Letters

Fatigue crack propagation of PMMA in organic solvents

In the present note, the effects of two typical organic solvents, ethanol (95%) and carbon tetrachloride (99%), on the fatigue crack propagation rates in PMMA are discussed.

Fracture mechanics which has been applied successfully to fatigue crack propagation in both metallic and polymeric sheet materials [1-4] in air is adopted in the present environmental fatigue analyses. The generalized fatigue crack growth law may be written as

\[
\frac{d(2L)}{dN} = C(AR)^n
\]

where \( L \) is the half crack length of a centre-notched specimen; \( N \) the number of elapsed cycles; \( \Delta R \) the crack extension resistance range; \( C \) a material constant depending on frequency, mean load and environmental fluid; and \( n \) an exponent to be determined from experiments.

Centre-notched plates were made from cast sheets of PMMA, the dimensions being 300 mm x 100 mm x (64 mm or 32 mm). Initial crack length \( (2L_0) \) was 32 mm. The cyclic load limits were chosen so that the lower limit \( (X_0 = 0 \sim 2 \text{ kg}) \) was negligible compared to the maximum load \( (X_m) \). The wave-form of the applied loading was approximately triangular. To avoid any significant rise in the temperature of the material in the crack tip vicinity, low frequencies of 2 to 20 cpm \((3.3 \times 10^{-2} \text{ to } 3.3 \times 10^{-1} \text{ Hz})\) were selected.

The crack extension resistance range is given by

\[
\Delta R = \frac{\Delta (\sigma^2)}{E} bW \tan \left( \frac{\pi L}{W} \right)
\]

where \( \Delta \sigma = (X_m - X_0)/(Wt) \), \( W \) and \( t \) being

![Figure 1](image1.png)  

**Figure 1** Fatigue crack growth of PMMA in 95% ethanol at 17 cpm.

*When quasi-static cracking [5] occurs, \( ER = bK^2 \), where \( K \) is the stress intensity factor, \( E \) the Young’s modulus and \( b = 1 \) or \( 1 - \nu^2 \) for plane stress and plane strain conditions of cracking, respectively. \( \nu \) is the Poisson’s ratio of the material.

![Figure 2](image2.png)  

**Figure 2** Fatigue crack growth of PMMA in 99% carbon tetrachloride.
respectively the width and thickness of the specimen. The fatigue crack growth rates were generated from the crack length \(2L\) versus elapsed cycles \(N\) records.

The data obtained from all the experiments indicate that fatigue crack propagation in these two organic solvents may be divided into three stages (see Figs. 1 and 2).

Stage I is associated with craze growth. It may be seen that the craze growth rate, \(d(2L)/dN\), decreases with increasing \(\Delta R\). This is due to the generation of minute crazes at the crack tip so that the effective craze speed is reduced. It has been noticed that under a constant applied stress range \(\Delta \sigma\), the appearance of crazes is independent of the initial crack length and hence of \(\Delta R\). When one of the crazes emerges as the main crack to propagate in the subsequent load cycles the crack eventually regains its normal growth rate. It is thought that such solvent crazes may be minimized. In fact some success has been obtained in minimizing the formation of crazes by properly annealing the specimen and by coating a layer of silicon oil on the specimen surfaces.

In stage II crack growth \((\Delta R < 0.40 \text{ kg cm}^{-1})\), where \(d(2L)/dN\) increases with \(\Delta R\), the rate is determined by the interaction of the stress and the absorbed solvent particles near the crack tip region. Obviously, the presence of the environmental fluid enhances the fatigue crack growth rate.

Before the onset of stage III crack growth \((0.40 \text{ kg cm}^{-1} \leq R < 0.87 \text{ kg cm}^{-1})\) where the environment-enhanced effect on fatigue is less significant, a slope transition is noted. This invariably occurs at a range of \(\Delta R\) values of 0.36 to 0.42 kg cm\(^{-1}\) which corresponds well to the fracture toughness of PMMA when tested under quasi-static conditions in air [5]. Experiments have shown that the component crack growth rate due to the solvents is diminished in magnitude, possibly indicating a lapse of solvent absorption at these high \(\Delta R\) values. Consequently, crack growth in this stage is more mechanically controlled than is environmentally enhanced. Thus, our results reflect that the magnitude of \(n\) of stage III is less than that of stage II in the fatigue crack propagation law.

Figs. 1 and 2 show typical results for fatigue crack growth rates of PMMA in ethanol and carbon tetrachloride respectively and at selected frequencies. The slope transitions are clearly indicated in these figures.

The magnitudes of the slopes \((n)\) and the constants \((C)\) for the last two stages of fatigue crack growth have been summarized in Table I. The exponent \(n\) in stage I is not shown because of its scattered variations and also because the crazes may be possibly eliminated. As a basis of comparison, the air results are also included. It may thus be seen that the presence of the organic solvents has greatly increased the fatigue crack growth rate of PMMA at comparable \(\Delta R\) values. Conversely, the fatigue life will be considerably shortened. This is reflected by the increases in magnitude of \(n\) and \(C\) in Equation 1 when compared with the air results. Also, by comparing the crack growth rate data presented here, it is surprising to find that carbon tetrachloride is more aggressive to PMMA than ethanol under cyclic loads. The absence of a slope transition in the fatigue crack propagation of PMMA in air is noteworthy.

The effects of cycling frequency on the crack growth of PMMA in 99% carbon tetrachloride solution may be seen from Figs. 2 and 3. At the same applied stress range, the time allowed for the stress-environment interaction per loading cycle is more for the lower frequencies, the crack growth rates will consequently be higher. For instance, at \(\Delta R = 0.56 \text{ kg cm}^{-1}\), the crack growth rates, for frequencies of 17 and 7 cpm (0.28 and 0.12 Hz) are respectively \(8.5 \times 10^{-2}\) and \(6.6 \times 10^{-2}\) cm per cycle. This represents an order of magnitude difference. The same effects have also been observed for ethanol.

Finally, for consideration of design against brittle fracture in PMMA it would be informative...

**Table I (a) Value of exponent \(n\) in fatigue crack propagation law**

<table>
<thead>
<tr>
<th>Environment</th>
<th>Stage II</th>
<th>Stage III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>2.0–2.50</td>
<td>2.0–2.50</td>
</tr>
<tr>
<td>Carbon tetrachloride (99%)</td>
<td>6.5–7.15</td>
<td>2.0–2.50</td>
</tr>
<tr>
<td>Ethanol (95%)</td>
<td>6.0–7.0</td>
<td>3.5–4.50</td>
</tr>
</tbody>
</table>

**Table I (b) Mean value of \(\log_{10} C\) in fatigue crack propagation law**

<table>
<thead>
<tr>
<th>Environment</th>
<th>Frequency</th>
<th>Stage II (\Delta R \leq 0.40 \text{ kg cm}^{-1})</th>
<th>Stage III (\Delta R &gt; 0.40 \text{ kg cm}^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>7 cpm</td>
<td>–3.29</td>
<td>–3.29</td>
</tr>
<tr>
<td>Carbon tetrachloride (99%)</td>
<td>7 cpm</td>
<td>1.331</td>
<td>–0.695</td>
</tr>
<tr>
<td>Ethanol (95%)</td>
<td>17 cpm</td>
<td>0.090</td>
<td>–1.518</td>
</tr>
<tr>
<td>Ethanol (95%)</td>
<td>7 cpm</td>
<td>0.061</td>
<td>–1.229</td>
</tr>
<tr>
<td>Ethanol (95%)</td>
<td>17 cpm</td>
<td>–0.50</td>
<td>–1.815</td>
</tr>
</tbody>
</table>
to know the fracture toughness at which unstable cracking ensues. This value may be calculated from the load and critical length measurements. At room conditions, PMMA has a fracture toughness of about 0.56 kg cm\(^{-1}\), whereas in the presence of these two organic solvents, a higher fracture toughness of 0.865 kg cm\(^{-1}\) was measured.

**Acknowledgement**

The author wishes to express his gratitude to Professor Charles Gurney for his helpful suggestions and encouragement during the course of this work which was performed at the University of Hong Kong.

**References**


Received 18 April and accepted 24 May 1974

Y. W. MAI

Department of Mechanical Engineering, University of Michigan, Ann Arbor, Michigan, USA

---

**An LVDT extensometer for tensile studies of composite materials**

The subject of this note is an economical, adaptable, high sensitivity extensometer system based on d.c./d.c. linear variable differential transformer (LVDT) transducers. These operate in the same way as a normal LVDT but the oscillator and detector circuits are built into the transducer body. This particular unit is designed to monitor the faces of glass-fibre-reinforced cement specimens, with nominal cross-section dimensions of 50 mm × 10 mm, in a uniaxial tensile test; it has also been used successfully for other similar materials and specimen sizes.

Lawley and Meakin [1] also employed a LVDT element in their extensometer for the study of microplasticity in metals. In a review of the alternative measuring methods for their application they rejected the use of resistance gauges, optical gauges and capacitance gauges mainly because of their limited range (maximum strain \(1 \times 10^{-2}\)). The capacitance gauge described by Roberts and Brown [2] Bacon and Cowling [3] and recently by Bonfield et al. [4] have a higher strain sensitivity (\(5 \times 10^{-7}\)) than gauges based on LVDT elements (\(2 \times 10^{-6}\)) but because they are concentric to the specimen, they average out any strains resulting from non-axial stresses. Non-axial stresses are introduced if the specimen is slightly curved initially or if it is anisotropic for any reason. The requirements for measuring bars and sheets of fibre reinforced cement composite are as follows: (1) a high initial sensitivity, better than a strain of \(1 \times 10^{-6}\), to allow study of the elastic behaviour; (2) a range up to a strain of \(1 \times 10^{-1}\) to enable monitoring of the pseudo-ductile behaviour resulting initially.