THE UNIVERSITY OF MICHIGAN

INDUSTRY PROGRAM OF THE COLLEGE OF ENGINEERING

CORRELATION OF BENDABILITY OF MATERIALS WITH THEIR TENSILE PROPERTIES

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Nomenclature

ε	True strain
ε_m'	True strain at maximum tensile load
ε_f	True strain at fracture
ε_o	True strain in outer fiber
ε_i	True strain in inner fiber
A_r	Percentage reduction of area in tensile specimen
ℓ_f	Final length
ℓ_o	Original length
A_f	Final cross-sectional area
A_o	Original cross-sectional area
t	Thickness of the bent bar
θ	Angle subtended by an arc length in a bent bar
R_o	Radius at outer fiber of the bent bar
R	Radius at inner fiber of the bent bar
R_n	Radius at the neutral axis of the bent bar
k	Ratio defined as $\frac{R}{t}$
W	Width of specimen
F	Force applied
D	Diameter of rolls in bending test
d	Distance between rolls in bending test
M	Bending moment

Introduction

In the fabrication of parts by plastically bending sheets or bars of a given material, it is necessary to know to what radius 'R' a piece of thickness 't' may be bent without fracture occurring during the bending process. This has always been done experimentally in sheet metal shops using various tables of '$\frac{R}{t}$' values for commonly used materials. However, with the advent of high-temperature resistant materials it is becoming increasingly important to treat the subject analytically.

About five years ago several researchers in the aircraft industries reported very good correlation between the true strain at the time that the maximum load was applied to a tensile specimen and the '$\frac{R}{t}$' ratio to which the same material could be bent. One of the more complete reports of this type is by Paulson, Anderson, and Roberts [1] . Fig. 1 of the present paper shows one of their experimentally determined curves which indicates good correlation between '$\frac{R}{t}$' and ε_m'. However, repeated tests conducted in the University's Mechanical Engineering Materials and Processes Laboratory for the past two years could not substantiate these published data and, therefore, that relationship is not believed to be very reliable.

This past year a new theory was developed by the authors and has been substantiated by many laboratory tests. This theory states that 'failure will occur in the outer fiber of a material being bent when the true-strain in the outer fiber is equal to the true-strain at the instant of fracture of a tensile test specimen of the same material'. Inasmuch as the true-strain at fracture can be very easily determined from the conventional percentage reduction of area, the application of this theory is greatly simplified as will be shown.

Analysis

The following assumptions were made:

1. The fracture strain in the outer fiber of bending specimen equals that in tensile test specimen.
2. The material is homogeneous and isotropic.
3. The bar bends in plain strain.

In the tensile specimen, see Fig. 4, the maximum true strain that the specimen can endure is the true strain in the reduced section at the instant prior to the fracture. This strain may be very accurately and very easily obtained from the relationship:

$$\varepsilon_f = \ln \left(\frac{100}{100 - A_r}\right) \qquad (1)$$

where A_r is percentage reduction in area.

This is apparent from the definition of true strain, which is, "the summation of changes in length divided by the length from which each change was produced". In mathematical form this is expressed as:

$$\varepsilon = \frac{\Delta l_1}{l_o} + \frac{\Delta l_2}{l_o + \Delta l_1} + \frac{\Delta l_3}{l_o + \Delta l_1 + \Delta l_2} + - - \text{etc.} \qquad (2)$$

By letting Δl become vanishingly small, the expression for true strain then reduces to:

$$\varepsilon = \ln \frac{l_f}{l_o} \qquad (3)$$

where ℓn is the natural logarithm, ℓ_f is the final length, and ℓ_o is the original length.

Also, if the volume of the material is assumed to remain constant with only its shape being changed, then the ratio of A_o/A_f must equal the ratio of ℓ_f/ℓ_o and then equation (3) may be expressed as:

$$\varepsilon = \ell n \left(\frac{wt}{w_f t_f}\right) = \ell n \ (A_o/A_f) \qquad (4)$$

Since, by definition, $A_r = \frac{A_o - A_f}{A_o} \times 100$, which may be rewritten as $\frac{A_o}{A_f} = \frac{100}{100 - A_r}$. If this latter expression is substituted in equation (4), and the strain ε is defined as the true strain at fracture, ε_f, then equation (1) results.

Consider next a flat sheet or specimen that is subjected to a bending moment as illustrated in Fig. 2, the outer fiber is strained in tension and the inner fiber is strained in compression. Inasmuch as failure will not occur on the compressive side, it is sufficient to consider only the strain in the outer fiber.

Case I. Neutral axis lies in the middle fiber $t_1 = \frac{1}{2} t$.

The arc-length along the neutral axis subtended by the angle θ is $(R + \frac{1}{2} t)\ \theta$ and arc-length of the outer fiber is $(R + t)\ \theta$. Therefore, the true strain in the outer fiber is given by:

$$\varepsilon_o = \ell n \left(\frac{\ell_f}{\ell_o}\right) = \ell n \frac{(R + t)\ \theta}{(R + \frac{t}{2})\ \theta} = \ell n \frac{(R + t)}{(R + \frac{t}{2})} \qquad (5)$$

This is based on the assumption that the neutral axis does not deviate appreciably from the half-thickness of the sheet, which is very valid for R/t ratios greater than 1-1/2. By equating ε_o in equation (5) to ε_f in equation (1), it is

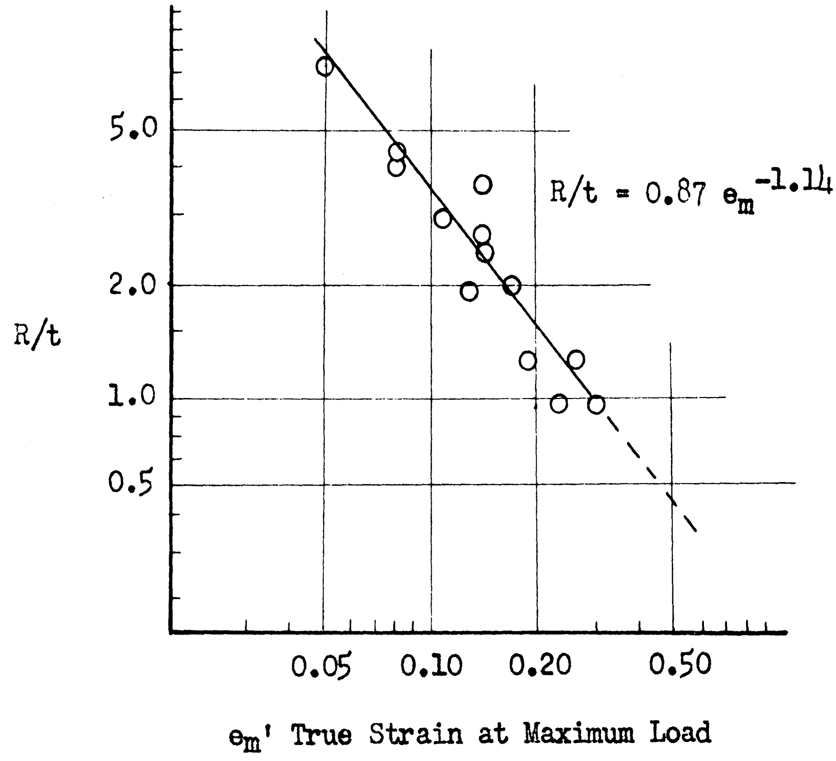


Figure 1. R/t vs. Critical Strain (True Strain) at the Maximum Load of a Tensile Test as reported by D. L. Paulson, W. E. Anderson and E. C. Roberts.

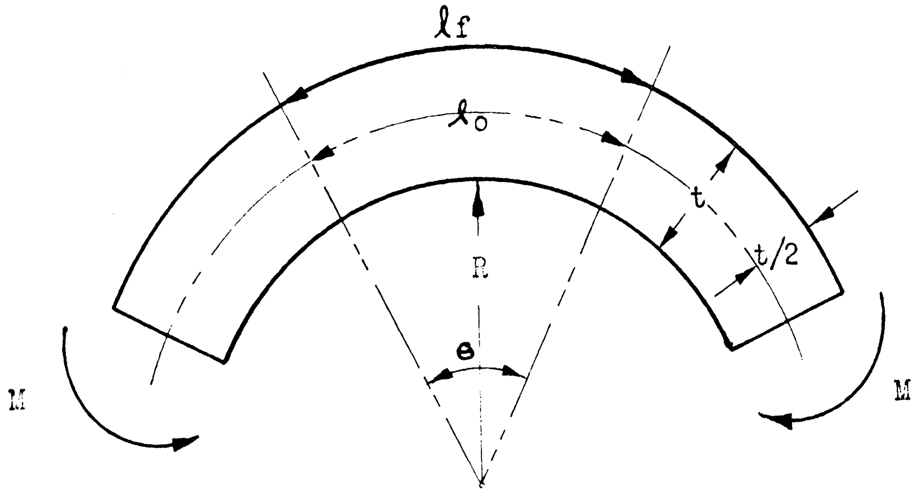


Figure 2. Simple Bending of a Plate.

apparent that

$$\frac{100}{100 - A_r} = \frac{R + t}{R + \frac{t}{2}} \quad \text{or} \quad \frac{R}{t} = \frac{50}{A_r} - 1 \qquad (6)$$

From the authors' experience it was found that this relationship is valid for materials having a percentage reduction in area (A_r) of less than 20. For materials having a percentage reduction in area greater than 20, the displacement of the neutral axis in bending is significant and will be discussed next.

Case II. Neutral axis is not at the mid layer.

$$\varepsilon_o = \ln\left(\frac{R_o}{R_n}\right) \qquad (7)$$

If k is defined as $\frac{R}{t}$, one has:

$$\left.\begin{aligned} R_o &= t(k + 1) \\ R &= kt \\ R_n &= t\sqrt{k^2 + k} \end{aligned}\right\} \qquad (8)$$

--- from Sachs and Hoffman [2].

Substituting equation (8) into equation (7) gives:

$$\varepsilon_o = \ln \frac{k + 1}{\sqrt{k^2 + k}}$$

or

$$\varepsilon_o = \ln\sqrt{1 + \frac{1}{k}} \qquad (9)$$

From equation (8)

$$k = \frac{R_o}{t} - 1 \qquad (10)$$

Combining equations (9) and (10), one obtains:

$$\frac{R_o}{t} = \frac{1}{e^{2\varepsilon_o} - 1} + 1 \quad \text{and} \quad \frac{R}{t} = \frac{1}{e^{2\varepsilon_o} - 1} \qquad (11)$$

From equation (1), $e^{\varepsilon_o} = \frac{100}{100 - A_r}$, where $\varepsilon_o = \varepsilon_f$.

By substituting e^{ε_o} in equation (11), the following equation results:

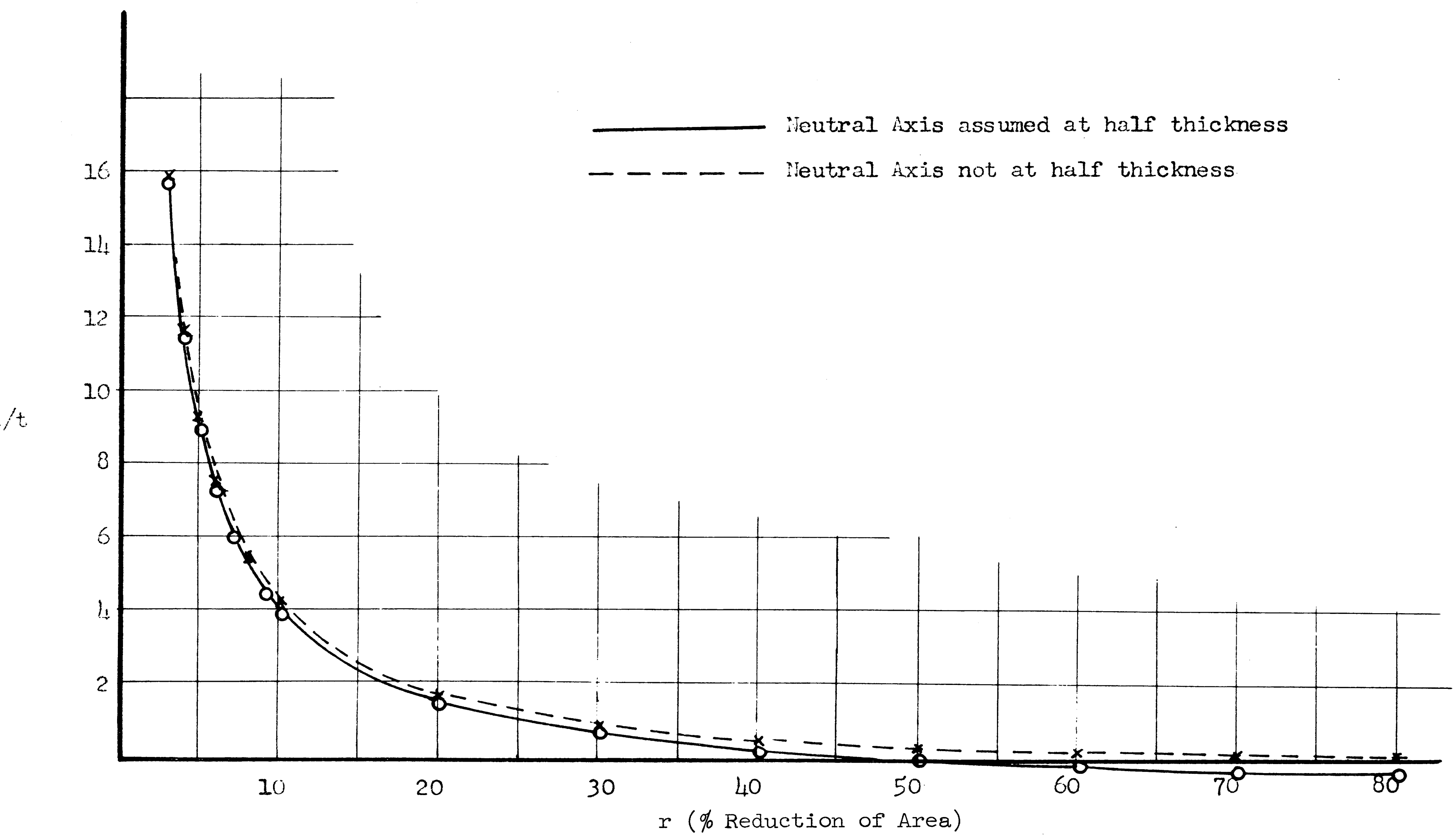


Figure 3. Curves from Analytical Solution for R/t vs. Percentage Reduction of Area.

$$\frac{R}{t} = \frac{(100 - A_r)^2}{200 A_r - A_r^2} \qquad (12)$$

The plotting of equations (6) and (12) are shown in Fig. 3.

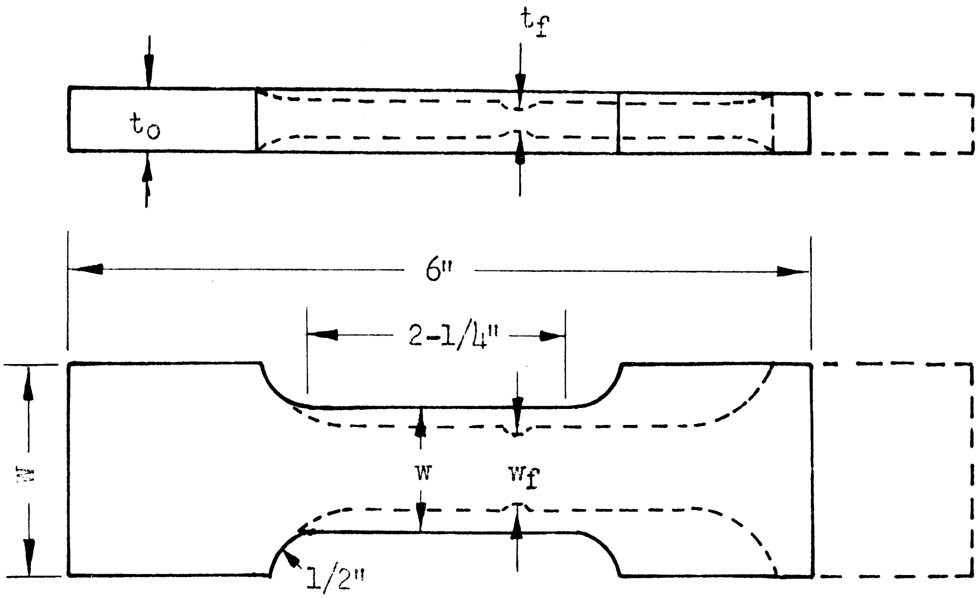
It is seen that both curves look hyperbolic and are asymtotic to the $\frac{R}{t}$ axis. In general for the same percentage reduction of area the solid curve gives lower value of $\frac{R}{t}$ than the dotted curve and the difference becomes smaller as A_r becomes smaller in value and vice versa. When A_r is less than 4, the two curves practically coincide.

Also, from laboratory experience, it has been found that the empirical relationship $\frac{R}{t} = \frac{60}{A_r} - 1$ (13)
is a very satisfactory relationship for the ductile materials that do have a shift in the position of the neutral axis.

Experimental Technique and Results

Flat tensile specimens were cut from bar stock of the following materials: magnesium, aluminum, brass, 1018 steel, titanium, cast iron and plastics. In nearly all cases, three specimens were tested and since the range of values obtained for each material was not large, only the averages are reported. The composition and code of each are shown in the 3rd column of Table 1, and the dimensions of the flat tensile specimens are shown in Fig. 4. To determine the agreement between the tensile properties obtained from flat and round tensile specimens of the same material, round tensile specimens for some metals were also tested, and the percentage reduction of area is also shown in Table 1.

The corresponding bending specimens were cut from the same bar stock to the dimensions shown in Fig. 5.



$W = w + 1/4"$

Figure 4. Dimensions of Flat Tensile Testing Specimens.

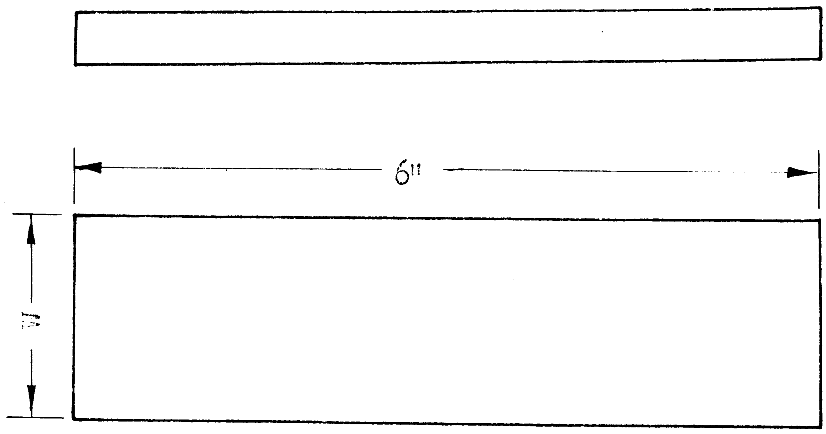


Figure 5. Dimensions of a Bending Specimen.

A 30 ton universal tensile testing machine was used to pull the tensile specimens until they fractured. The cross-sectional area of the specimens before testing and after fracture were measured on a tool-maker's microscope with an accuracy of 0.0001". The percentage reduction of area for each specimen was calculated and the results are listed in column 7 of Table 1.

In the bending test two methods were tried. In the first method the specimen was simply supported on two rolls and the load was applied through a third roll to deflect the flat specimen as shown in Fig. 6(a). The set of bottom rolls was selected such that $d = D + 2t$ (refer to Fig. 6(a)). Ten sets of such rolls ranging from 1/8" dia. to 1-1/2" dia. were prepared for the test. During the test it was necessary to stop loading and search for the small cracks on the outer fiber of the beam. Whenever the cracks appeared, the test was stopped and the inner radius of the bent beam was measured by means of a set of radius gages.

The second, more expedient, method of bending was found: namely, to clamp the flat bar in a vise vertically on one end as a cantilever beam and to apply a bending moment manually on the other end. The moment should be applied carefully so that the beam will be subjected to bending only and no shear forces exerted onto the bar. Fig. 6(b) shows this method schematically. During the process of bending, careful observation is also required to see the initiation of small cracks. Once the small crack appears, the loading should be stopped and the inner radius of the specimen measured as in the first method.

From the authors' experience, the two methods give very

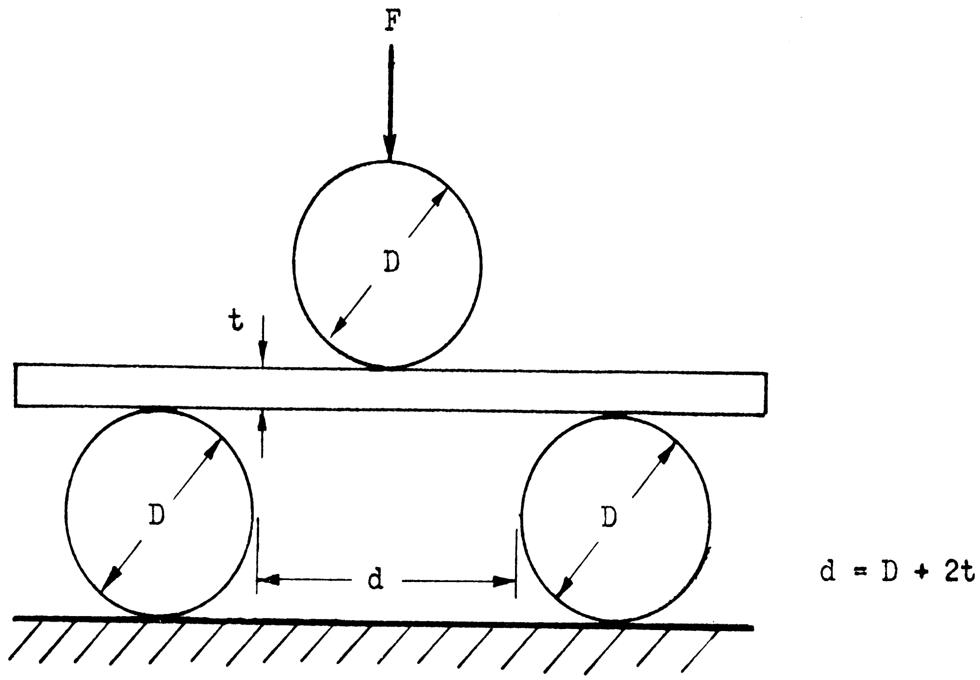


Figure 6(a). Bending Fixtures used for Testing.

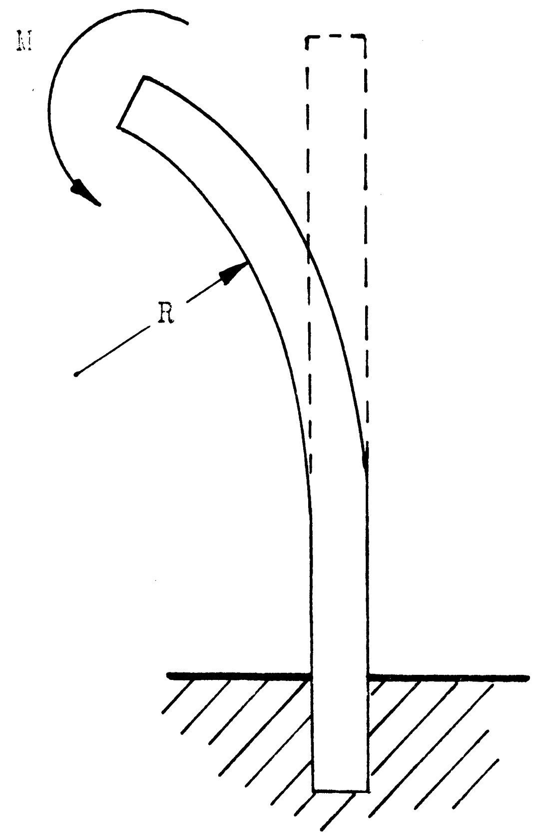


Figure 6(b). Bending Fixtures used for Testing.

similar results. However, the second method is much simpler to use and time-saving. Therefore, the second method was adopted in the tests and results obtained thereby were listed as shown in Table 1.

The results of '$\frac{R}{t}$' from bending tests are plotted against percentage reduction of area from corresponding tensile tests as shown in Fig. 7. The circled points are plotted from the experimental data and are superimposed on two curves calculated from the theoretical analysis which were also shown previously in Fig. 3.

It is at once apparent that the experimental data points fall along the theoretical curves, with the only material not giving close agreement being RC 130B titanium alloy and will be explained later. The most brittle material tested was a gray cast iron. With a percentage reduction of area of slightly less than one and a '$\frac{R}{t}$' ratio of 55, cast iron falls right on the theoretical curve. Polystyrene, a thermoplastic material, with a percentage reduction of area of 3 and a '$\frac{R}{t}$' of 16 fits the curve equally well. However, at this point it is well to point out the need, when working with a very elastic material that has no plasticity, to consider the elastic spring-back. For example, CR39 is a plastic that is commonly used in photoelasticity studies and as normally tested has a zero percentage reduction of area. Nevertheless, a 1/8" thick sheet can be bent to a 2" radius before it fractures. However, after breaking, the bend specimen is perfectly flat or straight indicating complete spring-back. But when a tensile specimen is tested and the actual area measured just prior to fracture, it is found to be nearly 3% less

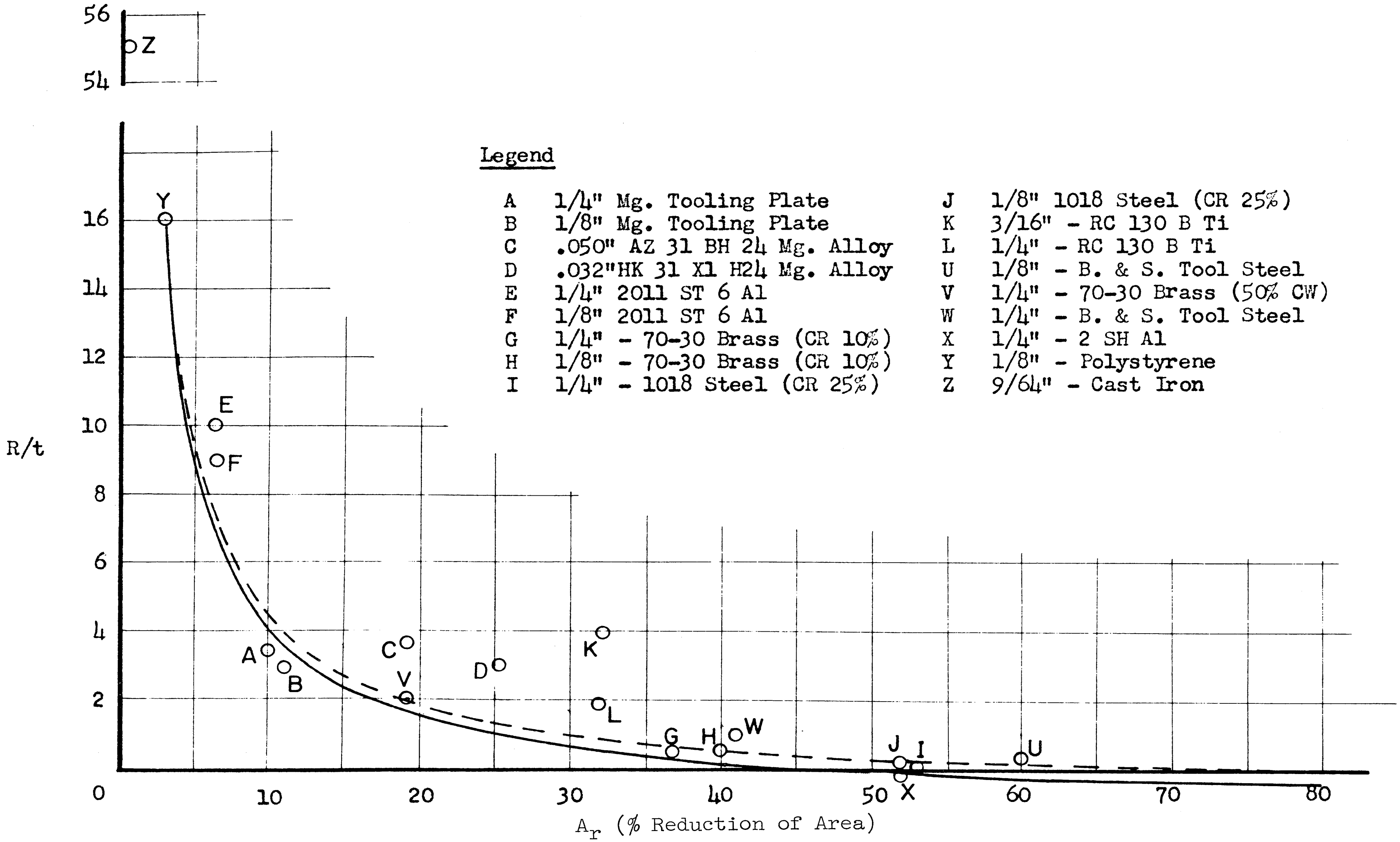


Figure 7. Experimental Data Super-imposed on Analytical Curves of R/t vs. Percentage Reduction of Area.

than the initial area, and with this correction CR39 also agrees with the theoretical curves.

Normally, aluminum is considered to be quite ductile and thoriated magnesium quite brittle. However, by comparing points E-F to C-D it is apparent that 2011 ST 6 aluminum can be bent to an '$\frac{R}{t}$' of 9 or 10 while the magnesium alloy can be bent to a '$\frac{R}{t}$' of only 3 to 4.

For the soft brasses, aluminum and steel, the percentage reduction of area is greater than 50 and these materials can be bent to a '$\frac{R}{t}$' ratio approaching 0. This is predicted from the theoretical equations and is substantiated in practice. Actually, these soft ductile materials can be bent over double with one face in contact with itself without any cracks appearing.

The experimental data of the 3/16" RC 130B titanium alloy is off the theoretical curve by an appreciable amount, much more so than the one piece of 1/4" thick titanium that was bent. It is believed by the authors that this is due to residual tensile stresses set up during the machining of the 3/16" thick specimens. These very shallow tensile stresses would be much more detrimental in the bend test than they would be in the determination of the percentage reduction of area in a tensile test. Before all of the titanium specimens were machined from 1/2" thick bars, one specimen was machined to a thickness of 1/4" to determine whether it would be feasible or possible to bend such a thick piece with the existing tooling. This piece was tested and is plotted as point 'L' in Figure 7 and fits the curve quite well. However, since considerable difficulty was encountered in bending this one trial piece, the standard size selected for the titanium alloy was 3/16". The 3 tensile and 3 bend specimens of this thickness are averaged as point 'K' which is somewhat above the curve. In talking to the machinist about this later in an attempt to find an explanation, it was learned that the first 1/4" thick piece was machined with light cuts whereas all of the 3/16" thick specimens were machined with just one heavier cut taken from each side, which could account for the presence of residual stresses in these latter specimens.

Conclusions

From the experimental data plotted in Fig. 7, it is obvious that there is definitely a correlation in theory and experiment between the '$\frac{R}{t}$' ratio in bending and a percentage reduction of area in a corresponding tension test. From the plotted data, it may be concluded that the equations $\frac{R}{t} = \frac{50}{A_r} - 1$, $\frac{R}{t} = \frac{(100 - A_r)^2}{200 A_r - A_r^2}$ or $\frac{R}{t} = \frac{60}{A_r} - 1$ can be used for the prediction of a minimum bend radius with respect to a particular plate thickness. Usually this property of a material is given in a handbook. Even if the data is not available in publications, it is quite simple to obtain from a tensile test.

In the derived equation of $\frac{R}{t} = \frac{50}{A_r} - 1$ the only property which affects the bend radius of a material is 'A_r' and nothing else. Therefore, this equation is valid for all materials, metal or non-metal. This is confirmed by the experimental data of two plastics which fall right onto the theoretical curve. However, the authors have some reservation for generalizing this theory because of the limited data for non-metals tested.

It is understood that ε_o (strain in outer fiber) and ε_i (strain in inner fiber) during bending are equal in magnitude only when the neutral axis is at the half thickness of the plate. If the beam is bent in the plastic region, the neutral axis is not at mid-fiber any more and, therefore, ε_0 and ε_1 are not the same. However, it can be proved that for the same $\frac{R}{t}$, ε_0 is much higher than ε_1. In fact, from practice it is found that compressive regions in bending hardly ever cause failure. Under such circumstances it is justified to use the extreme fiber in tension side to correlate with the tensile property.

Since plane strain is assumed in the analysis, the proposed equation $\frac{R}{t} = \frac{50}{A_r} - 1$, would give even closer results with experiment when the width to thickness ratio of the plate becomes large, which is usually the case in sheet metal work.

Table 1.

	Specimen #	Material	% R. A. Round	% R. A. Flat	$\frac{R}{t}$	% R. A.
A	1 - 3	1/4" Mg. Tooling Plate	--	9.9	3.5	9.9
B	4 - 6	1/8" Mg. Tooling Plate	--	11	3	11
C	7 - 9	.050 AZ 31 BH 24 Mg. Alloy	--	19	3.7	19
D	10 - 12	.032 HK 31 X1 H24 Mg. Alloy	--	25	3	25
E	13 - 15	1/4 2011 ST 6 Al	--	6.2	10	6.2
F	16 - 18	1/8 2011 ST 6 Al	--	6.4	9	6.4
G	19 - 21	1/4 - 70-30 Brass (CR 10%)	--	37	0.5	37
H	22 - 24	1/8 - 70-30 Brass (CR 10%)	--	40	0.55	40
I	26 - 28	1/4 - 1018 Steel (CR 25%)	--	53	0.2	53
J	29 - 31	1/8 - 1018 Steel (CR 25%)	--	52	0.2	52
K	32 - 33	3/16 - 130 B Ti	--	32	4.	32
L	34	1/4 - 130 B Ti	--	32	2	32
M	35 - 37	1/2" D. Mg. Tooling Plate	10.2	--	--	--
N	38 - 40	1/2" D. 2011 ST 6 Al	8	--	--	--
O	41 - 43	1/2" D. 70-30 Brass	50	--	--	--
P	44 - 46	1/2" D. 1018 Steel	57	--	--	--
Q	47 - 49	1/2" Drill Rod (C.T.S.)	60	--	--	--
R	50 - 52	1/2" D. 4340 Steel	35	--	--	--
S	53 - 55	1/2" - 130 B Ti	39	--	--	--
T	56	1/2" D. 70-30 Brass (50% CW)	41	--	--	--
U	57 - 58	1/8" - B. & S. Tool Steel	--	60	.4	60
V	59	1/4" - 70-30 Brass (50% CW)	--	19	2.0	19
W	60 - 61	1/4" - B. & S. Tool Steel	--	41	1.0	41
X	62 - 64	1/4" - 2 SH Al	--	52	0.06	52
Y	65	1/8" - Polystyrene	--	--	16	3
Z	66	9/64" - Cast Iron	--	--	55	3/4

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