y_1, x_3, y_3) = \text{constant} + \left[1 - \frac{g}{f}\right] \frac{\left(x_3^2 + y_3^2\right)}{2f} - \frac{x_1 y_3}{f} - \frac{y_1 y_3}{f}$$

Finally,

$$\hat{E}_3 = \left\{ \int\int \hat{E}_1 e^{-j\omega_x x_1} e^{-j\omega_y y_1} dx_1 dy_1 \right\} e^{j\beta(\omega_x, \omega_y)}$$

where

$$\omega_x = -\frac{2\pi x_3}{\lambda f}$$

$$\omega_y = -\frac{2\pi y_3}{\lambda f}$$

$$\beta = \left(1 - \frac{g}{f}\right)\left(\frac{x_3^2 + y_3^2}{2f}\right)$$

this demonstrates Equation 9a. Equation 9b follows immediately when one sets g = f.

Equation 9b can be obtained subject to weaker conditions than those required in the above analysis. It is assumed only that the lenses are aberration-free and together serve to image plane P_1 onto plane P_3 (Diagram 2). It is readily shown that each Fourier component $e^{-j\omega_x x + \omega_y y)}$ at P_1 produces a plane wave which is brought to a focus on plane P_2 at position

$$x_2 = -f\frac{\omega_x \lambda}{2\pi}, \quad y_2 = -f\frac{\omega_y \lambda}{2\pi}$$

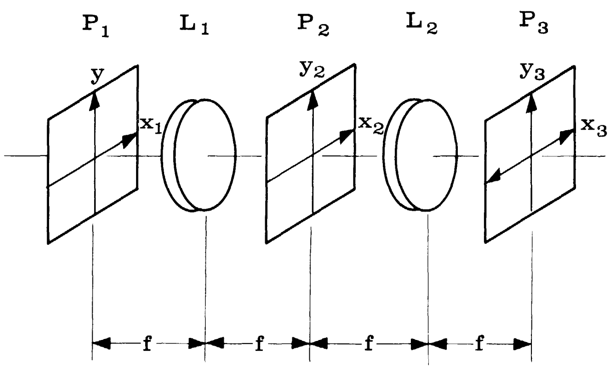


DIAGRAM 2

This implies that plane P_2 displays properly the two-dimensional power spectrum of the P_1 signal, but implies nothing about the relative phase of the spectral components. Thus, at plane

P_2, one has

$$\hat{E}_2 = \mathcal{F}\left[\hat{E}_1\right] \cdot e^{j\beta_1(x_2, y_2)}$$

where β_1 is the difference between the actual phase of $\hat{E}_2$ and the phase required by the transform relationship. Similarly,

$$\hat{E}_3 = \mathcal{F}\,\hat{E}_2\, e^{j\beta_2(x_3,\ x_3)}$$

If a real function is placed at plane P_1, i.e.,

$$S_1(x_1,\ y_1) = t(x_1,\ y_1)$$

then at plane P_3 an image of plane P_1 is observed, i.e.,

$$S_3(x_3,\ y_3) = S_1(-x,\ -y_1)$$

(The minus signs appear because successive Fourier transforms are taken with the kernel $e^{-j\omega_x x}$). One then has

$$t(-x,\ -y) = \mathcal{F}\left\{e^{j\beta_1} e^{j\beta_2} \mathcal{F}[t(x,\ y)]\right\}$$

which implies

$$\beta_1 = \text{a constant}$$

$$\beta_2 = \text{a constant}$$

hence Equation 9b is proved to within a constant phase factor. Since phase is measured relative to the phase at $x_3 = y_3 = 0$, this constant is zero.

That only the planes P_1 and P_2 can be Fourier transform planes is implied by the previous analysis. Again, however, this can be proved on the basis of a weaker assumption, namely that an aberration-free lens of arbitrary f-number is used. Let a point source of illumination be placed in plane P_1. The Fourier transform of the point (a spatial impulse function) is a uniform amplitude function with linear phase shift.[3] Therefore, if P_1 and P_2 constitute a Fourier transform pair, the light at plane P_2 must be collimated. Therefore, P_1 must be in the focal plane of the lens. Similarly, the point source may be placed in plane P_2, in which case it is obvious that P_2 must be a focal plane for the transfrom relation to hold. Therefore, the inequality 9c is proved.

[3] The linear phase shift is, in fact, a constant if the point source is on the optic axis.

Appendix B
RECORDING of COMPLEX FUNCTIONS

It is the purpose of this appendix to show how complex functions may be recorded as real functions and yet retain the essential features of the complex function. Equation 15 of the text will be derived.

Suppose $\hat{f}(t)$ is a complex function having no frequencies higher than ω_o. Denote the Fourier transform of $\hat{f}(t)$ as $\hat{F}(\omega)$. Let $\hat{f}(t)$ modulate a carrier and call the resulting function $\hat{g}(t)$; i.e.,

$$\hat{g}(t) = \hat{f}(t)\, e^{j\omega_o t}$$

The spectrum of $\hat{g}(t)$, namely $\hat{\zeta}(\omega)$, is given by

$$\hat{\zeta}(\omega) = \hat{F}(\omega + \omega_o)$$

Instead of recording $\hat{f}(t)$, a complex function, the Re $[\hat{g}(t)]$ is recorded

$$\mathrm{Re}[\hat{g}(t)] = f_o(t) \cos\left[\alpha(t) + \omega_o t\right]$$

if

$$\hat{f}(t) = f_o(t) e^{j\alpha(t)}$$

Note that if $\hat{f}(t)$ represents an electronic signal, then the Re[g(t)] can be obtained directly by first offsetting $\hat{f}(t)$ with a rotary phase shifter, thereby performing the operation

$$f_o(t) \cos\left[\omega t + \alpha(t)\right] \longrightarrow f_o(t) \cos\left[(\omega+\omega_o)t + \alpha(t)\right]$$

and then synchronously detecting the signal, producing

$$f_o(t) \cos\left[(\omega+\omega_o)\, t + \alpha(t)\right] \longrightarrow f_o(t)\left[\cos \alpha(t) + \omega_o t\right]$$

For positive frequencies, the spectrum of Re[g(t)] is the same as that for $\hat{g}(t)$. (Recall $\zeta(\omega) \equiv 0$ for $\omega < 0$.) Therefore, if the negative frequencies are rejected optically, the resulting function is $\hat{g}(t)$ to within a constant multiplier.

Suppose one would like to perform the following operation on the two complex functions $\hat{f}_1(t)$ and $f_2(t)$:

$$\int f_1(t) f_2^*(t) e^{j\omega t}\, dt$$

and suppose further that one would like not to record complex functions. It has been shown above that one can make available $\hat{f}_1(t)e^{j\omega_o t}$ under the restriction of recording real functions. In a similar manner $f_2^*(t)e^{-j\omega_o t}$ can be obtained, and since the optics introduces the kernel function $e^{j\omega t}$ automatically, the following operation is carried out:

$$\int \hat{f}_1(t)e^{j\omega_o t}\hat{f}_2^*(t)e^{-j\omega_o t}e^{j\omega t}\,dt = \int \hat{f}_1(t)\hat{f}_2^*(t)e^{j\omega t}\,dt$$

which is the desired operation. Equation 15 of the text is obtained by letting f_1 and f_2 be real.

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Willow Run Laboratories, U. of Michigan, Ann Arbor
OPTICAL DATA-PROCESSING AND FILTERING SYSTEMS by Louis J. Cutrona, Emmett N. Leith, Carmen J. Palermo, Leonard J. Porcello. Apr 60. 37 p. incl. illus., table, 9 refs.
(Technical report no. 2831-20-T/2900-123-T)
(Contracts AF 19(600)-38019 and DA-36-039 SC-78801)
Unclassified report

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1. Optical systems—Applications
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3. Data Processing

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