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A VEHICLE DYNAMICS HANDBOOK FOR SINGLE-UNIT AND ARTICULATED HEAVY TRUCKS

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16. Abstract

This handbook provides a compilation of the effects of the mechanical properties of vehicle components and configurations on the braking and steering of heavy trucks. It contains sections describing the braking and steering performance of straight trucks, tractor-semitrailers, truck-full trailers, B-trains, doubles, and triples. Performance signatures and performance measures are presented for driving situations involving constant deceleration braking, low- and high-speed offtracking, steady turning, initiation of curved paths, obstacle evasion (quick lane changes), and braking while turning.

The influences of component properties on braking and steering performance, as quantified by computerized analyses and simulations, are illustrated through the use of parameter sensitivity diagrams. The ranges of the mechanical properties used in these sensitivity studies are based upon previously measured data for components that have been employed in heavy trucks.

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PREFACE AND ACKNOWLEDGEMENTS

This document is a revised version of a preliminary handbook that was developed in the first phase of this project. The resulting handbook represents an initial effort to present a wealth of vehicle dynamics knowledge pertaining to heavy trucks.

Although we have had mathematical simulations of heavy trucks for many years, those simulations have not been previously used to develop a generalized knowledge base defining the braking and steering performances of heavy trucks. After exercising the simulations, developing specialized analysis procedures, and preparing this handbook, we believe that this project has made significant progress towards focusing future evaluations of the dynamics of heavy trucks on pertinent performance measures.

The preliminary draft of the handbook was reviewed by people who specify vehicles for fleets and by people who build trucks. We have made revisions in response to the comments of these reviewers. Even so, the reviewer's comments indicate that further work could be done to prepare a document that would be easier for the trucking industry to use. We wish to thank the following individuals for their comments and suggestions:

Mr. Blaine Johnson	Ryder Truck Rental, Inc.
Mr. Loren Swenson	PIE Nationwide, Inc.
Mr. Arthur Ball	Fruehauf Corporation
Mr. Bill Giles	Ruan Companies
Mr. C. F. Powell	Navistar International Corporation
Mr. Larry Strawhorn	American Trucking Associations
Mr. Gary Hu	PACCAR, Inc.
Mr. Donald Dawson	Roadway Express, Inc.

With regard to the maneuvering situations analyzed, we have covered situations that are related to avoiding accidents. One additional situation, that was suggested by the reviewers, has to do with the turning resistance produced by vehicle units with numerous axles. This subject has recently been studied in connection with vehicles used in Canada, and it could become important throughout the United States if we start to allow heavier vehicles with more axles. We think that the analysis of this subject would be a worthwhile addition to the handbook.

From another point of view, the reviewers pointed out that recommended levels of performance are not presented in the handbook. In the authors' view, the handbook is intended to provide the basis for synthesis studies in which designers or assemblers of heavy trucks set performance targets. Without targets, the exercise of evaluating performance lacks incentive and direction. Nevertheless, the matter of selecting levels of performance is beyond the scope of this document.

Another suggestion involves a condensed document of only a few pages summarizing the most important considerations. The general idea is expressed in the question, "••• how do we transition from this handbook to a handbook of simple recommended practices?"

Finally, in the preliminary draft, the diagrams used to display performance sensitivities were difficult for the reviewers to interpret. We have changed the labels on all of these diagrams to improve their readablilty. We believe that this effort has made the results easier to understand.

1.0 INTRODUCTION

1.1 Information Concerning the Development of This Handbook

This handbook has been prepared by The University of Michigan Transportation Research Institute (UMTRI) under sponsorship of the National Highway Traffic Safety Administration (NHTSA) during a project entitled "An Evaluation of Factors Influencing Heavy Truck Dynamic Performance."

The purposes of this handbook are (1) to summarize the effects of the mechanical properties of vehicle components and configurations on the braking and steering of heavy trucks and thereby, (2) to aid in predicting the improvements in dynamic performance that might be developed through the judicious selection of component properties.

The results presented here were obtained using mathematical models that provide a basic understanding of the dynamics of single-unit and articulated trucks [1]. These models range in complexity from very simple calculation procedures for offtracking and braking efficiency to simulations that include detailed representations of the pertinent components of the driver-vehicle system.

Parametric information describing the mechanical properties of heavy trucks is required in applying these mathematical models and simulations. Information concerning the mechanical properties of typical components currently employed in heavy trucks and combination vehicles operating in the United States may be found in a companion document [2] entitled "A Factbook of the Mechanical Properties of the Components for Single-Unit and Articulated Heavy Trucks." (The "Component Factbook" was also prepared during this project.)

1.2 Scope of the Handbook

The scope of this handbook encompasses the following vehicles and "driving" conditions:

Vehicles

1. Single-unit truck - no articulation points

2. Tractor-semitrailer - one articulation point

3. Truck-full trailer - two articulation points

4. Tractor-semitrailer-semitrailer (B-train or C-train) - two articulation points

5. Tractor-semitrailer-full trailer (Double) - three articulation points

6. Tractor-semitrailer-full trailer-full trailer (Triple) - five articulation points

Driving Conditions

(a) Constant-acceleration cases

1. braking - constant braking force
2. steady turning - tracking
3. steady turning - roll
4. steady turning - handling

(b) Transient maneuvers

5. turning a corner at low speed
6. initiating a curved path
7. change of lateral position - obstacle avoidance
8. braking while turning
9. response to external disturbances

1.3 Organization of the Handbook

The methodology used to derive and present results is described in Section 2. The results obtained by applying this methodology are presented in Sections 3 through 8, which correspond to the six vehicle types listed in Section 1.2. Results for a single-unit truck are presented first in Section 3, followed by sections that treat vehicles with increasing numbers of articulation points. In this way, material presented earlier can be referenced when considering articulated vehicles. (That is, the reader interested in articulated vehicles may need to read both Sections 2 and 3 in order to obtain a detailed understanding of the methodology employed in the Handbook.)

Within Sections 3 through 8, results for the various driving conditions are presented in a sequence going from those maneuvers whose analysis requires the least (but most fundamental) information on the individual units of a vehicle to those maneuvers that need detailed parametric data describing the mechanical properties of the vehicle's components. This arrangement of results is intended to facilitate a vehicle synthesis process in which a person specifying or assembling a heavy truck or combination vehicle can proceed from basic (first-order) considerations to matters requiring more sophisticated analyses.

Section 9, entitled "A Procedure for Using the Handbook to Evaluate a Proposed Vehicle Configuration," provides a discussion related to the process of specifying vehicles that are intended to satisfy selected levels of braking and steering performance.

Finally, several appendices document the simplified models employed in this study. The vehicle dynamicist may find these models useful for analyzing vehicles having components with mechanical properties that are substantially different from those treated here.

2.0 THE APPROACH USED IN SUMMARIZING THE BRAKING AND STEERING PERFORMANCE OF HEAVY TRUCKS

2.1 Synopsis of the Methodology Employed

As illustrated in Figure 1, the methodology for developing performance evaluations progresses conceptually from the first objective of the project (that is, summarizing the effects of mechanical properties) through (a) assembling data sets describing vehicles and maneuvers to (b) performing analyses, and then, to (c) processing and presenting results for each member of a set of selected driving conditions.

The data sets describing vehicles and their components were based on those available from previous research investigations [3] and laboratory tests [4]. In this project, suitable parametric data have been assembled into a component factbook [2].

The driving conditions and control inputs were selected from a review of past studies of braking and steering performance [1]. A family of braking and steering "maneuvers" was chosen to provide a basis for quantifying the sensitivity of performance to changes in pertinent mechanical properties of heavy trucks.

Computer-aided analysis techniques and computer simulations were then used to predict the performance of benchmark vehicles. Special-purpose models for tracking [5], braking [3], rolling [6], and directional response [7,8] were adapted, simplified, and refined for the purposes of this handbook. The resulting simplified models represent quantifiable "rules of thumb." Existing computer simulations [9,10,11] were used to make detailed analyses of transient situations.

The selected set of maneuvers provided the foundation for the types of analyses performed. For each maneuver, an output or set of outputs was selected to be a "performance signature" that characterized vehicle performance in that maneuver. The performance signatures were processed (examined) to derive safety-relevant "performance measures" for each driving situation.

Results of analyses of benchmark vehicles, but with variations in pertinent mechanical properties, were used to construct performance sensitivity diagrams that

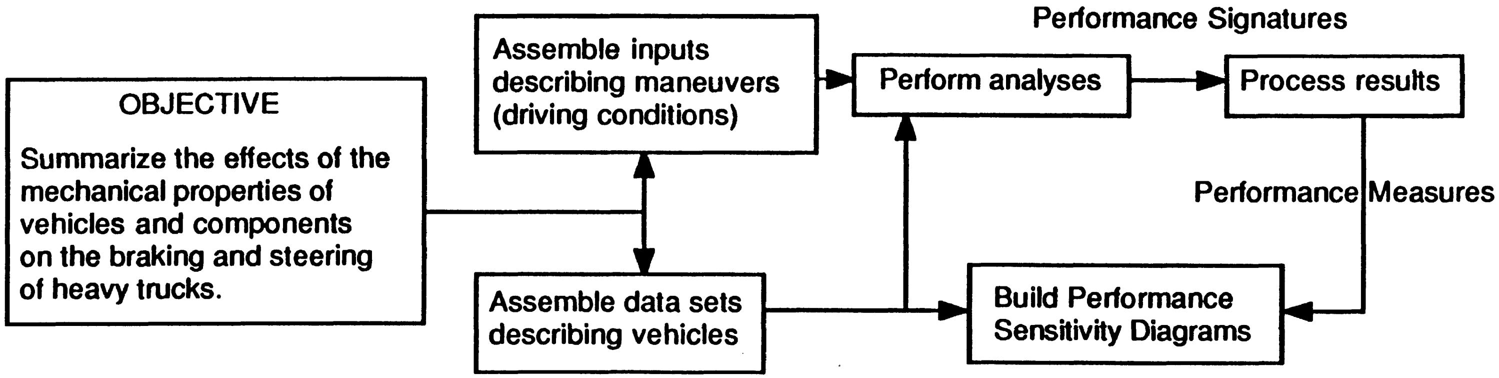


Figure 1. A methodology for summarizing the effects of mechanical properties on the braking and control of commercial vehicles

indicate the influence of feasible variations, corresponding to currently available mechanical properties, on the performance measures selected for each maneuver.

The remainder of this section provides more detailed explanations of the factors contributing to the methodology employed.

2.2 The Types of Mathematical Models and Simulations Used in Quantifying Braking and Steering Performance

The results presented herein are based on models developed during studies supported by the Motor Vehicle Manufacturers Association (MVMA), the Federal Highway Administration (FHWA), the State of Michigan, and the National Highway Traffic Safety Administration (NHTSA). Table 1 provides correspondences between open-loop vehicle maneuvers, computerized models, and published references. Simulated vehicle maneuvers have been used to make design evaluations similar to those that might have been obtained from typical vehicle tests [24,25,13].

2.2.1 Equilibrium Analyses. The equilibrium analyses used here are simplified calculation procedures that have been programmed in BASIC to operate on Apple Macintosh computers. (They have also been revised to operate on other personal computers.) These programs are operated interactively and they are "user friendly."

Three types of vehicle maneuvers are studied with the aid of these simplified analyses; namely, steady turning at constant velocity (and constant lateral acceleration), constant deceleration braking, and turning a tight corner at low speed. In the steady turning situation, three different analysis procedures are used to study tracking, rolling, and handling (steering). The procedures pertaining to braking and turning a tight corner address friction utilization (braking efficiency) and offtracking, respectively. Descriptions of the five available types of calculation procedures follow.

1. Low-Speed Offtracking. This type of procedure provides a simplified, quasi-static analysis of a vehicle turning a tight corner at low speed (see Figure 2). The first unit, the towing unit, is assumed to be steered such that the front axle follows a preselected path, typically a 90-degree or 180-degree segment of a circular arc with tangent sections preceding and following the curve.

There are two versions of this type of analysis. In one, which has been delivered to FHWA [28], the program can plot overlays of vehicle positions for use by highway

Table 1. Correspondence between Models, References and Maneuvers

Models	Background Reference *	Maneuvers	Model Number 1	2	3	4	5	6	7	8
Equilibrium Analyses		Steady turning								
1. Static roll	[10, 6]	a. Tracking	X						X	X
		b. Rolling		X					X	X
2. High-speed offtracking	[5, 20, 15]	c. Handling			X				X	X
3. Steady turning (Handling)	[7, 8, 15, 26]	Straight-line braking (Constant-deceleration braking)				X				X
4. Constant deceleration braking	[3]									
5. Low-speed offtracking (Tractrix)	[4, 27, 28]	Low-speed cornering (In-town cornering)					X		X	X
Simulations		Transient turning (Ramp-step steer)						X	X	X
6. Linear yaw plane analysis	[16, 9, 20]									
7. Yaw and roll (Constant velocity)	[3, 10]	Obstacle avoidance						X	X	X
8. Comprehensive braking and steering	[11, 29, 30]	Responding to external disturbances							X	X
		Braking in a turn								X

* Simplified versions of models 1 through 5 have been developed in this study

IN LOW-SPEED OFFTRACKING, EACH AXLE TRACKS INBOARD OF THE PRECEDING AXLE.

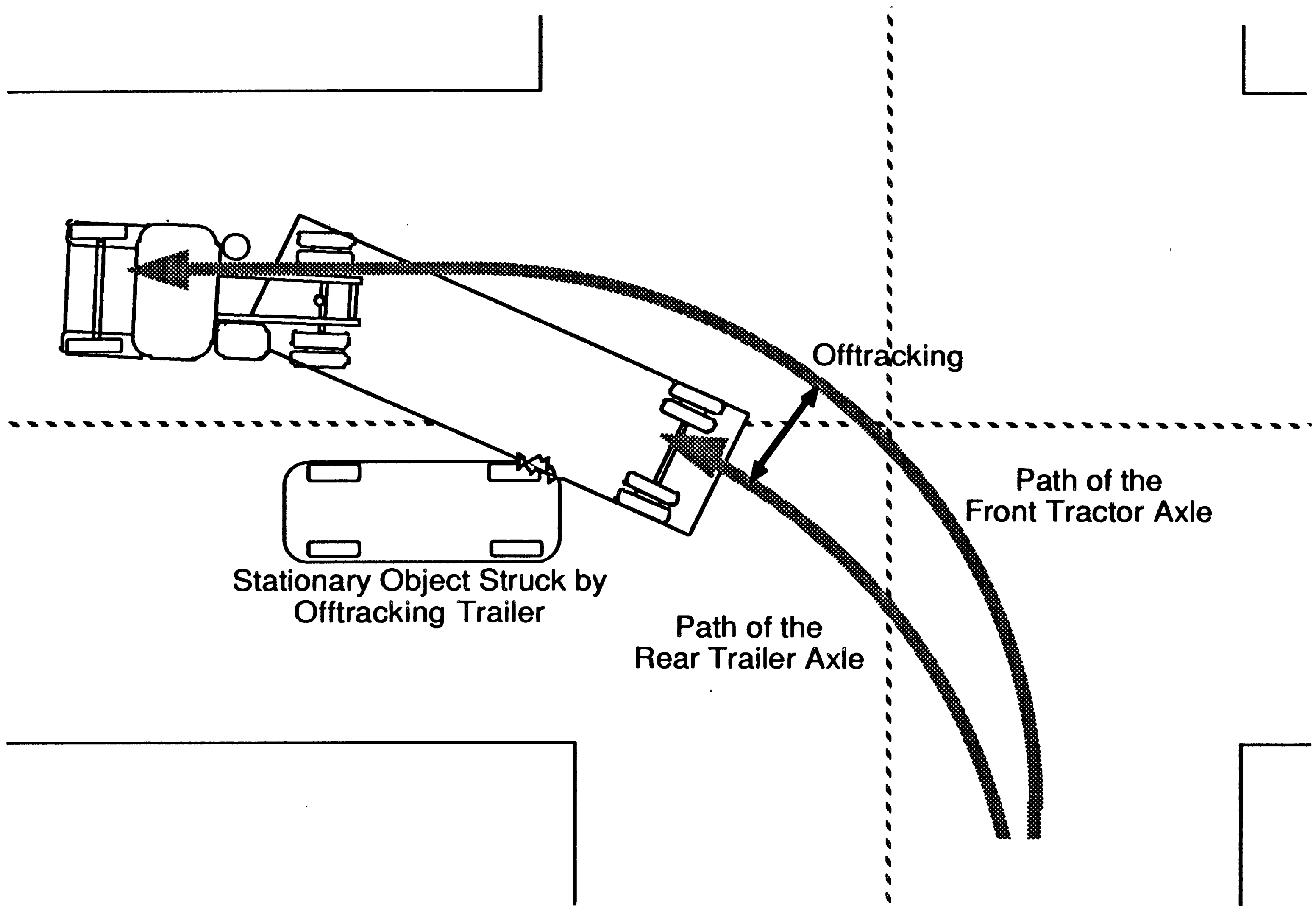


Figure 2. Low speed offtracking

engineers in evaluating intersections. In the other, which was used in developing this handbook, a simple algorithm is used to compute the offtracking of the various units of any vehicle that can be modeled as a train of semitrailers. Given wheelbases and hitch locations, the computer programs have been applied to tractor-semitrailers, doubles, triples, and truck/full-trailer combinations.

These programs are limited in scope in that they do not include scrubbing of wheels and the influences of low friction. Vehicle units with many axles and/or "belly" axles will require more complete models to obtain reasonably accurate predictions. One can use the yaw/roll simulation (to be discussed later) to handle those special situations. However, the representation of typical tandem-axle pairs by a single (centrally located) axle is a reasonable approximation for predicting the operation of conventional vehicles on good road surfaces.

2. *High-Speed Offtracking.* This analysis procedure applies to operation on highway curves at highway speeds. Besides offtracking dimensions, this procedure utilizes the cornering compliances applicable to each suspension location, that is, the ratio of vertical load on the tires to the sum of the cornering stiffnesses of all the tires mounted on that suspension's axles.

The result of this calculation is the offtracking at each axle as a function of lateral acceleration in steady turns at specified radii. At low speeds, the units of a combination vehicle will track towards the inside of the curve, but as speed increases the tires must operate at non-zero slip angles to generate the lateral forces required for equilibrium turning. In order to generate these slip angles, the trailing units of the vehicle tend to track towards the outside of the turn (see Figure 3). For typical vehicles, the transition from inside to outside offtracking often occurs at speeds less than 55 mph, such that at 55 mph the rear units tend to track outside of the path of the tractor.

The scope of this procedure is restricted by the assumptions made to obtain a simplified model. These restrictions include (1) working with large radius turns such that small angle assumptions apply (this is not a problem for analyzing the performance of heavy trucks on typical highway curves or interchange ramps) and (2) ensuring that the axles on a unit to be studied are closely spaced with a separation much less than the length of the unit. For vehicles with large axle spreads, the yaw/roll model may be used to calculate high-speed offtracking, however, much more information is required to use the yaw/roll model and a large computer is needed to perform the calculations.

WHEN CORNERING AT SPEED, TRAILERS MAY OFFTRACK OUTBOARD OF THE TRACTOR IF THE TIRE SLIP ANGLES ARE LARGE ENOUGH.

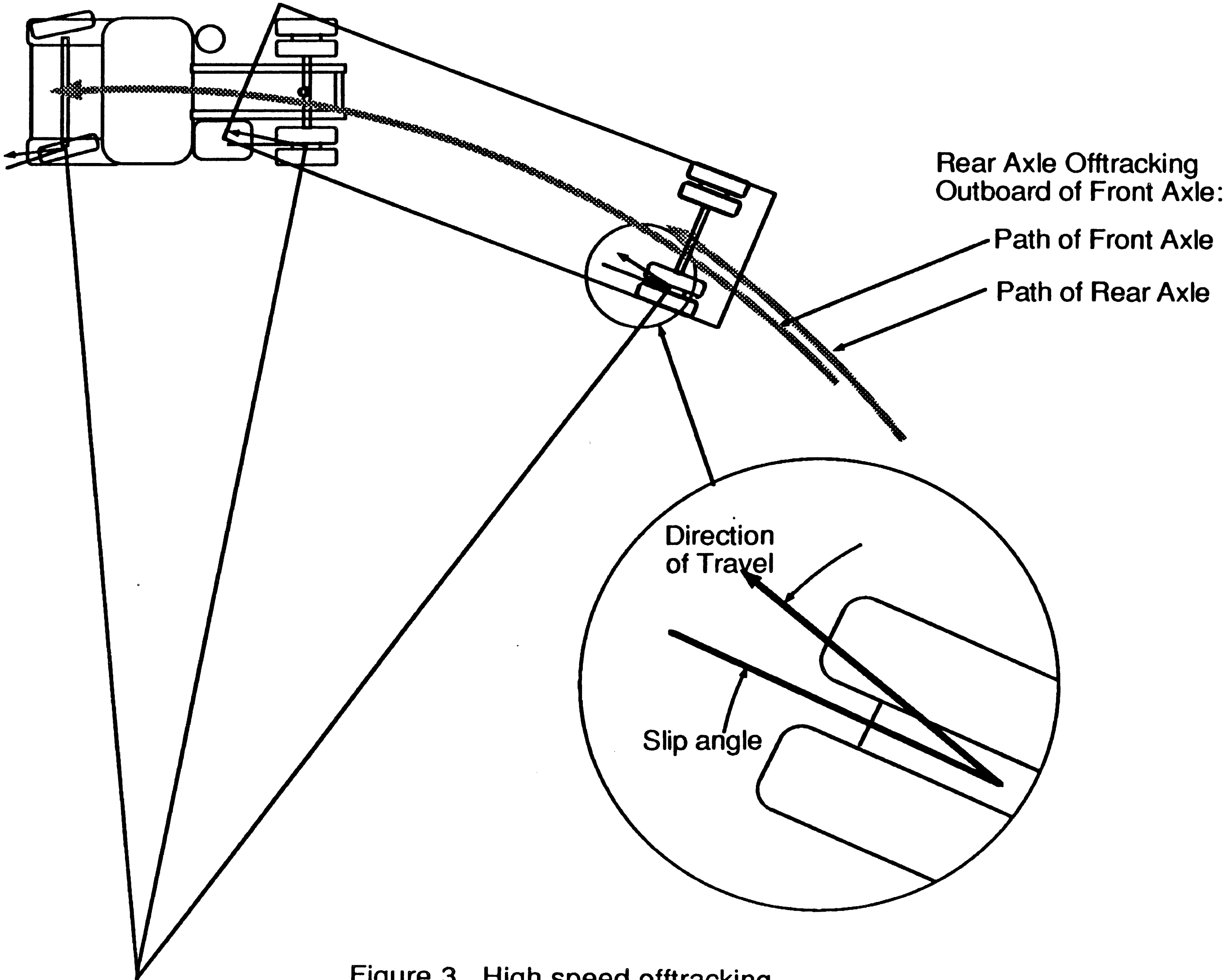


Figure 3. High speed offtracking

3. Constant Deceleration Braking. This procedure examines the proportioning of the braking system by calculating the friction level required at each axle to prevent wheel lock at that axle. The ratio of deceleration to the highest friction level, required at any axle, is the braking efficiency of the vehicle at that deceleration level.

This procedure uses the following information to predict friction utilization at each axle:

1) applied braking force as a function of treadle pressure

2) wheelbases

3) hitch locations

4) center of gravity heights for each unit of the combination vehicle

5) interaxle load transfer for tandem suspensions

Results may be obtained for a range of decelerations, for example, 0.1 to 0.5 g. These results are expressed in terms of (a) braking efficiency as a function of deceleration level and, also, (b) friction utilizations at each axle as a function of treadle pressure.

This is a very simple model of the braking process. It only considers first-order effects. Nevertheless, the results are very useful for illustrating braking arrangements and situations that will lead to low braking efficiencies, that is, poor deceleration performance compared to the frictional capability of the tire/road interface.

4. Static Roll. These calculations represent the rolling performance obtained during steady turning at various levels of lateral acceleration (see Figure 4). They represent analytical equivalents of tilt-table experiments.

There are two versions of this type of analysis. The comprehensive version includes representations of non-linear spring rates and spring lash, fifth wheel compliance and separation, frame compliance, and the influences of tire deflections. This version is written in FORTRAN and is too large to operate efficiently on our currently available personal computers. This comprehensive equilibrium model is often used for detailed studies of the influences of component characteristics on the rollover threshold because the results are much easier to interpret than those obtained from comprehensive simulations such as the yaw/roll or the comprehensive braking and steering models.

A HEAVY TRUCK IN A LEFT TURN.

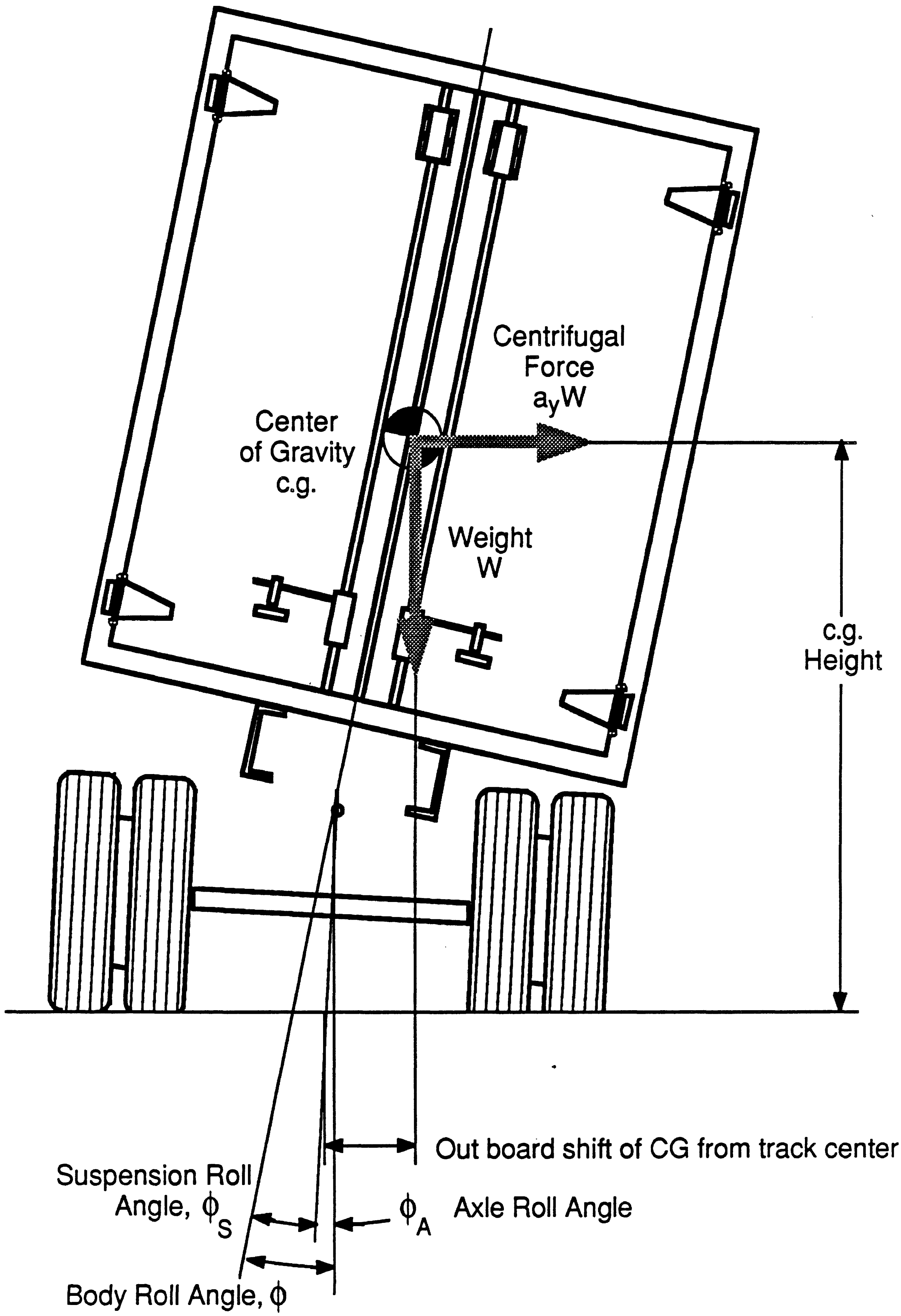


Figure 4. Static roll

The other version (the simplified static roll model) includes the primary factors influencing the ability of the vehicle to remain upright during severe turns, namely, it includes c.g. heights for sprung and unsprung masses, axle track widths, spring spreads, spring rates, suspension roll center heights, auxiliary roll stiffnesses for the suspensions, and tire vertical stiffnesses. The model provides the means for estimating the level of lateral acceleration at which rollover will occur. The calculations also compute axle and suspension roll angles and the conditions for wheel liftoffs.

The simplified static roll model is very useful for comparisons of major changes in vehicle components. Detailed effects, such as suspension lash, become important for vehicles with low rollover thresholds and, also, for improving the accuracy of predictions. Generally, each of the items omitted from the simplified model tends to lower the rollover threshold to the extent that simplified calculations may predict 5 to 10 percent higher rollover thresholds than those obtained from the more complete model.

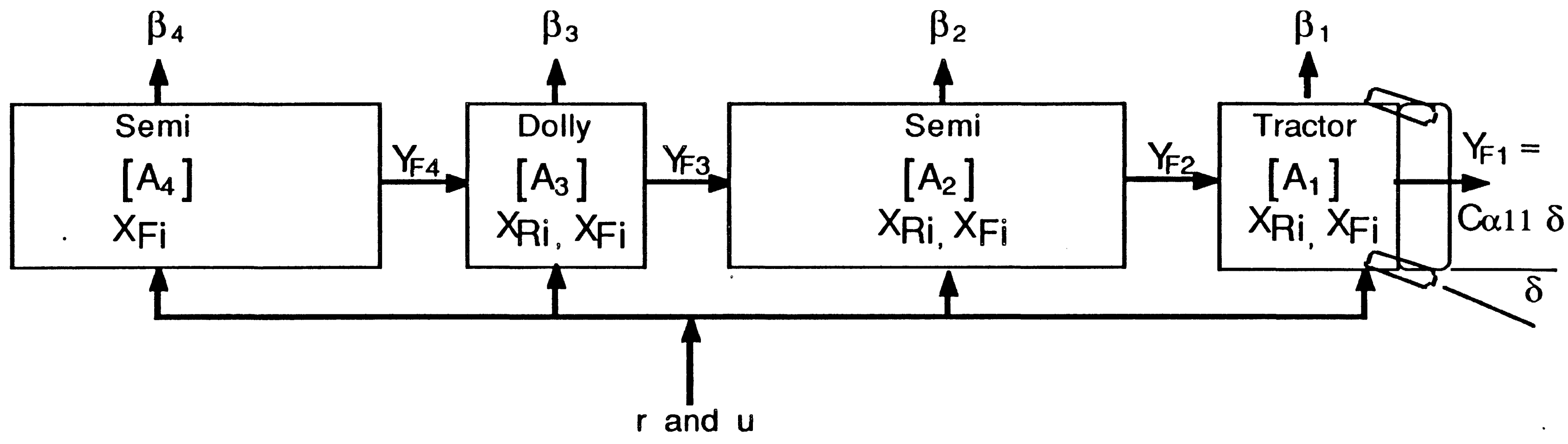
5. *Handling (Steady Turn).* "Handling" calculations are concerned with the steering angles required for a given type of steady turn (see Figure 5). In addition to the level of steering, these handling calculations indicate the possibility for the steering gain to become infinite, that is, the possibility for the vehicle to become statically unstable.

The previous tracking models have assumed that the vehicle is steered appropriately to perform the desired maneuvers. The tracking calculations indicate the extent to which the rear end of the vehicle follows the front end. In the handling context, the driver steers the towing unit and the rest of the combination vehicle is expected to follow the path of the towing unit. Handling is related to the ability of the driver to steer the lead unit of the vehicle.

For straight and articulated heavy trucks, the handling calculations are complex. The vehicle's response to steering may be linear only up to 0.15 g of lateral acceleration. Due to (a) nonlinearities in tire cornering stiffness as a function of vertical load and (b) the distribution of roll stiffnesses at the various suspensions, some heavy trucks may become directionally unstable at lateral acceleration levels above 0.15 g, but below their rollover thresholds.

The most important parametric quantities for this handling analysis pertain to the properties of the towing unit--its tire characteristics, suspensions, vertical axle loads, and geometric layout. The simplified static roll model is incorporated into this analysis in order to compute side-to-side load transfer at each axle. Hence, all of the parametric information

THE ELEMENTS INVOLVED IN CALCULATING STEERING ANGLE, δ



u: forward velocity

$[A_i]$: matrices of mechanical properties

X_{Fi}: distance between c.g. and forward hitch

X_{Ri}: distance between c.g. and rear hitch

Y_{Fi} : forces of constraint

β_i : unit side slip angle

$C\alpha 11$: cornering stiffness of the tires on the front axle

r : yaw rate

Figure 5. Handling analysis.

needed for the static roll model is required for the handling analysis. Information describing the layout, suspensions, and tire properties of the towed units of a combination are also used in this model.

The results from the handling analysis are presented in diagrams summarizing the influences of changes in velocity and lateral acceleration on the steer angles required for an equilibrium turn. In addition, if the vehicle can become unstable, a stability boundary is plotted in a graphical space defined by coordinates representing lateral acceleration and velocity.

This handling analysis applies to straight trucks, tractor-semitrailers, and tractor-semitrailer-semitrailer vehicles. It also applies to doubles and triples in which the pintle hitch connections used in these vehicles "analytically uncouple" their full trailers from the tractor-semitrailer units towing these full trailers.

2.3.2 Simulations. A hierarchy of simulation models is currently used to study the directional performance of heavy trucks. These models progress in complexity depending upon the number of modes of motion allowed in them. The least complex simulation (called the "yaw-plane" model) is a linear model that allows constant velocity movements of the vehicle in a horizontal plane; rolling effects and side-to-side load transfer are not treated in the model. The "yaw/roll" model, which is considerably more complex than the linear yaw-plane model, includes both yawing and rolling motions of the vehicle. It is again a constant velocity model, but nonlinearities in tires, suspensions, and the equations of motion are carefully represented. The comprehensive braking and steering model (often referred to as "Phase 4") is capable of simulating all types of vehicle maneuvers. It adds braking and longitudinal deceleration to the factors considered in the yaw/roll model.

The simulation programs are all written in FORTRAN and they are structured to be operated on a large computer system. Brief summaries of these models follow.

1. Linear Yaw-Plane Model. This model offers a first look at the response of combination vehicles to rapid changes in steering. Since the model is linear, computational procedures have been developed for both the time and frequency domains. In the time domain, a quick obstacle-avoidance maneuver is used to assess the amount of rearward amplification (that is, "whipping") that occurs for a particular combination vehicle (see Figure 6). Frequency-domain calculations are also used to study rearward amplification, which is technically defined as the ratio of the lateral acceleration of the last unit to the

In a rapid lane change maneuver, rearward amplification results in "crack-the-whip" action of the trailer, sometimes resulting in rear tailer rollover.

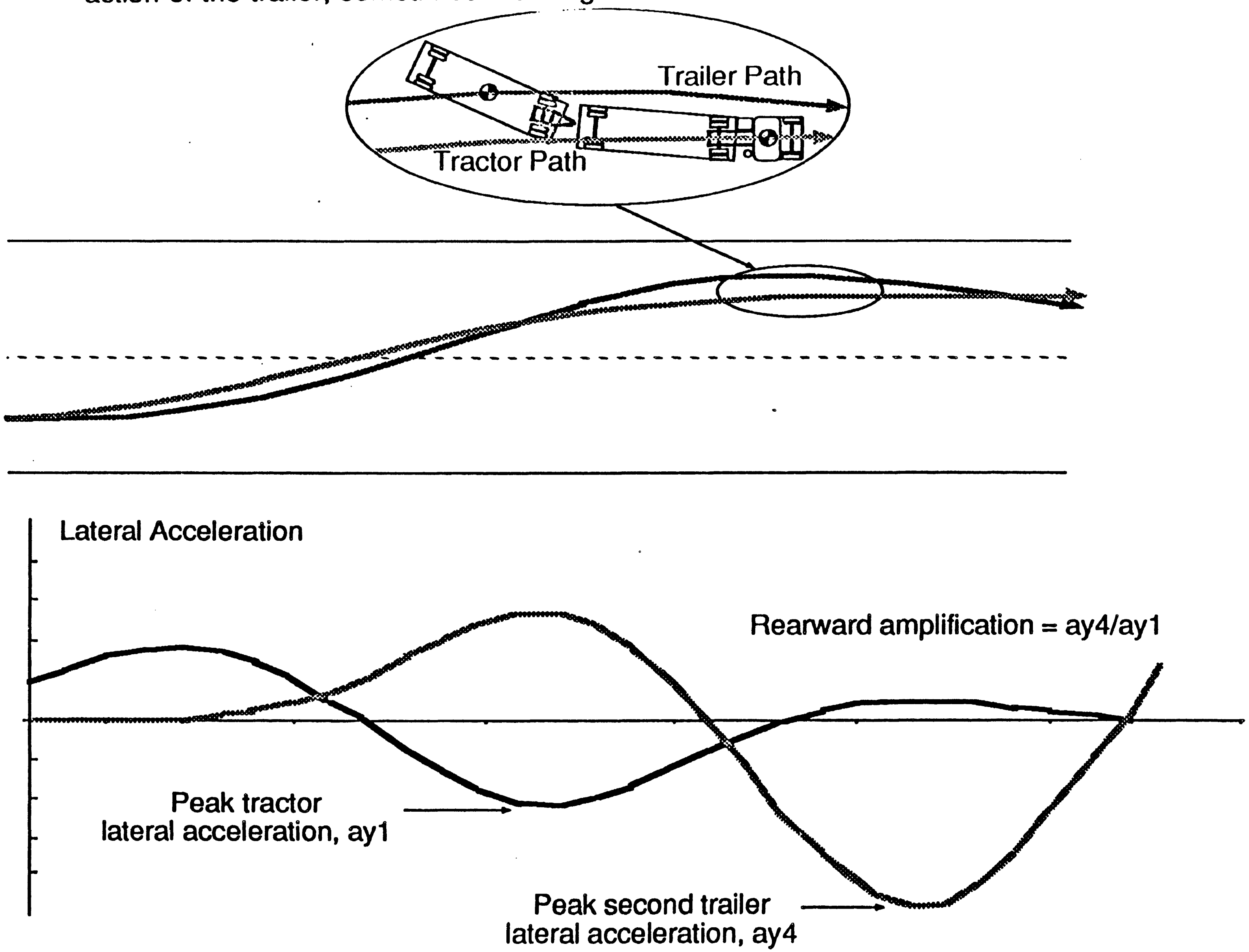


Figure 6. Rearward amplification

lateral acceleration of the first unit of a combination vehicle. In this context, the lateral acceleration of the first unit may be viewed as the independent input variable used in evaluating the extent to which the motion of the last unit exceeds that of the first unit.

This type of analysis has been applied primarily to the study of rearward amplification in doubles combinations. Since the maximum rearward amplification is a function of the period of the steering input, or equivalently, the frequency content of the input, calculations are made for a range of periods or frequencies, for example, for frequencies ranging from 0.1 to 0.5 Hz. Experiments have shown that drivers can easily apply steering inputs at 0.5 Hz (2-second period), however, analyses indicate that drivers only need to apply these rapid inputs in nearly emergency situations. In this sense, rearward amplification is an obstacle-evasion problem.

If the last trailer has a heavy load with a high center of gravity, the ultimate outcome of an avoidance maneuver, involving high levels of rearward amplification, is likely to be the rollover of the last trailer. The yaw/roll model is used to study the rollover phenomena involved in rearward amplification situations.

A simplified version of the frequency-domain analysis derived from the yaw-plane model has been developed. This analysis procedure is implemented on personal computers. The fact that the lateral forces at pintle hitches are practically negligible allows us to "analytically decouple" the influences of full trailers upon the units towing them, thereby allowing individual analyses of the contribution of each vehicle unit to the overall rearward amplification.

2. *Yaw/Roll Model.* This model can simulate the directional and roll responses of straight and articulated vehicles during steering maneuvers up to those that approach the rollover threshold of the vehicle. Typical steering maneuvers that have been simulated range from low-speed turning of a tight corner to sudden obstacle-avoidance maneuvers at high speed. A maneuver of current interest is a gradually tightening turn from which one can determine high-speed offtracking, the rollover threshold, and handling characteristics--all from one computer run.

This model requires a complete description of all of the vehicle's components except the brakes. It contains provisions for including multiple-axle suspensions and special hitching arrangements between units (features not available in the comprehensive braking and steering model). The model treats combination vehicles as a "train" of semitrailers. For example, it can be used to simulate a B-train without any special modifications to the computer code.

The results of the simulation are time histories of response variables such as yaw rates, lateral accelerations, roll angles, wheel loads, etc. To interpret these results, one desires performance measures that indicate the quality of selected responses--for example, the level of lateral acceleration at which rollover occurs, the amount of offtracking of the rear unit, or the maximum amount of rearward amplification. Clearly, the processing of simulated time histories is comparable to the processing of data obtained in vehicle tests.

3. Comprehensive Braking and Steering Model (Phase 4). This is the largest and most complex model. It simulates the braking and steering performance of straight trucks, tractor-semitrailers, doubles, and triples combinations.

Since the yaw/roll model provides a convenient means for simulating constant velocity maneuvers, the Phase 4 model is currently applied to situations in which braking is involved, for example, braking-in-a-turn maneuvers, antilock braking on slippery surfaces, or examining jackknifing rates when tractor rear wheels are locked.

Besides the input information needed for predicting directional response, the following braking-related items can be represented in the simulation: (1) interaxle load transfer in tandem suspensions, (2) brake proportioning, timing, and hysteresis, (3) antilock control logic, and (4) thermal properties of brake drums and linings.

As with the yaw/roll model, the direct results of the simulation are time histories of pertinent response variables. The basic outputs of the simulation are comprehensive tables of response variables. These outputs need to be processed into graphical form or into derived performance measures to provide convenient means for evaluating the results.

The computerized model contains many degrees of freedom including (a) rotational degrees of freedom for up to 26 wheels, (b) vertical and roll degrees of freedom for up to 13 axles, (c) 6 degrees of freedom of the tractor/truck sprung mass, and (d) degrees of freedom for the semitrailer and each full trailer.

2.3 General Goals for Braking and Steering Performance

Now consider the application of the vehicle models, just described, to the evaluation of the braking and steering performances of heavy trucks.

Stated in practical, everyday terms, the goals pertaining to the braking and steering performances of heavy trucks are that:

1. the rear end of the vehicle should follow (track) the motion of the front end with adequate fidelity;

2. the vehicle should attain a desirable level of deceleration during braking (without losing directional control or stability);

3. the vehicle should remain upright on its tires (not rollover) during severe maneuvers; and

4. the vehicle should be controllable and stable enough to follow a desired path in response to steering.

The vehicle should be capable of performing acceptably with respect to these goals over appropriate ranges of loading, roadway "friction," speed, tire wear, brake work-history, and other operational factors.

2.4 Descriptions of Maneuvering Conditions, Performance Signatures, and Performance Measures

2.4.1 Discussion of the Selected Maneuvers. The maneuvering and operating conditions given in Table 2, have been selected for use in predicting how well vehicles will perform relative to the four general goals stated in Section 2.3.

Three of these maneuvers (turning a corner at low speed; steady turn, tracking; and obstacle avoidance) challenge the ability of the trailing units to follow the motion of the towing unit.

Rollover immunity in a steady turn has been chosen as the primary test of roll stability for all heavy trucks. However, during obstacle-avoidance maneuvers the rearmost trailers in multi-articulated combinations are susceptible to rolling over if the motion of the towing unit is greatly amplified at the rear of the vehicle. Vehicles that are not overly susceptible to rolling over in either the steady-turn or the obstacle-avoidance maneuver are not as likely to have rollover accidents as those that do poorly in these situations.

Table 2. Performance Signatures and Measures for various maneuvers

Maneuvers	Performance Signatures (or Operating Condition)	Performance Measure
1. Low Speed Cornering (In-Town Cornering)	(41 ft radius, 90° corner) Trajectory of the rear axle-Tractrix	Maximum Offtracking
2. Constant-deceleration stopping	Friction utilization and deceleration versus pressure	Braking efficiency at 0.2 and 0.4 g's
3. Steady Turning		
a. Tracking	(1200 ft radius at 55 mph) Trajectory of the rear of the vehicle	Offtracking
b. Rolling	Lateral acceleration versus roll angle	Rollover threshold
c. Handling	Handling curve and critcal speed versus lateral acceleration	1. Steering gain at 55 mph and 0.3 g's 2. Critical speed at 0.3 g's
4. Transient turning (Ramp-step steer or lange change)	(Steering wheel angle 200°/sec to 28°) Lateral acceleration time history	Lateral acceleration response times (50% steering to 90% of steady state) or average time lag between steering input and lateral acceleration output
5. Obstacle avoidance (Rearward amplification)	Transfer function: lateral acceleration of last unit to that of first unit	Maximum rearward amplification (steering frequency < 0.5 Hz)
6. Braking in a turn	(0.8-second braking pulse while following a 1200 ft turn at 55 mph) Yaw rate and sideslip angle time histories	Open loop: maximum changes in yaw rate and sideslip. Closed loop: deviation from a reference yaw rate.
7. Responding to External Disturbances	Transfer function: steering control to equivalent disturbance input	Maximum closed-loop steering gain

Heavy-vehicle handling and stability are evaluated by performance in steady turning, initiating curved paths, braking while turning, and in response to external disturbances. The gain of the response to steering in a steady turn is an indication of the stability margin that the driver has in negotiating a highway curve. The response time of the vehicle in initiating a turn is important in determining the performance of the driver-vehicle system. The quality of driver control depends upon the response time of the vehicle. External disturbances, for example, wind gusts, road bumps, etc. may excite vehicle motions that are difficult for the driver to damp out. Braking-while-turning situations can be very difficult to control if wheels become locked or approach lockup. All of these maneuvers provide information as to vehicle controllability.

Braking performance can be evaluated from a constant-deceleration analysis. The results from this type of analysis cover the entire range of tire/road friction and should be performed for the vehicle in both its fully laden and empty condition. The important matter here is to stop quickly without locking any wheels. If wheels lock, the vehicle will become either unsteerable if the front wheels lock or directionally unstable if the wheels on other axles lock. Braking performance in terms of stopping distance is much the same in braking in a turn as it is in straightline braking. However, the control challenge is much greater in braking while turning. The braking-while-turning maneuver is the ultimate challenge of vehicle design characteristics with regard to the driver's ability to maintain directional control.

There may be reason to argue that this set of maneuvers could well be expanded to include other tests of vehicle performance. On the other hand, only low-speed offtracking and straightline braking have received attention in vehicle standards. Operators know that a vehicle must not cut corners by such a large amount that the vehicle is not usable in town or in loading areas. Highway engineers design roads with offtracking in mind. Vehicle safety standards address braking performance, but, even in this seemingly straightforward context, universal agreement and acceptance of braking requirements has not been achieved in the United States. The set of maneuvers and operating conditions selected for this handbook are intended to provide the basis for developing vehicles with good overall braking and steering performance. If heavy trucks do well in the selected maneuvers, it is expected that they will be safer if they are driven prudently.

Some of the maneuvers are open loop such that a "test-driver" would perform predetermined control actions which are independent of the instantaneous position or path of the vehicle. Constant-acceleration maneuvers are inherently of this type. Transient maneuvers (in that they involve changing from one path or operating state to another) may have either open- or closed-loop versions. In this case, the ramp-step and low-speed cornering maneuvers are treated in an open-loop fashion; obstacle avoidance and braking in a turn are investigated using both open- and closed-loop analyses; and the response to external disturbances is studied through closed-loop calculations.

Open-loop results serve to define the accident-avoidance capabilities of the vehicles. In this analysis, a "driver" representation [12] is used to steer the vehicles to attempt to follow preselected paths in simulated closed-loop situations. These closed-loop analyses aid in understanding the influences of open-loop vehicle properties on the predicted performance of the driver/vehicle system.

2.4.2 Performance Signatures and Measures. For the maneuvers employed here, a performance "signature" is obtained for each type of vehicle. Then, performance "measures" are evaluated at safety-relevant levels of the performance signatures of the various vehicles. For example, in a steady-turning maneuver, the roll angles of the vehicle's units increase as the lateral acceleration of the turn increases. At the limit of performance, one of the vehicle's units rolls over at a level of lateral acceleration called the "rollover threshold." In this case, the roll angle versus lateral acceleration graph is the performance signature and the rollover threshold is the safety-relevant performance measure. The following subsections present descriptions of the performance signatures and measures used in examining vehicle capabilities in the selected maneuvers.

Low-Speed Cornering. The term "tractrix" pertains to the path of the axles of a semitrailer while it is turning a corner at low speed. To evaluate transient offtracking at low speed, articulated heavy vehicles are treated as a train of semitrailers in which the path of each hitch is the general curve followed by the attached semitrailer.

For example, the trajectories (tractrices) of the various axle sets of a truck-full trailer are shown in Figure 7. The center of the steering axle is assumed to follow a 90-degree turn with a radius of 41 ft. The rear axles of the truck are treated as a single axle. The dolly axle does not offtrack far from the path of the truck's rear axle (see Figure 7). The rear axle of the trailer has a maximum inboard offtracking of 9.6 ft which occurs when the rear axle has turned through 59 degrees.

The maximum offtracking of the rear axle of the last unit has been used to quantify low-speed turning performance of the prototypical vehicles (see Table 3, row 1).

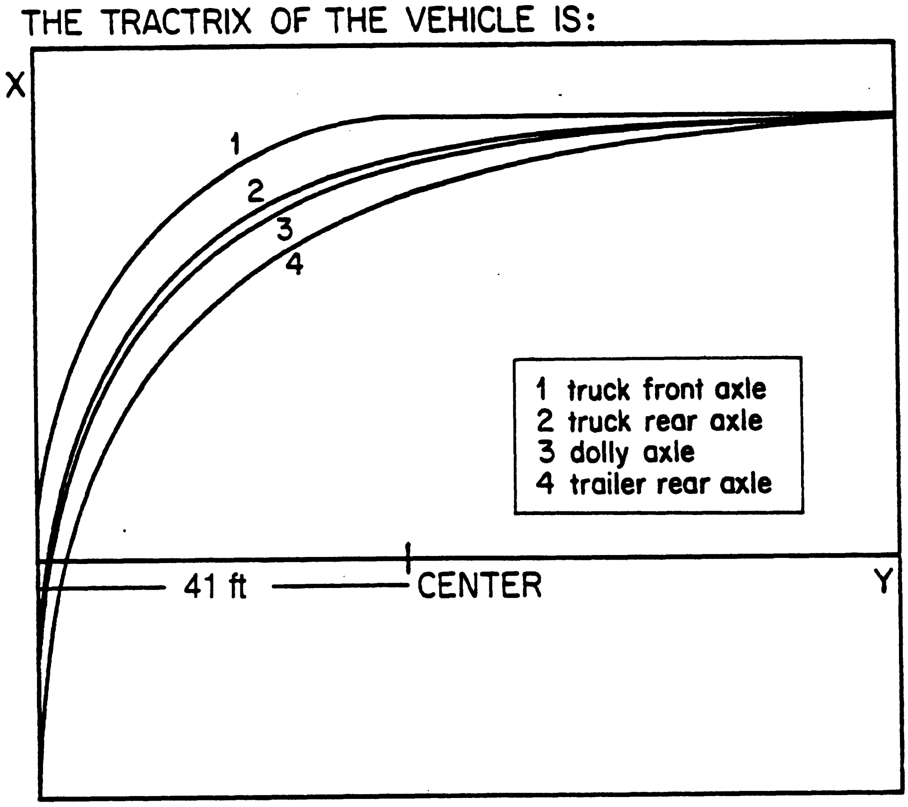


Figure 7. Tractrices for a truck-full trailer

Table 3. Summary of Performance Measures for Benchmark Vehicles

Performance Measure	Straight Truck (3)	Tractor and Semitrailer (3-S2)	Truck and Full Trailer (3-2)	B-Train (2-S2-S1)	Double (2-S1-2)	Triple (2-S1-2-2)
1. Maximum transient (low-speed) offtracking (ft) - 41 ft and 90°	4.94	14.36	9.71	-	11.56	15.90
2. Braking efficiency at 0.4 g's						
- Loaded	0.826	0.890	0.780	-	0.840	0.830
- Empty	0.514	0.590	0.550	-	0.590	0.660
3. High-speed offtracking (ft) - 1200 ft at 55 mph						
- At last axle	0.370	0.650	1.270	-	1.210	1.830
- At end of last unit	0.510	0.732	1.360	-	1.290	1.920
4. Rollover threshold (g's)	0.368	0.368	0.399	0.395	0.394	0.394
5.a. Critical speed at 0.3 g's (mph)	221.7	Nonexistent	Nonexistent	95.3	97.5	97.5
5.b. Steering sensitivity at 0.3 g's and 55 mph (radians/g)	0.095	0.097	0.146	0.029	0.034	0.034
6. Lateral acceleration response times - ramp step (sec)	0.744	0.800	0.790	-	0.980	0.980
7. Maximum rearward amplification		1.080	2.030	-	2.070	2.960

Constant-Deceleration Braking. The performance signature selected for constant deceleration braking is a family of curves (one for each axle) showing the friction, required to avoid wheel lock (that is, the "friction utilization"), displayed as a function of treadle pressure (see Figure 8). Also superimposed on this plot is a graph of deceleration in g's versus treadle pressure. *Braking efficiency is the ratio of deceleration divided by the highest required friction coefficient (i.e., the results for axle 4 in Figure 8).* The braking efficiency at 0.4 g has been used to provide the performance measures listed in row 2 of Table 3. These results indicate that empty heavy vehicles have braking efficiencies that are less than 0.6.

In the example shown in Figure 8, the brake torque acting on the tandem suspension of the tractor does not cause interaxle load transfer. Hence, the curves labeled 2 and 3 coincide in Figure 8. Curves 4 and 5 do not coincide because, in this example, a significant amount of load is transferred from axle 4 to axle 5 during braking. The effect of this interaxle load transfer is to reduce the braking efficiency at 0.4 g from 0.59 for a vehicle without interaxle load transfer to 0.43 for a vehicle with a semitrailer whose tandem suspension has a large amount of interaxle load transfer.

Tracking. Ideally, the trailing units in an articulated vehicle would be expected to follow exactly the path of the front axle of the towing unit. Practical vehicles come close to achieving this type of performance at *highway speeds* on highway curves. However, small deviations from the path of the front axle have caused trailer wheels to strike curbs and other roadside obstacles and thereby precipitated rollovers or control difficulties. Recognition of this danger has prompted the recommendation of a tracking test [13].

In this maneuver, a complete performance signature has not been sought. Rather, a specific turn has been selected as being representative of a high-speed exit ramp. The simulated maneuver is performed at a velocity of 55 mph on a flat turn with a radius of 1200 ft. *The outboard offtracking attained by the rear axle of the last trailer has been selected as the performance measure.*

Figure 9 is a performance sensitivity diagram showing the influences of (a) the cornering stiffnesses of the tires and (b) the wheelbase of the third trailer on the high-speed offtracking of a triples combination employing 27-ft trailers. The abscissa values correspond to deviations from the baseline values of wheelbase and cornering stiffness. These deviations cover a representative range of values.

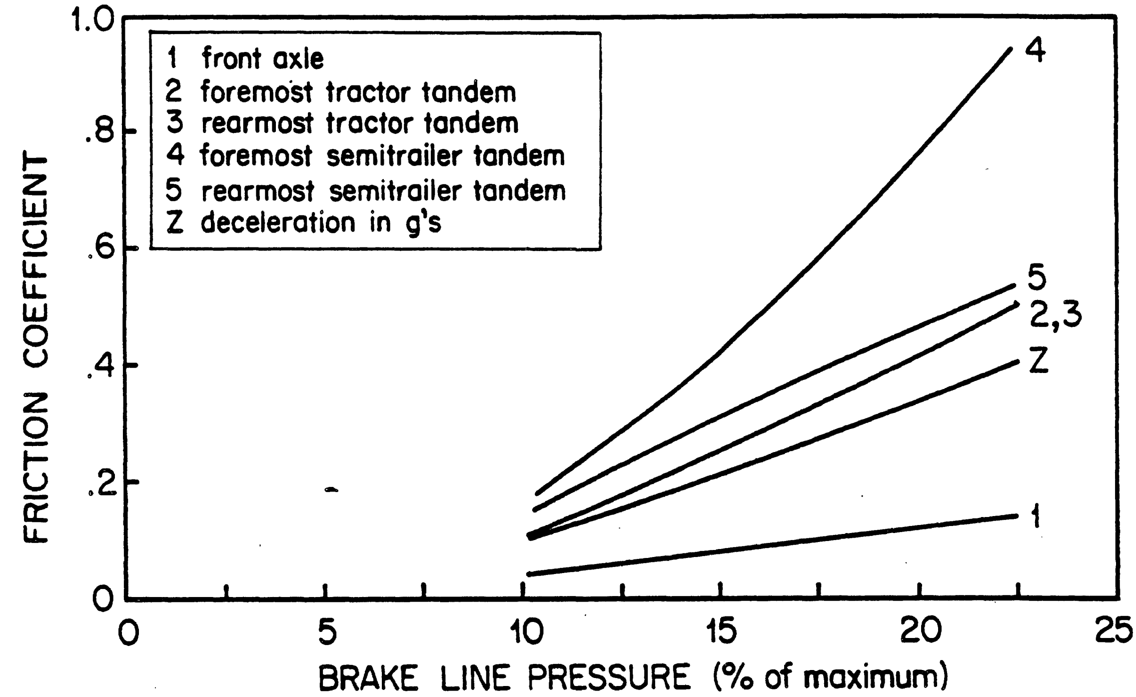


Figure 8. Friction utilization and deceleration, empty tractor-semitrailer (interaxle load transfer for the semitrailer's tandem is equal to 20% of the braking torque divided by the tandem spread.)

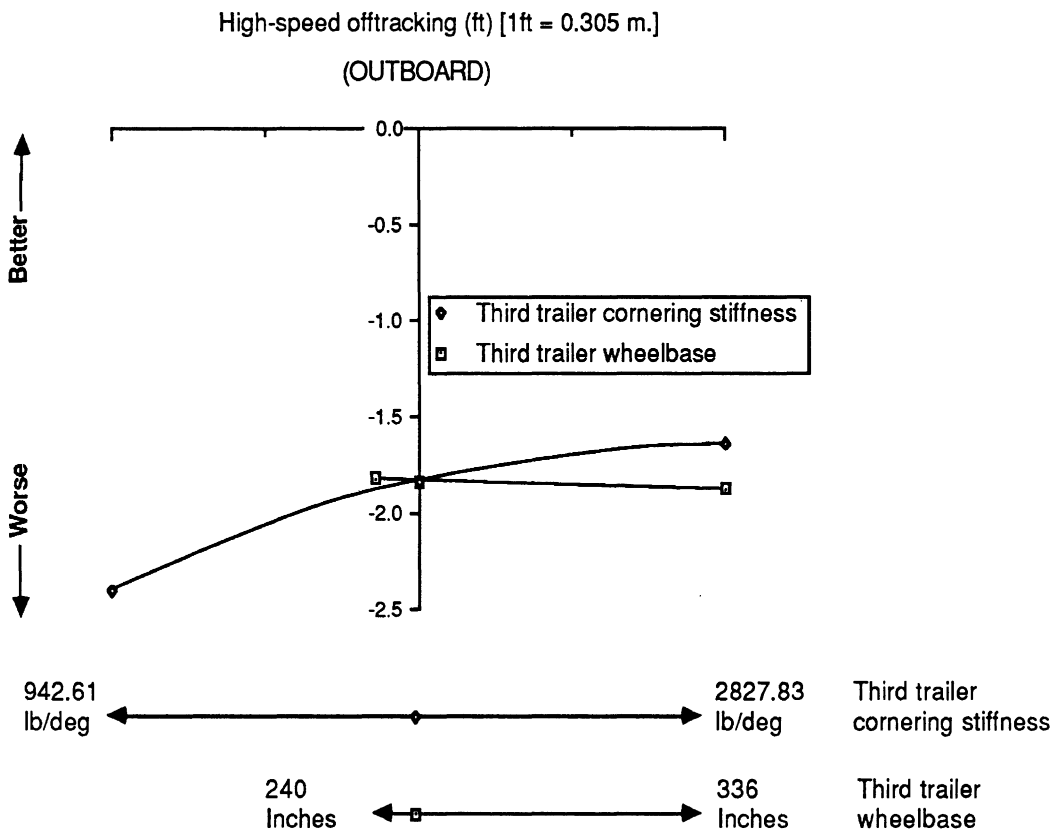


Figure 9. High-speed offtracking, performance sensitivity diagram.

For example, the large value represents a cornering stiffness typical of radial tires, the middle (baseline) value corresponds to bias-ply tires, and the small value represents lug-type tires. Although lug tires are not usually used on trailers, the performance sensitivity diagram shows the adverse effect of low cornering stiffness. (Specifically, the tire data involved here are based on measurements of a Michelin XZA 10x20G radial tire with rib tread, a Goodyear Super Hi Miler 10x20 bias ply tire with rib tread, and a Uniroyal Fleetmaster 10x20 bias ply tire with lug tread.)

The triple tracks outboard of the double by an amount that is equal to the contribution of a full trailer, since the double and triple are made up of identical units. The triple has the largest offtracking amongst the benchmark vehicles (see the third row of Table 3).

Roll. Equilibrium values of roll angle are a function of lateral acceleration, as illustrated in Figure 10. This performance signature has a discontinuity in slope at the point (0.36 g) where the semitrailer's inside wheels lift off the ground. The maximum value of lateral acceleration, the rollover threshold, occurs at 0.39 g when the tractor's inside rear wheels lift. Above 0.08 radians of roll angle, the slope of the curve (lateral acceleration versus roll angle) becomes negative, indicating points of unstable equilibrium.

The roll performance signatures of vehicles with full trailers include additional acceleration versus roll angle curves for each full trailer. Non-linear spring characteristics and free play when leaf springs go into tension cause more complicated-looking characteristics than those shown in Figure 10. *Nevertheless, the rollover threshold is readily identified as the maximum attainable level of lateral acceleration before rollover of any unit of the vehicle.*

The fourth row of Table 3 provides first-order estimates of the rollover thresholds of fully laden versions of various types of articulated commercial vehicles. In addition to the ratio of c.g. height to track width, these estimates are influenced by those conditions which allow the c.g. of the sprung mass to translate laterally; specifically, low suspension roll rates, free play, and low roll center heights [7]. Accident data have been used to show that small changes in rollover threshold can have a large influence on the number of rollover accidents for heavy vehicles having values of rollover thresholds in the vicinity of those given in Table 3 [3].

Handling. In this context (i.e., steady turning), handling refers to the response of the towing unit to steering inputs. In the initial phase of this project, handling diagrams

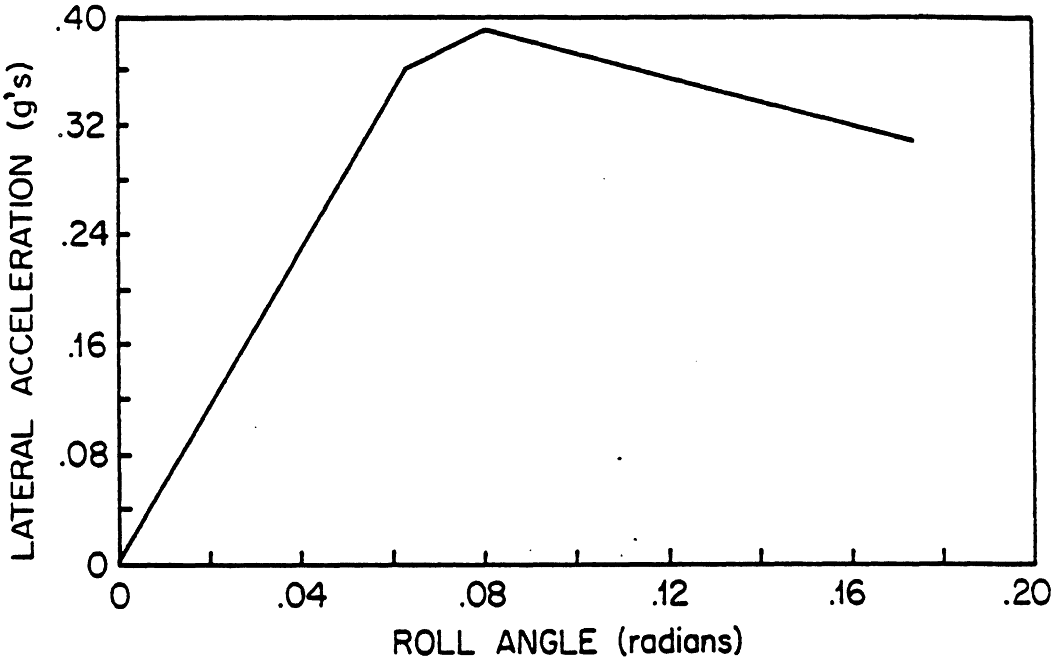


Figure10. Roll performance signature

[14] (see Figure 11a) were constructed at 50 mph to obtain performance signatures. The handling diagram shown in Figure 6a contains a handling curve which displays steady-turning properties as a function of lateral acceleration, yaw rate,r, forward velocity,U, a reference wheelbase, L, and a reference front-wheel angle, Delta (in this case, steering-wheel angle divided by steering-gear ratio).

Vehicles with tandem axles on the tractor or the first semitrailer do not have a unique handling curve that is applicable at all speeds [15]. However, vehicles without tandem axles do have unique handling diagrams analogous to those used in the study of passenger cars. Figure 11a contains handling curves for a truck-full trailer and a doubles combination. The handling curve for the double is applicable to all speeds since this double has single-axle suspensions. The truck in the truck-full trailer combination has tandem axles at the rear of the truck. For the truck-full trailer, the handling curve only applies at 50 mph because, due to the tandem axles, the effective wheelbase changes with velocity [15]. (The wheelbase used in Figure 11a is the distance from the front axle to the center of the rear suspension of the towing unit (i.e., the truck).)

Although handling diagrams have been used in previous experimental studies of truck dynamics [8], they have been judged to be more complicated than desired for the purposes of this Handbook. Conceptually simpler graphs, showing steering wheel angle as a function of lateral acceleration at a velocity of 55 mph, have been selected as the performance signature for handling. (See Figure 41a for an example of a typical performance signature.) *The rate of change of steering angle with respect to lateral acceleration (that is, the "steering sensitivity") evaluated at 0.3g's of acceleration and at 55 mph has been selected as a performance measure indicating the margin of directional stability.* (Figure 41b contains a graph illustrating how steering sensitivity changes with speed and lateral acceleration.) If the steering sensitivity is zero or less, the vehicle is statically unstable with an exponentially divergent directional response. A steering sensitivity equal to zero corresponds to the situation in which the vehicle becomes divergently unstable and the driver will need to continuously adjust steering to maintain control of the system.

For vehicles that exhibit divergent instability at lateral acceleration levels below their rollover thresholds, critical speeds (the speeds at which instability commences) have been computed. For these vehicles, a stability boundary can be plotted in a space defined by critical speed and lateral acceleration (see Figure 11b). This stability boundary is a special

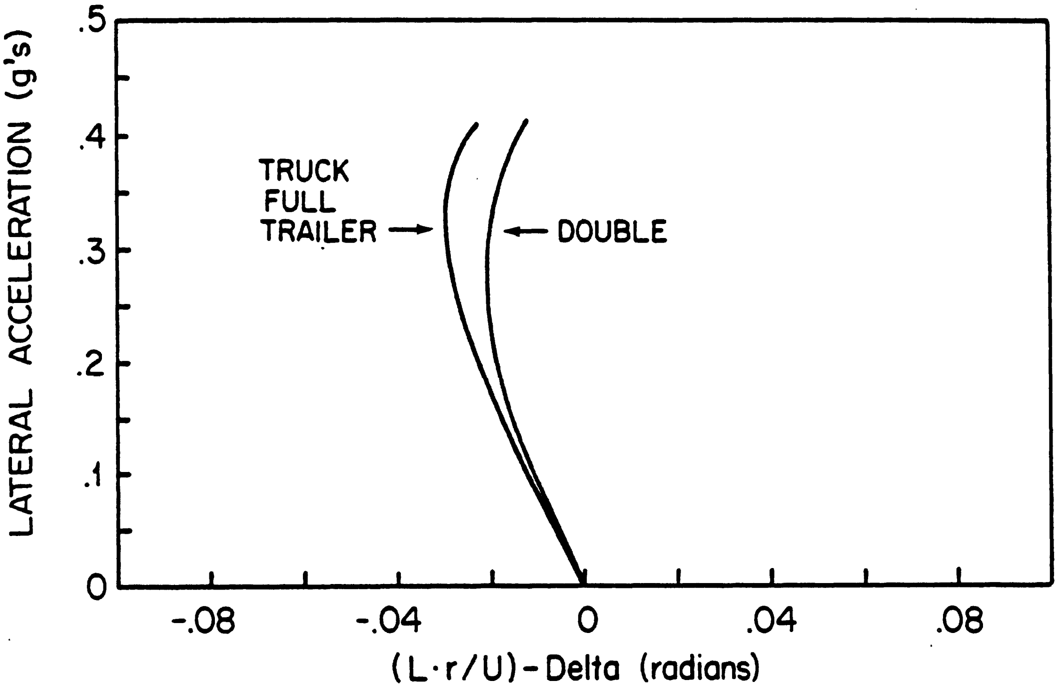


Figure11a.Handling diagram

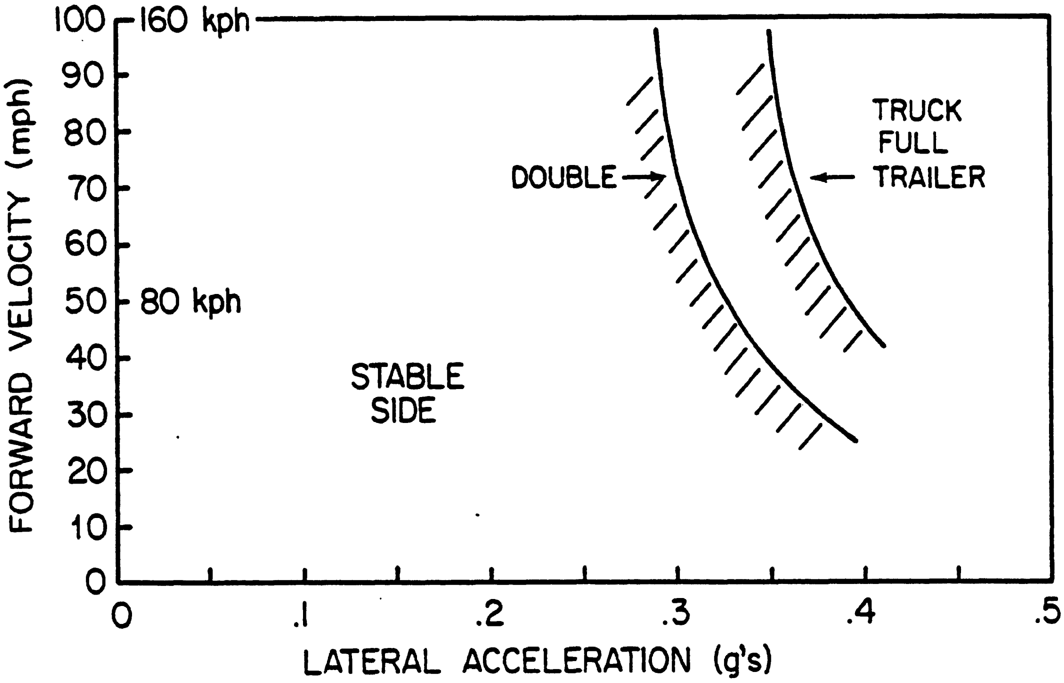


Figure11b.Critical speed versus acceleration

type of performance signature that provides an indication of the conditions under which the driver would have to control an unstable vehicle.

For the baseline (benchmark) vehicles, the results indicate that all of them remain stable at speeds up to 55 mph and lateral acceleration levels below 0.3 g. Row 5a of Table 3 lists the critical speeds obtained at 0.3 g of lateral acceleration. Row 5b lists steering sensitivity levels in a severe turn at 55 mph and 0.3 g.

For vehicles with full trailers, the handling results are similar to those that would have been obtained if the full trailers had been removed. That is, conventional dollies nearly "decouple" the full trailer from the unit towing it, because the lateral force at the pintle hitch is very small compared to the tire forces acting on the towing unit [14]. For example, the handling results for the double and triple are practically identical to those obtained if only the tractor-semitrailer portion of these vehicles were to be analyzed. (The handling results for the double and triple are identical to each other since these vehicles employ the same tractor-semitrailer as their towing unit.)

At turning levels above 0.15 g, lateral load transfer has an important influence on truck tire characteristics. The curvature of tire cornering stiffness with respect to vertical load becomes especially important for the drive axles of the lead unit since a large portion of the lateral load transfer takes place at these axles on typically suspended heavy vehicles in the U.S. Given a bias in roll stiffness distribution, the level of oversteer at high g levels depends to a large extent upon the curvature of the tire characteristics. The baseline results presented here are for vehicles with typical bias-ply truck tires, having a moderate amount of curvature in their characteristics. However, sudden transitions to large oversteer may take place for vehicles with tires that have considerable curvature in the relationship between cornering stiffness and vertical load.

Ramp Step Steer. This maneuver is used to establish the quickness of the lateral acceleration response of the first unit in a combination vehicle. Response times are measured between the time when a rapid steering input reaches 50% of its final value and the time when the lateral acceleration response reaches 90% of its steady-state value. The magnitude of this response time depends upon vehicle loading, speed, and the amplitude of the steering input. The results given in Table 3, row 6 are for low-amplitude steering-wheel inputs (28 degrees) applied to fully laden vehicles traveling at 50 mph on high friction surfaces. The response times of the basic vehicles range from 0.79 to 0.98 sec.

The lateral acceleration response time is believed to relate to the manner in which drivers correct for *external disturbances*. A computational method, similar to one used in vehicle testing [17], has been developed by MacAdam [18] for assessing closed-loop response to external disturbances. Results from these computations for the basic vehicles indicate that drivers, represented by a delay time of 0.25 sec and a preview time of 1.5 sec, will increase, by a factor of approximately 2, the magnitude of the influences of external disturbances, occurring at approximately 3 rad/sec. These results are sensitive to driver-control characteristics (delay time and preview time). Shorter delay times and/or longer preview times will reduce the gain of the closed-loop response.

Obstacle Avoidance. The obstacle-avoidance maneuver is based on traffic conflicts in which another vehicle stops or suddenly pulls out in the path of a heavy truck. The truck driver is assumed to attempt to avoid a collision by suddenly swerving into another lane. Vehicle performance in this type of situation depends upon the period of the maneuver and the forward velocity of the vehicle. Quick maneuvers, in which the major steering activity occurs within 2 seconds, have been found to excite amplified responses at the last units of combination vehicles with full trailers [9].

These amplified responses, referred to as "rearward amplification," have been studied in both the time and frequency domains. The frequency-domain approach has been found to be effective because (1) the "worst" frequency, the one causing maximum amplification, can be readily observed and (2) the magnitude of the amplification determined by frequency domain methods has proven to be a fairly good indicator of the magnitude of amplification predicted by time domain analyses (simulations) that include nonlinearities in the vehicle system [9].

Rearward amplification not only has tracking or swept-path implications, it also indicates situations in which full trailers with high c.g. loads are likely to roll over. Since the rollover threshold is expressed in terms of lateral acceleration, the ratio of the lateral acceleration of the last unit divided by the lateral acceleration of the first unit of a combination vehicle has been used to quantify rearward amplification. In the frequency domain, this ratio is displayed as the amplitude of the transfer function between the motion of the center of gravity (c.g.) of the towing unit and the motion of the c.g. of the last unit (see Figure 12). The maximum value of this transfer function has been selected as a performance measure for this maneuver.

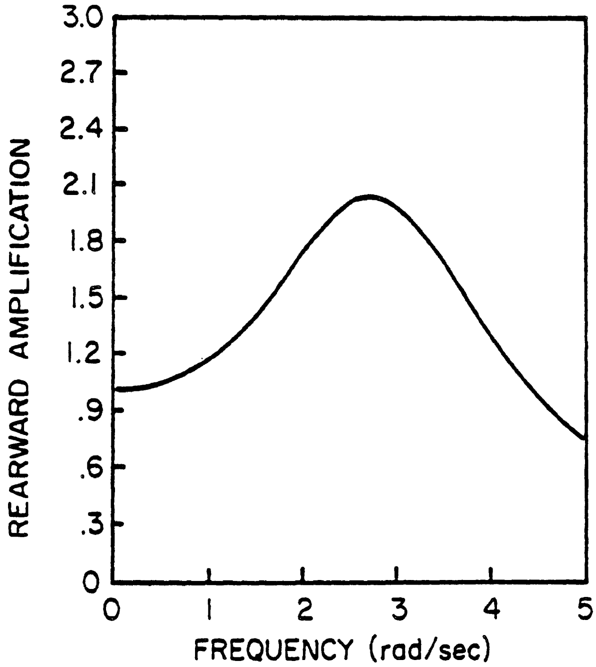


Figure12. Rearward amplification for a basic U.S. double

The results, given in row 7 of Table 3, indicate that tractor-semitrailers have small amounts of rearward amplification compared to vehicles with full trailers. To first approximation, rearward amplification is a cumulative property consisting of the product of transfer functions between (a) unit c.g.'s and hitch points and (b) hitch points and unit c.g.'s from the front to the rear of the vehicle [19,20]. Hence, vehicles with more units tend to have higher amplification.

For example, a basic triple is obtained by attaching a full trailer to the basic double. The rearward amplification of the triple is approximately equal to the rearward amplification of the double multiplied by (a) the transfer function from the c.g. of the first full trailer to the pintle hitch between the first and second full trailers and (b) the transfer function from that pintle hitch to the c.g. of the last full trailer [20]. The rearward amplification of the triple exceeds that of the double by a multiplicative factor that depends primarily upon where the last pintle hitch is located, the length of the last trailer, and the ratio of the weight of the last trailer divided by the sum of the cornering stiffnesses of all of the tires installed on the last trailer [20].

Sensitivity analyses have shown that rearward amplification may be reduced significantly by (a) reducing speed (the results given here are at 50 mph), (b) increasing the wheelbases of full trailers, (c) reducing the distance from the center of gravity of a unit to the pintle hitch installed at the rear of that unit, and (d) increasing the cornering stiffnesses of the tires.

To produce a closed-loop version of the obstacle-evasion maneuver, a path is selected to represent the choice made by the driver in an attempt to avoid the obstacle. "Driver-controlled" path following is then used in the simulations [12]. Preliminary results for a maneuver in which the "driver" attempts to suddenly translate the basic double by a lateral distance of 4 ft while traveling at 50 mph show a rearward amplification of approximately 1.5 if the driver has a 2-second preview and is allowed an additional 2 seconds (a total of 4 seconds) for completing the 4 ft displacement. However, the last unit will not experience more than 0.1 g of lateral acceleration in this case where the driver can use a long preview time. If the preview is shortened to 1.0 second and the total time for clearing the obstacle is 1.6 seconds, corresponding to a maximum lateral acceleration at the tractor of 0.27 g., the lateral acceleration of the last unit exceeds 0.7 g during the rollover of the last trailer. These closed-loop results are interpreted to mean that rearward amplification is not a problem if the driver can foresee the obstacle to be avoided.

However, if drivers are forced to take emergency evasive actions to resolve traffic conflicts at highway speeds, fully laden (high c.g.) full trailers are likely to rollover.

Braking While Turning. Combined braking and steering maneuvers are difficult to control on a poor, wet road. When braking is applied while turning, the driver may lose directional control momentarily or wheels on individual axles may lock up, leading to jackknifes or trailer swings.

From an analytical point of view, braking in a turn is difficult to treat because it involves all of the dynamic modes of vehicle motion. The tires are required to produce both longitudinal and lateral force. On even moderately slippery surfaces, demand for high deceleration may result in a lack of side force. The critical levels of performance, where loss of control may occur, are significantly altered by many vehicle characteristics. The interaction between longitudinal and lateral tire forces is clearly critical, but data describing the influences of longitudinal slip on lateral force and slip angle are not generally available.

Not only are the basic results difficult to predict, but also, suitable performance signatures and performance measures are difficult to select. In open-loop testing, the disturbances in yaw rate and sideslip shortly after braking have been used to quantify the magnitude of the directional control problem presented to the driver [21,22].

Closed-loop results have been predicted for vehicles equipped with antilock brakes [23]. In that situation, the simulated driver does not need to modulate brake pressure. However, information is not available to use in predicting how drivers will modulate brake pressure when the vehicle does not have an antilock system.

Open-loop calculations have been performed for empty and fully-laden vehicles turning at 0.11g at 50 mph on poor, wet roads (skid number at 40 mph is 28). (See [23] for a discussion of tire properties applicable to turning and braking on a poor, wet road.) The maximum deviations in yaw rate, sideslip angle, and articulation angles (where appropriate) are used to quantify the influences of the "disturbances" caused by braking. The brakes are applied suddenly and fully. At the end of 2 seconds fromthe initiation of braking, the brake pressure is released. The maximum deviations occurring (a) when the brakes are applied and (b) after the brakes are released are used as performance measures.

Given the above braking "disturbance," closed-loop simulations are run to study driver/vehicle system performance during braking in a turn.

2.5 The Vehicle Properties Required to Ascertain Performance in the Selected Types of Maneuvers

The right-hand column of Table 4 lists pertinent mechanical properties that must be known for each of the selected maneuvers. For example, wheelbases and hitch locations (referred to as the "offtracking dimensions") are all the information needed to determine offtracking in turning a corner at low speed.

In order to execute a first-order analysis of braking performance, one needs to know the offtracking dimensions, the heights and longitudinal positions of the centers of gravity of each of the major units comprising the vehicle, the heights of each of the hitches, and the brake torque versus air pressure relationships applicable to each of the brakes. In addition, for vehicles with tandem suspensions, large amounts of interaxle load transfer will have a significant influence on "wheels-unlocked" braking performance.

Tire cornering stiffness is the only parameter (not already mentioned) required for studying "high"-speed offtracking in a steady turn. Tracking in a steady turn can be predicted by a very simple procedure [5] in which the lateral position of the wheels is determined by (1) the forces the tires must generate to perform the steady turn and (2) the cornering stiffnesses of those tires.

For the first three maneuvers listed in Table 4, the amount of descriptive information required to be able to evaluate braking and steering performance is relatively small compared to that required for the other maneuvers. In the first three maneuvers it is not necessary to consider the effects of rolling the vehicle. Roll is important in handling, obstacle avoidance, and, to some extent, in initiating curved paths. In order to include roll, the vehicle is described in terms of sprung and unsprung masses. Furthermore, suspension roll stiffnesses, roll center heights, and tire vertical spring rates are needed.

Suspension roll stiffnesses and roll center heights should be carefully selected to reduce the likelihood of rollover of vehicles with high centers of gravity.

In a steady turn, the compliance of the steering system reduces the influence of front tire cornering stiffness. With regard to handling, roll properties and steering system properties can be used to adjust the influences of tire side force characteristics in ways that can either degrade or improve static stability. The distributions of the cornering stiffnesses and suspension roll stiffnesses from axle to axle influence the handling performance of the vehicle. The tire and suspension characteristics of the *tractors* or *trucks* (the lead towing

Table 4. Maneuvers and Corresponding Descriptive Information

Sequence of "maneuvers"	Corresponding sequence of additional descriptive information
1. Turning a corner at low speed	- wheelbases and hitch locations
2. Constant deceleration braking	- brake effectiveness (Torque versus pressure, for example, see SAE J1505) - wheel loads (vehicle weights) - center of gravity heights and longitudinal locations - hitch heights - interaxle load transfer
3. Steady turn, Tracking	- tire cornering stiffness including the influences of vertical load
4. Steady turn, Rolling	- suspension roll stiffness - suspension roll center heights - tire vertical stiffness - sprung and unsprung masses
5. Steady turn, Handling	- same as 3. and 4. combined - steering system stiffness
6. Initiating curved paths	- moments of inertia - tire lateral force characteristics
7. Obstacle avoidance	- all of the above
8. Braking while turning	- all of the above, plus - combined longitudinal and lateral tire force characteristics - advanced systems for modulating brake pressure
9. Response to external disturbance	- all of the above

units) are the most important factors in determining the handling performance of combination vehicles.

The analyses of "transient" maneuvers (numbers 6 through 9 in Table 4) require moments of inertia to be able to produce time histories of vehicle motions. They also may require detailed tire shear force properties to represent longitudinal and/or lateral tire forces in extreme operating conditions. The total amount of parametric information used in a comprehensive vehicle simulation is very large. The need to condense this data into a set of pertinent mechanical properties is urgent if one is to develop a basic understanding of the influence of vehicle properties on vehicle performance. The following list of mechanical properties is used here in giving first-order descriptions of benchmark vehicles:

- offtracking dimensions (locations of axles and hitches)
- axle loads (empty and loaded)
- total weight
- brake gains for each axle
- interaxle load transfer for each tandem suspension
- total cornering stiffness for each axle
- reduction in front cornering stiffness due to steering system stiffness
- roll stiffness of each axle
- roll center height of each suspension
- distance from the front axle or an articulation point to the center of gravity of each unit
- center of gravity height for each unit.

2.6 The Use of Benchmark Vehicles

Heavy trucks come in a great range of sizes and lengths depending upon their vocational requirements. There is no such thing as a standard truck. However, in order to control the size of the factbook, "benchmark" vehicles have been chosen as a baseline or reference condition for studying the influences of variations in the mechanical properties of vehicle components. To the extent that other vehicles are not fundamentally different from the benchmark vehicles, the parametric sensitivities obtained for the benchmark vehicles will be useful.

On the other hand for example, there are many varieties of single-unit trucks and it is not clear as to what is a suitable benchmark. Nevertheless, the principles illustrated by examining the results for specific benchmarks are worth noting, however, if critical differences are suspected, additional calculations may be needed for vehicles that differ substantially from those chosen as benchmarks.

The benchmark vehicles are specified by their pertinent mechanical properties. These properties are listed at the beginnings of Sections 3 through 8, respectively, for straight trucks, tractor-semitrailers, truck-full trailers, doubles, triples, and C-trains (tractor-semitrailer-semitrailer combinations). For convenient reference, the pertinent mechanical properties of all of the benchmark vehicles are listed in Table 5.

The list of generic types of heavy trucks could be expanded, but suitable models and analytical techniques are not ready nor are there major trends toward other types of vehicles now. When new types of vehicles are developed, this handbook will need to be augmented. Furthermore, since the handbook is based on "current" benchmarks, it may become outdated as changes in component characteristics take place. Although the general trends illustrated in the handbook will remain important, the detailed results will need to be updated in the future.

2.7 The Rationale for the Sequence of Performance Evaluations

The results in Sections 3 through 8 are presented in an order that is intended to aid in specifying vehicles with desirable steering and braking performances. The first performance property that is checked is whether the vehicle will be able to negotiate confined spaces and not become immobilized due to offtracking. Then the basic braking capability in both loaded and empty conditions is evaluated. The performance sensitivity diagrams presented for these two situations are to be used to ensure that a proposed vehicle will be able to turn and stop up to the expectations of the vehicle designer or assembler.

The sequence of analyses and the presentation of results has been arranged to allow one to start from very fundamental information describing the vehicle. The wheelbases and hitch locations may have been set by vocational considerations. The axle loads may be dependent on road-use laws and vocational requirements. Given these fundamental factors

Table 5. Pertinent Mechanical Characteristics of Benchmark Vehicles

Mechanical Properties	Straight Truck	Tractor & Semi-Trailer	Truck & Full-Trailer	Doubles	Triples	C-Train*
TRACTOR/TRUCK - First Unit						
Wheelbase (in)	240.00	144.00	235.00	120.00	120.00	120.00
Weight (lb)	46,000.00	15,500.00	42,000.00	14,000.00	14,000.00	14,000.00
Front suspension load (lb)	12,000.00	12,000.00	10,500.00	9,950.00	9,950.00	9,950.00
Rear suspension load (lb)	34,000.00	34,000.00	31,500.00	18,550.00	18,550.00	18,550.00
Front axle brake gain (in.lb/psi)	2,000.00	2,000.00	2,000.00	2,000.00	2,000.00	2,000.00
Rear suspension - leading axle brake gain (in.lb/psi)	3,000.00	3,000.00	3,000.00	3,000.00	3,000.00	3,000.00
Rear suspension - trailing axle brake gain (in.lb/psi)	3,000.00	3,000.00	3,000.00			
Interaxle load transfer on rear tandem	0.00	0.00	0.00			
Front suspension cornering stiffness (lb/deg)	1,047.00	1,047.00	1,016.00	998.00	998.00	998.00
Rear suspension cornering stiffness (lb/deg)	3,746.00	3,746.00	3,611.00	1,945.00	1,945.00	1,945.00
Reduction in cornering stiffness due to steering system (%)	32.16	32.16	32.01	31.92	31.92	31.92
Front suspension roll stiffness (in.lb/deg)	21,000.00	21,000.00	21,000.00	21,000.00	21,000.00	21,000.00
Rear suspension roll stiffness (in.lb/deg)	160,000.00	140,000.00	140,000.00	70,000.00	70,000.00	70,000.00
Front suspension roll center height (in)	20.00	20.00	20.00	20.00	20.00	20.00
Rear suspension roll center height (in)	29.00	29.00	29.00	29.00	29.00	29.00
Height of the center of gravity (in)	69.05	34.83	64.00	37.50	37.50	37.50
Distance of the center of gravity from the front axle (in)	177.40	60.85	176.25	47.15	47.15	47.15
SEMI-TRAILER/FULL-TRAILER - Second Unit						
Wheelbase (in)		432.00	222.00	252.00	252.00	292.00
Weight (lb)		64,500.00	38,000.00	31,000.00	31,000.00	33,500.00
Front suspension load (lb)			19,000.00			
Rear suspension load (lb)		34,000.00	19,000.00	16,500.00	16,500.00	34,250.00
Front axle brake gain (in.lb/psi)			3,000.00			
Rear suspension - leading axle brake gain (in.lb/psi)		3,000.00	3,000.00	3,000.00	3,000.00	3,000.00
Rear suspension - trailing axle brake gain (in.lb/psi)		3,000.00				3,000.00
Interaxle load transfer on rear tandem		0.00				
Front suspension cornering stiffness (lb/deg)			1,963.00			
Rear suspension cornering stiffness (lb/deg)		3,746.00	1,963.00	1,847.00	1,847.00	3,756.00
Front suspension roll stiffness (in.lb/deg)			80,000.00			
Rear suspension roll stiffness (in.lb/deg)		160,000.00	80,000.00	80,000.00	80,000.00	160,000.00
Front suspension roll center height (in)			29.00			
Rear suspension roll center height (in)		29.00	29.00	29.00	29.00	29.00
Height of the center of gravity (in)		81.44	73.50	78.40	78.40	74.75
Distance of the center of gravity from the front axle or kingpin (in)		227.70	111.00	134.15	134.15	151.60

Table 5. Pertinent Mechanical Characteristics of Benchmark Vehicles

Mechanical Properties	Straight Truck	Tractor & Semi-Trailer	Truck & Full-Trailer	Doubles	Triples	C-Train*
CONVERTER DOLLY - Third Unit						
Wheelbase (in)				80.00	80.00	
Weight (lb)				2,500.00	2,500.00	
Dolly suspension load (lb)				17,750.00	17,750.00	
Dolly axle brake gain (in.lb/psi)				3,000.00	3,000.00	
Dolly suspension cornering stiffness (lb/deg)				1,909.00	1,909.00	
Dolly suspension roll stiffness (in.lb/deg)				80,000.00	80,000.00	
Dolly suspension roll center height (in)				29.00	29.00	
Height of the center of gravity (in)				29.50	29.50	
Distance of the center of gravity from the axle (in)				0.00	0.00	
SEMI-TRAILER - Third or Fourth Unit						
Wheelbase (in)				252.00	252.00	252.00
Weight (lb)				32,500.00	32,500.00	32,500.00
Rear suspension load (lb)				17,250.00	17,250.00	17,250.00
Rear axle brake gain (in.lb/psi)				3,000.00	3,000.00	3,000.00
Rear suspension cornering stiffness (lb/deg)				1,885.00	1,885.00	1,885.00
Rear suspension roll stiffness (in.lb/deg)				80,000.00	80,000.00	80,000.00
Rear suspension roll center height (in)				29.00	29.00	29.00
Height of the center of gravity (in)				78.50	78.50	78.50
Distance of the center of gravity from the kingpin (in)				133.75	133.75	133.75

* In the case of the C-train the B-dolly and the lead trailer have been combined into a single unit.

that constrain the nature of design possibilities, the low-speed offtracking and constant-deceleration braking calculations can be used to see if the vehicle will meet basic requirements for maneuvering in tight places and stopping quickly.

If the braking and low-speed maneuverability of the vehicle are satisfactory, then the designer has established the "form" of the vehicle. The next recommended step is to examine the high-speed offtracking in a steady turn. This will establish the first constraint on the cornering stiffnesses of the tires because the tire stiffnesses are the only additional mechanical properties that influence high-speed offtracking. Handling analyses and transient performance will also be influenced by tire cornering stiffnesses, but these are much more complex situations. Very tight requirements on high-speed offtracking will mean stiff tires and this will be compatible with good directional performance in general.

Vehicle roll is the next subject addressed. Roll is primarily influenced by the heights of the centers of gravity of the vehicle's units and the roll stiffnesses of the suspensions. The center of gravity height may be fixed by the application of the vehicle. At this stage in evaluating a vehicle design, one should be concerned with the selection of adequate roll stiffness to provide as much roll stability as practical for steady-turning maneuvers.

The roll characteristics of the vehicle will have an important influence on handling. The distribution of roll stiffnesses from axle to axle is significant in determining the side-to-side load transfer during a turn. The side-to-side load transfer interacts with tire properties to establish the stability of the vehicle in steady turns. Once roll properties are chosen, the tire stiffnesses can be selected to meet desired handling requirements.

At this point, the vehicle's properties will have been fairly well established and the remaining calculations serve as checks to ensure that the vehicle will not exhibit undesirable response qualities in special transient or complex maneuvering situations. If the vehicle has a difficulty with slow response times, braking in a turn, or rearward amplification, the parametric values selected earlier may need to be adjusted to eliminate the difficulty. The necessary changes could involve restructuring the entire vehicle or, if the problem is rearward amplification, increasing the stiffness of the trailer tires may improve performance.

The parametric influences displayed in the performance sensitivity diagrams provide indications of the types and amounts of adjustment that are needed to achieve particular levels of performance. If the desired levels of performance can be achieved without changing mechanical properties that are important to the simpler maneuvers, a vehicle that meets preselected requirements can be designed. On the other hand, one may find that their initial goals for vocational requirements and performance levels may have been set too high. There may be no reasonable compromise without going back and reconsidering the original levels of performance. In that case, the vehicle itself may end up being an unexpected compromise, but at least the designer is aware of the difficulty and can recommend countermeasures such as restricting velocity under certain operating conditions.

2.8 The Meaning of "Performance Sensitivity Diagrams"

The "performance sensitivity diagram" utilized in this study consists of a single plot that is used to display the *individual* influences of various mechanical properties on vehicle performance. The vertical axis of the plot displays values of the performance measure for the maneuver under study. The horizontal axis is used for the mechanical properties to be compared. In order to display different types of mechanical properties on the same graph, multiple scales have been used on the horizontal axes of these graphs. For example, using this arrangement of sensitivity results (see Figure 40 pertaining to rollover thresholds), one can compare the influences of changes in c.g. height to the influences of changes in suspension roll stiffness. The ranges of the changes used in the sensitivity diagrams have been selected to correspond approximately to the ranges of mechanical properties found in measurements of heavy truck components and characteristics.

These diagrams are used to provide graphical indications of the importance of pertinent mechanical properties in various maneuvering situations. They constitute the means for portraying the basic information presented in this handbook.

2.9 The Ranges of Pertinent Mechanical Properties

The component factbook [2] presents parametric data describing the mechanical properties of vehicle units and components. The following figures (numbers 13 through 24), which are taken from the factbook, illustrate the ranges of properties that represent the current heavy-truck fleet. These ranges form the basis for the parametric variations selected for use in developing performance sensitivity diagrams.

The "Component Factbook" contains (1) descriptions and definitions of pertinent mechanical properties, (2) qualitative discussions of the importance of these properties to the braking and steering of heavy trucks,and (3) ranges of values corresponding to the pertinent mechanical properties that have been measured or can be estimated. The information in the factbook describes components in a manner that is independent of any particular vehicle in which these components may be installed. In that sense, the information in the component factbook is general rather than vehicle specific. This handbook complements the factbook in that it uses the information in the factbook to provide quantitative results for specific vehicle applications.

Sample of Cornering Coefficient Values Measured at Rated Load

$$\left(\frac{C\alpha}{F_z}\right)_{F_z = \text{Rated Load}}, \ \deg^{-1}$$

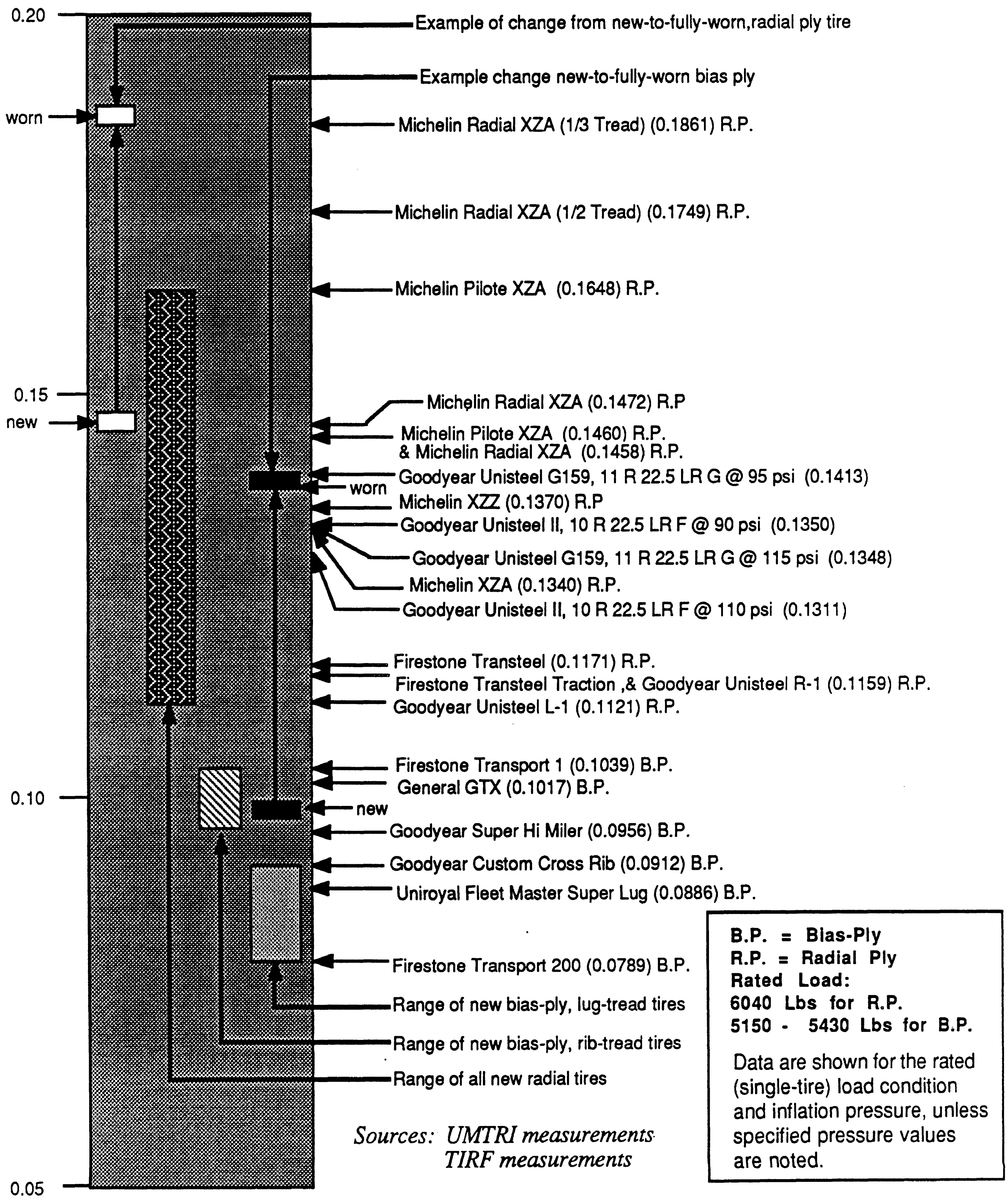
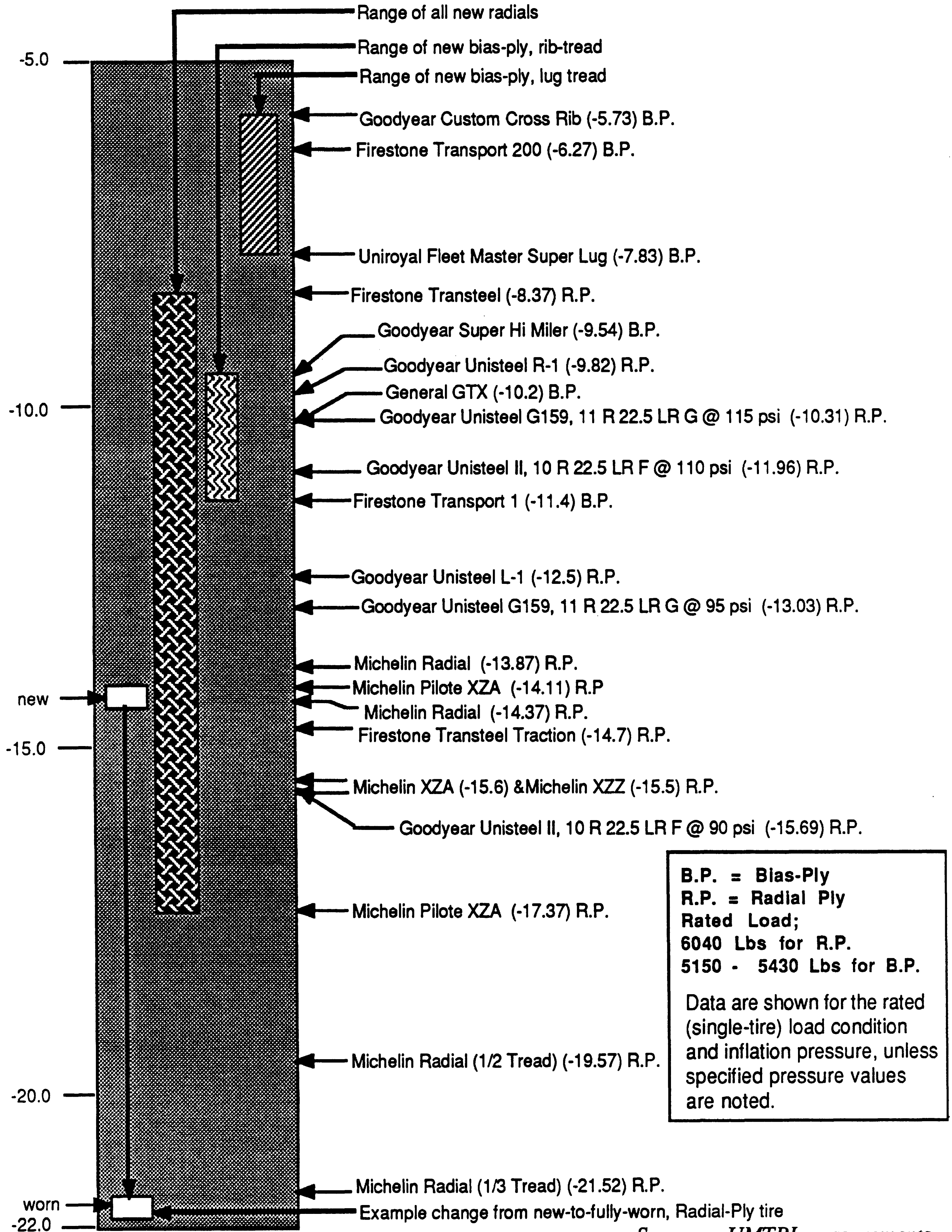


Figure 13. Cornering coefficient

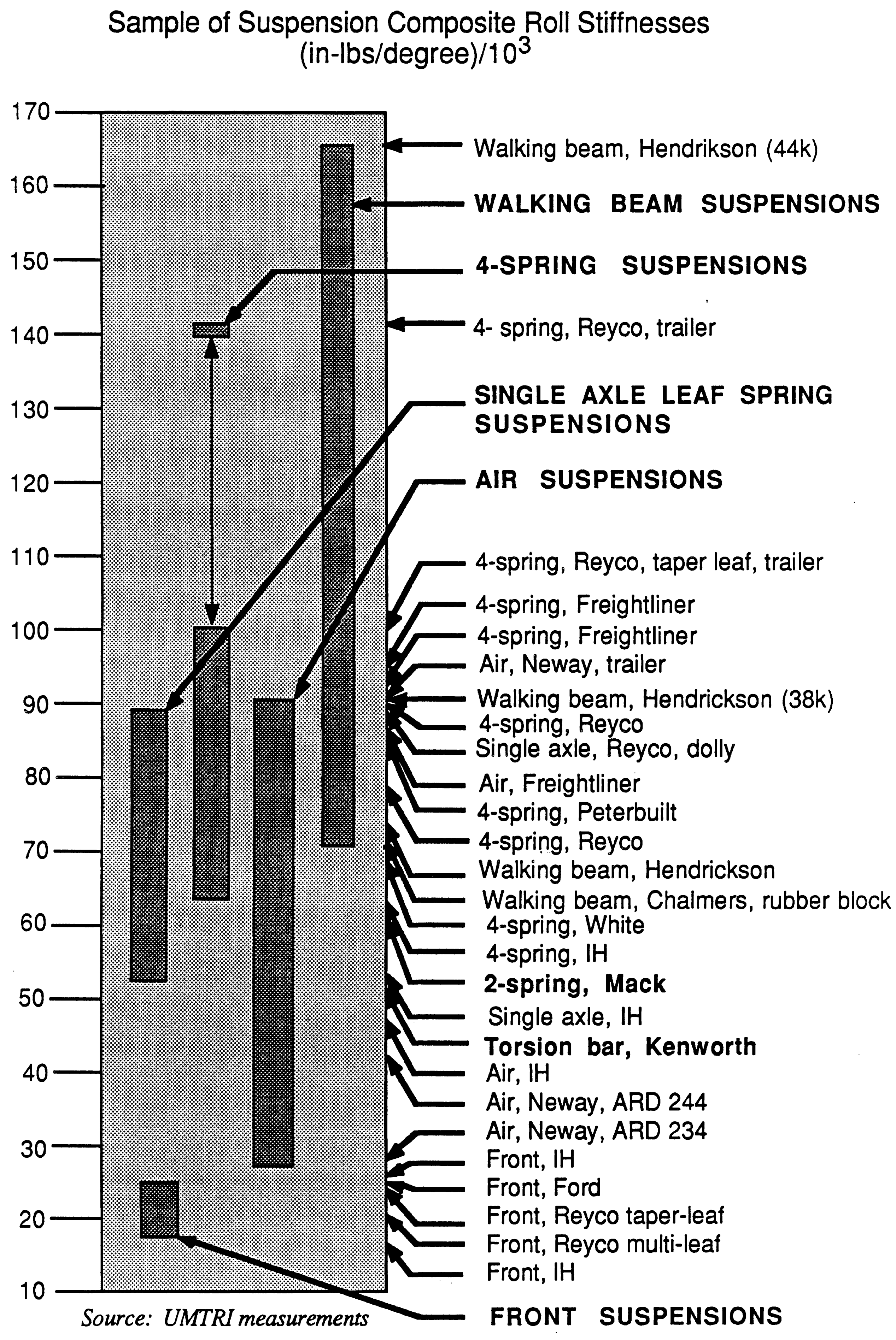
Sample of Curvature Coefficient Values Measured at Rated Load

$C_2 \times 10^6$, (lb-deg)$^{-1}$



Sources: UMTRI measurements
TIRF measurements

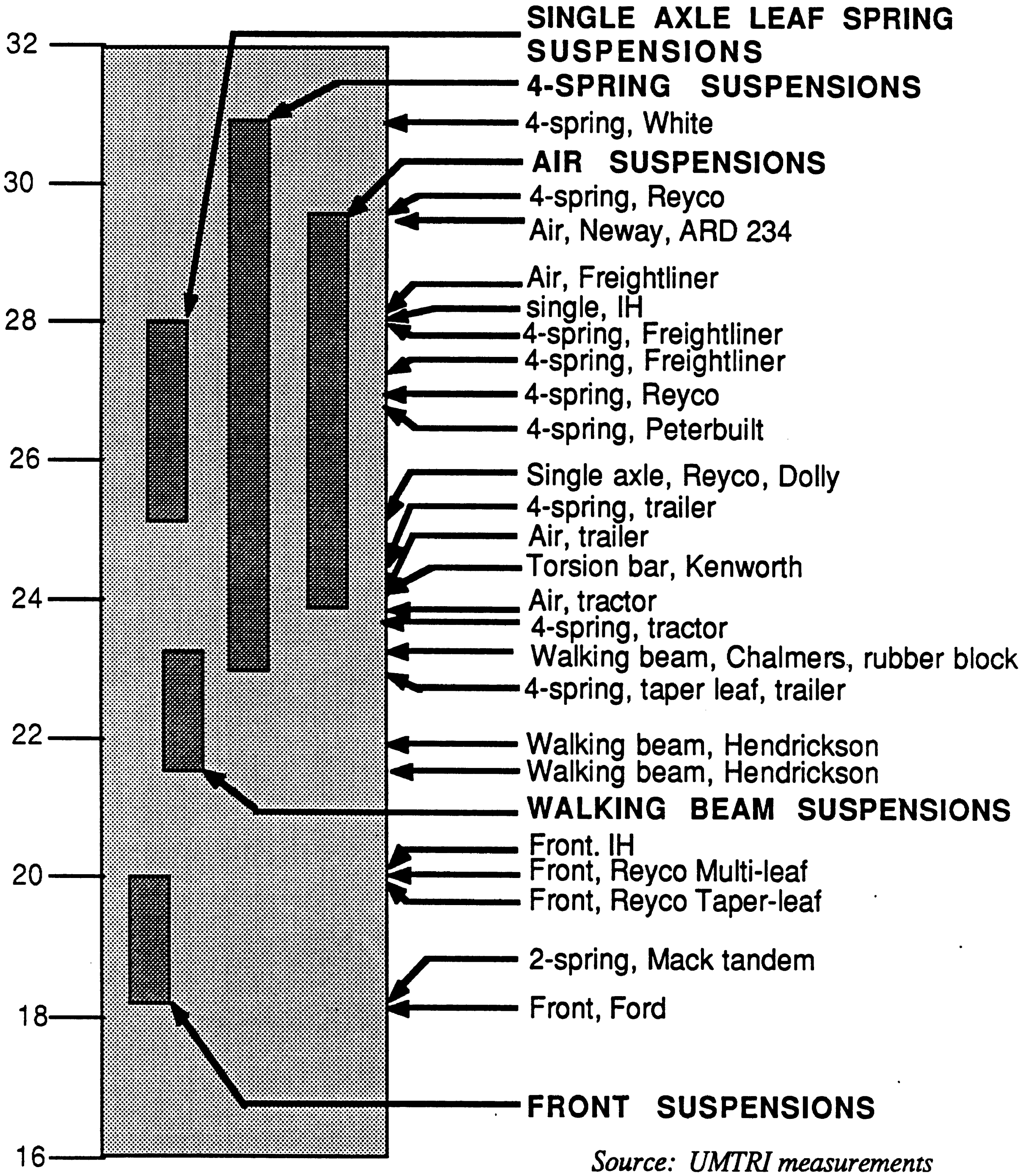
Figure 14. Curvature coefficient



Note: All values given are on a per axle basis. For tandem suspensions, the value presented is for the average of the two axles.

Figure 15. Suspension composite roll stiffness

Sample of Suspension Roll Center Heights
(Inches above the ground)



Source: UMTRI measurements

Note: All values given are on a per axle basis. For tandem suspensions, the value presented is for the average of the two axles.

Figure 16. Suspension roll center heights

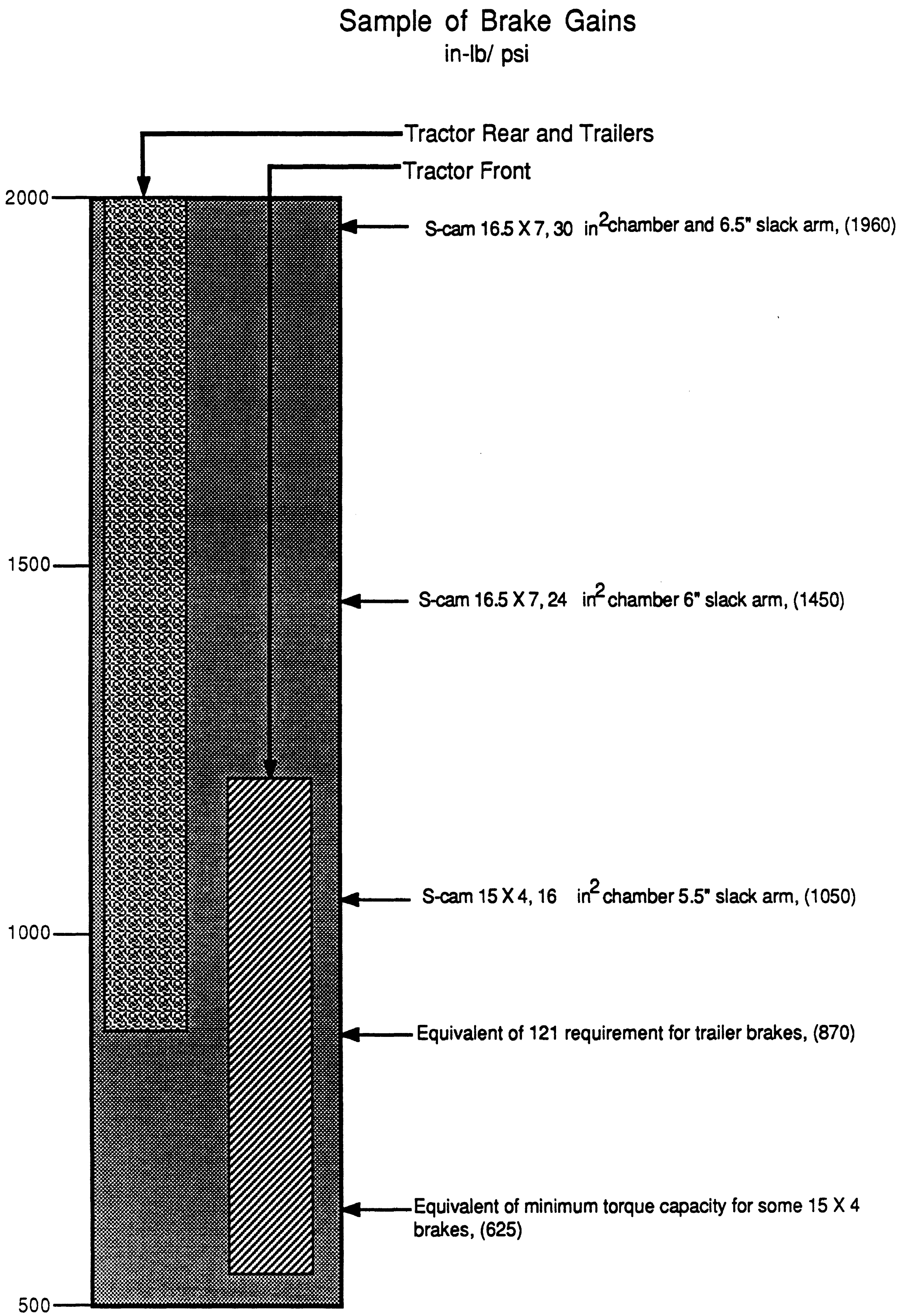


Figure 17. Estimates of brake gain approximating effectiveness at high pressure and 50 mph initial velocity

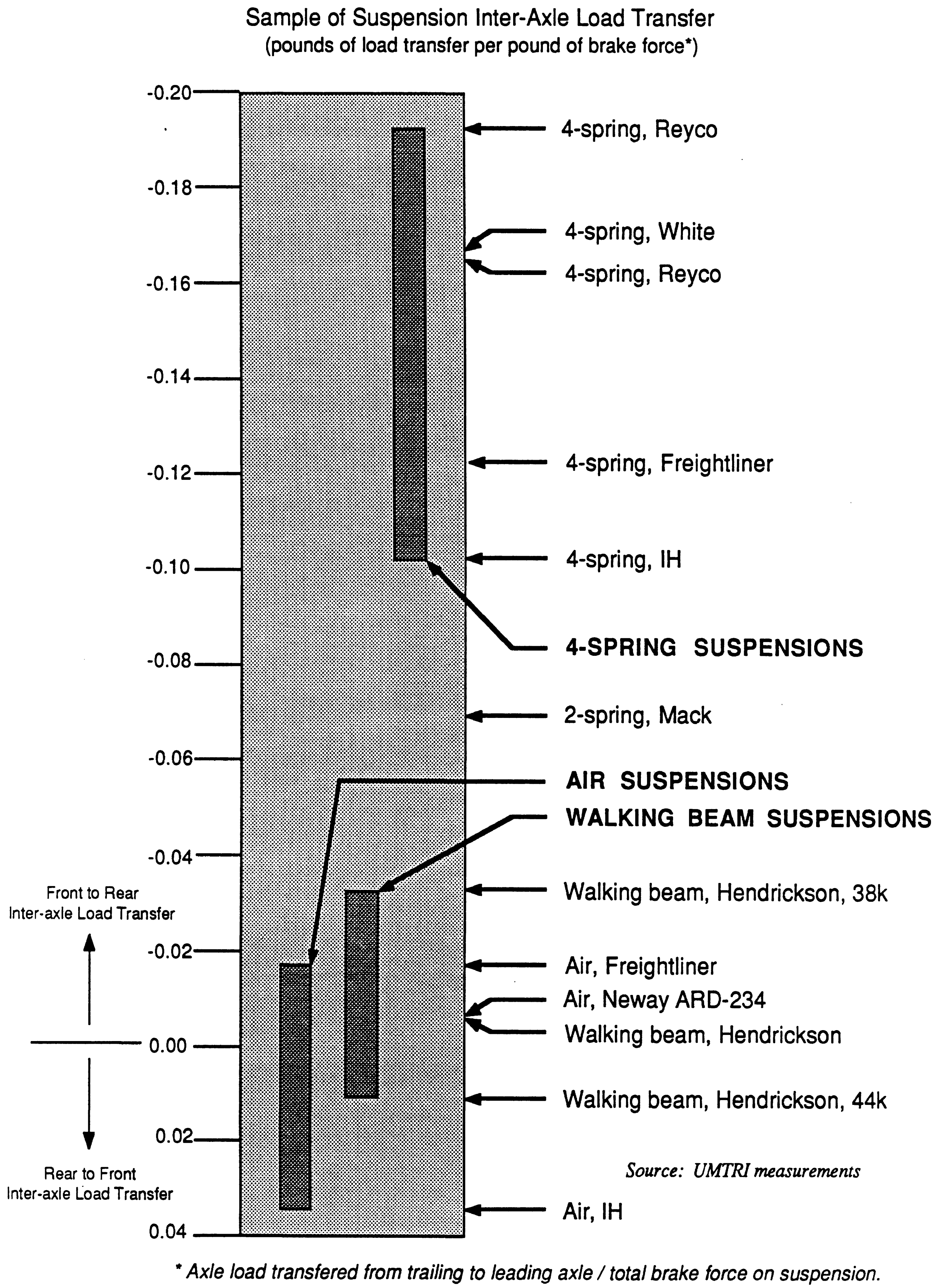


Figure 18. Suspension inter-axle load transfer

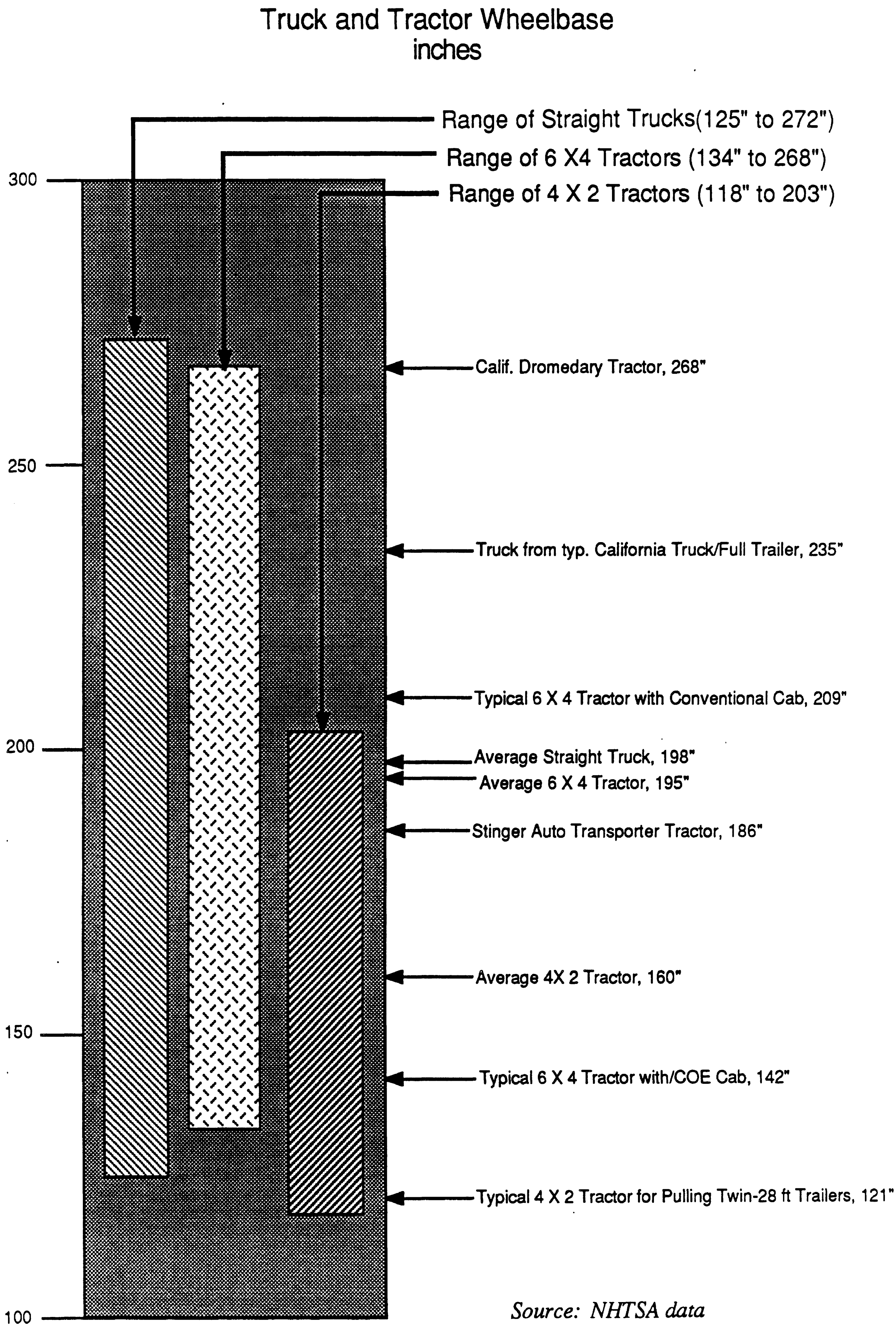


Figure 19. Truck and tractor wheelbase

Sample of Trailer Wheelbase- Kingpin-to-rear Axle (or Tandem) Center
feet

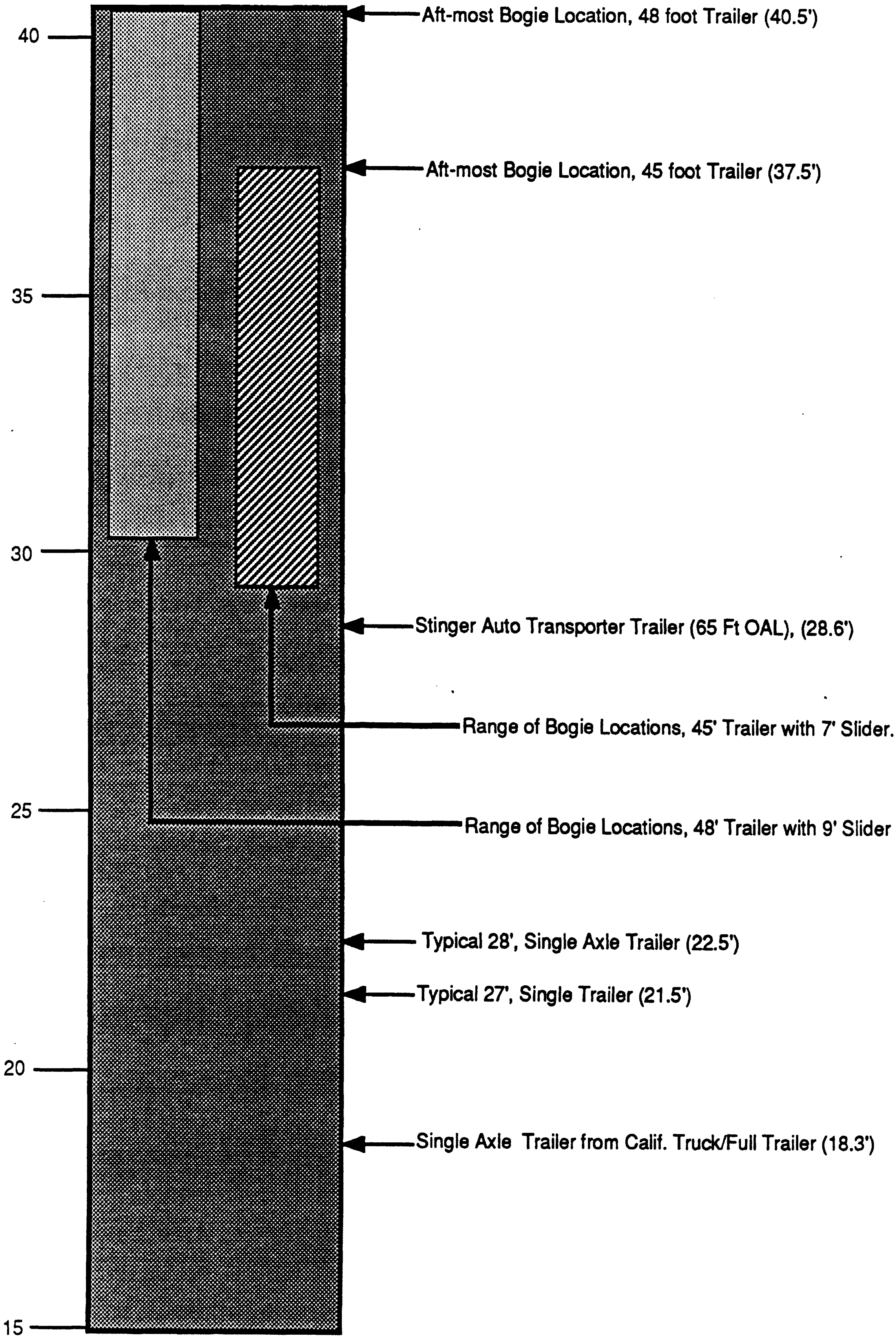


Figure 20. Trailer wheelbase- kingpin-to-rear axle (or tandem) center

Sample of Tractor and Straight Trucks
Yaw and Pitch Moments of Inertia
($in\text{-}Lbs\text{-}sec^2$)

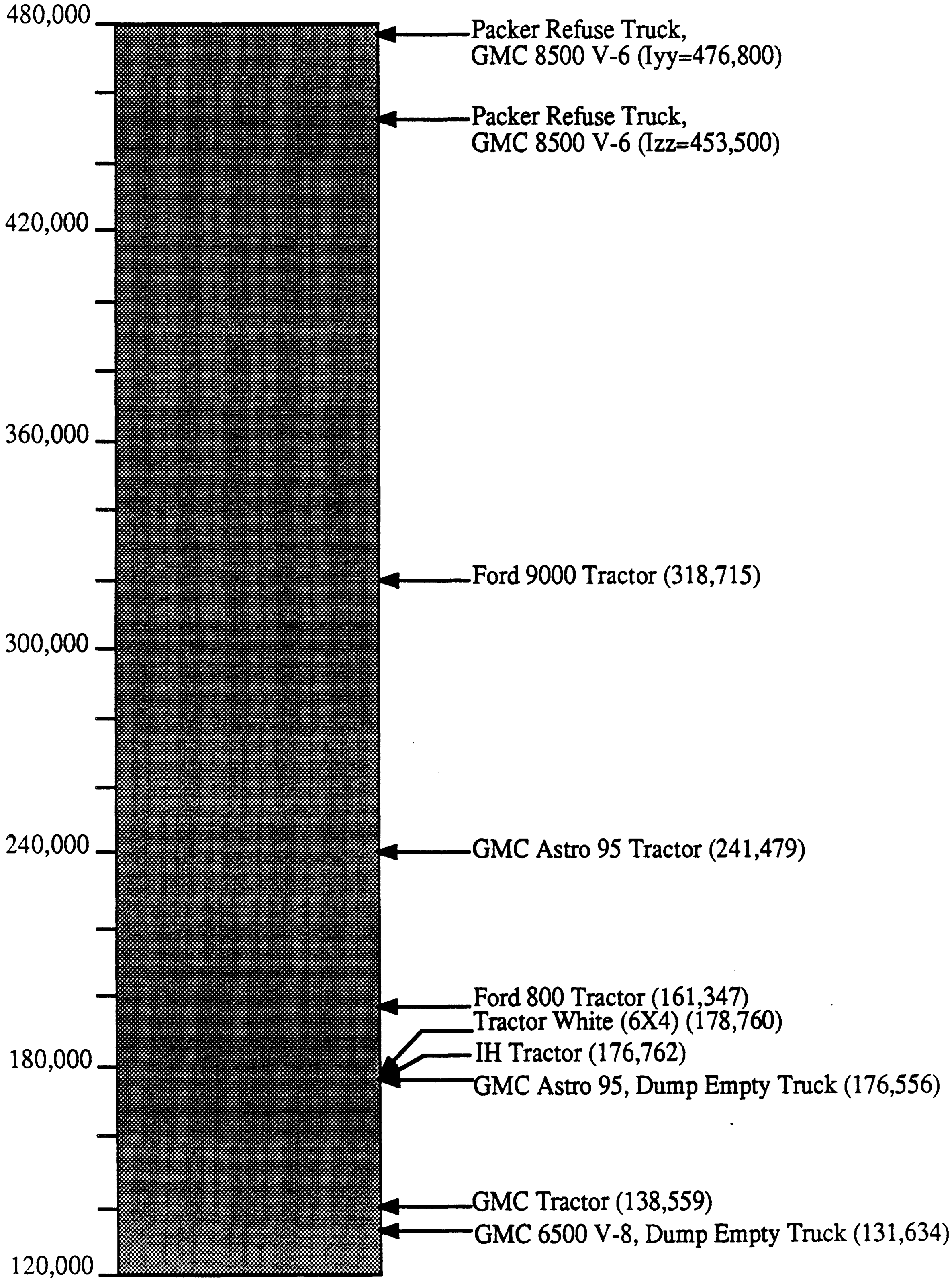
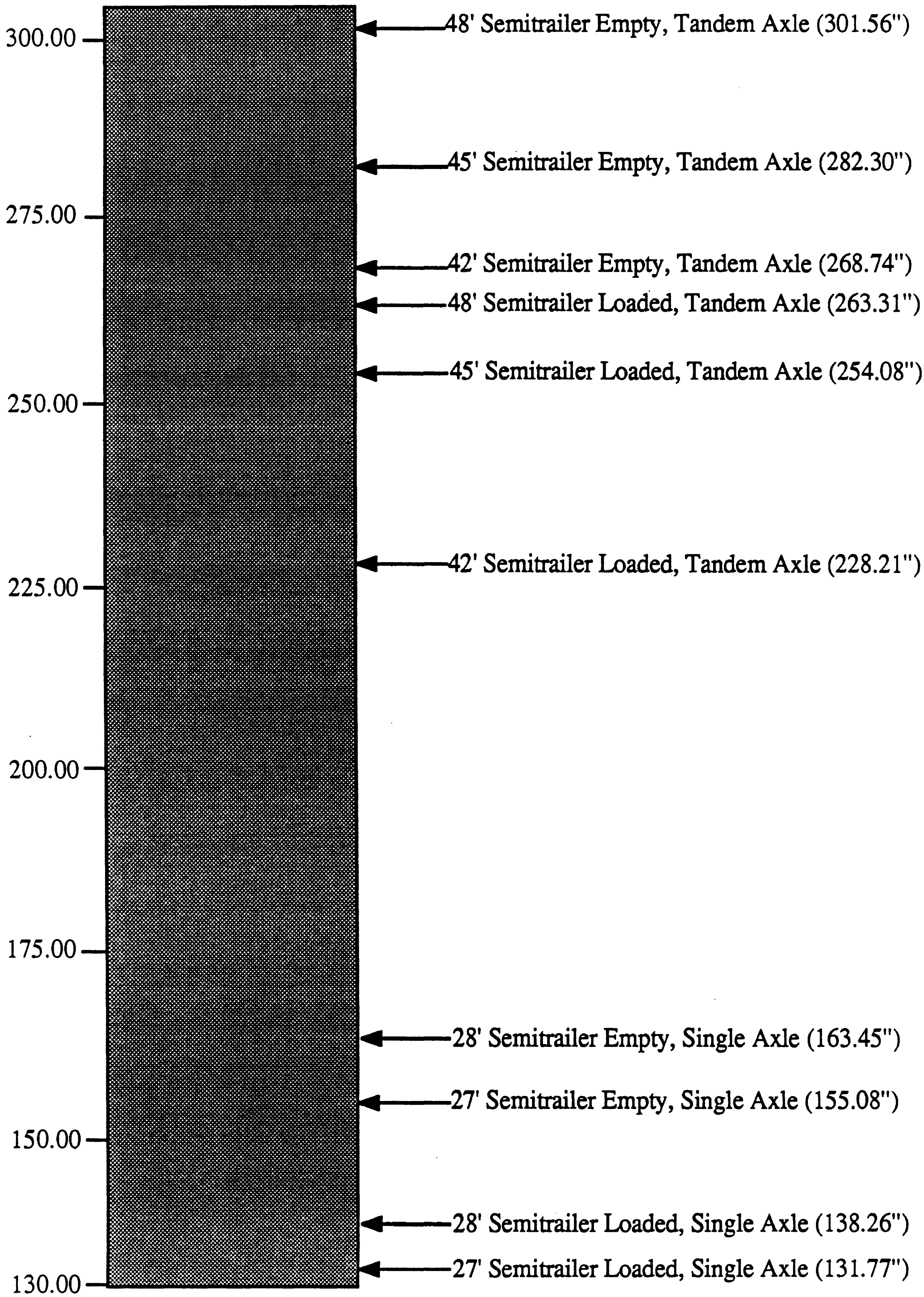


Figure 21. Tractor and straight trucks yaw and pitch moments of inertia about axes through total c.g. (unit unladen)

Sample of Semitrailer Fore-aft C.G. Location
(inches behind the kingpin)



Note: Estimated values taking a Uniformly Homogeneous Freight with a density of; $\rho = 14.0 \frac{\text{Lbs}}{\text{Ft}^3}$

Figure 22. Semitrailers fore-aft c.g. location (inches behind the kingpin)

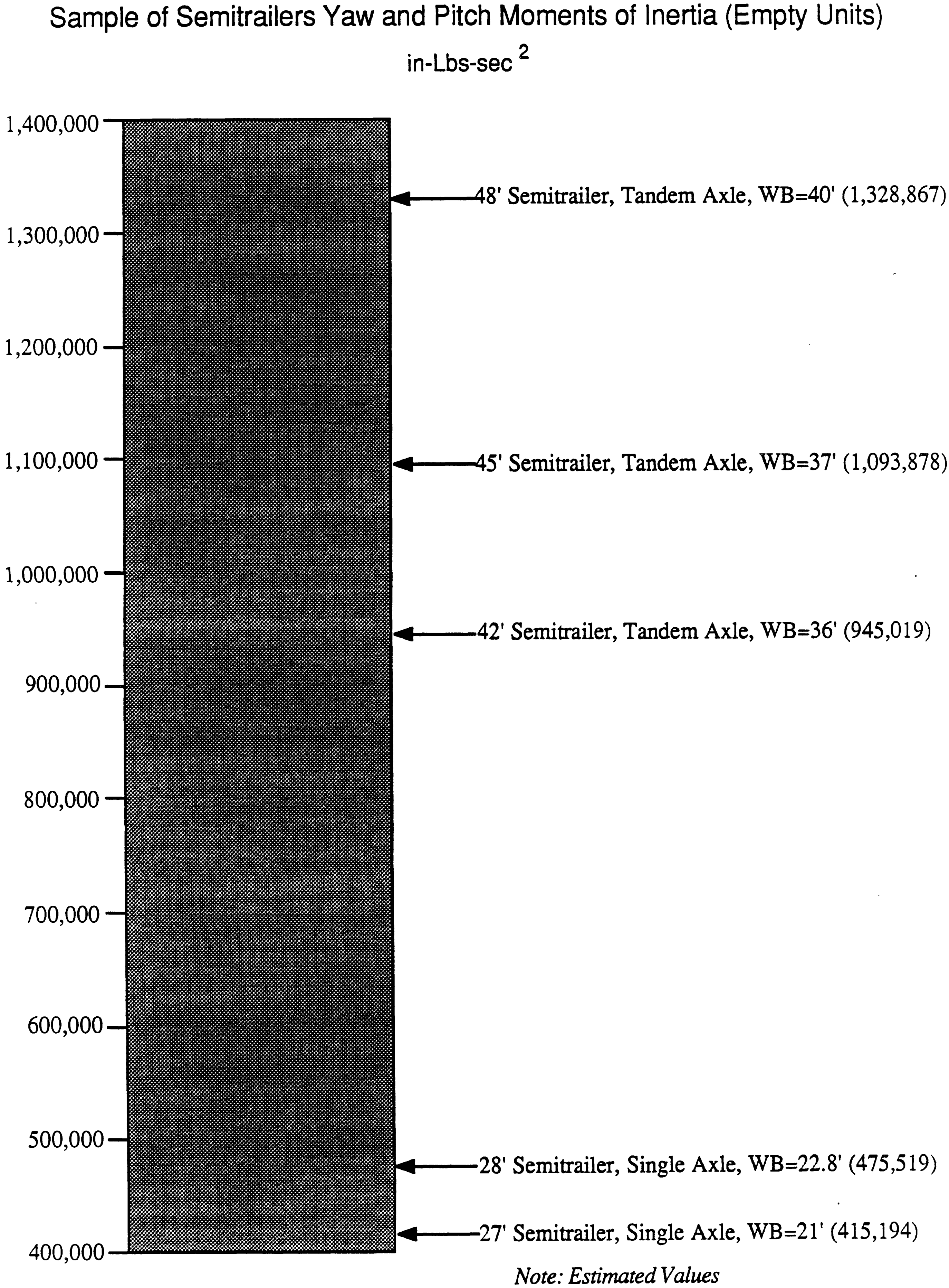
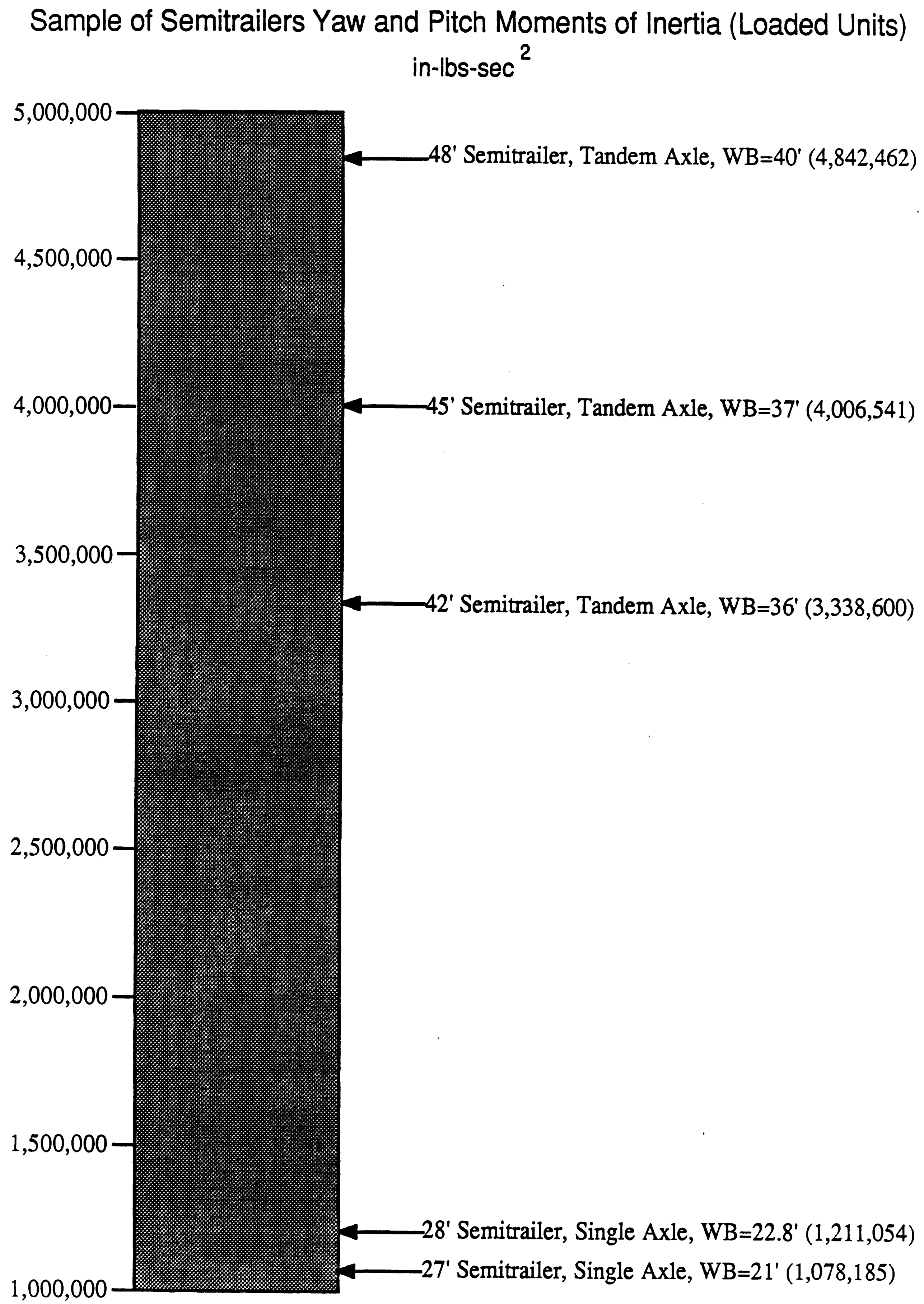


Figure 23. Semitrailers yaw and pitch moments of inertia (empty units)



Note: Estimated values taking a Uniformly Homogeneous Freight with a density of ; $\rho = 14.0 \frac{Lbs}{Ft^3}$

Figure 24. Semitrailers yaw and pitch moments of inertia (loaded units)

PART TWO--PERFORMANCE SENSITIVITIES

3.0 SINGLE-UNIT (STRAIGHT) TRUCKS

3.1 Baseline Values of Pertinent Mechanical Properties.

Although many of the single-unit trucks utilized for delivering goods are medium trucks, this handbook deals with heavy trucks, and hence the benchmark vehicle is heavier (and longer) than many typical straight trucks.

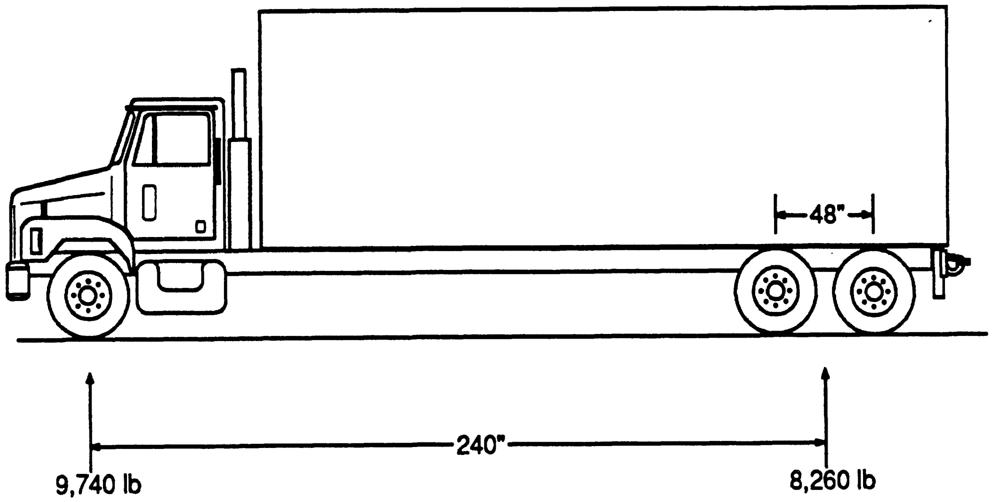
The benchmark straight truck weighs 46,000 pounds when fully laden. It has a wheelbase of 240 inches from the front axle to the center of the rear tandem axle pair. The geometric layout and axle loads are illustrated in Figure 25. The values of its basic mechanical properties are listed in Table 6. These values represent the baseline condition for the performance sensitivity diagrams presented in this section.

3.2 Low-Speed Cornering - Tractrix

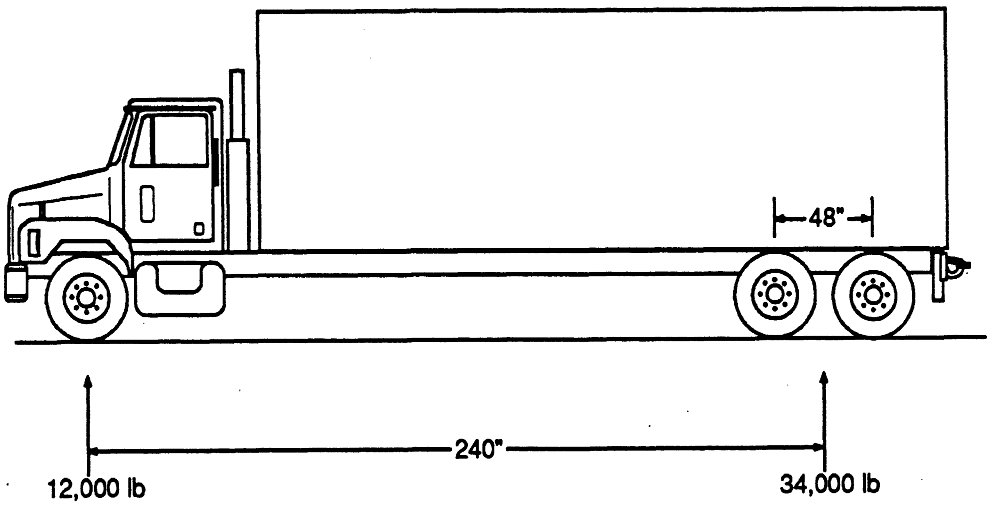
Benchmark performance. The performance signature for the benchmark straight truck is the tractrix shown in Figure 26. This "signature" is the path of the center of the rear tandem in a 90-degree turn with a radius of 41 feet to the center of the front axle. The tandem center offtracks the path of the center of the front axle by a maximum amount of 4.94 feet during this maneuver. This is the value of the performance measure used to represent the benchmark vehicle in the performance sensitivity diagram, Figure 27.

Parametric sensitivities. For the straight truck the only parameter of concern here is the wheelbase. As shown in Table 7, the range from 125 inches to 272 inches is examined. (Although there is no other mechanical property to compare with in this case, a standard sensitivity diagram is presented to be consistent with the presentation for other vehicle types.)

The diagram clearly indicates the well-known sensitivity of offtracking to wheelbase. *In general, for all vehicles, the longest wheelbase in the combination is the most important parameter with respect to low-speed offtracking. The basic method for improving low-speed offtracking is to reduce the longest wheelbase that can be changed reasonably.*



EMPTY STRAIGHT TRUCK



LOADED STRAIGHT TRUCK

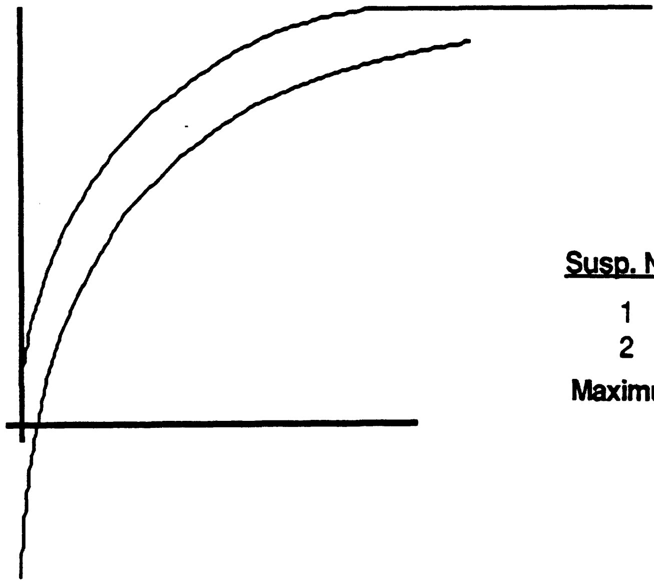
Figure 25. Geometric layout, benchmark truck

Table 6. Basic Mechanical Properties (Straight Truck)

Mechanical Property	Empty	Loaded
Weight (lb)	18,000.00	46,000.00
Front axle brake gain (in.lb/psi)	2,000.00	2,000.00
Rear suspension - leading axle brake gain (in.lb/psi)	3,000.00	3,000.00
Rear suspension - trailing axle brake gain (in.lb/psi)	3,000.00	3,000.00
Interaxle load transfer on rear tandem	0.00	0.00
Front suspension cornering stiffness (lb/deg)	991.00	1,047.00
Rear suspension total cornering stiffness (lb/deg)	1,550.00	3,746.00
Reduction in cornering stiffness due to steering system (%)	31.88	32.16
Front suspension roll stiffness (in.lb/deg)	21,000.00	21,000.00
Rear suspension roll stiffness (in.lb/deg)	160,000.00	160,000.00
Front suspension roll center height (in)	20.00	20.00
Rear suspension roll center height (in)	29.00	29.00
Height of the center of gravity (in)	44.25	69.05
Distance of the center of gravity from the front axle (in)	110.10	177.40

Table 7. Straight Truck - Low-Speed Offtracking

Parameter Name	Benchmark Value	Minimum Value	Maximum Value	Notes
Wheelbase	240 inches	125 inches	272 inches	



MINIMUM TRACK

Susp. No.	Radius (ft)	Angle (deg)
1	41.00	0.00
2	36.07	68.91

Maximum Offtracking = 4.94 ft.

Figure 26. Tractrices, straight truck, 41 foot turn

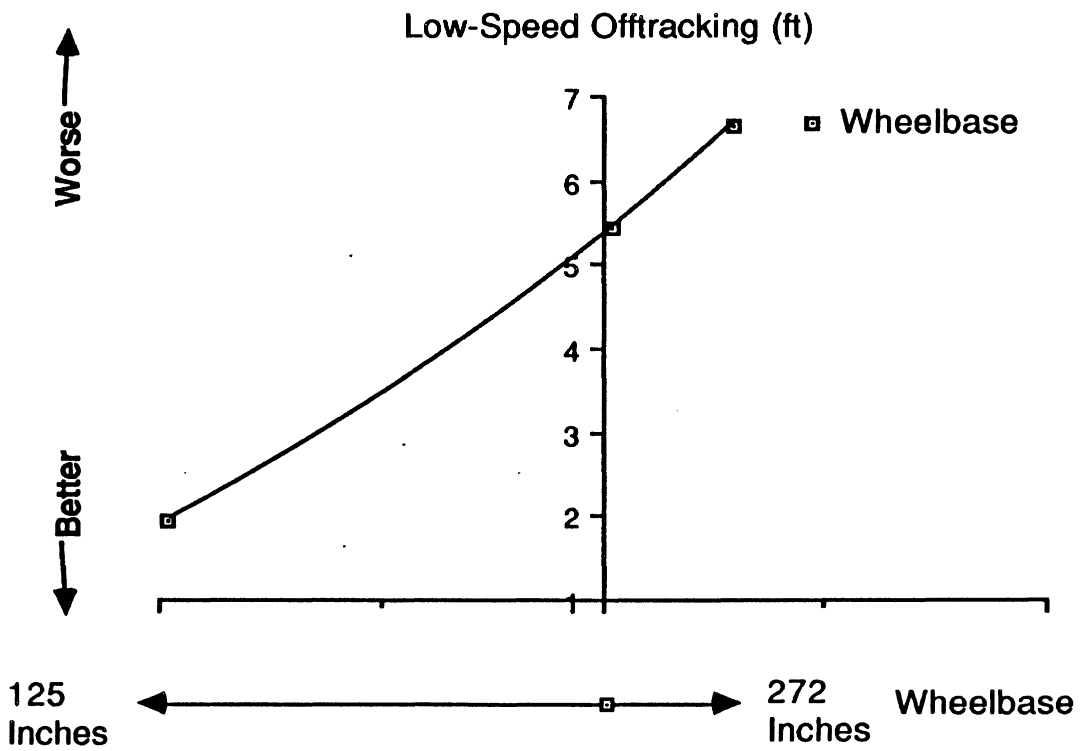


Figure 27. Parameter sensitivity diagram: low-speed offtracking, straight truck.

3.3 Constant Deceleration Braking

Benchmark performance. Braking performance differs greatly between that of loaded and empty vehicles. The braking efficiency of the empty vehicle is much poorer than that of the loaded vehicle because brake proportioning is typically arranged in the United States to favor the loaded vehicle. The difference in performance is evident in the performance signatures presented in Figures 28 and 29.

These diagrams contain graphs of friction utilization and deceleration (as in Figure 8) plus braking efficiency as a function of pressure at the treadle valve. The friction utilization curves indicate the amount of tire/road friction needed to avoid wheel lock on each axle. The braking efficiency indicates the relationship of the deceleration attained to the friction needed to prevent lockup at any axle. The axle that has the highest friction utilization is the one that is used in determining the efficiency.

The braking efficiencies in 0.4 g stops are 0.51 for the empty vehicle and 0.83 for the loaded vehicle. The values of these performance measures indicate that the empty vehicle has a low wheels-unlocked braking capability. The driver of the empty vehicle may be in danger of locking wheels and thereby initiating a spin or loss of steering control. *High braking efficiency for both empty and loaded conditions can only be provided with advanced braking systems such as those with antilock and/or load-sensing proportioning.*

Parametric sensitivities. The influences of wheelbase, cg height, brake gains, and interaxle load transfer are examined here. The baseline values and the amounts of deviations from the baseline values are given in Table 8. Performance sensitivity diagrams are presented for both the empty and loaded vehicle at 0.2 g and 0.4 g (see Figures 30 through 33). *Examination of these figures shows that the mechanical property causing the greatest loss in performance is interaxle load transfer.*

Interaxle load transfer is a property of tandem suspensions that represents the influence of braking torque on the distribution of the vertical loads carried by a tandem set of axles. The maximum deviation of 0.2 (see Table 8) means that 20 percent of the braking force is reacted through interaxle load transfer. Currently produced tandem suspensions often have brake reaction rods that effectively reduce interaxle load transfer to zero, but a value of approximately 0.2 is not unreasonably large.

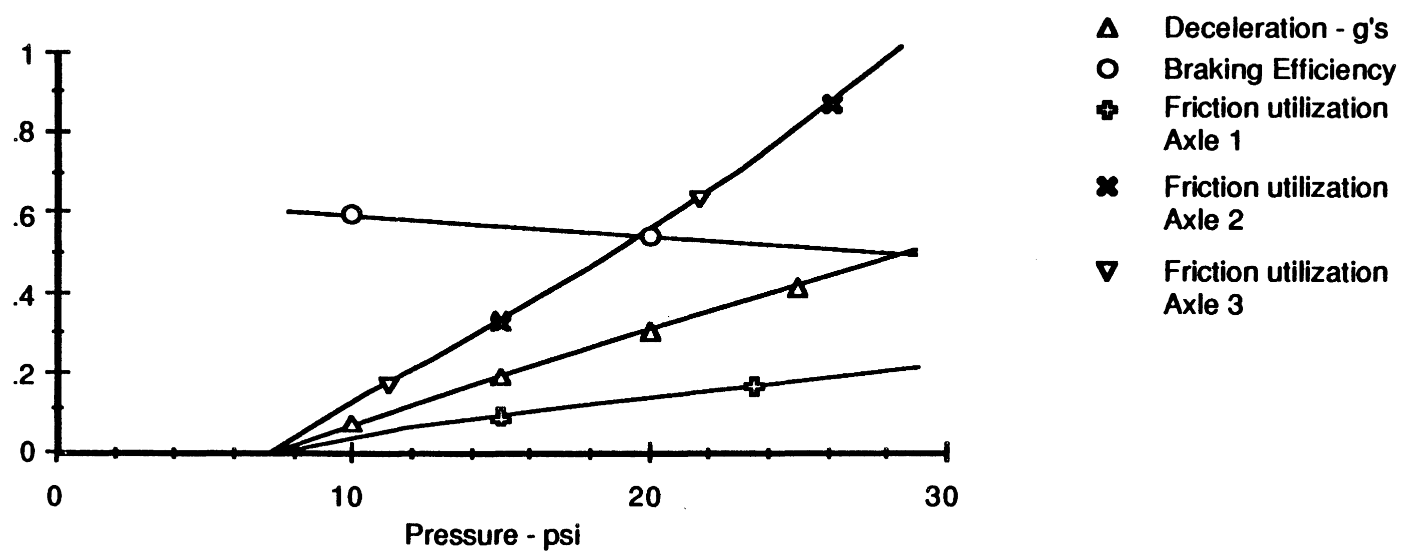


Figure 28. Performance signature, braking, empty straight truck

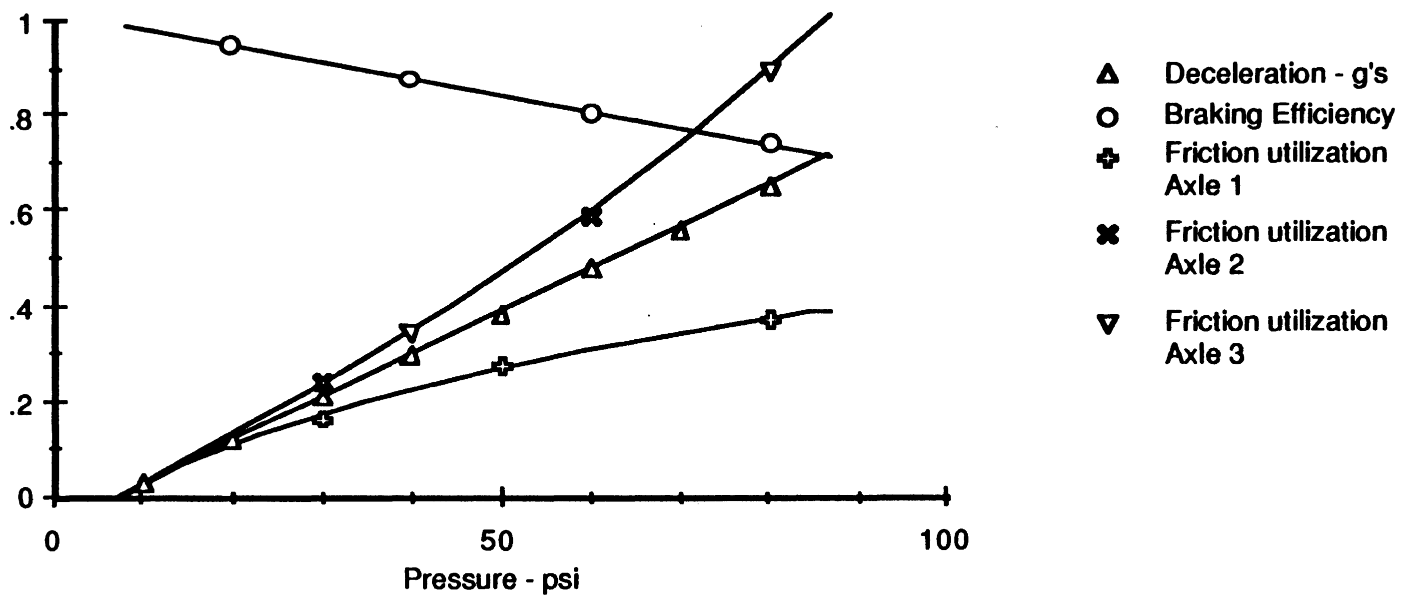


Figure 29. Performance signature, braking, loaded straight truck

Table 8. Straight Truck - Braking

Parameter Name	Benchmark Value	Minimum Value	Maximum Value	Notes
Wheelbase	240 inches	125 inches	272 inches	
Interaxle load transfer	0	-0.2	0.2	
* C.G. height	69.05 inches	35 inches	100 inches	
Front brake gain	2000 in.lb/psi	2000 in.lb/psi	3000 in.lb/psi	Both brakes combined

* For the loaded vehicle

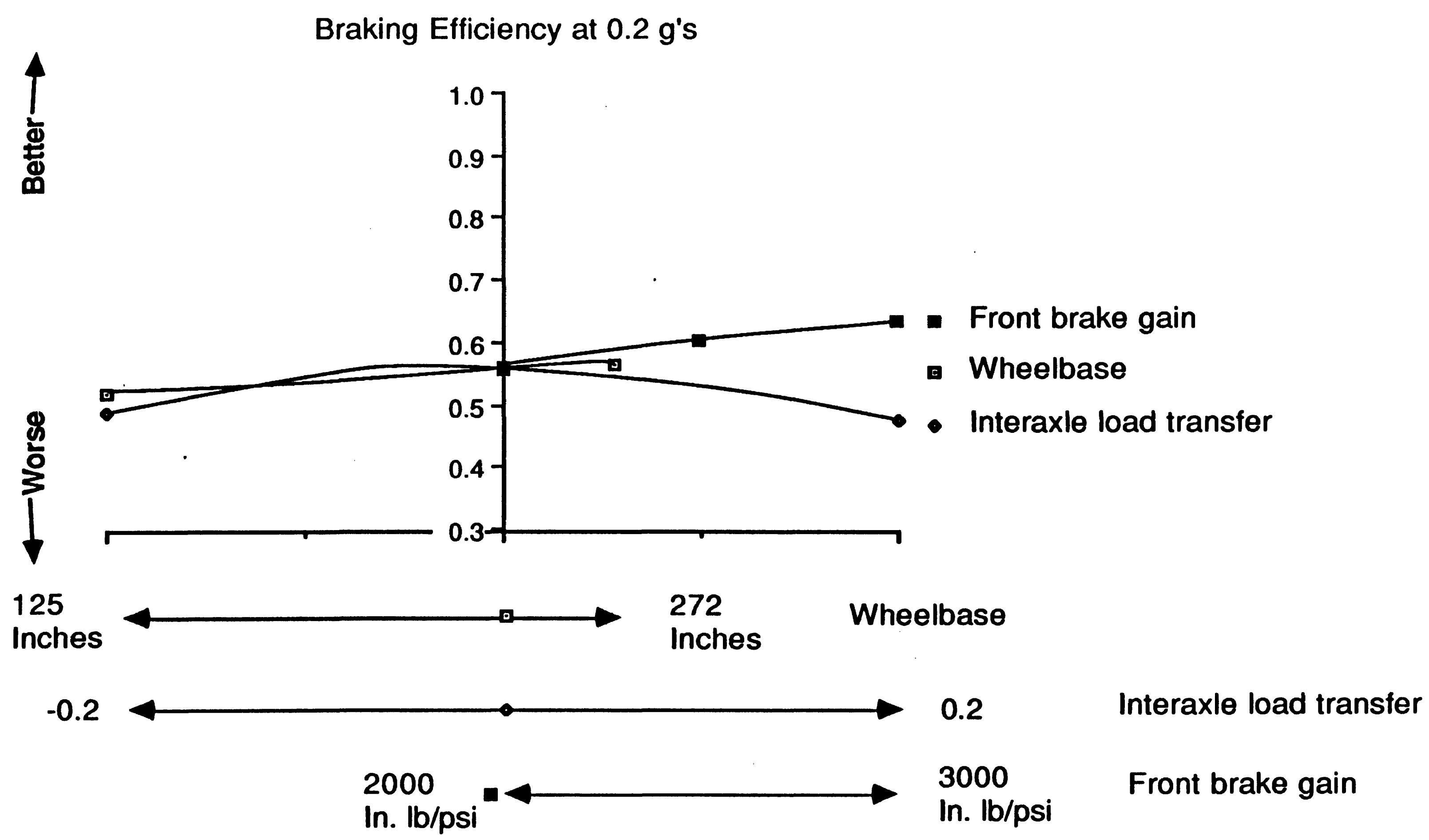


Figure 30. Parameter sensitivity diagram: braking, empty straight truck.

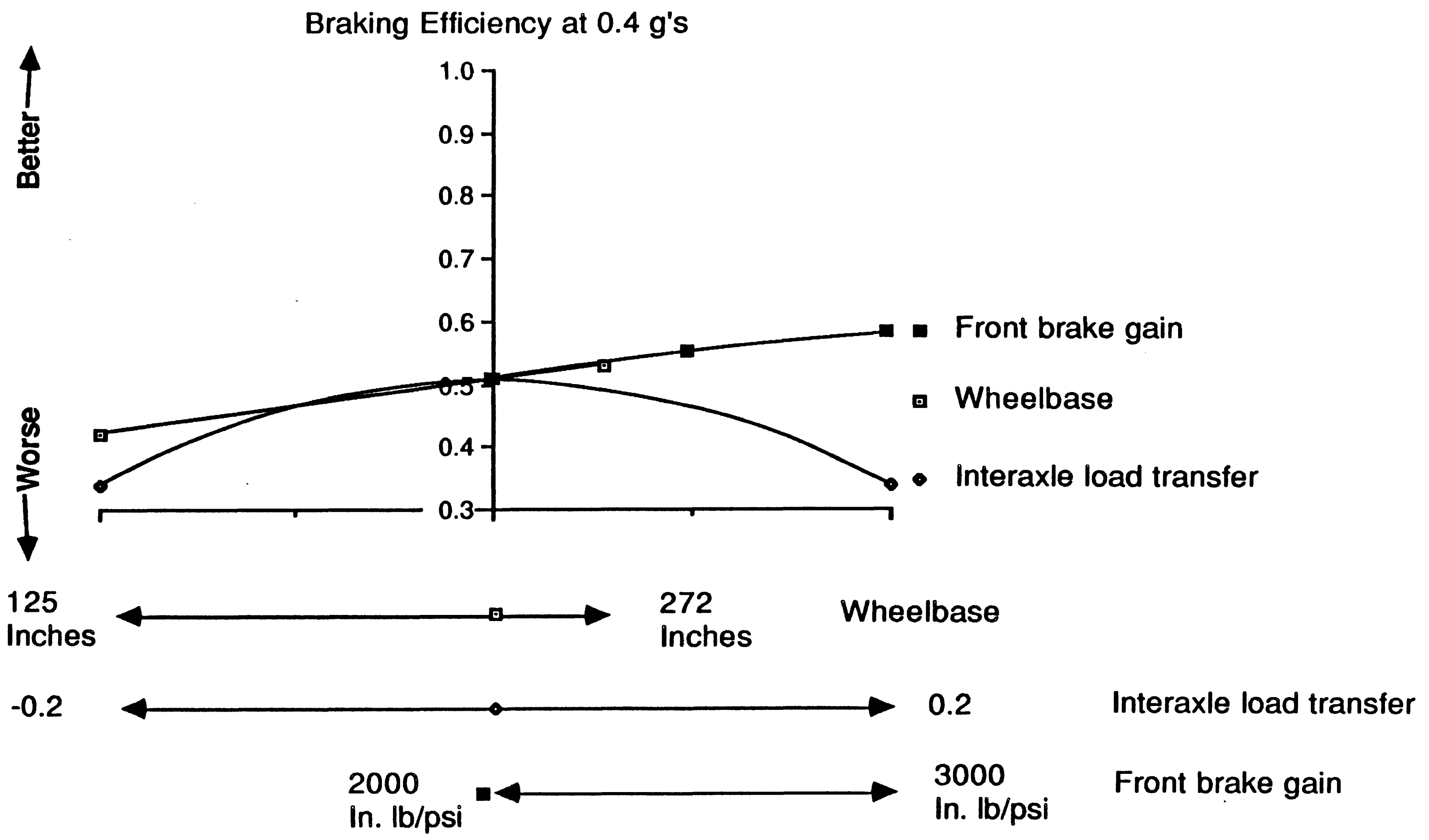


Figure 31. Parameter sensitivity diagram: braking, empty straight truck.

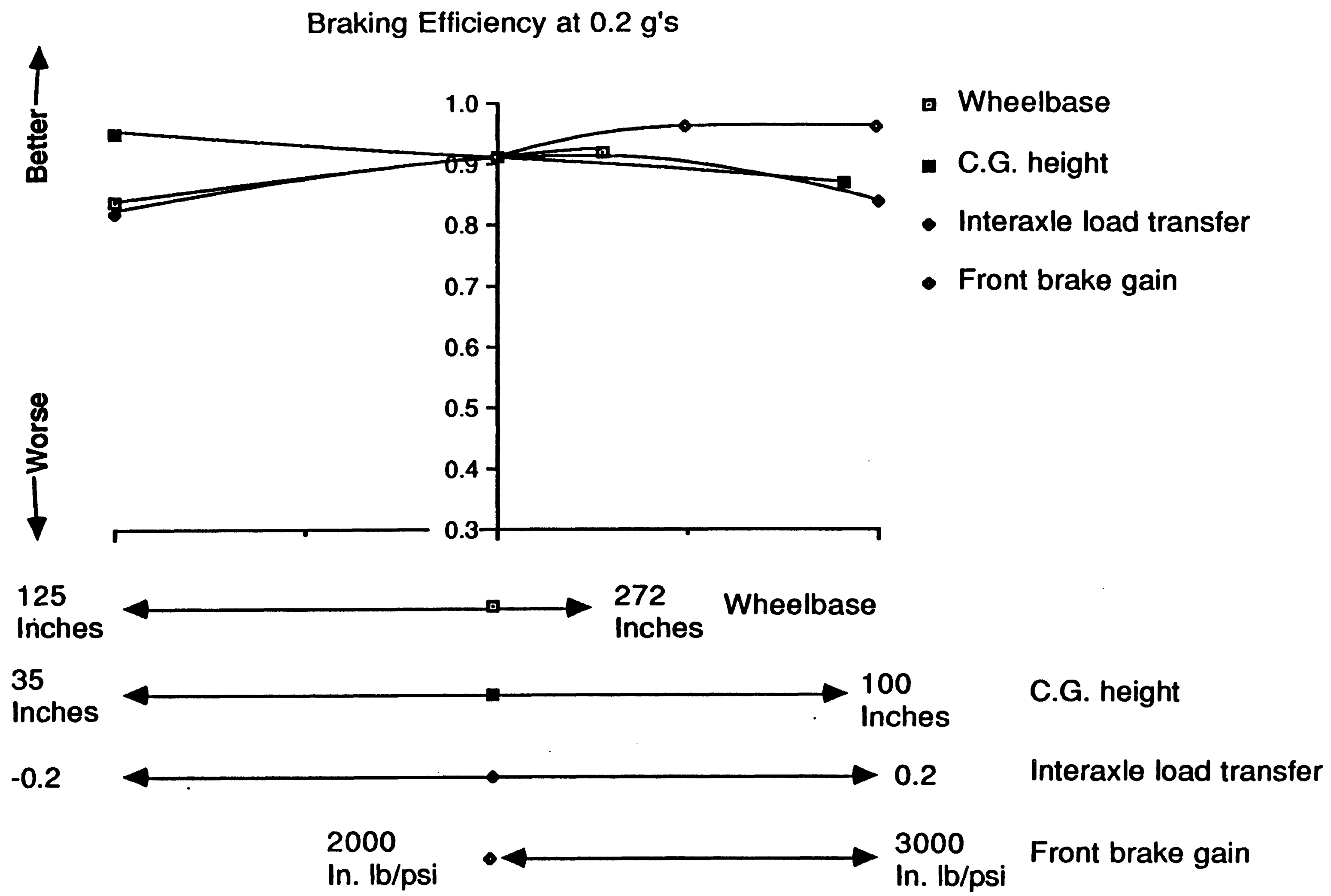


Figure 32. Parameter sensitivity diagram: braking, loaded straight truck.

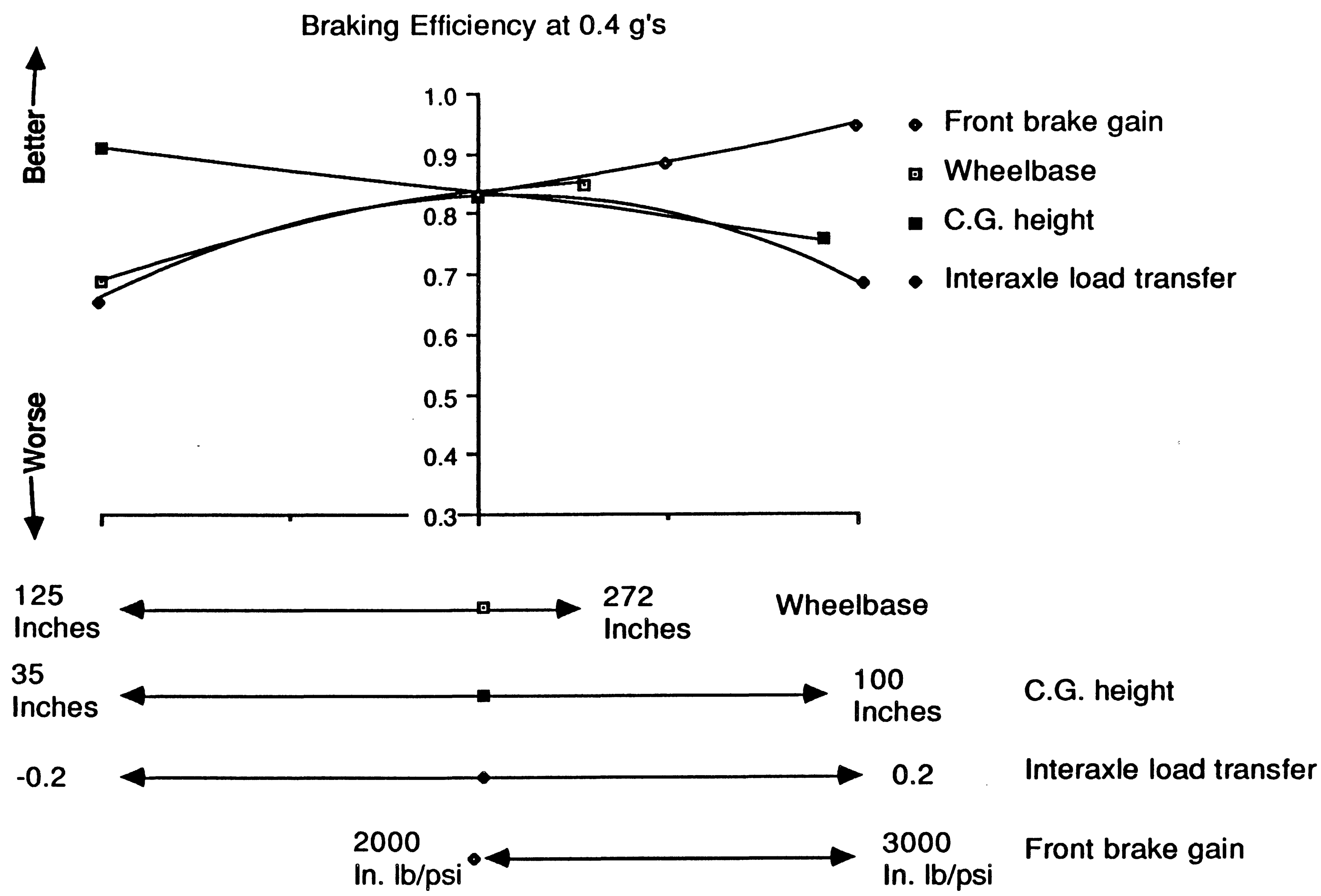


Figure 33. Parameter sensitivity diagram: braking, loaded straight truck.

Gains in braking efficiency can be made by increasing front brake gain, increasing wheelbase, and lowering c.g. height. However, changes in these properties will not solve the discrepancy between empty and loaded performance.

3.4 High-Speed Offtracking (Steady Turn - Tracking)

Benchmark performance. The specific turns treated here are specified by a 1,200-foot radius being negotiated at 55 mph and a 600-foot radius negotiated at 38 mph. For these steady turning situations the performance signature reduces to a single number, which is the offtracking of the center of the rearmost suspension. This quantity is also the performance measure for this case.

The benchmark offtracking for the 1,200-foot radius is 0.37 feet and it is 0.18 feet for the 600-foot radius. This amount of offtracking is to the outside of the turn in the opposite direction from the inboard offtracking that would occur at low speed. (See Figures 34 and 35.)

Figure 36 illustrates the transition from inboard to outboard offtracking as the velocity increases from low to high speed. At "zero" speed, the wheel plane (in this drawing all the wheels on the suspension are treated as a single centrally located wheel) is aligned with its path--the slip angle is zero. As shown in Figure 36, the radius of the wheel is inside of the radius of the center of the front axle at zero speed. Also, as indicated in the diagram, there is a speed, V_o, at which the offtracking is zero. Interestingly, this speed does not depend upon the radius of the turn, only on the pertinent mechanical properties of the vehicle (see the equations presented in Figure 36). For the benchmark vehicle, V_o is 45 feet/second, i.e., 30.7 mph. *At speeds above 30.7 mph, the offtracking of the benchmark vehicle will be toward the outside of the turn.*

Parametric sensitivities. The primary properties influencing high-speed offtracking are the wheelbase and the ratio of cornering stiffness to vertical load. In this maneuver, the rear overhang of the vehicle will offtrack more than the rear axle does (in contrast to the low-speed case where the rear axle has the maximum steady-state offtracking). Hence, the rear end of the truck is included in Table 3.

The performance sensitivity diagram presented in Figure 35 indicates that 38 mph is nearly the zero offtracking speed for vehicles with higher than baseline cornering stiffnesses, i.e., stiffnesses corresponding to stiff radial tires. These results are in keeping with the equation for V_o given in Figure 36.

Table 9. Straight Truck - High-Speed Offtracking

Parameter Name	Benchmark Value	Minimum Value	Maximum Value	Notes
Wheelbase	240 inches	125 inches	272 inches	
Truck's cornering stiffness	3743.4 lb/deg	1871.7 lb/deg	5615.1 lb/deg	Cornering stiffness is determined from loaded vehicle data

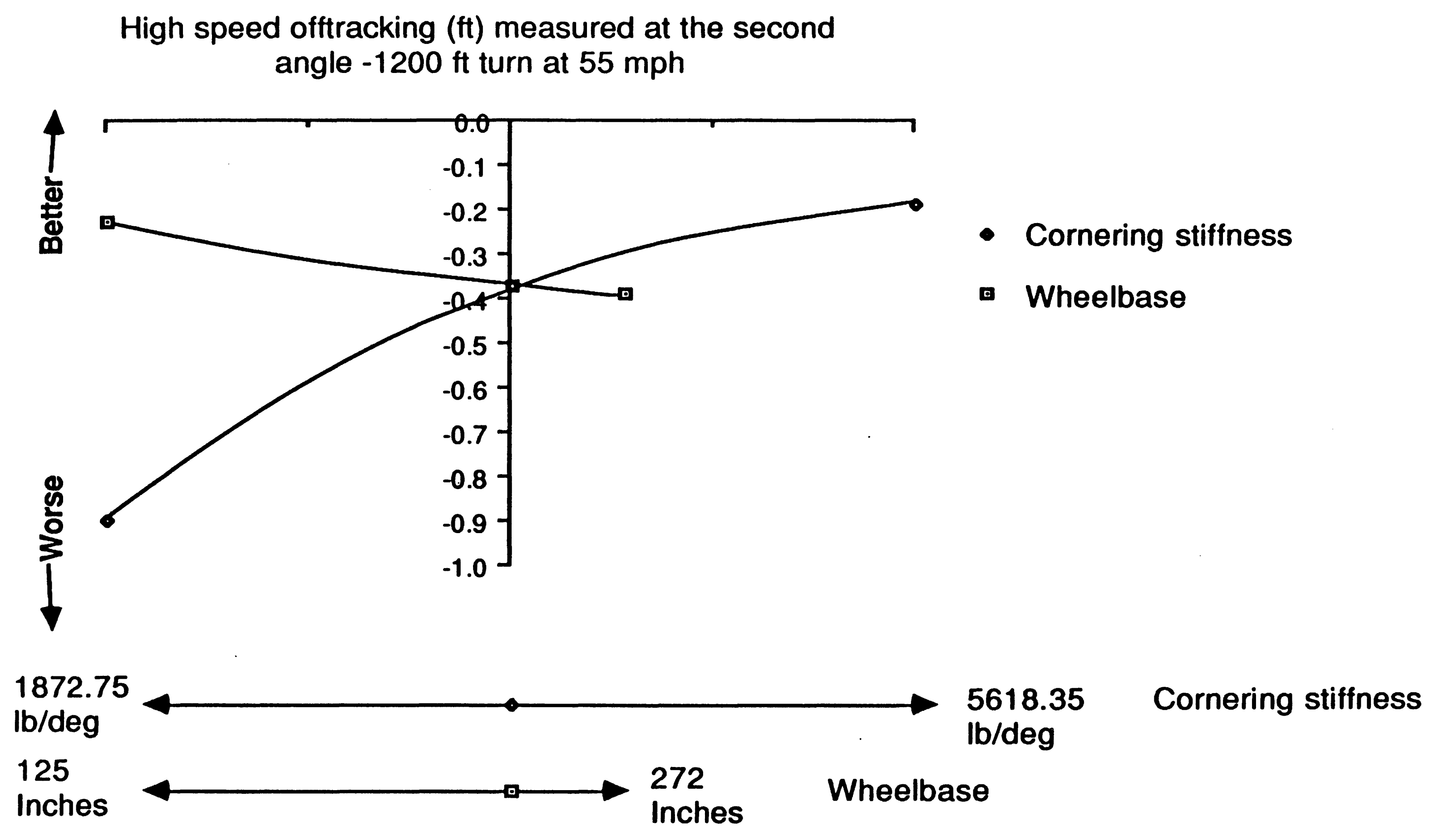


Figure 34. Parameter sensitivity diagram: high-speed offtracking, straight truck.

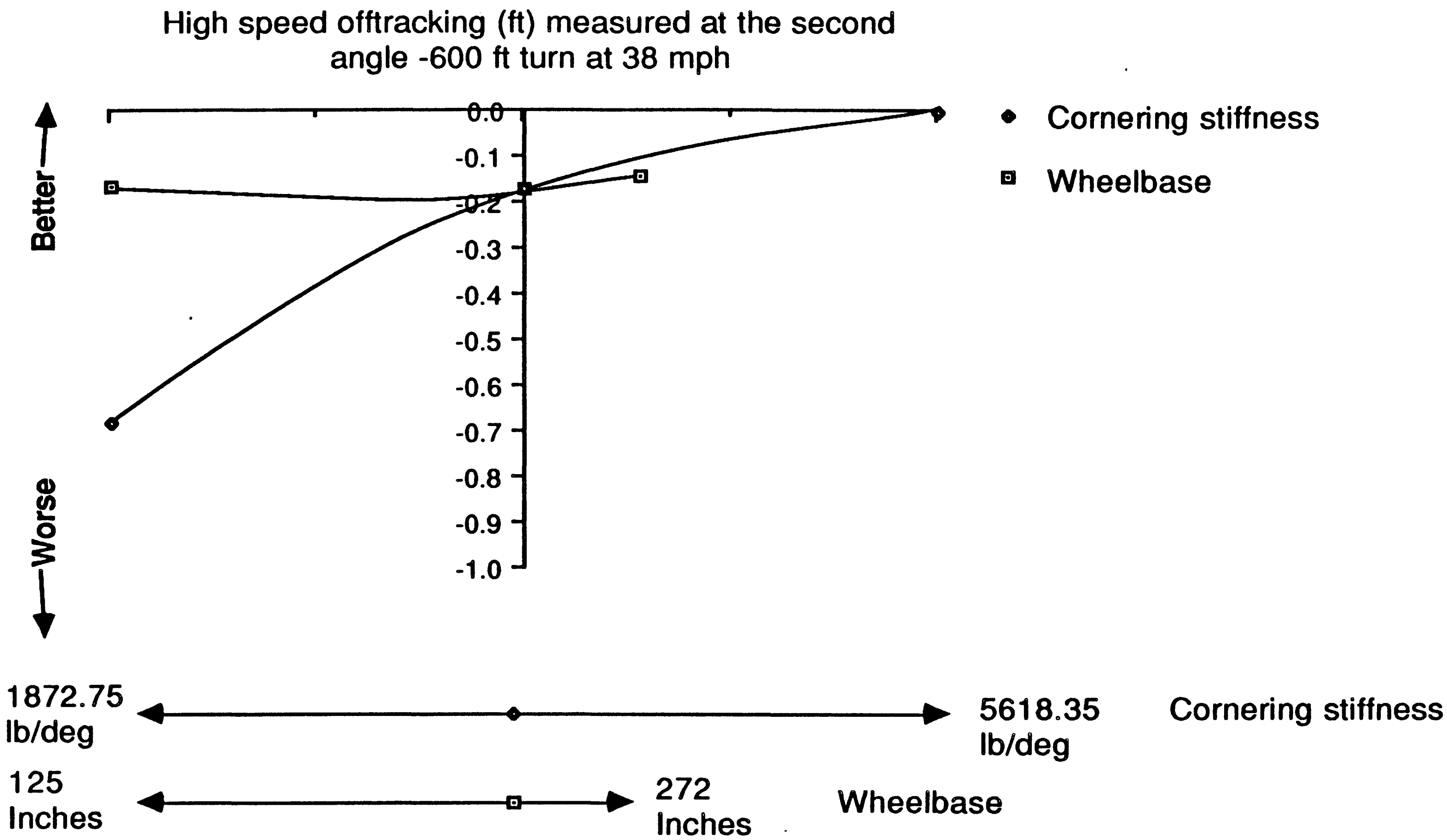


Figure 35. Parameter sensitivity diagram: high-speed offtracking, straight truck.

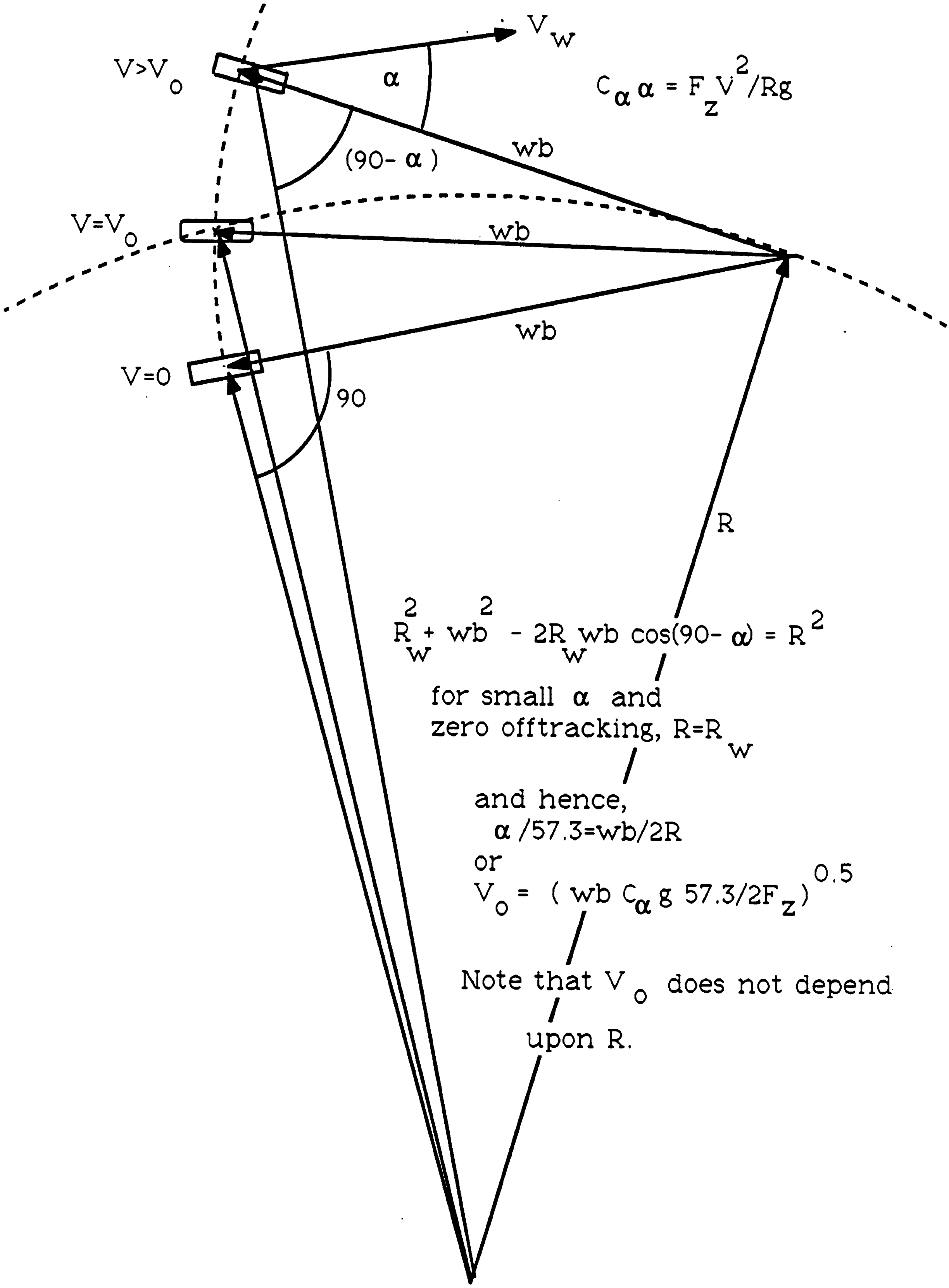


Figure 36. Illustration of V_o, the speed for zero offtracking

For both the 1,200-foot and the 600-foot turns, an increase in cornering stiffness will reduce the high-speed offtracking. However, by comparing the sensitivities given in Figures 34 and 35, it can be seen that an increase in wheelbase may either improve or degrade high-speed offtracking, depending upon the circumstances of the turn. Wheelbase influences both the zero-speed and the high-speed (slip-angle-dependent) components of high-speed offtracking. Since the zero-speed factor tends to reduce high-speed offtracking and the slip-angle-dependent factor tends to increase high-speed offtracking, the overall effect of increasing wheelbase depends upon which factor prevails. *On the other hand, cornering stiffness only influences the slip-angle-dependent factor and, hence, an increase in cornering stiffness always serves to decrease high speed offtracking.*

3.5 Steady Turn - Roll

Benchmark performance. The performance signature for the rolling performance of a straight truck has a very simple appearance because it is simply a straight line indicating the increase of roll angle with lateral acceleration up to the point where the rear wheels lift off (see Figure 37). Once the rear wheels on the inside of the turn have lifted, the front suspension does not have enough roll stiffness to prevent rollover.

The performance measure for this analysis is the rollover threshold. This occurs at the maximum lateral acceleration level reached by the performance signature. In this case the rollover threshold is at 0.368 g and a roll angle of 0.082 radians, that is, 4.7 degrees. The roll angle referred to here is the angle between a line perpendicular to the road surface and a line between the center of the wheel track and the center of gravity of the sprung mass. This angle and the height of the c.g. determine the outboard shift of the c.g. and the contribution of this shift to the rollover process. The benchmark vehicle is fairly stiff in roll such that the roll angle attained at the threshold of rollover is not very large, nevertheless, the c.g. height is high enough to result in rollover at 0.368 g.

Parametric sensitivities. The list of parametric changes given in Table 10 is extensive because it contains items that are both important and relatively unimportant. The important mechanical properties are c.g. height, track width of the rear axles, roll stiffness of the rear suspension, and the roll center height. The most fundamental parameters are c.g. height and track width. Although c.g. height can vary considerably, depending upon the density and shape of the load, these calculated results are for variations to 65 and 75 inches around the baseline value of 69.05 inches. This choice of variation yields levels of change in rollover threshold that are convenient for comparison with those obtained by varying other mechanical properties. Track width is varied from -3 to +1 inches to provide allowance for wider vehicles and small variations in wheel locations. This amount of variation is again a useful basis for comparison with other parametric changes.

Figure 38 presents results for variations in tire vertical stiffness, tire radius, and front axle roll stiffness. The likely changes in available tires have little influence on rollover threshold. In general, front suspensions are usually kept "soft" for ride purposes and, hence, they are not as important a contributor to roll stability as the rear suspension.

The influences of axle weights and front track width are illustrated in Figure 39. These quantities have negligible influences on the rollover threshold.

The mechanical properties of the rear suspension have an important influence on roll stability. All of the factors that go into determining the roll stiffness of the rear suspension are important. These include spring rate, auxiliary roll stiffness, and spring spread. The roll center height is also important. The influences of these mechanical properties are shown in Figure 40. As illustrated in the figure, these influences are comparable to those caused by changes in c.g. height of -4 and +6 inches. *Since small increases in rollover threshold have an important influence on the likelihood of rollover accidents, attention to roll stiffness, roll center height, and spring spacing is warranted, with higher roll centers and wider spring spacing being preferable.*

3.6 Steady Turn - Handling

Benchmark performance. This section starts with further background concerning the presentation of results from handling analyses. As discussed in Section 2.2, this handbook uses the types of curves shown in Figure 41 rather than those presented in a classical handling diagram. The steer angles required for equilibrium at various levels of velocity and lateral acceleration are presented in Figure 41a (in this case, for an example of a straight truck with tandem rear axles). To provide insight into the geometry of these turns, dashed lines corresponding to 1,000- and 2,000-foot radii have been superimposed in the figure. Inspection of the figure shows that at each velocity (40, 55, and 70 mph) the curves "hook over" to a zero slope in the range from 0.3 to 0.35 g.

The slopes of the curves presented in Figure 41a are plotted in Figure 41b. These slopes are referred to as the "steering sensitivity" in this handbook. The curves indicate that the steering sensitivity, at a given velocity, remains fairly constant up to about 0.1 g. At 0.2 g, the steering sensitivity is starting to decrease rapidly as acceleration increases. When the steering sensitivity approaches zero, the steady state response to steering wheel inputs is characterized by a very high

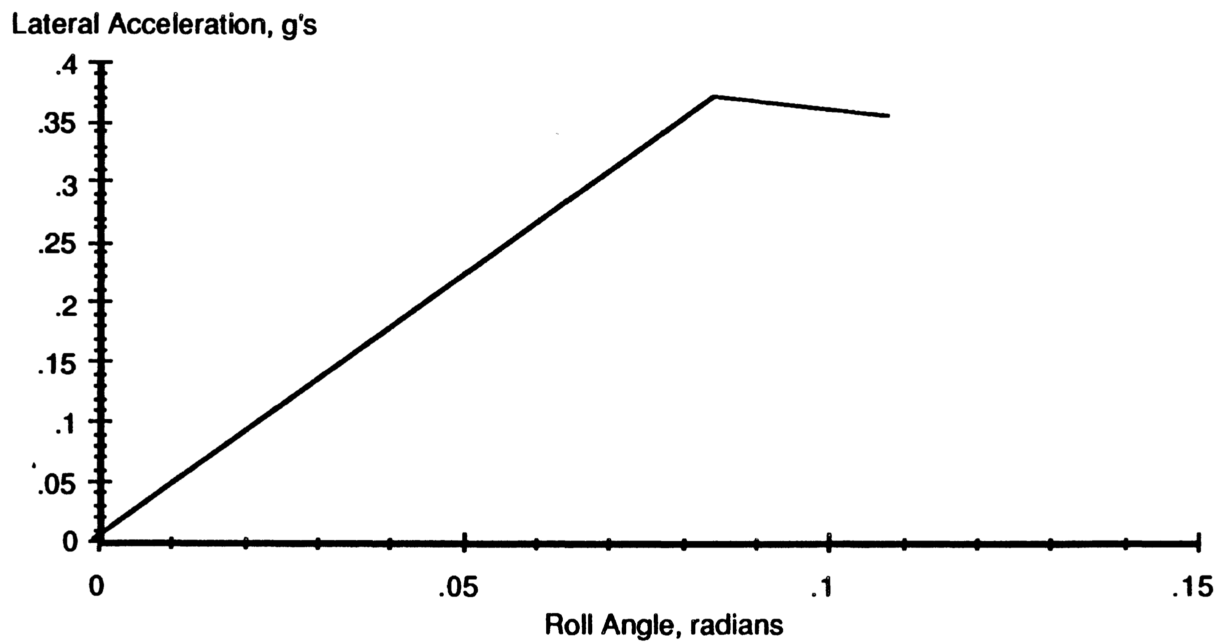


Figure 37. Performance signature, static roll, straight truck

Table 10. Straight Truck - Static Roll

Parameter Name	Benchmark Value	Minimum Value	Maximum Value	Notes
Total C.G. height	69.05 inches	65 inches	75 inches	Much larger deviations used for braking
Rear axle roll center heights	29 inches	21 inches	31 inches	
Tire vertical stiffness	4500 lb/in	4000 lb/in	5000 lb/in	
Tire radius	19.5 inches	18.5 inches	20.5 inches	
Front axle roll stiffness	21,000 in.lb/deg	17,000 in.lb/deg	25,000 in.lb/deg	
Front axle track width	80 inches	77 inches	81 inches	
Front axle weight	1200 lb	1150 lb	1250 lb	
Rear suspension roll stiffness	160,000 in.lb/deg	60,000 in.lb/deg	330,000 in.lb/deg	Total roll stiffness on rear tandem
Rear axle track widths	72 inches	71 inches	78 inches	
Rear axle weights	2300 lb	2250 lb	2350 lb	Per axle

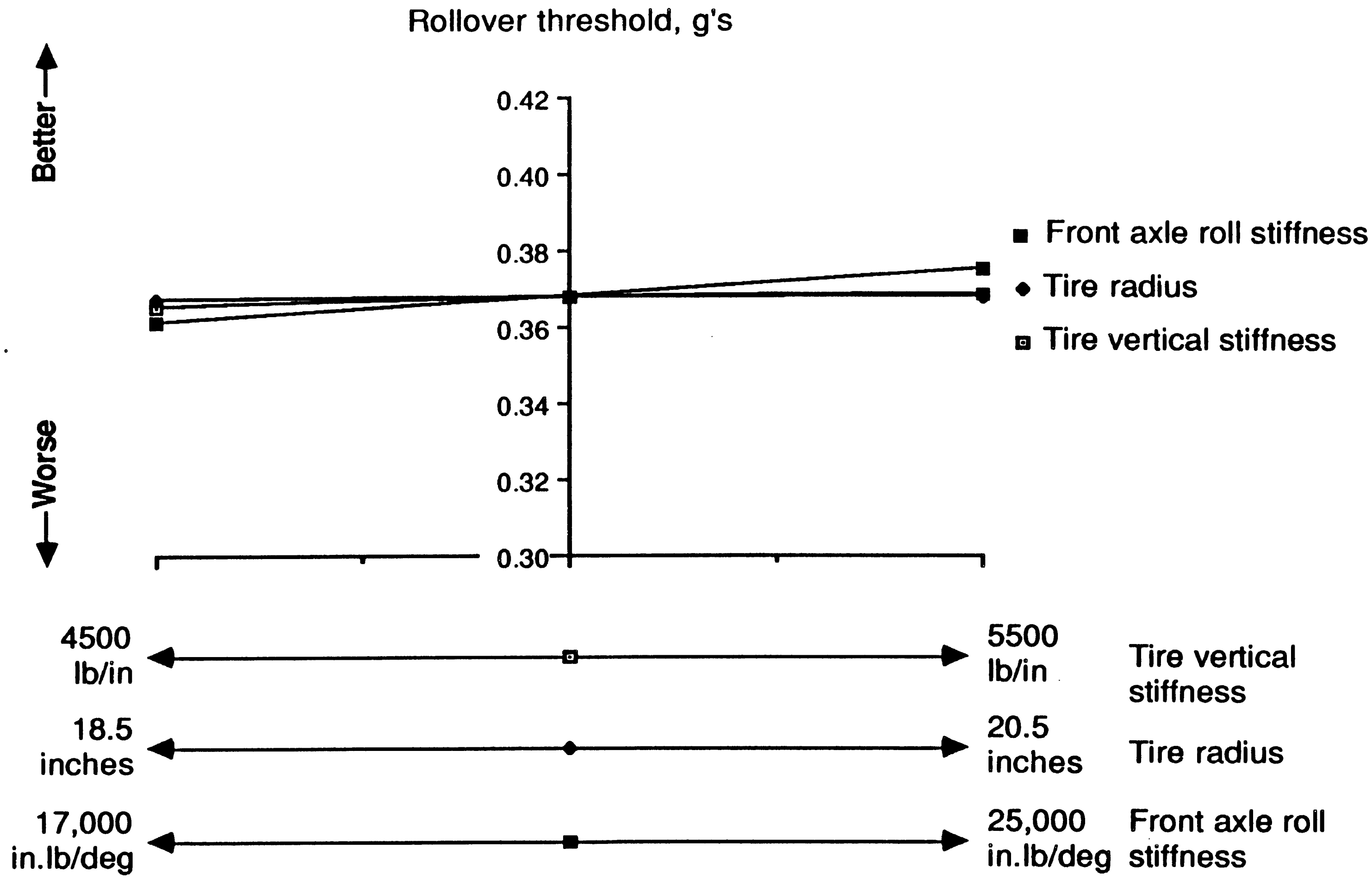


Figure 38. Parameter sensitivity diagram, static roll, straight truck.

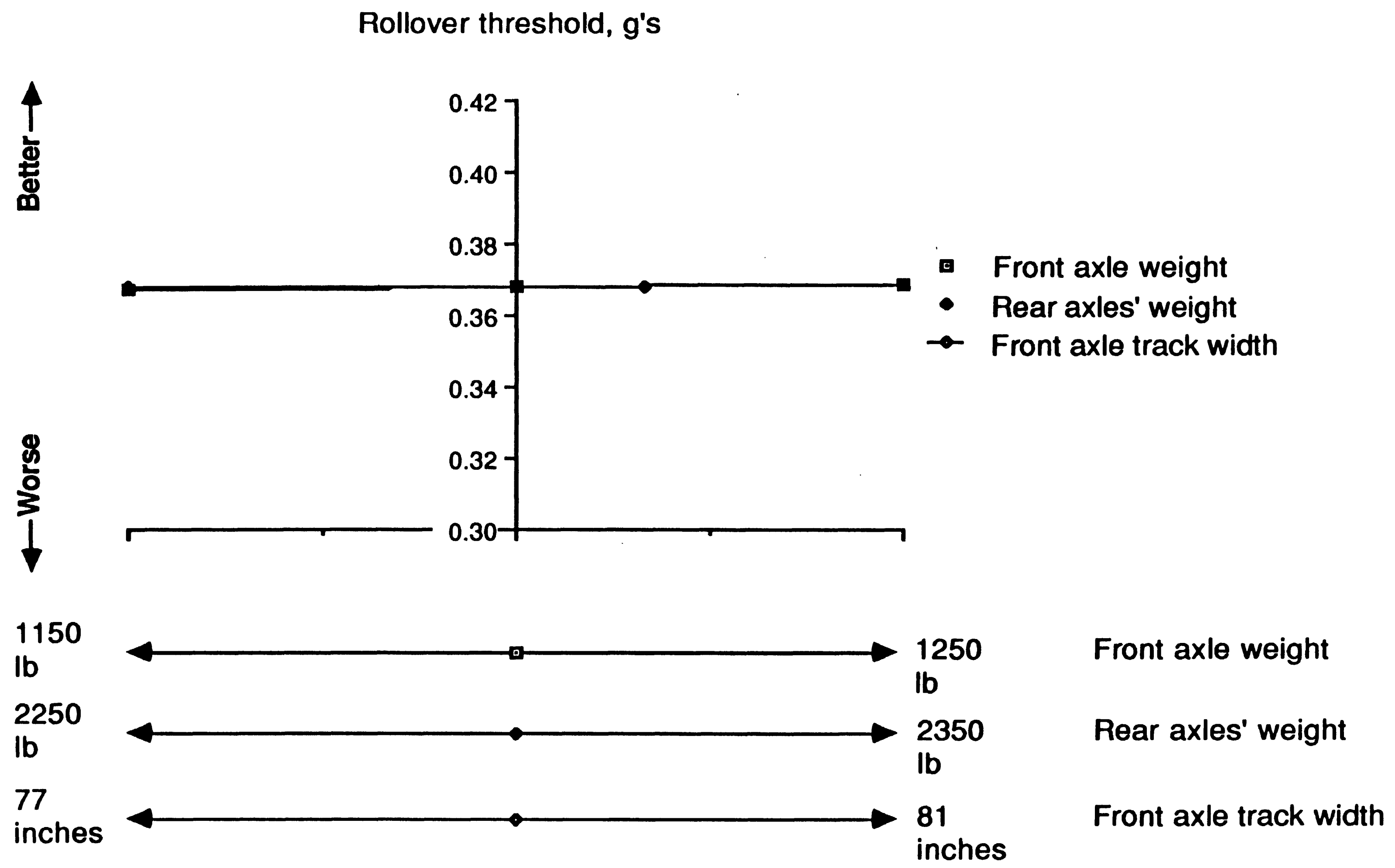


Figure 39. Parameter sensitivity diagram: static roll, straight truck.

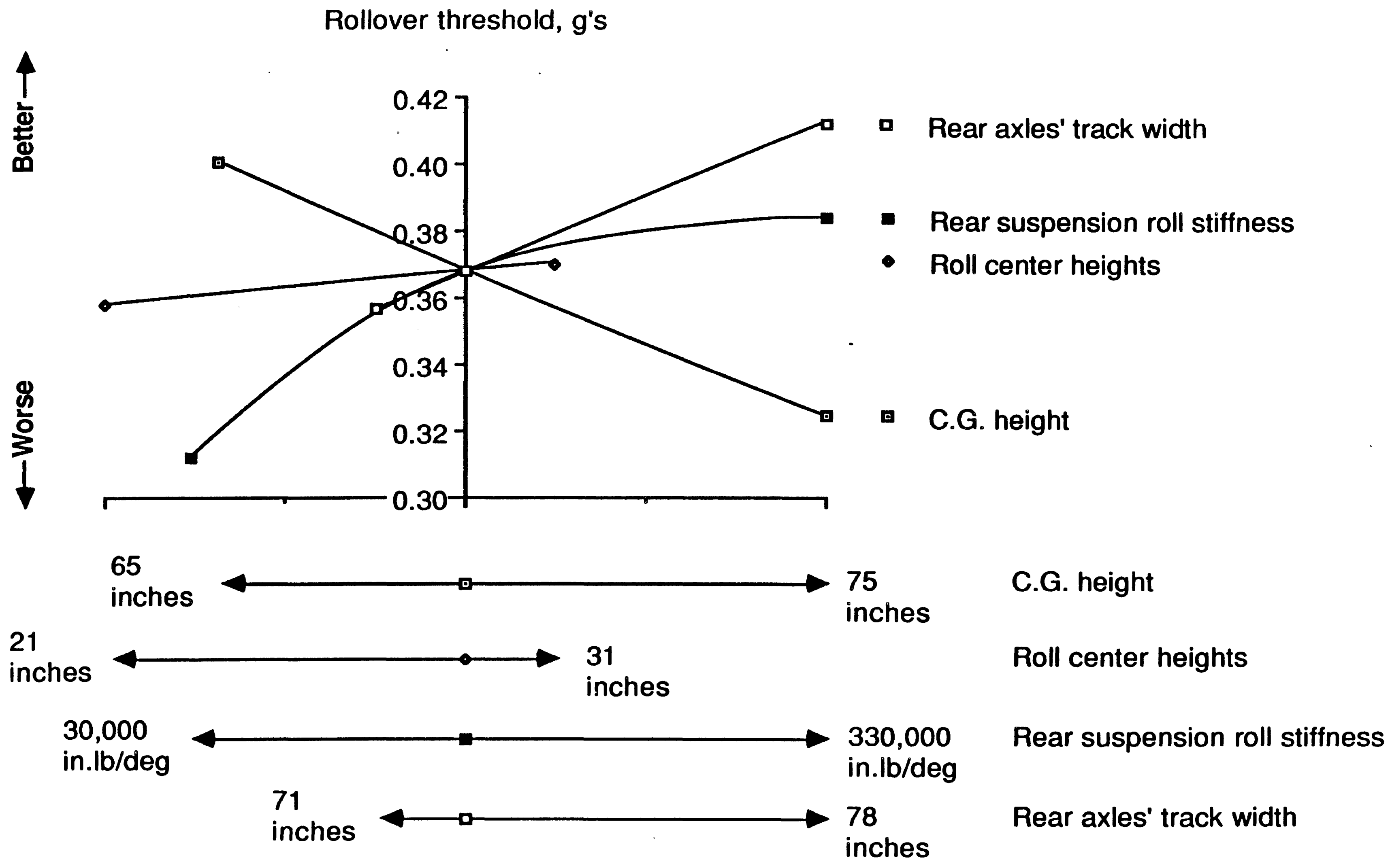
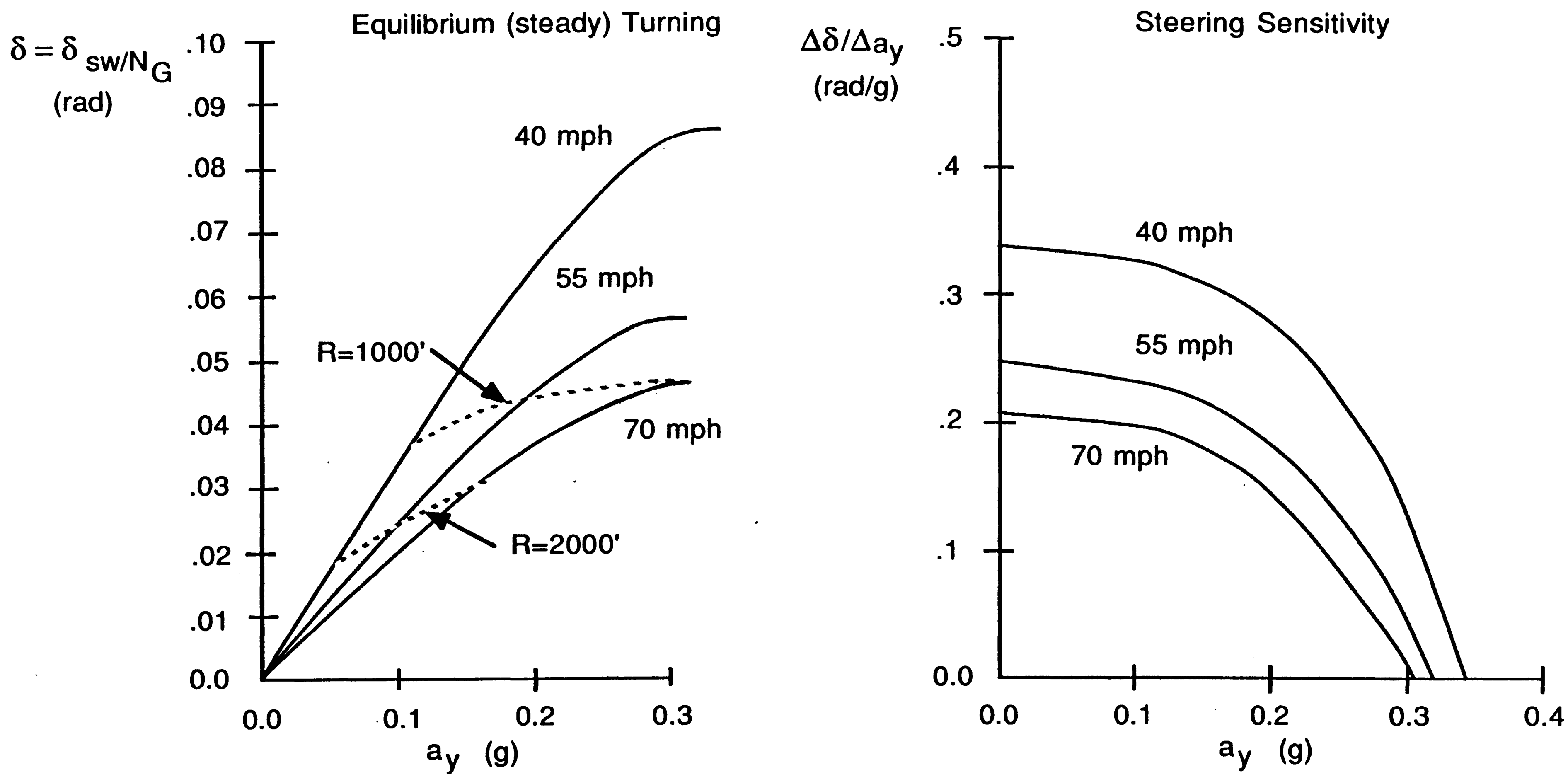


Figure 40. Parameter sensitivity diagram: static roll, straight truck.



41 a.

41 b.

Figure 41. Steady turning characteristics and steering sensitivity

Simple Models:T.Str.Truck

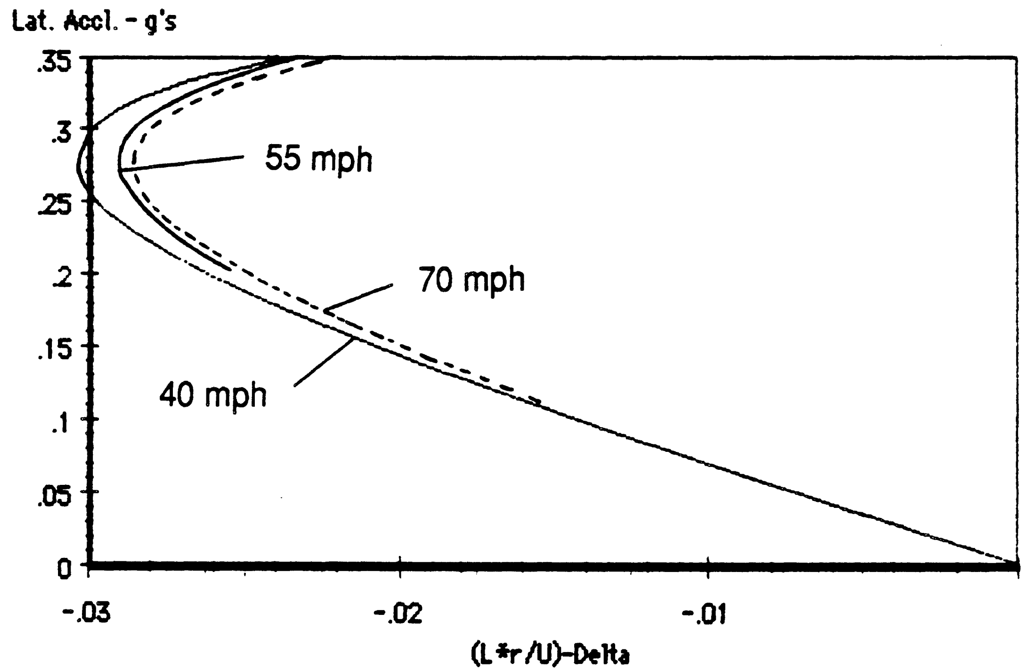
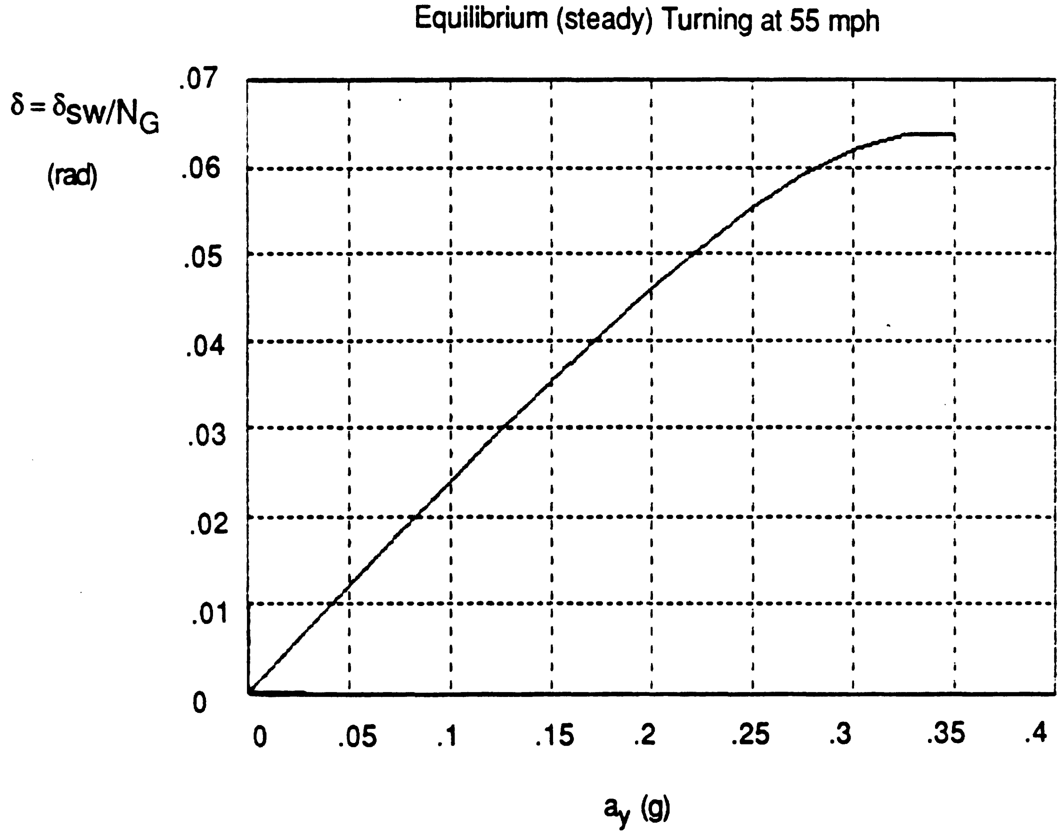


Figure 42. Handling diagrams at 40, 55, and 70 mph for the benchmark vehicle



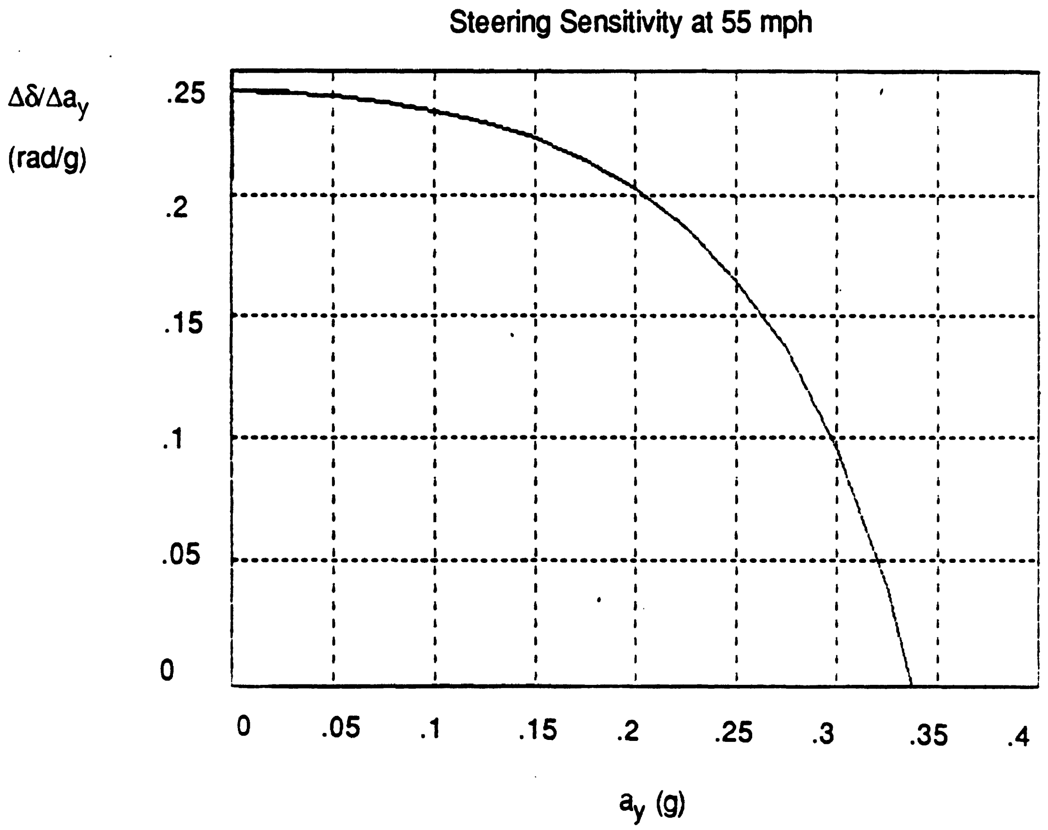


Figure 43. Performance signature, handling, straight truck

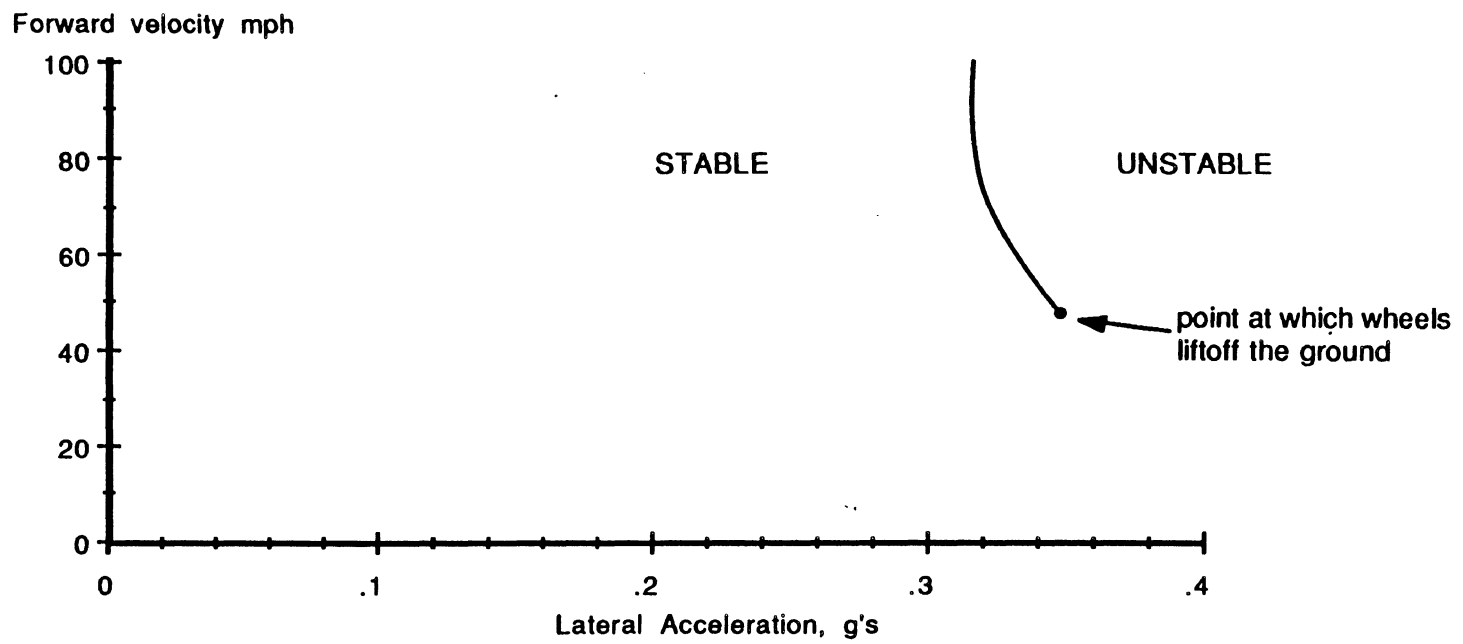


Figure 44. Yaw stability boundary, performance signature, straight truck

gain . Those points where the steering sensitivity curves intersect the horizontal axis in Figure 41b correspond to the boundary between stable and yaw divergent performance of the vehicle.

For example as shown in Figure 41a, if a driver were attempting to follow a 1,000-foot radius at 70 mph (approximately 0.3 g's of lateral acceleration), the equilibrium steering input would be approximately 0.046 radians (or roughly 80 degrees at the steering wheel if the steering ratio were about 30). However, if the driver were to apply slightly more steering, the steering sensitivity would pass through zero (see Figure 41b). There would not be a stable equilibrium point and there would not be any constant steering input that could be used to maintain a steady turn. In this case, the driver must continually modulate the steering to stabilize the vehicle. (Of course, the driver could slow down, thereby reducing the lateral acceleration to a stable level for the radius of this turn.)

In heavy trucks the response to steering wheel inputs may be characterized by a very high gain depending upon the distributions of tire forces and suspension stiffnesses and the nature of truck tire characteristics. Results of the type presented in Figure 41b can be used for these vehicles to see if the steering sensitivity can become zero and, if so, to determine the speeds and accelerations which correspond to infinite gain. As mentioned previously, these speed-acceleration points define the boundary between stable and yaw divergent turns. A plot of this boundary provides a secondary performance signature for those vehicles which, at highway speeds, become yaw divergent before they roll over.

(The data presented in Figure 41a can also be presented in a handling diagram as illustrated in Figure 42. Since the truck used in this example has a pair of axles on the rear, there are three handling curves instead of one as there would be for a passenger car. Nevertheless, the influence of velocity is not large, particularly between the speeds of 55 and 70 mph. In this handbook, and for vehicles with multiple (yaw redundant) axles, results at 55 mph will be used as the performance signature. (For vehicles without redundant axles the handling curve is independent of velocity.))

Per the above discussion, the handling performance signature for the *benchmark* vehicle (see Figure 43) contains plots of required steering angle and steering sensitivity evaluated at a forward speed of 55 mph. The stability boundary between yaw stable and divergent operating conditions is displayed in the second performance signature for this heavy truck (see Figure 44). The stability boundary is plotted in a space that has lateral acceleration and velocity as its coordinates.

Table 11. Straight Truck - Handling

Parameter Name	Benchmark Value	Minimum Value	Maximum Value	Notes
Total C.G. height	69.05 inches	65 inches	75 inches	Much larger deviations used for braking
Wheelbase	240 inches	125 inches	272 inches	
Rear axle roll center heights	29 inches	21 inches	31 inches	
Front axle roll stiffness	21,000 in.lb/deg	17,000 in.lb/deg	25,000 in.lb/deg	
Front axle track width	80 inches	77 inches	81 inches	
Tires on the front axle	Bias-ply, rib tread	Bias-ply, rib tread	Radial	
Rear suspension roll stiffness	160,000 in.lb/deg	60,000 in.lb/deg	330,000 in.lb/deg	Total roll stiffness on rear tandem
Rear axle track widths	72 inches	71 inches	78 inches	
Tires on the rear axles	Bias-ply, rib tread	Bias-ply, lug tread	Radial	
Steering stiffness	11,000 in.lb/deg	11,000 in.lb/deg	40,000 in.lb/deg	

Parametric sensitivities. *The mechanical characteristics of primary importance in determining the yaw-divergent tendencies of straight trucks are:*

(1) fore/aft cornering stiffness distribution

(2) the sensitivity of cornering stiffness to changes in vertical load

(3) wheelbase

(4) fore/aft roll stiffness distribution

(5) c.g. height

(6) steering system compliance

From a phenomenological point of view, divergent yaw response (spinout) is caused by more side force from the front tires than that which the rear tires will stabilize. Equilibrium is attained by a moment balance between the forces generated by the front and rear tires. For loaded heavy trucks, large amounts of side-to-side load transfer take place at lateral acceleration levels of 0.2 g and above. This load transfer will be reacted more at the axles that are stiffer in roll, and the side force capability of those axles will be reduced. Specifically, for the benchmark truck the load transfer effect is greater at the rear axles and anything that tends to increase this load transfer will degrade performance.

In summary, if front tire forces are increased relative to the rear (or, if rear forces are decreased relative to the front), stability will be decreased.

Table 11 lists the variations that have been used to illustrate the parametric sensitivities of the benchmark vehicle.

Figure 45 indicates the performance sensitivities of variations in wheelbase, c.g. height, rear roll center height, and rear tandem roll stiffness. The wheelbase of any unit has an important influence on the damping in yaw, and longer wheelbases will improve the stability margin available for avoiding yaw divergence. C.g. height, rear tandem roll stiffness, and roll center height all influence roll and, thereby, load transfer. When the c.g. height is increased to 75 inches, the vehicle becomes directionally unstable as indicated by the slightly negative value shown in Figure 45. Although the variations in roll center height have a small effect, the decrease in rear tandem roll stiffness provides a dramatic increase in the yaw stability margin. (This is achieved at the expense of a reduced rollover threshold, however.)

The influences of parameters associated with the front end of the truck are illustrated in Figure 46. As shown, stiffening the front suspension in roll will increase the stability margin. The changes in track width at the front axle are not large enough to have significant influences on the results. Stiffening the *effective* cornering stiffness at the front will decrease the stability margin.

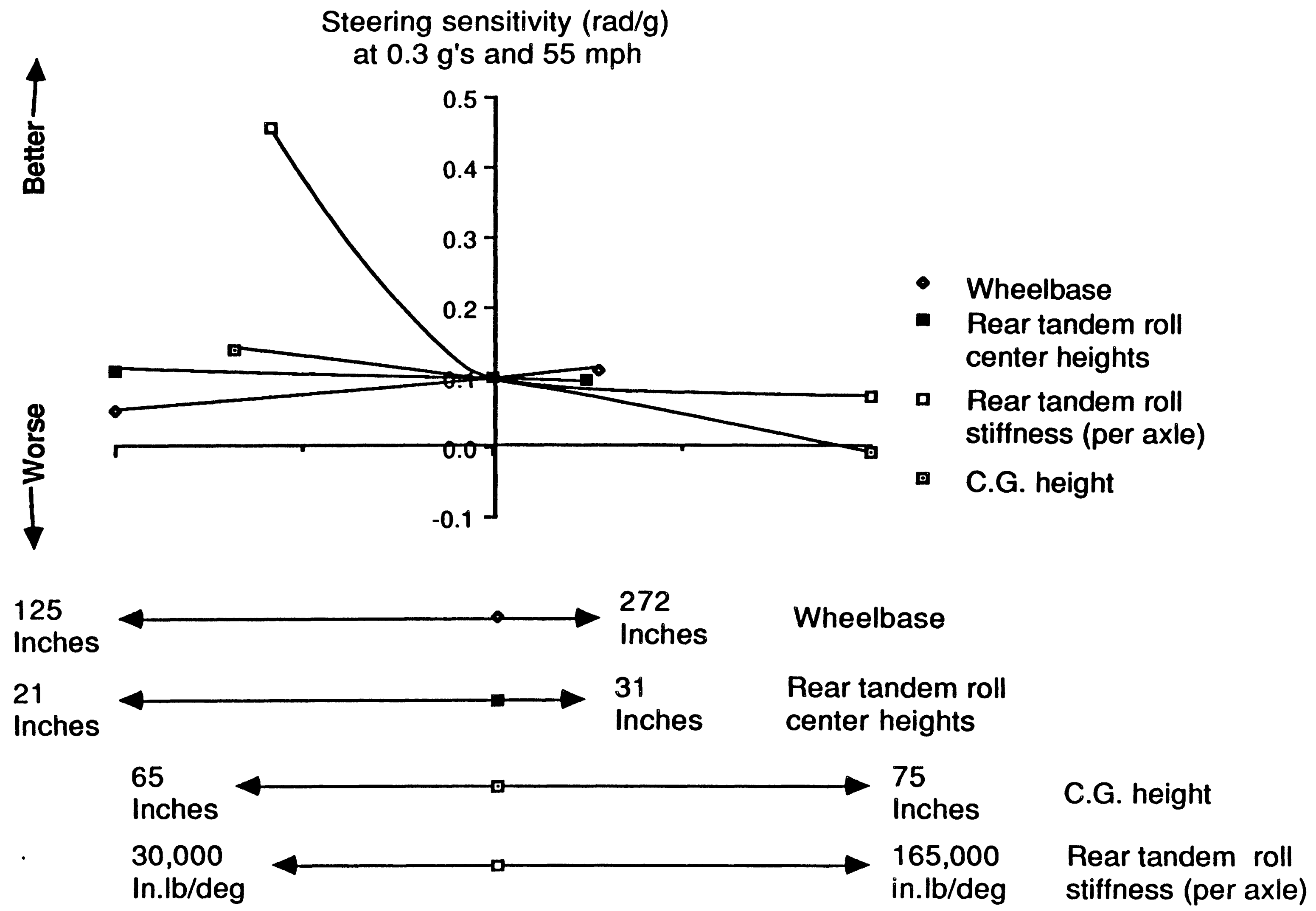


Figure 45. Parameter sensitivity diagram: steering sensitivity, straight truck.

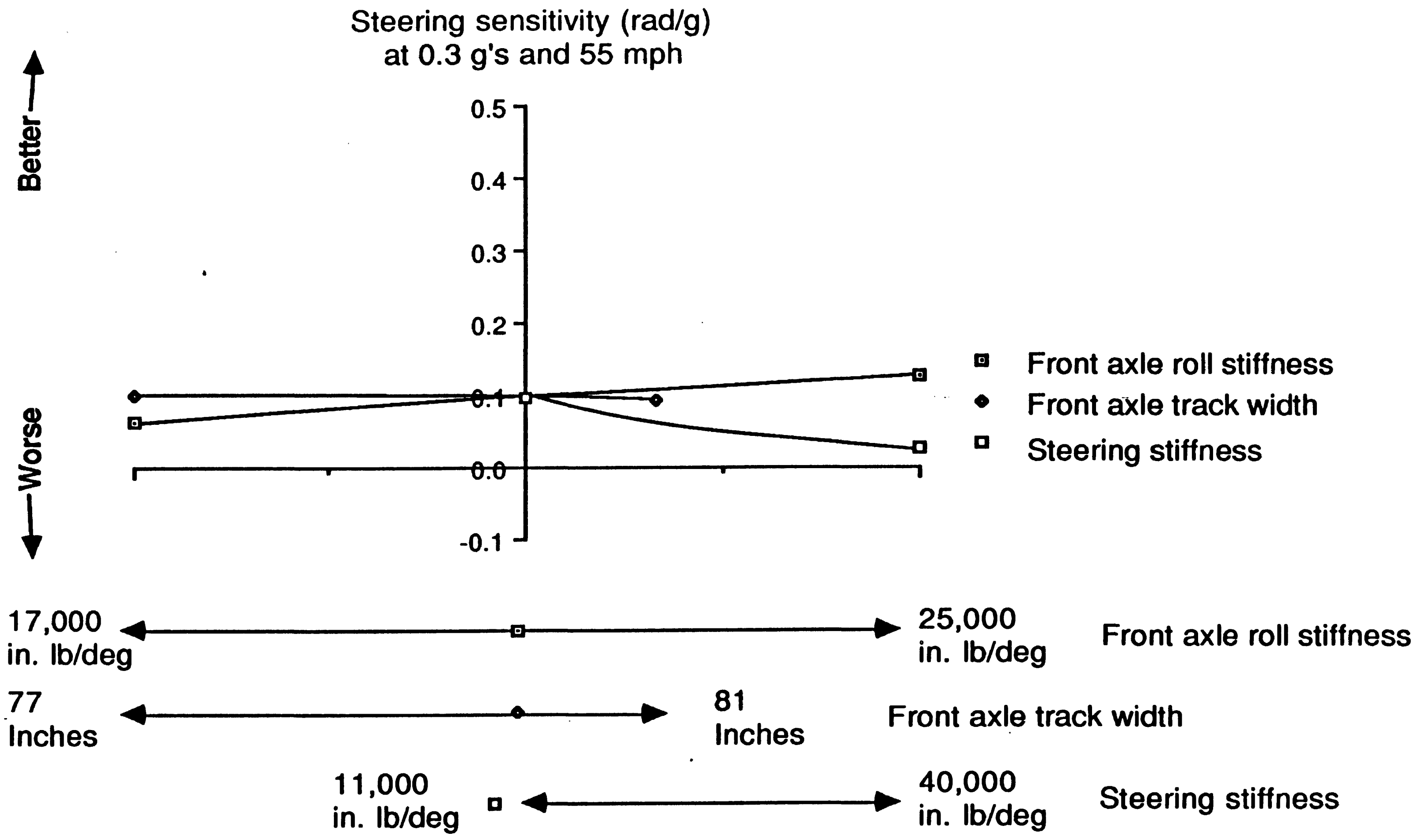


Figure 46. Parameter sensitivity diagram: steering sensitivity, straight truck.

This effective stiffening can be achieved by either increasing the cornering stiffness of the tires or increasing the stiffness of the steering system. The steering system stiffness acts like a spring in series with the tires, and if this stiffness is increased, the front tires will become more effective in generating side force at a given level of lateral acceleration. *To improve handling performance of the benchmark vehicle, one would like more load transfer and less cornering stiffness at the front end.*

(Also to improve handling performance, one would like less load transfer at the rear tires and more cornering stiffness--just the opposite of the situation for the front. One might think that reducing the roll stiffness at the rear suspension would necessarily cause less load to be transferred at the rear suspension and thereby improve handling. However, reducing the rear roll stiffness could move the performance of the vehicle closer to the rollover threshold and could increase the load transfer at the rear tires. If this happens, the stability margin could decrease.)

Included in Figure 47 are results for various tire arrangements. These results illustrate findings that merit explanation. For example, the case corresponding to stiff radial tires all around is roughly equivalent (very slightly worse) than cases with various combinations of bias-ply rib or lug tires on the front and rear of the vehicle. For this vehicle, the worst arrangement is to have stiff radial tires on the front axle with considerably more compliant bias ply tires on the rear axles. The arrangement with the greatest stability margin is one with radial tires on the rear wheels and bias ply tires on the front wheels.

All of these results concerning tires are in keeping with the idea of having stiffer tires (per unit load carried) on the rear as compared to the cornering stiffness per unit load carried on the tires installed on the front. Nevertheless, there can be situations in which reductions in tire cornering stiffnesses due to load transfer effects will be large enough to reduce the stability margin by a surprising amount, even though the nominal level of cornering stiffness is quite high.

Detailed Discussion of Tire Properties. Heretofore, the designation of tire properties has been by generic names such as radial, bias ply rib, and bias ply lug. The analyses have been based on representative samples of these types of tires. The following discussion provides engineering definitions and values for the pertinent mechanical properties of the tires employed in these examples.

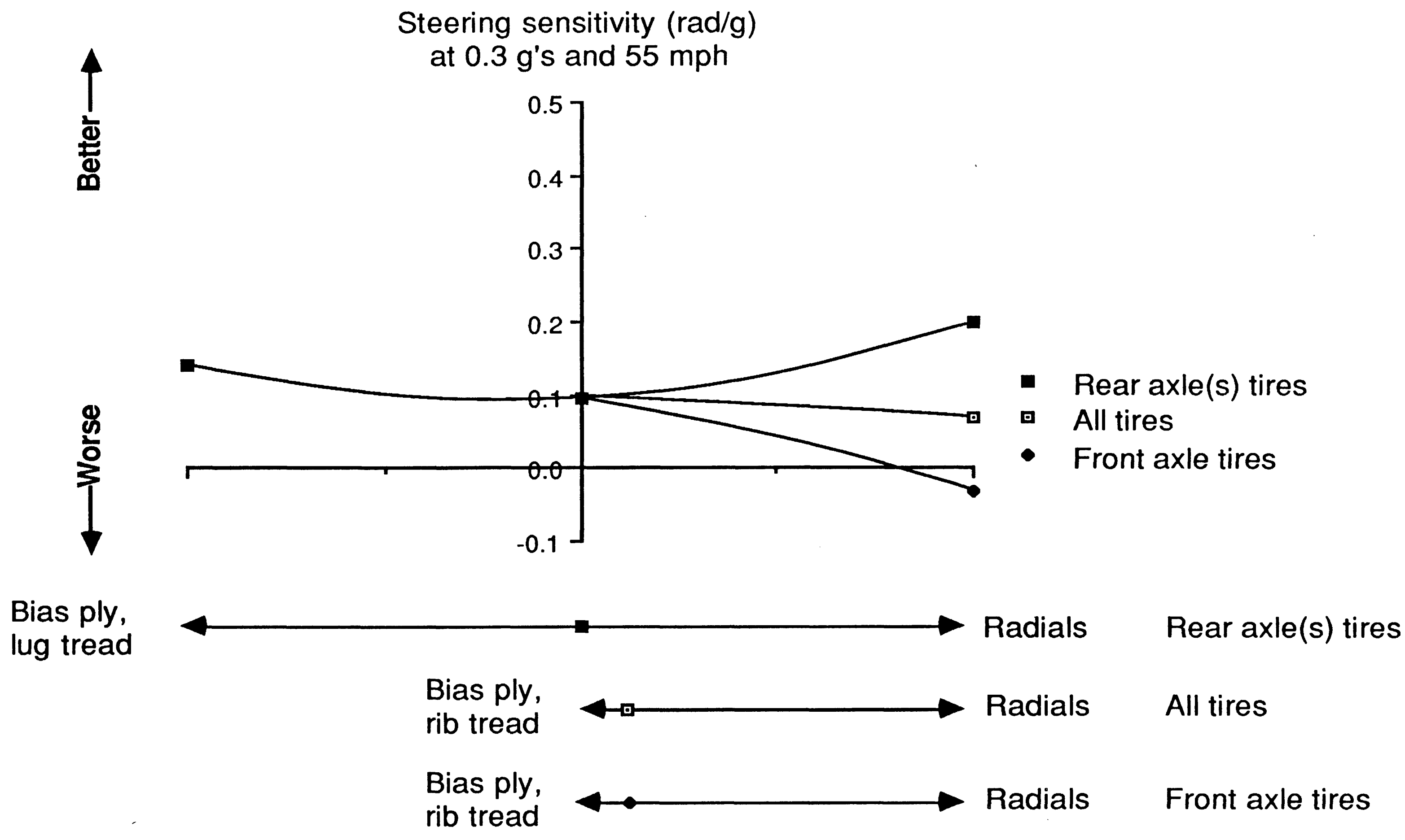


Figure 47. Parameter sensitivity diagram: steering sensitivity, straight truck.

To a good approximation, the relationship of cornering stiffness to vertical load is expressed by the following equation for a single tire:

$$C_\alpha = C_0 + C_1 (F_Z - F_N) + C_2 (F_Z - F_N)^2 \qquad (1)$$

where C_0 is the nominal cornering stiffness evaluated at F_N,
C_1 is the linear coefficient of the load transfer $(F_Z - F_N)$,
C_2 is the curvature coefficient,
and F_Z is the vertical load.

Since the load is transferred equally from side to side, the linear term, C_1, does not influence the change in cornering stiffness due to side to side load transfer. That is, the total cornering stiffness on an axle, C_T, is given by the following equation:

$$C_T = N(C_0 + C_1(F_Z - F_N) + C_2(F_Z - F_N)^2 + C_2 \Delta F_Z^2) \qquad (2)$$

where $N(C_0 + C_1(F_Z - F_N) + C_2(F_Z - F_N)^2$ is the cornering stiffness evaluated at the static load, $\pm\Delta F_Z = \pm (F_Z - F_N)$ is the load transferred from one side to the other, and where N is the number of tires on the axle.

Inspection of equation 2 indicates that two tire properties, C_α evaluated at static load and C_2 (see Figures 13 and 14), influence the total cornering stiffness acting at an axle during a steady turn.

The values of these properties for the tires used in this analysis are presented in the following table:

Table 12. Cornering Stiffness Parameters

Generic Name	C_α at 5430 lbs (lb/deg)	$C_1 (10)^2$ $(deg.)^{-1}$	$C_2 (10)^6$ $(lb. deg.)^{-1}$
Bias Ply Rib	500	2.49	-10.83
Radial	800	5.65	-18.06
Bias Ply Lug	481	3.32	-7.83

The values of the curvature coefficients are all negative, indicating a loss in side force with an increase in load transfer. Note that the radial tire has a much greater curvature coefficient than either of the bias ply tires. Hence axles equipped with radial tires stand to lose more side force in a 0.3 g turn than axles equipped with bias or lug tires. The loss for rear axles equipped with radial tires may be large enough to nearly offset the high level of cornering stiffness available at 5,430 lbs.

The tire influences illustrated in Figure 47 may be understood by applying the following observations: (1) load transfer is small at the front axle, and hence the side force capability of the tires installed on the front axle is dependent primarily on the value of tire cornering stiffness at the static load and not on the curvature coefficient and (2) the curvature coefficient determines the amount of destabilization that takes place due to load transfer at the rear axles. For example, for the benchmark vehicle equipped with radial tires on all axles, the front tires have high side force capability because the radial tires are very stiff at the loads carried by the front tires, but the total side force capability of the tires installed on the rear axles is relatively low, approaching that of a lug or bias tire, because there is a large load transfer at the rear axles and the radial tires are very sensitive to load transfer, that is, they have a large negative curvature coefficient.

Clearly, these results imply that knowing the generic types of tires is not sufficient to make even qualitative estimates of handling performance in severe turns approaching rollover conditions. The influence of load on cornering stiffness, as well as the static load level of cornering stiffness, needs to be known. This means that detailed tire information is required if one attempts to specify tires that will provide a stability margin for safe handling at highway speeds.

3.7 Response Times in Steering Maneuvers

Benchmark performance. Response times have been studied using two maneuvers, namely, ramp-step steer and closed-loop obstacle evasion. The ramp-step steer simulates a proving grounds test in which a predetermined level of steer angle is achieved very quickly. An idealized input waveform is illustrated in Figure 48. Also, the time required to reach 90 percent of the steady-state lateral acceleration is indicated in Figure 48. This measure of response time is the performance measure for this maneuver.

The results for the closed-loop evasive maneuver are exemplified by the time histories presented in Figure 49. The dashed-line superimposed on the graph of lateral acceleration is a time-shifted plot of the steering waveform. In this example, the steering waveform needs to be shifted by 0.32 seconds in order to obtain an "optimum" fit between the input (the steering waveform) and the output (the lateral acceleration waveform). This optimum fit is determined by a cross-correlation calculation which finds the best value of time shift, herein referred to as the "response time." *In a well-behaved vehicle maneuver, the shape of the lateral acceleration response will be the same as that of the steering input and the amount of time shift, that is,*

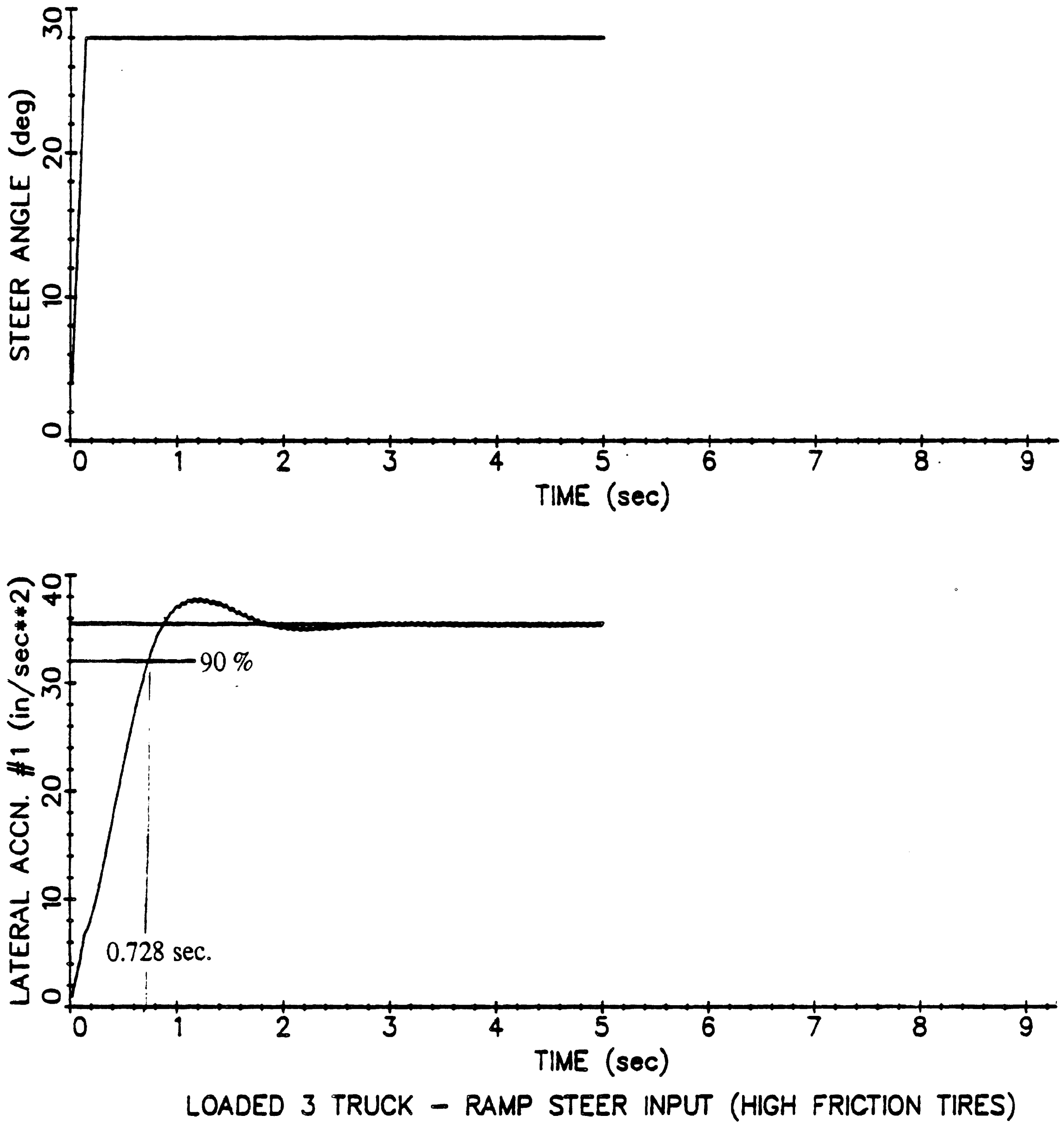


Figure 48. Response time in a ramp-step steer maneuver

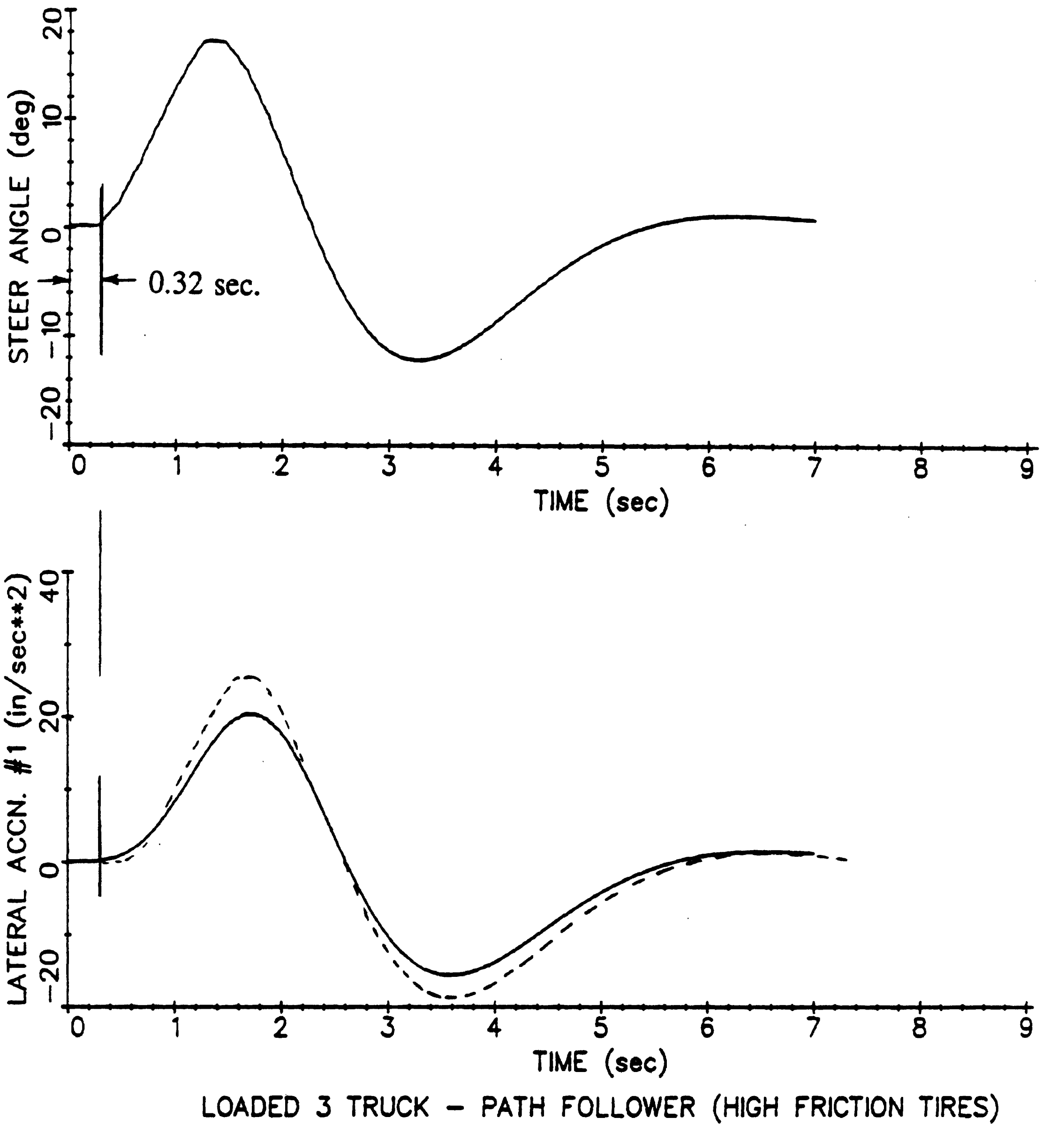


Figure 49. Response time in a closed-loop evasive maneuver

response time, will be small. (The cross-correlation between the steering and acceleration waveforms ought to be greater than 0.99.)

In the closed-loop maneuver, the simulated "driver" is programmed to follow a path that requires that the vehicle move four feet laterally in a period of one second while travelling at 50 mph. This turns out to be a quick, but mild, maneuver that does not approach rollover conditions for typical vehicles.

Parametric variations. The usual types of parametric variations have not been performed in this case. Instead the response times of the benchmark vehicle have been studied in empty and fully loaded conditions and with high and low levels of tire/road friction corresponding to a good, dry road and a poor, wet road. Parametric data illustrating the differences between the empty and loaded truck are presented in Table 13 and the two levels of tire lateral force capability are described by the data given in Table 14.

The differences between empty and fully loaded conditions are characterized by (1) loaded yaw moments of inertia that are approximately nine times the unladen value, (2) rear axle loads that change from 4,130 lbs. to 17,000 lbs., and (3) a sprung mass c.g. height that varies from 56 inches to 76.2 inches.

The influences of the tire variations are not as obvious as it may seem. The tire data for the two surfaces show large differences at slip angles from 4 to 12 degrees. However, the data for the high-friction surface at 4 degrees and above indicates tire forces that would be sufficient to roll over this truck. In this sense, the high friction data at 4 degrees and above are irrelevant with regard to response times. In the case of the low friction data, the forces are large enough to be irrelevant at slip angles above 6 degrees. In terms of the response-time maneuvers, these differences mean that in the low-friction situation the vehicle's tires must achieve larger slip angles to obtain forces comparable to those that would be attained on the high-friction surface. However, as shown by the results that follow in Tables 15 and 16, the differences between performance on the high- and low-friction surfaces are only important for one situation, specifically, when the empty vehicle is performing a moderately severe (76.6 degree) ramp-step maneuver.

The results presented in Table 15 indicate that the vehicle is slower (to achieve 90 percent of its steady-state response) in severe maneuvers than it is in mild (28 degree) maneuvers. The differences in response times between mild and severe maneuvers are more pronounced on the low-friction surface than they are on the high-friction surface. The loaded vehicle is somewhat slower to respond than the empty vehicle in this open-loop maneuver.

Table 13. Parametric Differences between Empty and Laden Conditions

Parameter	Empty 3-Axle Straight Truck			
	Sprung Mass	Axle #1	Axle #2	Axle #3
Weight of the sprung mass (lb)	12,200.00			
Roll moment of inertia of the sprung mass (in.lb.sec.sec)	40,000.00			
Pitch moment of inertia of the sprung mass (in.lb.sec.sec)	105,000.00			
Yaw moment of inertia of the sprung mass (in.lb.sec.sec)	105,000.00			
Height of the sprung mass c.g. above ground (in)	56.00			
Load on each axle (lb)		9,740.00	4,130.00	4,130.00
Axle weight (lb)		1,200.00	2,300.00	2,300.00
Axle roll moment of inertia (in.lb.sec.sec)		3,719.00	4,458.00	4,458.00
Longitudinal distance from sprung mass c.g. (in)		72.00	-144.00	-192.00

Parameter	Loaded 3-Axle Straight Truck			
	Sprung Mass	Axle #1	Axle #2	Axle #3
Weight of the sprung mass (lb)	40,200.00			
Roll moment of inertia of the sprung mass (in.lb.sec.sec)	128,515.00			
Pitch moment of inertia of the sprung mass (in.lb.sec.sec)	909,825.00			
Yaw moment of inertia of the sprung mass (in.lb.sec.sec)	921,315.00			
Height of the sprung mass c.g. above ground (in)	76.20			
Load on each axle (lb)		12,000.00	17,000.00	17,000.00
Axle weight (lb)		1,200.00	2,300.00	2,300.00
Axle roll moment of inertia (in.lb.sec.sec)		3,719.00	4,458.00	4,458.00
Longitudinal distance from sprung mass c.g. (in)		175.52	-40.48	-84.48

Table 14. Tire Cornering Forces in High and Low Friction

CORNERING FORCE TABLE - LOW FRICTION

Lateral Force vs. Slip angle and Vertical Load

	Slip Angle, deg					
Vertical Load, lbs per tire	1.00	2.00	4.00	6.00	8.00	12.00
3000	532.05	886.27	1,072.32	1,134.44	1,165.57	1,196.86
6000	880.09	1,590.89	2,009.84	2,149.73	2,219.84	2,290.28
9000	976.10	1,934.01	2,729.24	2,994.76	3,127.84	3,261.56

CORNERING FORCE TABLE - HIGH FRICTION

Lateral Force vs. Slip angle and Vertical Load

	Slip Angle, deg					
Vertical Load, lbs per tire	1.00	2.00	4.00	6.00	8.00	12.00
3000	532.05	1,064.43	1,922.53	2,259.16	2,427.87	2,597.41
6000	880.09	1,760.72	3,363.72	4,092.07	4,457.14	4,823.97
9000	976.10	1,952.80	3,910.36	5,223.72	5,882.02	6,543.53

Table 15. Ramp-Step Response Times

Loading Condition	Road Surface	Response Times Ramp Steer Input 28°	76.6°
Empty Vehicle	Dry	0.620	0.744
	Wet	0.597	0.918
Loaded Vehicle	Dry	0.728	1.094
	Wet	0.713	1.167

Table 16. Closed-Loop Response Times

Loading Condition	Road Surface	Path Follower Cross Correlations Lag	Value
Empty Vehicle	Dry	0.10	0.99862
	Wet	0.10	0.99887
Loaded Vehicle	Dry	0.32	0.99854
	Wet	0.32	0.99861

In the closed-loop maneuver, tire friction has no observable influence in a four-foot lateral evasion (see Table 16). In contrast, loading is a key factor, with the loaded vehicle responding three times slower than the empty vehicle. The greater yaw moment of inertia undoubtedly contributes to the slowness of the loaded vehicle, although further study is needed to understand the nuances of closed-loop performance.

3.8 Concluding Remarks on Straight Trucks

The material presented in Sections 3.1 through 3.7 comprises the information available on straight trucks. Future editions of this handbook could be expanded to include sections on braking-in-a-turn and the response to external disturbances. More information on the influences of parametric variations on response times could be included in Section 3.7. In general, the study of transient maneuvering situations merits further study, although the results of the simplified analyses go a long way towards understanding the nature of the expected performance qualities of heavy trucks.

4.0 TRACTOR-SEMITRAILER

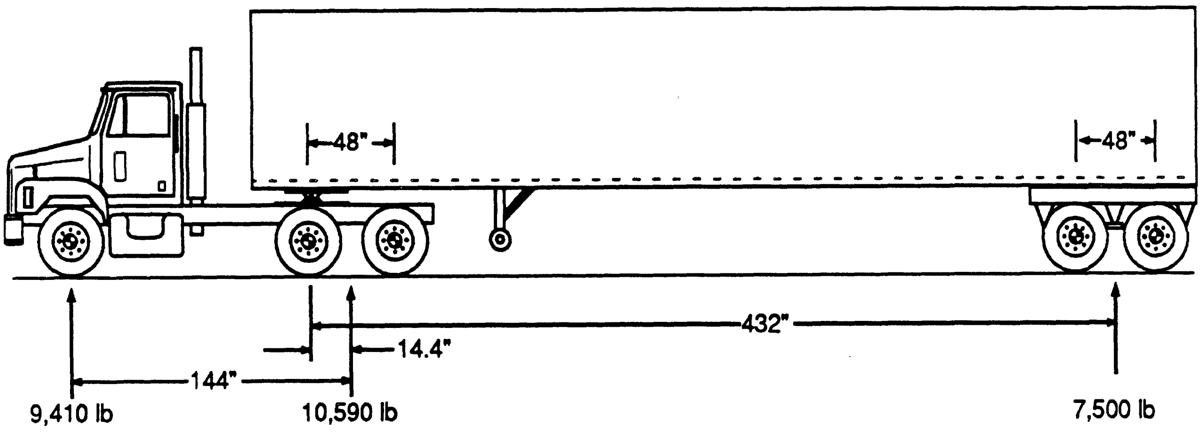
4.1 Baseline Values of Pertinent Mechanical Properties

The benchmark tractor-semitrailer is comprised of a three-axle tractor and a two-axle semitrailer (see Figure 50). (The "shorthand" designation for this vehicle is 3S2.) The gross combination weight is 80,000 lbs. The tractor wheelbase is 144 inches and the trailer wheelbase (from tandem center to the kingpin) is 432 inches (36 feet). The axle loads in fully laden and empty conditions are illustrated in Figure 50, along with the basic geometric layout of the benchmark 3S2. Table 17 lists the values of the basic mechanical properties of this vehicle. These values represent the baseline conditions for the performance sensitivity diagrams presented in this section.

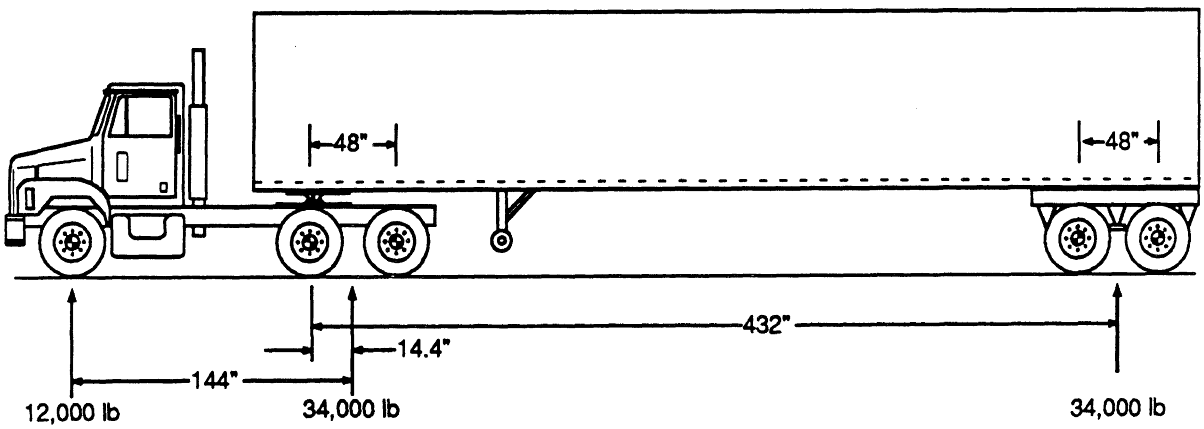
4.2 Low-Speed Cornering - Tractrix

Benchmark performance. The performance signature for the benchmark 3S2 is the set of tractrices presented in Figure 51. The upper curve in Figure 51 is the path of the center of the front axle when the vehicle is making a right angle turn with an outside radius of 45 feet. The next lower curve is the path of the center of the rear tandem axle set. This path is the tractix of the center of the tractor's suspension. The path of the fifth wheel is used in determining the path (tractrix) of the center of the trailer's tandem axle set. Clearly, the offtracking of the semitrailer is much larger than that of the tractor, which is to be expected since the semitrailer is much longer than the tractor. The maximum offtracking of the semitrailer is used as the performance measure for this maneuver. The maximum offtracking is 14.36 feet for the benchmark 3S2.

Parametric sensitivities. Parametric variations are examined for the tractor and trailer wheelbases and the fifth wheel location (see Table 18 and Figure 52). Overall, the influence of the tractor's wheelbase is smaller than that of the trailer's wheelbase (that is, king pin to the center of the rear suspension). However, if the tractor's wheelbase is increased to 268 inches, the offtracking is increased from 14.4 feet to 17.4 feet. In this case, an extremely long tractor has been connected to a 42-foot trailer. In contrast, if the trailer wheelbase is only increased from 432 inches to 450 inches, the offtracking is increased to 15.17 feet. This trailer variation corresponds to moving the slider to the rearmost location on a 42-foot trailer. *As illustrated by these examples, although the longest wheelbase in the combination dominates the offtracking results, major variations in tractor wheelbase can cause significant increases in low speed offtracking.*



TRACTOR AND EMPTY SEMITRAILER

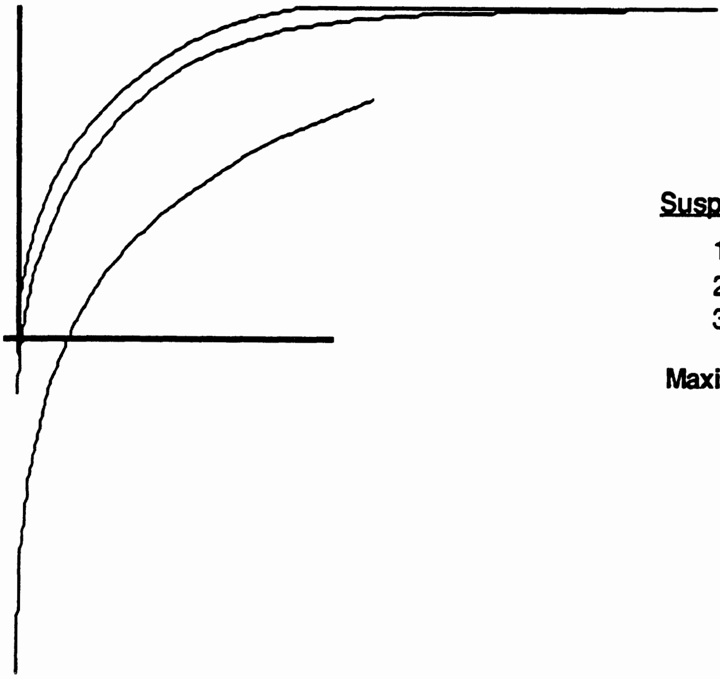


TRACTOR AND LOADED SEMITRAILER

Figure 50. Geometric layout, benchmark 3S2

Table 17. Basic Mechanical Properties (Tractor and Semitrailer)

Mechanical Property	Empty	Loaded
Tractor		
Weight (lb)	15,500.00	15,500.00
Front axle brake gain (in.lb/psi)	2,000.00	2,000.00
Rear suspension - leading axle brake gain (in.lb/psi)	3,000.00	3,000.00
Rear suspension - trailing axle brake gain (in.lb/psi)	3,000.00	3,000.00
Interaxle load transfer on rear tandem	0.00	0.00
Front suspension cornering stiffness (lb/deg)	978.00	1,047.00
Rear suspension total cornering stiffness (lb/deg)	1,822.00	3,746.00
Reduction in cornering stiffness due to steering system (%)	31.81	32.16
Front axle suspension stiffness (in.lb/deg)	21,000.00	21,000.00
Rear suspension roll stiffness (in.lb/deg)	140,000.00	140,000.00
Front axle roll center height (in)	20.00	20.00
Rear suspension roll center height (in)	29.00	29.00
Height of the center of gravity (in)	34.83	34.83
Distance of the center of gravity from the front axle (in)	60.85	60.85
Semitrailer		
Weight (lb)	12,000.00	64,500.00
Rear suspension - leading axle brake gain (in.lb/psi)	3,000.00	3,000.00
Rear suspension - trailing axle brake gain (in.lb/psi)	3,000.00	3,000.00
Interaxle load transfer on rear tandem	0.00	0.00
Rear suspension total cornering stiffness (lb/deg)	1,457.00	3,746.00
Rear suspension roll stiffness (in.lb/deg)	160,000.00	160,000.00
Rear suspension roll center height (in)	29.00	29.00
Height of the center of gravity (in)	50.00	81.44
Distance of the center of gravity from the kingpin (in)	270.00	227.70



MINIMUM TRACK

Susp. No.	Radius (ft)	Angle (deg)
1	41.00	0.00
2	39.08	73.82
3	26.64	61.57

Maximum Offtracking =14.36 ft.

Figure 51. Tractrices, tractor and semitrailer, 41 foot turn

Table 18. Tractor and Semitrailer - Low-Speed Offtracking

Parameter Name	Benchmark Value	Minimum Value	Maximum Value	Notes
Tractor wheelbase	144 inches	134 inches	268 inches	
Fifth wheel offset	14.4 inches	0 inches	24 inches	
Trailer wheelbase	432 inches	348 inches	450 inches	

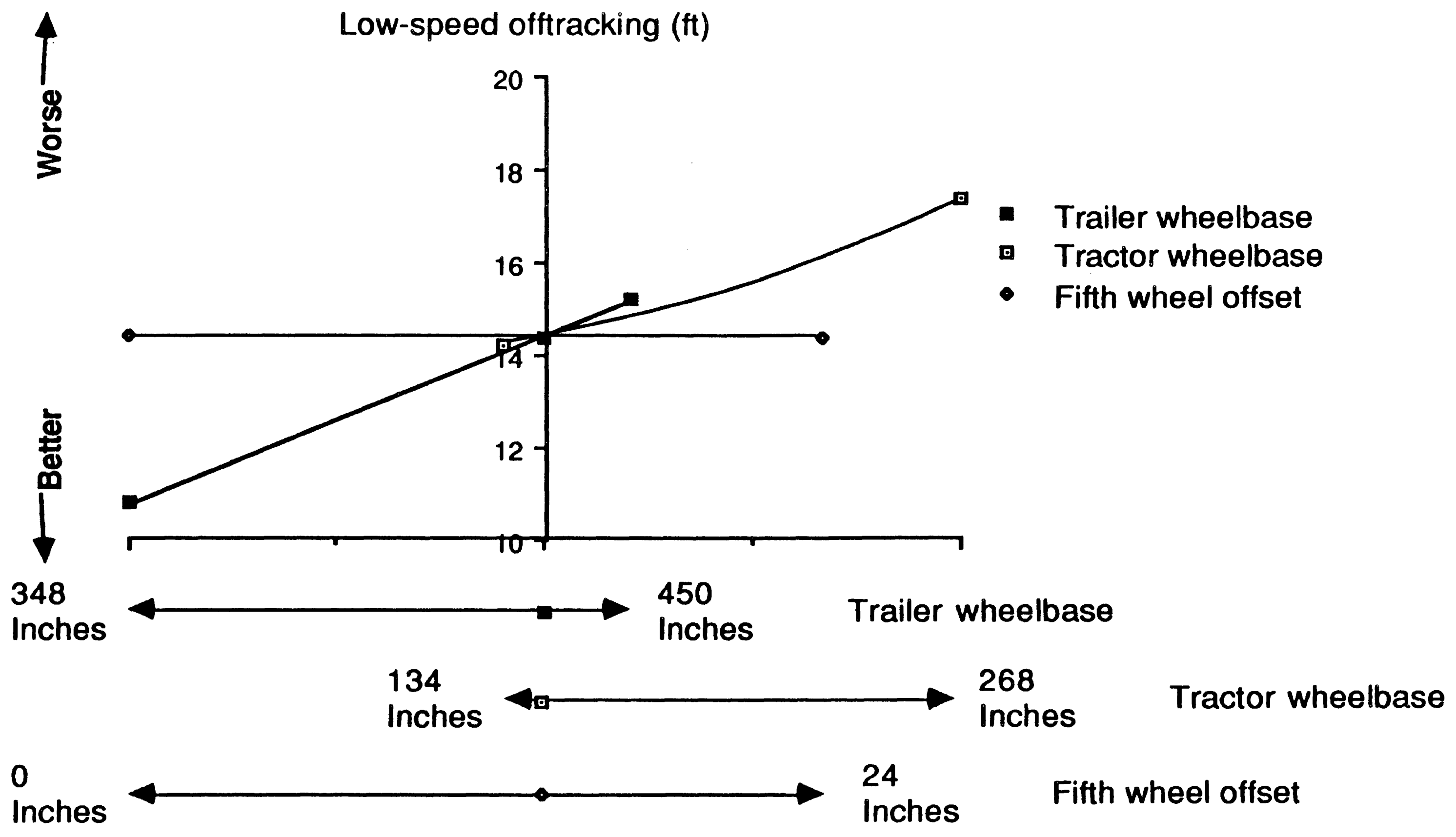


Figure 52. Parameter sensitivity diagram: low-speed offtracking, tractor and semitrailer.

The influence of fifth wheel location is minor for the 3S2.

4.3 Constant Deceleration Braking

Benchmark performance. Brake proportioning in the United States is arranged to favor the fully laden 3S2. Results for empty vehicles show much lower braking efficiencies than those for loaded vehicles.

Figure 53 presents the friction utilization at each axle, deceleration, and braking efficiency as functions of treadle valve pressure for the benchmark 3S2 in the empty condition. These quantities constitute the braking performance signature. From these graphs one can read the friction required (at each axle) for various levels of deceleration. The braking efficiency at a selected level of deceleration is determined by dividing the highest level of friction required at any axle (at the selected level of deceleration) by that deceleration. The braking efficiency provides a performance measure that indicates how well the vehicle can utilize tire/road friction in performing constant deceleration stops in which no wheels lock up on any axle. For example, at 0.4 g deceleration, the results presented in Figure 53 show that the empty benchmark 3S2 requires a friction level of 0.68 to prevent locking the wheels on the semitrailer axles and this corresponds to a braking efficiency of 0.59.

At 0.2 g deceleration, the braking efficiency of the empty vehicle is only slightly better with a value of 0.61. Since the axles in danger of locking up (over the range from 0 to 0.4 g) are the trailer axles, the performance difficulty is likely to be trailer-swing in an emergency braking situation.

Figure 54 is the performance signature for the fully laden 3S2. In this case the braking efficiencies are 0.89 at 0.4 g and 0.94 at 0.2 g. These are excellent values of braking efficiency.

Parametric sensitivities. The influences of changes in the tractor and the semitrailer are treated separately in the following performance sensitivity diagrams. The influences of quantities that contribute to fore-aft and interaxle load transfer during braking are studied here (see Table 19). (Interaxle load transfer is defined in Section 3.3.) In addition, increases in front brake gain are examined. Results are presented for the empty and loaded 3S2 at deceleration levels of 0.2 and 0.4 g.

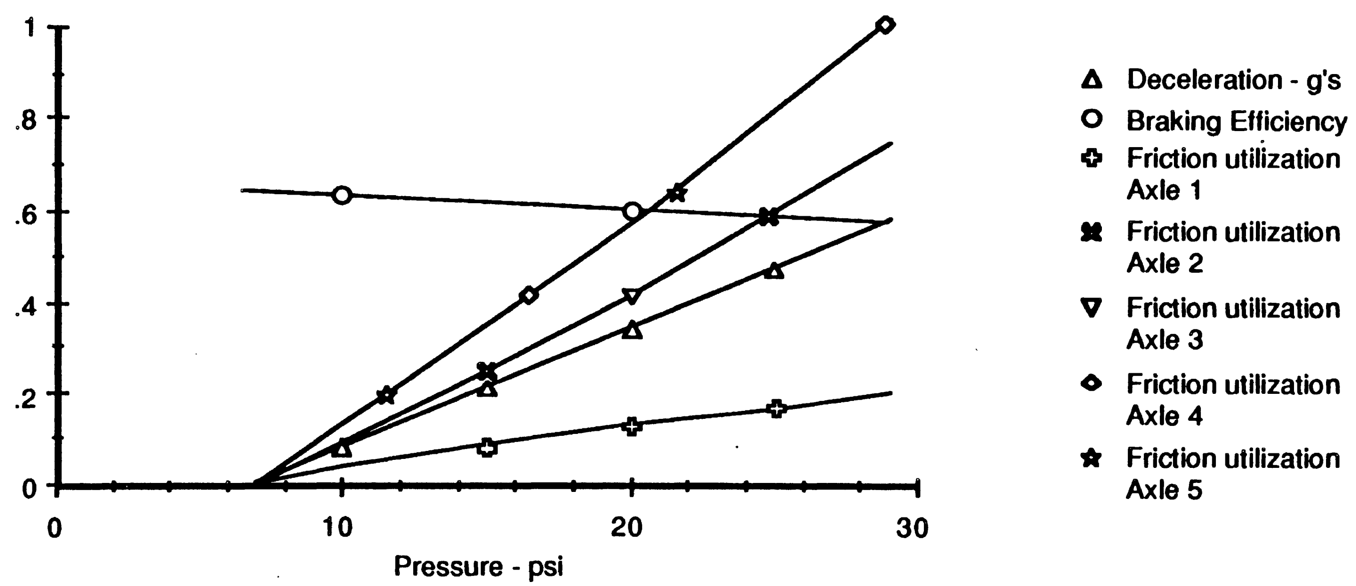


Figure 53. Performance signature, braking, tractor and empty semitrailer

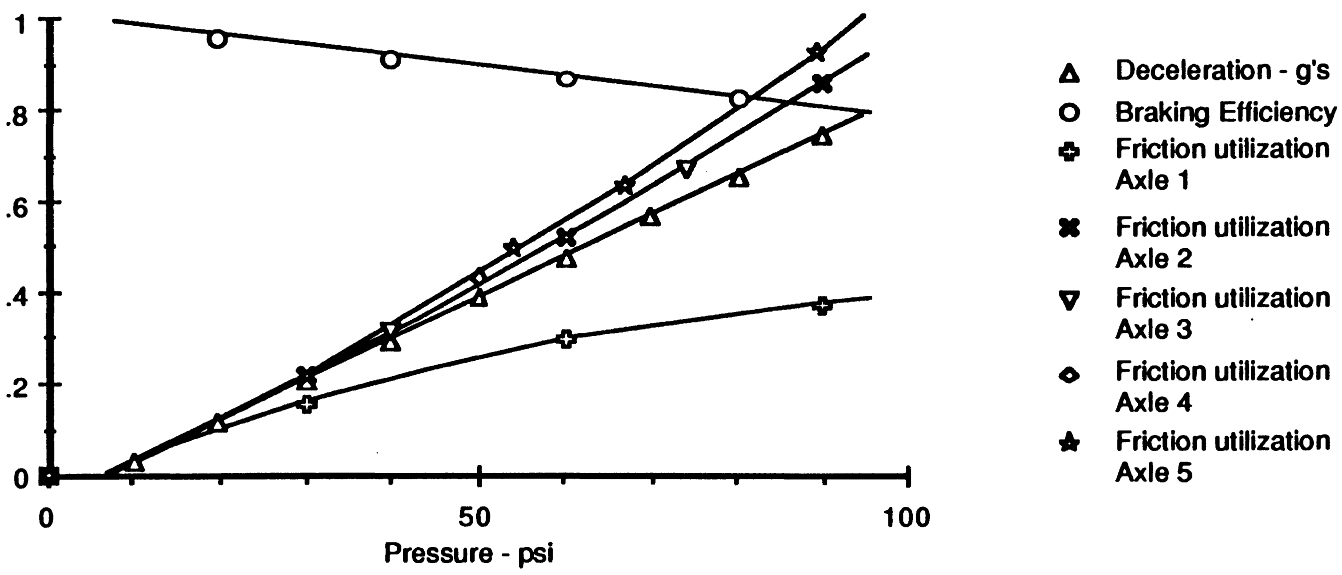


Figure 54. Performance signature, braking, tractor and loaded semitrailer

Table 19. Tractor and Semitrailer - Braking

Parameter Name	Benchmark Value	Minimum Value	Maximum Value	Notes
Tractor wheelbase	144 inches	134 inches	268 inches	
Interaxle load transfer (tractor)	0	-0.2	0.2	
Front brake gain	2000 in.lb/psi	2000 in.lb/psi	3000 in.lb/psi	Both brakes combined
Trailer wheelbase	432 inches	348 inches	450 inches	
Interaxle load transfer (trailer)	0	-0.2	0.2	
* C.G. height	81.44 inches	60 inches	105 inches	

* For the loaded vehicle

At 0.2 g the braking performance of the empty 3S2 is insensitive to changes in tractor properties that influence tire loads through load transfer (see Figure 55). This insensitivity is simply because the friction at the trailer wheels is the controlling factor in determining the braking efficiency in these cases. However in this situation, *noticeable improvements in braking efficiency can be obtained by increasing the gain of the front brakes*. This is because an increase in front brake torque relative to the trailer braking means that not as much trailer braking is needed to attain a given level of deceleration, and trailer braking turns out to be the controlling element in this case.

The results for *tractor* variations for the empty 3S2 at 0.4 g (included in Figure 56) are much the same as those at 0.2 g.

Changes in trailer properties have an important influence on the braking efficiency of the empty 3S2, as evidenced by the results presented in Figures 56 and 57. The loads on the trailer wheels are critical in determining whether wheel lock will occur. *Either positive or negative coefficients of interaxle load transfer will degrade braking efficiency.* (A positive coefficient is typical of some types of walking-beam (Hendrickson) suspensions and a negative coefficient is typical of some four-spring suspensions.)

As shown in Figure 57, braking efficiency is only improved by a very slight amount by a small increase in trailer wheelbase. Increasing trailer wheelbase by a large amount (thereby significantly decreasing the amount of load transferred off of the trailer wheels) would have a more noticeable influence .

When the benchmark 3S2 is fully laden, its braking efficiency is high. This means that all wheels will lock up at approximately the same level of tire/road friction. *As illustrated in Figures 58 and 59, large amounts of interaxle load transfer at the tractor tandem axles cause reductions in braking efficiency.* In these cases, wheels on the tractor's rear axles are approaching lock up, that is, the lightly loaded member of the tractor's tandem set requires more friction than that required at the trailer axles. In contrast, the changes in tractor wheelbase do not cause a large enough effect to change the braking efficiency at either 0.2 or 0.4 g (see Figures 58 and 59).

At 0.2 g (see Figure 58), increasing the front brake gain results first in an increase in efficiency and then a decrease in efficiency as the gain is increased further. This change in trend is brought about by a transition from trailer wheel locking to tractor front wheel locking when the gain exceeds 2,500 inch-lbs per psi for the total gain pertaining to both front brakes. This trend is not present at 0.4 g because more load is transferred off the

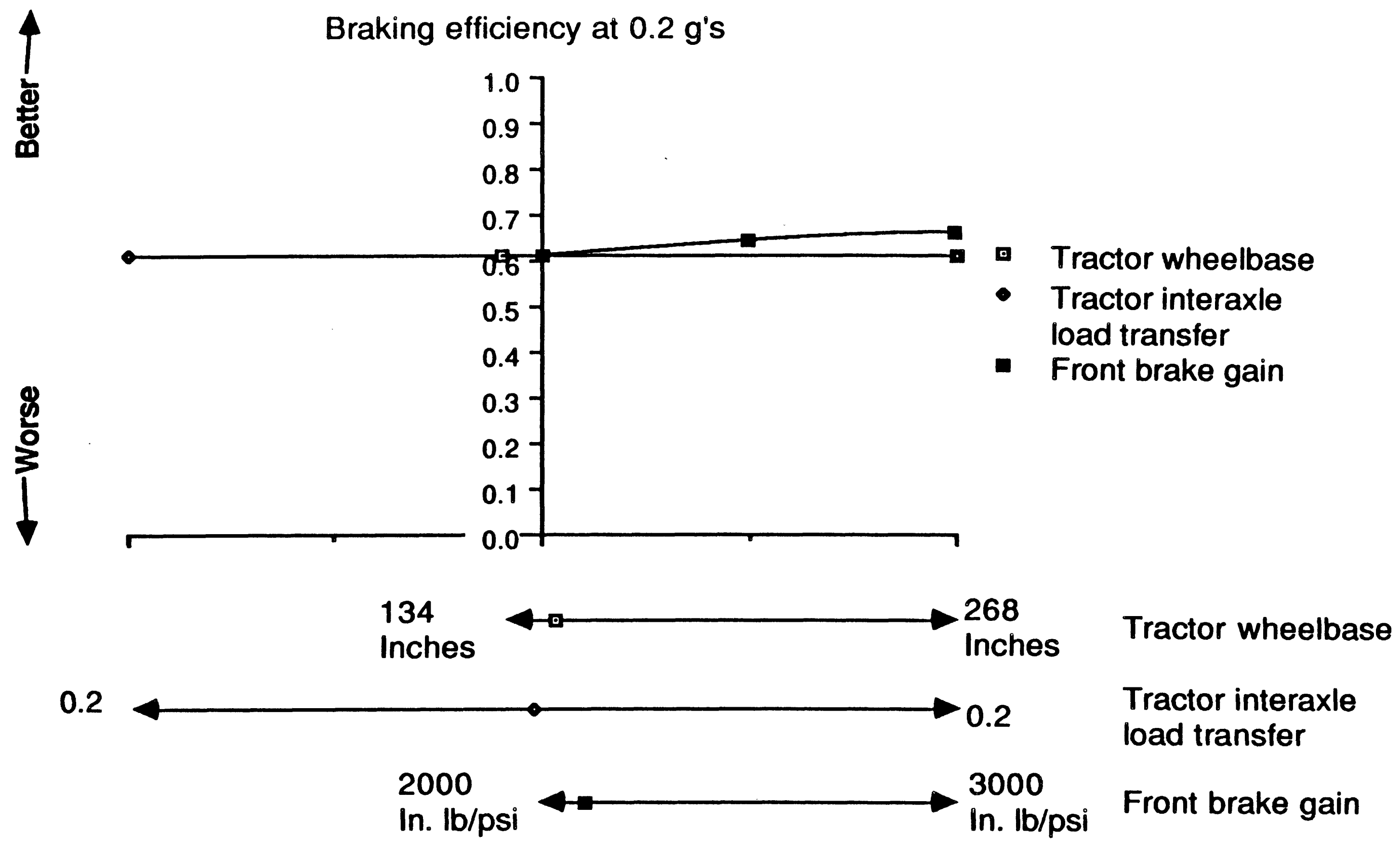


Figure 55. Parameter sensitivity diagram: braking, empty tractor and semitrailer.

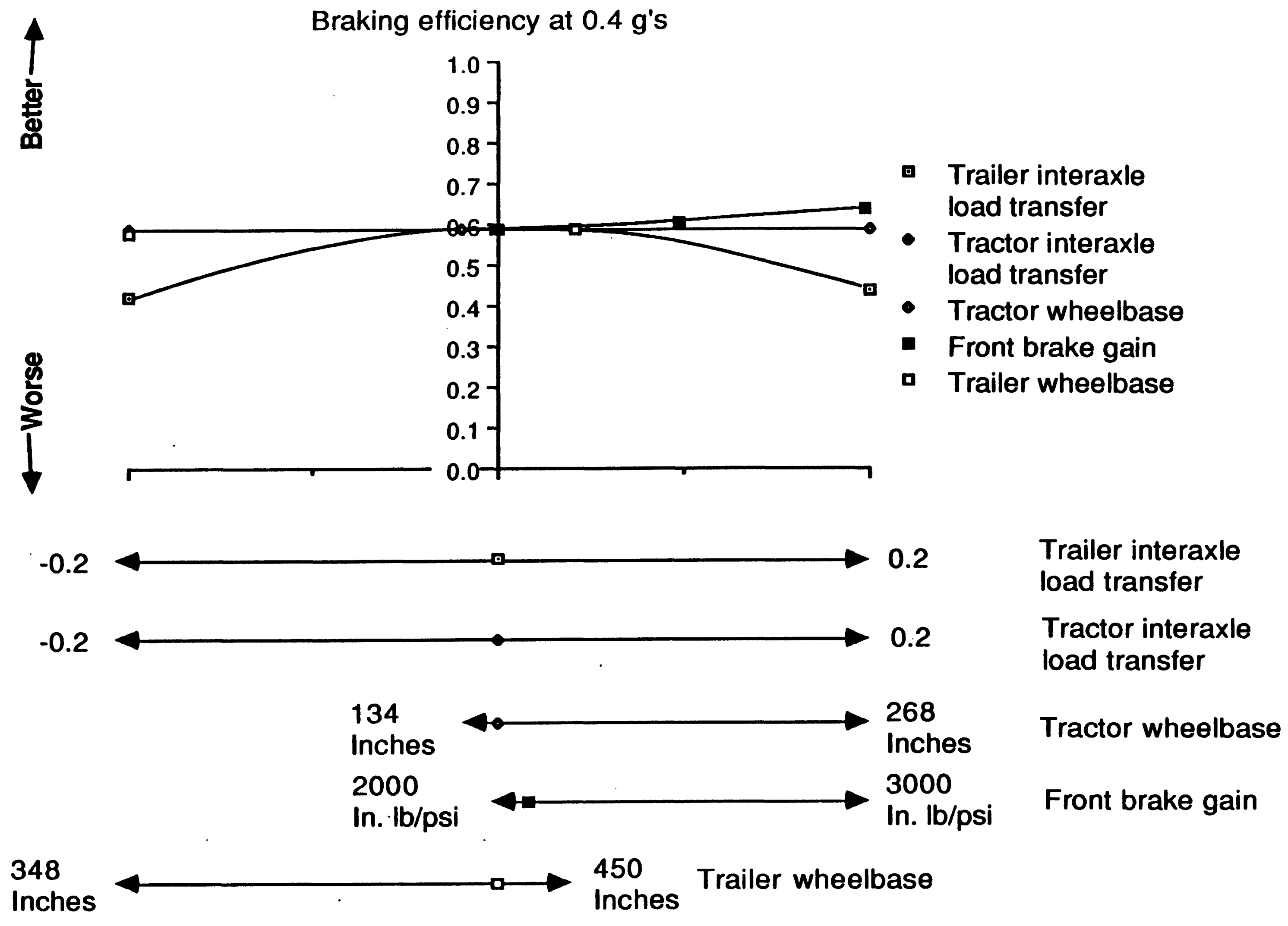


Figure 56. Parameter sensitivity diagram: braking, empty tractor and semitrailer.

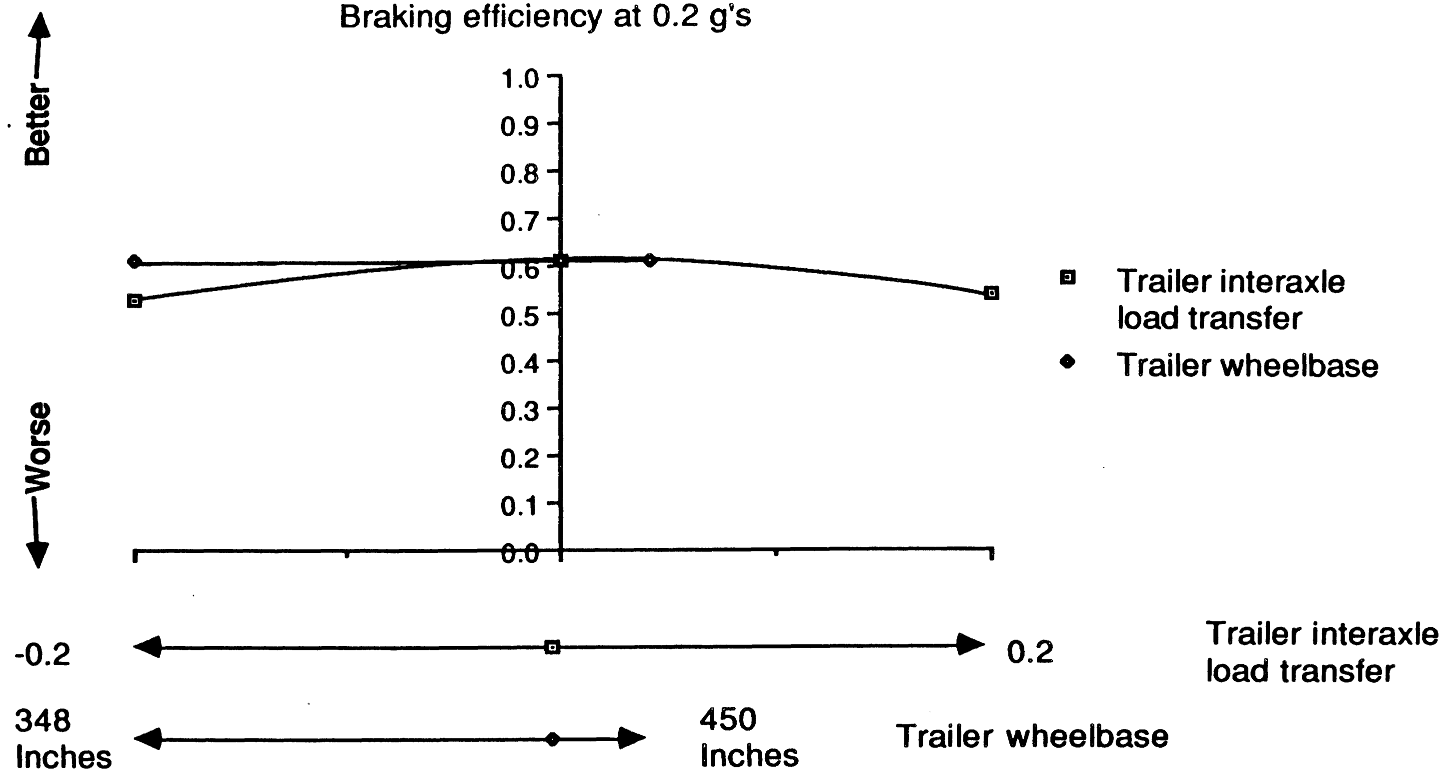


Figure 57. Parameter sensitivity diagram: braking, empty tractor and semitrailer.

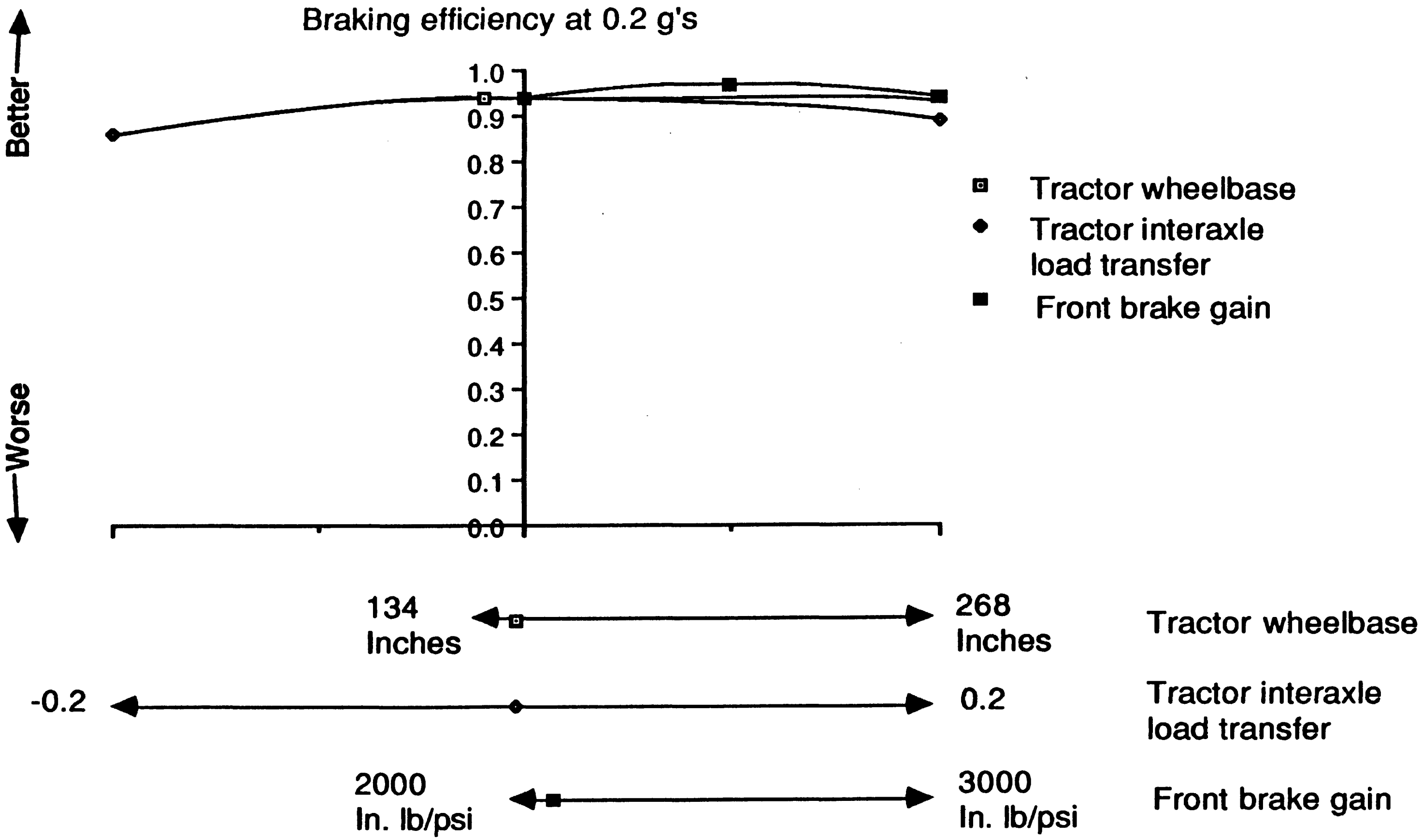


Figure 58. Parameter sensitivity diagram: braking, loaded tractor and semitrailer.

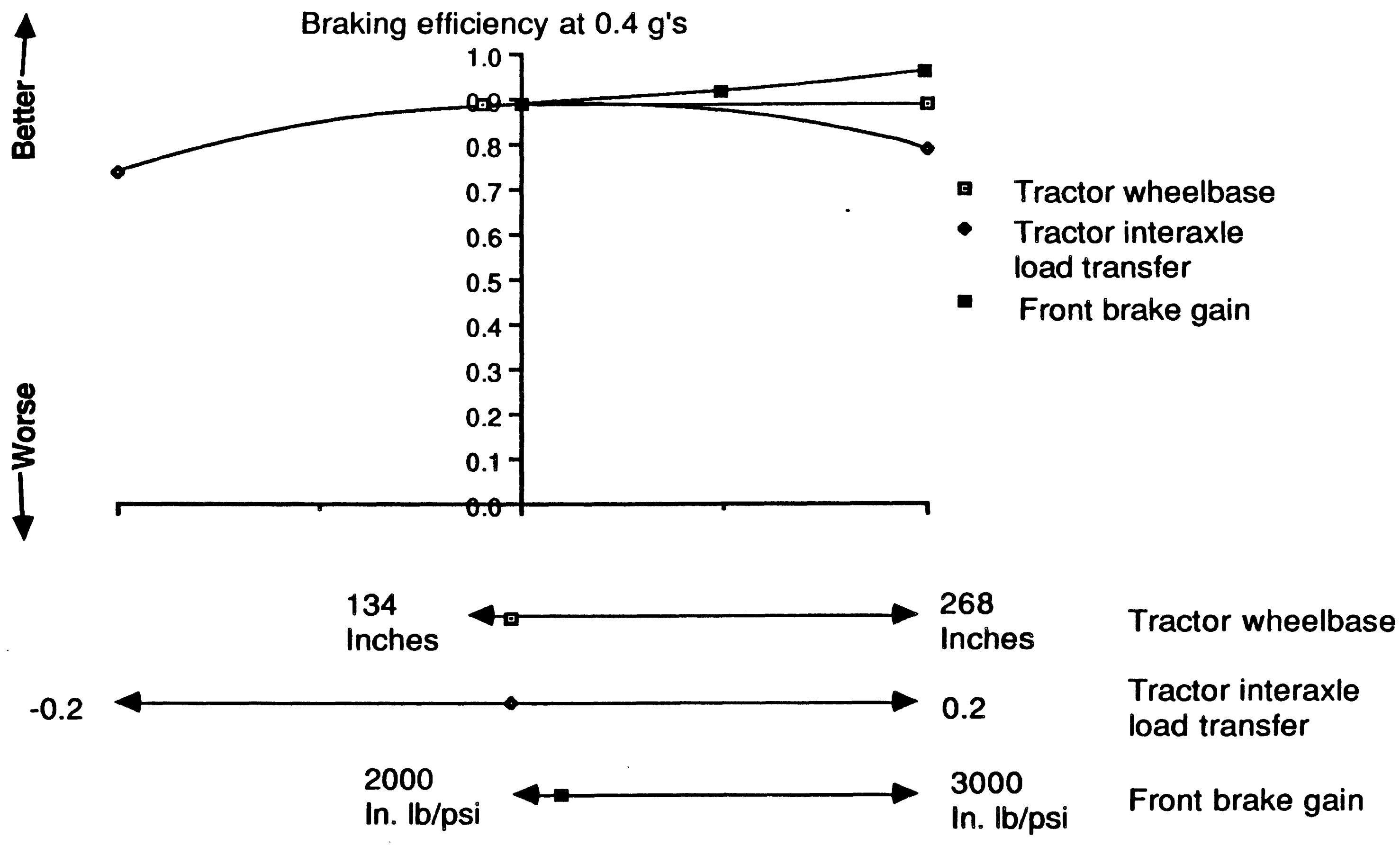


Figure 59. Parameter sensitivity diagram: braking, loaded tractor and semitrailer.

trailer wheels and onto the tractor's front wheels at the higher deceleration level. In that case (see Figure 59), increases in front brake gain produce progressive improvements in braking efficiency.

The trailer parameters examined in the loaded vehicle situation are interaxle load transfer, wheelbase, and c.g. height (see Figures 60 and 61). The results for each of these parameters indicate that each of them have optimum (maximum) values on these performance sensitivity diagrams. All of these optima can be explained by examining the load transfer and friction utilization at each axle. For example, when the c.g. height of the trailer is decreased, less load is transferred from the trailer axles and the load on the tractor's rear axles is not as large as it would have been if the baseline amount of load had been transferred. Consequently, the vehicle's performance is limited by lockup of the tractor tandems rather than by lockup of the trailer tandems. In this case, this results in a decrease in efficiency. However, if the c.g. height of the trailer is increased, the efficiency will also drop because more load will be transferred off the trailer axles. These types of changes in trend are to be expected when the baseline efficiency of the vehicle is high.

The differences between laden and unladen braking efficiencies present a problem that can not be completely resolved by static proportioning of the brakes. For some vehicles, efficiencies no less than 0.7 (either loaded or empty) can be attained through appropriately increasing the gain of the front brakes. However, operators of heavy trucks have traditionally been opposed to "aggressive" front brakes. In Europe, load-sensing proportioning is used to increase braking efficiency and to attempt to cause front wheels to lock before rear wheels (just the opposite of the traditional American view). Load-sensing proportioning optimizes the friction utilization at the axles controlled by the proportioning system if the capabilities of all the brakes are taken into account. If semitrailers had load sensing and their brake gains were specified, tractor brakes could be proportioned to obtain good efficiencies throughout the vehicle's range of loading conditions. *The ultimate solution would be a reliable and rapidly acting antilock braking system that prevented wheel lock and provided a margin of tire side force for directional control.*

4.4 High-Speed Offtracking (Steady Turn - Tracking)

Benchmark performance. As in Section 3.4, the specific turns treated here are (1) a 1,200-foot radius being negotiated at 55 mph and (2) a 600-foot radius being negotiated at 38 mph. For these steady turning situations, the performance signature and measure is a single quantity, namely, the offtracking of the center of the rearmost suspension. The radii

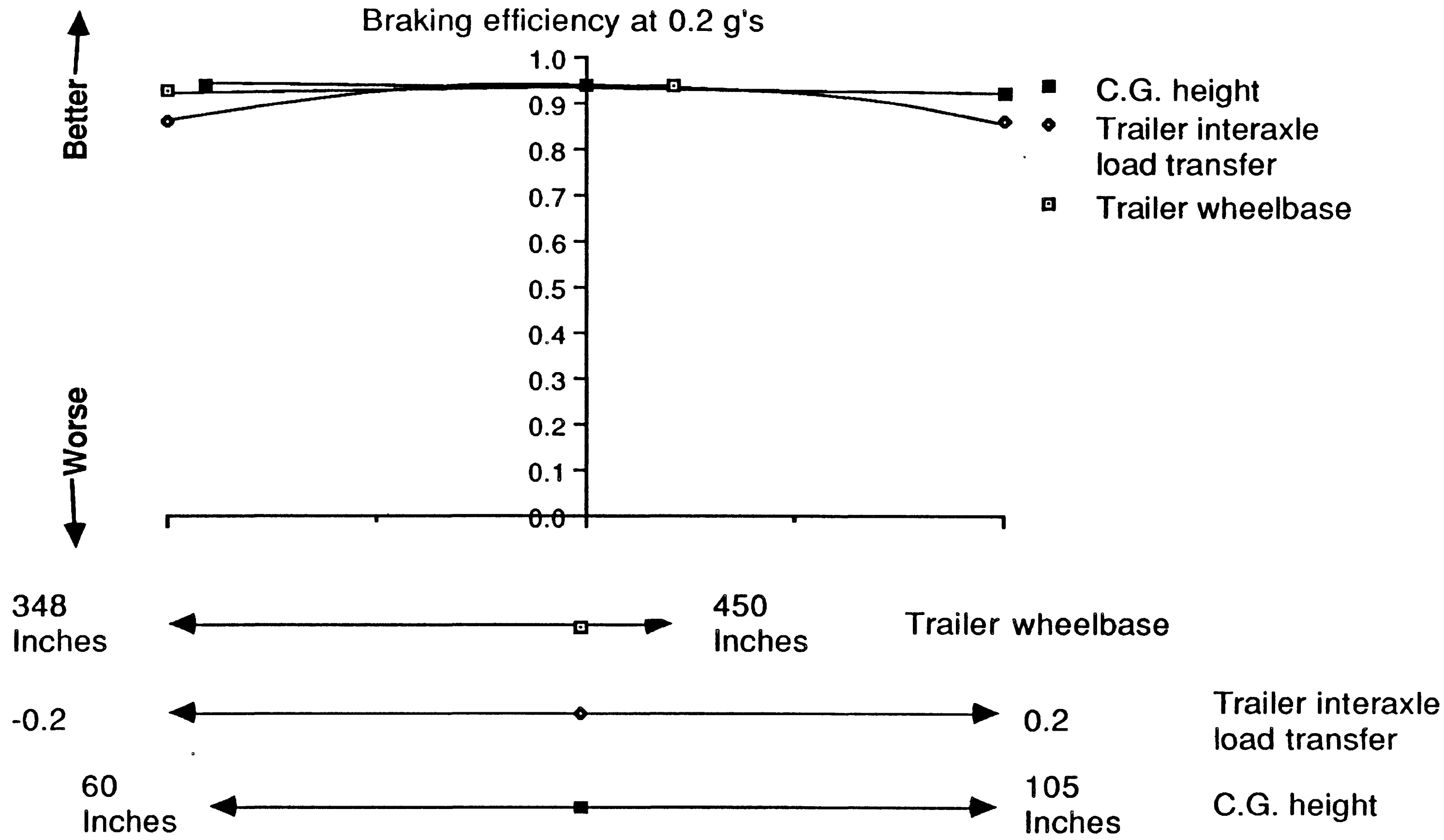


Figure 60. Parameter sensitivity diagram: braking, loaded tractor and semitrailer.

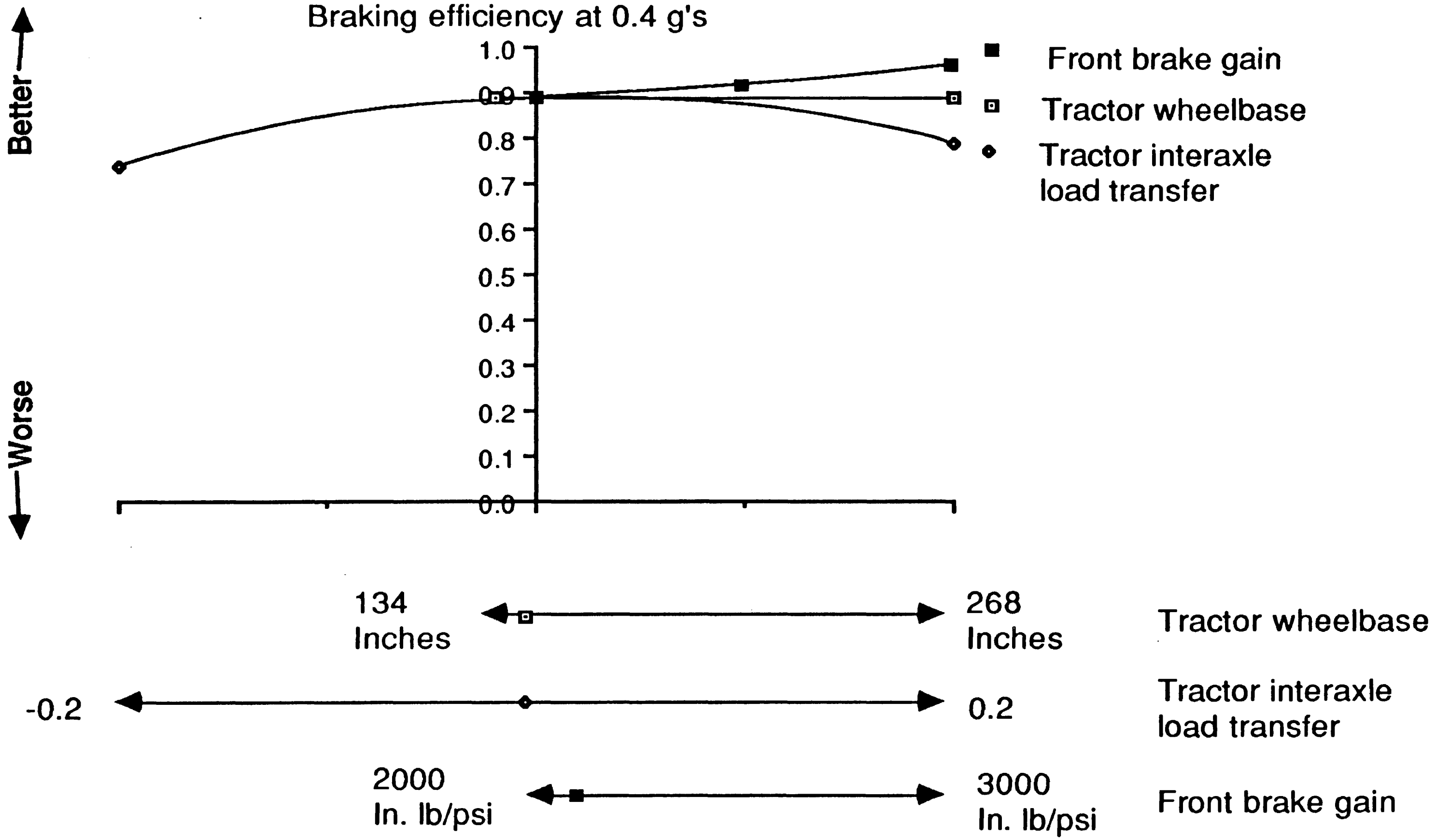


Figure 61. Parameter sensitivity diagram: braking, loaded tractor and semitrailer.

attained by various locations on the benchmark 3S2 in a 1,200-foot turn are listed in Table 20. Examination of this table indicates that the center of the rear suspension offtracks the path of the front axle by 0.65 feet towards the outside of the turn (the opposite direction from that occurring at low speed). On the 600-foot turn, the offtracking of the rear suspension center is approximately zero for the benchmark 3S2.

In Section 3.4 we explained that for the straight truck there is a speed at which the high-speed offtracking is zero regardless of the radius of the turn. For articulated vehicles the situation is more complicated, but the same general ideas apply. By examining Table 20, one can see that the fifth wheel ("Rear Articulation Point on Unit #1") is tracking to the outside of the front axle by 0.16 feet when the vehicle is traveling at 38 mph. However, the trailer suspension (#3) tracks far enough inside of the fifth wheel that the net offtracking at the rearmost suspension is only 0.01 feet. In this case we happened to be investigating a situation in which all axles of the benchmark vehicle have little offtracking. At higher speeds each unit will track outside of its towing point, that is, each unit has a speed at which it will track its towing point and that speed is determined by the same rules as those given in Figure 36 when discussing the straight truck.

Parametric sensitivities. Offtracking is towards the inside of the turn at low speed, and it works toward the outside as speed increases. *The high-speed effect is due to tire slip angles whose magnitudes are related to the ratio of tire cornering stiffness to vertical load*. Hence, parametric variations in this ratio are examined. In addition, those wheelbase and hitch location parameters that are important to low-speed offtracking are important here (see Table 21).

The factors that decrease low-speed offtracking will increase offtracking at high speed because low-speed offtracking is towards the inside of the turn while high-speed offtracking is towards the outside of the turn. However, at a given speed, there is a "worst case wheelbase" that will lead to the maximum high-speed offtracking. This value of wheelbase, wbw, can be estimated by using the following equation:

$$wbw = [\, V^2 / (g(57.3)\,(C_\alpha / F_z))] \qquad (4)$$

where V is velocity,
g is the gravitational constant,
and C_α / F_z is the cornering stiffness to vertical load ratio in 1/degrees.

For the 3S2 travelling at 55 mph, wbw is approximately 385 inches for the semitrailer equipped with the benchmark tires. Upon inspecting Figure 62, one can barely

Table 20. Path Radii for the Benchmark 3S2

Velocity=55 mph
Radius of the turn=1200 feet

Suspension No.	Rear Articulation Point on Unit No.	Zero Speed Radius (ft)	High Speed Radius (ft)
1		1,200.00	1,200.00
2		1,199.94	1,200.26
	1	1,199.94	1,200.23
3		1,199.40	1,200.65
	2	1,199.40	1,200.73

Velocity=38 mph
Radius of the turn=600 feet

Suspension No.	Rear Articulation Point on Unit No.	Zero Speed Radius (ft)	High Speed Radius (ft)
1		600.00	600.00
2		599.88	600.19
	1	599.88	600.16
3		598.80	599.99
	2	598.79	600.08

Table 21. Tractor and Semitrailer - High-Speed Offtracking

Parameter Name	Benchmark Value	Minimum Value	Maximum Value	Notes
Tractor wheelbase	144 inches	134 inches	268 inches	
Fifth wheel offset	14.4 inches	0 inches	24 inches	
Tractor's cornering stiffness	3743.4 lb/deg	1871.7 lb/deg	5615.1 lb/deg	Cornering stiffness is determined from loaded vehicle data
Trailer wheelbase	432 inches	348 inches	450 inches	
Trailer's cornering stiffness	3743.4 lb/deg	1871.7 lb/deg	5615.1 lb/deg	Cornering stiffness is determined from loaded vehicle data

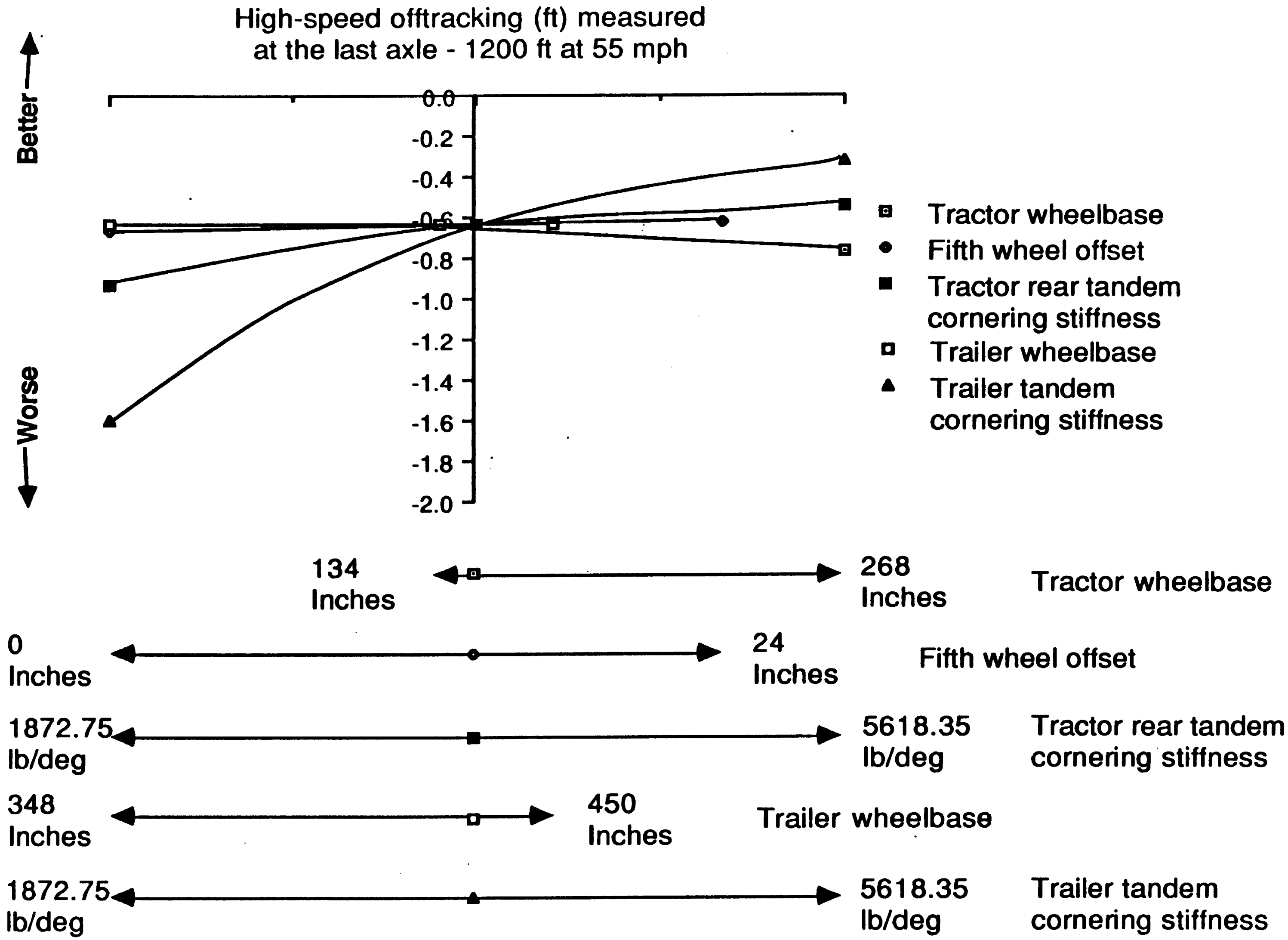


Figure 62. Parameter sensitivity diagram: high-speed offtracking, tractor and semitrailer.

see this effect in the results obtained by varying the trailer wheelbase. The baseline condition of the benchmark vehicle produces nearly the same offtracking as the cases when either the trailer wheelbase is increased or decreased by the maximum amounts shown in Figure 62.

The influence of varying the cornering stiffness of the trailer tires is seen to have a large influence upon high-speed offtracking with low values being particularly detrimental to performance on the 1,200-foot turn. The influence of the tractor's tires is not as important because the tractor has a shorter wheelbase than the trailer. (See Figure 62.)

On the tighter turn (see Figure 63), the performance trends are all monotonic with the stiffer tires and longer wheelbases tending to eliminate offtracking to the outside of the turn.

As was the case for the straight truck, stiffer tires will reduce the tendency of wheels to track towards the outside of the turn regardless of the geometry of the turn or the speed of the vehicle.

4.5 Steady Turn - Roll

Benchmark performance. The performance signature for the rolling performance of the 3S2 is characterized by breakpoints indicating, first, when the trailer axles lift, and then, when the tractor's rear axles lift off. These breakpoints occur at the changes in slope shown in Figure 64. The rollover threshold occurs when the slope of the performance signature becomes negative, that is, when the tractor's rear wheels lift off of the ground. There is a small increment in lateral acceleration between when the inside trailer wheels lift off and when the tractor rear wheels lift off. With only the inside trailer wheels off the ground the vehicle will not roll over, however, once the tractor rear wheels have lifted there is no stopping the rollover. The benchmark rollover threshold is 0.37 g and it occurs at approximately 0.08 radians (4.6 degrees).

Parametric sensitivities. Table 22 presents the list of mechanical properties varied to study the roll properties of the 3S2. These properties include c.g. heights, suspension roll stiffnesses, tire lateral spacings (track widths), and suspension roll center heights, plus a few less important quantities. The performance sensitivity diagrams (Figures 65 through 68) include a diagram showing the influence of variations from 75 in. to 85 in. in the c.g. height of the semitrailer. These results pertaining to c.g. height (see Figure 68) provide a

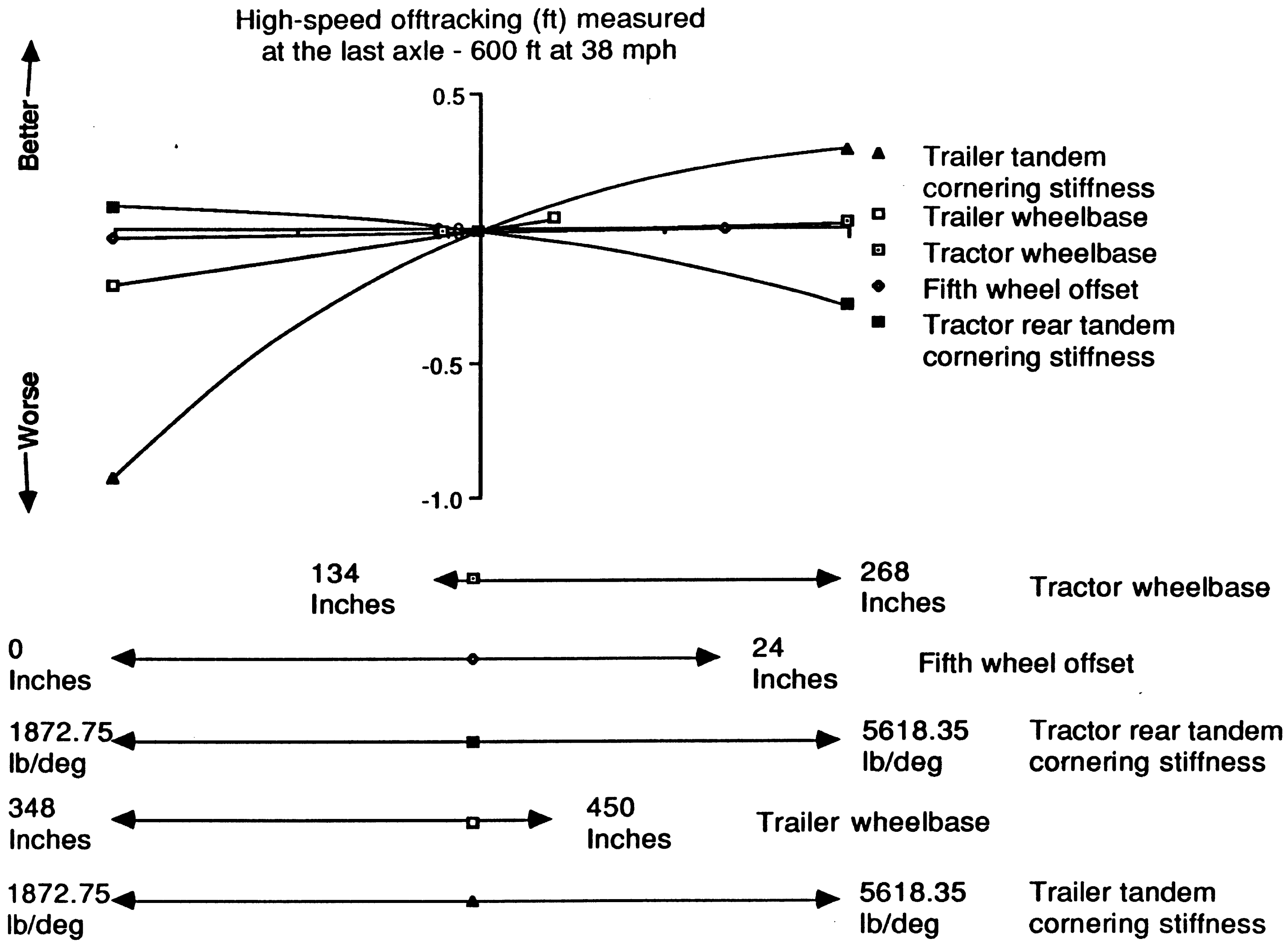


Figure 63. Parameter sensitivity diagram: high-speed offtracking, tractor and semitrailer.

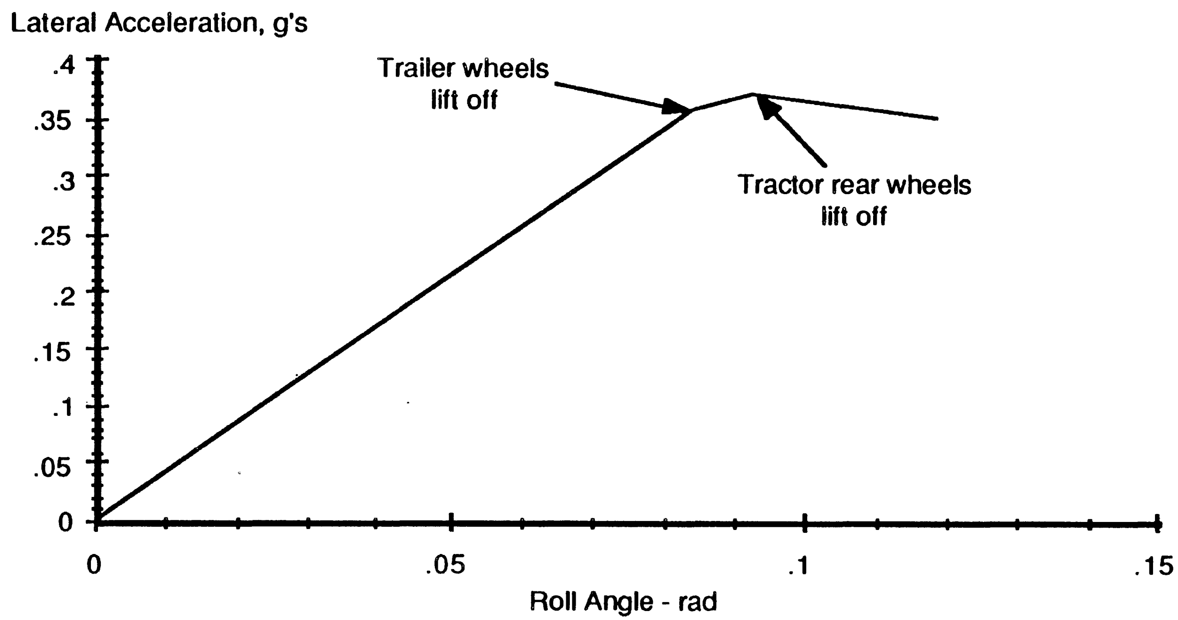


Figure 64. Performance signature, static roll, tractor and semitrailer

Table 22. Tractor and Semitrailer - Static Roll

Parameter Name	Benchmark Value	Minimum Value	Maximum Value	Notes
Rear axle roll center heights	29 inches	21 inches	31 inches	
Tire vertical stiffness	4500 lb/in	4000 lb/in	5000 lb/in	
Tractor's front axle roll stiffness	21,000 in.lb/deg	17,000 in.lb/deg	25,000 in.lb/deg	
Tractor's front axle track width	80 inches	77 inches	81 inches	
Tractor's front axle weight	1200 lb	1150 lb	1250 lb	Negligible effect
Tractor's rear suspension roll stiffness	140,000 in.lb/deg	60,000 in.lb/deg	330,000 in.lb/deg	Total roll stiffness on rear tandem
Tractor's rear axle track widths	72 inches	71 inches	78 inches	
Tractor's rear axle weights	2300 lb	2250 lb	2350 lb	Per axle - Negligible effect
Trailer's c.g. height	81.44 inches	75 inches	85 inches	Much larger deviations used for braking
Trailer's rear suspension roll stiffness	160,000 in.lb/deg	60,000 in.lb/deg	330,000 in.lb/deg	Total roll stiffness on rear tandem
Trailer's rear axle track widths	72 inches	71 inches	78 inches	
Trailer's rear axle weights	1500 lb	1450 lb	1550 lb	Per axle - Negligible effect

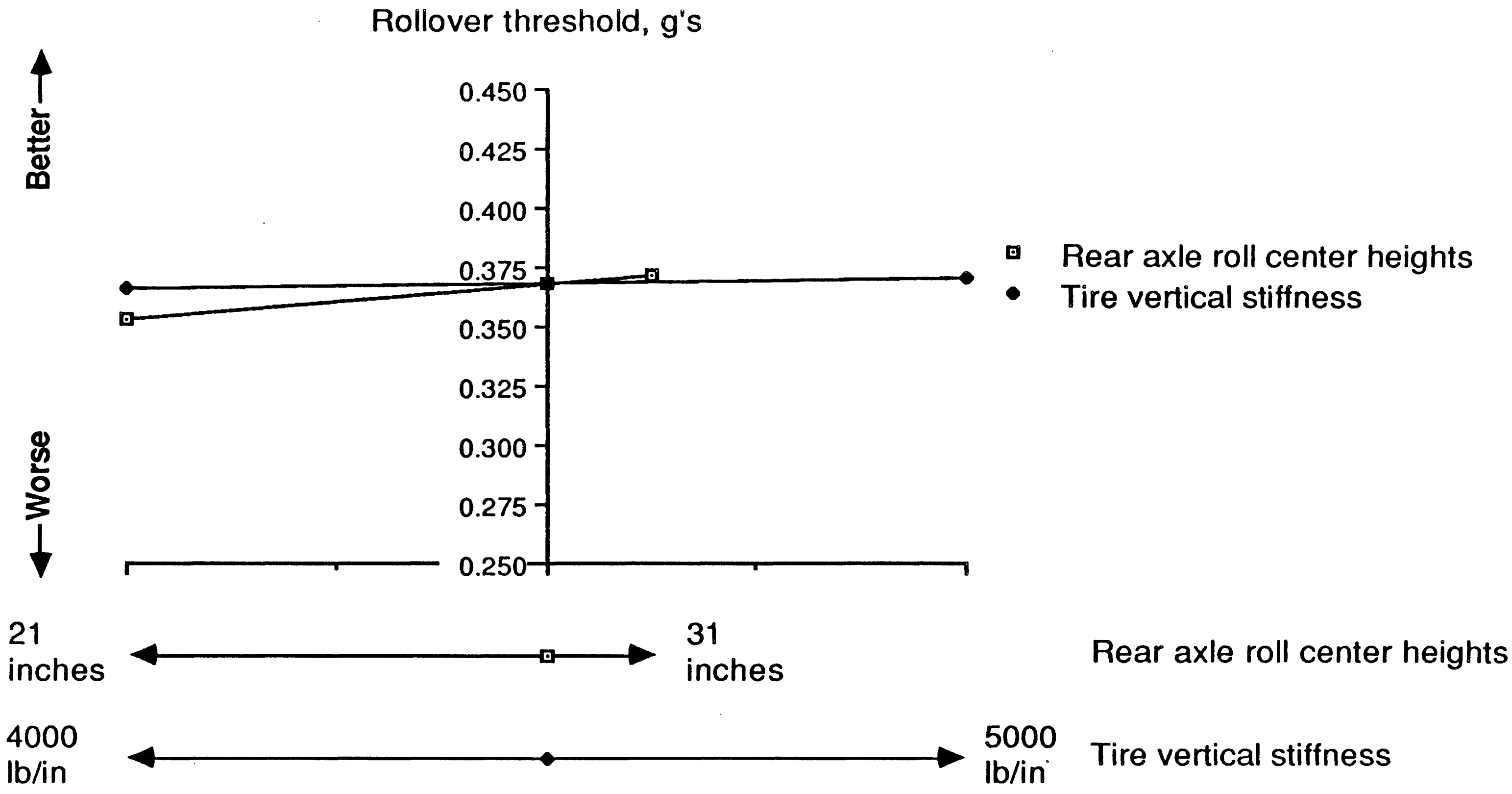


Figure 65. Parameter sensitivity diagram: static roll, tractor and semitrailer

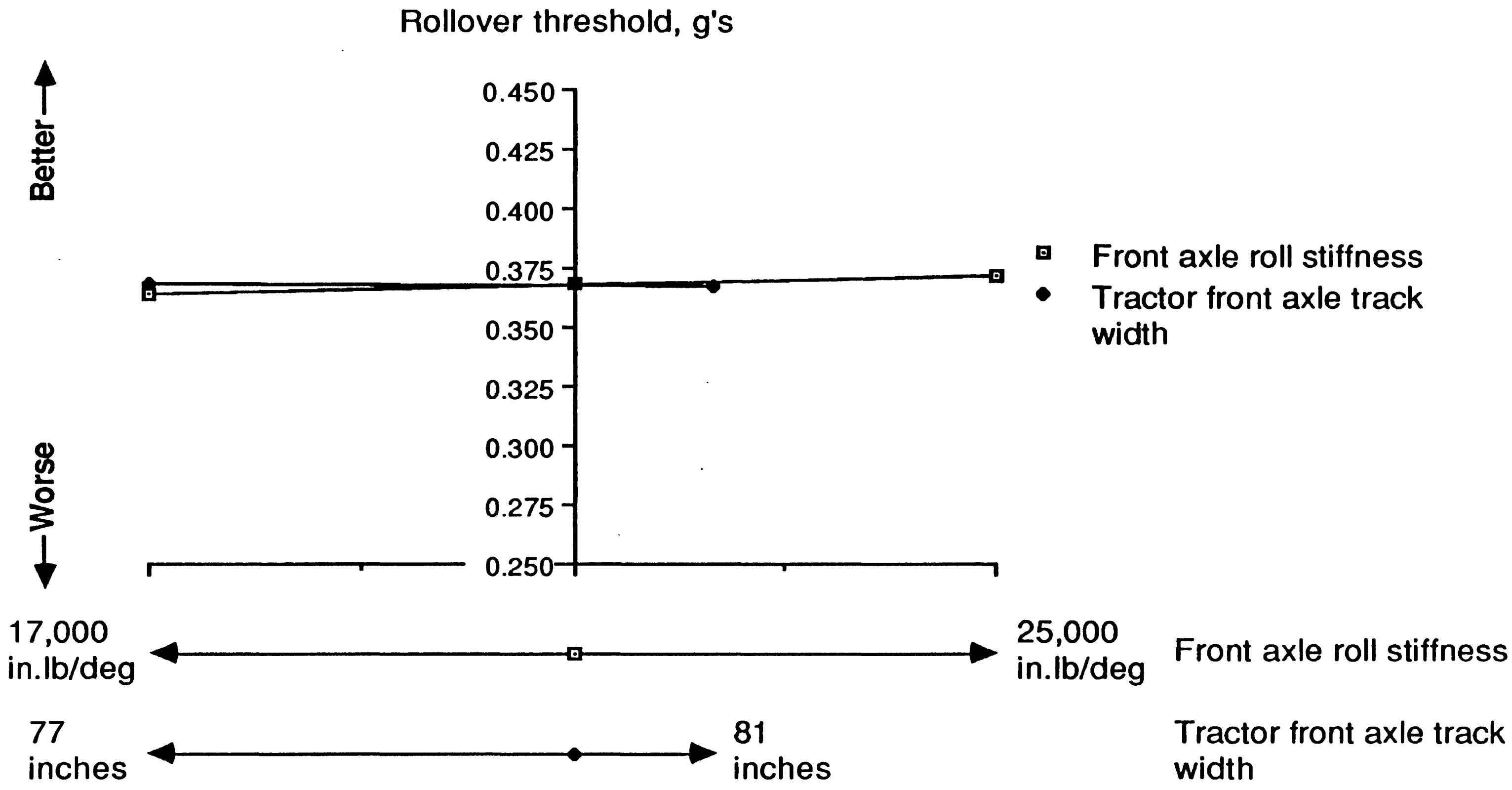


Figure 66. Parameter sensitivity diagram: static roll, tractor and semitrailer

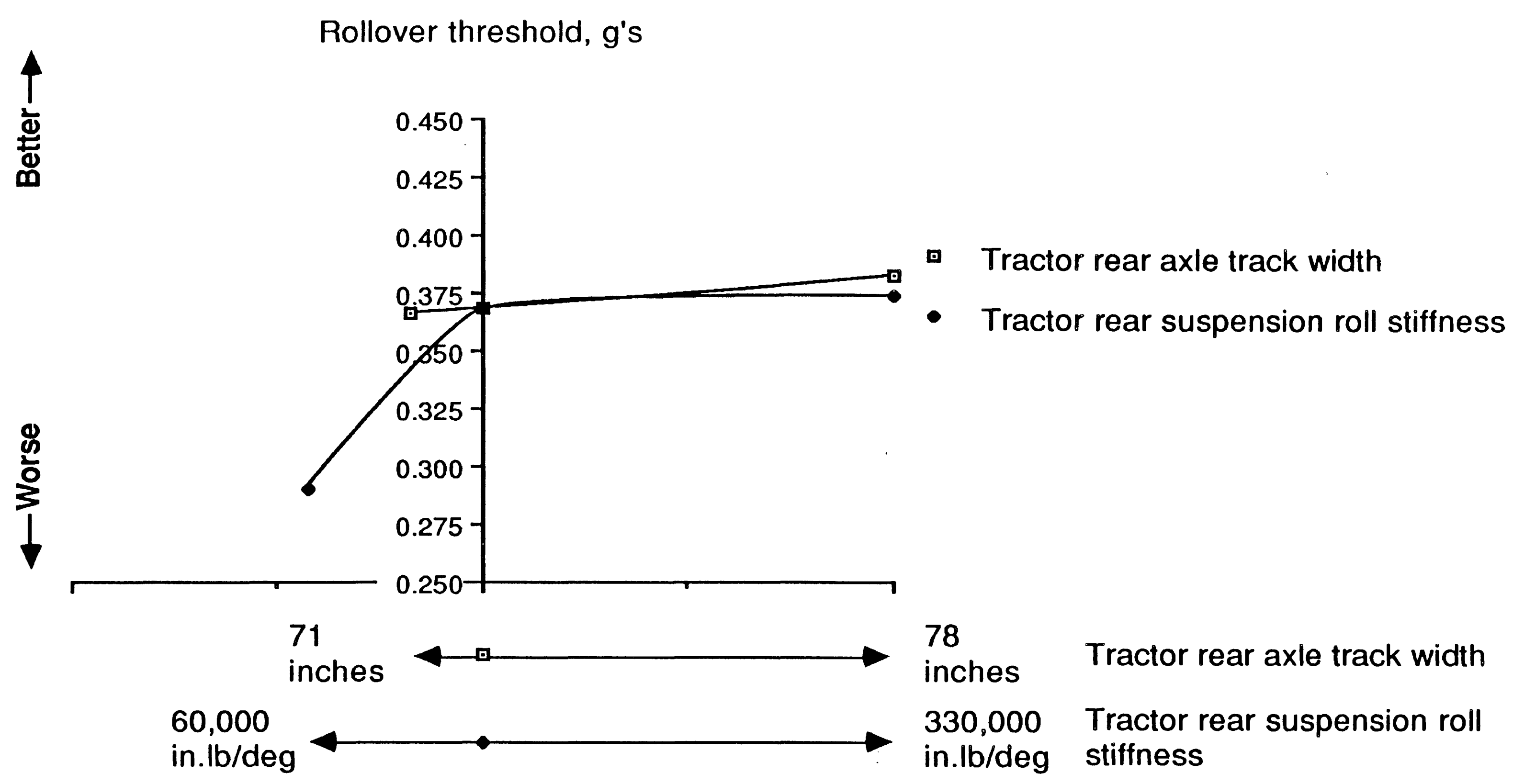


Figure 67. Parameter sensitivity diagram: static roll, tractor and semitrailer.

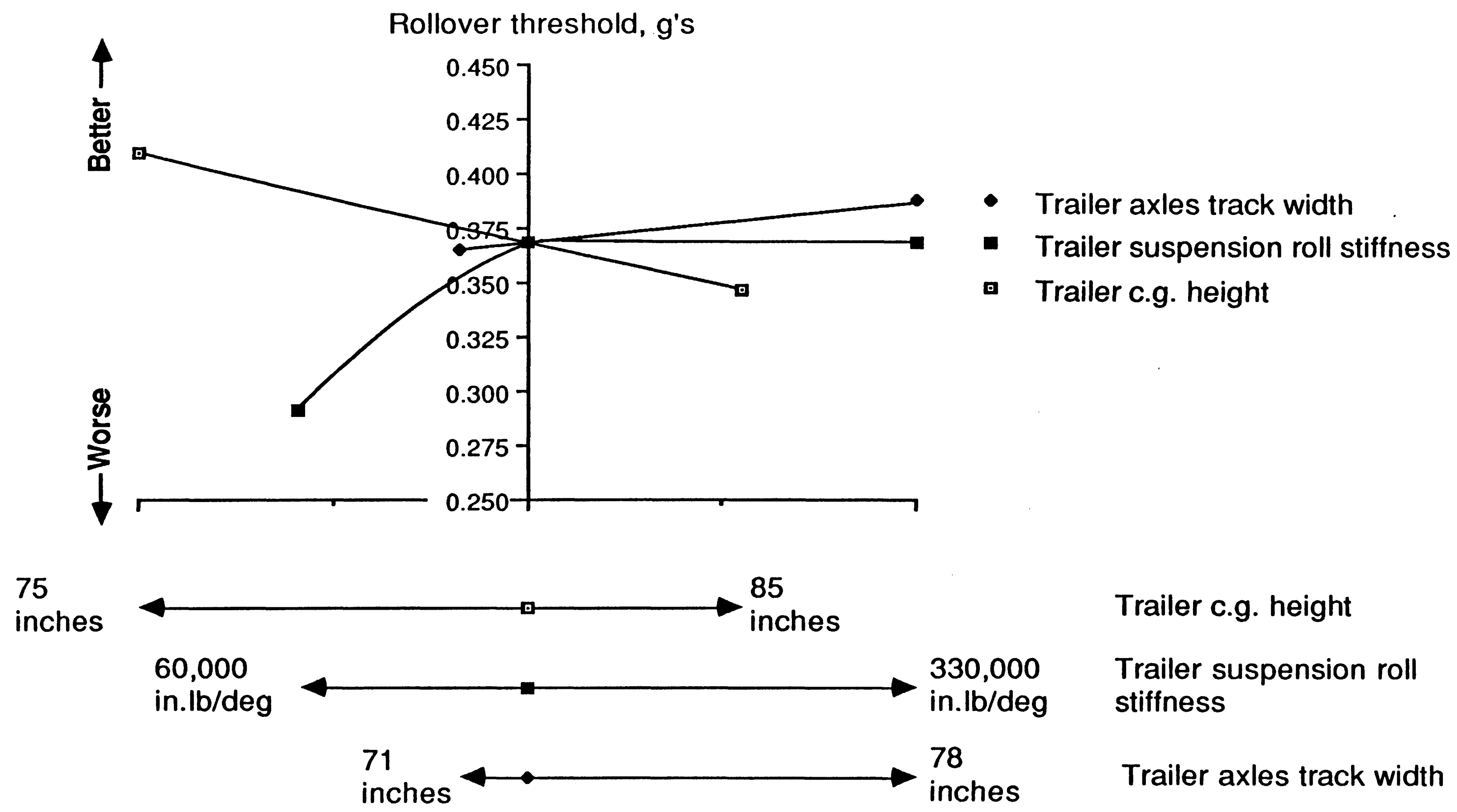


Figure 68. Parameter sensitivity diagram: static roll, tractor and semitrailer.

convenient reference for assessing the importance of the results presented in the other diagrams.

Rollover is resisted by keeping the ratio of c.g. height to track width low and by employing suspensions that are stiff in roll. Properties that reduce the tendency of the c.g.'s of the masses to move laterally will also have a beneficial effect on roll stability.

Figure 65 presents results for variations in tire vertical stiffnesses, and roll center heights. The changes in tire vertical stiffnesses influence the overall roll resistance at the various axles. An increase in roll center height contibutes to a reduction in the lateral translation of the c.g. of the sprung masses. The results presented in Figure 65 are for cases in which the changes are made throughout the entire vehicle. In this way, the effects of these changes become more noticeable than if they had been made one at a time for each unit of the vehicle.

The results displayed in Figure 66 show the influences of changes in the front suspension and axle. The front suspension is relatively "soft" compared to the other suspensions. In fact, the results indicate that the likely changes in front suspension and axle properties will have little influence on roll performance unless the roll stiffness is substantially increased.

In contrast to the previous results, changes in the tractor's rear suspension will have a large influence on roll stability (see Figure 67). The two quantities having the greatest effect on the results given in Figure 67 are the roll stiffnesses of the suspensions and the track widths of the tandem axles. A six-inch increase in track width results in an important increase in roll stability. In this case, the influence is only due to changing the spacing of the wheels on the tractor's rear axles.

The influence of changing the stiffness of tractor's tandem suspension has a discontinuity in trend depending upon whether the stiffness is increased or decreased. For a small decrease in stiffness, a large decrease in roll stability is achieved. While, for a relatively large increase in stiffness, only a relatively small increase in roll stability is achieved.

In order to explain this phenomenon, it is convenient to introduce the concept of "stiff and soft suspensions" [7]. The idea of this concept is that a relative distinction between "stiff" and "soft" suspensions can be made on the basis of whether the vehicle would roll over when the wheels associated with a given suspension lift off. For example, the trailer wheels on the benchmark 3S2 will lift off first, but the vehicle will not roll over until the tractor rear wheels lift off. In this case the trailer suspension is called a "stiff" suspension and the tractor rear suspension is a "soft" suspension. The front suspension is also a soft suspension in this concept. The importance of this distinction is that it does not do any good (with respect to roll stability) to increase the stiffness of a stiff suspension. The vehicle will roll over at the same lateral acceleration level and roll angle as it would have if the stiff suspension had not been stiffened.

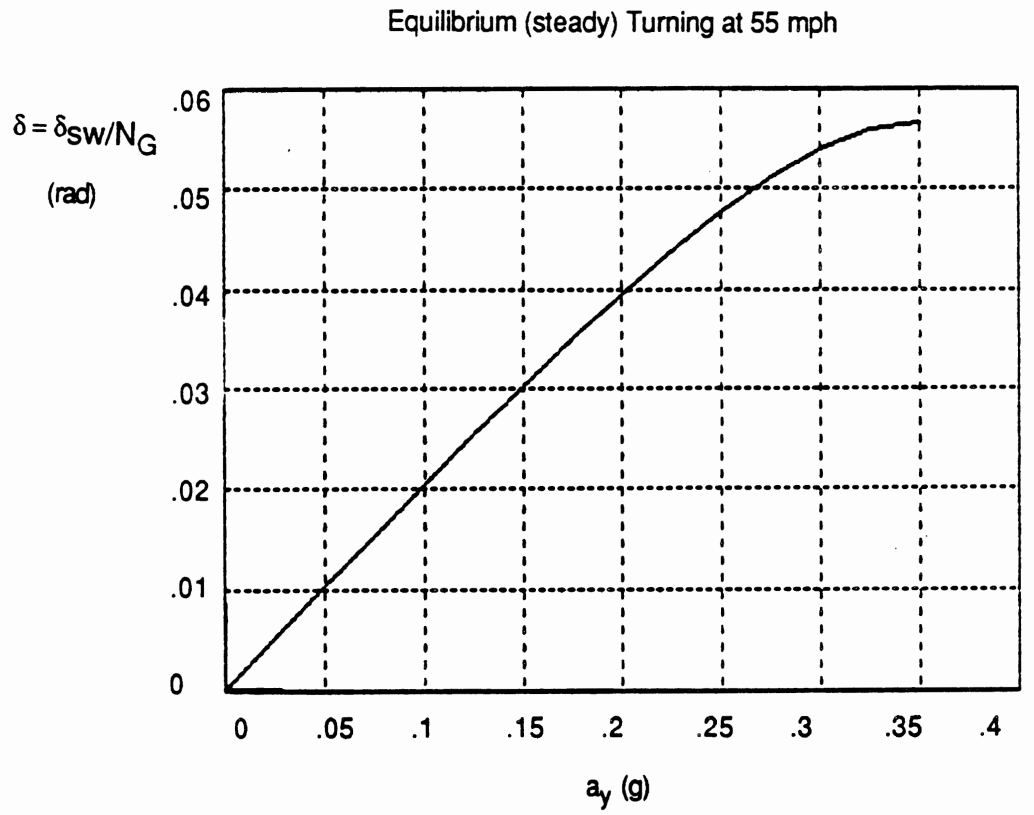
The situation for the variations in the tractor's rear suspension is a little bit complicated because this suspension goes from a soft to a stiff suspension when its roll stiffness is increased above the baseline value. This happens because the tractor rear and trailer suspensions are close enough in baseline stiffness that a reasonable change in the stiffness of the tractor rear suspension will make it stiffer than the trailer suspension. The tractor rear wheels will lift off before the trailer wheels and the vehicle will not roll over until the trailer wheels lift off. (Possibly, all of this will seem easier to understand if one notes that, once wheels lift off, the roll-restoring moment coming from that suspension saturates at the level of moment achieved at lift off.)

However, it is important to observe that any reduction in roll stiffness is going to be detrimental to roll stability if the suspension involved is a soft suspension or if that suspension will become a soft suspension due to the reduction in roll stability. The major loss in roll stability attributed to decreasing the stiffness of the tractor rear suspension (see Figure 67) is a prime illustration of this point.

The results, obtained when trailer axle and suspension parameters are varied, are similar to those obtained when the tractor rear suspension and axle parameters are varied (see Figure 68). When the trailer suspension stiffness is reduced to the point where this suspension becomes a soft suspension, roll stability degrades, as indicated in Figure 68. It does not do any good to increase the stiffness of the trailer suspension, given the baseline values of the stiffnesses of the tractor suspensions. A major increase in roll stability can be achieved by increasing the track widths of the trailer axles.

4.6 Steady Turn - Handling

Benchmark performance. The handling signatures were introduced and explained in Section 3.6. They are simply a means for portraying the steering performance of a vehicle executing a steady turn. The handling signature for the 3S2 is given in Figure 69. These diagrams are only applicable at 55 mph, but, as discussed previously, the influence



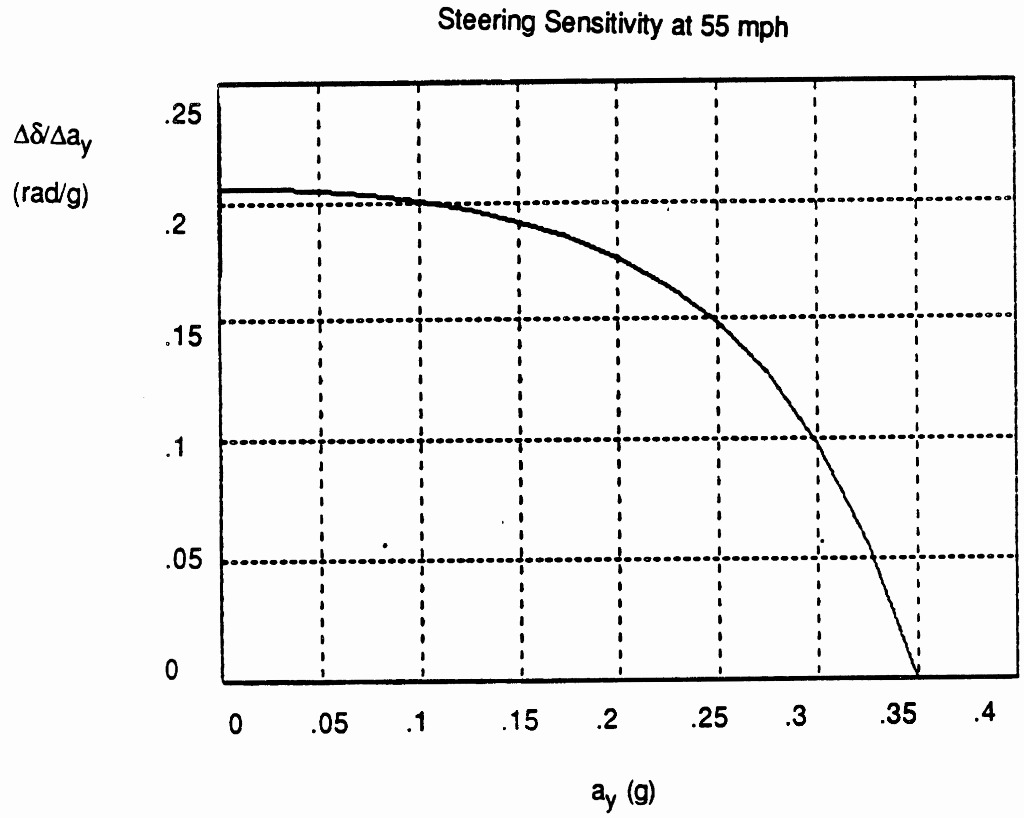


Figure 69. Performance signature, handling, tractor semitrailer

of the tandem axles on the effective wheelbase is small and it does not affect the results appreciably at speeds above 55 mph.

In a combination vehicle operating at highway speeds, the driver steers the tractor and, by so doing, expects to have the rest of the vehicle follow the path of the tractor. If the high-speed offtracking is low, the driver's expectations will be met in steady-turning situations. In this sense, handling is primarily a tractor-related matter and at low levels of lateral acceleration this is the case, with the static loading and stiffnesses of the tractor tires being dominant factors. However, due to the large amounts of side-to-side load transfer occurring at moderate levels of lateral acceleration, *the suspension roll properties of the trailer can have a large influence on the load transfer taking place at the tractor tires and thereby have an important influence on handling.*

The handling performance signature for the 3S2 (Figure 69) is ended at the lateral acceleration level corresponding to the point where the inside trailer tires lift off of the ground. This vehicle is yaw stable up to this point. The steering sensitivity at 0.3 g and 55 mph is 0.097 radian/g. This level of steering sensitivity is well above zero, which is the condition for yaw instability.

Parametric sensitivities. Table 23 lists the variations that have been used to illustrate the parametric sensitivities of the benchmark vehicle. The five sensitivity diagrams (see Figures 70 through 74) used to display the results of these variations have been arranged to show the influences of changes in tire and suspension properties at each of the three suspensions plus two other diagrams that contains results for changes in wheelbases,trailer c.g. height, fifth wheel location, and roll center heights. The steering sensitivity in radians/g at 0.3 g and 55 mph is used here in developing the performance sensitivity diagrams. The value of this performance measure provides an indication of relative yaw stability (with a zero value indicating yaw divergence).

The results presented in Figure 70 show that variations in the fifth wheel location are the most important of the variations made in the general properties of the vehicle. *This vehicle is made unstable (the steering sensitivity is made less than zero) by moving the fifth wheel back to the center of the tractor's rear suspension.* The stability margin of the vehicle can be increased by increasing the tractor's wheelbase or decreasing the c.g. height of the trailer.

The influences of front tire and suspension stiffnesses are illustrated in Figure 71. As shown, the stiffness increase associated with going to radial tires on the front axle

Table 23. Tractor and Semitrailer - Handling

Parameter Name	Benchmark Value	Minimum Value	Maximum Value	Notes
Tractor wheelbase	144 inches	134 inches	268 inches	
Fifth wheel offset	14.4 inches	0 inches	24 inches	
Roll center heights on rear axles	29 inches	21 inches	31 inches	
Tractor's front axle roll stiffness	21,000 in.lb/deg	17,000 in.lb/deg	25,000 in.lb/deg	
Tires on tractor's front axle	Bias-ply, rib tread	Bias-ply, lug tread	Radials	
Tractor's rear suspension roll stiffness	140,000 in.lb/deg	60,000 in.lb/deg	330,000 in.lb/deg	Total roll stiffness on rear tandem
Tires on tractor's rear axles	Bias-ply, rib tread	Bias-ply, lug tread	Radials	
Trailer's c.g. height	81.44 inches	75 inches	85 inches	
Trailer's wheelbase	432 inches	348 inches	450 inches	
Trailer's rear suspension roll stiffness	160,000 in.lb/deg	60,000 in.lb/deg	330,000 in.lb/deg	Total roll stiffness on rear tandem
Tires on trailer's axles	Bias-ply, rib tread	Bias-ply, lug tread	Radials	

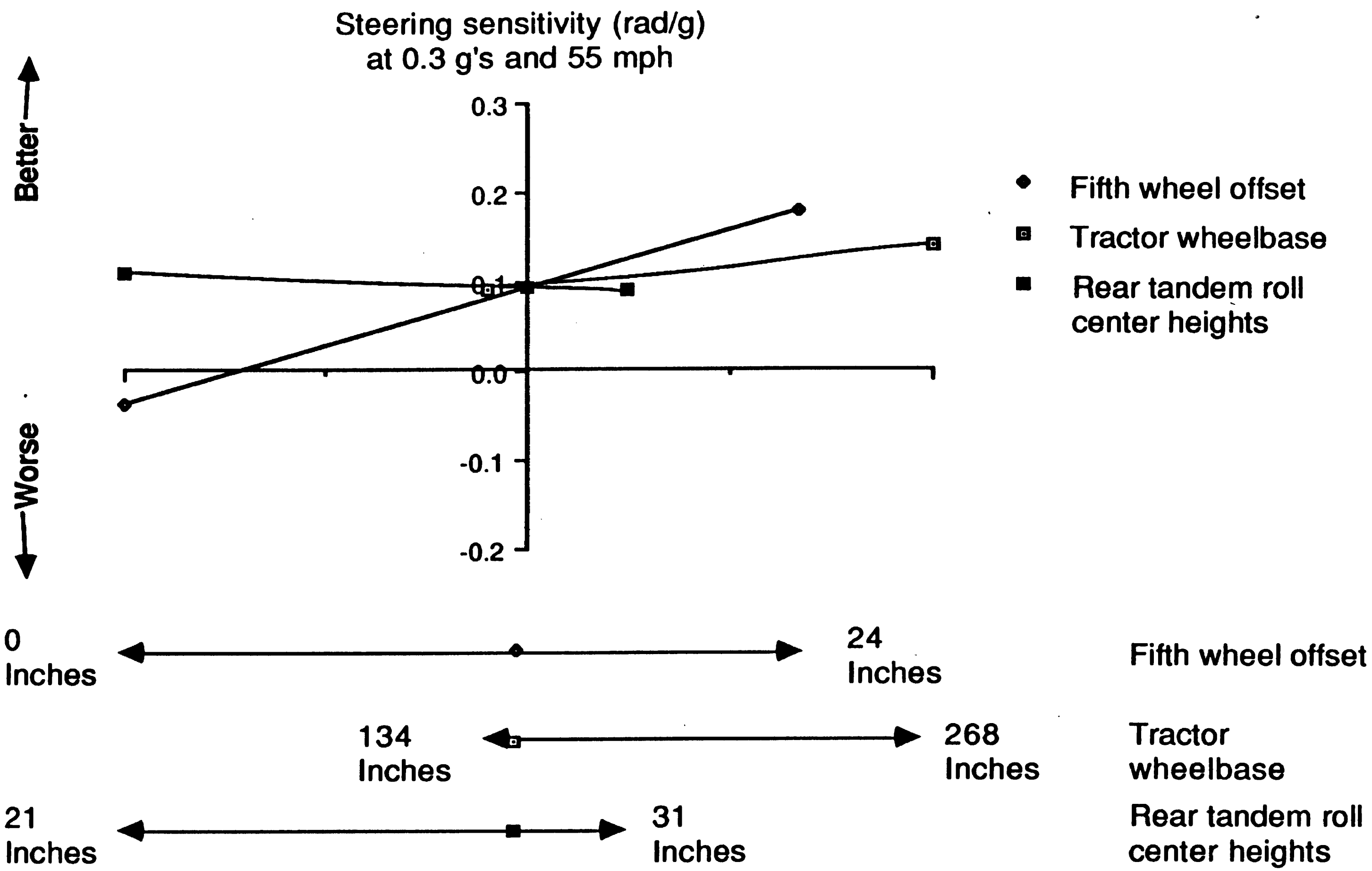


Figure 70. Parameter sensitivity diagram: steering sensitivity, tractor and semitrailer.

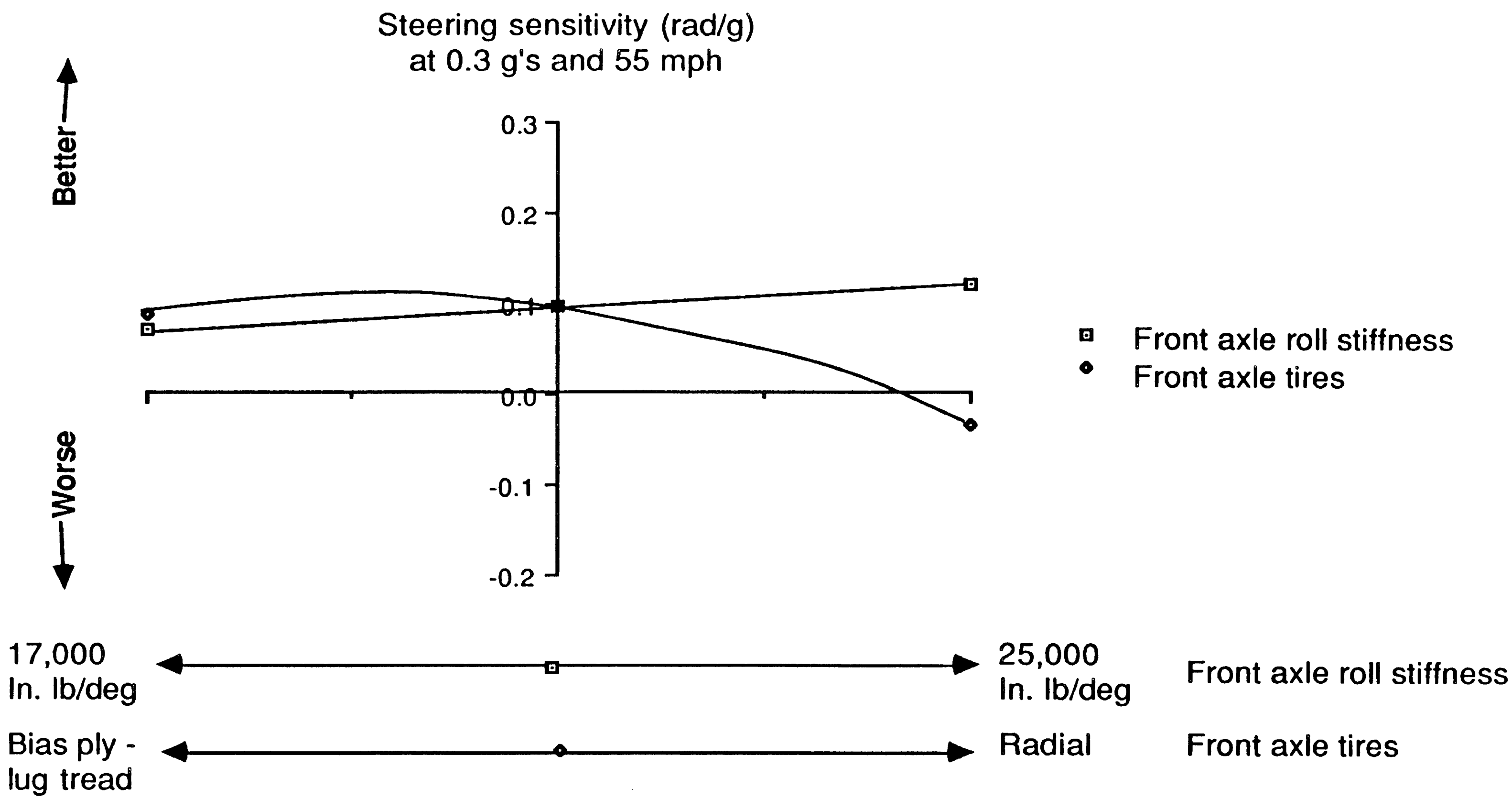


Figure 71. Parameter sensitivity diagram: steering sensitivity, tractor and semitrailer.

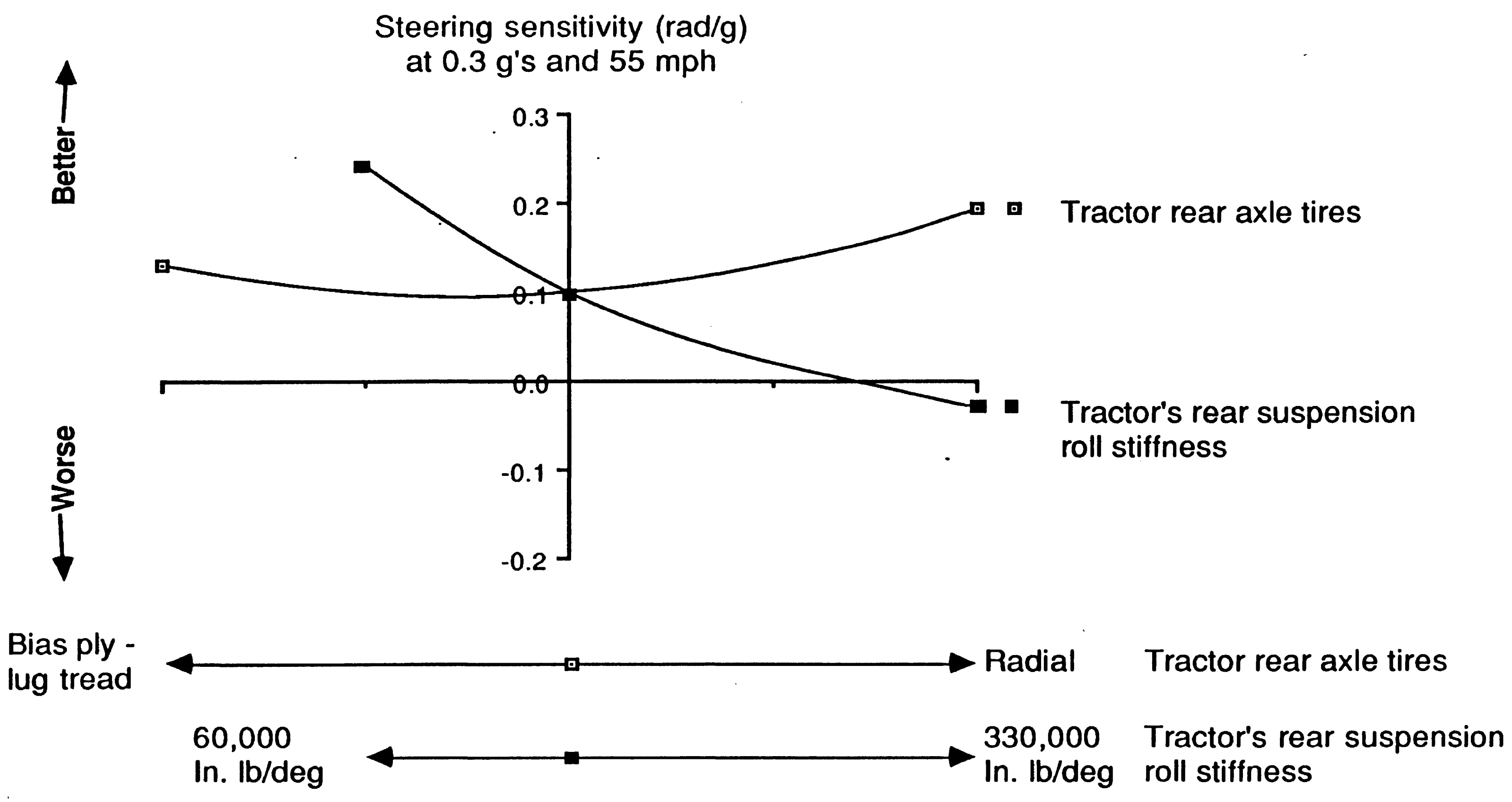


Figure 72. Parameter sensitivity diagram: steering sensitivity, tractor and semitrailer.

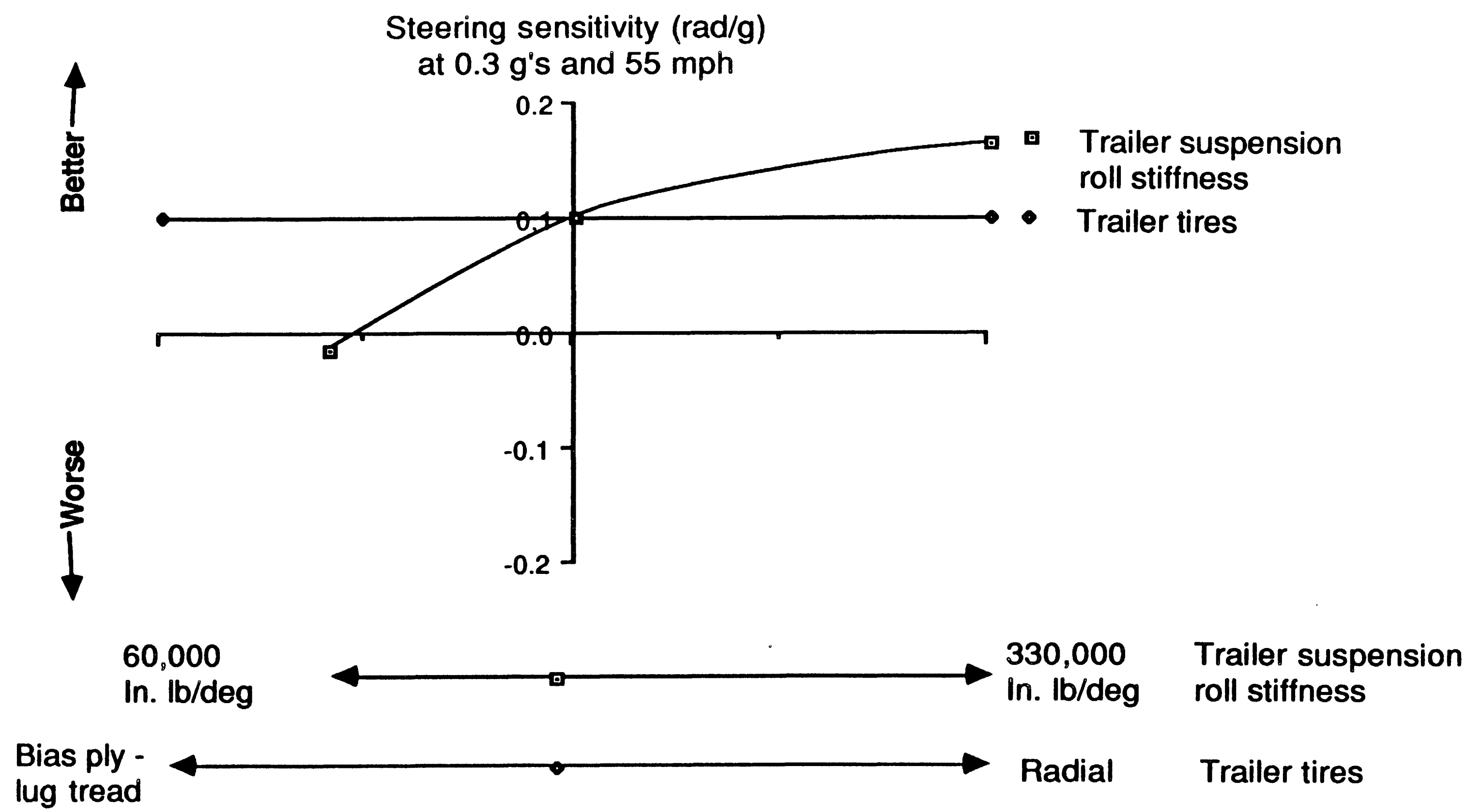


Figure 73. Parameter sensitivity diagram: steering sensitivity, tractor and semitrailer.

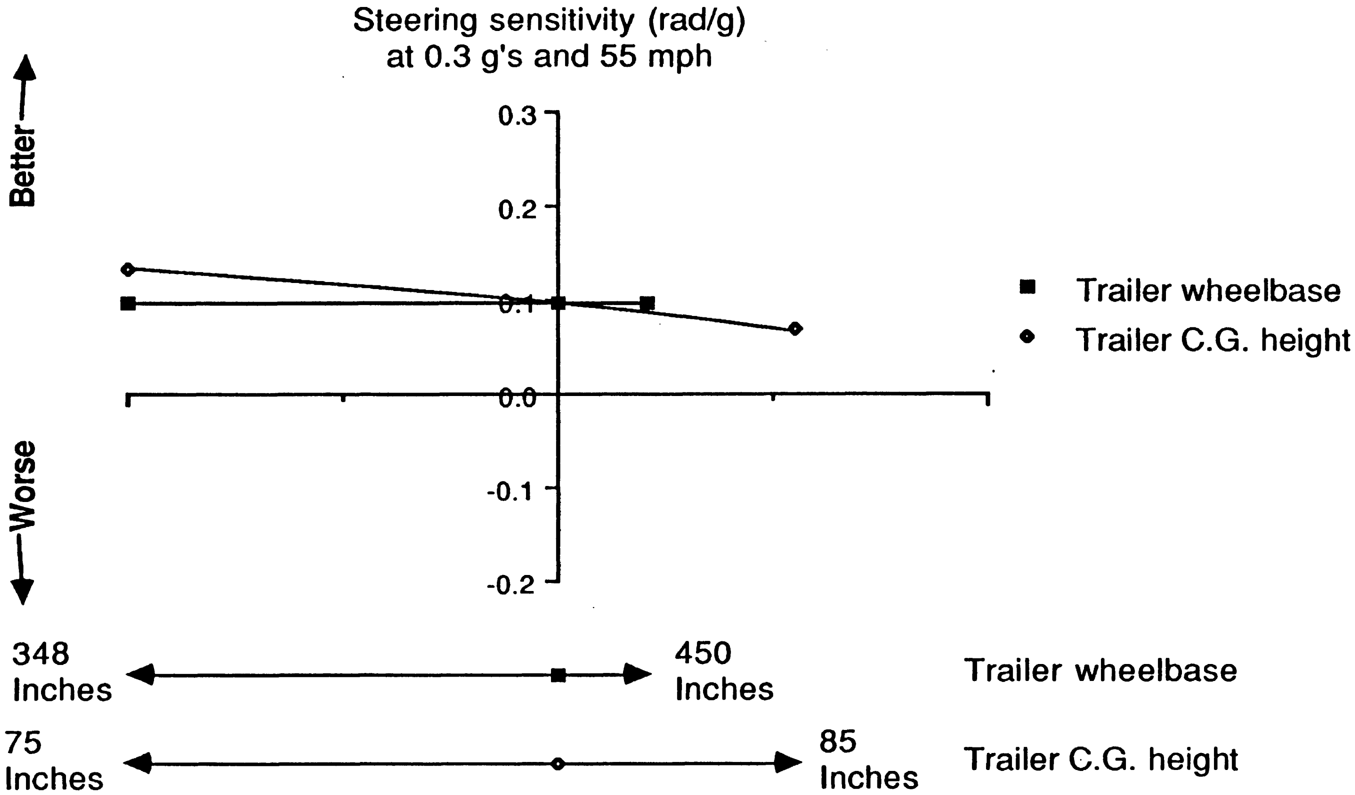


Figure 74. Parameter sensitivity diagram: steering sensitivity, tractor and semitrailer.

results in a negative stability margin. The vehicle is yaw divergent when radial tires are installed on the front wheels and the other wheels are equipped with bias-ply tires.

The changes in front suspension stiffnesses have little effect on handling performance because very little load is transferred at the front. The front roll stiffness is only a small part of the total roll stiffness even when the front stiffness is increased up to the maximum amount used in current practice.

The situation at the tractor's rear axles is entirely different from that at the front axle. As shown in Figure 72, the maximum expected increase in the stiffness of the tractor's rear springs results in a sizeable reduction in stability margin (enough to make the vehicle yaw divergent). This sensitivity to suspension stiffness occurs because the amount of load transferred from side to side at the rear wheels of the tractor is highly dependent upon the magnitude of the rear suspension stiffness. *Yaw stability is reduced when more load is transferred at the tractor's rear tires because (1) those tires provide the side forces needed for directional stability and (2) load transfer results in a net loss in side force capability.*

The influences of changing tire characteristics at the rear of the tractor have just the opposite effect from those obtained by changing the front tires. In this case stiffening the rear tires results in an increase in stability. However, when lug tires are installed on the tractor's rear wheels, stability is not decreased even though the stiffness is decreased some. The reason for this seemingly anomalous behavior is that the lug tire has a low load transfer sensitivity which more than makes up for its reduction in cornering stiffness. (See Section 3.6 and the discussion associated with Equations 2 and 3 for an explanation of the tire's load transfer sensitivity.) The results in Figure 72 show that changes in suspension properties are more influential than changes in tire type in this situation.

The steering sensitivity of combination vehicles is not influenced by the properties of the trailer tires, as indicated by the results given in Figure 73. (The properties of these tires are important to high-speed offtracking.)

Trailer properties do influence handling through their influence on the vehicle's roll stiffness distribution. The amount of load transferred at the *tractor's* rear wheels is as dependent upon *trailer* roll stiffness as it is on *tractor* roll stiffness. When the trailer's roll stiffness is reduced, more load is transferred at the tractor's rear wheels than that transferred in the baseline situation. It is this effect that accounts for the negative stability margin shown in Figure 73 for the situation in which the roll stiffness of the trailer's rear suspension is reduced to that of some air suspensions with very low roll stiffnesses.

In summary, the 3S2 is seen to be stable in handling, although its stability margin can be significantly reduced by reasonable changes in mechanical properties. *Individual changes of the amounts used here for (a) fifth wheel offset, (b) front axle tires, (c) tractor rear suspension roll stiffness, and (d) trailer suspension roll stiffness result in a slightly yaw divergent vehicle.* Combinations of several unfavorable changes could result in a very unstable configuration.

4.7 Response Times in Steering Maneuvers

Benchmark performance. Response times are studied in two maneuvers, namely, ramp step steer and closed-loop obstacle evasion. The measures of response time employed in evaluating performance are illustrated in Figures 48 and 49. (The discussions associated with these figures provide definitions of these maneuvers and their performance measures. See Section 3.7.)

Figure 75 presents time histories of steer angle and lateral acceleration calculated for the benchmark 3S2 performing a ramp step maneuver. The lateral acceleration response of the tractor's c.g. is characterized by a rapid increase up to approximately 30 in./sec.^{2} and then a transition to a slower rise rate as the semitrailer picks up acceleration. For this case, the time to reach 90 percent of the steady-state acceleration is 0.8 sec.

Results for a closed-loop obstacle-evasion maneuver show a striking similarity between the steer angle and lateral acceleration waveforms (see Figure 76). The lateral acceleration of the baseline fully laden 3S2 operating on a good road surface is seen to lag the steering input by approximately 0.1 to 0.2 seconds. As measured by a "cross-correlation" computation, the "average" lag is found to be 0.14 seconds.

Parametric sensitivities. The response times of the 3S2 are studied in the empty and loaded conditions and on a good, dry road and on a poor, wet road. The results obtained at a modest steering level, corresponding to a maneuver of about 0.1 g, and in a more severe maneuver of approximately 0.25 g are listed in Table 24 for ramp step inputs of 28 and 65.6 degrees. These response times to 90 percent of the steady-state response are longer when the vehicle is loaded and when the severity of the turn is larger. The longest response time given in Table 24 occurs on a good road with the fully laden vehicle performing a 0.25 g turn. In this case the system is more damped than in the other situations leading to a slow

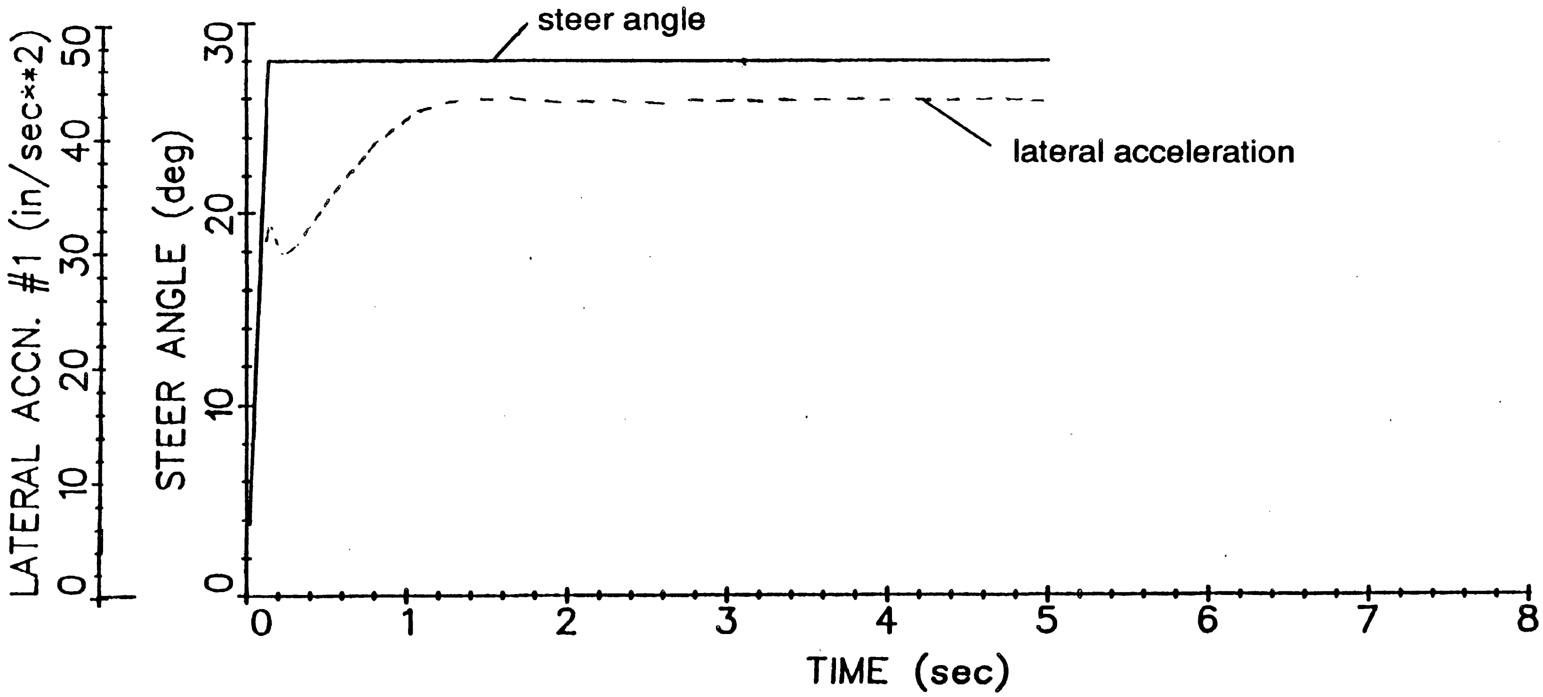


Figure 75. Loaded 5-axle tractor-semitrailer, ramp steer input (high friction tires)

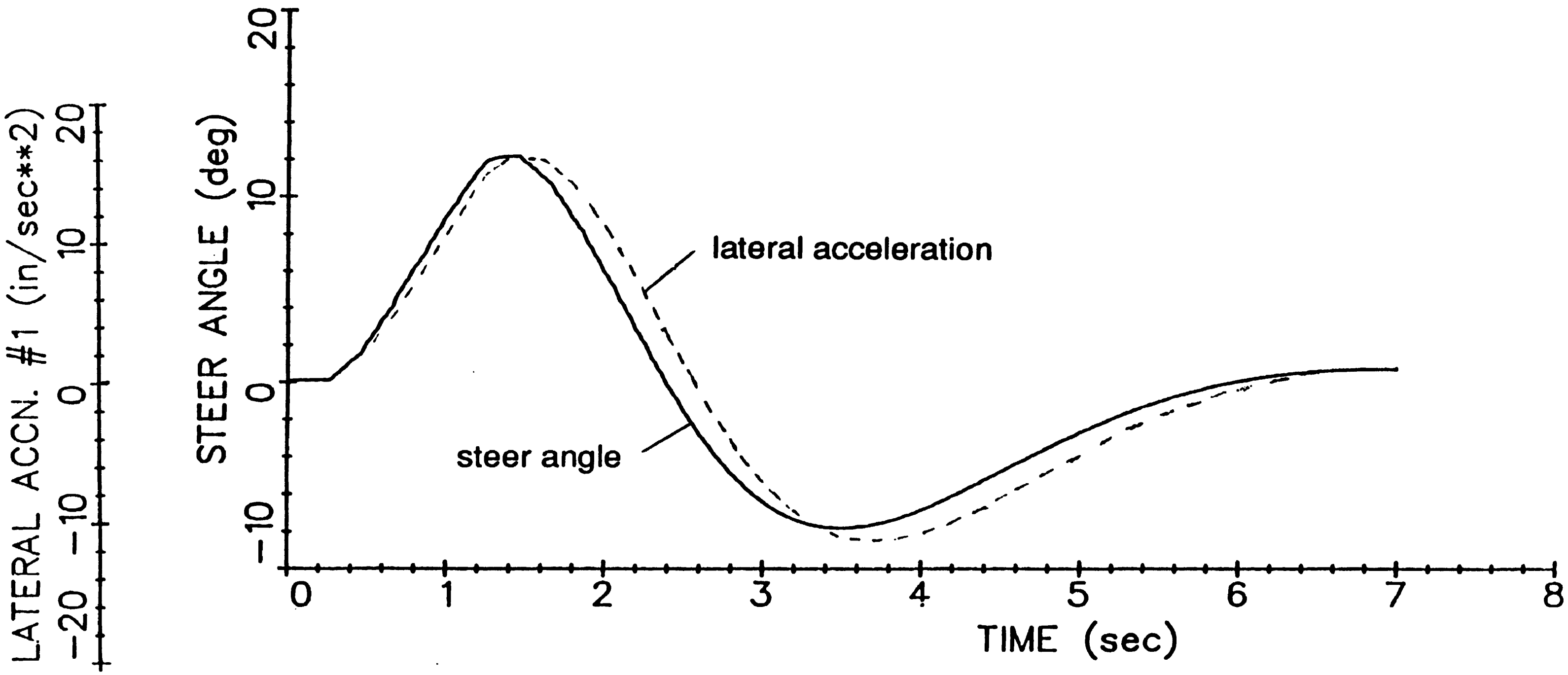


Figure 76. Loaded 5-axle tractor-semitrailer, path follower (high friction tires)

Table 24. Ramp-Step Response Times - Tractor and Semitrailer (Benchmark)

Loading Condition	Road Surface	Response Times Ramp Steer Input	
		28°	65.6°
Empty Vehicle	Dry	0.689	0.797
	Wet	0.652	0.755
Loaded Vehicle	Dry	0.799	1.259
	Wet	0.723	0.871

increase in lateral acceleration as steady state is approached. In all the cases studied, the vehicle responds more quickly on the low-friction road than it does on the high-friction road under comparable operating conditions.

The results from the obstacle-evasion maneuver can be interpreted on a point-by-point basis. For example, the times of maximums (peaks) and minimums (troughs) can be examined (see Table 25). The left most columns of Table 25 provide an indication of time lags determined between peaks and troughs. If a cross-correlation calculation is made to find the average lag that produces the best fit between the steering and acceleration waveforms, the results presented in Table 26 are obtained for this maneuver. The results presented in Table 26 show very high correlations (perfect correlation is 1.0) indicating that the steering input and lateral acceleration outputs are nearly identical in "shape." The time lags (response times) range from 0.18 to 0.14 seconds over the set of variations studied in a 4-foot obstacle-evasion maneuver. In this closed-loop maneuver, the loaded vehicle is slightly quicker than the empty vehicle. The greatest lag is obtained when the vehicle is empty and operating on the high-friction surface. Although only a limited understanding of closed-loop performance is currently available, the calculated results appear to indicate that drivers should not have any special problems in any of the situations considered here.

So far, we have been treating the tractor's response to steering. Table 27 presents time lags between when the tractor maneuvers and when the trailer maneuvers. Since the trailer's c.g. is approximately 26.5 feet behind the tractor's c.g. and the vehicle is traveling at 73.33 feet/sec. (50 mph), the desired time shift between tractor response and trailer response is 0.36 seconds. As shown by the results in Table 27, the semitrailer is a wonderful device that provides excellent tracking (near perfect lag). The loaded semitrailer's longitudinal error, corresponding to 0.02 seconds of "error" in response time, is less than 1.5 feet.

4.8 Rearward Amplification

Benchmark performance. The semitrailer in a 3S2 follows the path of the tractor with very little amplification between the motion of the tractor and that of the semitrailer. Rearward amplification is primarily a problem for vehicles that employ full trailers and conventional dollies. Nevertheless, the rearward amplification of the 3S2 is examined here in order to quantify the magnitude of the amplification between the tractor's c.g. and the trailer's c.g. As indicated in the previous section, the semitrailer's acceleration is almost identical to that of the tractor in an obstacle evasion maneuver. In this section, we will

Table 25. Closed-Loop Response Times - Tractor and Semitrailer (Benchmark)

Loading Condition	Road Surface	Response Times Path Follower									
		Lateral acceleration				Steer Angle				Time Lag	
		Peak		Trough		Peak		Trough		Peak	Trough
		Value	Time	Value	Time	Value	Time	Value	Time		
Empty Vehicle	Dry	17.430	1.540	-12.200	3.640	10.810	1.420	-7.110	3.420	0.120	0.220
	Wet	17.310	1.540	-12.000	3.660	11.780	1.430	-7.700	3.440	0.110	0.220
Loaded Vehicle	Dry	16.360	1.460	-11.400	3.720	12.460	1.420	-7.830	3.510	0.040	0.210
	Wet	16.170	1.460	-11.300	3.260	12.950	1.460	-8.390	3.560	0.000	0.200

Table 26. Closed-Loop Response Times - Tractor and Semitrailer (Benchmark)

Loading Condition	Road Surface	Path Follower Cross Correlations	
		Lag	Value
Empty Vehicle	Dry	0.180	0.997
	Wet	0.160	0.998
Loaded Vehicle	Dry	0.140	0.997
	Wet	0.140	0.997

Table 27. Closed-Loop Rearward Amplification - Tractor and Semitrailer (Benchmark)

Loading Condition	Road Surface	Path Follower Cross Correlations	
		Lag	Value
Empty Vehicle	Dry	0.360	0.998
	Wet	0.360	0.998
Loaded Vehicle	Dry	0.380	0.999
	Wet	0.380	0.999

generalize the previous results by using calculations in the frequency domain to study the open-loop transfer function between the motion of the tractor and that of the trailer.

The performance signature chosen to evaluate rearward amplification is a graph of amplification versus the frequency of sinusoidal excitation (for example, see Figures 77 and 78 which are the performance signatures for the benchmark 3S2 in the loaded and empty conditions). The figures show that the loaded vehicle has a small amount of amplification (less than 1.1) at the trailer's c.g. at a frequency of approximately 1.5 radians/sec. The empty vehicle has a gain (amplification) of 1.0 at close to zero frequency and a rapid decrease in amplification, actually an attenuation, as frequency increases. (A gain of one out to a frequency suitable for performing emergency maneuvers and then a sudden attenuation in response is an idealized goal that one might use to evaluate rearward amplification.)

Parametric sensitivities. Figures 79 and 80 show that, if stiff radial tires are installed on all axles of the 3S2, there is no amplification in either the loaded or empty condition and the gain only decreases to about 0.9 at 2 radians/sec. This type of performance is rated as good.

No other performance sensitivities have been examined because, in general, 3S2 configurations are very well behaved with regard to rearward amplification.

4.9 Braking in a Turn

Benchmark performance. "Braking in a turn" could refer to many different arangements of braking and steering inputs. In this analysis, the vehicle is put into a moderate turn of 1,500-foot radius at an initial speed of 50 mph. At the time the brakes are applied, the tractor has reached approximately 0.15 g of lateral acceleration. The treadle valve is applied suddenly and then released as shown in Figure 81. This type of braking input represents a situation in which a driver over brakes (because the road may have been slipperier than it appeared to be) and then releases the brakes quickly in an attempt to regain directional control. The effect of this action on lateral acceleration is also illustrated in Figure 81. At 4 seconds into the maneuver the lateral acceleration has nearly reached the appropriate steady-state value for the turn. When the brakes are applied (1) the wheels lock up or approach lockup, (2) the tire side forces fall, and consequently (3) the lateral acceleration drops to nearly zero. The c.g. of the vehicle starts to proceed straight ahead rather than turning. The driver releases the brakes and, because the tires have now

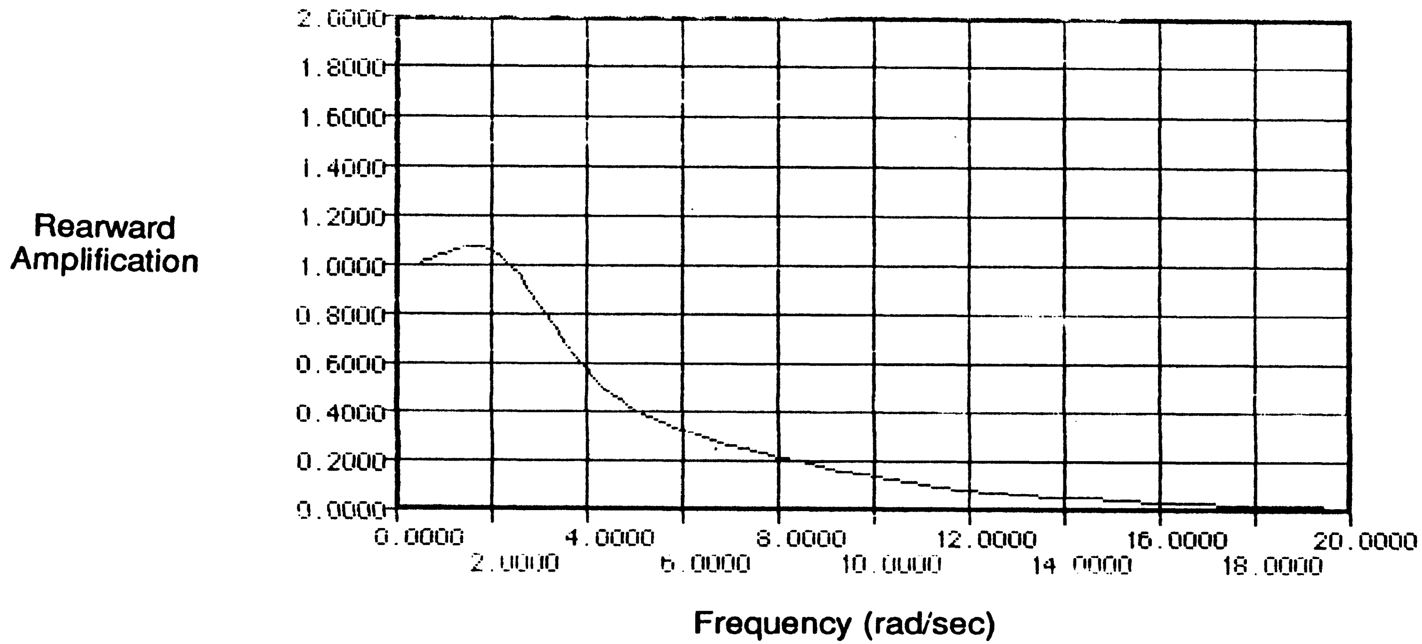


Figure 77. Performance signature, rearward amplification, loaded 3S2

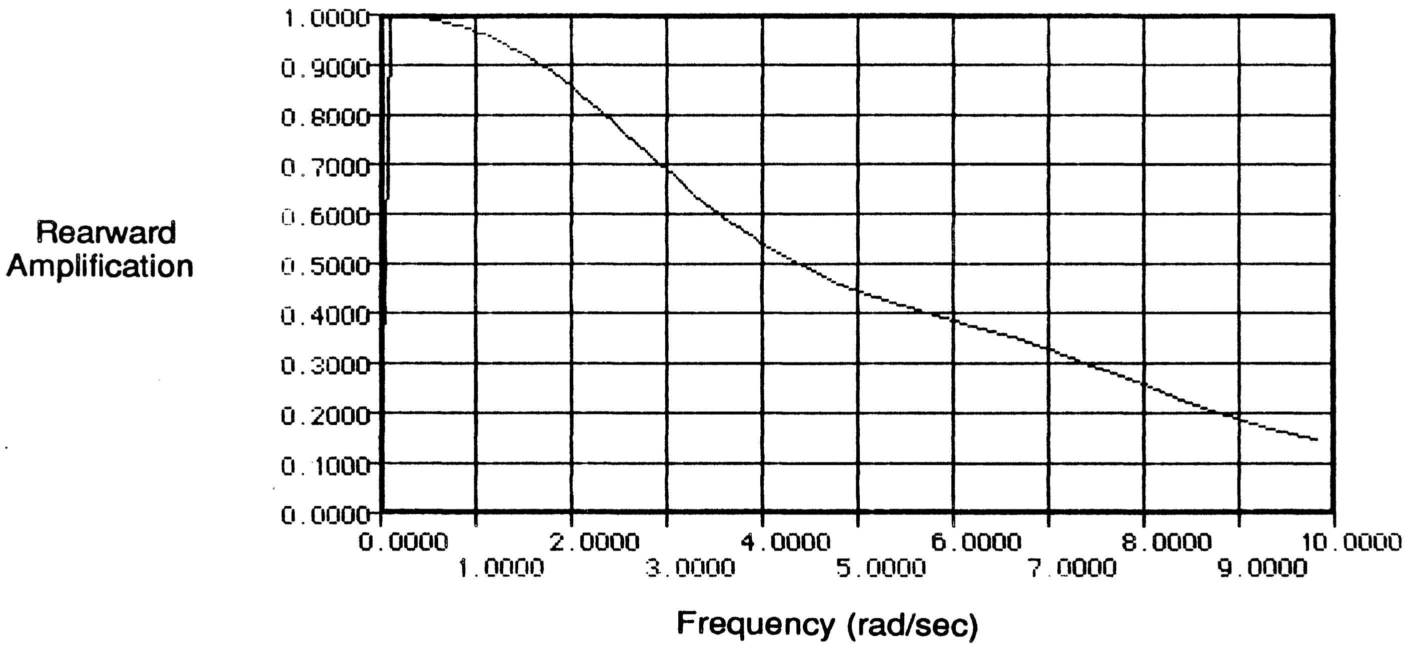


Figure 78. Performance signature, rearward amplification, empty 3S2

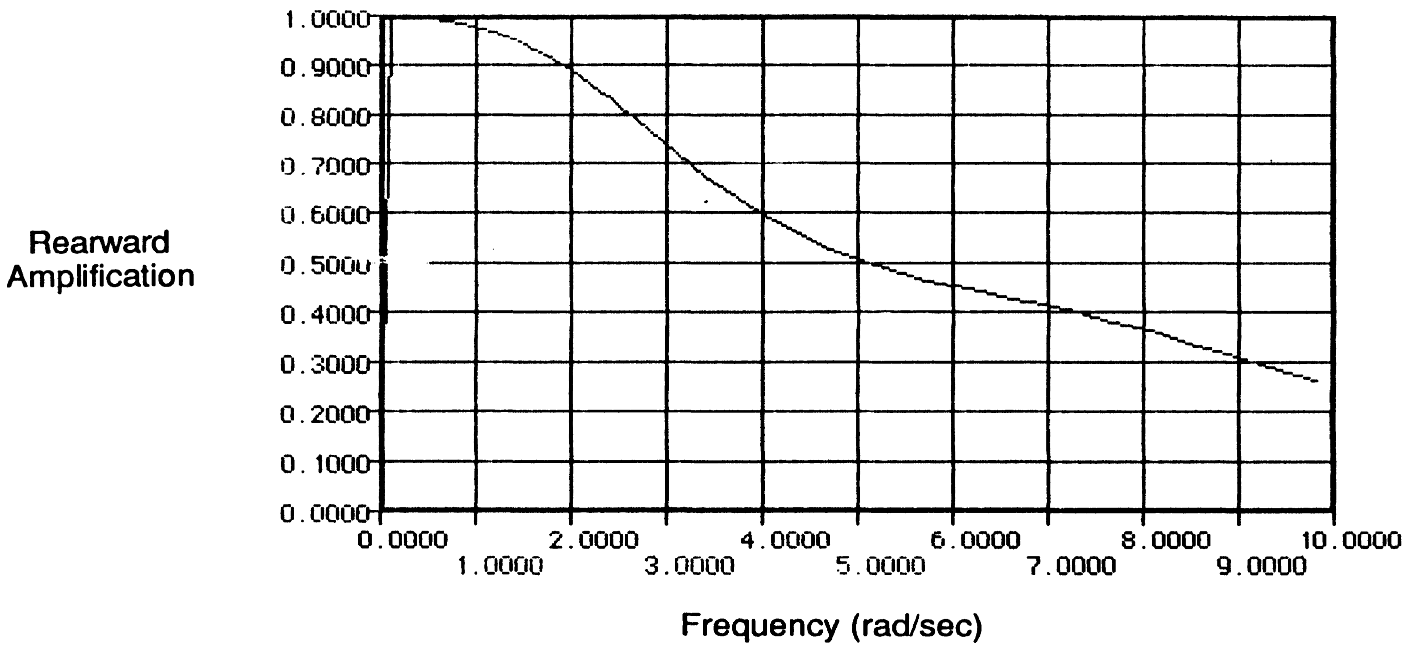


Figure 79. Rearward amplification of a loaded 3S2 equipped with radial tires

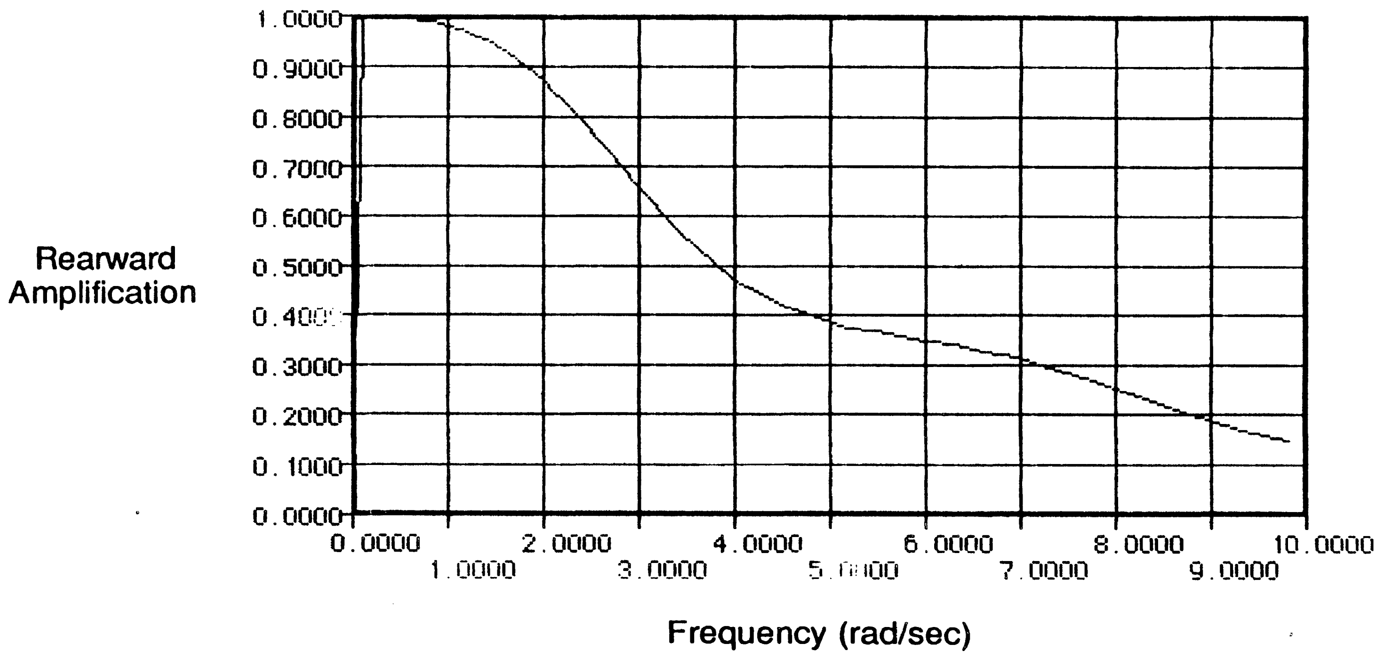
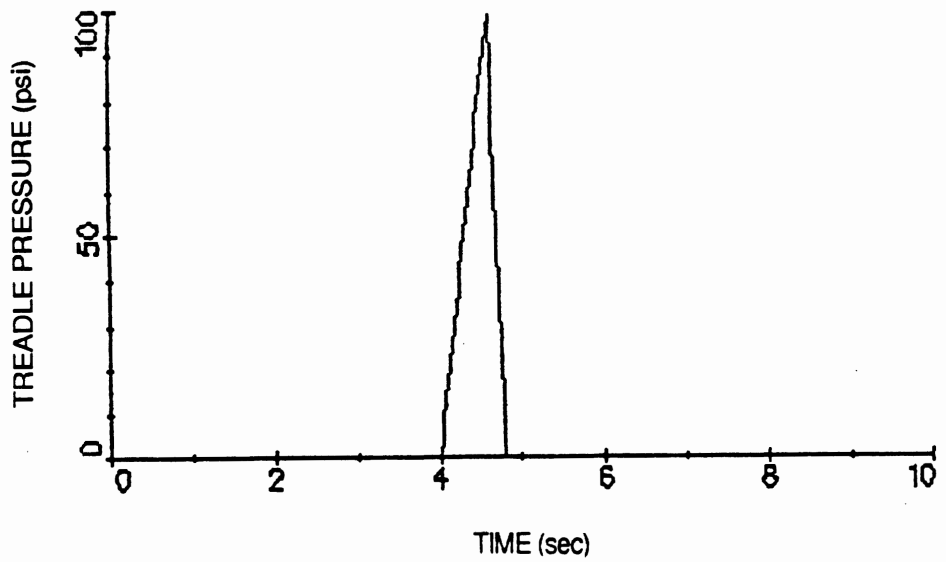
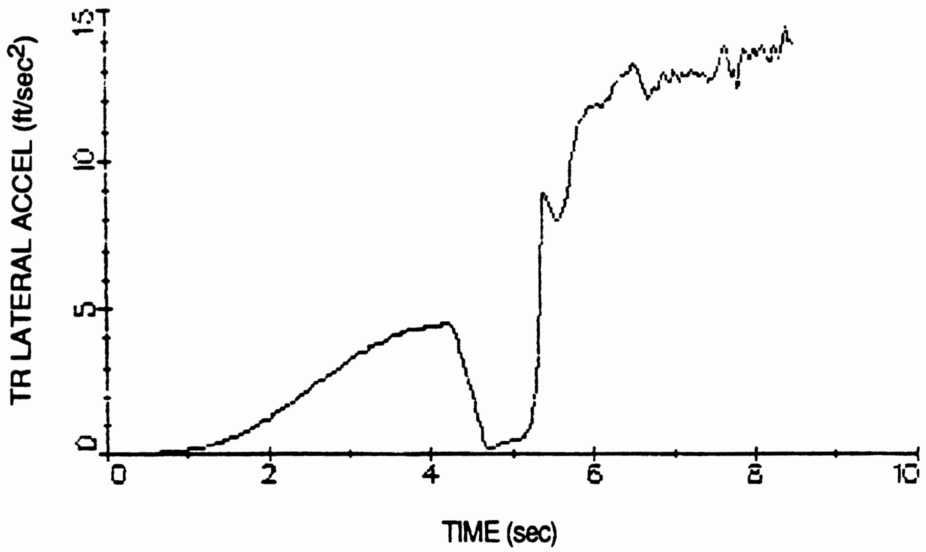


Figure 80. Rearward amplification of an empty 3S2 equipped with radial tires



EMPTY TRAC/SEMI (TIRE MODEL WET SURFACE): BRAKING IN A 1500' TURN

81 a. Braking pulse



EMPTY TR-SEMI (WET) BRAKING W/O ANTILOCK CLOSED LOOP

81 b. Lateral acceleration

Figure 81. Braking in a turn, braking pulse, and lateral acceleration response

developed large slip angles, the tires produce large side forces causing the lateral acceleration to increase to a level that is much larger than that required for the original turn.

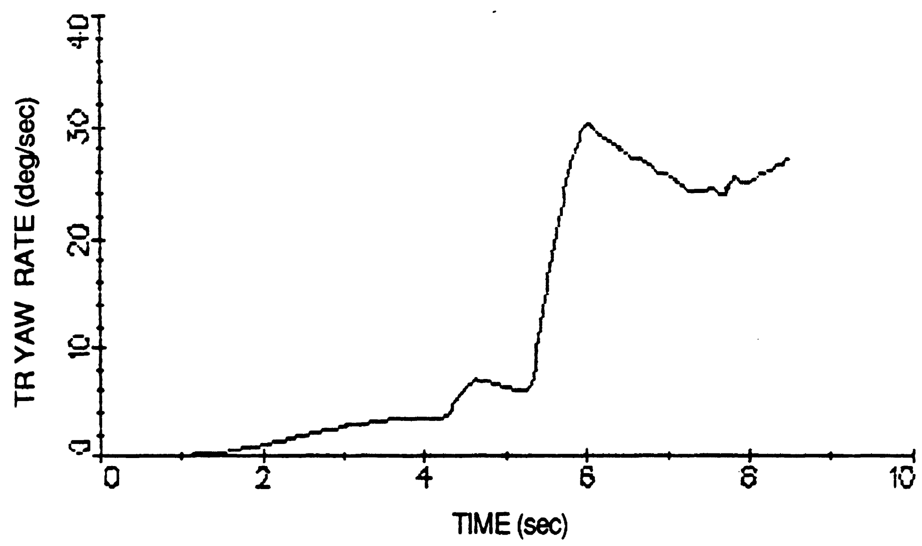
Furthermore, the side forces at the front and rear of the tractor do not return simultaneously. The side forces return first at the front tires because the front wheels unlock much more quickly than the rear wheels. In terms of directional response, this means that the tractor will rotate (yaw) rapidly and, since the trailer does not rotate as much as the tractor, the vehicle jackknifes (see Figure 82). The jackknife is indicated by the large increase in the articulation angle between the tractor and the trailer.

Parametric sensitivities. The results presented in Figures 81 and 82 are for the empty benchmark 3S2, operating on a poor, wet road surface. This same type of maneuver could be performed for the loaded vehicle and/or on a good road. In all of these operating states the braking pulse will lead to a violent and undesirable result. On a good high-friction surface, this maneuver will lead to large lateral accelerations and, consequently, to large roll angles--even the empty vehicle is in danger of rolling over. The same factors that lead to the low braking efficiency of the empty vehicle cause the possibility of jackknifing to be greater for the empty vehicle than it is for the loaded vehicle. Specifically, in the unladen condition the vehicle has too much braking at the tractor rear and not enough braking at the front of the tractor.

These results represent a new discovery with regard to defining maneuvering situations that can be difficult or impossible for truck drivers to control with existing hardware. Possibly, adjustments in the timing of brake releases can be arranged to reduce the tendency to develop large articulation angles. Other possibilities are to try load-sensing proportioning and antilock systems, however, more research is needed to determine how best to deal with this type of problem. Until more work is done, one can attempt to avoid situations and configurations in which large side forces are inadvertently present at the front wheels while the rear wheels are incapable of generating the side forces necessary to stabilize the vehicle.

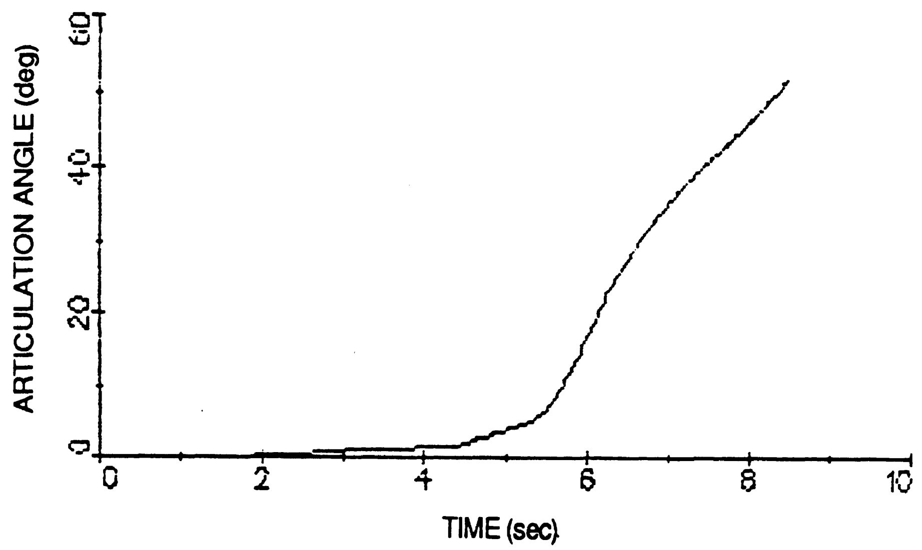
4.10 Concluding Remarks on the Tractor-Semitrailer

The material presented in Sections 4.1 through 4.9 provides analyses of the 3S2 that range in complexity from the simplest offtracking considerations to a complicated and very complex braking-in-a-turn situation. These analyses are all more extensive than those applied to the straight truck because the articulation point essentially adds another vehicle to the system being analyzed. In offtracking matters, the semitrailer is the dominant unit,



EMPTY TR-SEMI (WET) BRAKING W/O ANTILOCK CLOSED LOOP

82 a. Yaw rate



EMPTY TR-SEMI (WET) BRAKING W/O ANTILOCK CLOSED LOOP

82 b. Articulation angle between tractor and semitrailer

Figure 82. Braking in a turn, jackknife response

while in handling, the characteristics of the tractor usually dominate. In rollover situations, either tractor or trailer properties can be important. In addition to the maneuvers considered for the truck, rearward amplification and braking in a turn were introduced here. Although typical 3S2's do not have rearward amplification problems, rearward amplification was discussed to provide background for the treatment of vehicles that employ full trailers. The braking-in-a-turn analysis was added to study jackknifing tendencies, however, this same maneuver could be used to study the spinout of straight trucks.

5.0 TRUCK/FULL TRAILER

5.1 Baseline Values of Pertinent Mechanical Properties

The benchmark vehicle examined in this section consists of the truck and full trailer illustrated in Figure 83. The loaded truck weighs 42,000 lbs and the loaded trailer weighs 38,000 lbs. The wheelbase of the truck is 235 inches and the wheelbase of the trailer is 222 inches. The truck employs a tandem suspension at its rear. The trailer employs single axles front and rear. (The shorthand notation for this vehicle is 3-2.) The dolly is a "fixed" dolly--not a converter dolly. That means that the dolly tongue does not transmit vertical load to the truck. The dolly has a pintle hitch at the first articulation point and a turntable at the rear articulation point. (The dimensions of this vehicle are based on those of gasoline tankers used in California.)

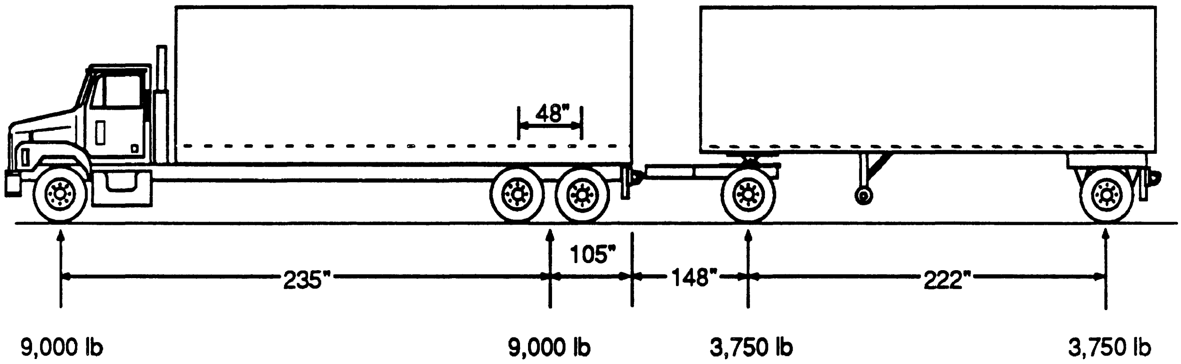
Table 28 lists the values of the basic mechanical properties characterizing this 3-2. These values represent the baseline condition for the performance sensitivity diagrams presented here.

5.2 Low-Speed Cornering - Tractrices

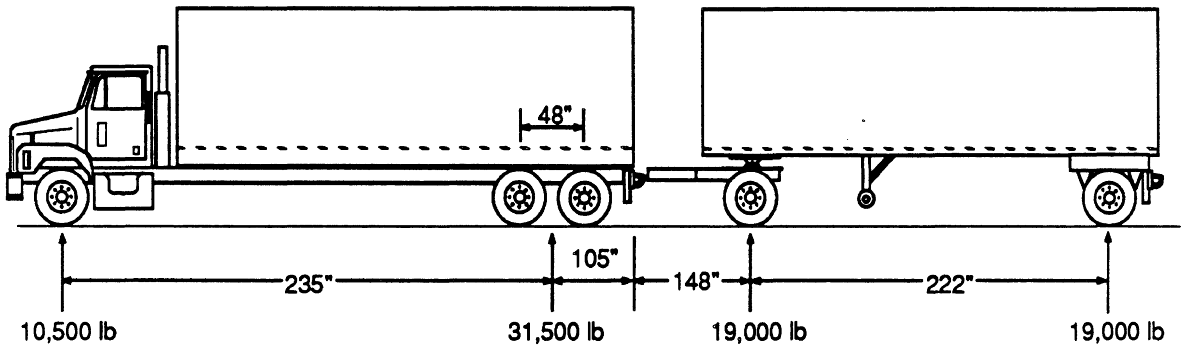
Benchmark performance. The performance signature of the benchmark vehicle, presented in Figure 84, shows the paths of each of the four suspension centers. The two "center" curves, corresponding to the paths of the tractor rear and the dolly axles, are relatively close together (compared to the path of the rearmost axle) because the 105-in overhang of the pintle hitch behind the truck's tandem axle set almost compensates for the influence of the length of the dolly (148 in between the pintle hitch and the dolly's axle). *Overhang decreases in board offtracking since the tandem set is at the minimum radius for any point on the tractor*. The maximum offtracking of the center of the rearmost axle is 9.71 feet in a right turn with a radius of 41 feet to the center of the front axle. This is the baseline value used in the following performance sensitivity diagrams.

Parametric sensitivities. The quantities varied for studying both low- and high-speed offtracking are listed in Table 29. Only geometric dimensions (the offtracking dimensions) are considered in the evaluation of low-speed offtracking.

Figures 85, 86, and 87 show, respectively, the influences of changing truck, dolly, and semitrailer dimensions. The pertinent truck dimensions, wheelbase and pintle hitch overhang, have opposing influences on low-speed offtracking. *A decrease in wheelbase will decrease offtracking at low speed while a decrease in overhang will increase low-speed offtracking.* As shown in Figure 85, the wheelbase changes have a larger influence than the changes in the



EMPTY TRUCK AND FULL TRAILER

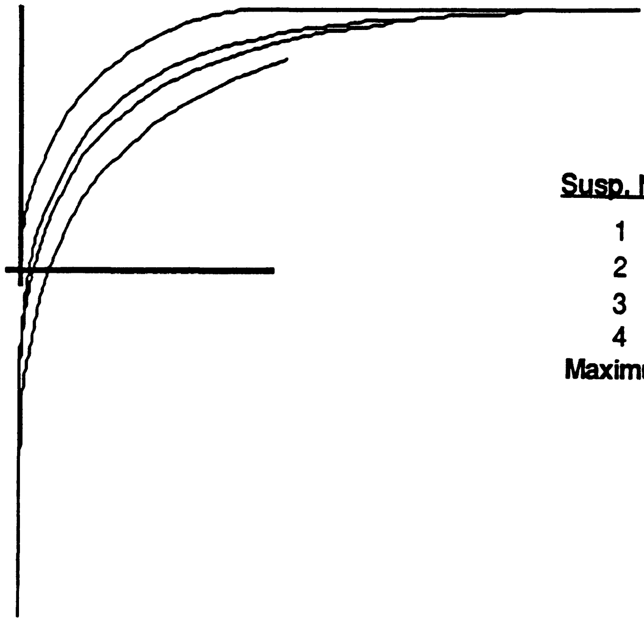


LOADED TRUCK AND FULL TRAILER

Figure 83. Geometric layout, truck and full trailer

Table 28. Basic Mechanical Properties (Truck and Full Trailer)

Mechanical Property	Empty	Loaded
Truck		
Weight (lb)	18,000.00	42,000.00
Front axle brake gain (in.lb/psi)	2,000.00	2,000.00
Rear suspension - leading axle brake gain (in.lb/psi)	3,000.00	3,000.00
Rear suspension - trailing axle brake gain (in.lb/psi)	3,000.00	3,000.00
Interaxle load transfer on rear tandem	0.00	0.00
Front axle cornering stiffness (lb/deg)	960.00	1,016.00
Rear suspension total cornering stiffness (lb/deg)	1,638.00	3,611.00
Reduction in cornering stiffness due to steering system (%)	31.70	32.00
Front suspension roll stiffness (in.lb/deg)	21,000.00	21,000.00
Rear suspension roll stiffness (in.lb/deg)	140,000.00	140,000.00
Front suspension roll center height (in)	20.00	20.00
Rear suspension roll center height (in)	29.00	29.00
Height of the center of gravity (in)	42.25	64.00
Distance of the center of gravity from the front axle (in)	117.50	176.25
Full Trailer		
Weight (lb)	7,500.00	38,000.00
Front axle brake gain (in.lb/psi)	3,000.00	3,000.00
Rear axle brake gain (in.lb/psi)	3,000.00	3,000.00
Front suspension total cornering stiffness (lb/deg)	729.00	1,963.00
Rear suspension total cornering stiffness (lb/deg)	729.00	1,963.00
Front suspension roll stiffness (in.lb/deg)	80,000.00	80,000.00
Rear suspension roll stiffness (in.lb/deg)	80,000.00	80,000.00
Front suspension roll center height (in)	29.00	29.00
Rear suspension roll center height (in)	29.00	29.00
Height of the center of gravity (in)	55.00	73.50
Distance of the center of gravity from front suspension (in)	111.00	111.00



MINIMUM TRACK

Susp. No.	Radius (ft)	Angle (deg)
1	41.00	0.00
2	36.25	68.09
3	34.94	65.33
4	31.29	60.11

Maximum Offtracking =9.71 ft.

Figure 84. Tractrices, truck and full trailer, 41 foot turn

Table 29. Truck and Full Trailer - Low- and High-Speed Offtracking

Parameter Name	Benchmark Value	Minimum Value	Maximum Value	Notes
Truck wheelbase	235 inches	125 inches	272 inches	
Truck's pintle overhang	105 inches	0 inches	105 inches	
Truck's cornering stiffness	3606.75 lb/deg	1803.4 lb/deg	5410.1 lb/deg	Cornering stiffness is determined from loaded vehicle data
Dolly's wheelbase	148 inches	80 inches	148 inches	
Dolly's cornering stiffness	1964.6 lb/deg	982.3 lb/deg	2946.9 lb/deg	Cornering stiffness is determined from loaded vehicle data
Trailer wheelbase width	222 inches	219 inches	258 inches	
Trailer's cornering stiffness	1964.6 lb/deg	982.3 lb/deg	2946.9 lb/deg	Cornering stiffness is determined from loaded vehicle data

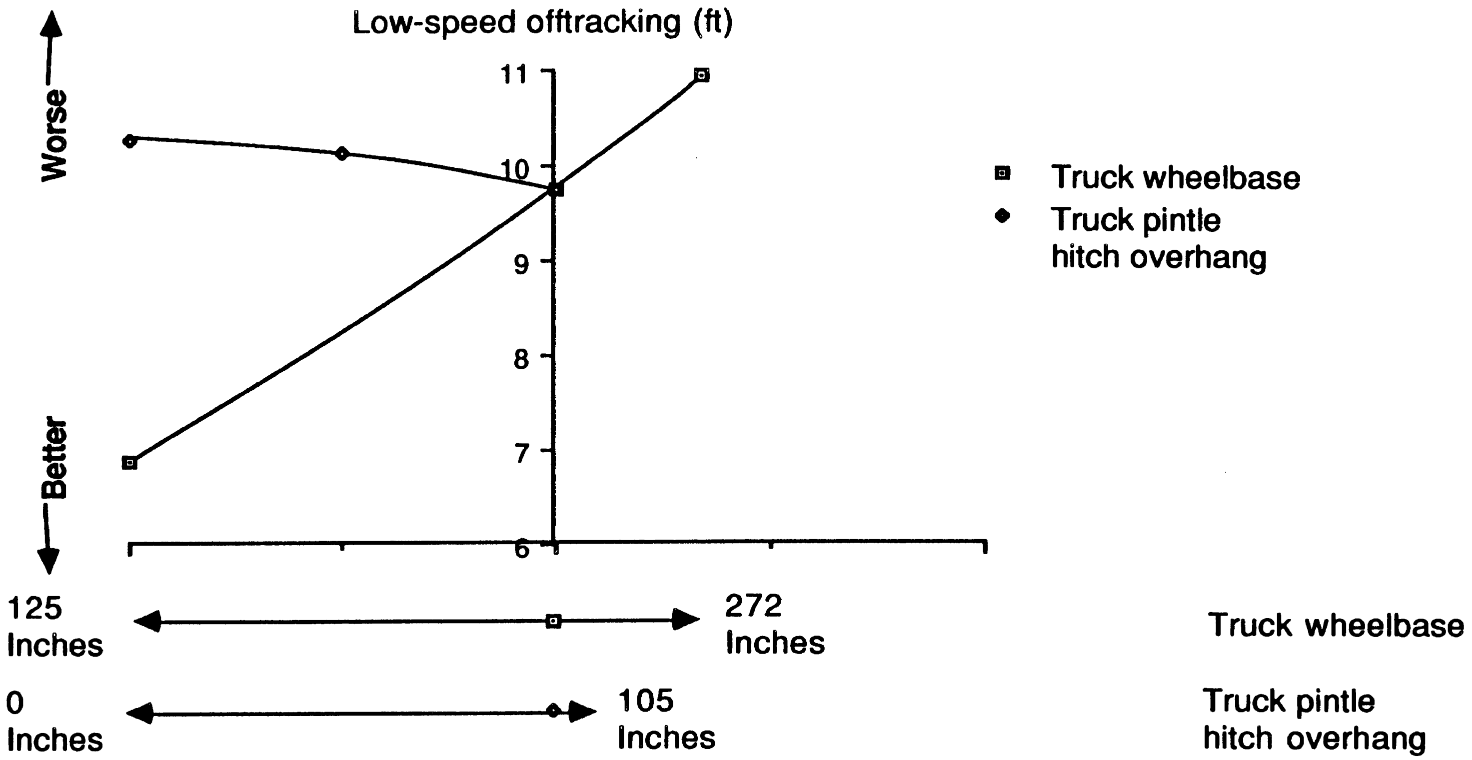


Figure 85. Parameter sensitivity diagram: low-speed offtracking, truck and full trailer.

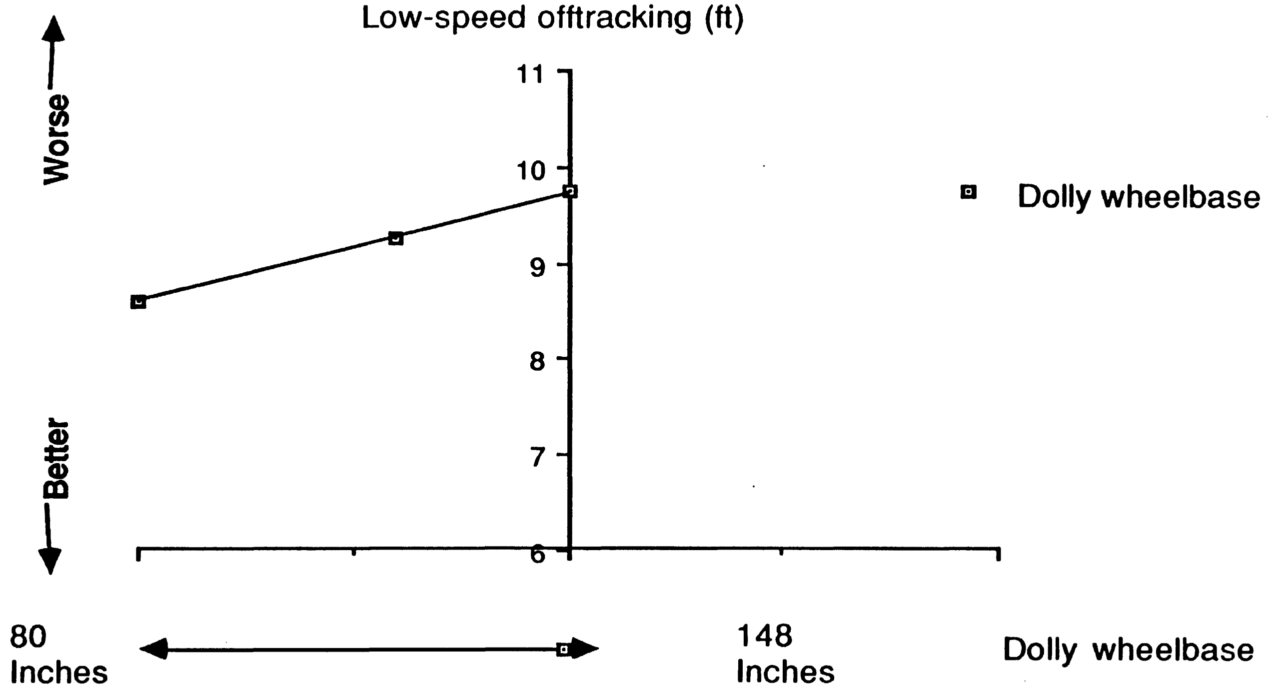


Figure 86. Parameter sensitivity diagram: low-speed offtracking, truck and full trailer.

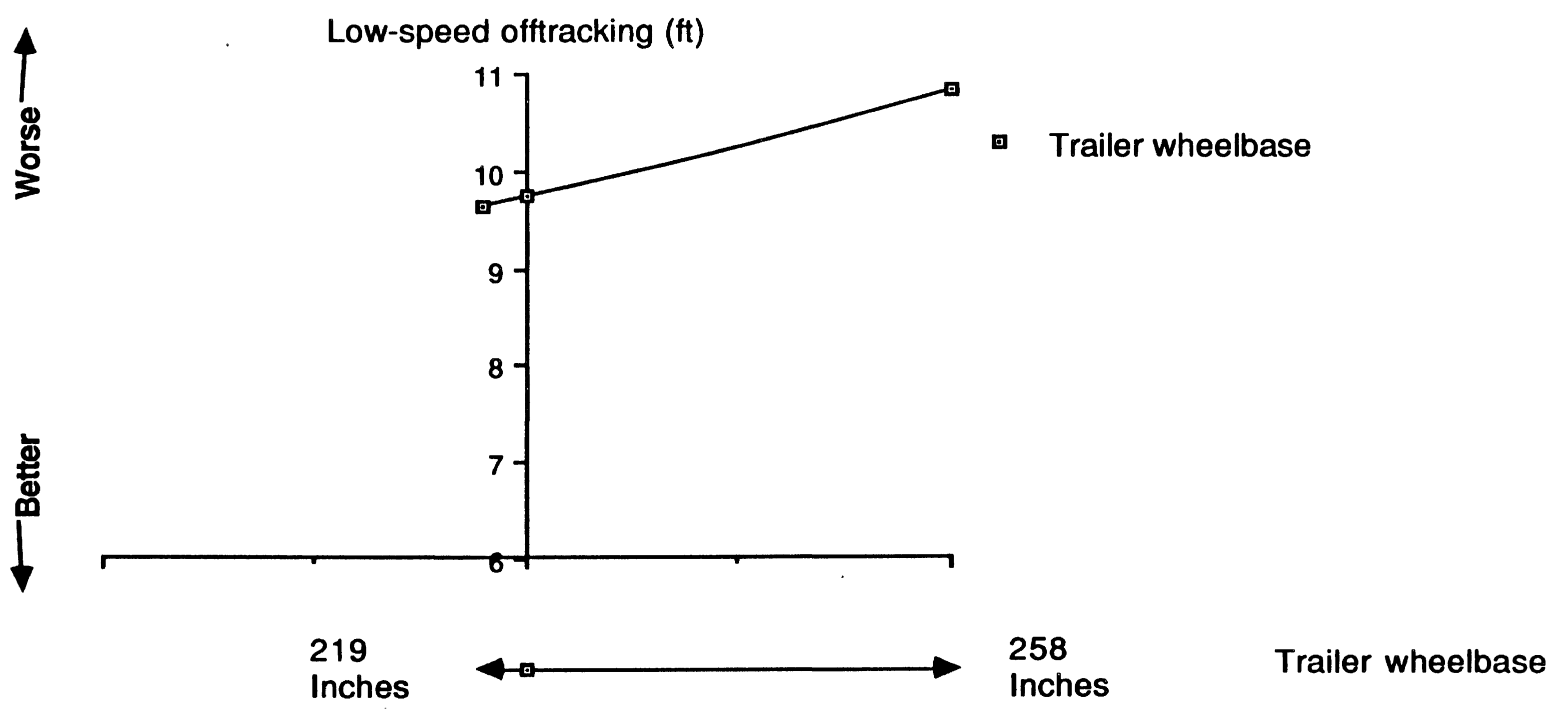


Figure 87. Parameter sensitivity diagram: low-speed offtracking, truck and full trailer.

overhang of the hitch (as measured from the center of the tandem axles on the tractor to the pintle hitch).

Since the turntable and dolly axle are located in nearly the same place (within a few inches), the influence of likely changes in these quantities is small. As shown in Figure 86, the change in dolly "tongue length" (referred to as "dolly's wheelbase" in the figure) is the important dolly parameter for this maneuver. Shortening the tongue length will decrease the low-speed offtracking.

The results given in Figures 85 and 87 show that trailer and truck wheelbases have approximately equivalent effects on offtracking. This is to be expected in this case because comparable lengths are involved here. Both of these wheelbases are important to the results.

5.3 Constant Deceleration Braking

Benchmark performance. The performance signatures for the 3-2 in the loaded and empty conditions are presented in Figures 88 and 89. In the benchmark condition, the interaxle load transfer taking place at the tractor's tandem axles is zero which means that the friction utilization curves for axles 2 and 3 are superimposed in Figures 88 and 89.

In the loaded state, the braking efficiencies at 0.2 and 0.4 g are 85 and 78 percent, respectively. For the empty vehicle, these efficiencies reduce to 62 and 55 percent. In the loaded state, braking efficiency is determined by the friction utilization at the truck's tandem axles. However, when the vehicle is empty, the friction utilization of the rearmost axle on the trailer determines the braking efficiency.

Parametric sensitivities. Table 30 lists the truck and trailer parameters varied to study the braking efficiencies of the 3-2. The quantities influencing braking efficiency through fore/aft load transfer are c.g. heights and wheelbases of the truck and the trailer. The analysis performed is specific to the fixed type of dolly meaning that no vertical load is transferred from the trailer to the truck. The other quantities studied are interaxle load transfer in the tandem suspension and increases in the front brake gain of the truck.

Figures 90 through 97 are arranged to show the influences of truck and trailer parameters at 0.2 g and 0.4 g with the vehicle in empty and loaded conditions. Hence, there are eight figures used to cover all of these combinations. *In general, the results show that (1) the loaded vehicle is much more efficient than the empty vehicle and (2) the performance sensitivities are greater at 0.4 g than those found at 0.2 g.*

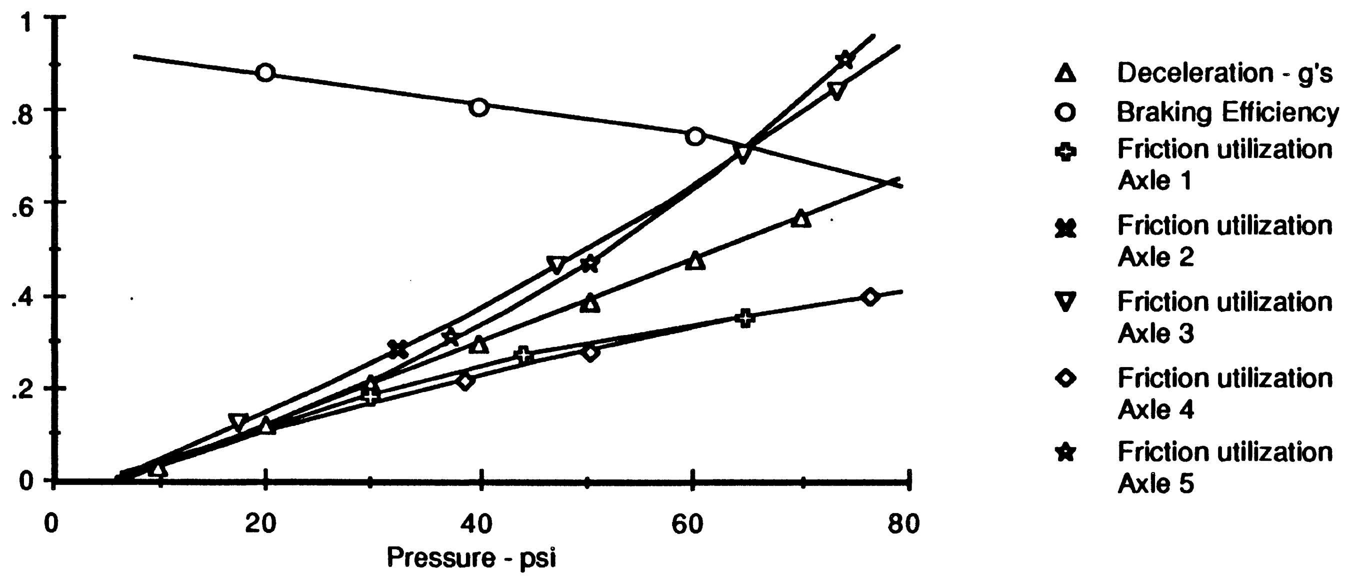


Figure 88. Performance signature, braking, loaded truck and full trailer

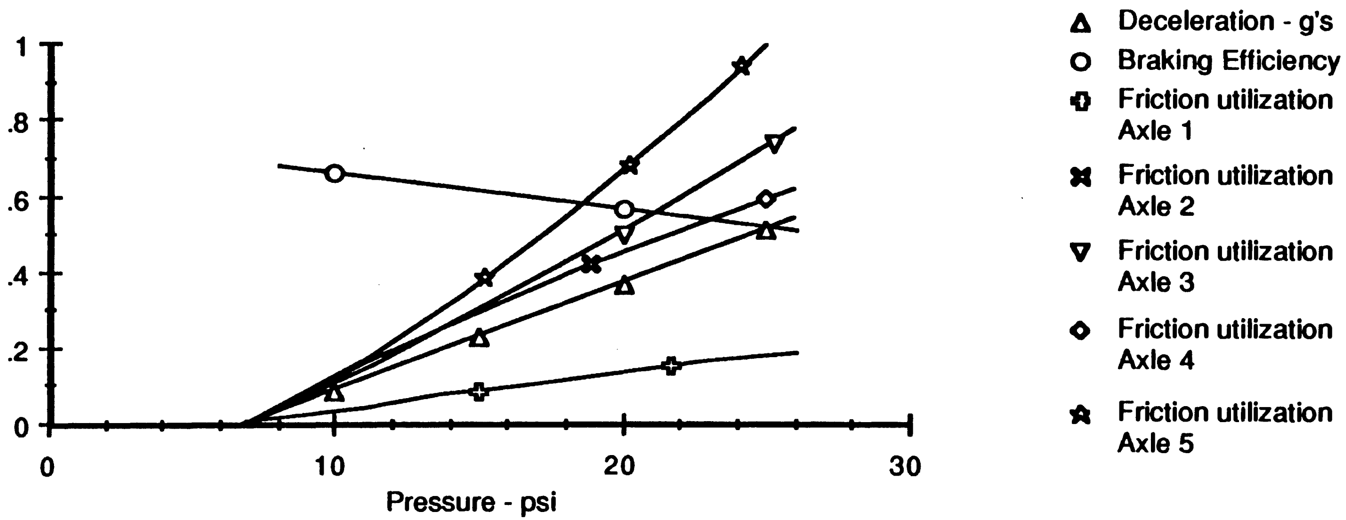


Figure 89. Performance signature, braking, empty truck and full trailer

Table 30. Truck and Full Trailer - Braking

Parameter Name	Benchmark Value	Minimum Value	Maximum Value	Notes
Truck wheelbase	235 inches	125 inches	272 inches	
*Truck's c.g. height	63.93 inches	36 inches	100 inches	
Interaxle load transfer (truck)	0	-0.2	0.2	
Front brake gain	2000 in.lb/psi	2000 in.lb/psi	3000 in.lb/psi	Both brakes combined
Trailer wheelbase	222 inches	219 inches	258 inches	
*Trailer's c.g. height	73.5 inches	60 inches	105 inches	

* For the loaded vehicle

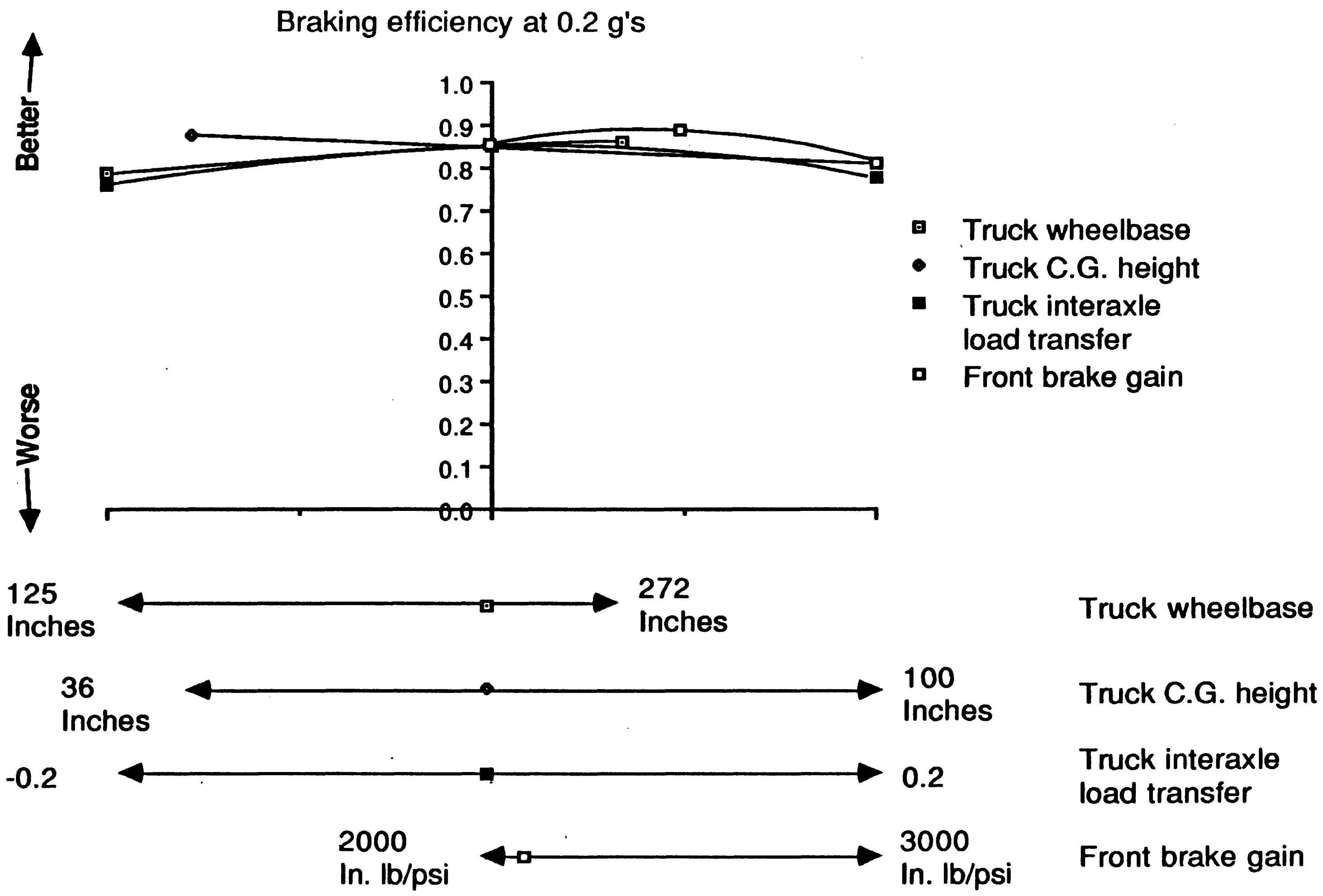


Figure 90. Parameter sensitivity diagram: braking, loaded truck and full trailer.

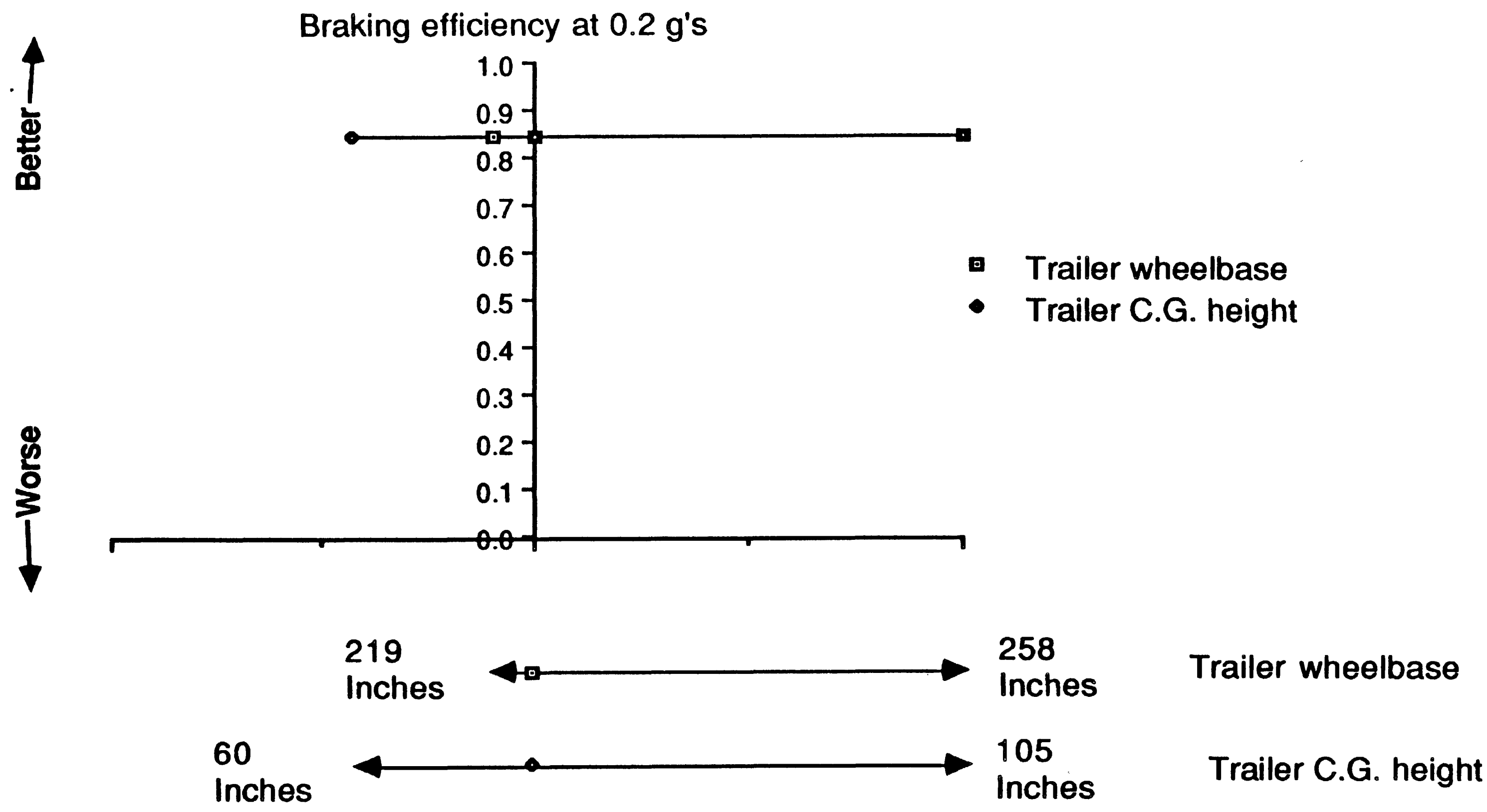


Figure 91. Parameter sensitivity diagram: braking, loaded truck and full trailer.

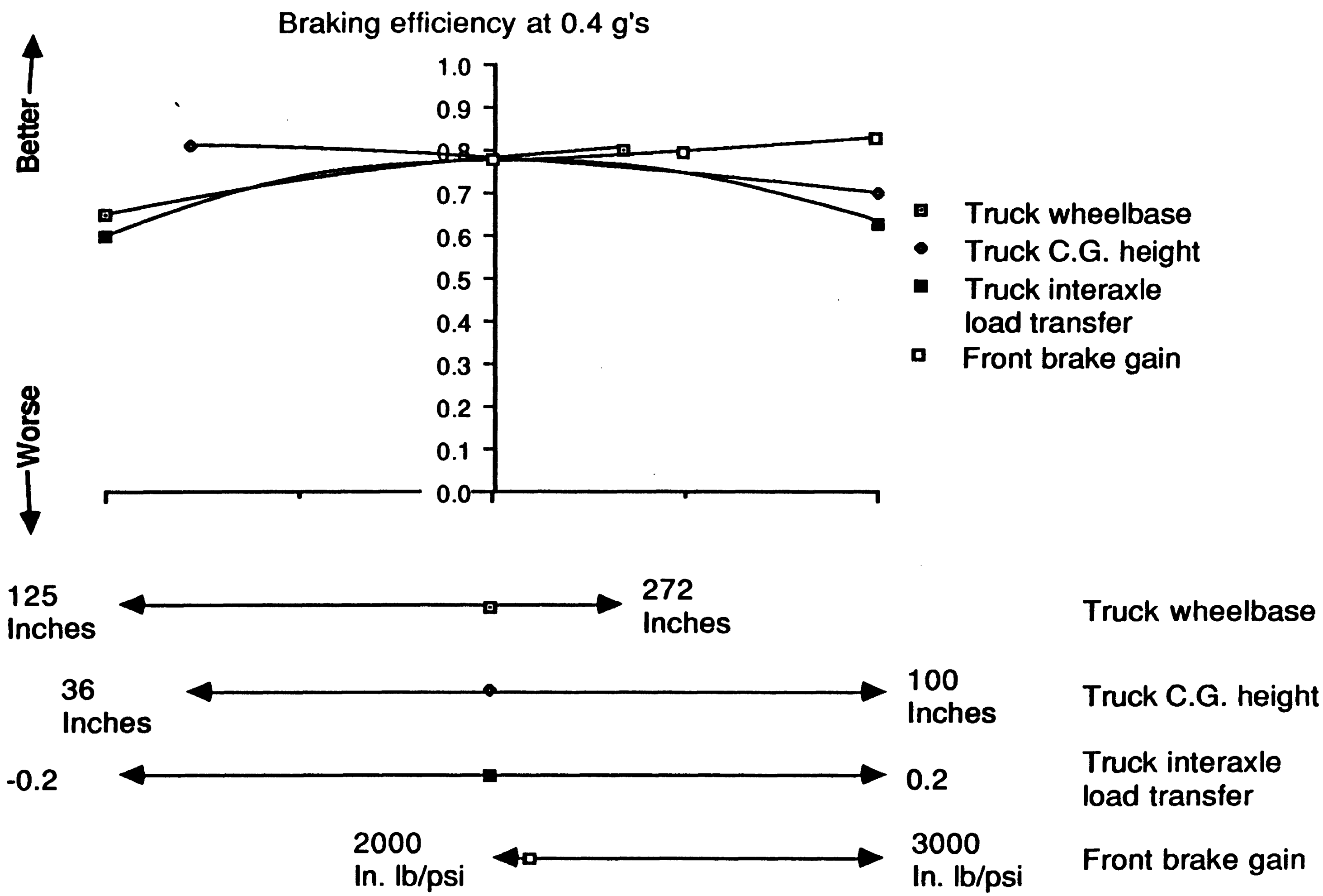


Figure 92. Parameter sensitivity diagram: braking, loaded truck and full trailer

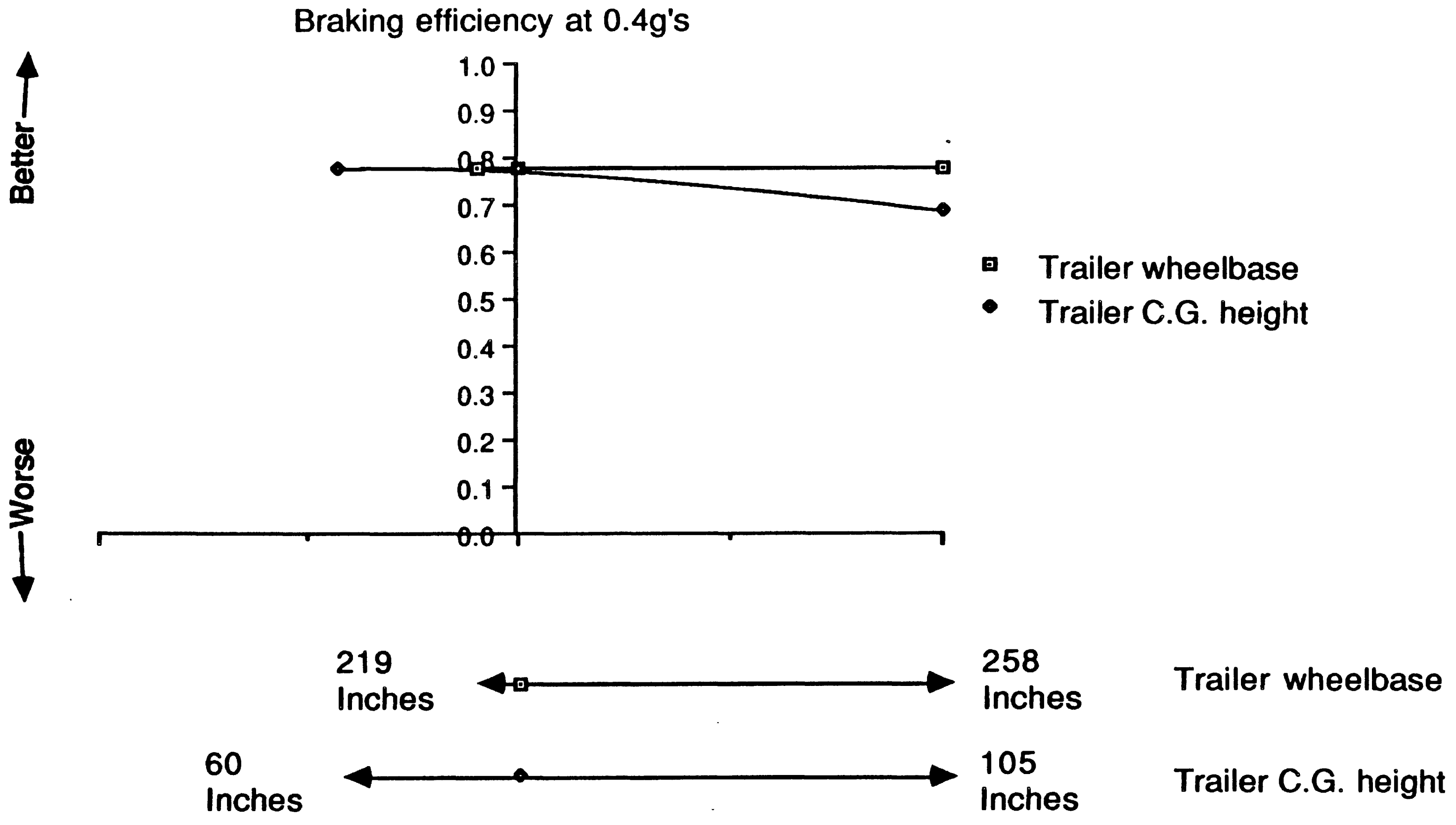


Figure 93. Parameter sensitivity diagram: braking, loaded truck and full trailer.

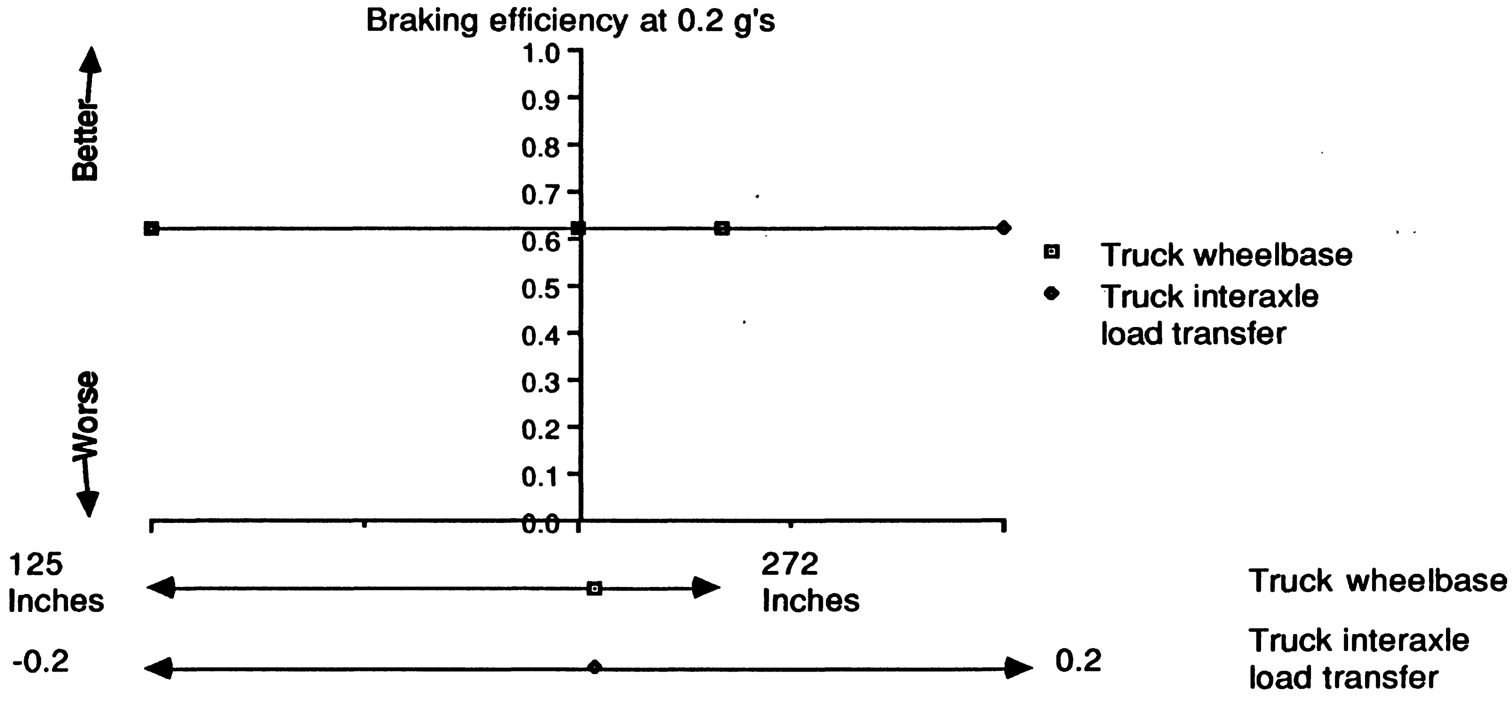


Figure 94. Parameter sensitivity diagram: braking, empty truck and full trailer.

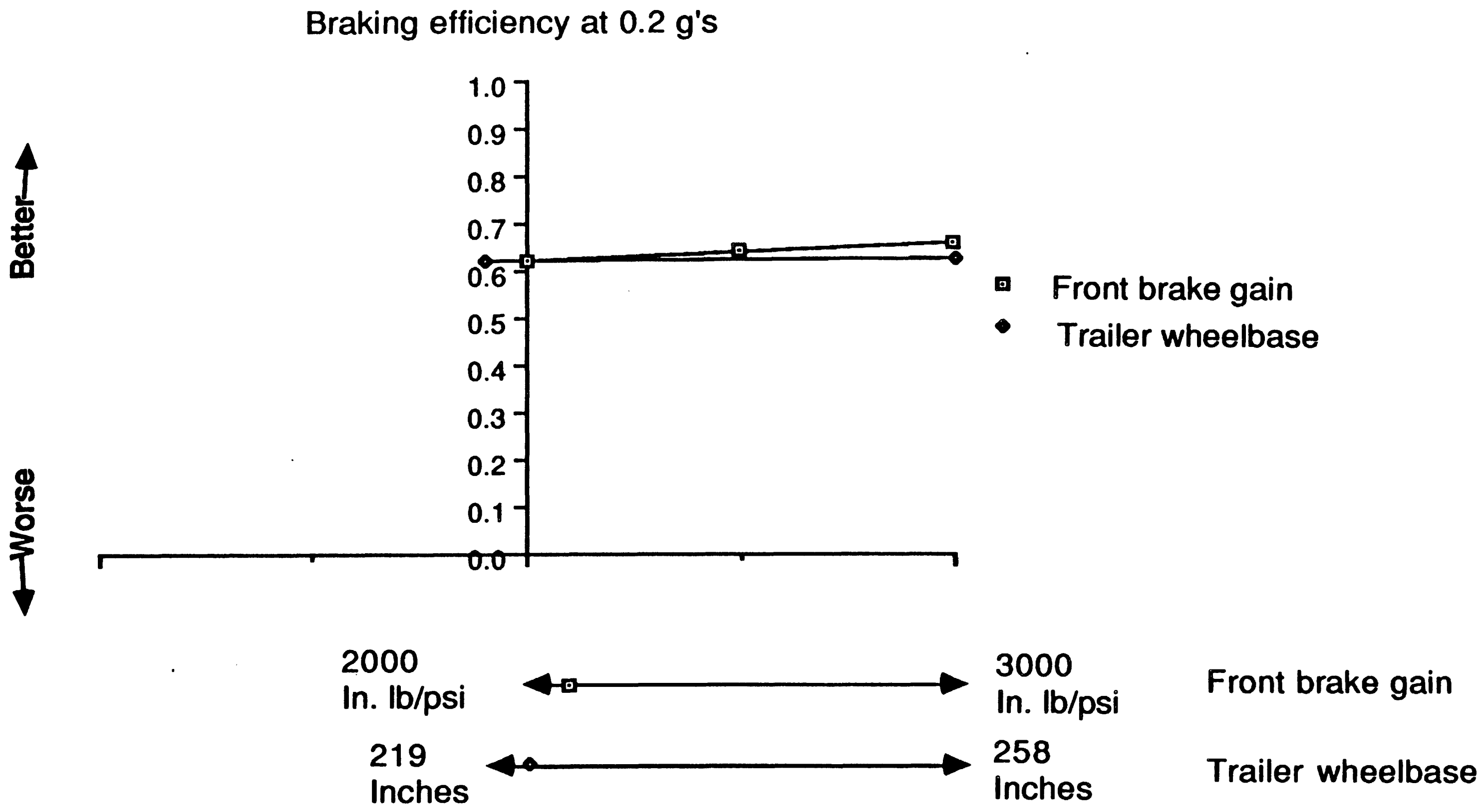


Figure 95. Parameter sensitivity diagram: braking, empty truck and full trailer.

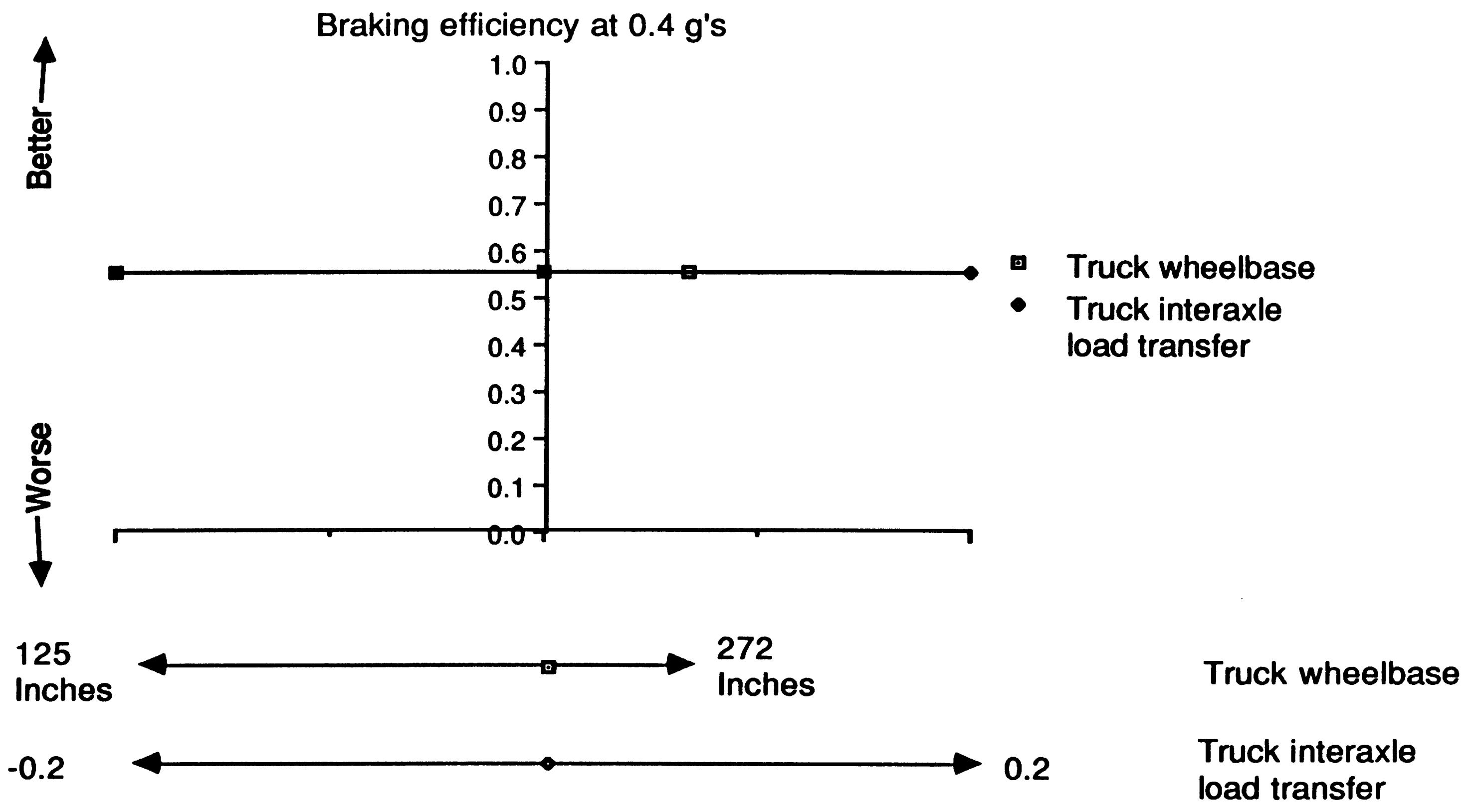


Figure 96. Parameter sensitivity diagram: braking, empty truck and full trailer.

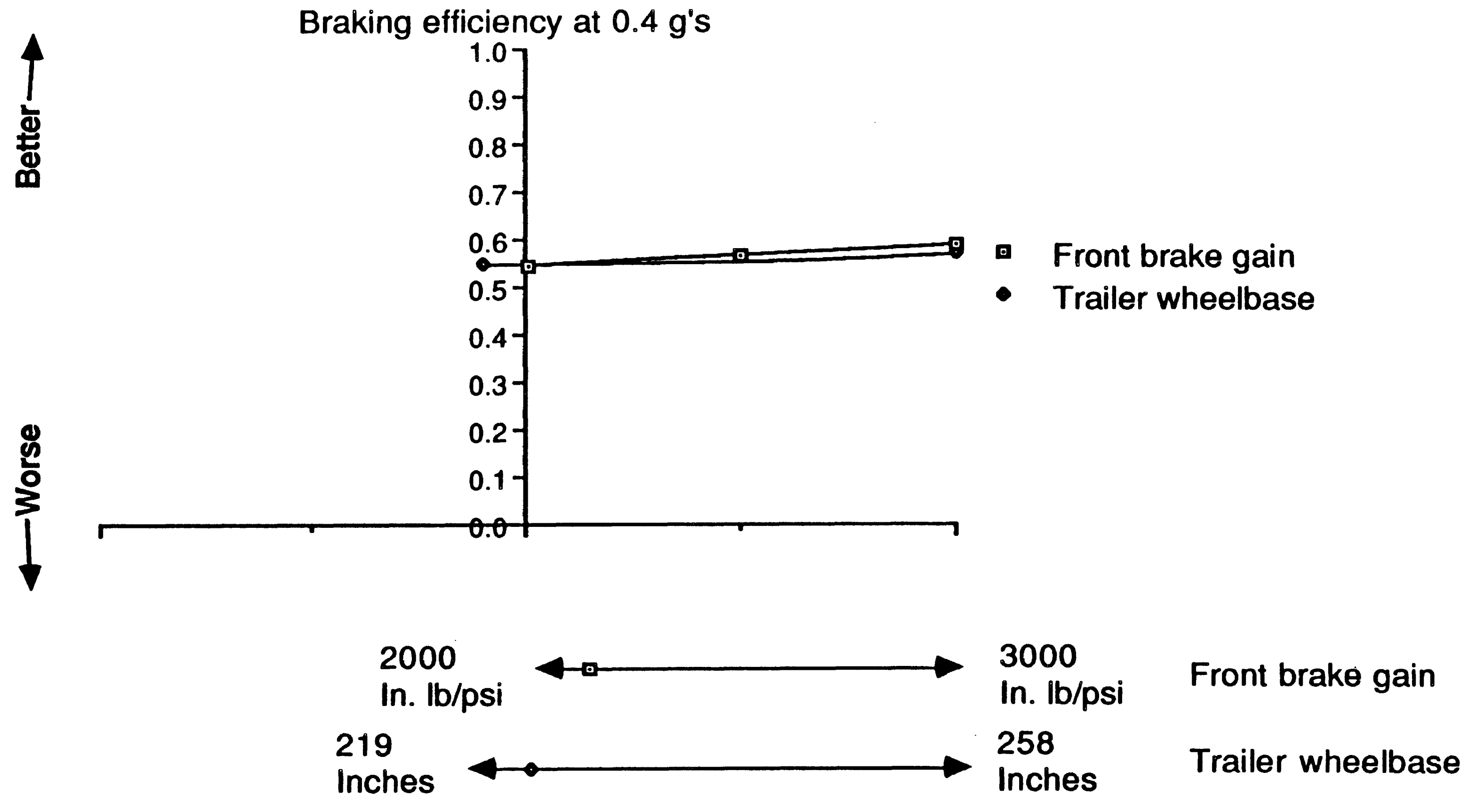


Figure 97. Parameter sensitivity diagram: braking, empty truck and full trailer.

At 0.2 g and with the vehicle loaded (Figures 90 and 91) the results are straightforward with two notable exceptions. First, upon increasing the front brake gain to the maximum deviation we see that the efficiency first increases then decreases. This is because the controlling wheels change from those on the truck tandem to those on the front axle. For these operating conditions, the efficiency is lower when the front wheels are approaching lockup. The second exception is that the variations in trailer properties have little effect. This results because (1) there is no vertical load transferred from the trailer to the truck and (2) the longitudinal force between the trailer and the truck is small in this situation.

At 0.4 g, the braking performance of the loaded 3-2 is susceptible to the influences of changes in both truck and trailer properties. Changes in trailer wheelbase and interaxle load transfer at the truck's tandem suspension have greater influences than any of the other changes. To keep efficiencies high, one wants to reduce fore/aft load transfer for a vehicle that is proportioned to match the static loading (as is the case for the loaded 3-2).

When the 3-2 is empty, the performance can be improved by increasing the front brake gain and by increasing the trailer wheelbase. These influences have a more pronounced effect at 0.4 g than they do at 0.2 g (see Figures 94 through 97). Increasing the front brake gain improves the proportioning because, in the empty state, the front wheels carry a higher proportion of the weight than they carry when the vehicle is loaded. The wheelbase of the trailer is important because the rear wheels of the trailer are the first wheels to lock. If the wheelbase of the trailer is increased, less load is transferred off of the rear axle, thereby increasing the frictional force that can be utilized and hence increasing the efficiency.

5.4 High-Speed Offtracking (Steady Turn - Tracking)

Benchmark performance. The baseline performance of the benchmark 3-2 is specified by the offtracking results given in Table 31 for a steady turn at 55 mph and at a radius of 1,200 feet to the center of the front axle. The results indicate that offtracking to the outside of the turn increases as one progresses from considering points near the front to points toward the rear of the vehicle. In particular, the rear of the 3-2 offtracks by 1.36 feet while the rear axle offtracks by 1.27 feet. With regard to sideswiping adjacent objects, the rear of the vehicle is the important consideration. With regard to "tripping" on curbs, the location of the rear axle is what matters. In the following parametric analysis, we have chosen to use the rear of the vehicle simply because this will show the effects of a large overhang between the rear axle and the end of the vehicle.

Parametric sensitivities. The changes in mechanical properties examined here were listed previously in Table 29. These variations include wheelbases, hitch locations, tire stiffnesses, and

Table 31. High-Speed Offtracking Performance, Truck and Full Trailer

Velocity = 80.6667 ft/sec (55 mph)
Radius of the turn = 1200 ft

Suspension #	Rear Articulation Point on Unit #	Zero Speed Radius	High Speed Radius
1		1200.00	1200.00
2		1199.84	1200.34
	1	1199.87	1200.60
3		1199.81	1200.88
	2	1199.81	1200.88
4		1199.67	1201.27
	3	1199.67	1201.36

Zero Speed Offtracking= .33 ft
High Speed Offtracking =-1.36 ft

rear overhang with respect to the rear axle. The influences of truck, dolly, and semitrailer properties are presented in several figures (i.e., Figures 98, 99, and 100 for a 1,200-foot radius and Figures 101, 102, and 103 for a 600-foot radius).

In every case tire cornering stiffnesses are important properties pertaining to each unit--truck, dolly, or semitrailer. Wheelbases have a small and somewhat mixed influence, with the benchmark wheelbases being fairly close to the "worst-case wheelbase" discussed previously in Section 4.3. The location of the pintle hitch with respect to the rear tandem center of the truck has an important influence on high-speed offtracking. *At low speed a decrease in overhang will increase offtracking towards the inside of the turn, but at high speed just the opposite is true. For high-speed performance, zero overhang is the ideal situation.*

5.5 Steady Turn - Roll

Benchmark performance. The performance signature for the rolling response of the 3-2 consists of two curves, one for the truck (see Figure 104) and the other for the full trailer (see Figure 105), because the pintle hitch serves to decouple these two units with respect to roll. The truck has a rollover threshold of 0.419 g and the full trailer has a rollover threshold of 0.391 g.

Parametric sensitivities. We have chosen to examine the rollover performance of the trailer because the parametric sensitivities of trucks were treated in Section 3.5. This choice is reasonable to make because truck properties do not influence trailer rollover and visa versa. *Nevertheless, one should remain cognizant of the fact that both units of a truck/full trailer combination need to be checked for roll stability and changes in one unit may improve or degrade its performance to the point where that unit becomes less or more susceptible to rollover than the other unit.*

The changes in trailer properties (see Table 32) have been divided into three categories, namely, (1) general properties, including c.g. heights and roll center heights, (2) dolly roll stiffness and track width, and (3) semitrailer roll stiffness and track width. In the general properties category the change in c.g. height is seen to be significant (Figure 106). As shown in Figure 107, increasing the track width of the dolly is important for providing increased roll stability. Increasing the track width of the semitrailer's wheels is also important (see Figure 108). The results show that changes in roll stiffnesses are relatively important for this full trailer, if the roll stiffnesses are reduced. (This is explained by the concept of "soft versus stiff suspensions" discussed earlier in Section 4.5.)

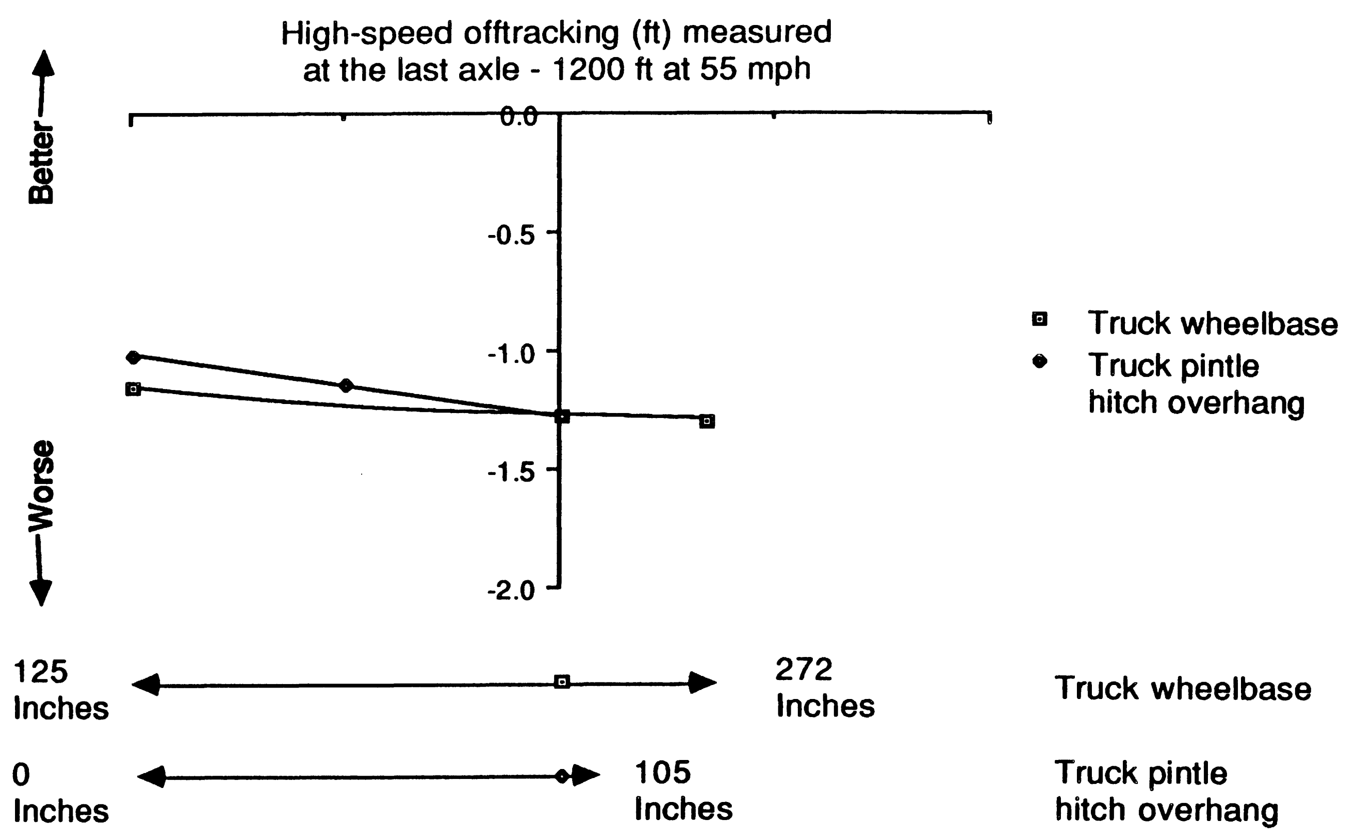


Figure 98. Parameter sensitivity diagram: high-speed offtracking, truck and full trailer.

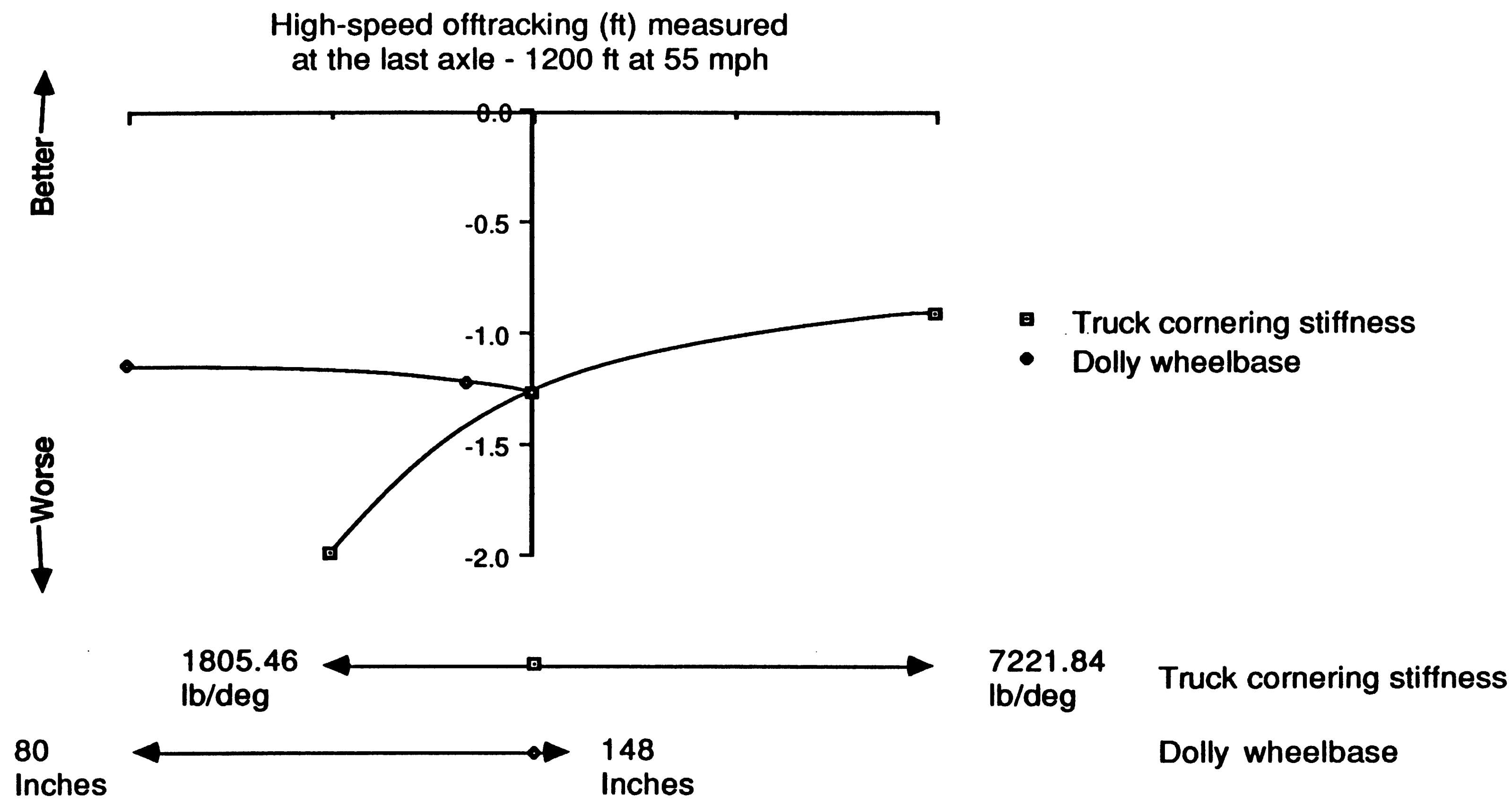


Figure 99. Parameter sensitivity diagram: high-speed offtracking, truck and full trailer.

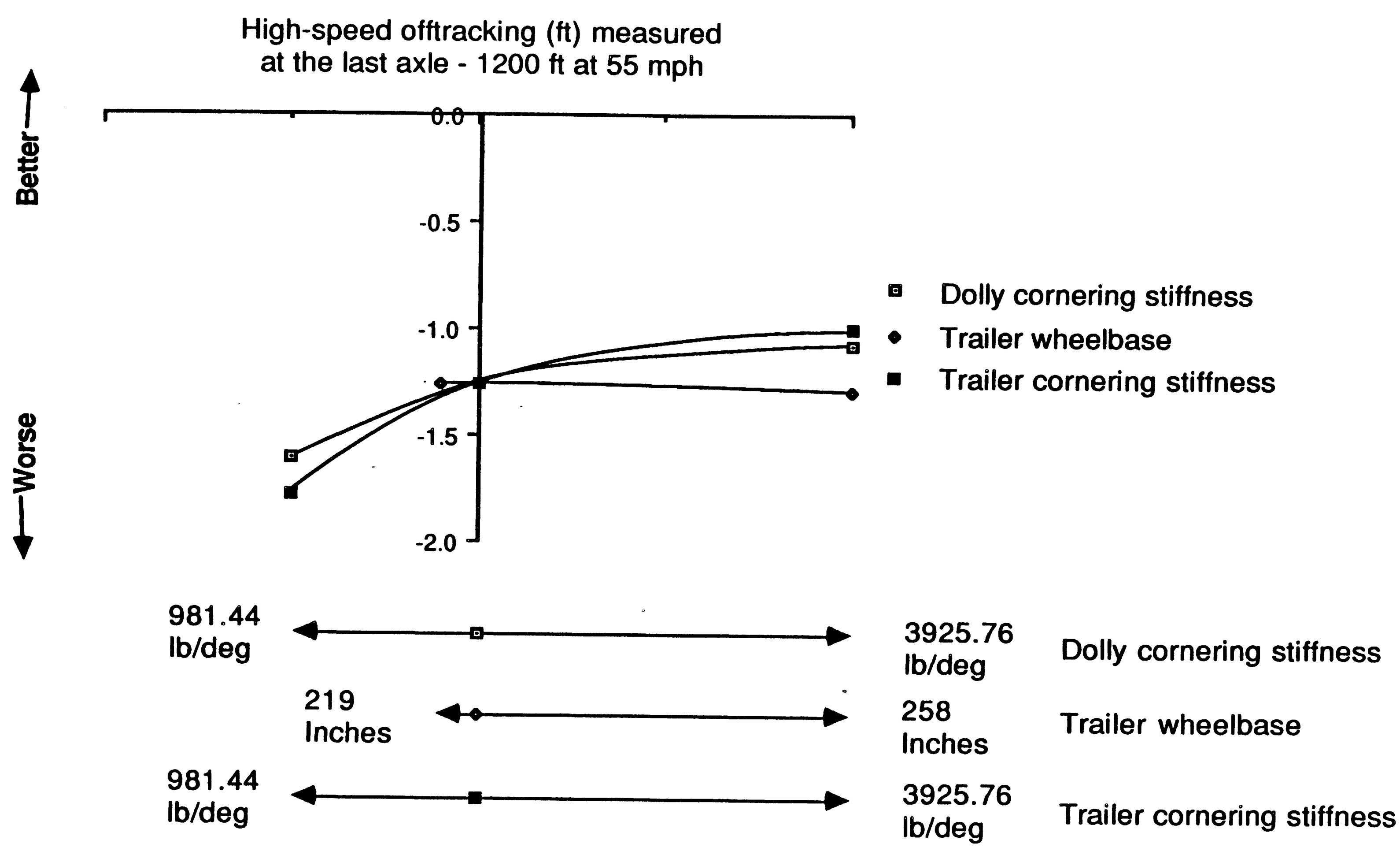


Figure 100. Parameter sensitivity diagram: high-speed offtracking, truck and full trailer.

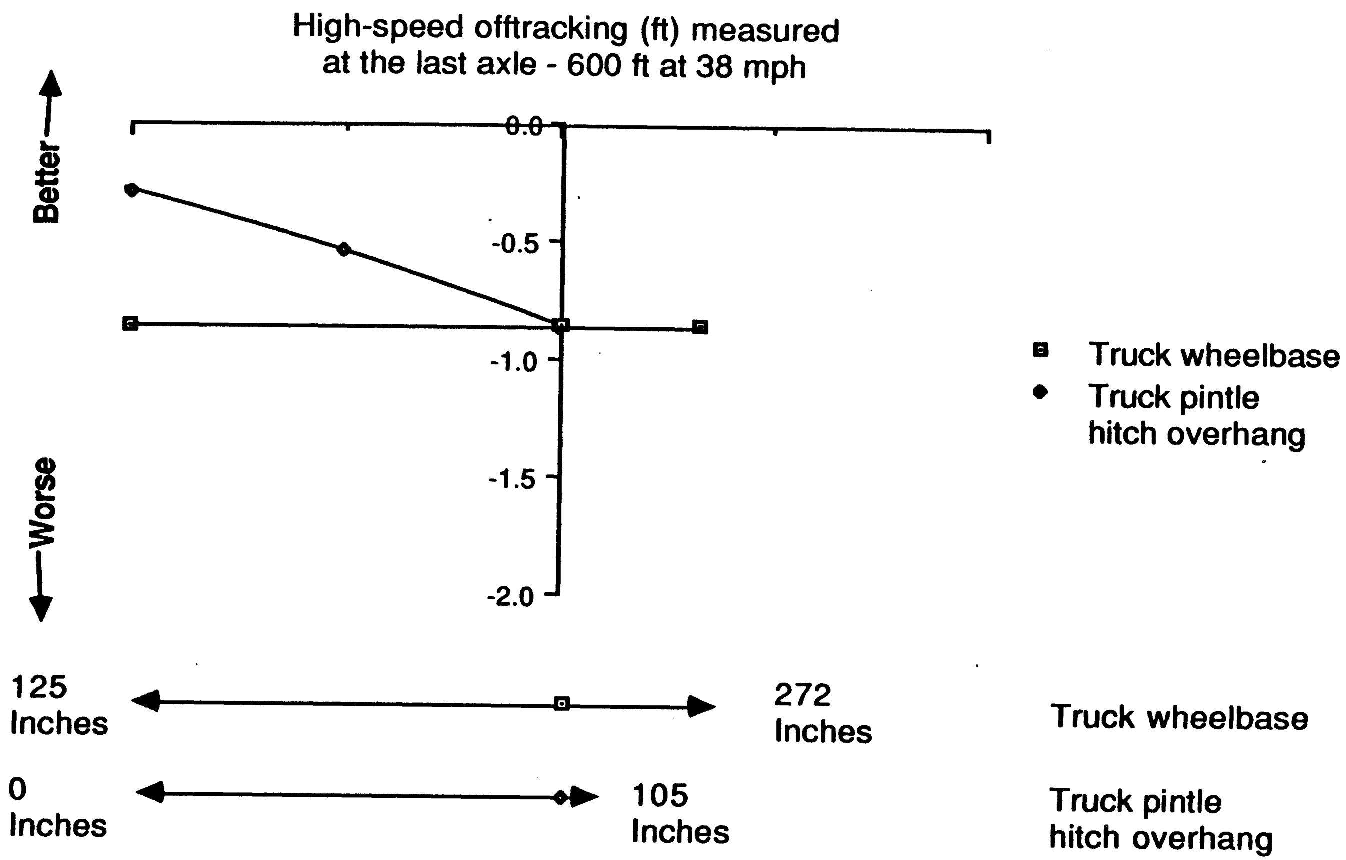


Figure 101. Parameter sensitivity diagram: high-speed offtracking, truck and full trailer.

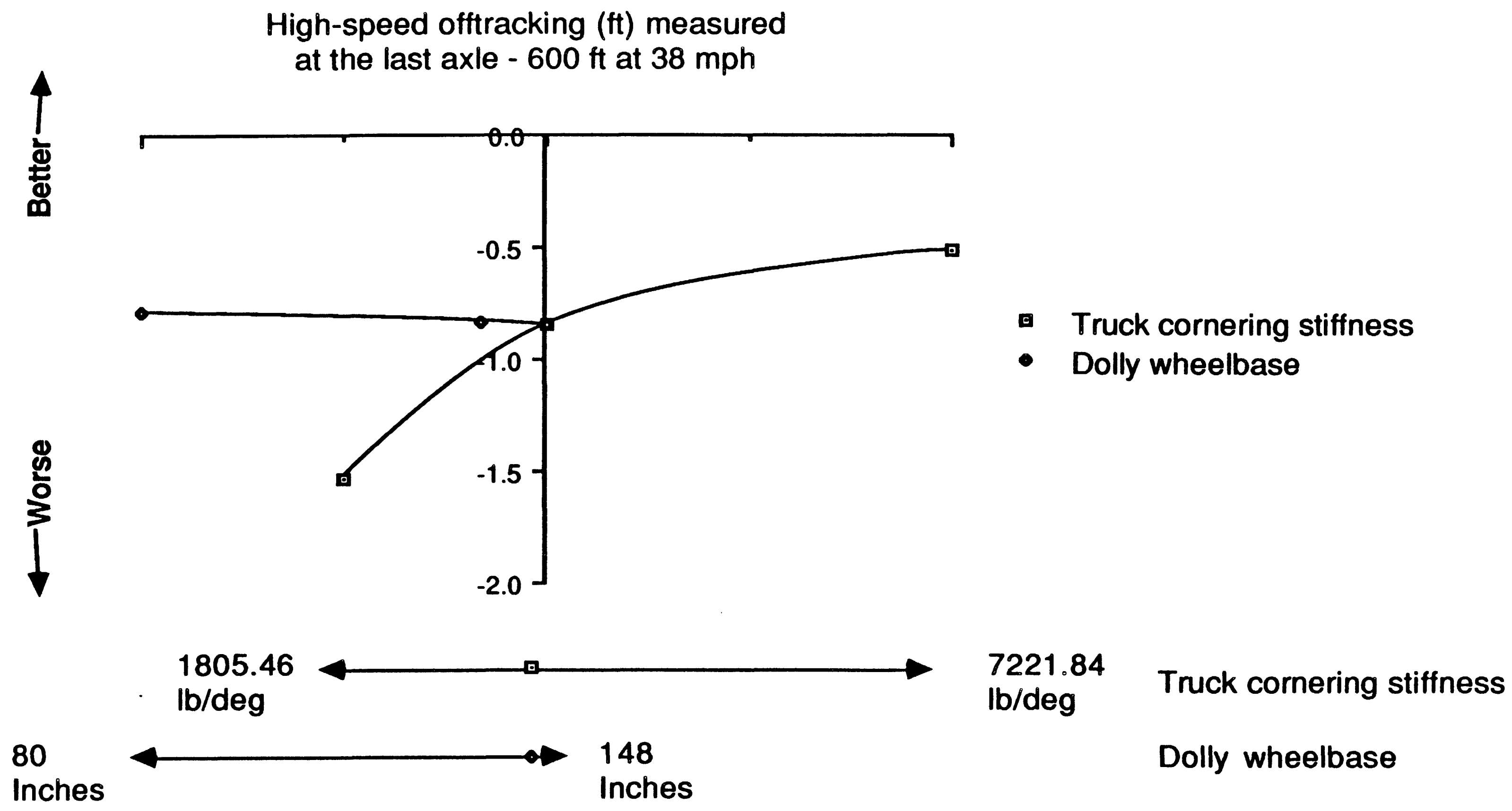


Figure 102. Parameter sensitivity diagram: high-speed offtracking, truck and full trailer.

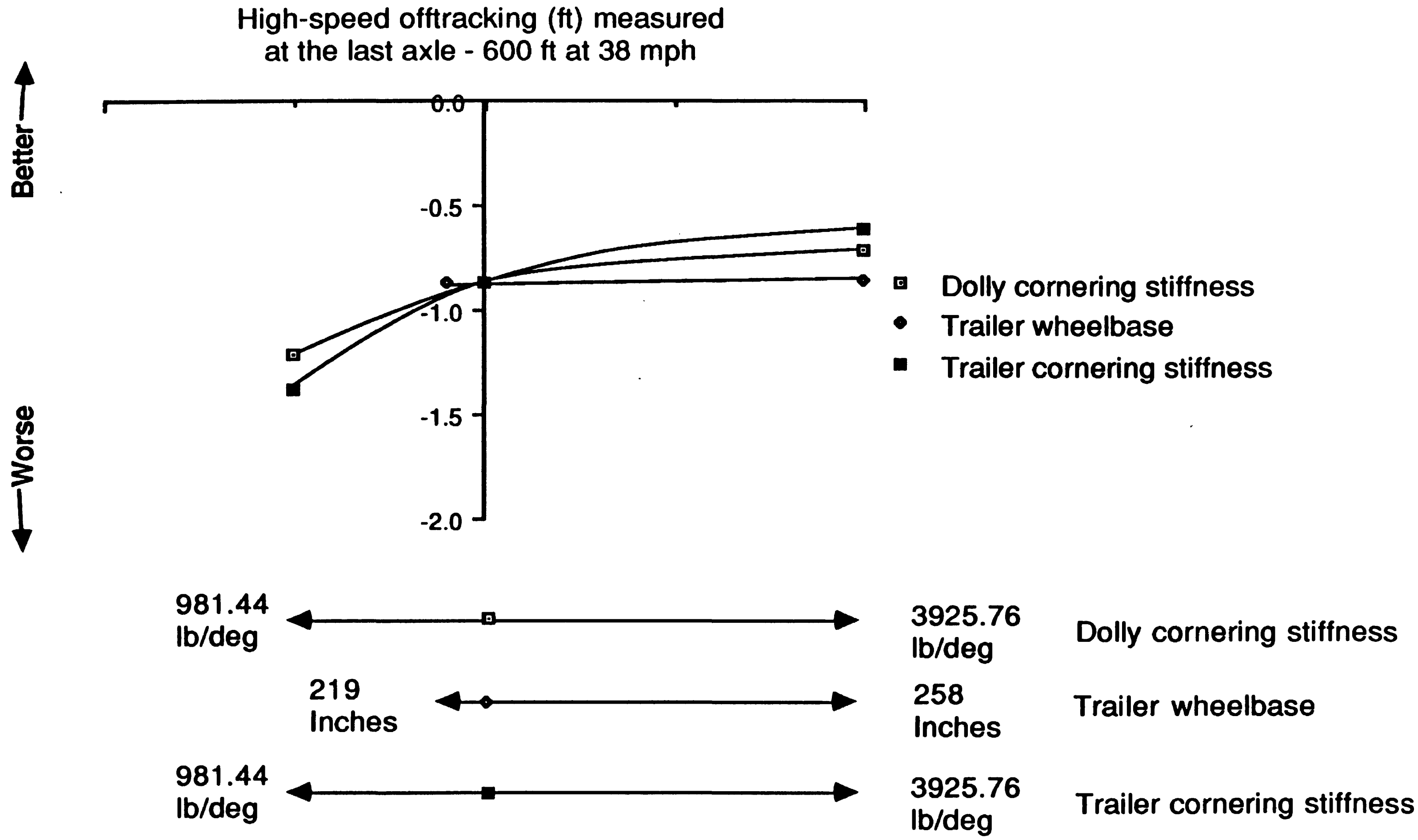


Figure 103. Parameter sensitivity diagram: high-speed offtracking, truck and full trailer.

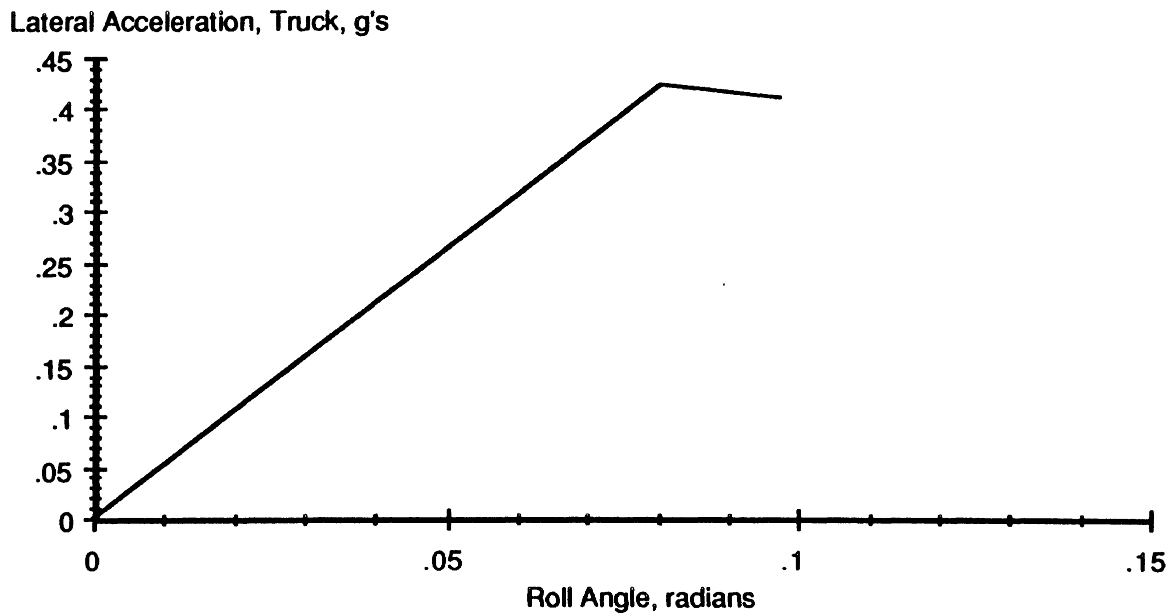


Figure 104. Performance signature, static roll, truck and full trailer

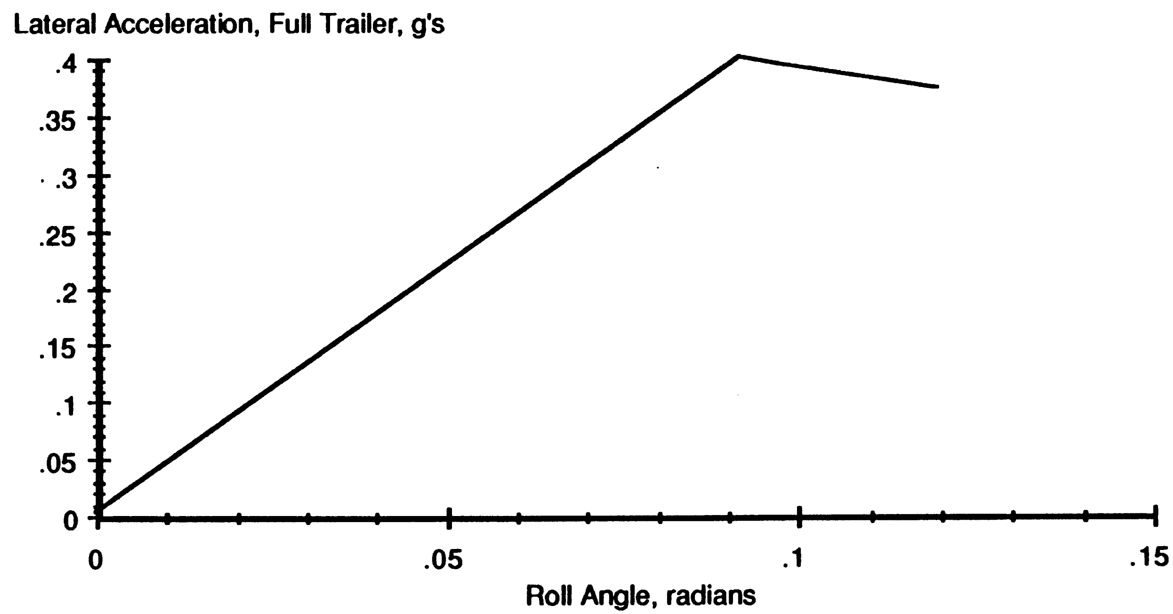


Figure 105. Performance signature, static roll, truck and full trailer

Table 32. Truck and Full Trailer - Static Roll

Parameter Name	Benchmark Value	Minimum Value	Maximum Value	Notes
Trailer's c.g. height	76.8 inches	70 inches	80 inches	Much larger deviations used for braking
Rear axle roll center heights	29 inches	21 inches	31 inches	
Dolly's axle roll stiffness	80,000 in.lb/deg	30,000 in.lb/deg	165,000 in.lb/deg	
Dolly's axle track width	72 inches	71 inches	78 inches	
Trailer's axle roll stiffness	80,000 in.lb/deg	30,000 in.lb/deg	165,000 in.lb/deg	
Trailer's axle track width	72 inches	71 inches	78 inches	

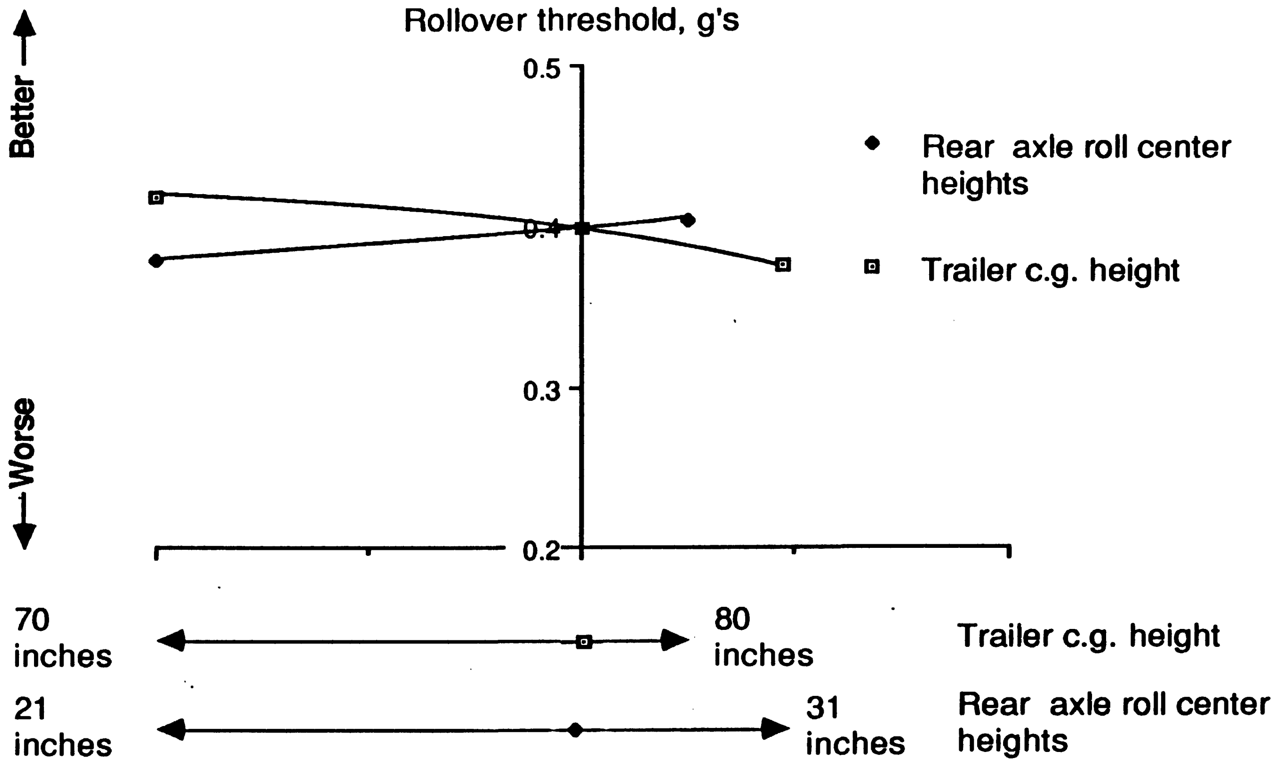


Figure 106. Parameter sensitivity diagram: static roll, truck and full trailer.

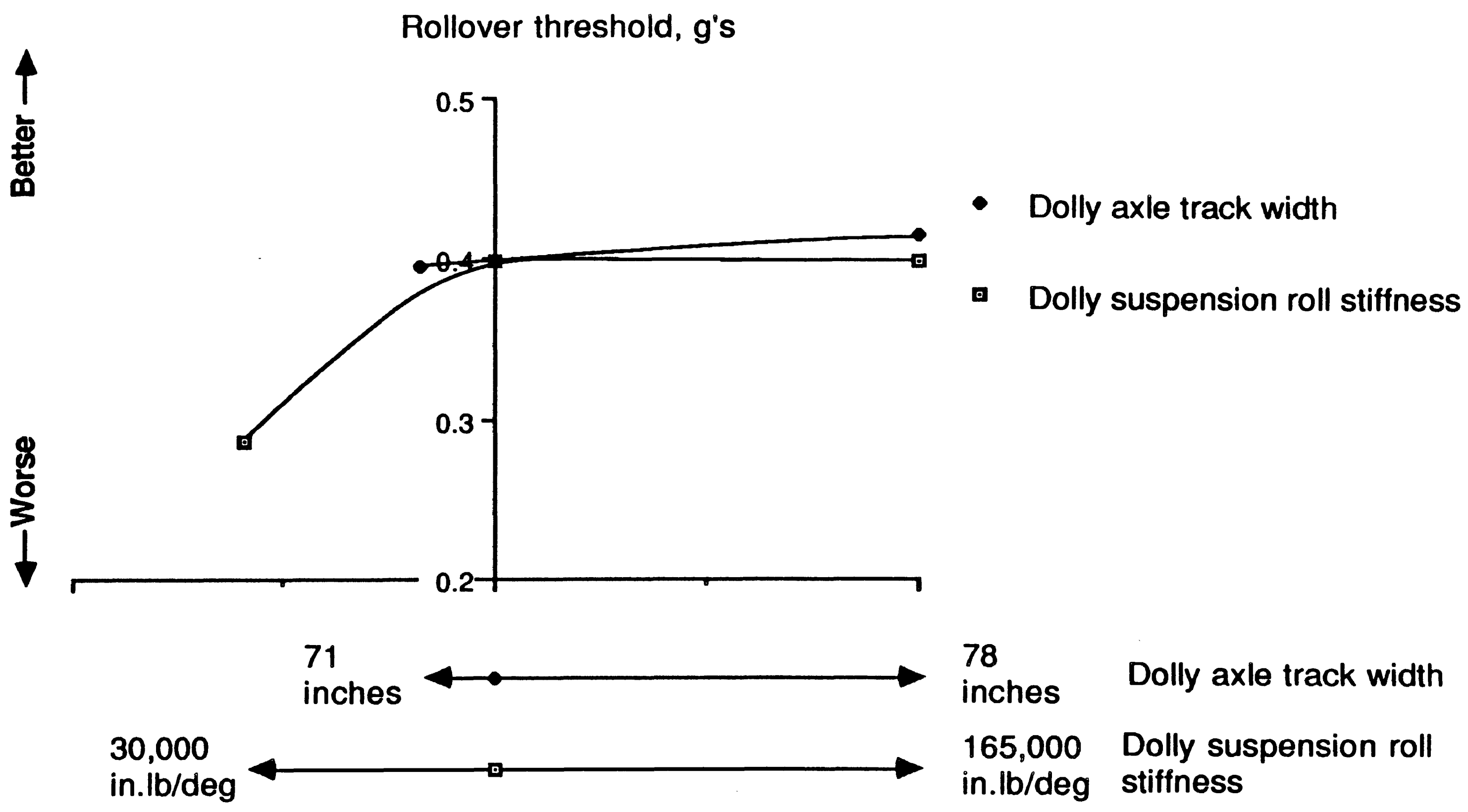


Figure 107. Parameter sensitivity diagram: static roll, truck and full trailer.

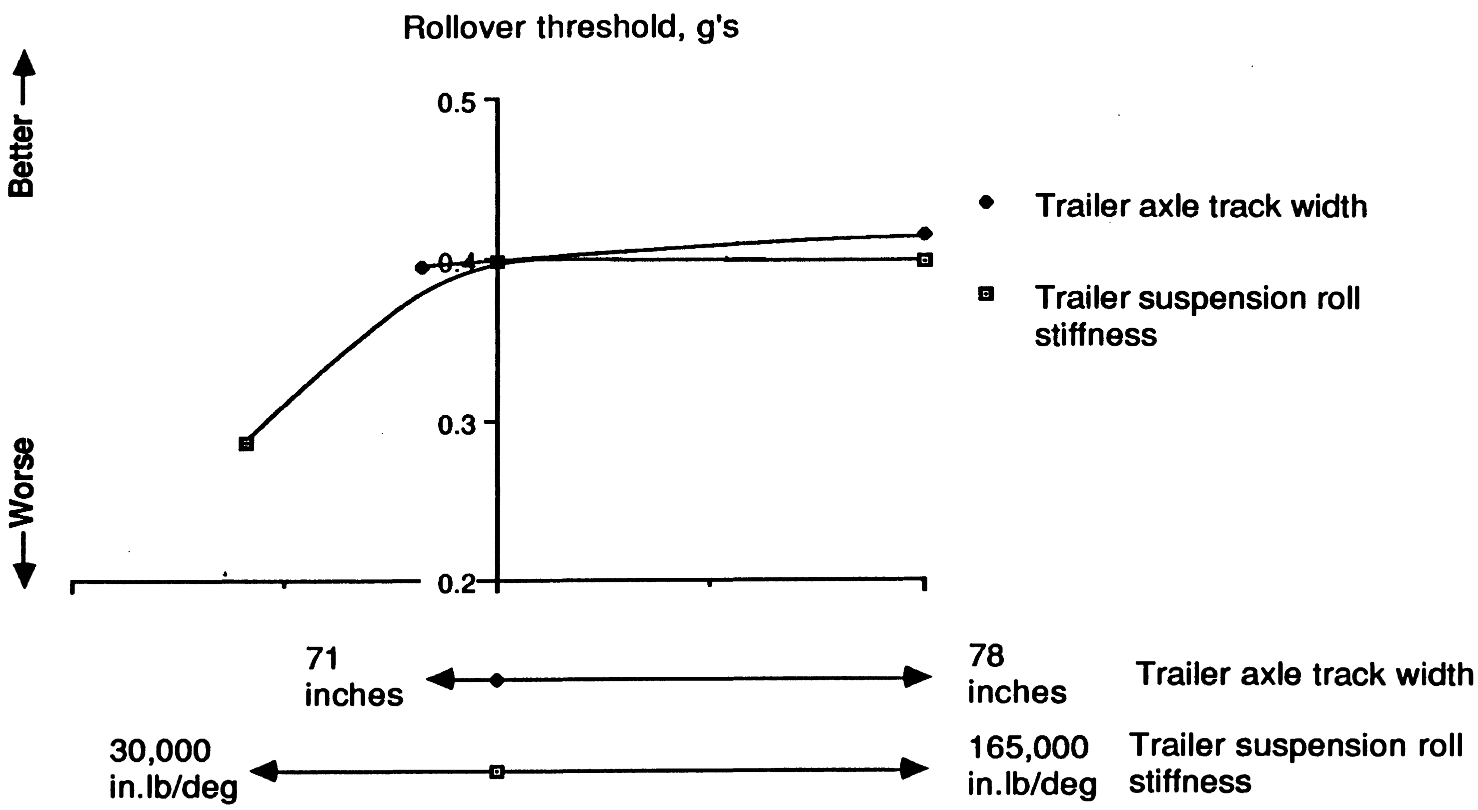


Figure 108. Parameter sensitivity diagram: static roll, truck and full trailer.

5.6 Steady Turn - Handling

Benchmark performance. Figure 109 is the handling signature for the benchmark 3-2 traveling at 55 mph. In the linear range at low g-levels, the steering sensitivity is 0.22 radians per g. At 0.3 g and 55 mph the steering sensitivity is 0.146 radians per g.

Parametric sensitivities. The pintle hitch effectively decouples the full trailer from the truck in that the lateral force at the pintle hitch is practically zero for steady-turning situations. This means that trailer properties do not influence handling performance. It is only necessary to consider truck properties. The truck properties studied here are listed in Table 33. The results are presented in three performance sensitivity diagrams (Figures 110, 111, and 112) corresponding to variations in (1) general truck characteristics such as c.g. height, wheelbase, and roll center heights, (2) front spring and tire characteristics, and (3) rear spring and tire characteristics.

Since the vehicle is stable up to 0.3 g, the steering sensitivity, that is, the rate of change of reference front-wheel angle with respect to lateral acceleration, is used as the performance measure for comparing the influences of variations in mechanical properties. As illustrated in Figure 110, steering sensitivity is slightly sensitive to changes in wheelbase and c.g. height, although none of these changes is large enough to approach yaw divergence. (Yaw divergence is indicated by a zero value of steering sensitivity.) The closest approaches to yaw divergence occur when the truck's wheelbase is reduced to 125 inches and when the truck's c.g. height is increased to 70 inches. This extreme condition results in a steering sensitivity of approximately 0.11 radians/g.

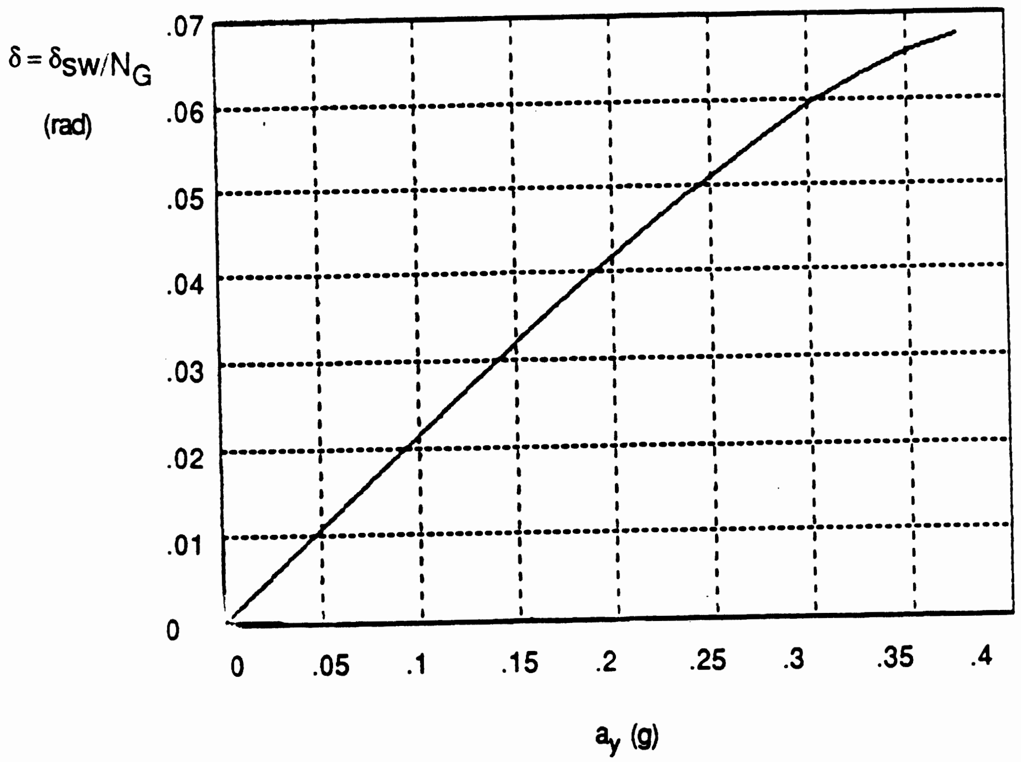
The results given in Figure 111 indicate that this vehicle would become less stable if the front tires are replaced with stiff radial tires. An increase in rear tire stiffness would improve the stability margin, as indicated by the results presented in Figure 111.

The results in Figure 112 show that a decrease in rear suspension roll stiffness will increase the stability margin, however, this increase in stability margin is achieved at the expense of a low rollover threshold (see Section 5.5).

5.7 Rearward Amplification

Benchmark performance. The full trailer in the 3-2 has an amplified performance with respect to that of the truck. The full trailer is steered by the dolly tongue which is attached to the rear of the truck. This arrangement steers the full trailer to attempt to follow the motion of the rear of the truck, which the trailer does very well in normal maneuvering at highway speed. However, in sudden maneuvers such as those required to avoid an unexpected obstacle, the lateral

Equilibrium (steady) Turning at 55 mph



Steering Sensitivity at 55 mph

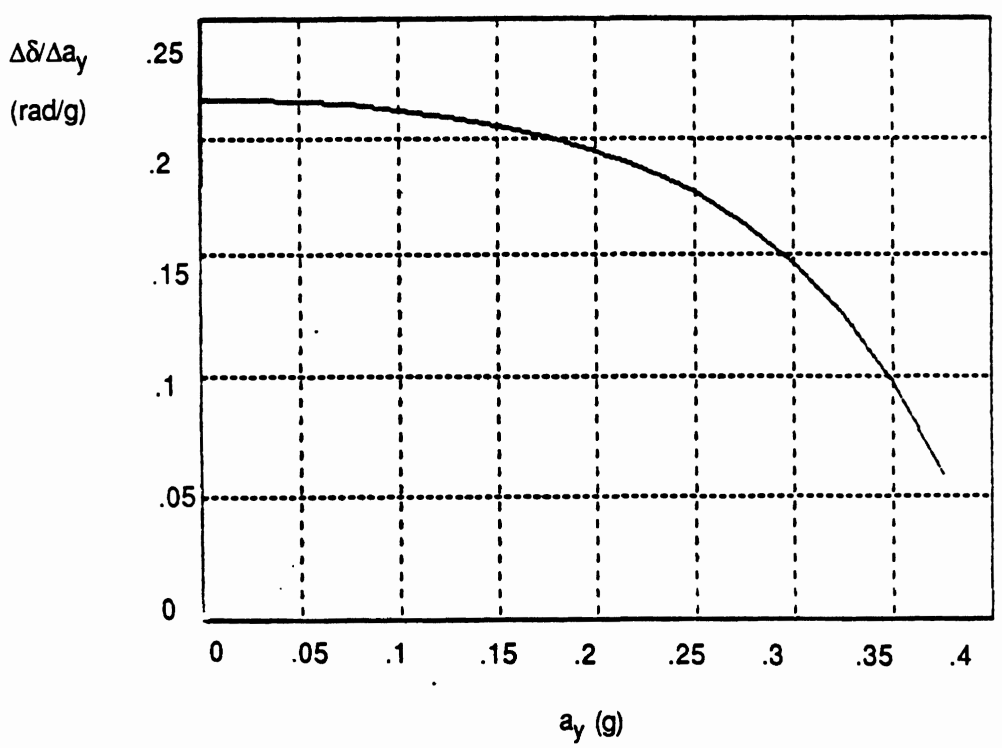


Figure 109. Performance signature, handling, truck and full trailer

Table 33. Truck and Full Trailer - Handling

Parameter Name	Benchmark Value	Minimum Value	Maximum Value	Notes
Truck's c.g. height	63.9 inches	60 inches	70 inches	Much larger deviations used for braking
Truck's wheelbase	235 inches	125 inches	272 inches	
Rear axle roll center heights	29 inches	21 inches	31 inches	
Truck's front axle roll stiffness	21,000 in.lb/deg	17,000 in.lb/deg	25,000 in.lb/deg	
Tires on truck's front axle	Bias-ply, rib tread	Bias-ply, lug tread	Radials	
Truck's rear suspension roll stiffness	140,000 in.lb/deg	60,000 in.lb/deg	330,000 in.lb/deg	Total roll stiffness on rear tandem
Tires on truck's rear axles	Bias-ply, rib tread	Bias-ply, lug tread	Radials	

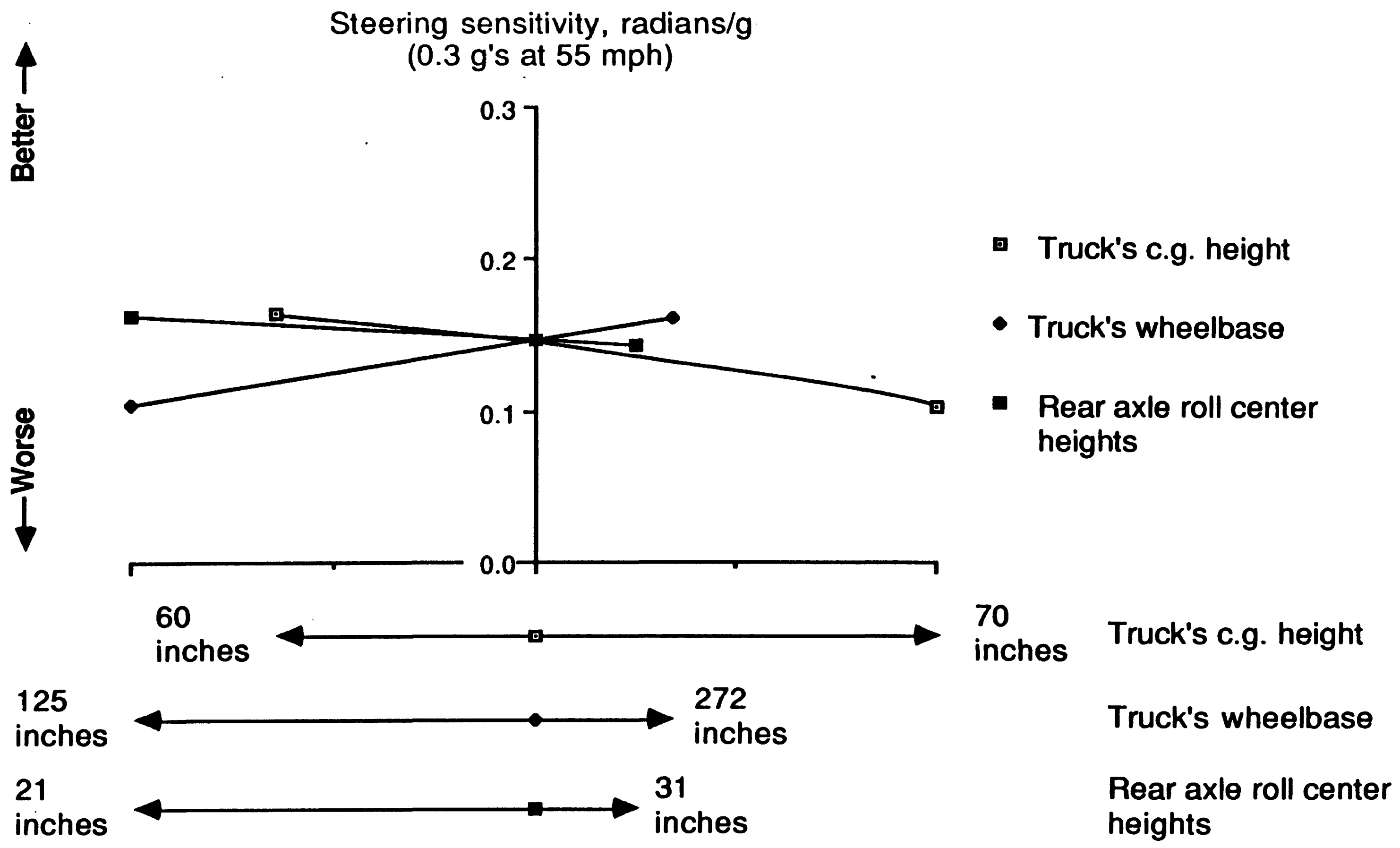


Figure 110. Parameter sensitivity diagram: handling, truck and full trailer.

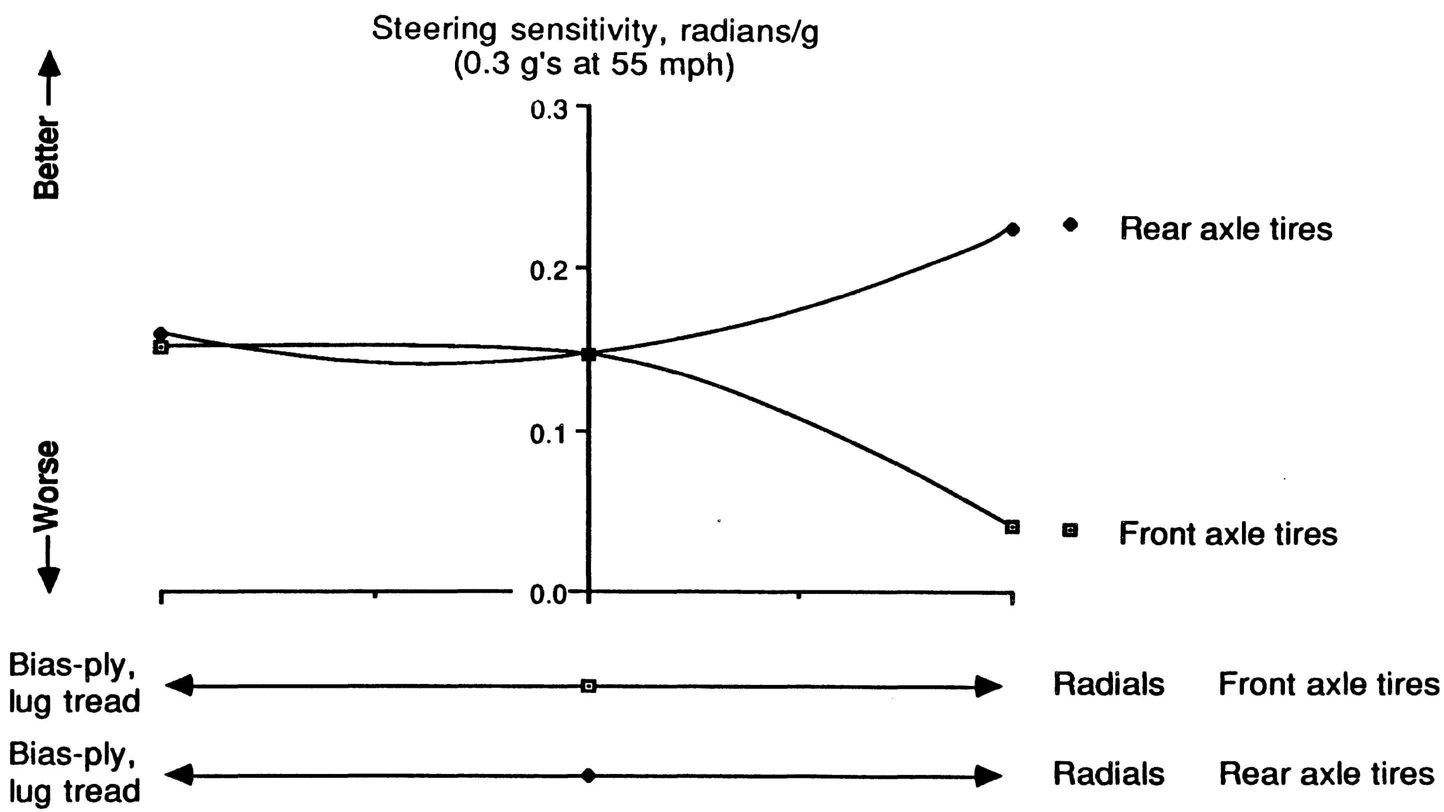


Figure 111. Parameter sensitivity diagram: handling, truck and full trailer.

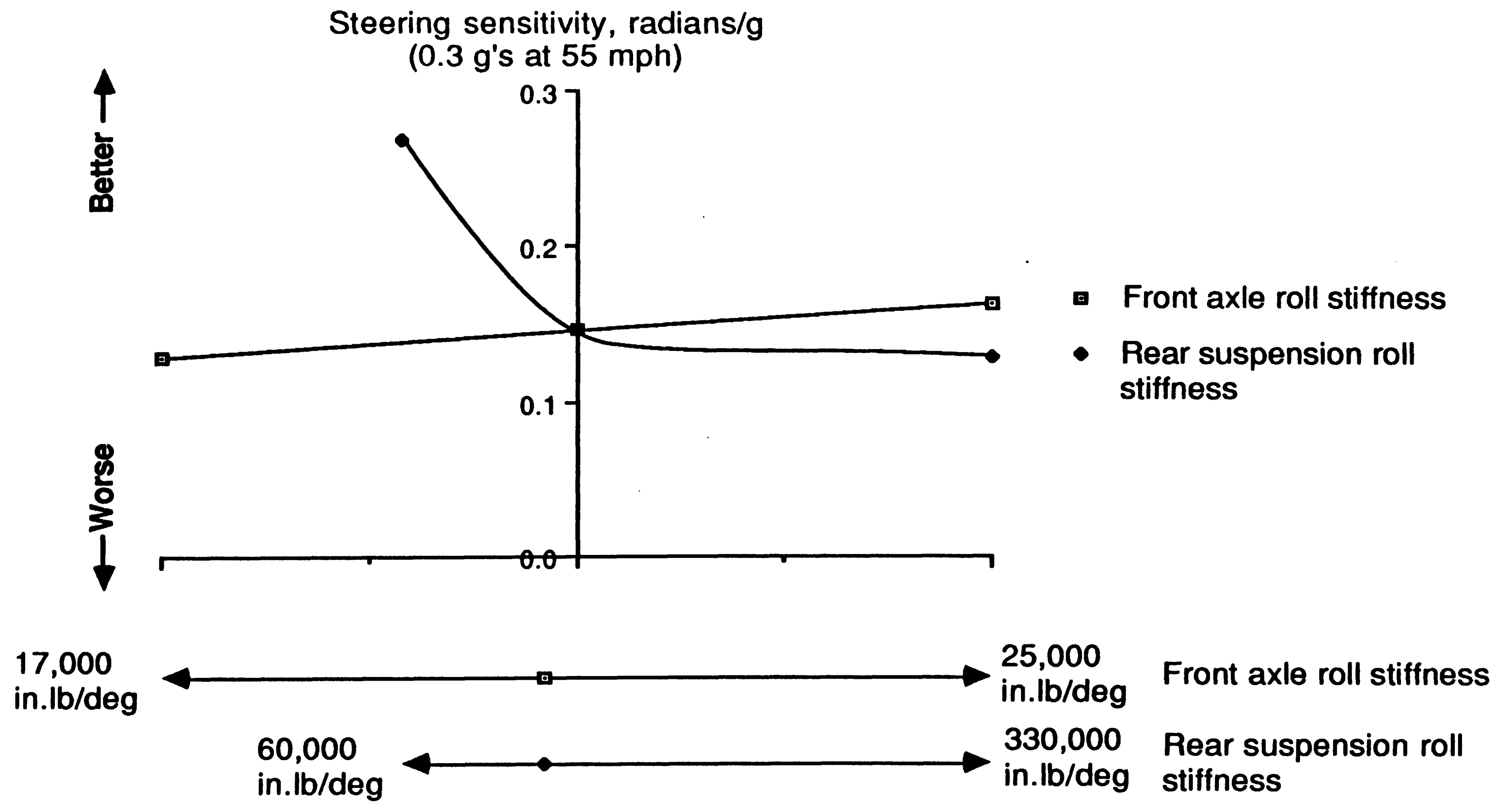


Figure 112. Parameter sensitivity diagram: handling, truck and full trailer.

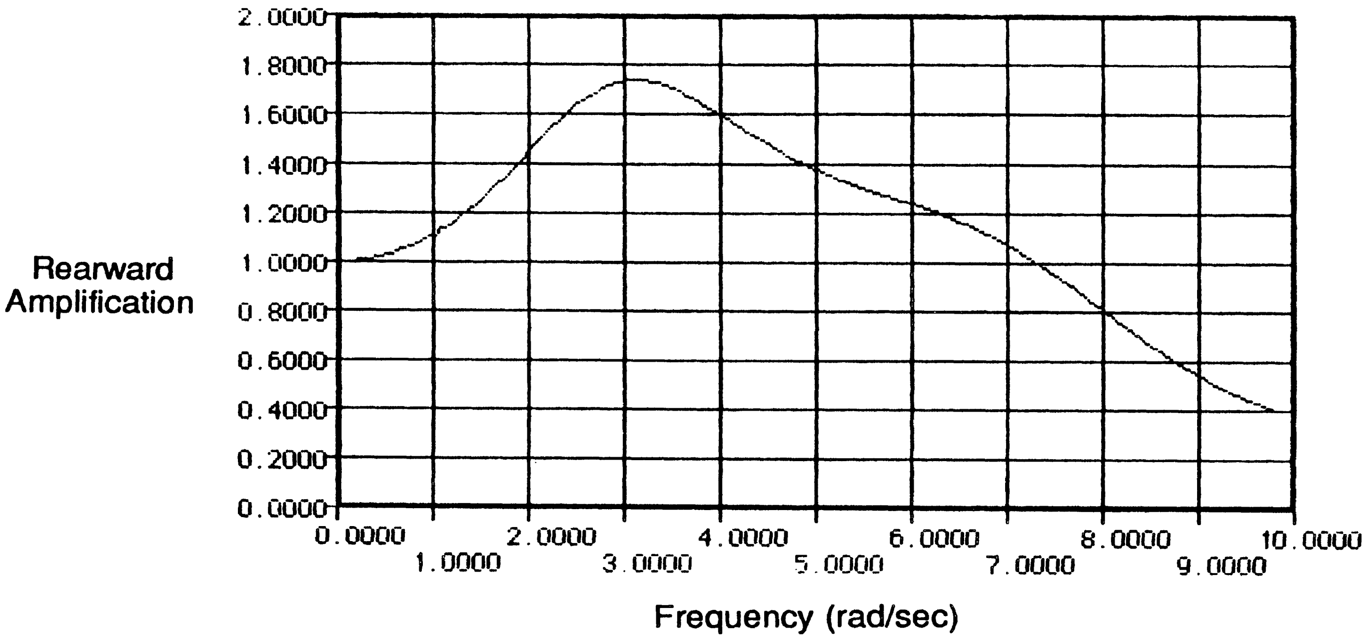


Figure 113. Performance signature, rearward amplification, benchmark truck/full trailer

acceleration of the trailer can be roughly twice as large as that of the truck. Specifically, the performance signature of the benchmark 3-2 (see Figure 113) has a maximum amplification of 1.73 at a frequency of 3.1 radians/sec. That frequency corresponds to a rapid maneuver, but one that is well within driver capabilities and one which is required to avoid unforeseen obstacles when the vehicle is traveling at highway speed. A maximum amplification of 1.73 means that the trailer is much more prone to rolling over than the tractor in this type of maneuver. *To first approximation, the steady-turning rollover threshold of the trailer can be used to estimate the level of tractor maneuver that will roll over the trailer when avoiding an obstacle.* In this case the result is 0.23 g obtained by dividing the rollover threshold, 0.4 g, by the rearward amplification, 1.73.

Parametric sensitivities. Rearward amplification is influenced by hitch location and tire cornering stiffnesses in addition to the longitudinal distances between axle sets (tires). The pintle hitch is installed well behind the rear axle of the benchmark vehicle. *If the pintle hitch connection were to be moved closer to the rear axle of the truck, the maximum rearward amplification could be reduced to 1.5, per the results displayed in Figure 114.* In this case, the trailer would be trying to follow a point on the truck that was closer to the truck's c.g. and therefore not as influenced by the yawing motion of the truck.

Switching to "stiff" tires will also reduce rearward amplification, as illustrated in Figure 115. In this case, the maximum amplification is reduced to 1.6 at a frequency that is slightly higher than that observed in the baseline case. This is only a small improvement compared to the influences that tire cornering stiffness can have in other cases.

The influence of dolly tongue length is a difficult matter to treat in a simple fashion. In many situations involving conventional doubles configurations, tongue length is not a major factor in determining rearward amplification. However, for this particular 3-2, tongue length (that is, the distance from the pintle hitch to the c.g. of the dolly) changes not only the amplitude, but also the "bandwidth" of the rearward amplification performance signature. The results presented in Figure 116 show that when the tongue length is reduced from 148 inches to 108 inches, the maximum amplification increases to almost 1.9 and the amplification remains above 1.5 out to a frequency of 7 radians/sec. Although inputs above approximately 4 radians/sec. are probably beyond the capabilities and frequencies usable by drivers, the increased bandwidth is a disadvantage because it indicates responsiveness to undesirable types of excitation. Figure 117 shows an even more exaggerated result when the tongue length is reduced to 80 inches. In this case, the maximum amplification actually occurs at a frequency that may be beyond those that drivers will attempt to use.

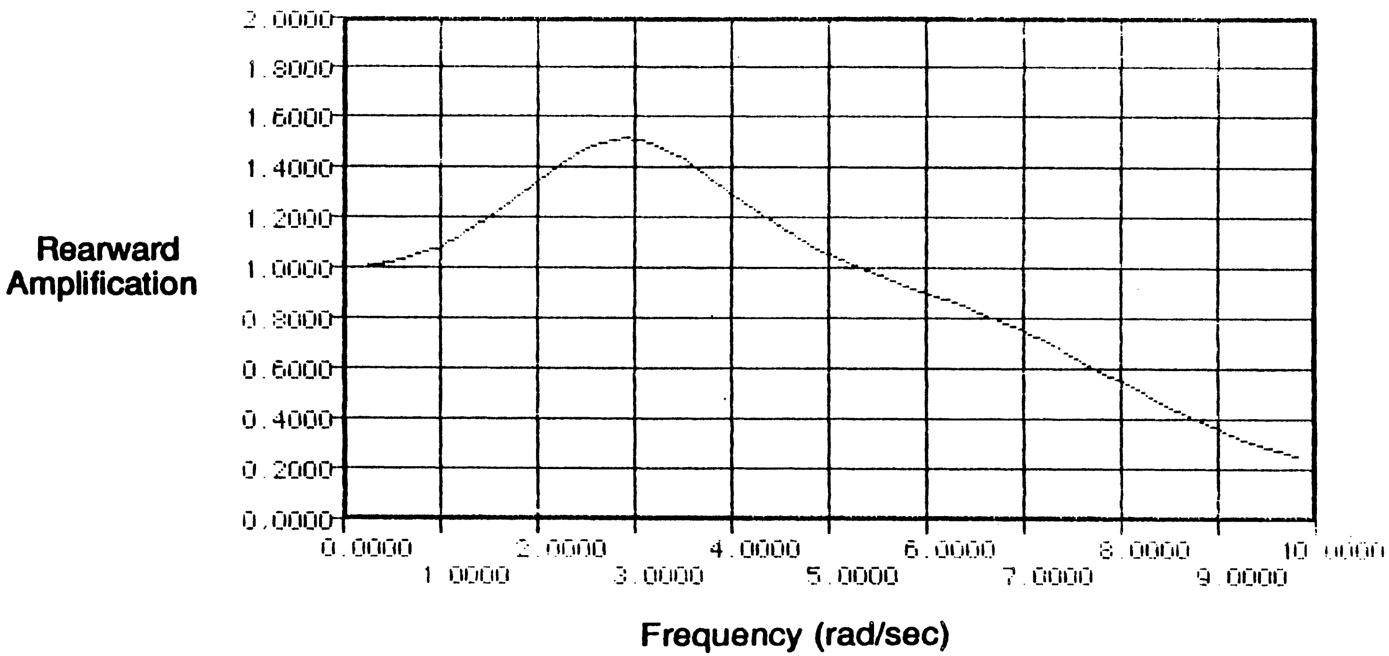


Figure 114. Performance signature, rearward amplification, 3-2, 113 inch truck c.g. to pintle hitch

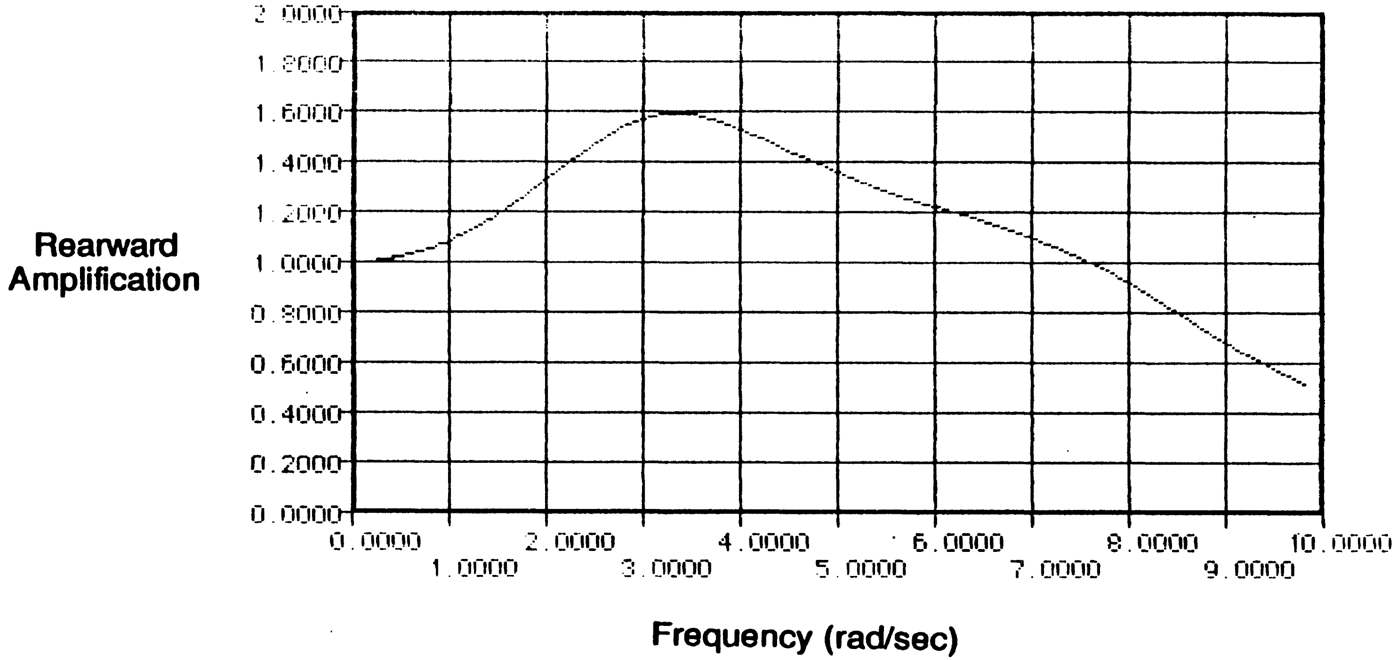


Figure 115. Performance signature, rearward amplification, 3-2, stiff radial tires on all axles

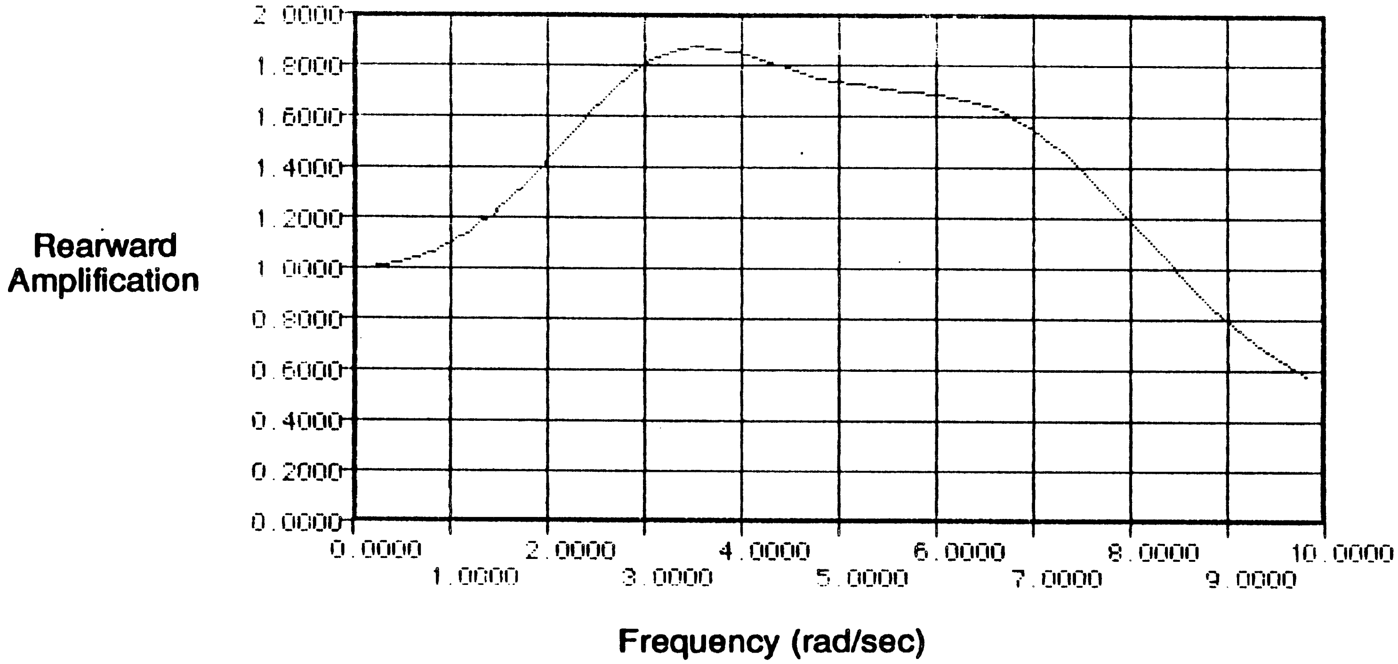


Figure 116. Performance signature, rearward amplification, 3-2, 108 inch dolly tongue length

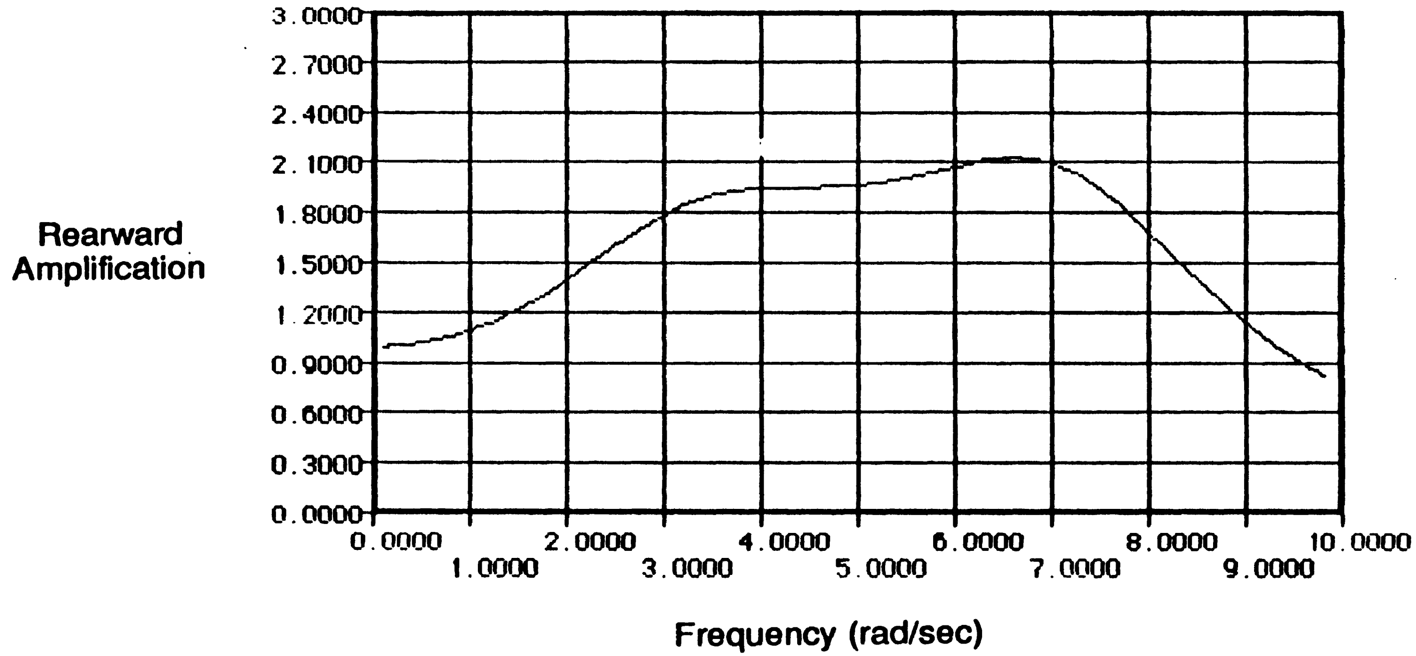


Figure 117. Performance signature, rearward amplification, 3-2, 80 inch dolly tongue length

In summary, the results presented here indicate that reducing the amount of the overhang of the pintle hitch behind the rear axle is an effective means of reducing rearward amplification for the benchmark 3-2. Changes in pertinent trailer properties had less effect than that obtained by relocating the pintle hitch. Significant changes in rearward amplification are hard to come by for this particular arrangement of 3-2.

5.8 Concluding Remarks on the Truck Full Trailer

With this vehicle the influences of pintle hitch properties have been introduced into the analyses. The pintle hitch effectively decouples the towing unit from the towed unit with respect to rolling constraints and lateral forces acting between the truck and the full trailer. This means that (1) rollover of each unit must be evaluated individually, (2) handling is independent of trailer properties, and (3) rearward amplification can be large if (a) the truck must yaw agressively to obtain lateral acceleration and (b) the trailer has an amplified response with respect to the motion of the pintle hitch. Pintle hitch location has an important influence on low- and high-speed offtracking and rearward amplification. To improve high-speed offtracking and rearward amplification, the pintle hitch should be located close to the rear axle. To improve low-speed offtracking, the pintle hitch should be located well away from the rear axle. For safety on the highway, one needs to be aware of these tradeoffs and one should not introduce a safety hazard in an attempt to gain a small improvement in low-speed offtracking.

6.0 B-TRAINS OR C-TRAINS

6.1 Baseline Values of Pertinent Mechanical Properties

B-trains and C-trains are popular vehicle configurations in Canada. They are not currently used to any significant amount in the United States. They are treated briefly in this handbook to introduce some of the performance aspects of these types of vehicles. The benchmark vehicle or the "C-train" (as it is referred to here) is a fabricated vehicle that might be obtained by using a double draw bar dolly in a Western double. The basic dimensions and axle loads of the hypothesized vehicle are presented in Figure 118.

The vehicle employs two fifth wheels--one on the tractor and the other over the rear axle that is appended to the first semitrailer. With regard to directional response, the C-train is a tractor-semitrailer-semitrailer vehicle. As such, it has one less articulation point than the double and it employs no full trailers. It has the same number of articulation points as the truck-full trailer, but the performance of this vehicle is not like that of the 3-2 because the C-train is made up entirely of semitrailers.

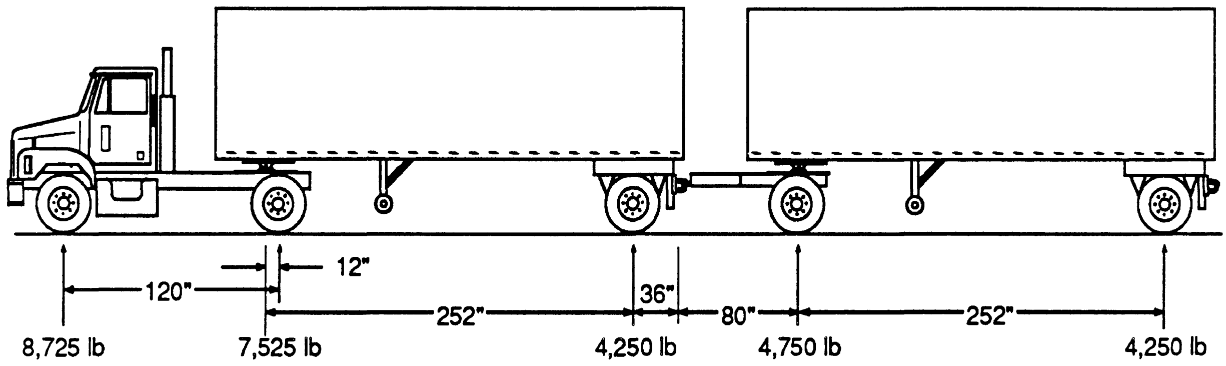
The pertinent mechanical properties of the benchmark C-train are listed in Table 34. The two-axle tractor is a rather standard unit. The semitrailers are like those used in the Western double except that the axle which would have been the dolly axle is appended to the first semitrailer.

For this vehicle we examine performance in steady turning (rollover and handling) and rearward amplification because the results are different from those for a conventional double and like those for a B-train.

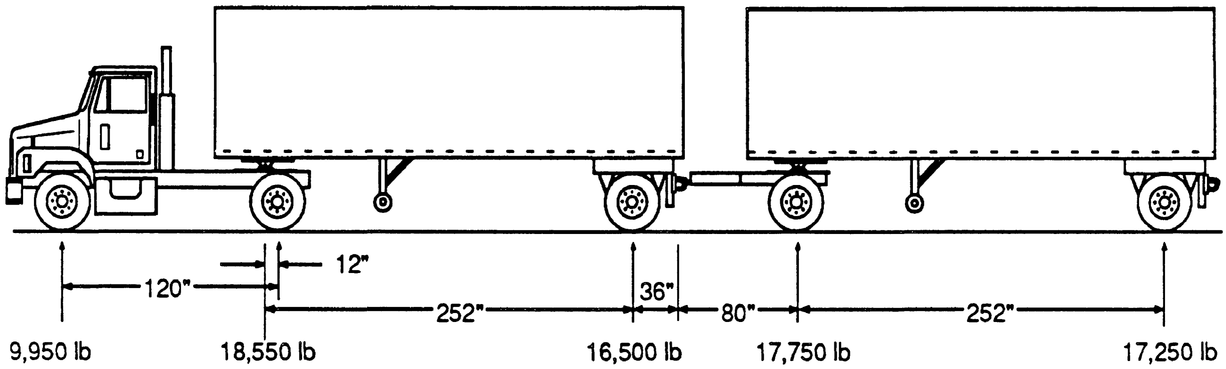
6.2 Steady Turn - Roll

Benchmark performance. The entire C-train rolls as a single unit. The performance signature is a straight line up to the point where the first axle on the first semitrailer lifts off. After that the other rear axles lift off at only slightly higher levels of lateral acceleration. These effects are illustrated by the performance signature presented in Figure 119. The rollover threshold for the benchmark vehicle is 0.395 g.

Parametric sensitivities. The individual variations that are considered here are presented in the same order as they are listed in Table 35.



EMPTY C-TRAIN



LOADED C-TRAIN

Figure 118. Geometric layout, C-train

Table 34. Basic Mechanical Properties (C-Train)

Mechanical Property	Empty	Loaded
Tractor		
Weight (lb)	14,000.00	14,000.00
Front axle brake gain (in.lb/psi)	2,000.00	2,000.00
Rear axle brake gain (in.lb/psi)	3,000.00	3,000.00
Front suspension cornering stiffness (lb/deg)	948.00	998.00
Rear suspension cornering stiffness (lb/deg)	1,152.00	1,945.00
Reduction in cornering stiffness due to steering system (%)	31.66	31.92
Front suspension roll stiffness (in.lb/deg)	21,000.00	21,000.00
Rear suspension roll stiffness (in.lb/deg)	70,000.00	70,000.00
Front suspension roll center height (in)	20.00	20.00
Rear suspension roll center height (in)	29.00	29.00
Height of the center of gravity (in)	37.50	37.50
Distance of the center of gravity from the front axle (in)	47.15	47.15
First Trailer and Double Drawbar Dolly		
Weight (lb)	9,000.00	34,300.00
Rear suspension - leading axle brake gain (in.lb/psi)	3,000.00	3,000.00
Rear suspension - trailing axle brake gain (in.lb/psi)	3,000.00	3,000.00
Interaxle load transfer on rear tandem	0.00	0.00
Rear suspension cornering stiffness (lb/deg)	1,638.00	3,758.00
Rear suspension roll stiffness (in.lb/deg)	160,000.00	160,000.00
Rear suspension roll center height (in)	29.00	29.00
Height of the center of gravity (in)	44.30	74.80
Distance of the center of gravity from kingpin (in)	221.20	151.60
Second Trailer		
Weight (lb)	6,500.00	32,500.00
Rear axle brake gain (in.lb/psi)	3,000.00	3,000.00
Rear suspension cornering stiffness (lb/deg)	790.00	1,885.00
Rear suspension roll stiffness (in.lb/deg)	80,000.00	80,000.00
Rear suspension roll center height (in)	29.00	29.00
Height of the center of gravity (in)	50.00	78.50
Distance of the center of gravity from kingpin (in)	164.75	133.75

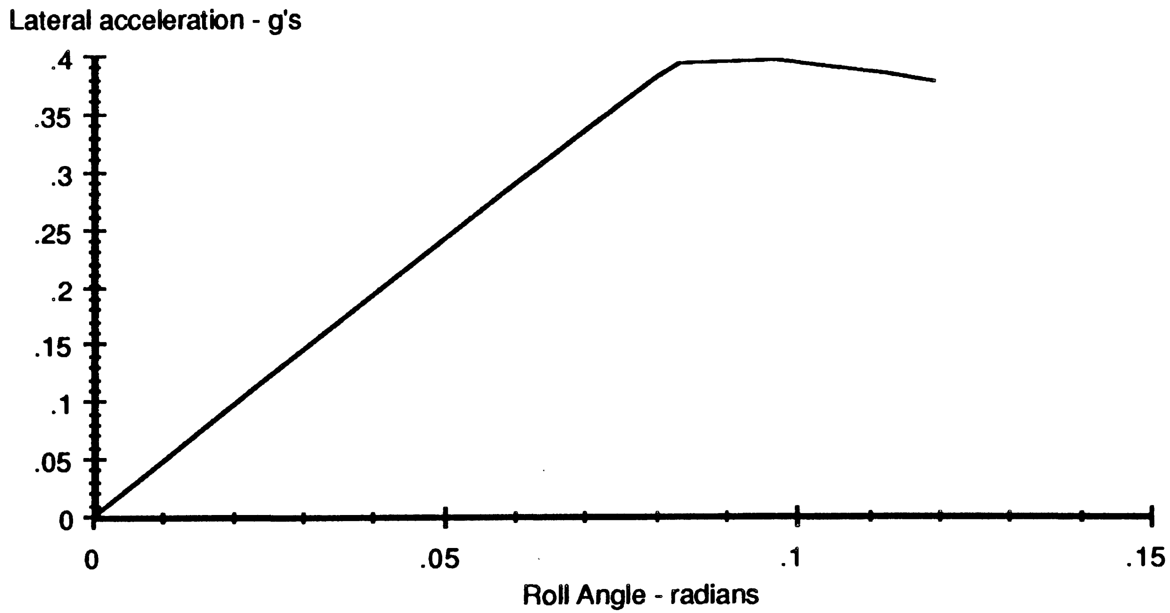


Figure 119. Performance signature, static roll, C-train

Table 35. C-Train - Static Roll

Parameter Name	Benchmark Value	Minimum Value	Maximum Value	Notes
Trailer #1 c.g. height	74.75 inches	70 inches	80 inches	
Trailer #2 c.g. height	78.5 inches	70 inches	80 inches	
Rear axle roll center heights	29 inches	21 inches	31 inches	
Tractor's front axle roll stiffness	21,000 in.lb/deg	17,000 in.lb/deg	25,000 in.lb/deg	
Tractor front axle track width	80 inches	77 inches	81 inches	
Tractor rear suspension roll stiffness	70,000 in.lb/deg	30,000 in.lb/deg	165,000 in.lb/deg	
Tractor rear axle track width	72 inches	71 inches	78 inches	
Trailer #1 axle roll stiffness	160,000 in.lb/deg	60,000 in.lb/deg	330,000 in.lb/deg	Both axles on the lead trailer and the dolly
Trailer #1 axle track width	72 inches	71 inches	78 inches	
Trailer #2 axle roll stiffness	80,000 in.lb/deg	30,000 in.lb/deg	165,000 in.lb/deg	
Trailer #2 axle track width	72 inches	71 inches	78 inches	

The influences of trailer c.g. heights (see Figure 120) are to degrade the rollover threshold if the c.g. height is increased. Since the trailer boxes are identical, the influences of increasing the c.g. height of either trailer are nearly the same.

Figure 121 indicates that changing all of the roll center heights on all of the rear axles has more influence on the rollover threshold than changing the front suspension's roll stiffness and the track of the front axle. The influence of changing any one roll center height by the amounts indicated would not be very large.

The results in Figures 122, 123, and 124 show that major reductions in rollover threshold will be obtained if the roll stiffnesses of the tractor rear, the first trailer, or the last trailer suspensions are significantly reduced. In each of these cases, the suspension becomes a "soft" suspension, and reductions in roll stiffness become very important.

6.3 Steady Turn - Handling

Benchmark performance. The performance signature for the handling of the benchmark C–train is presented in Figure 125. The signature "bends down," indicating the possibility of yaw divergence at high speeds and g levels. The stability boundary (see Figure 126) does not place the point at 0.3 g and 55 mph in the unstable region. At 0.3 g and 55 mph the steering sensitivity is 0.029 radians/g. This value is the baseline condition for the following performance sensitivity diagrams.

Parametric sensitivities. An extensive set of variations in mechanical properties have been examined in this case (see Table 36). The results are presented in Figures 127 through 135. Examination of these figures indicates four cases in which the variations cause the vehicle to become yaw divergent (yaw divergence is indicated by a negative value of steering sensitivity). Specifically, the vehicle becomes yaw divergent under the following circumstances: (1) when the baseline front tires on the tractor are replaced with stiff radial tires, (2) when the tractor's rear suspension is made very stiff, (3) when the suspensions on the first trailer (including the dolly) are made very soft in roll, and (4) when the suspension on the last trailer is made soft in roll. In general, the benchmark vehicle has a small margin of yaw stability in a steady turn, and the changes that brought about yaw divergence are those that contribute to excessive side force at the front axle or to a lack of side force at the rear axle of the tractor due to a large amount of load transfer at the rear axle of the tractor.

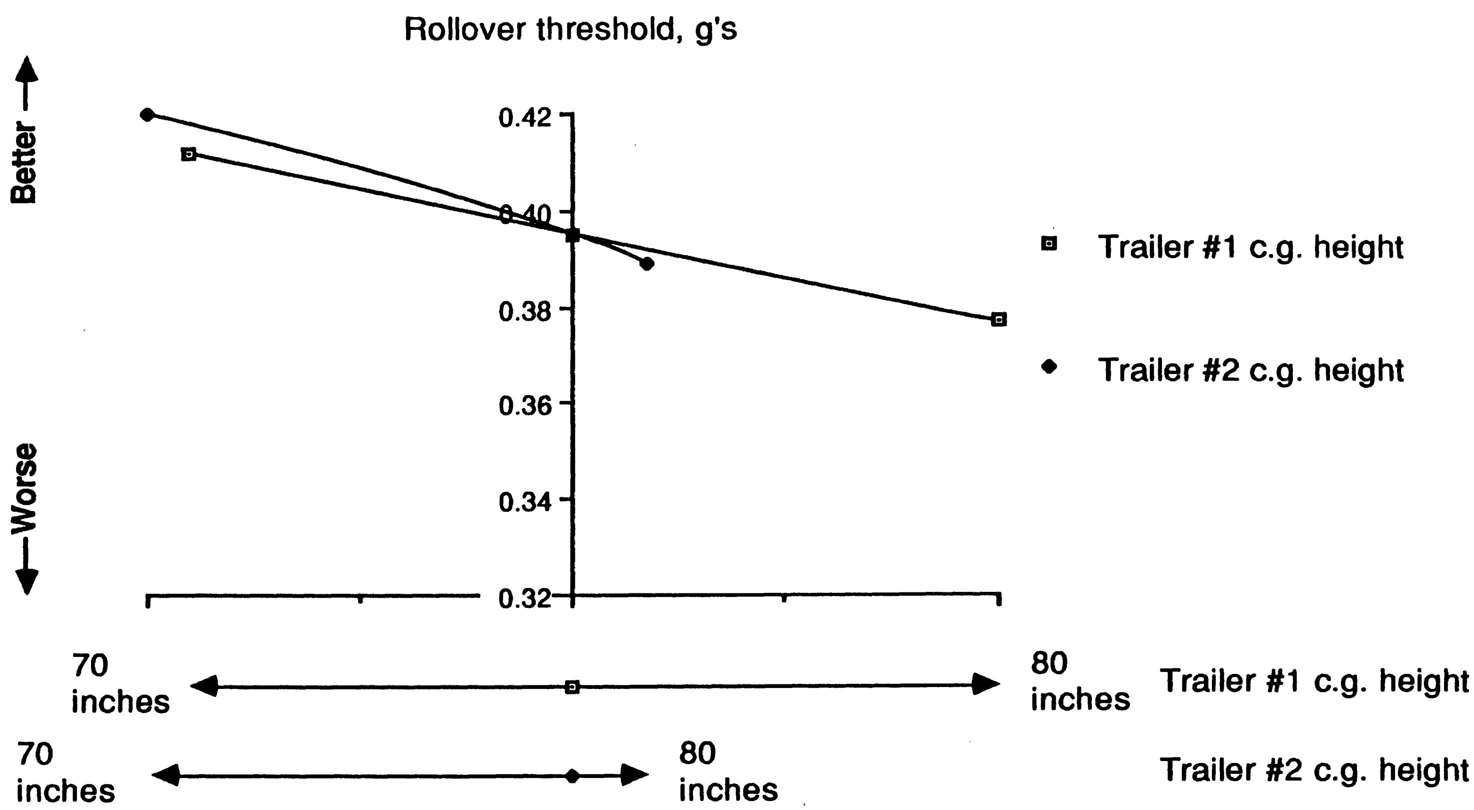


Figure 120. Parameter sensitivity diagram: static roll, C-train.

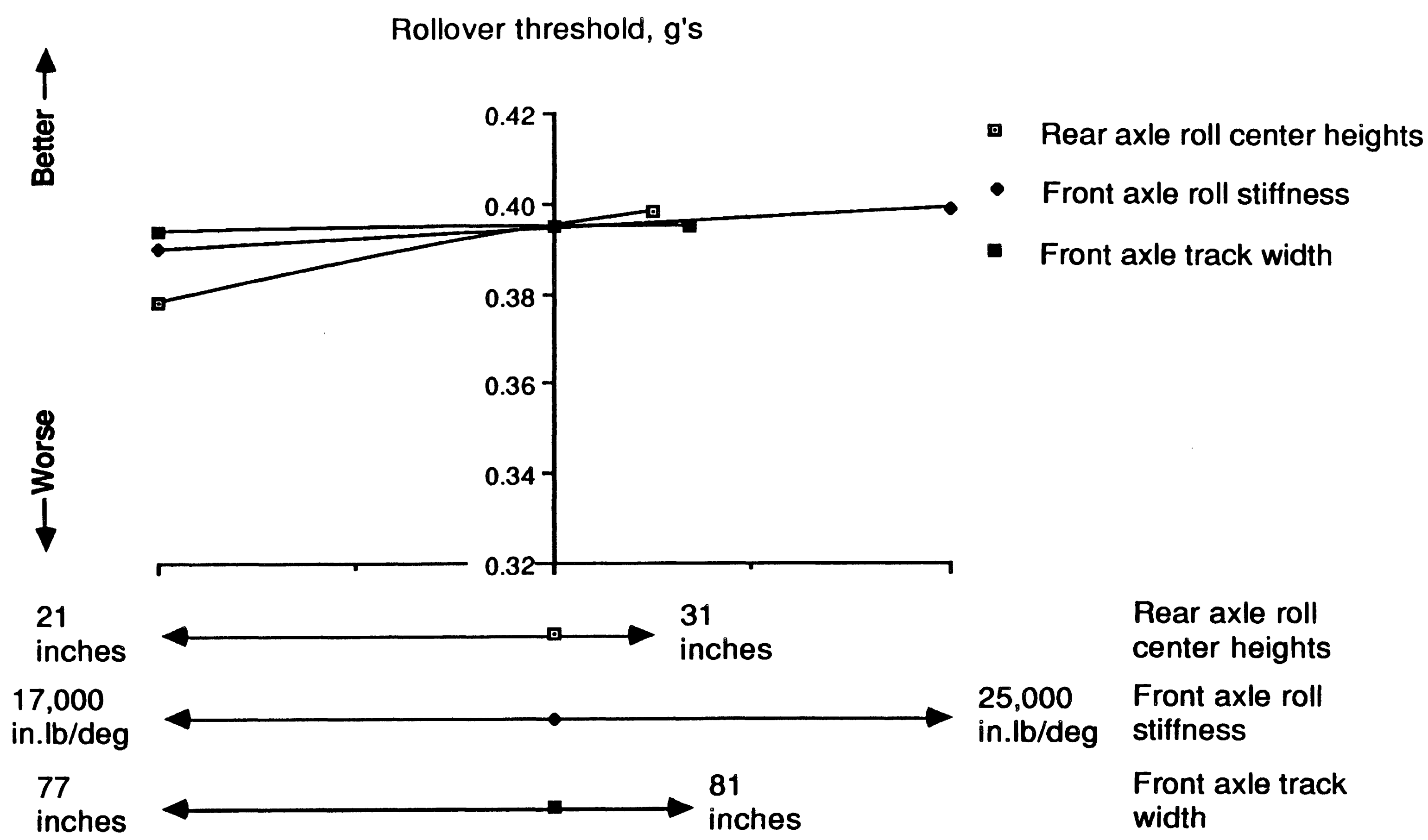


Figure 121. Parameter sensitivity diagram: static roll, C-train.

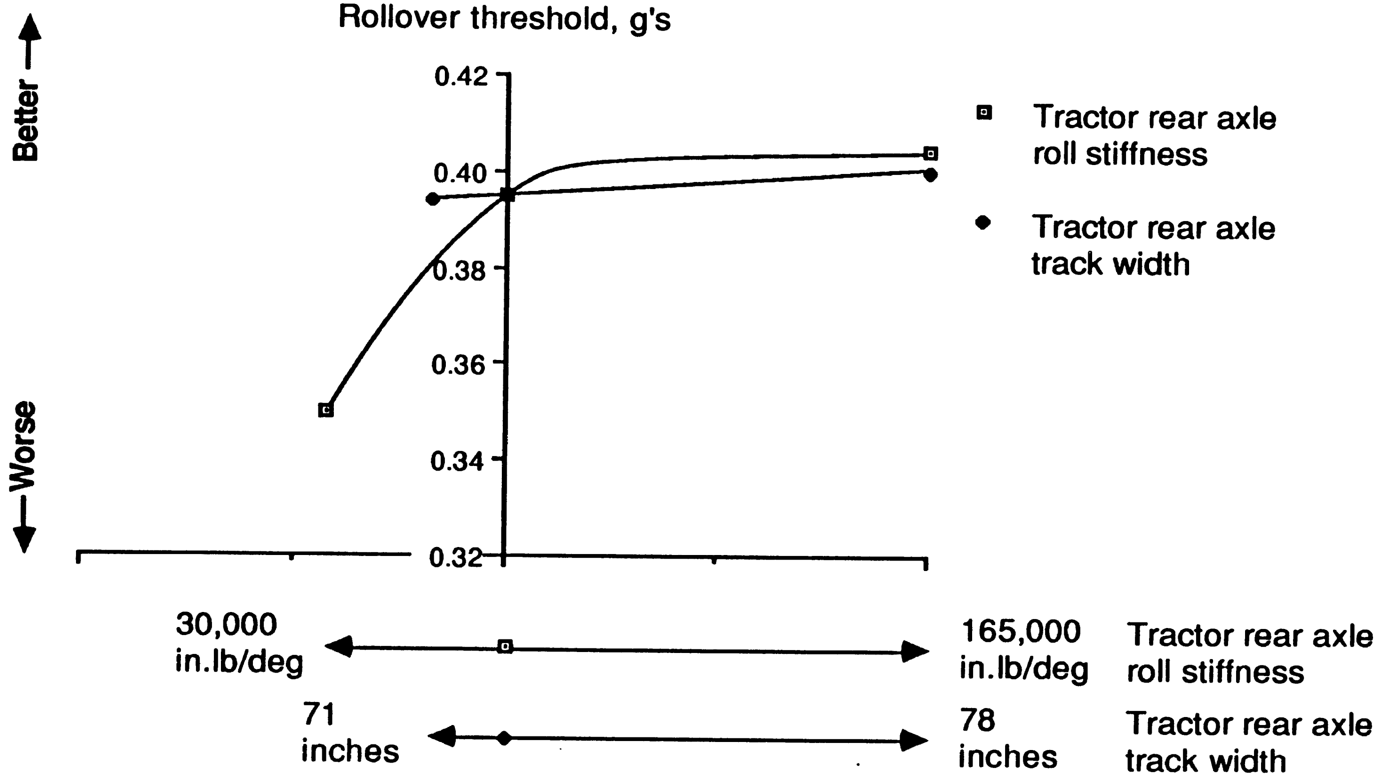


Figure 122. Parameter sensitivity diagram: static roll, C-train.

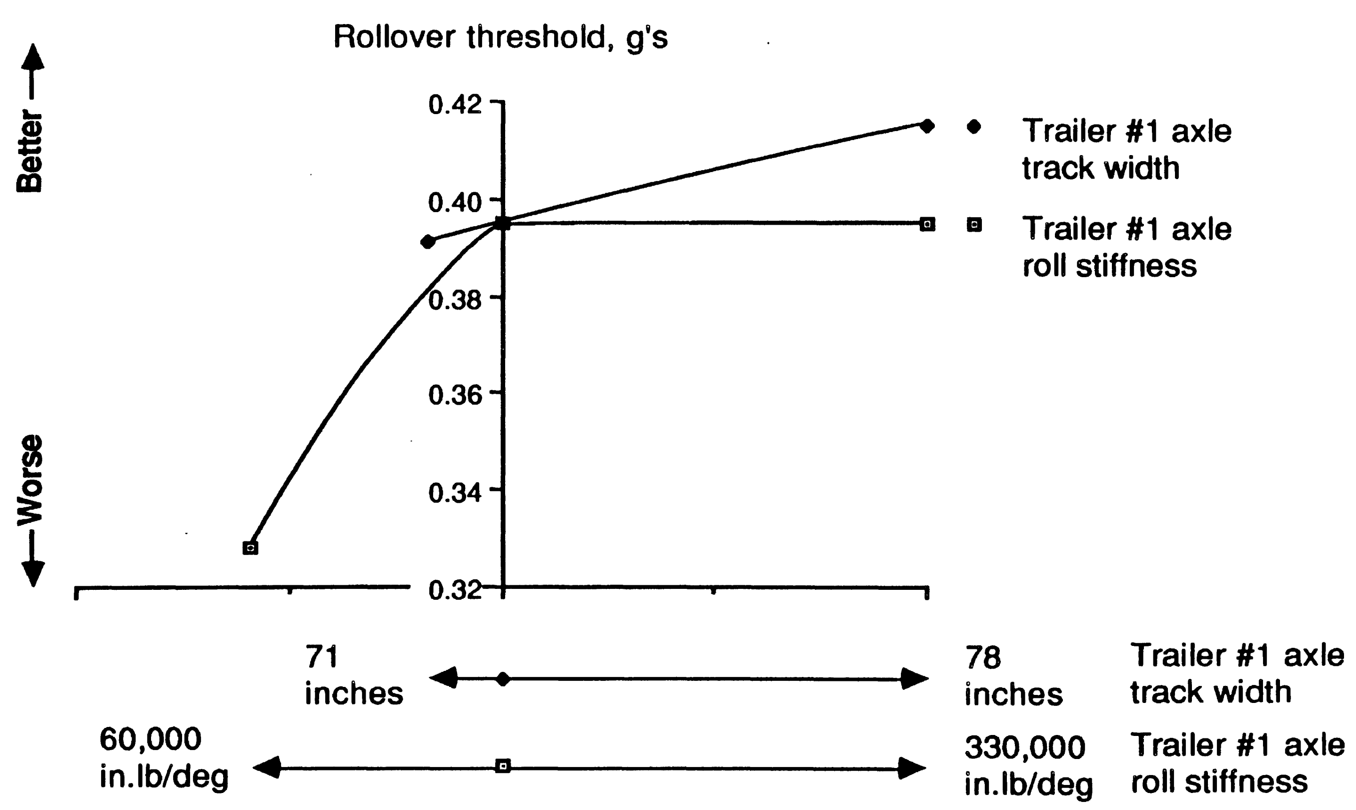


Figure 123. Parameter sensitivity diagram: static roll, C-train.

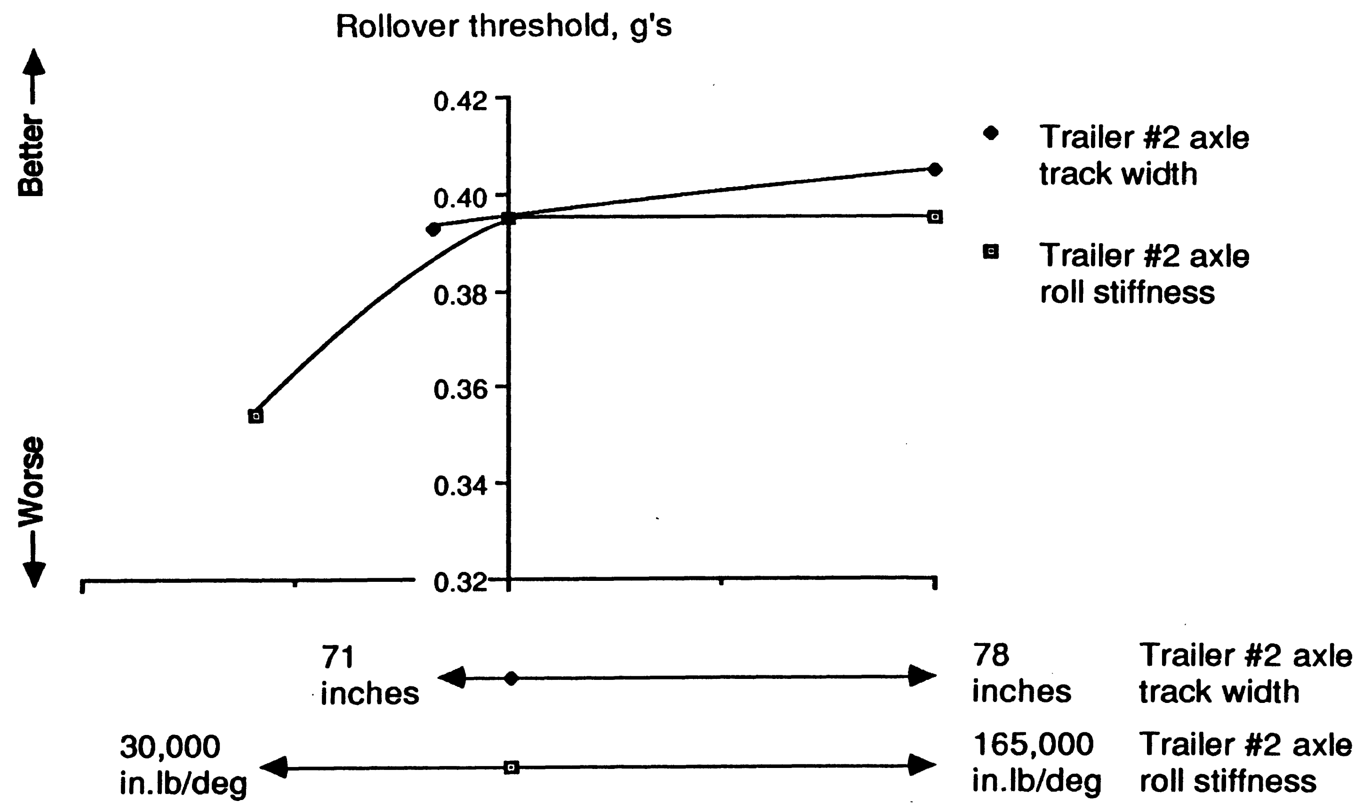
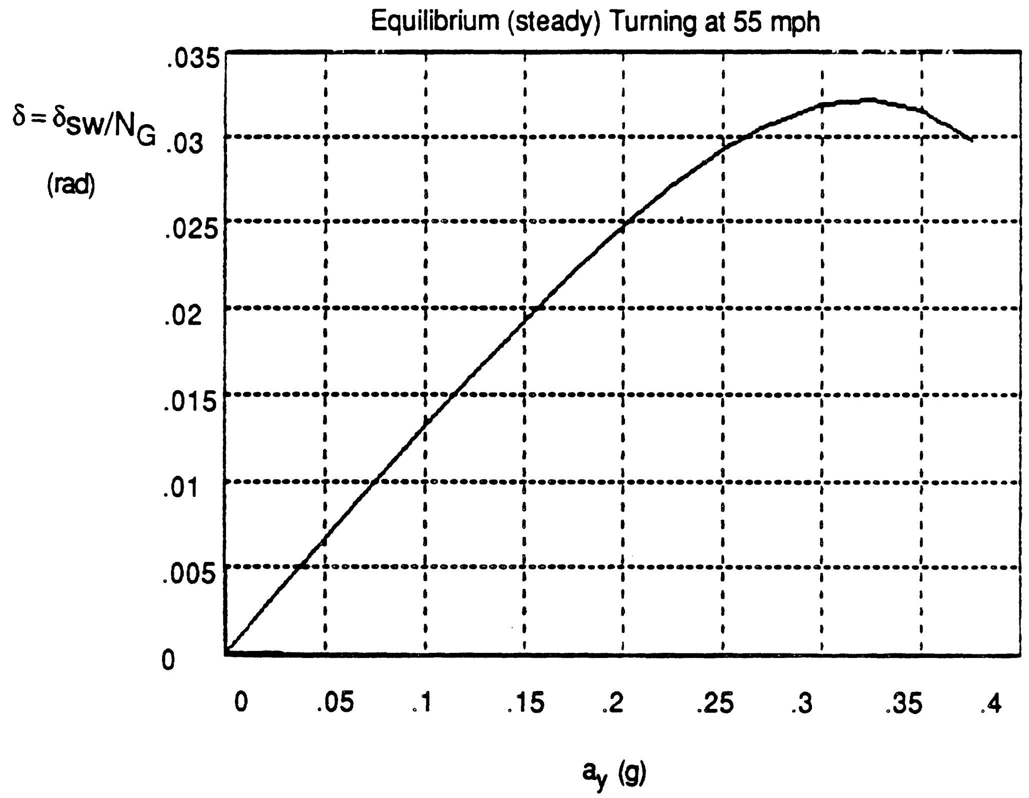


Figure 124. Parameter sensitivity diagram: static roll, C-train.



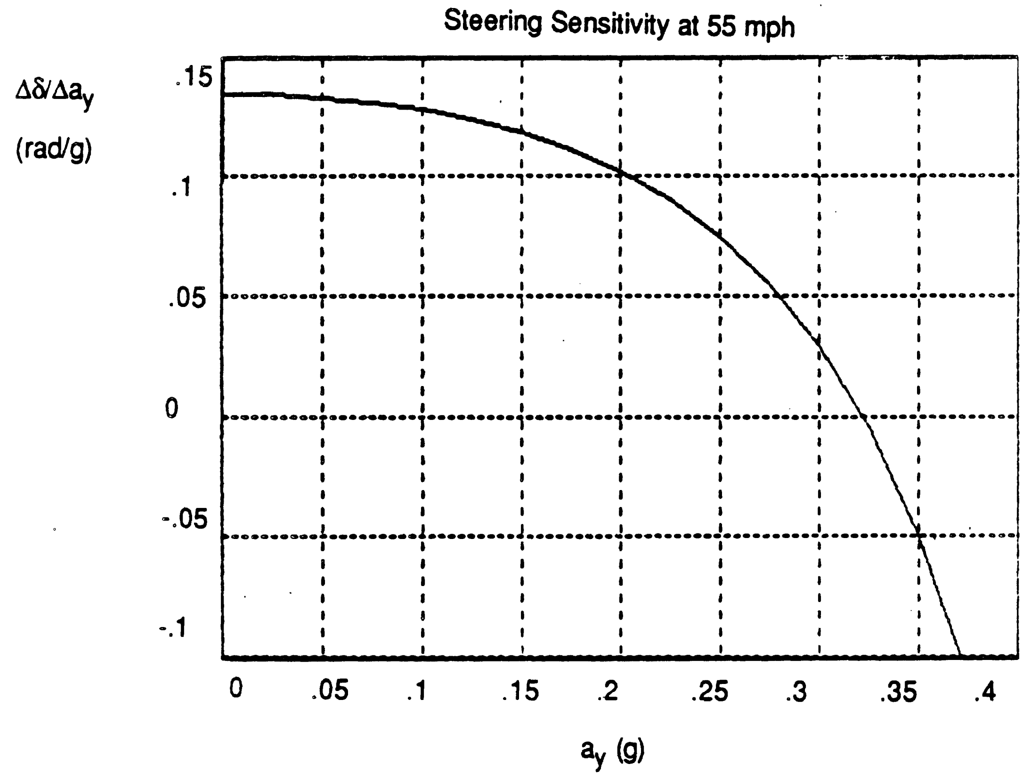


Figure 125. Performance signature, handling, C-train

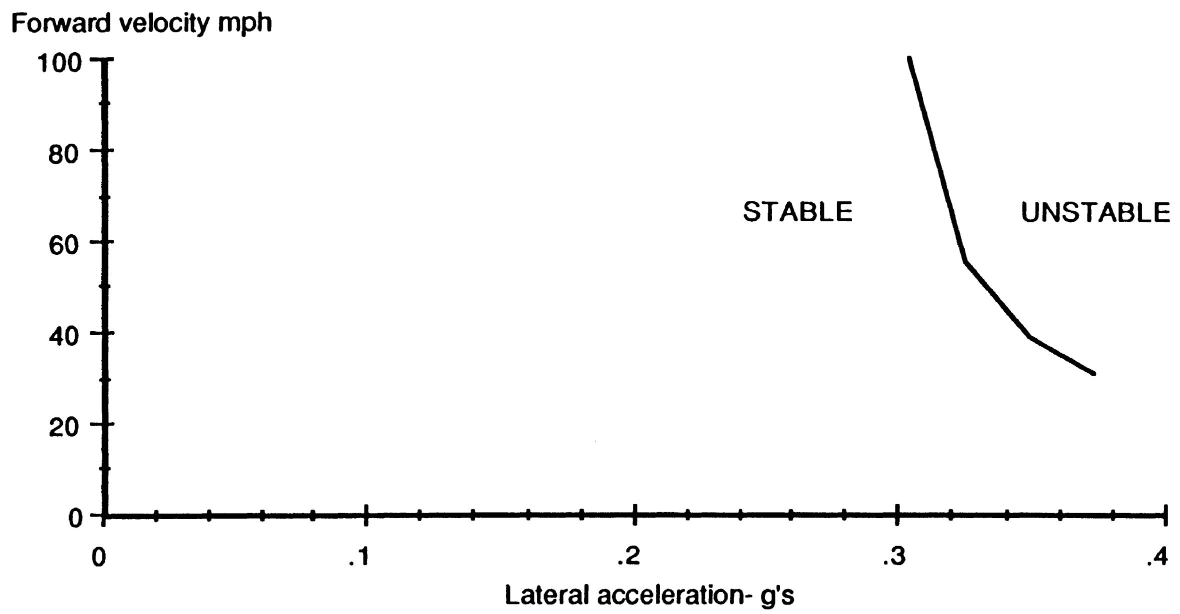


Figure 126. Yaw stability boundary, performance signature, C-train

Table 36. C-Train - Handling

Parameter Name	Benchmark Value	Minimum Value	Maximum Value	Notes
Tractor wheelbase	120 inches	118 inches	203 inches	
Rear axle roll center heights	29 inches	21 inches	31 inches	
Tractor's front axle roll stiffness	21,000 in.lb/deg	17,000 in.lb/deg	25,000 in.lb/deg	
Tires on tractor's front axle	Bias-ply, Rib tread	Bias-ply, Lug tread	Radial	
Tractor front axle track width	80 inches	77 inches	81 inches	
Tractor rear suspension roll stiffness	70,000 in.lb/deg	30,000 in.lb/deg	165,000 in.lb/deg	
Tires on tractor's rear axle	Bias-ply, Rib tread	Bias-ply, Lug tread	Radial	
Tractor rear axle track width	72 inches	71 inches	78 inches	
Trailer #1 c.g. height	74.75 inches	70 inches	80 inches	
Trailer #1 axle roll stiffness	160,000 in.lb/deg	60,000 in.lb/deg	330,000 in.lb/deg	
Trailer #1 axle track width	72 inches	71 inches	78 inches	
Trailer #1 tires	Bias-ply, Rib tread	Bias-ply, Lug tread	Radial	
Trailer #2 c.g. height	78.5 inches	70 inches	80 inches	
Trailer #2 axle roll stiffness	80,000 in.lb/deg	30,000 in.lb/deg	165,000 in.lb/deg	
Trailer #2 axle track width	72 inches	71 inches	78 inches	

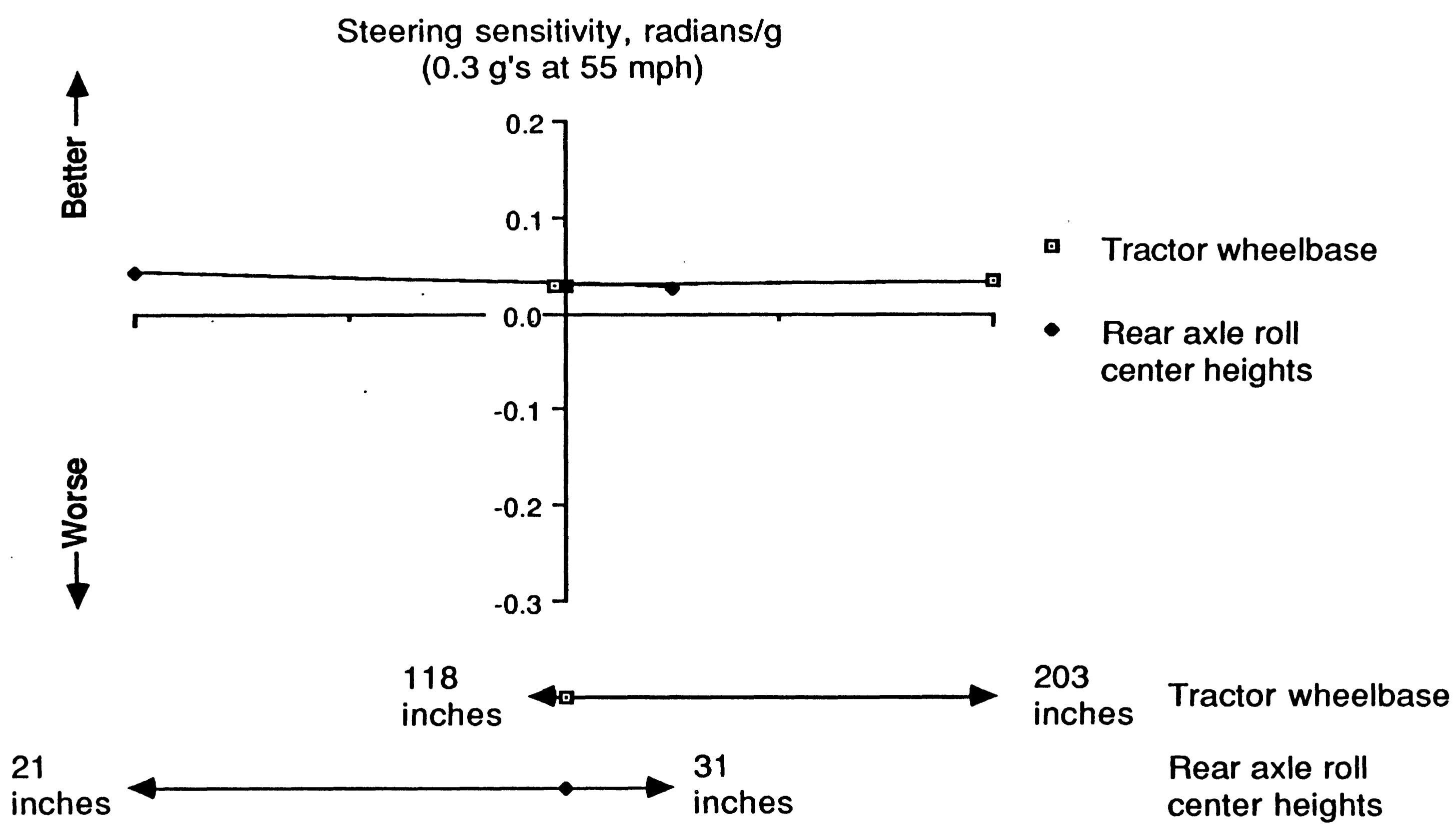


Figure 127. Parameter sensitivity diagram: handling, C-train.

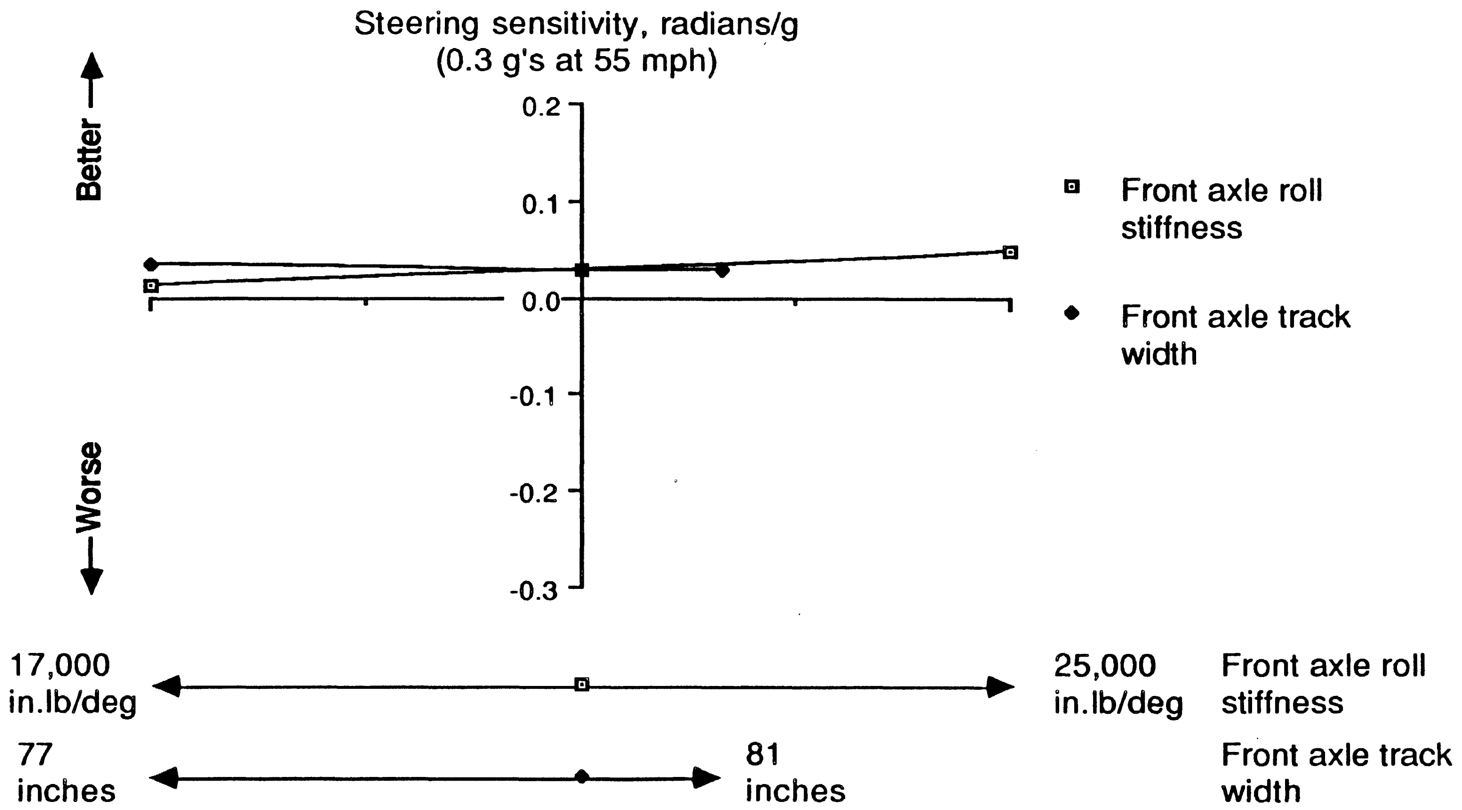


Figure 128. Parameter sensitivity diagram: handling, C-train.

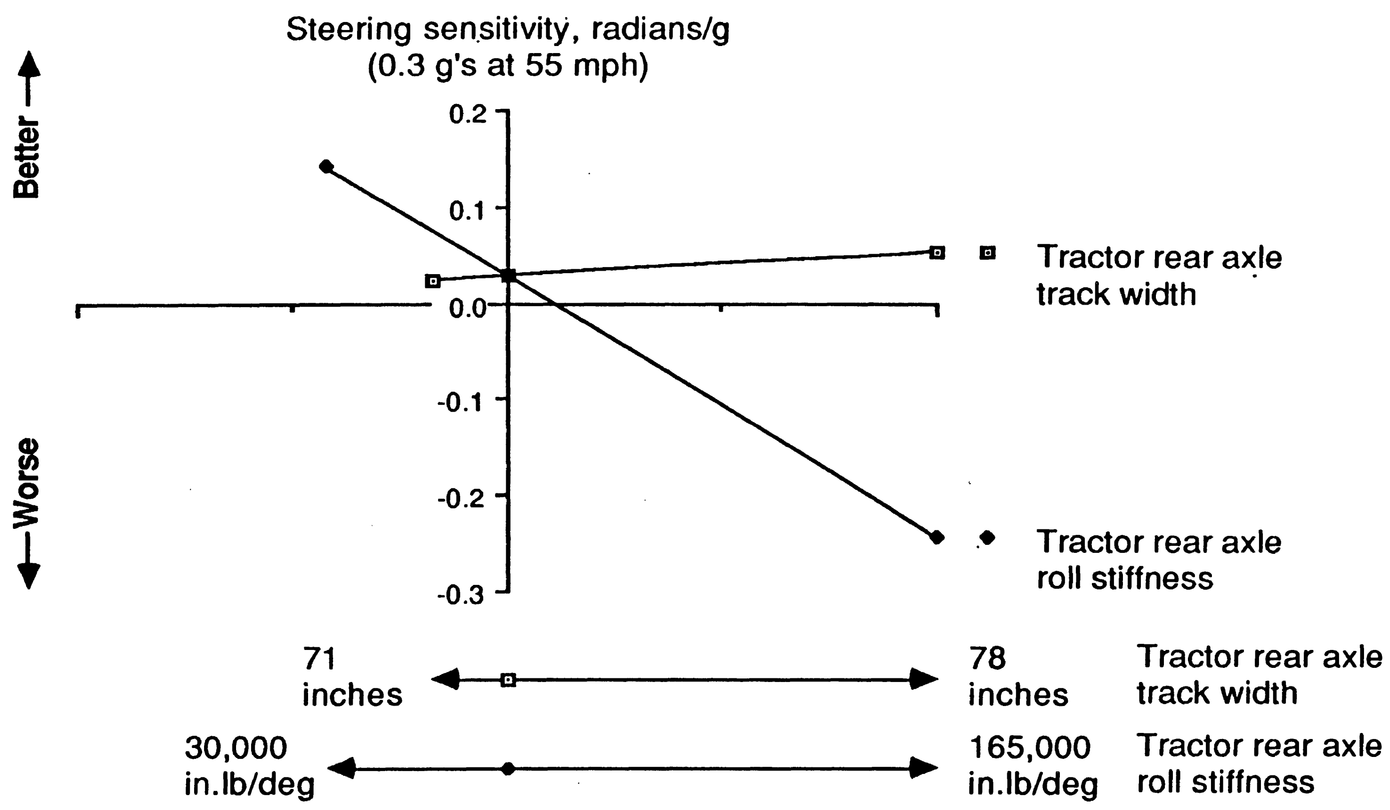


Figure 129. Parameter sensitivity diagram: handling, C-train.

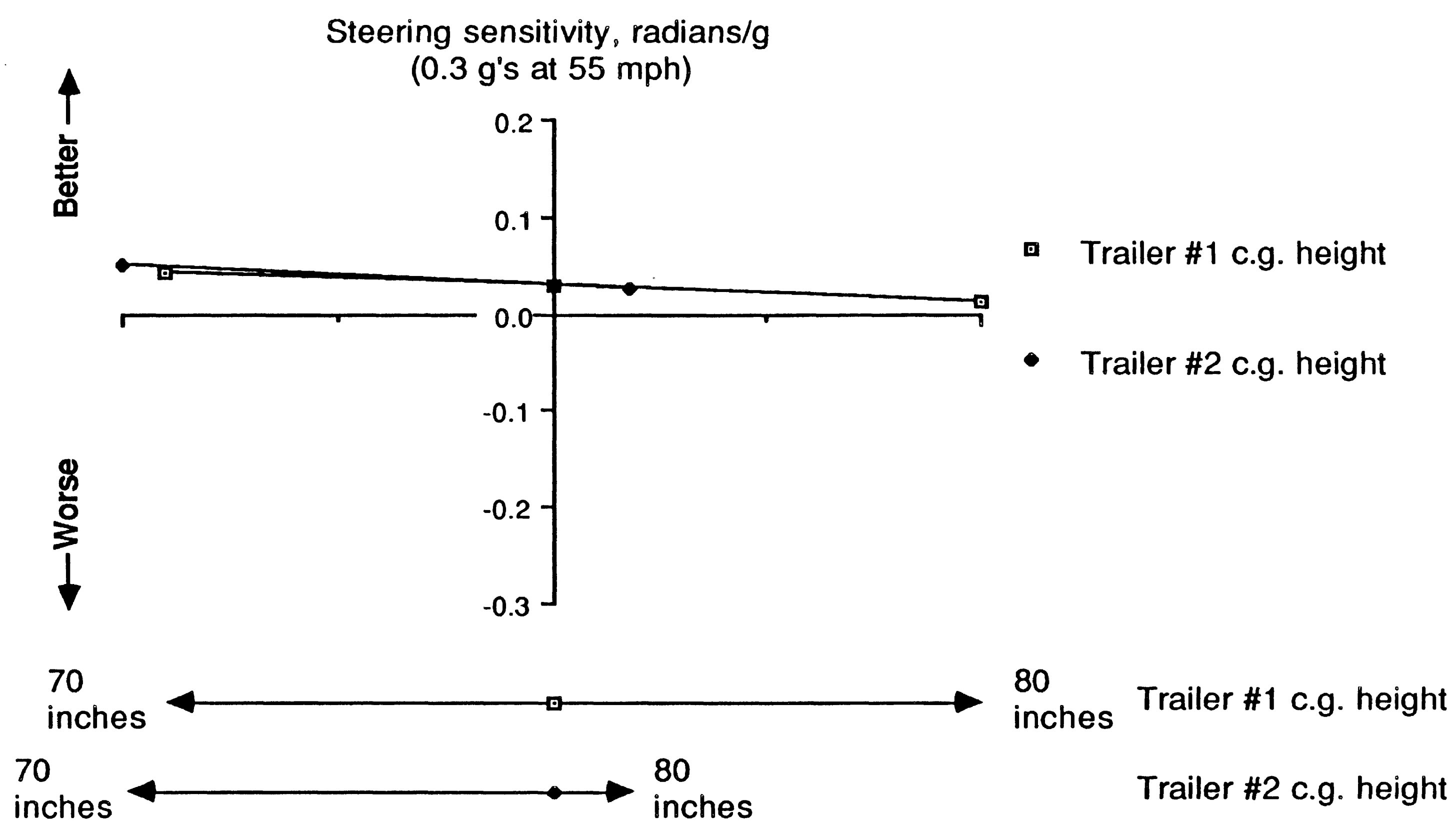


Figure 130. Parameter sensitivity diagram: handling, C-train.

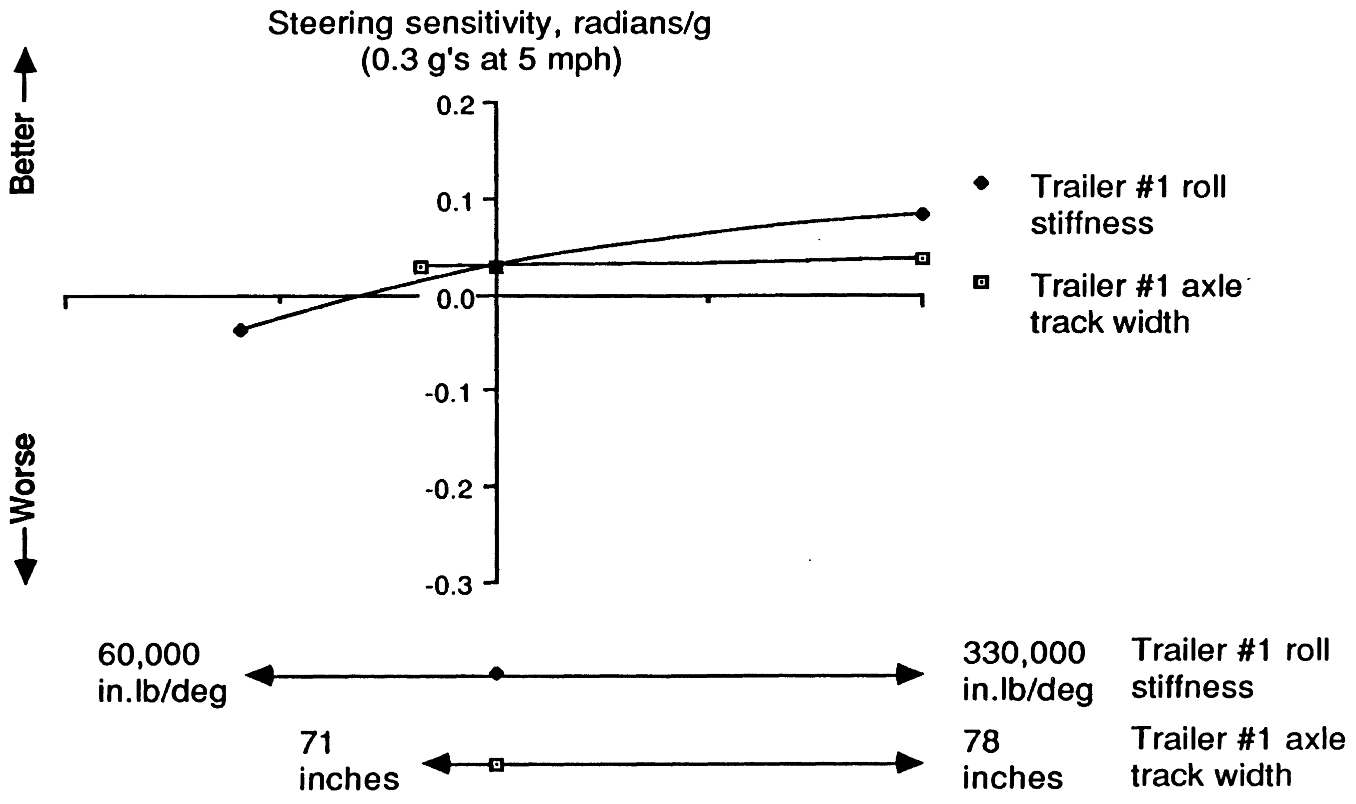


Figure 131. Parameter sensitivity diagram: handling, C-train.

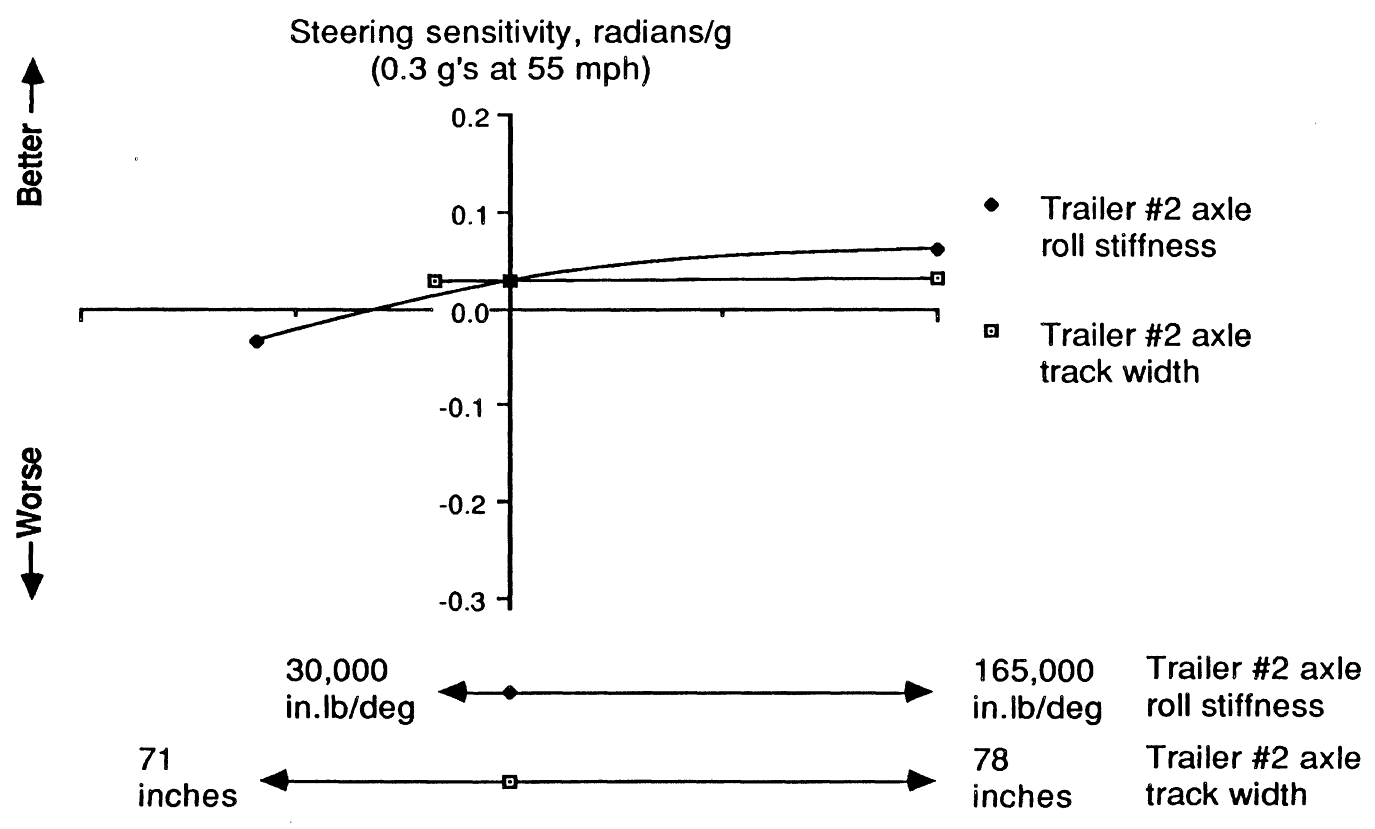


Figure 132. Parameter sensitivity diagram: handling, C-train.

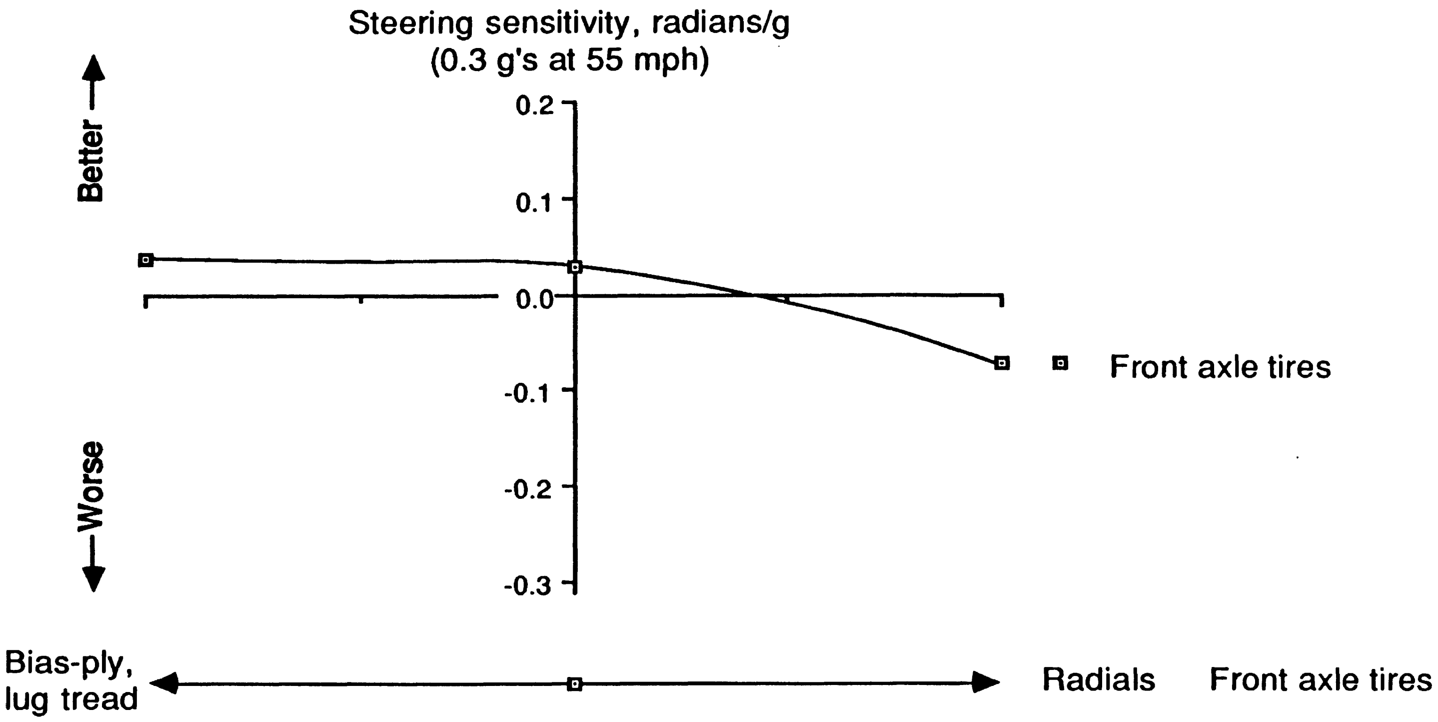


Figure 133. Parameter sensitivity diagram: handling, C-train.

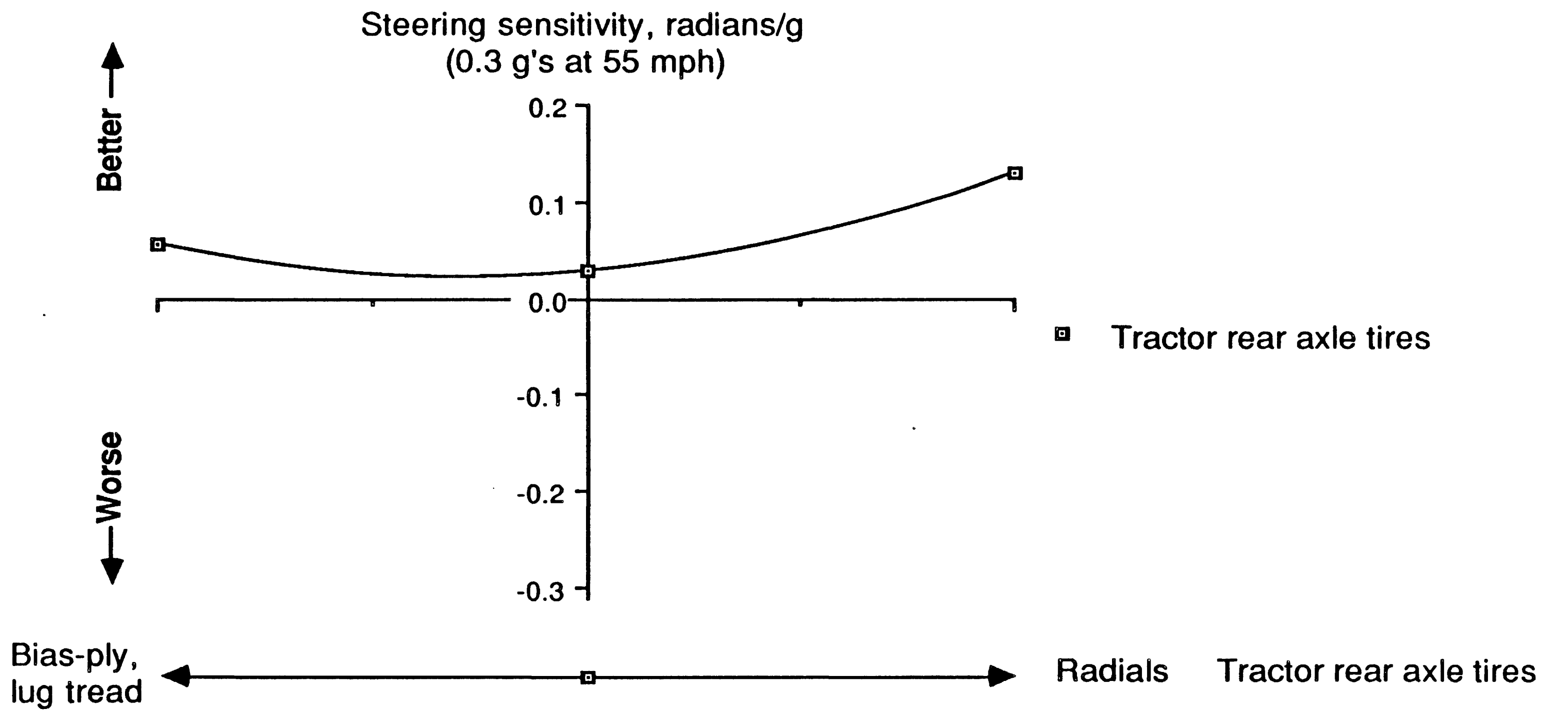


Figure 134. Parameter sensitivity diagram: handling, C-train.

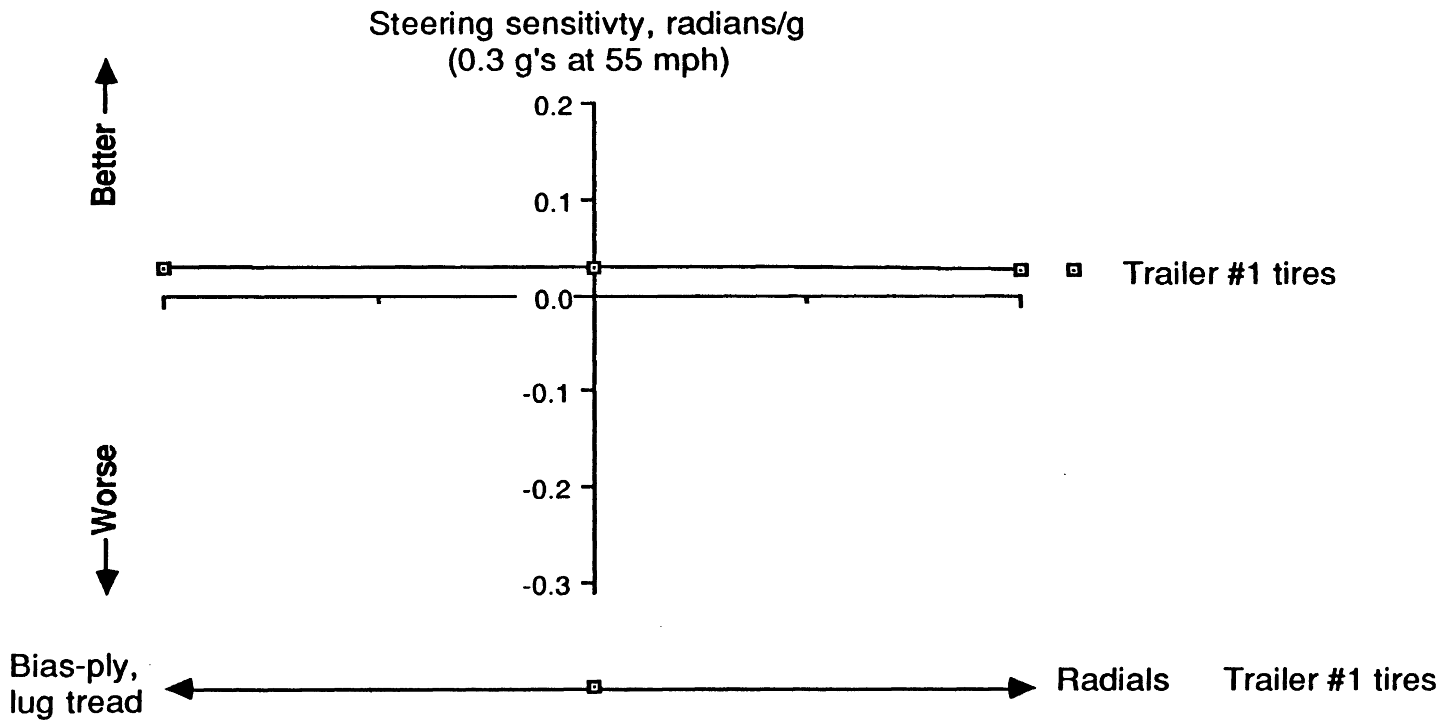


Figure 135. Parameter sensitivity diagram: handling, C-train.

6.4 Rearward Amplification

The performance signature for the C-train is presented in Figure 136 and it shows that the maximum amplification is just over 1.4 at a frequency of approximately 2 radians/sec. The benchmark B-train has much less amplification than the benchmark double, which is discussed in the next section.

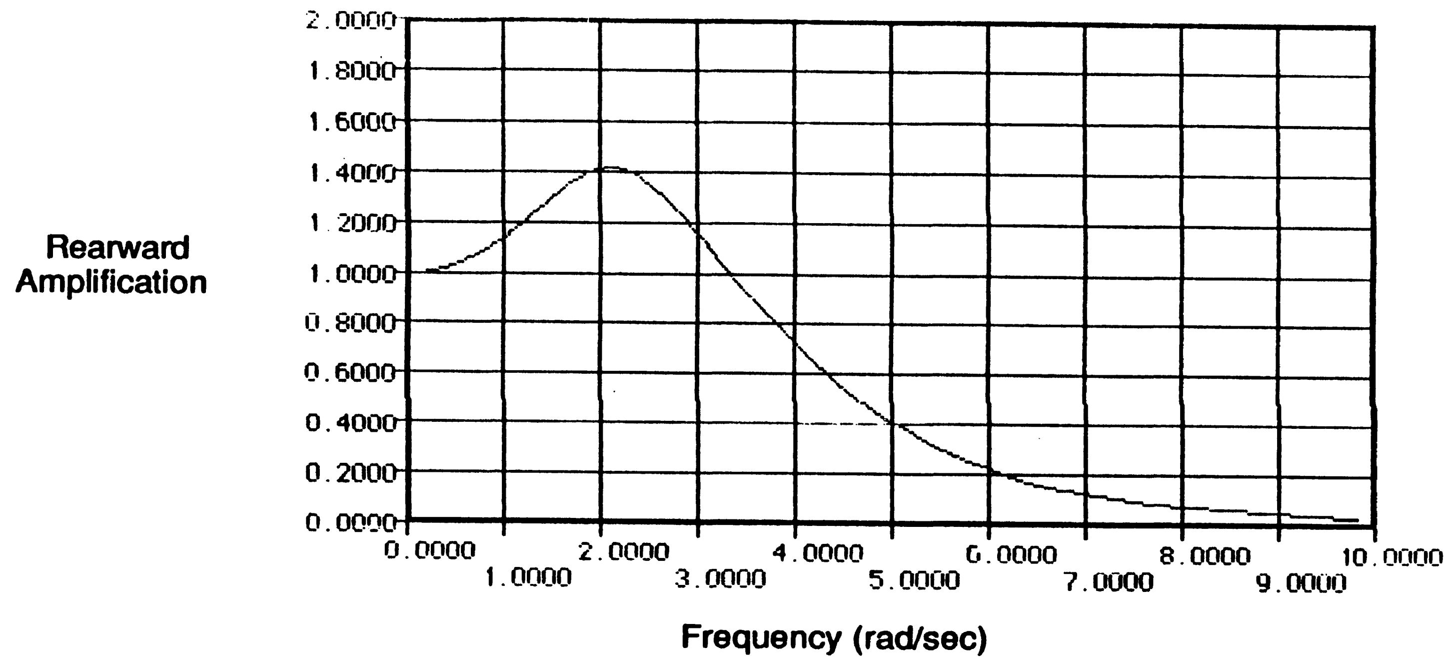


Figure 136. Performance signature, C-train, rearward amplification

7.0 DOUBLES

7.1 Baseline Values of Pertinent Mechanical Properties.

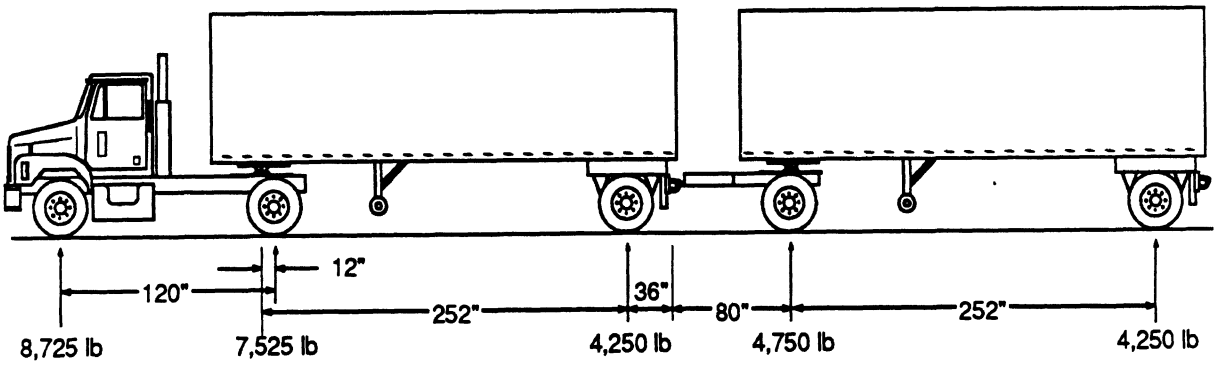
The benchmark double is patterned after the Western double that is currently legal throughout the interstate highway system in the United States. As shown in Figure 137, the vehicle consists of a tractor (the tractor might be better illustrated by a cab over style) and two identical semitrailers connected by a converter dolly. (Incidentally, the benchmark triple consists of this double plus another identical dolly and semitrailer. Much of this discussion applies to the triple, also.)

The loaded double weighs 80,000 lbs and the distance from the front axle to the rearmost axle is 60.67 feet. Pertinent details concerning axle loads and the geometric layout of the vehicle are illustrated in Figure 137. Additional information concerning the tires, brakes, steering, mass distribution, and suspensions are presented in Table 37.

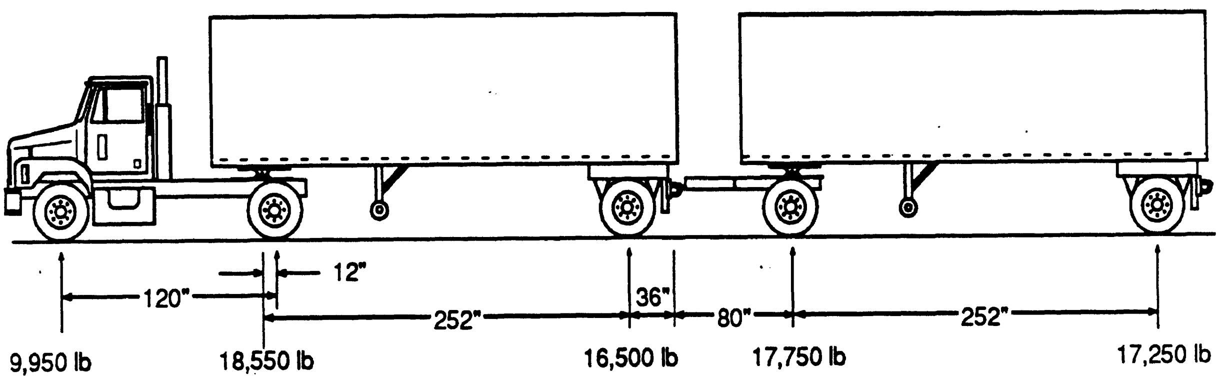
7.2 Low-Speed Cornering - Tractrices

Benchmark performance. The performance signature consists of the paths of the centers of the vehicle's five axles during a 41-foot-radius 90-degree turn (see Figure 138). The tractor's rear axle follows the path of the front axle with little offtracking because the tractor's wheelbase is relatively short compared to the wheelbases of the semitrailers. Since the dolly is short, the dolly axle closely tracks the path of the rear axle of the first semitrailer. Most of the offtracking is due to the lengths of the semitrailers. In this maneuver the maximum offtracking of the benchmark double is 11.56 feet. This compares to an offtracking of 14.36 feet for the benchmark tractor-semitrailer. Even though the double is longer than the 3S2, the additional articulation points make the maximum offtracking dimensions smaller for the double.

Parametric sensitivities. The mechanical properties of the full trailer have been varied to obtain the results presented here (see Table 38). Variations in tractor and semitrailer properties were studied in connection with the treatment of the 3S2. The results presented in Figure 139 show that the changes made in dolly dimensions are not large enough to significantly influence low-speed offtracking. As shown in Figure 140, the length of the wheelbase from hitch to rear axle on the second trailer has a significant effect on offtracking with an increase in length of 1.33 times resulting in an increase of over 2.8



EMPTY DOUBLE



LOADED DOUBLE

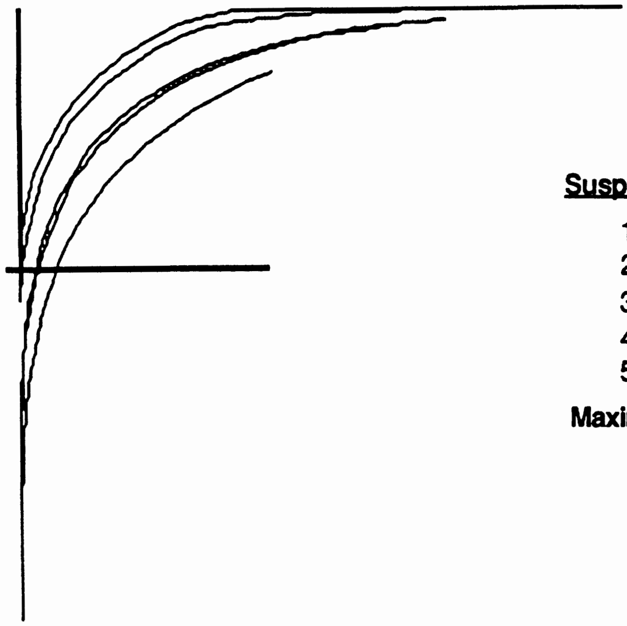
Figure 137. Geometric layout, double

Table 37. Basic Mechanical Properties (Doubles)

Mechanical Property	Empty	Loaded
Tractor		
Weight (lb)	14,000.00	14,000.00
Front axle brake gain (in.lb/psi)	2,000.00	2,000.00
Rear axle brake gain (in.lb/psi)	3,000.00	3,000.00
Front suspension cornering stiffness (lb/deg)	948.00	998.00
Rear suspension cornering stiffness (lb/deg)	1,152.00	1,945.00
Reduction in cornering stiffness due to steering system (%)	31.66	31.92
Front suspension roll stiffness (in.lb/deg)	21,000.00	21,000.00
Rear suspension roll stiffness (in.lb/deg)	70,000.00	70,000.00
Front suspension roll center height (in)	20.00	20.00
Rear suspension roll center height (in)	29.00	29.00
Height of the center of gravity (in)	37.50	37.50
Distance of the center of gravity from the front axle (in)	47.15	47.15
First Trailer		
Weight (lb)	6,500.00	31,800.00
Rear axle brake gain (in.lb/psi)	3,000.00	3,000.00
Rear suspension cornering stiffness (lb/deg)	789.00	1,847.00
Rear suspension roll stiffness (in.lb/deg)	80,000.00	80,000.00
Rear suspension roll center height (in)	29.00	29.00
Height of the center of gravity (in)	50.00	78.40
Distance of the center of gravity from front kingpin (in)	155.00	134.15
Converter Dolly		
Weight (lb)	2,500.00	2,500.00
Dolly axle brake gain (in.lb/psi)	3,000.00	3,000.00
Dolly suspension cornering stiffness (lb/deg)	848.00	1,847.00
Dolly suspension roll stiffness (in.lb/deg)	80,000.00	80,000.00
Dolly suspension roll center height (in)	29.00	29.00
Height of the center of gravity (in)	29.50	29.50
Distance of the center of gravity from pintle hitch (in)	0.00	0.00

Table 37. Basic Mechanical Properties (Doubles)

Mechanical Property	Empty	Loaded
Second Trailer		
Weight (lb)	6,500.00	32,500.00
Rear axle brake gain (in.lb/psi)	3,000.00	3,000.00
Rear suspension cornering stiffness (lb/deg)	790.00	1,885.00
Rear suspension roll stiffness (in.lb/deg)	80,000.00	80,000.00
Rear suspension roll center height (in)	29.00	29.00
Height of the center of gravity (in)	50.00	78.50
Distance of the center of gravity from kingpin (in)	155.00	133.75



MINIMUM TRACK

Susp. No.	Radius (ft)	Angle (deg)
1	41.00	0.00
2	39.65	75.32
3	34.43	63.93
4	33.94	61.92
5	29.44	56.73

Maximum Offtracking =11.56 ft.

Figure 138. Tractrices, double, 41 foot turn

Table 38. Double - Low Speed Offtracking

Parameter Name	Benchmark Value	Minimum Value	Maximum Value	Notes
Dolly's wheelbase	80 inches	72 inches	96 inches	
Second trailer wheelbase	252 inches	240 inches	336 inches	

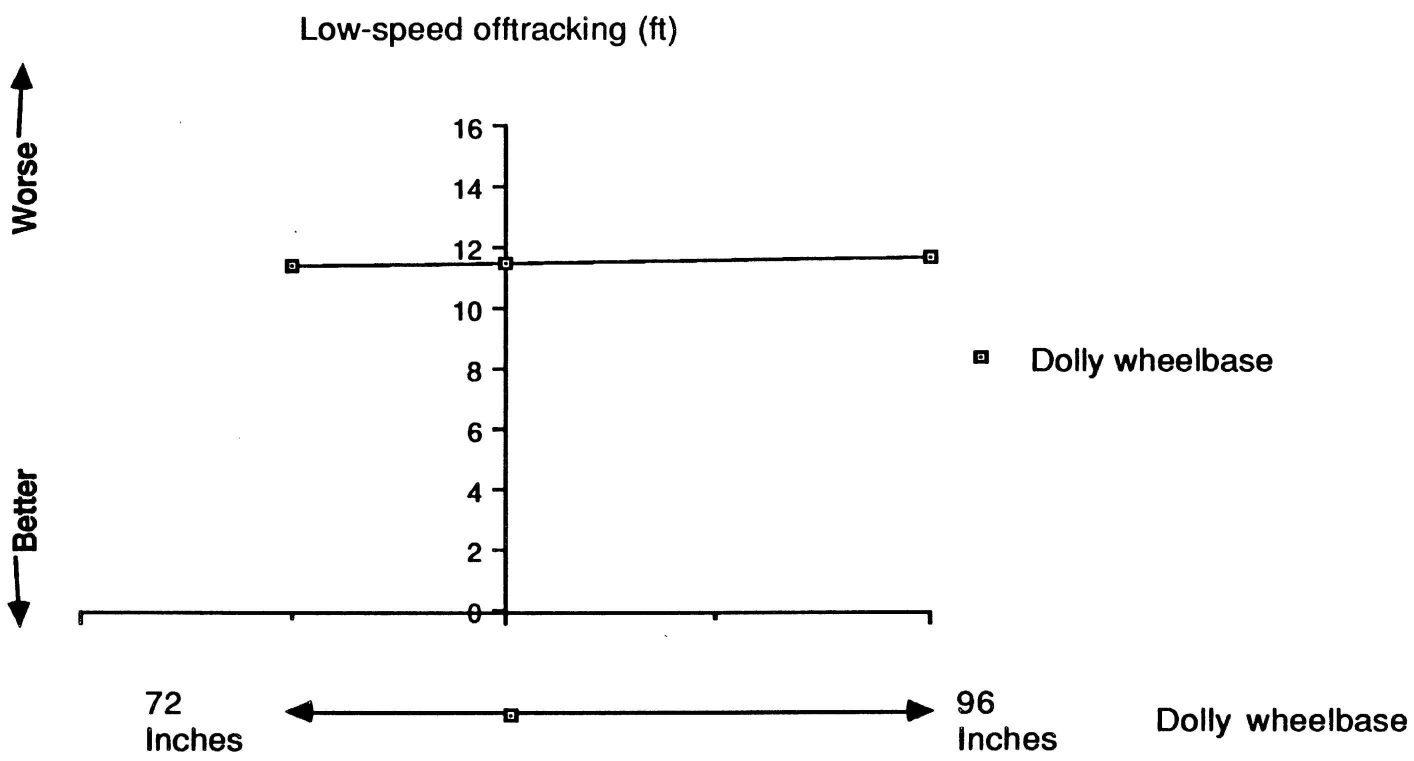


Figure 139. Parameter sensitivity diagram: low-speed offtracking, double.

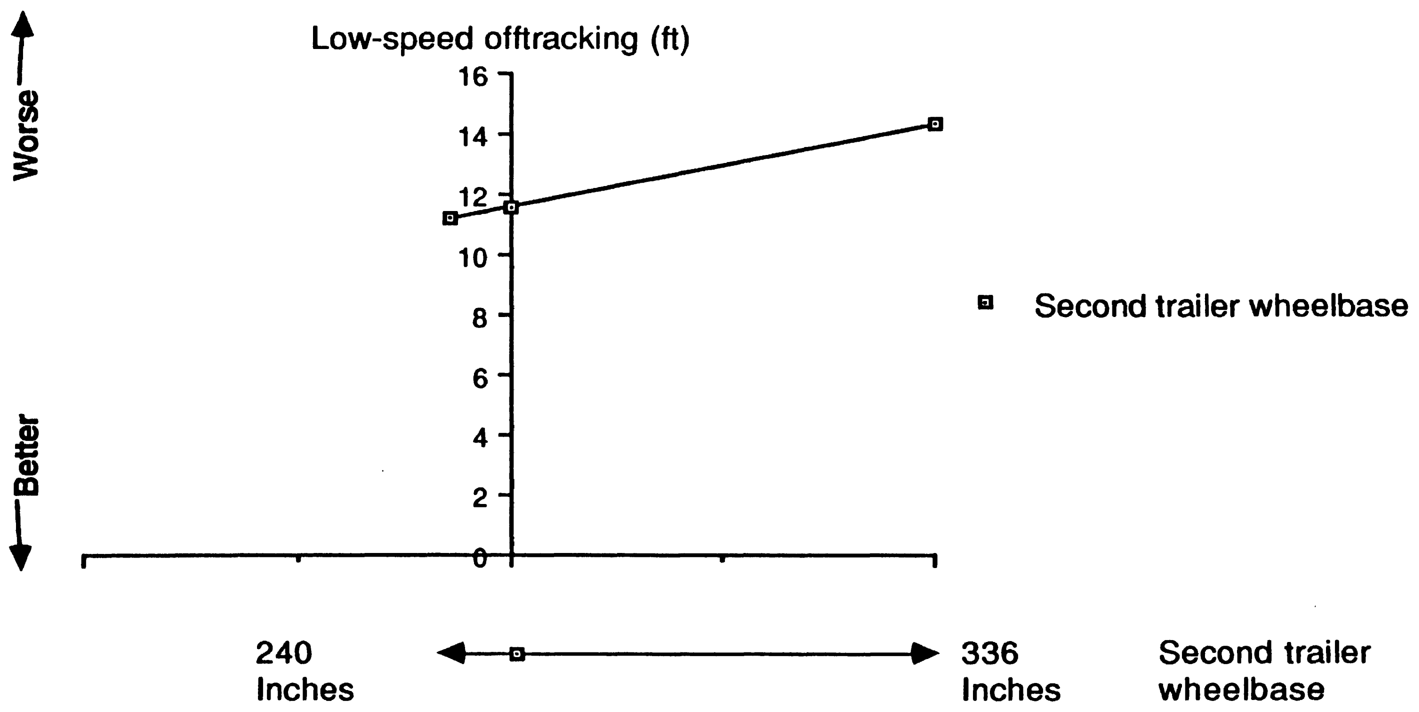


Figure 140. Parameter sensitivity diagram: low-speed offtracking, double.

feet in low-speed offtracking. A similar increase in low-speed offtracking would be obtained if the wheelbase of the first semitrailer were to be increased by 1.33 times.

7.3 Constant Deceleration Braking

Benchmark performance. Performance signatures for the loaded and empty double are presented in Figures 141 and 142. This double employs a converter dolly which means that during braking vertical load is transferred from the dolly to the rear of the first semitrailer. Since the dolly is short, the amount of load transfer is large enough to make the friction utilization of the third axle (the rear axle of the first semitrailer) less than that of the dolly axle (#4) when the vehicle is loaded. Nevertheless, the rearmost axle (#5) is the axle requiring the greatest friction utilization in either the loaded or empty condition, and hence, it is the axle controlling braking efficiencies.

For the loaded vehicle, the braking efficiencies are 0.84 at 0.4 g and 0.92 at 0.2 g. When the vehicle is empty, the efficiencies are 0.59 at 0.4 g and 0.63 at 0.2 g. These efficiencies are in line with those typically found on heavy trucks in this country and they show that the braking efficiency of the empty double is poor.

Parametric sensitivities. Table 39 only lists three variations for studying the braking performance of this type of double. Three variations are all that are needed because the rearmost axle on the vehicle is usually the controlling axle. At a given level of treadle pressure, the primary parameters that affect the load on the rearmost axle are the c.g. height and wheelbase of the rear semitrailer. Nevertheless, any change in the gain of one of the brakes will change brake proportioning, thereby changing braking efficiency. In this case the front brake gain is increased because this will improve the braking performance of the empty vehicle. Expected variations in the geometric and inertial properties of the other units will not change the braking efficiency because they would not be large enough to cause another axle to become the controlling axle.

Results for the loaded and empty vehicle at 0.2 and 0.4 g are given in Figures 143 through 146. In the case where the loaded vehicle is decelerating at 0.2 g, the maximum increase in front brake gain leads to a sudden loss in efficiency. This is because the level of front braking becomes large enough to make the front axle the controlling axle. Under these circumstances, the front axle load is low enough to allow the front wheels to lock prematurely with a decrease in efficiency. At 0.4 g this change in controlling axle does not occur. Improvements in efficiency accompany (1) increases in the front brake gain, (2) lowering of the last trailer's c.g. height, and (3) increasing the wheelbase of the last trailer.

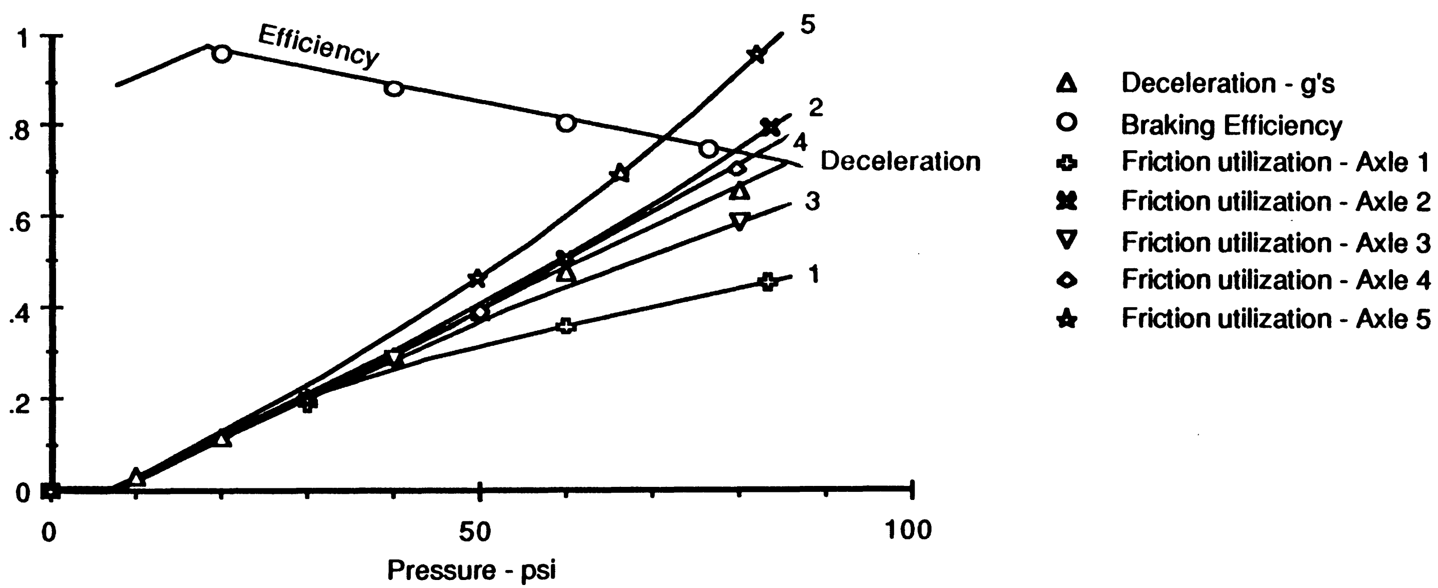


Figure 141. Performance signature, braking, loaded double

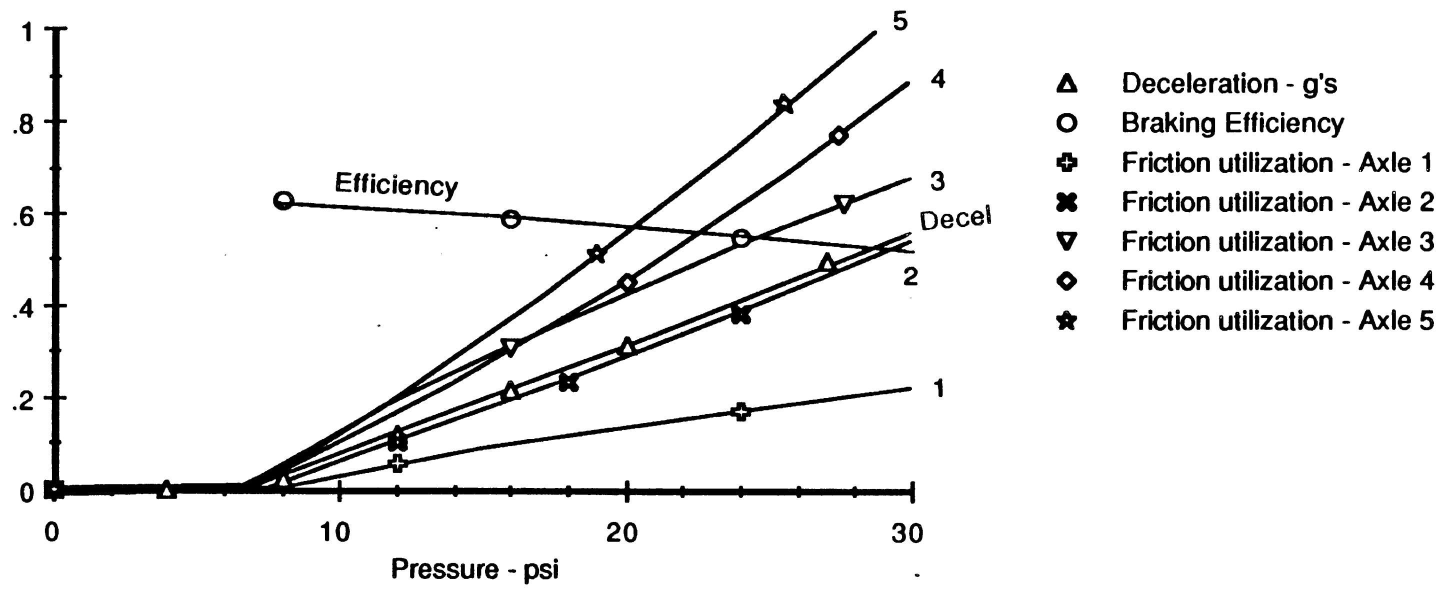


Figure 142. Performance signature, braking, empty double

Table 39. Double - Braking

Parameter Name	Benchmark Value	Minimum Value	Maximum Value	Notes
Front brake gain	2000 in.lb/psi	2000 in.lb/psi	3000 in.lb/psi	Both brakes combined
Second trailer wheelbase	252 inches	240 inches	336 inches	
*Second trailer c.g. height	78.5 inches	60 inches	105 inches	

* For the loaded vehicle

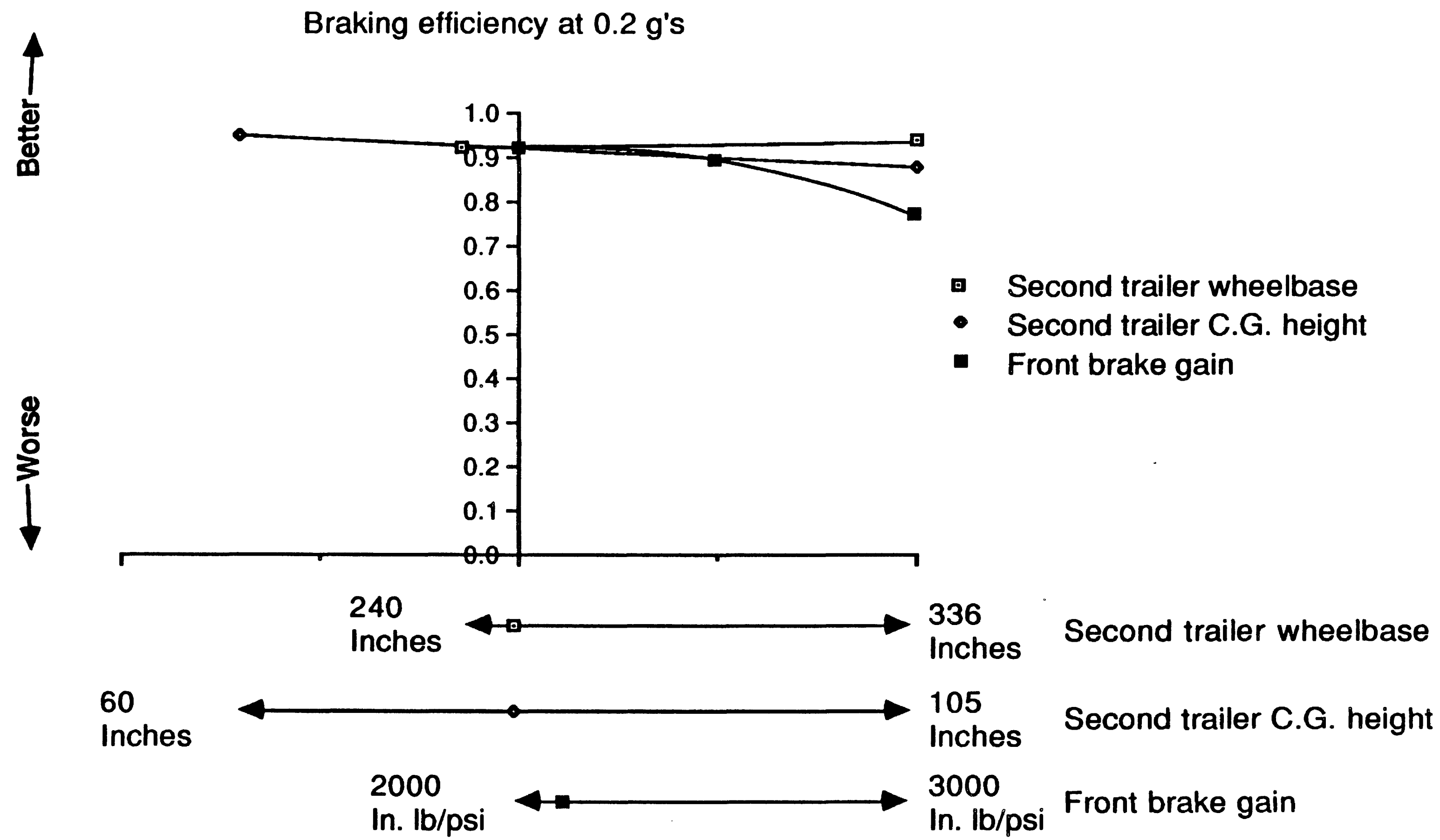


Figure 143. Parameter sensitivity diagram: braking, loaded double.

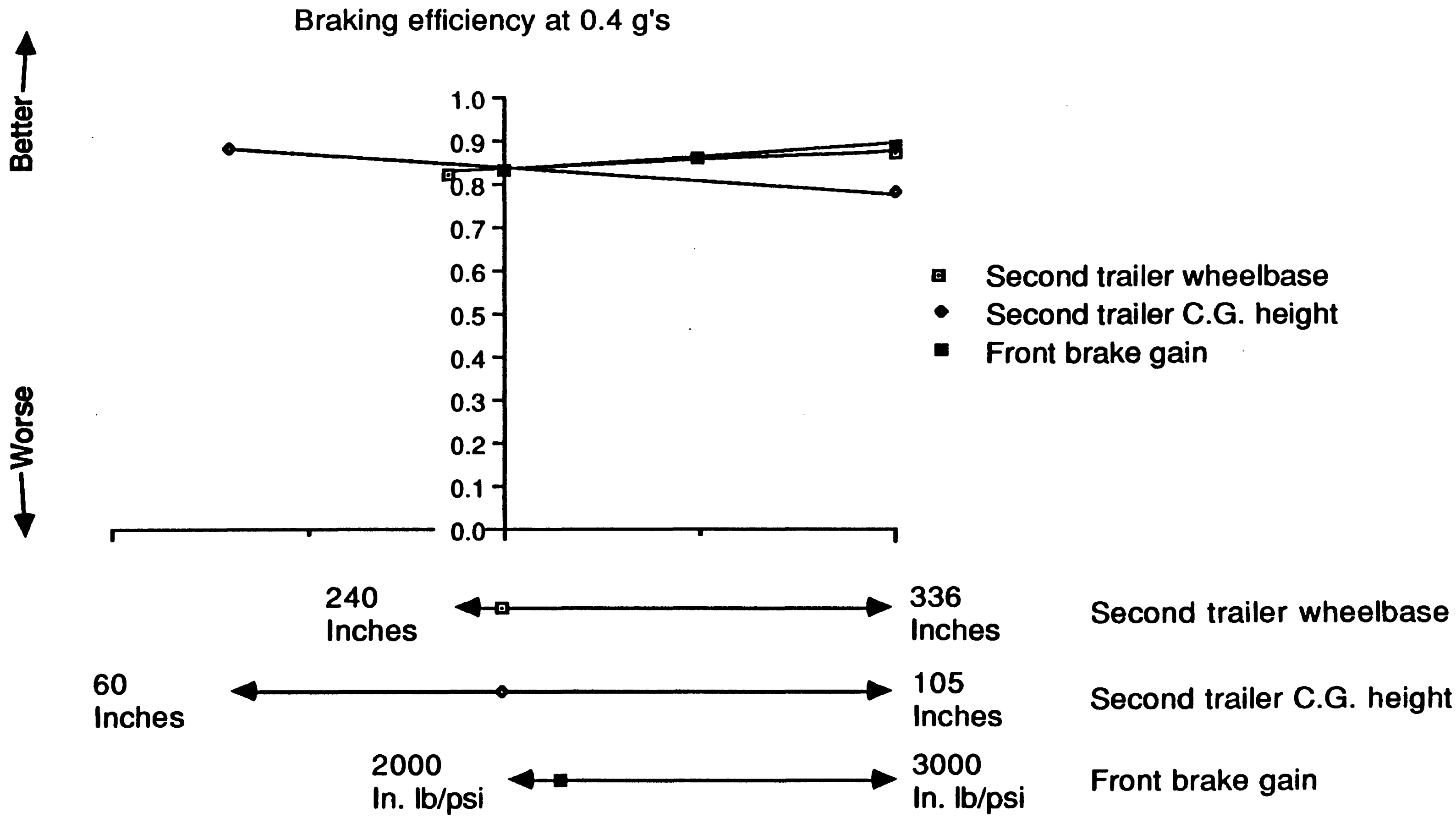


Figure 144. Parameter sensitivity diagram: braking, loaded double.

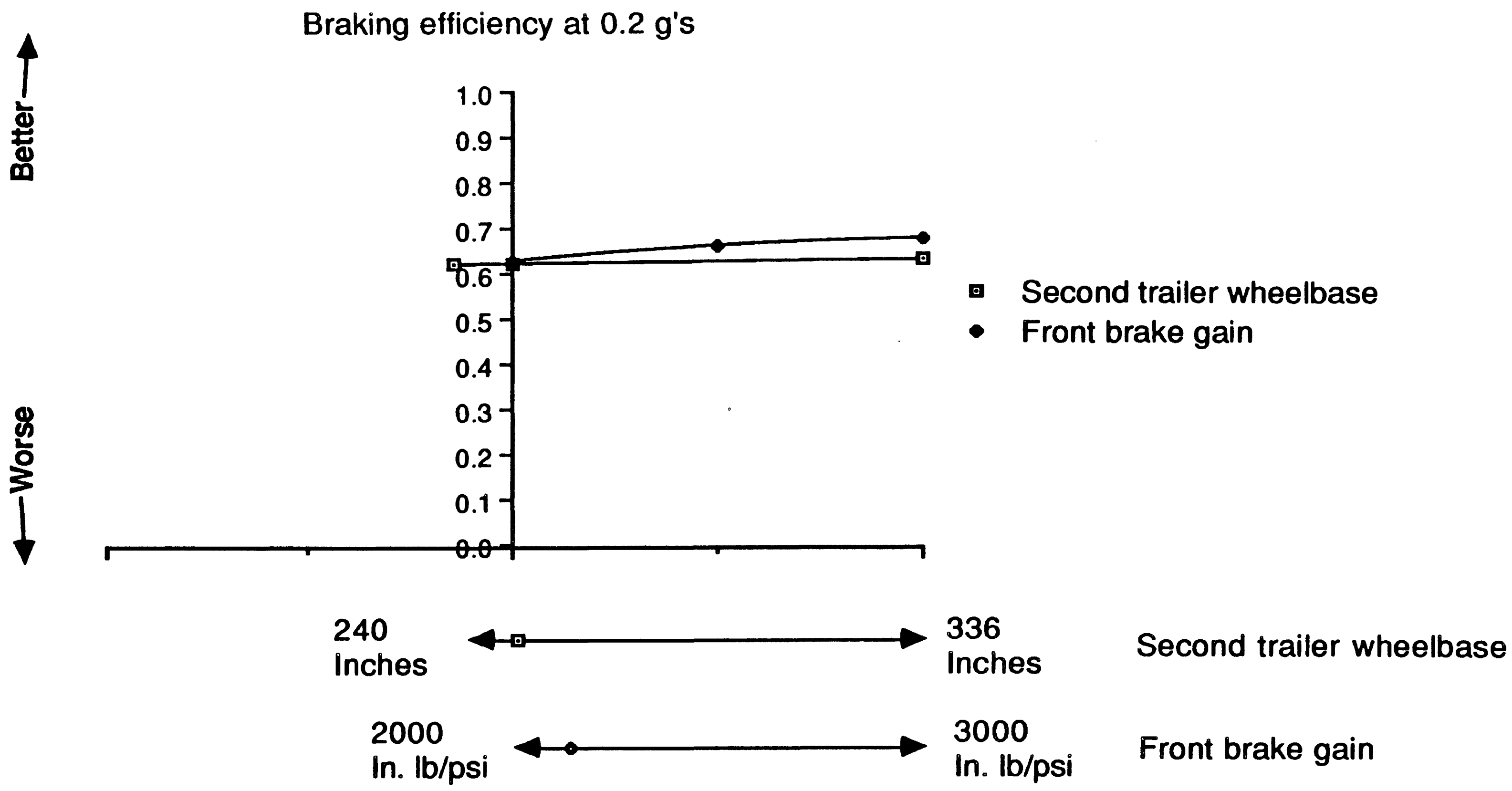


Figure 145. Parameter sensitivity diagram: braking, empty double.

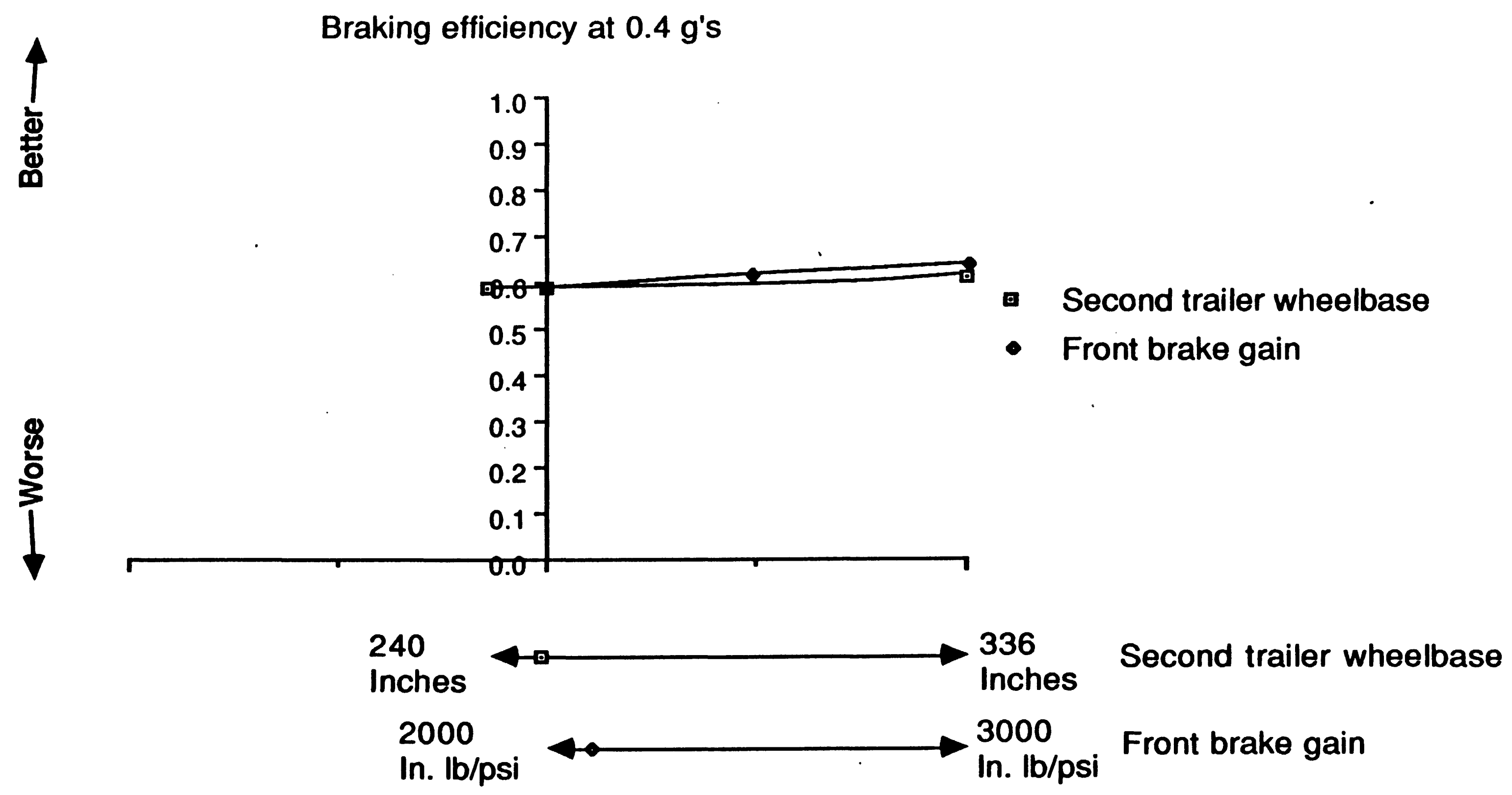


Figure 146. Parameter sensitivity diagram: braking, empty double.

The performance of the empty vehicle is particularly insensitive to likely changes in vehicle properties. Advance braking systems employing load sensing proportioning or antilock control are probably needed if braking performance is to be improved.

7.4 High-Speed Offtracking (Steady Turn - Tracking)

Benchmark performance. The offtracking at various locations along the centerline of the benchmark double are given in Table 40 for this vehicle traveling at 55 mph on a turn with a radius of 1,200 feet. The offtracking of the rearmost axle is 1.21 feet toward the outside of the turn. If the vehicle were traveling at 38 mph on a 600-foot turn, the offtracking of the rearmost axle would be 0.71 feet.

Parametric sensitivities. The influences of changes in dolly and last trailer properties (see Table 41) are illustrated in Figures 147 through 150. The results show that on either the 1,200- or the 600-foot turn, the cornering stiffnesses of the tires installed on the last axle have a greater influence on high-speed offtracking than the influences of any of the other properties examined. As observed for other vehicles, an increase in cornering stiffness always reduces high-speed offtracking.

7.5 Steady Turn - Roll

Benchmark performance. With regard to roll, the tractor and semitrailer perform as one unit and the full trailer performs as an independent unit. This is because the pintle hitch does not transmit roll moments between the first semitrailer and the dolly. Accordingly, the performance signature for the double consists of a graph for the tractor-semitrailer, Figure 151, and a graph, Figure 152, for the full trailer.

As seen by examining Figures 151 and 152, the rollover threshold of the tractor-semitrailer portion of the double is higher than the rollover threshold of the full trailer. The rollover threshold of the tractor-semitrailer is 0.408 g and the rollover threshold of the full trailer is 0.394 g. The axle on the first semitrailer is a "stiff" axle. This axle is the first axle to lift on the tractor-semitrailer portion. The tractor-semitrailer portion does not roll over until the tractor's rear axle lifts off.

Parametric sensitivities. Pertinent properties of each unit have been varied in this case (see Table 42). Since the front 2S2 portion of the benchmark double (which is coded as a 2S2-2 vehicle) has the higher rollover threshold, variations in 2S2 properties would not indicate any influence on the rollover threshold unless they made the 2S2 roll over

Table 40. High-Speed Offtracking Performance, Double

Simple Models:O.Double

Velocity = 80.6667 ft/sec
Radius of the turn = 1200 ft

Suspension #	Rear Articulation Point on Unit #	Zero Speed Radius	High Speed Radius
1		1200.00	1200.00
2		1199.95	1200.25
	1	1199.95	1200.23
3		1199.77	1200.61
	2	1199.77	1200.70
4		1199.75	1200.86
	3	1199.75	1200.86
5		1199.57	1201.21
	4	1199.57	1201.29

Zero Speed Offtracking = 0.43 ft
High Speed Offtracking = -1.29 ft

Table 41. Double - High-Speed Offtracking

Parameter Name	Benchmark Value	Minimum Value	Maximum Value	Notes
Dolly's wheelbase	80 inches	72 inches	96 inches	
Dolly's cornering stiffness	1909.1 lb/deg	954.55 lb/deg	2863.65 lb/deg	Cornering stiffness is determined from loaded vehicle data
Second trailer wheelbase	252 inches	240 inches	336 inches	
*Second trailer's cornering stiffness	1885.22 lb/deg	942.61 lb/deg	2827.83 lb/deg	Cornering stiffness is determined from loaded vehicle data

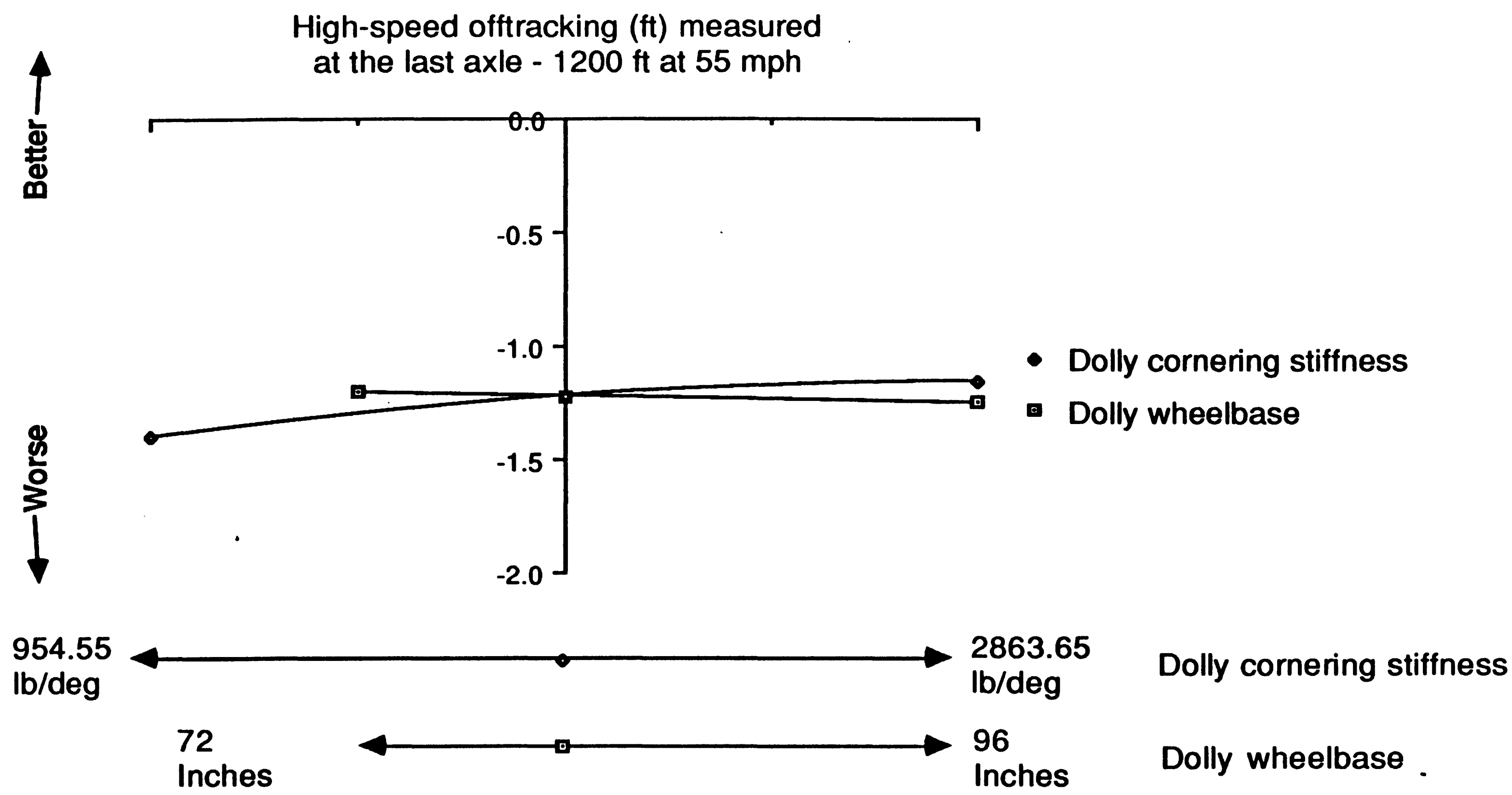


Figure 147. Parameter sensitivity diagram: high-speed offtracking, double.

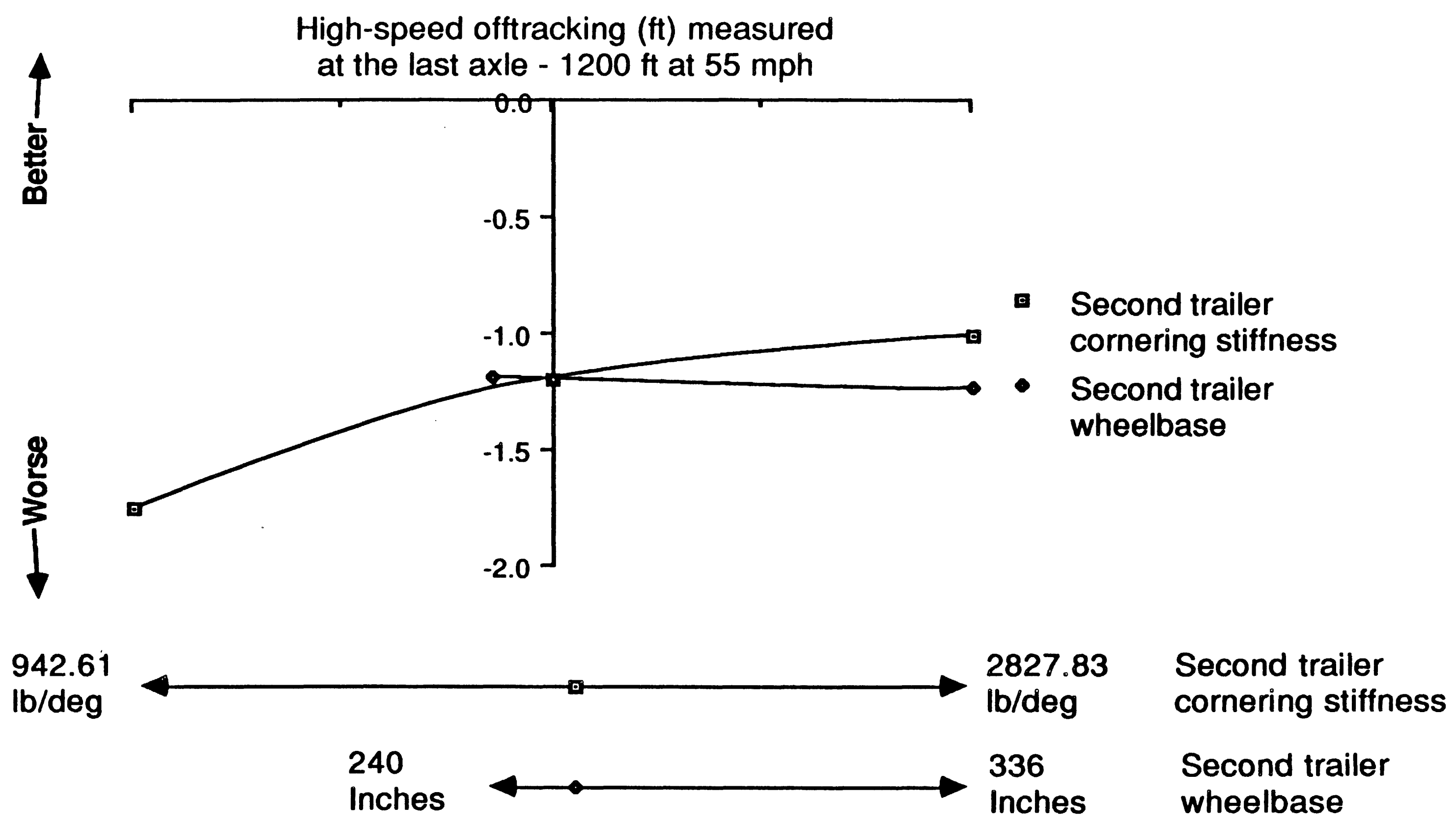


Figure 148. Parameter sensitivity diagram: high-speed offtracking, double.

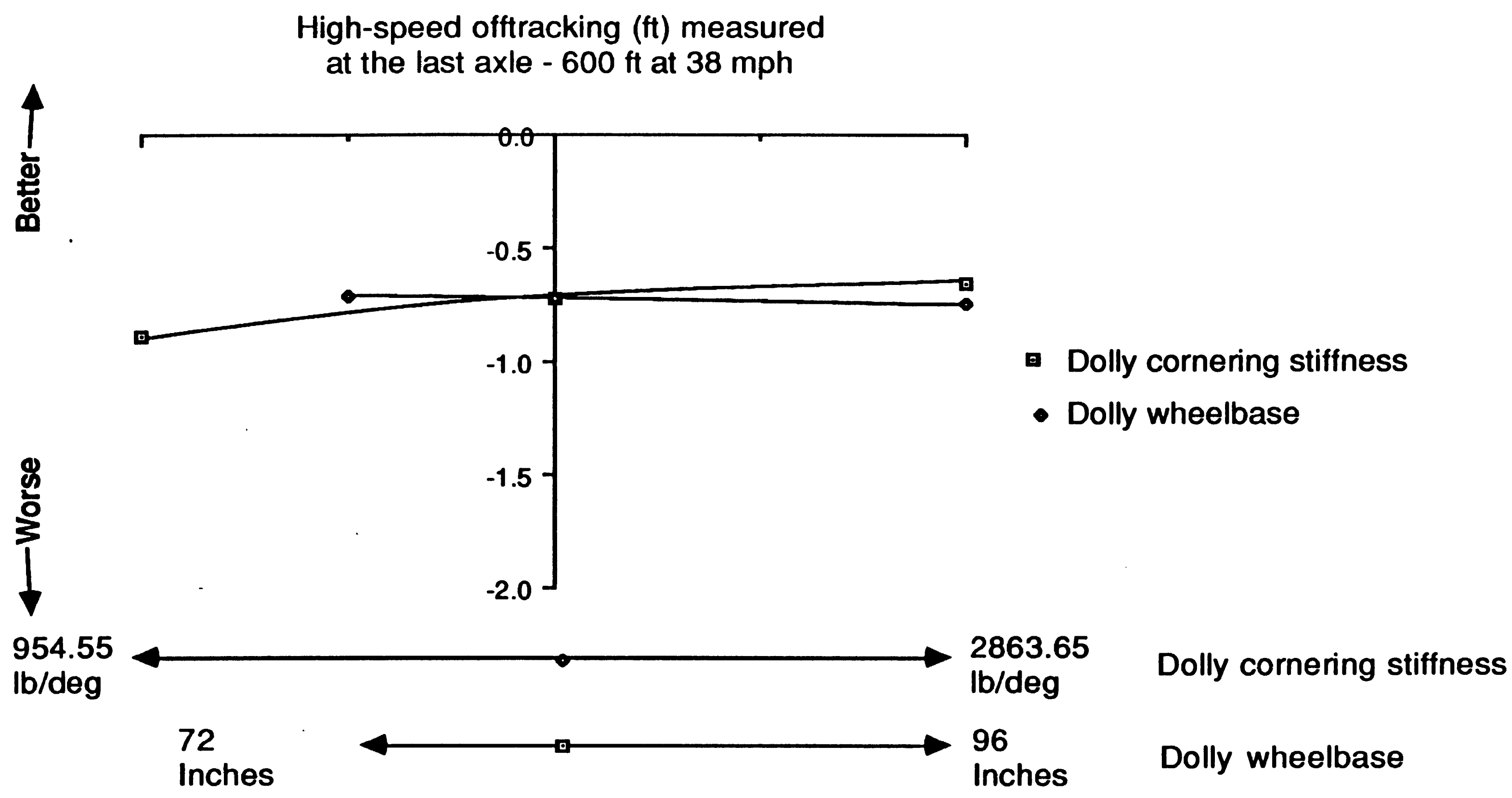


Figure 149. Parameter sensitivity diagram: high-speed offtracking, double.

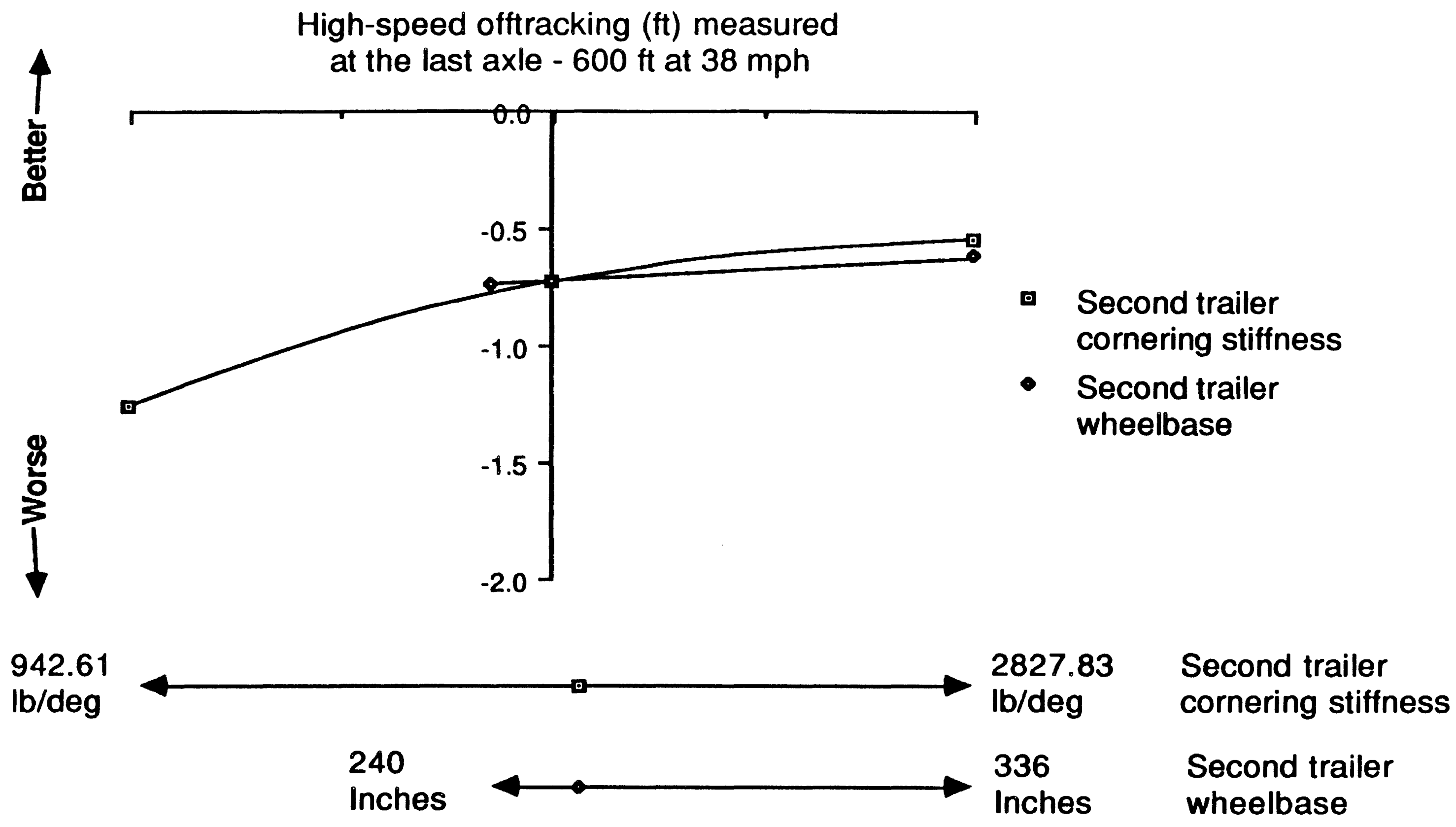


Figure 150. Parameter sensitivity diagram: high-speed offtracking, double.

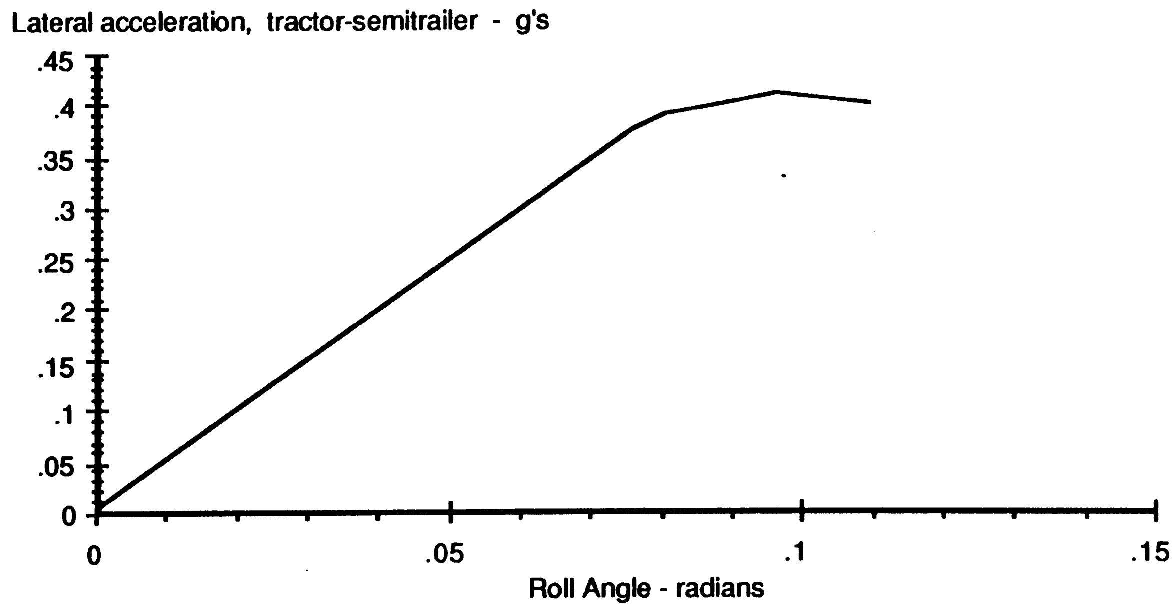


Figure 151. Performance signature, static roll, double

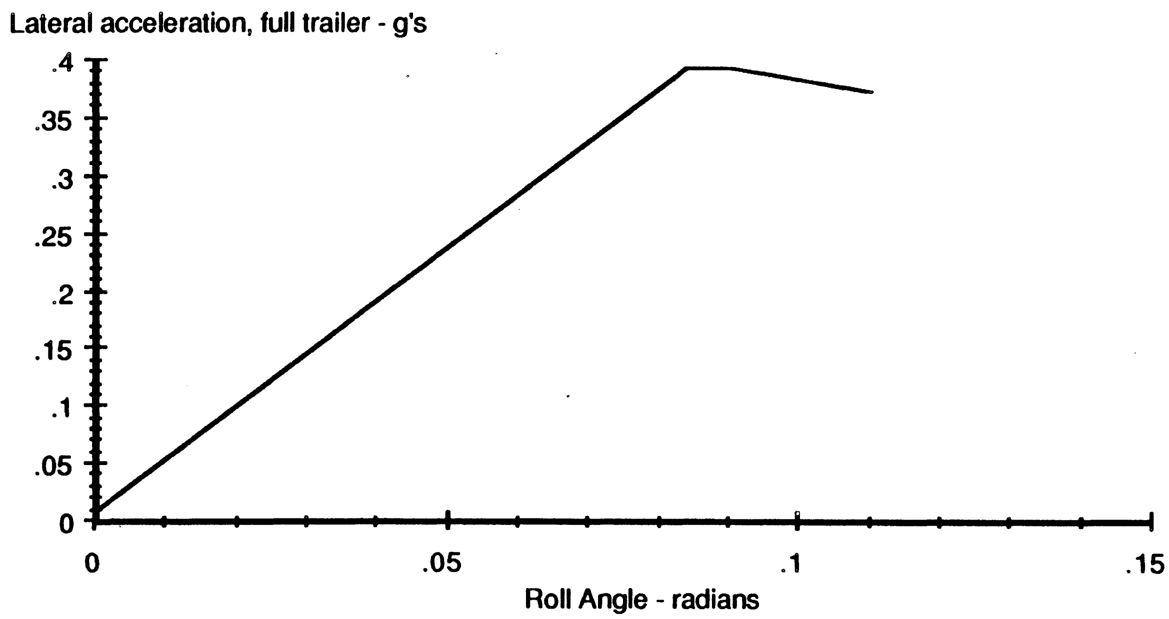


Figure 152. Performance signature, static roll, double

Table 42. Double - Static Roll

Parameter Name	Benchmark Value	Minimum Value	Maximum Value	Notes
Trailer #1 c.g. height	78.4 inches	70 inches	80 inches	Much larger deviations used for braking
Trailer #2 c.g. height	78.5 inches	70 inches	80 inches	Much larger deviations used for braking
Rear axle roll center heights	29 inches	21 inches	31 inches	
Front axle roll stiffness	21,000 in.lb/deg	17,000 in.lb/deg	25,000 in.lb/deg	
Front axle track width	80 inches	77 inches	81 inches	
Tractor rear suspension roll stiffness	70,000 in.lb/deg	30,000 in.lb/deg	165,000 in.lb/deg	
Tractor rear axle track widths	72 inches	71 inches	78 inches	
Trailer #1 axle roll stiffness	80,000 in.lb/deg	30,000 in.lb/deg	165,000 in.lb/deg	
Trailer #1 axle track width	72 inches	71 inches	78 inches	
Dolly axle roll stiffness	80,000 in.lb/deg	30,000 in.lb/deg	165,000 in.lb/deg	
Dolly axle track width	72 inches	71 inches	78 inches	
Trailer #2 axle roll stiffness	80,000 in.lb/deg	30,000 in.lb/deg	165,000 in.lb/deg	
Trailer #2 axle track width	72 inches	71 inches	78 inches	

before the full trailer. For example, lowering the c.g. height of the second trailer increases the rollover threshold, as indicated in Figure 153, to the level of the rollover threshold of the 2S2 — (increasing the rollover threshold of the full trailer beyond that of the 2S2 does not provide any help to the 2S2 and visa versa). In this case the 2S2 rolls over first, since the rollover threshold of the full trailer is now greater than that of the 2S2.

The results presented in Figures 153 through 158 do not illustrate any new concepts. As long as one keeps track of which unit is rolling over first, then basic notions apply. For example, widening axle track is always beneficial and it does not help to increase the roll stiffness of a stiff axle. Results pertaining to roll stiffnesses and track widths of each axle appear in Figures 154 through 158. In general, with the exception of the front axle, the roll stiffness distribution of this vehicle is well arranged such that the influences of changes in the individual properties of any one suspension will have minimal effect on overall performance unless the roll stiffness of an axle is greatly reduced.

Logically, one needs to improve both the 2S2 and the full trailer to improve the overall rollover threshold, but one can lower the rollover threshold by degrading the performance of either the 2S2 or the full trailer.

7.6 Steady Turn - Handling

Benchmark performance. The negative slope in the handling signature presented in Figure 159 shows that the benchmark double will become unstable before any wheels lift off. The stability boundary in the space defined by lateral acceleration and velocity is given in Figure 160. At 100 mph, the vehicle would become yaw divergent at slightly above 0.3 g. Since the vehicle is stable at 0.3g and 55 mph, the steering sensitivity at this operating condition is used as the baseline in the following performance sensitivity diagrams. This baseline value is 0.034 radians/g.

Parametric sensitivities. Handling performance is practically independent of the properties of the second trailer because the pintle hitch effectively decouples the 2S2 portion from the full trailer with respect to roll moments and side forces. Tractor and first semitrailer properties are varied to examine the sensitivity of handling performance to changes in vehicle properties (see Table 43).

As shown in Figure 161, the location of the tractor's fifth wheel has a large influence on handling (as long as the vertical loads on the tractor's tires are kept the same). As indicated in Figures 162 through 166, other significant changes are brought about by (a)

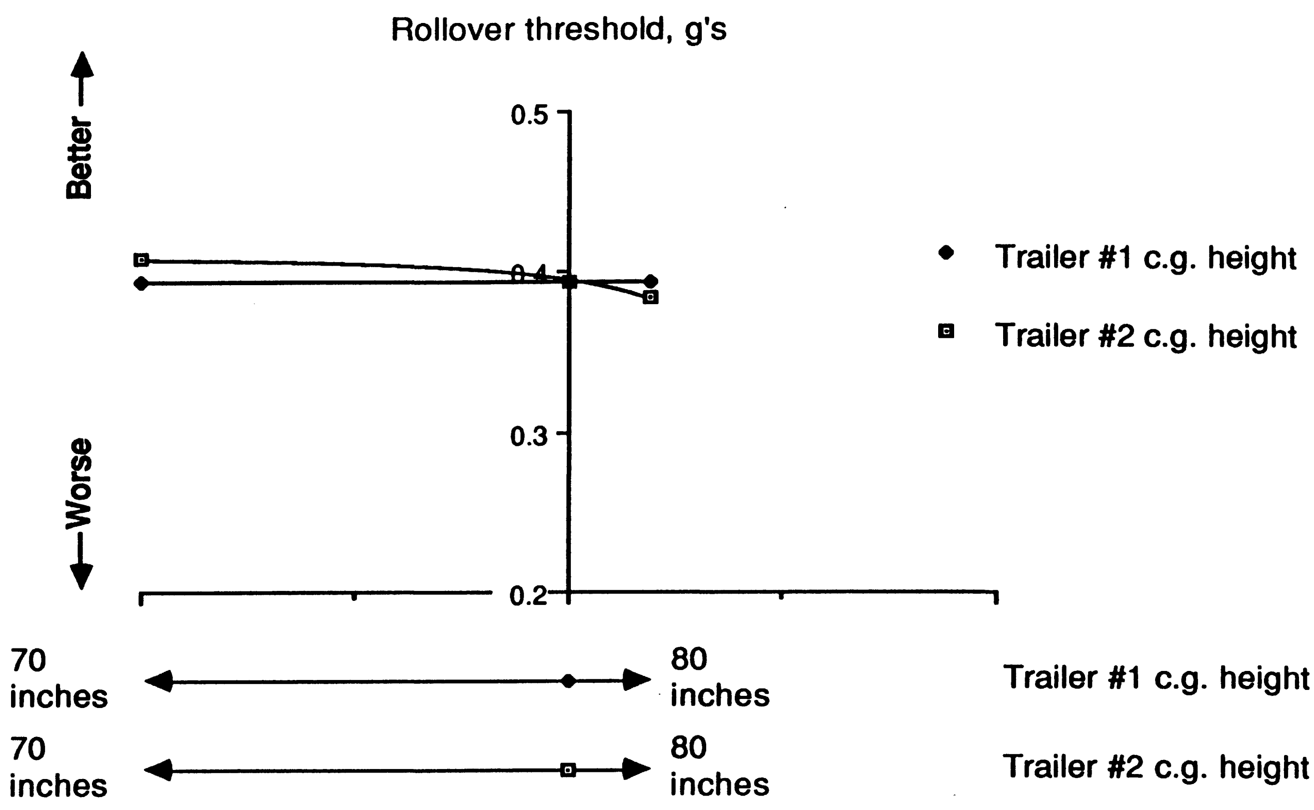


Figure 153. Parameter sensitivity diagram: static roll, double.

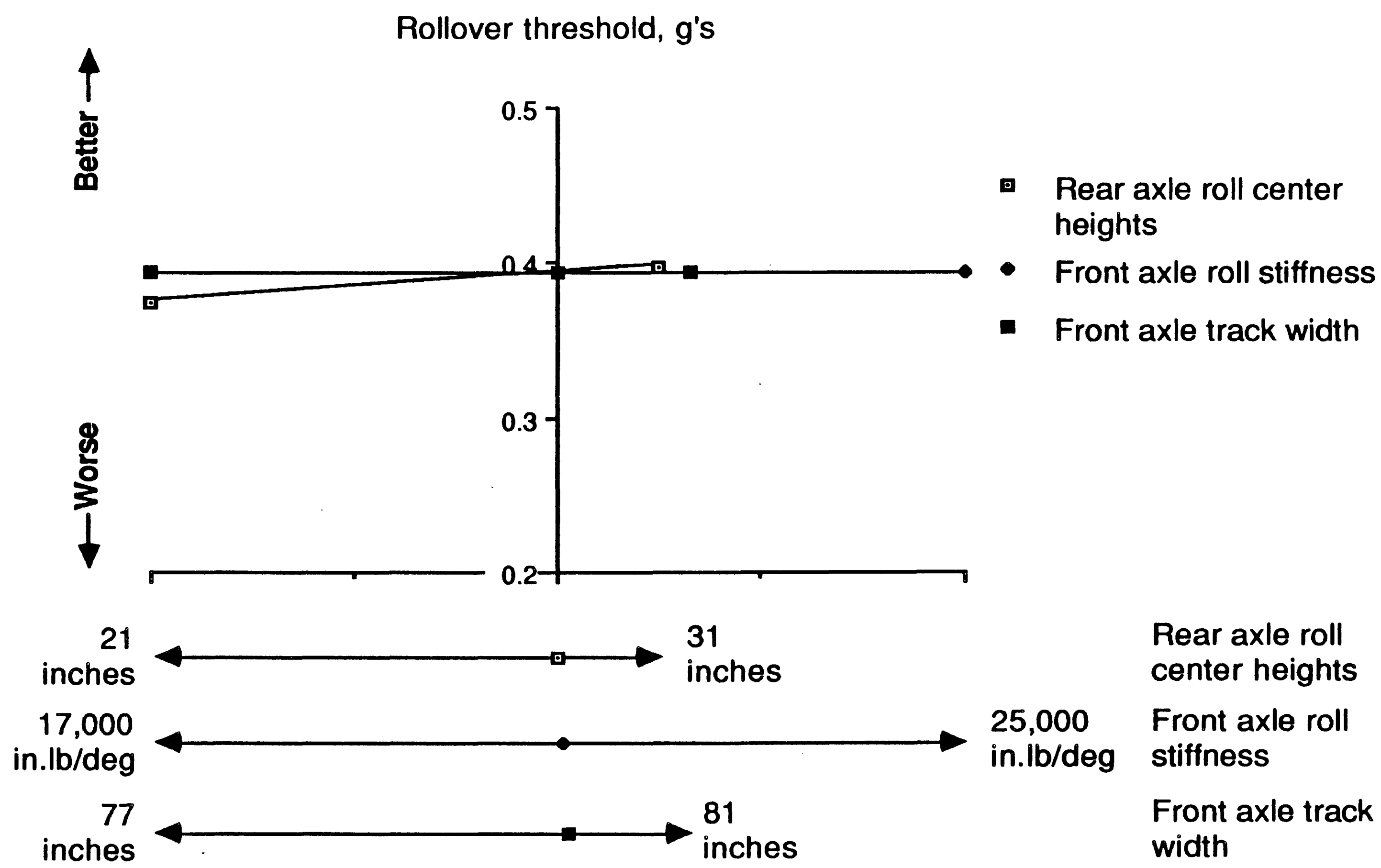


Figure 154. Parameter sensitivity diagram: static roll, double.

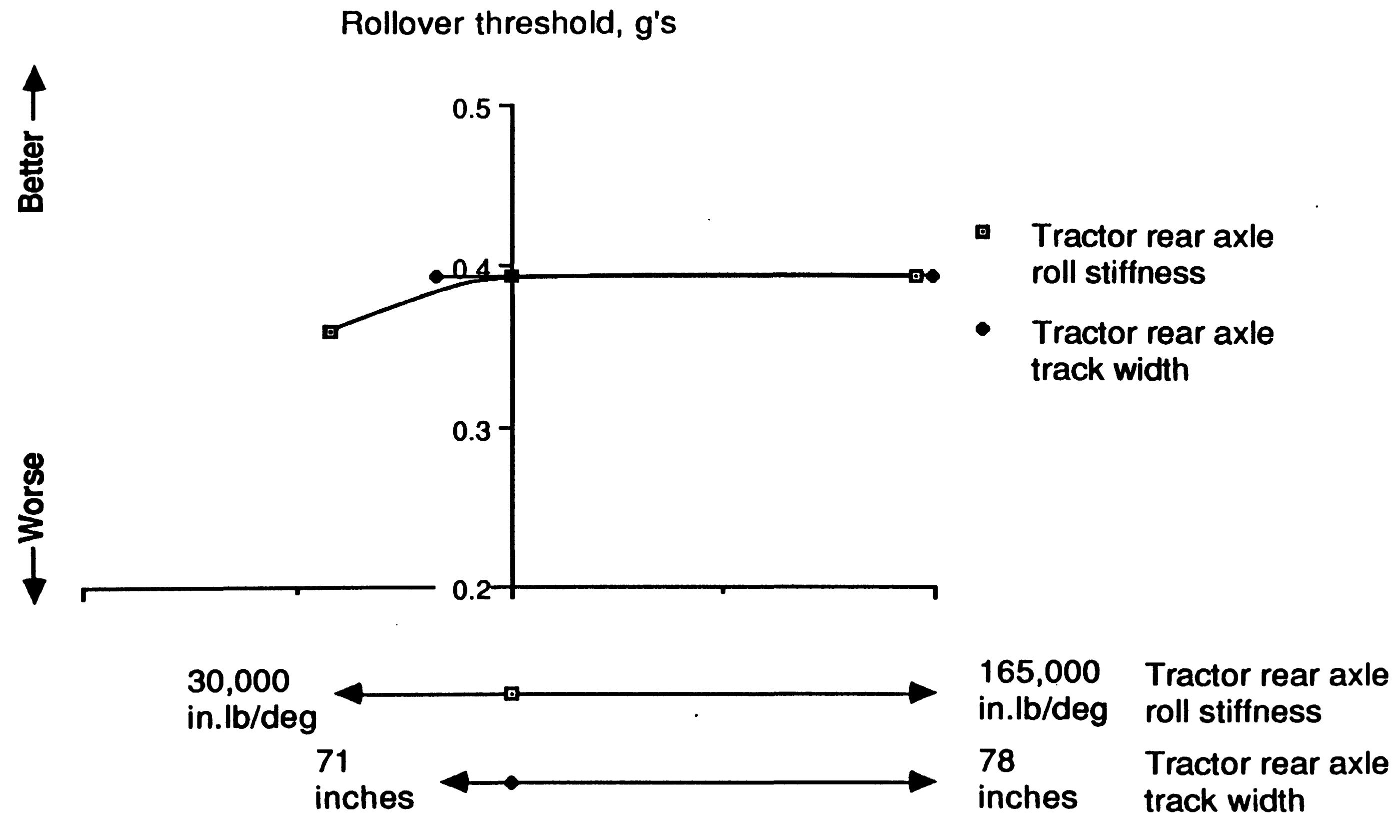


Figure 155. Parameter sensitivity diagram: static roll, double.

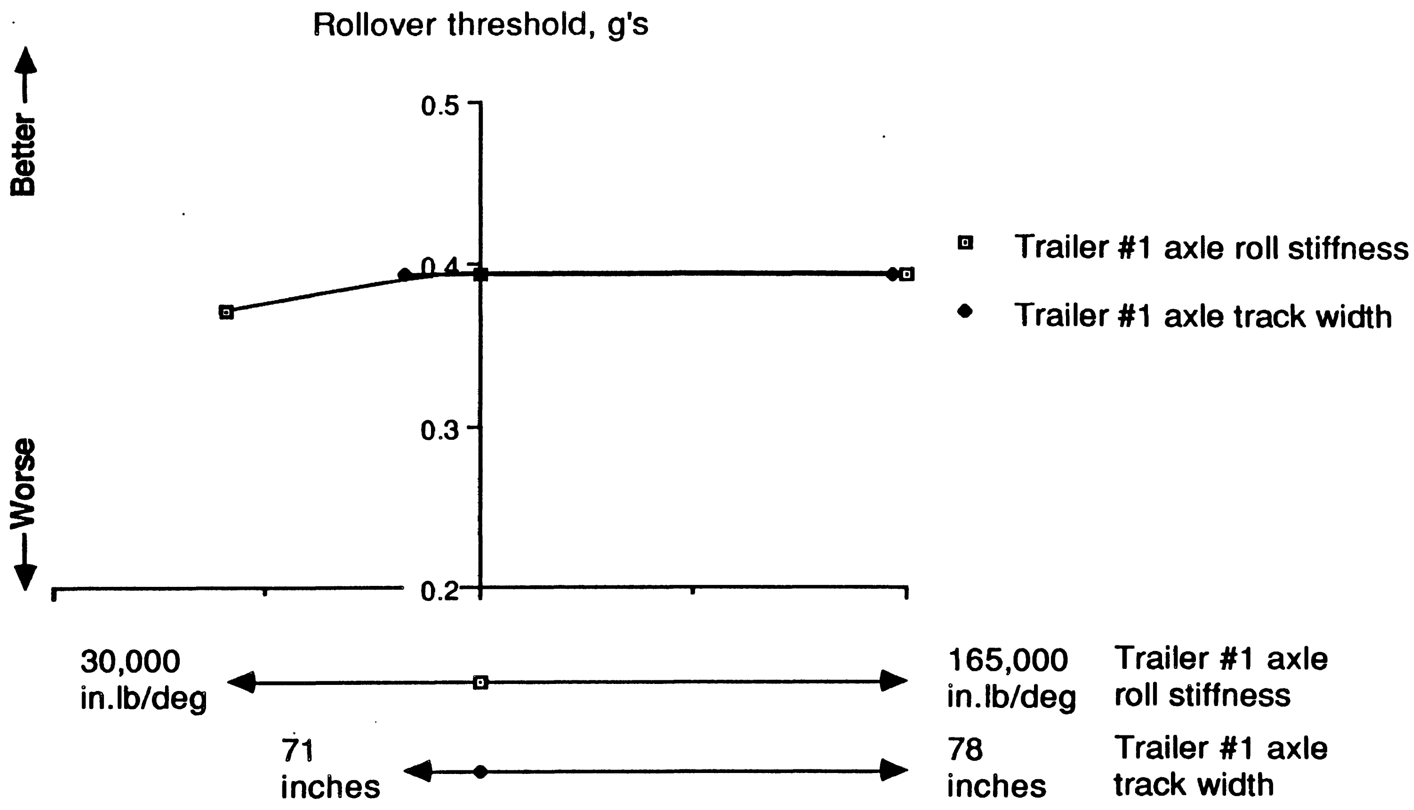


Figure 156. Parameter sensitivity diagram: static roll, double.

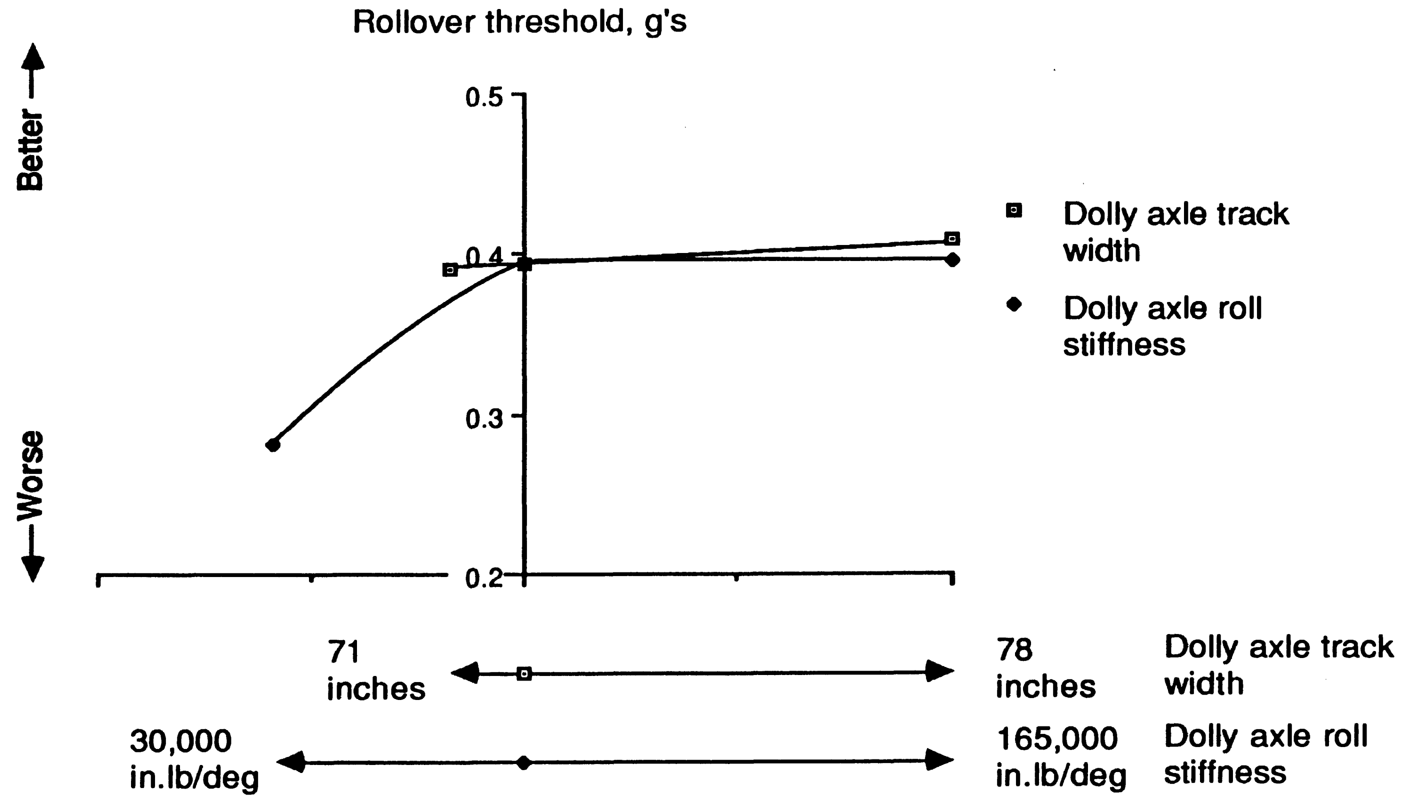


Figure 157. Parameter sensitivity diagram: static roll, double.

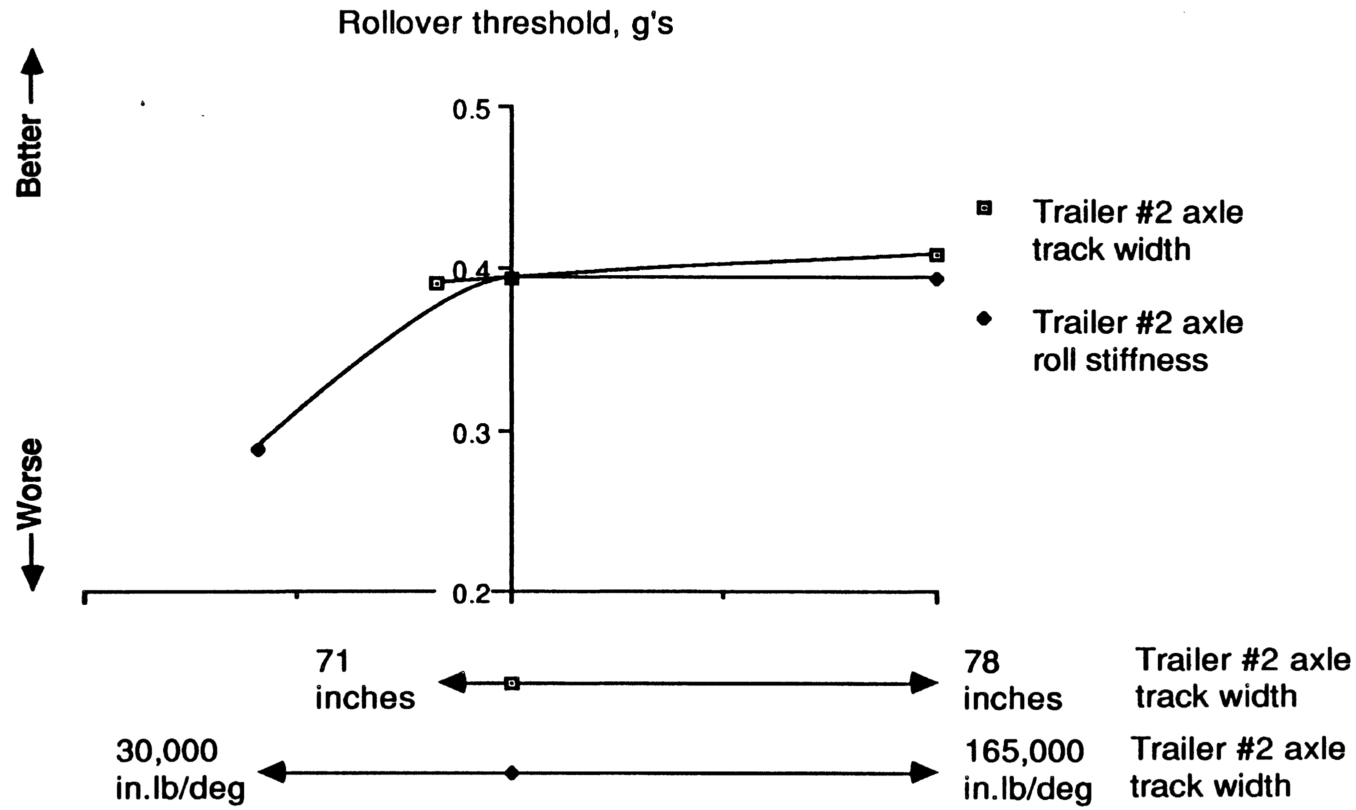
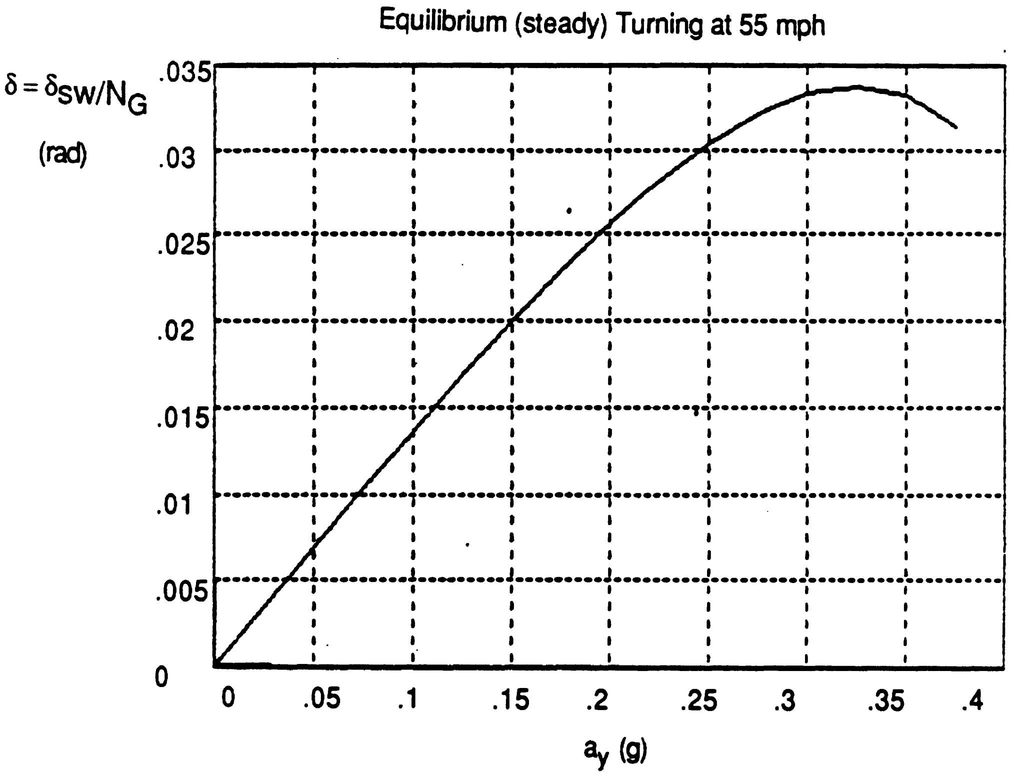


Figure 158. Parameter sensitivity diagram: static roll, double.



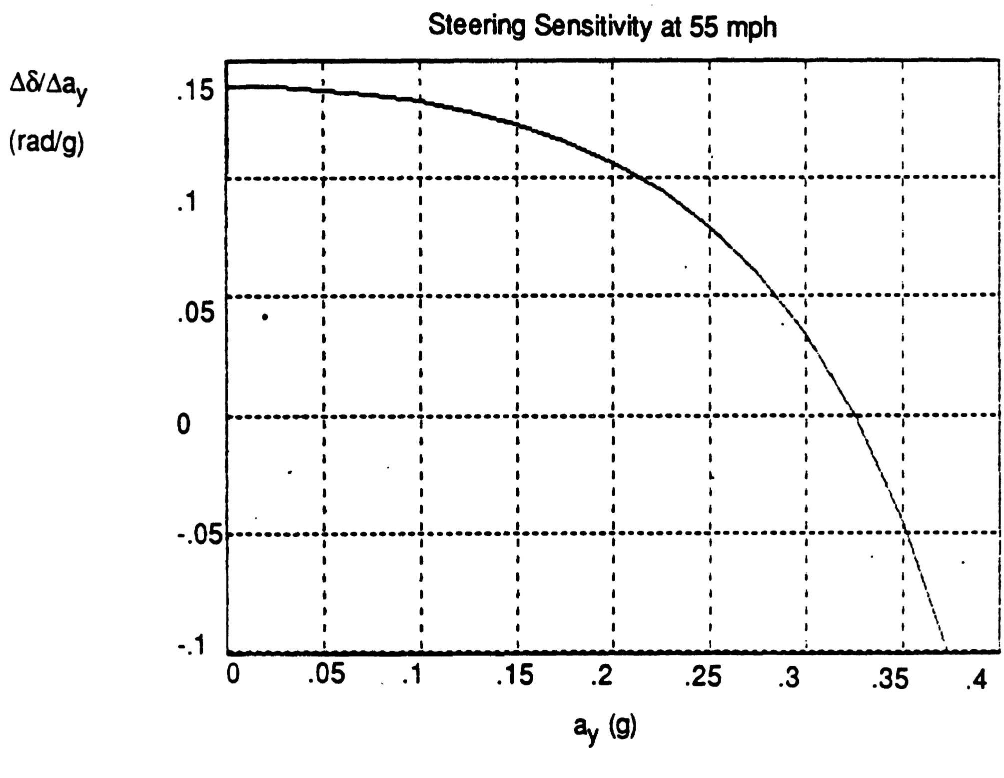


Figure 159. Performance signature, handling, double

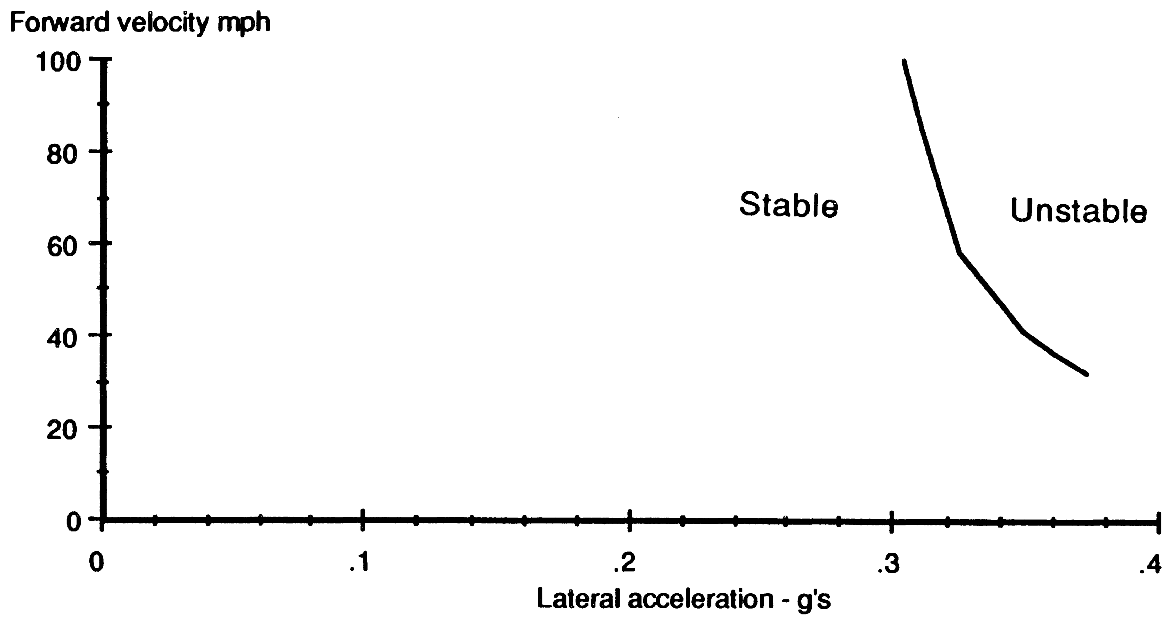


Figure 160. Yaw stability boundary, performance signature, double

Table 43. Double - Handling

Parameter Name	Benchmark Value	Minimum Value	Maximum Value	Notes
Tractor wheelbase	120 inches	118 inches	203 inches	
Fifth wheel offset	12 inches	0 inches	24 inches	
Rear axle roll center heights	29 inches	21 inches	31 inches	
Tractor's front axle roll stiffness	21,000 in.lb/deg	17,000 in.lb/deg	25,000 in.lb/deg	
Tires on tractor's front axle	Bias-ply, Rib tread	Bias-ply, Lug tread	Radial	
Tractor front axle track width	80 inches	77 inches	81 inches	
Tractor rear suspension roll stiffness	70,000 in.lb/deg	30,000 in.lb/deg	165,000 in.lb/deg	
Tires on tractor's rear axle	Bias-ply, Rib tread	Bias-ply, Lug tread	Radial	
Tractor rear axle track width	72 inches	71 inches	78 inches	
Trailer #1 c.g. height	78.4 inches	70 inches	80 inches	Much larger deviations used for braking
Trailer #1 axle roll stiffness	80,000 in.lb/deg	30,000 in.lb/deg	165,000 in.lb/deg	
Trailer #1 axle track width	72 inches	71 inches	78 inches	
Trailer #1 tires	Bias-ply, Rib tread	Bias-ply, Lug tread	Radial	

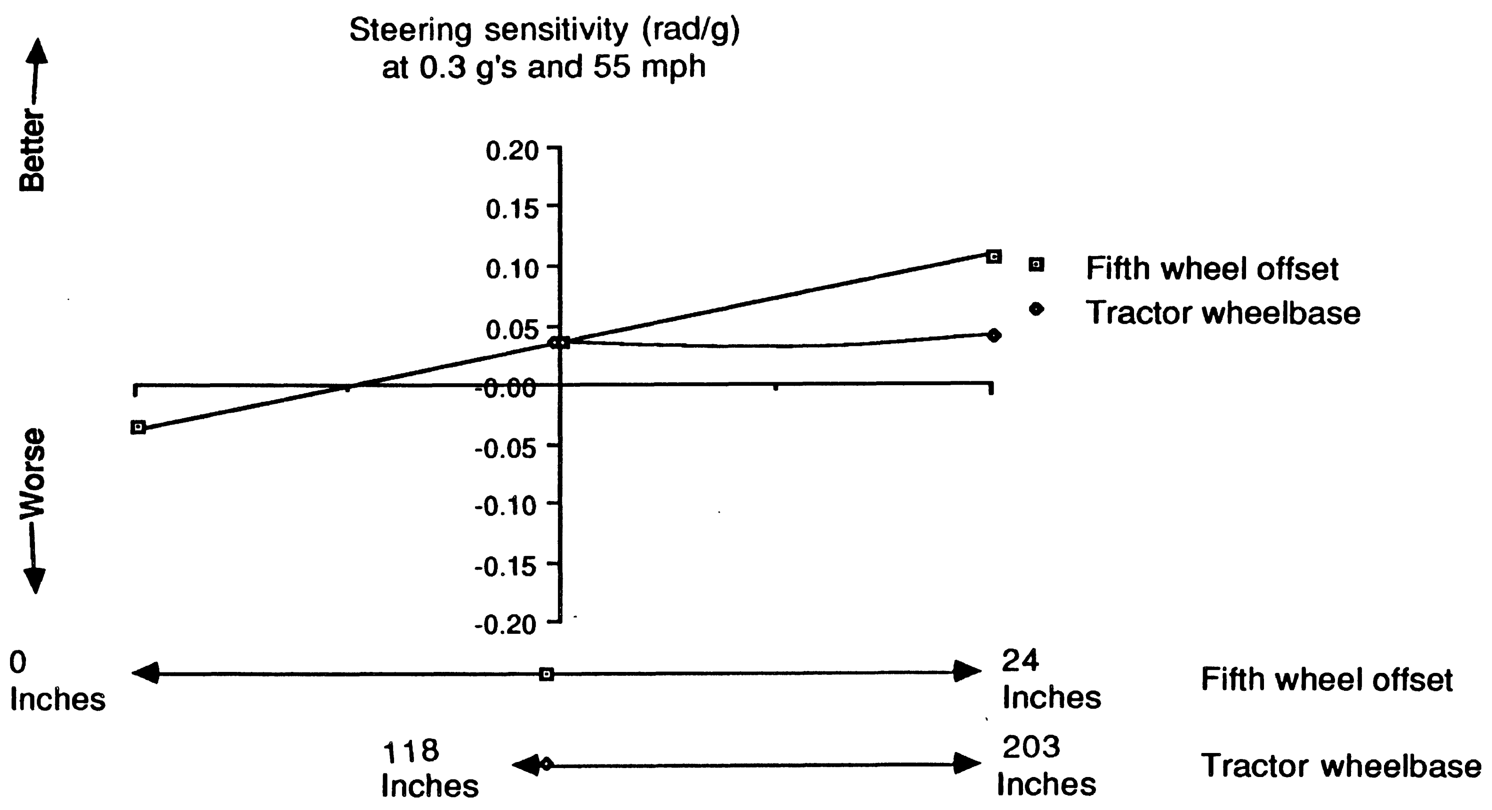


Figure 161. Parameter sensitivity diagram: handling, double.

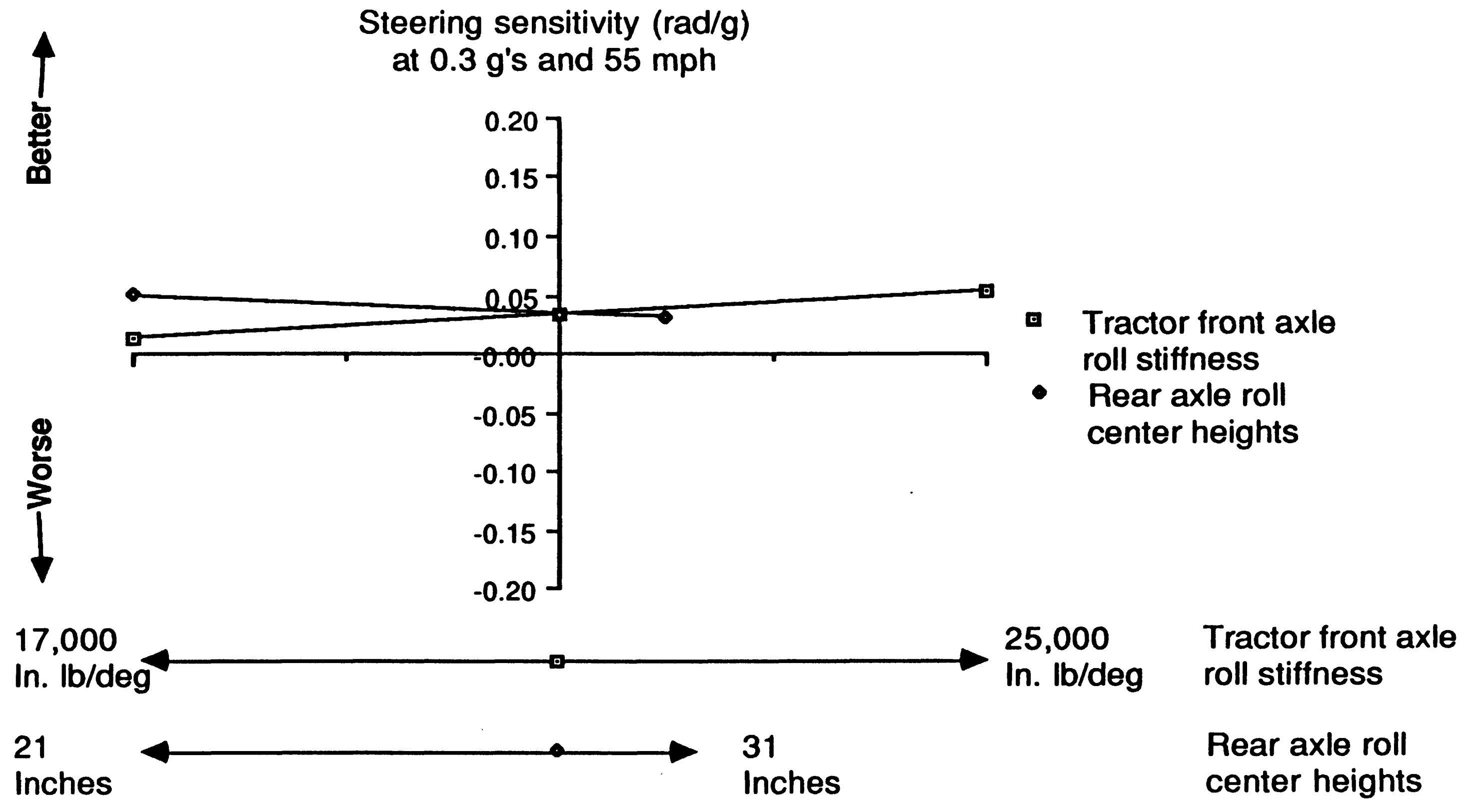


Figure 162. Parameter sensitivity diagram: handling, double.

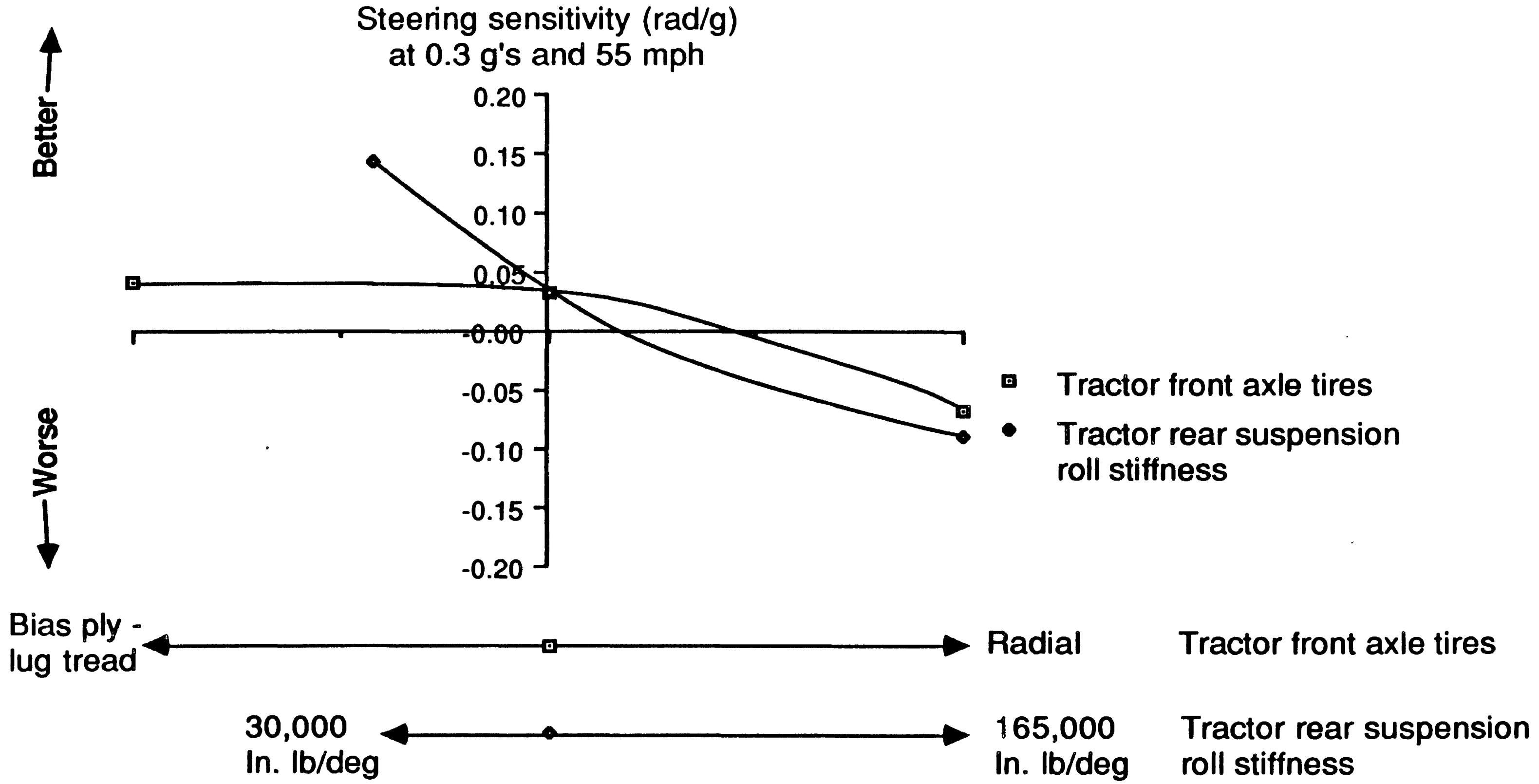


Figure 163. Parameter sensitivity diagram: handling, double.

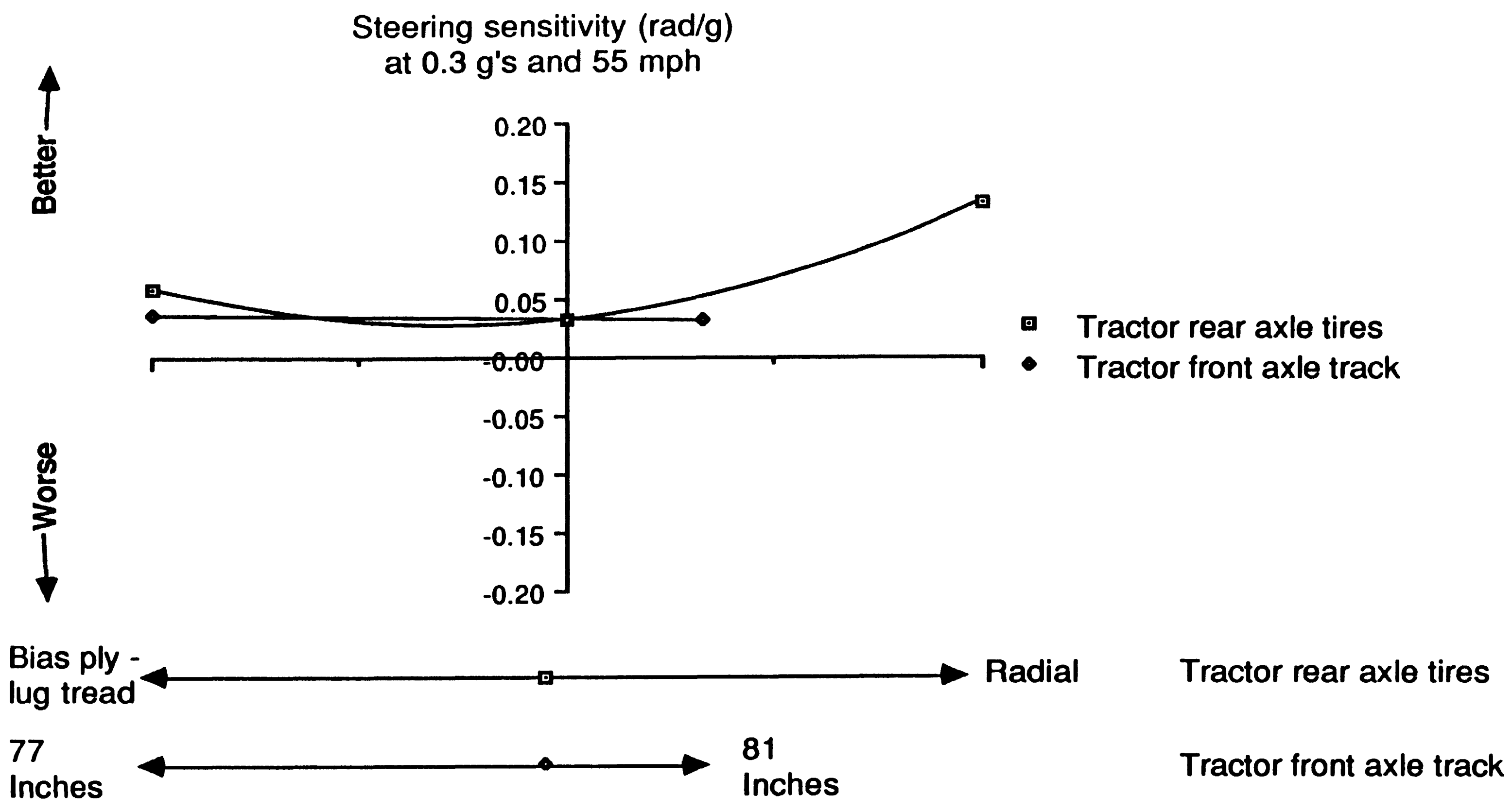


Figure 164. Parameter sensitivity diagram: handling, double.

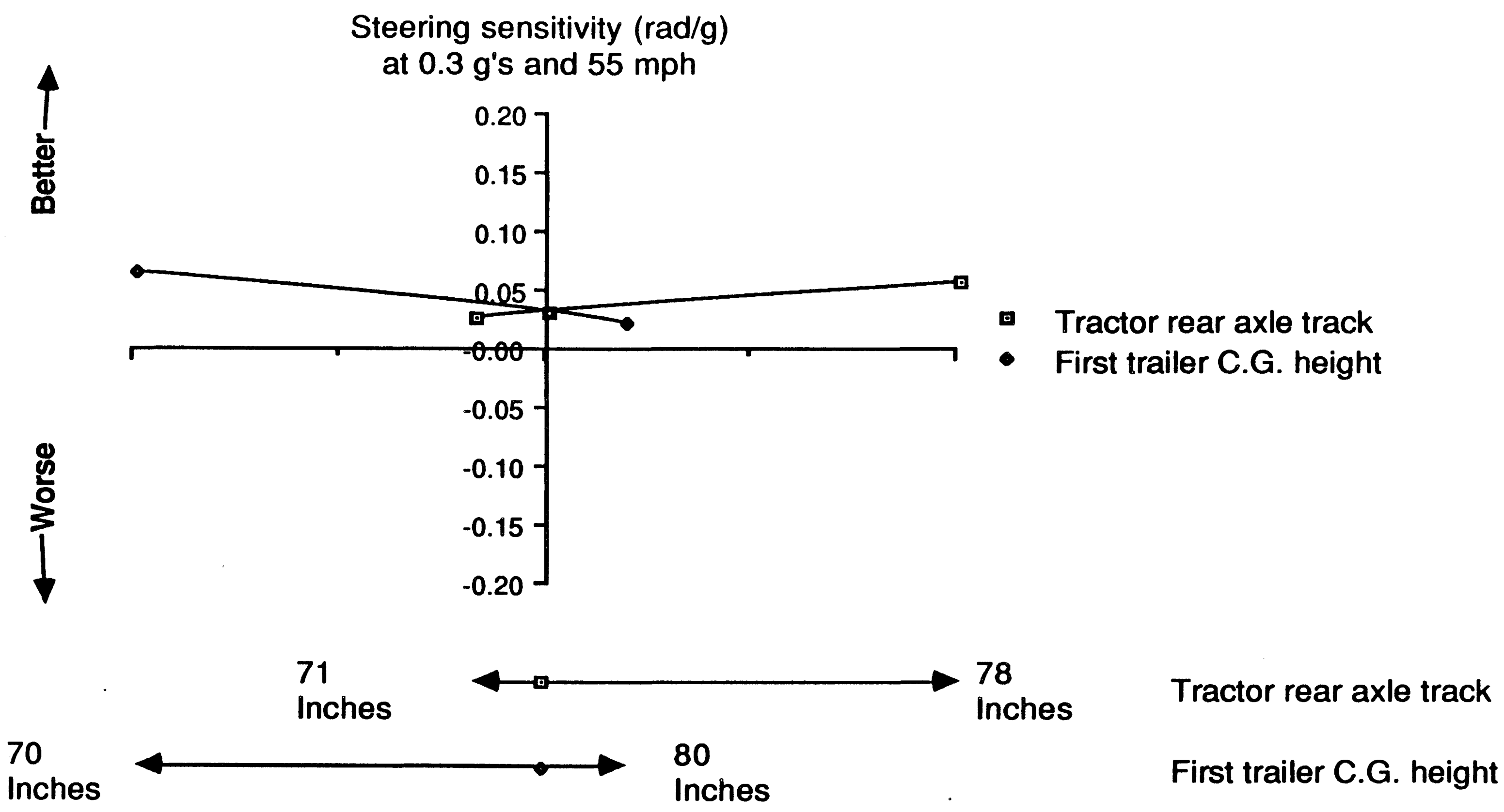


Figure 165. Parameter sensitivity diagram: handling, double.

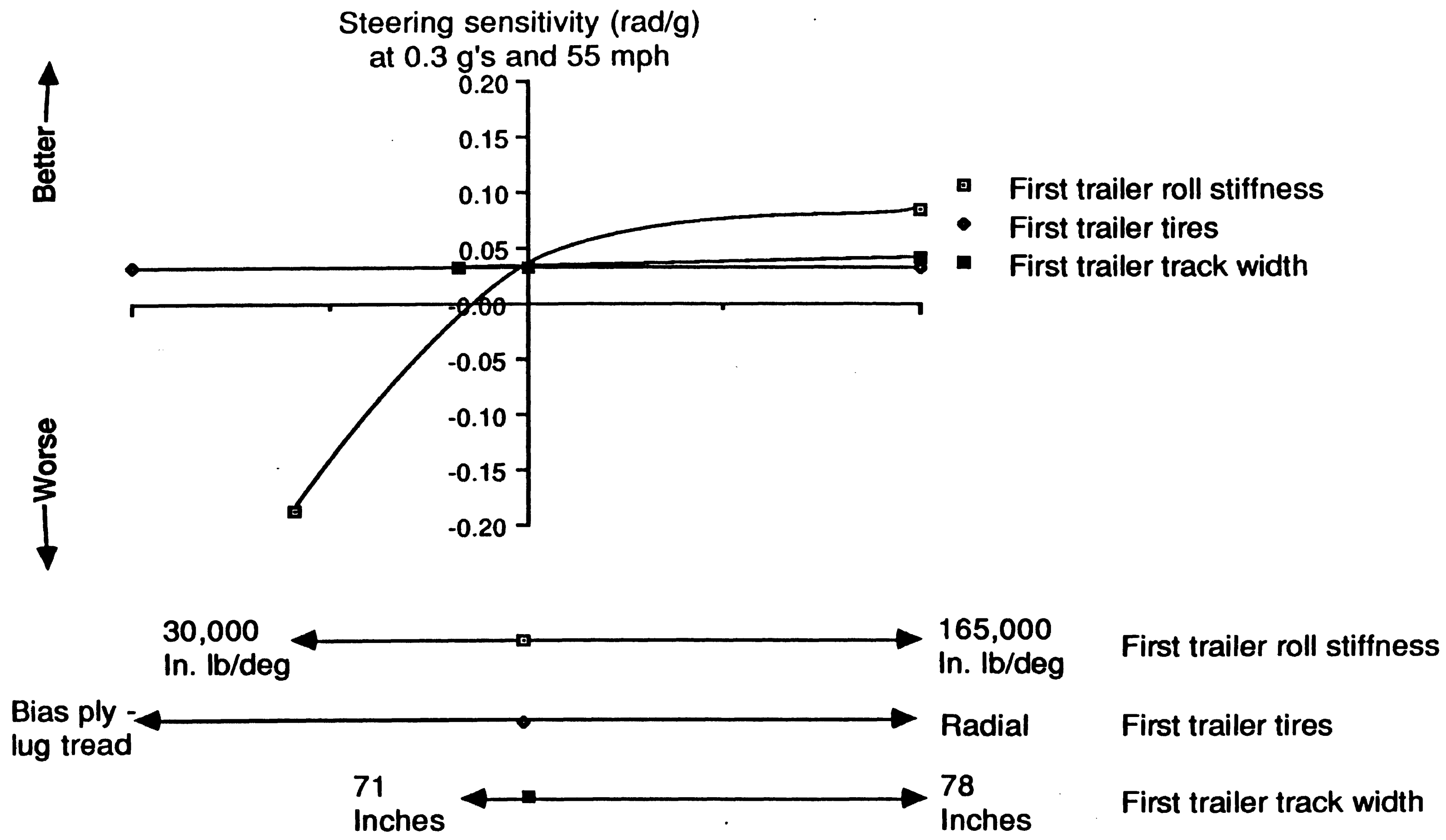


Figure 166. Parameter sensitivity diagram: handling, double.

decreasing the roll stiffness of the front axle, (b) increasing the cornering stiffnesses of the tires installed on the front axle, (c) increasing the roll stiffness of the tractor's rear suspension, (d) increasing the cornering stiffnesses of the tires installed on the tractor's rear axles, and (e) decreasing the roll stiffness of the suspension on the first semitrailer. Of these variations, the stability margin is only increased in the case in which the cornering stiffnesses of the tires installed on tractor's rear axles are increased.

7.7 Response Times in Steering Maneuvers

Benchmark performance. Response times are evaluated for ramp-step and closed-loop obstacle-avoidance maneuvers. The performance measures used in evaluating vehicle responses in these maneuvers are illustrated in Figures 48 and 49. In essence, these measures are (1) the time for the lateral acceleration to reach 90 percent of its steady-state value for a steady turn and (2) the time lag between the steering and the lateral acceleration waveforms obtained in an evasive maneuver, when the vehicle is traveling at 50 mph.

Tables 44 and 45 present results for the empty and loaded vehicle operating under good and poor road surface conditions. In this case, the performance of the loaded vehicle on the good road might be taken as the baseline, but the concept of a baseline is not absolutely necessary for presenting these results.

Parametric sensitivities. Ramp-step results, showing that the loaded vehicle is much slower to respond than the empty vehicle, are given in Table 44. In low-level turns (28 degrees) the vehicle responds more quickly on the slippery surface than on the good road. However, in severe maneuvers the vehicle responds more slowly on the poor road with 2.99 seconds being required for the loaded vehicle to reach 90 percent of steady-state acceleration on the poor surface.

In the closed-loop maneuver, the maximum cross correlations are very high indicating an excellent fit between the time shifted steering input and the lateral acceleration response. Both the ramp-step and the closed-loop response times for the double are noticeably longer than those obtained for the 3S2 (compare the results in Table 24 with those in Table 44 and also those in Table 26 with those in Table 45). These longer response times are not caused by the addition of the full trailer. Even in the more severe cases, the lateral forces at the pintle hitches are small compared to the tire forces. The differences in response times are mainly due to differences between the properties of the 3S2 and the 2S2 portion of the double.

Table 44. Ramp-Step Response Times - Double (Benchmark)

Loading Condition	Road Surface	Response Times Ramp Steer Input	
		28°	48.9°
Empty Vehicle	Dry	0.720	0.780
	Wet	0.680	1.030
Loaded Vehicle	Dry	0.980	2.010
	Wet	0.900	2.990

Table 45. Closed-Loop Response Times - Double (Benchmark)

Loading Condition	Road Surface	Path Follower Cross Correlations	
		Lag	Value
Empty Vehicle	Dry	0.200	0.997
	Wet	0.180	0.998
Loaded Vehicle	Dry	0.240	0.994
	Wet	0.240	0.995

The closed-loop response times range from 0.20 to 0.24 sec. The influences of load and surface friction are not large in an evasive maneuver requiring the driver to steer so that the vehicle will translate 4 feet laterally while moving forward 73 feet.

7.8 Rearward Amplification

Benchmark performance. The performance signature for the benchmark double is presented in Figure 167. The results show that the lateral acceleration of the last trailer starts to noticeably exceed that of the tractor at a frequency of approximately one radian/sec. Between two and three radians/sec, the rearward amplification reaches a maximum of about 2.1. This is a large amplification in the range of frequencies required in accident-avoidance maneuvers. To a rough approximation, this means that since the trailer has a rollover threshold of 0.415 g, the trailer is on the verge of rolling over when the tractor performs an avoidance maneuver with only a maximum acceleration of 0.2 g.

Parametric sensitivities. The factors having an important influence on rearward amplification are (1) the cornering stiffnesses of the tires, (2) the wheelbases of the trailers, and (3) the location of the pintle hitch with respect to the rear axle of the first semitrailer.

Figure 168 illustrates the improvement that can be made by installing stiff tires on all wheels. In this case the maximum rearward amplification is reduced to 1.43 which is a sizeable improvement that would greatly reduce the risk of rollover.

The wheelbases of the trailers in a double have an important influence on rearward amplification. Doubles with long wheelbase trailers have little amplification. For example, if the distance from the articulation point to the axle is increased from 252 inches to 300 inches on each trailer, rearward amplification is reduced from 2.1 to 1.75 for the western double (see Figure 169). This trend continues as wheelbase is increased such that a turnpike double has almost no amplification at 55 mph.

The location of the hitch on the towing unit, that is, the 2S2 in this case, is another important determinant of the amount of rearward amplification. As shown in Figure 170, an increase from 36 inches to 60 inches in the distance from the axle of the first semitrailer to the pintle hitch causes an increase in amplification from 2.1 to 2.25. Very large amounts of overhang of the pintle hitch contribute to a large factor that multiplies whatever amplification the vehicle would otherwise possess. In this sense, the convenience of locating the pintle hitch as far back as possible can be counterproductive with respect to safety.

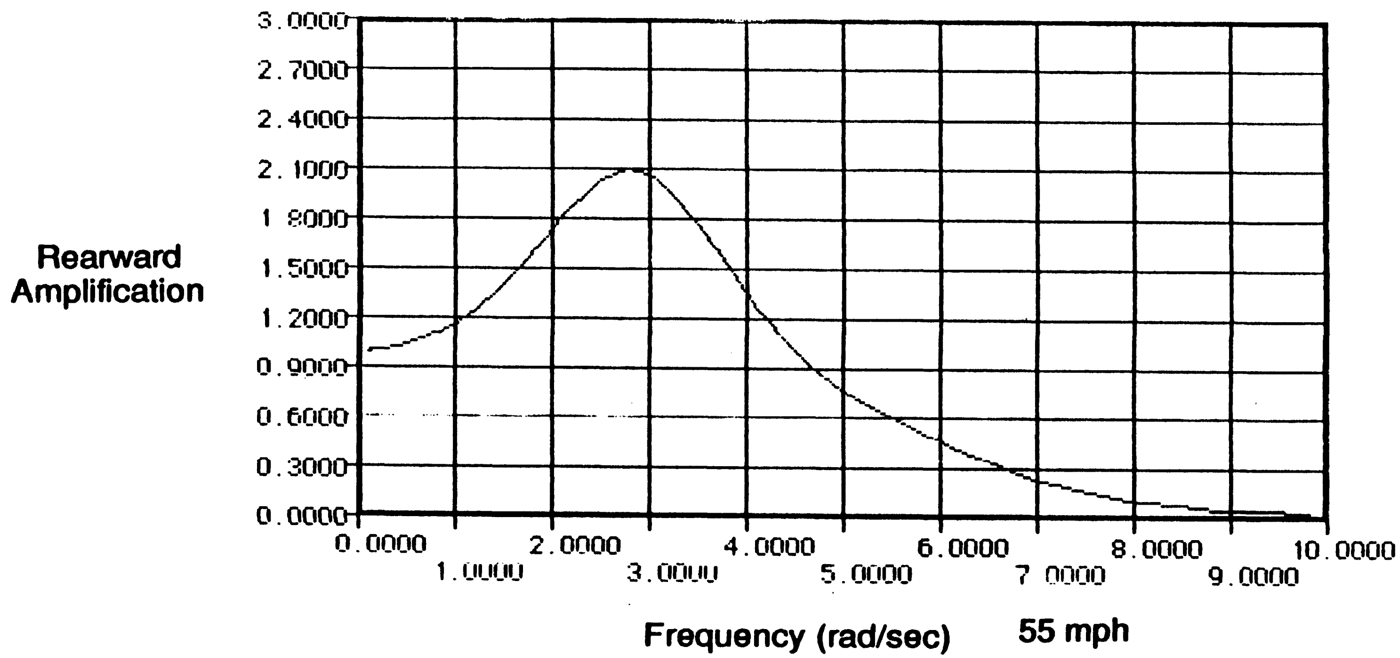


Figure 167. Performance signature, rearward amplification, benchmark double

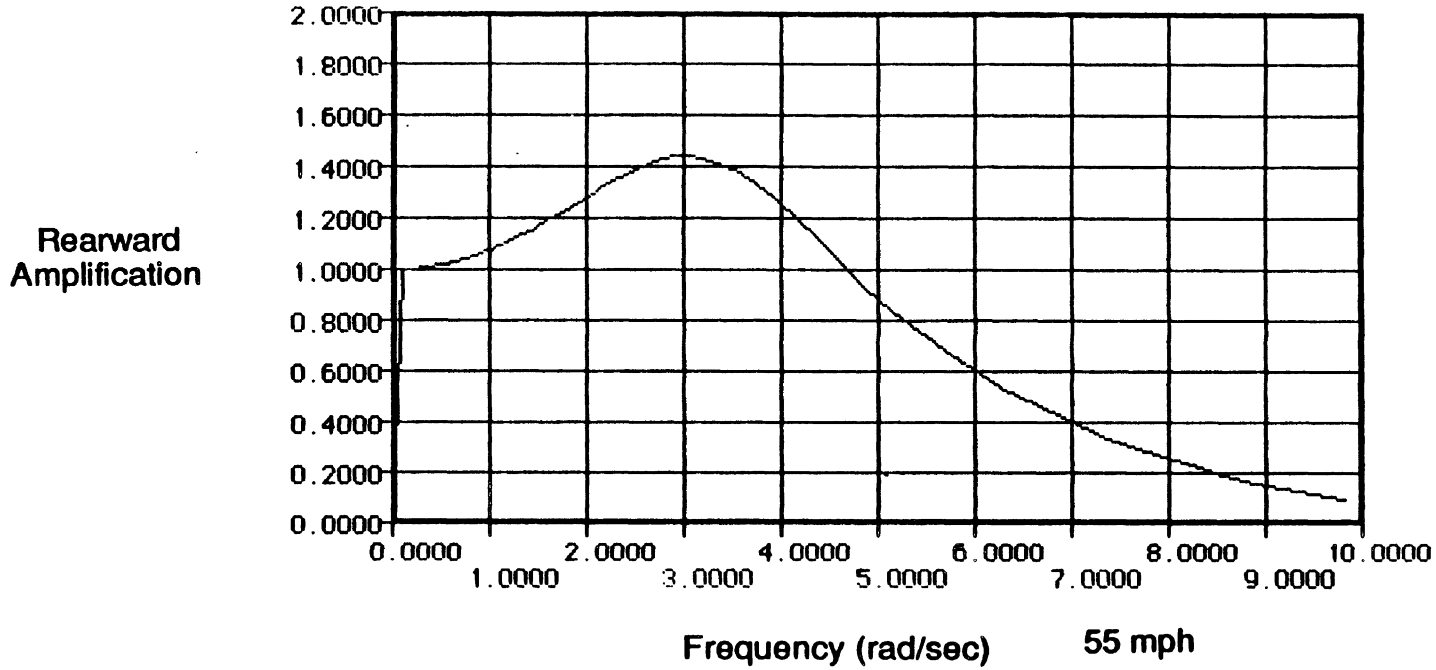


Figure 168. Performance sensitivity, loaded double with radial tires

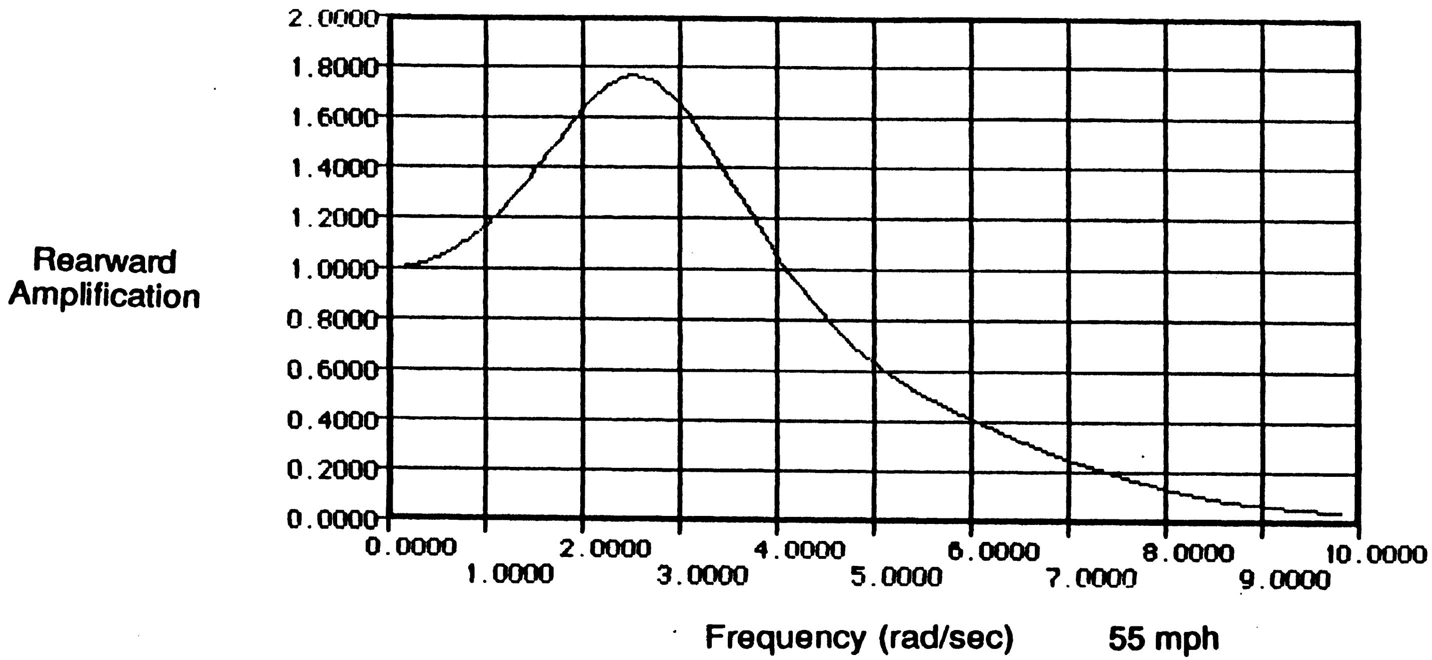


Figure 169. **Performance sensitivity, loaded double with 300 inch trailer wheelbase**

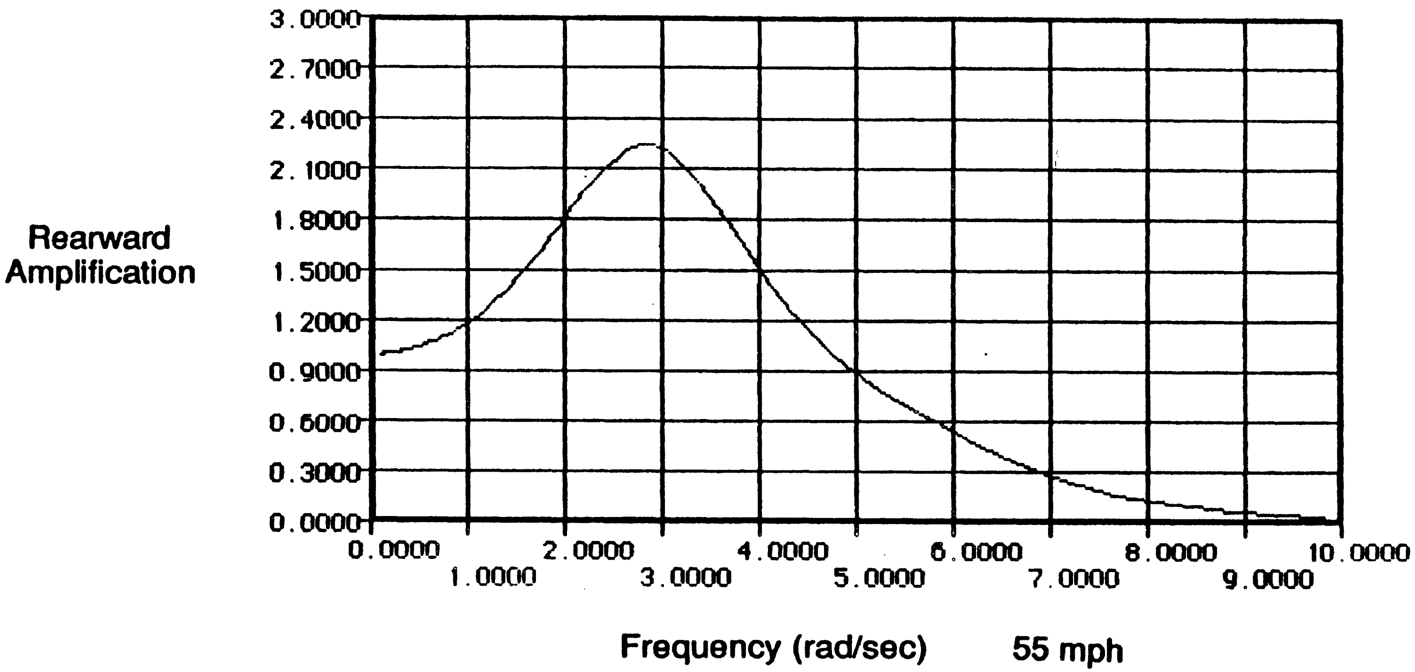


Figure 170. Performance sensitivity, loaded double with pintle hitch overhang of 60 inches

7.9 Braking in a Turn

Benchmark performance. The general description of vehicle response to a pulse of braking during a steady turn is presented in Section 4.9. In this maneuver, the vehicle is traveling along at 50 mph on a turn with a radius of 1,500 feet. Then a pulse of braking is applied. The brake pressure quickly rises to 100 psi and then it is released. The effect of this braking action is to start directional instability and then to make it worse when the brakes are released and the tires are again capable of producing large side forces.

For the loaded double the vehicle response in this maneuver is substantial, but the modelled driver is able to maintain control. The following Figures (171 a through e) show the timing of the braking pulse and the effect of this pulse on the yaw rates of the tractor, the first semitrailer, and the second semitrailer. The yaw rate graphs show the turning levels of yaw rates before the brakes are applied, then the changes in yaw rates caused by the braking, and finally the remaining directional motions after the brakes are released and the simulated driver is regaining directional control. Figure 171e shows the simulated steering actions used to maintain control.

Parametric sensitivities. The only variation considered here is to unload the vehicle and to repeat the same maneuver. The results are presented in Figure 172 a through e. In this case, the yaw response of the tractor to the release of braking is very large and the effects of this yaw divergence is to set up a violent directional disturbance throughout the vehicle system. As shown in Figure 172e, the simulated driver has lost control and a very large steering wheel angle is reached. The vehicle does not jackknife at the first articulation joint, rather the second trailer takes on a very large angle with respect to the first trailer.

The properties that can influence this situation are discussed in Section 4.9. Advanced braking systems with load sensing, good timing, and no hysteresis in the brakes appear to be a possible solution.

7.10 Concluding Remarks on Doubles

The double is a unique vehicle with its own special idiosyncracies, nevertheless, many of its performance characteristics can be related to those of simpler vehicle configurations. For example, handling performance is determined largely by the cornering stiffnesses of the front tires and the load transfer taking place at the rear tires of the tractor. In some respects, the pintle hitch effectively decouples the full trailer from the front 2S2 portion of the double. Roll moments are not coupled through the pintle hitch, so the

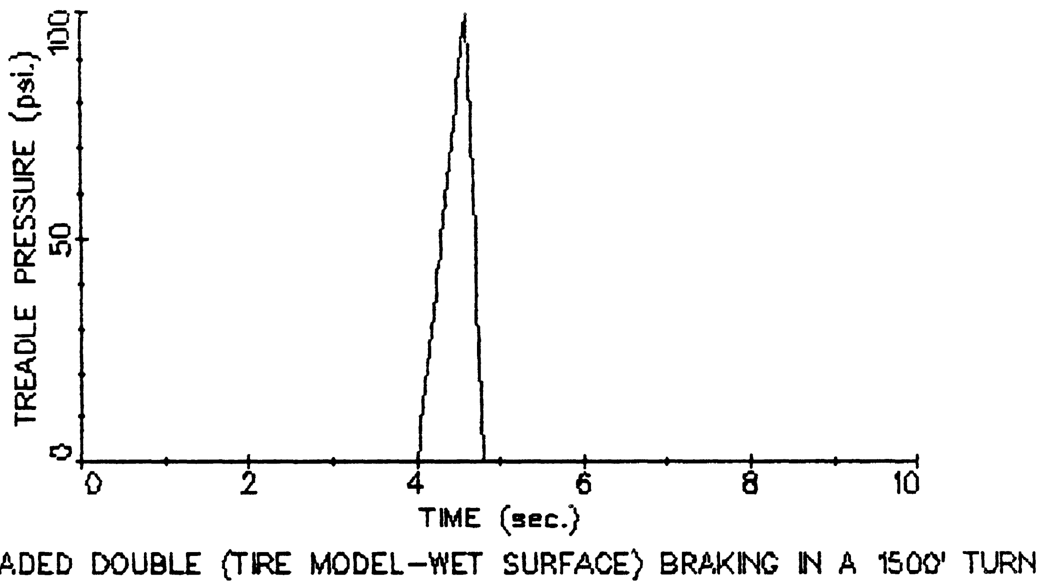


Figure 171a. Braking in a turn, loaded double

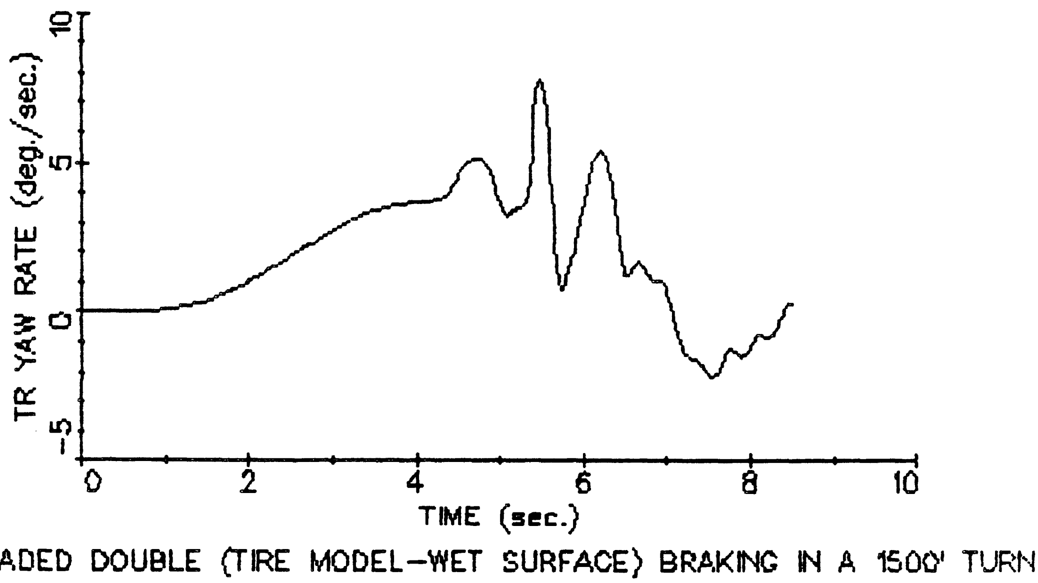


Figure 171b. Braking in a turn, loaded double

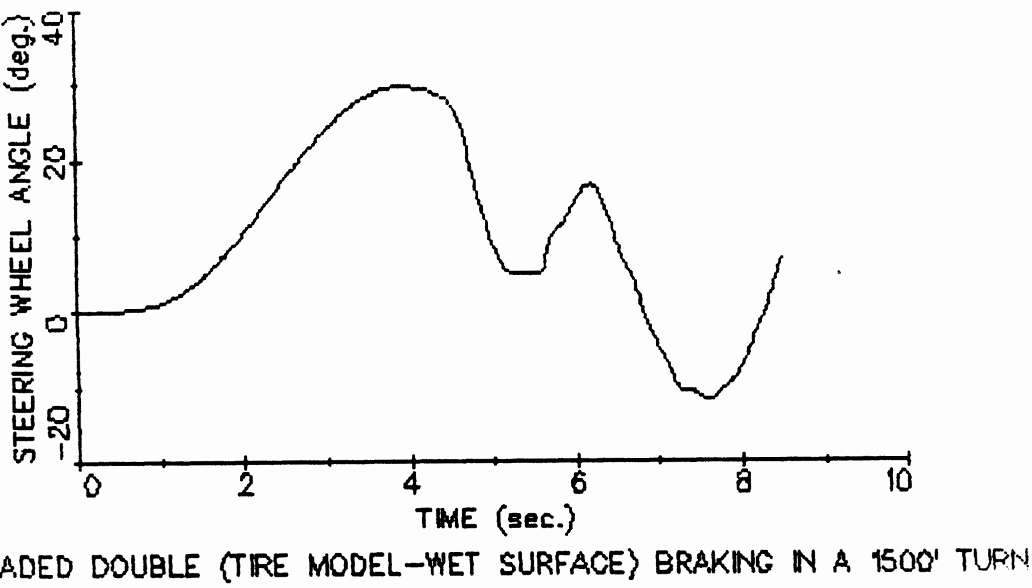


Figure 171e. Braking in a turn, loaded double

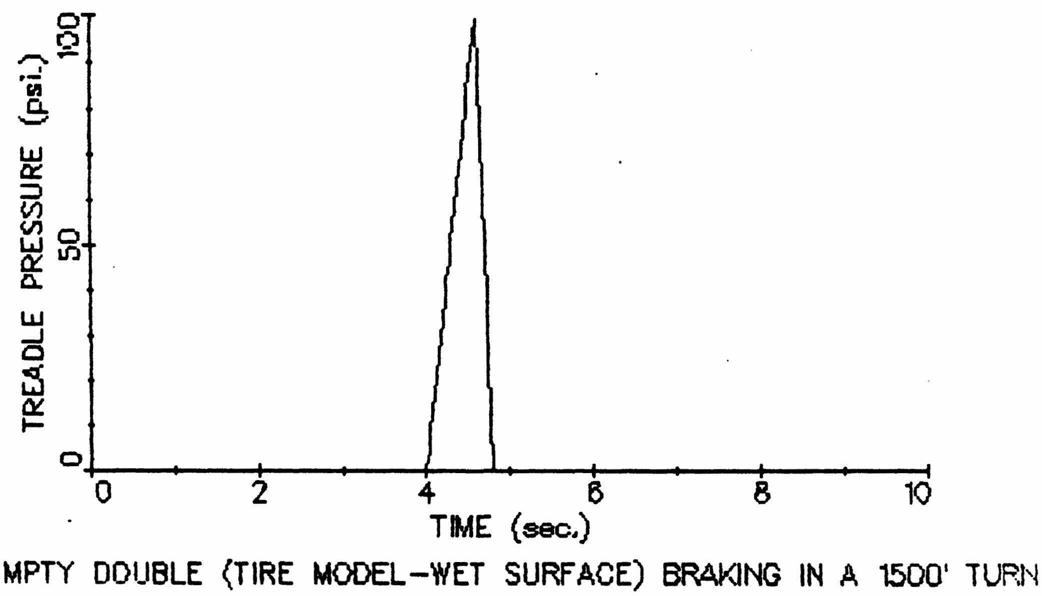


Figure 172a. Braking in a turn, empty double

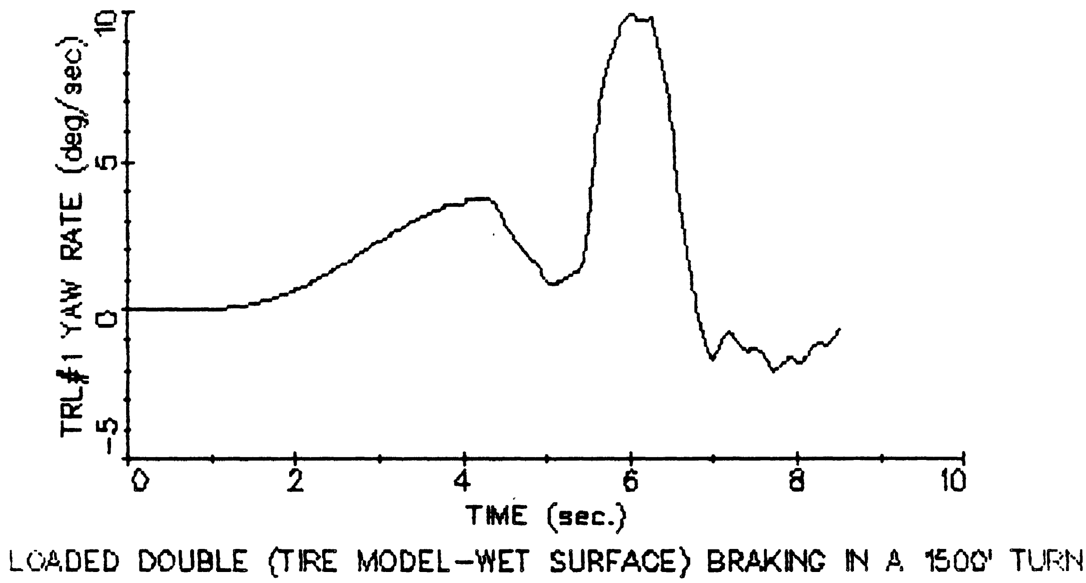


Figure 171c. Braking in a turn, loaded double

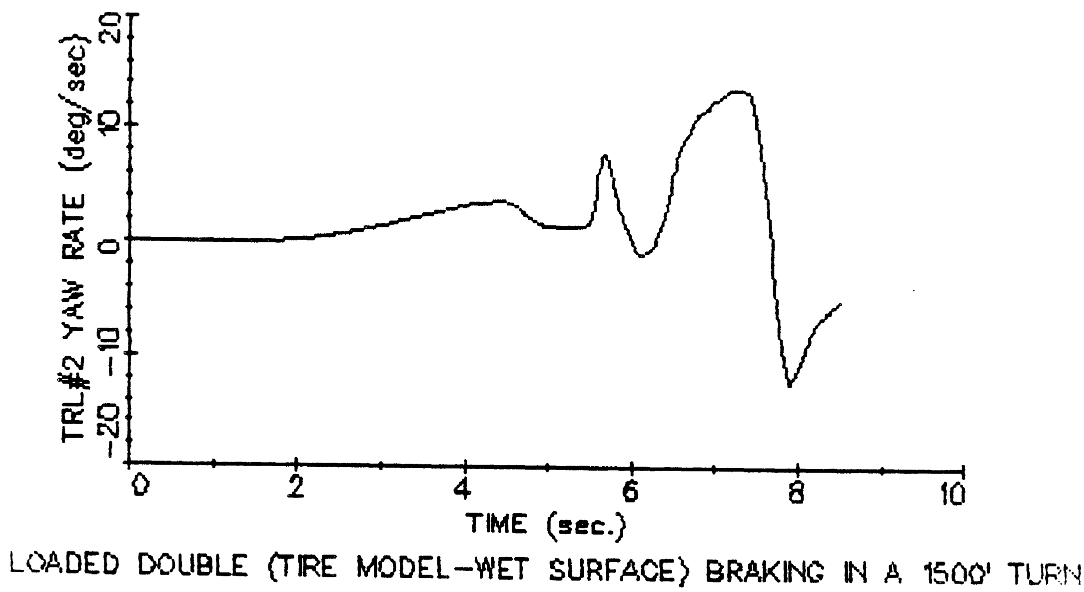
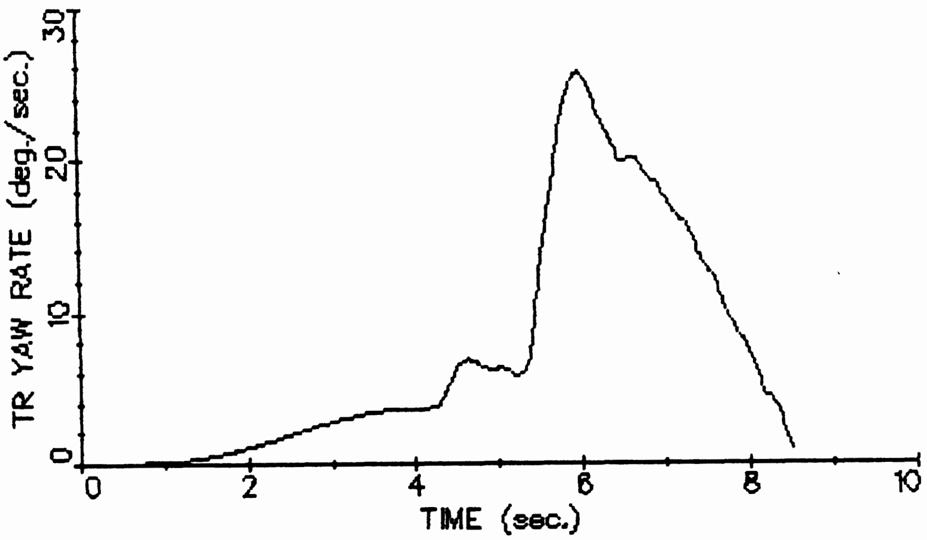
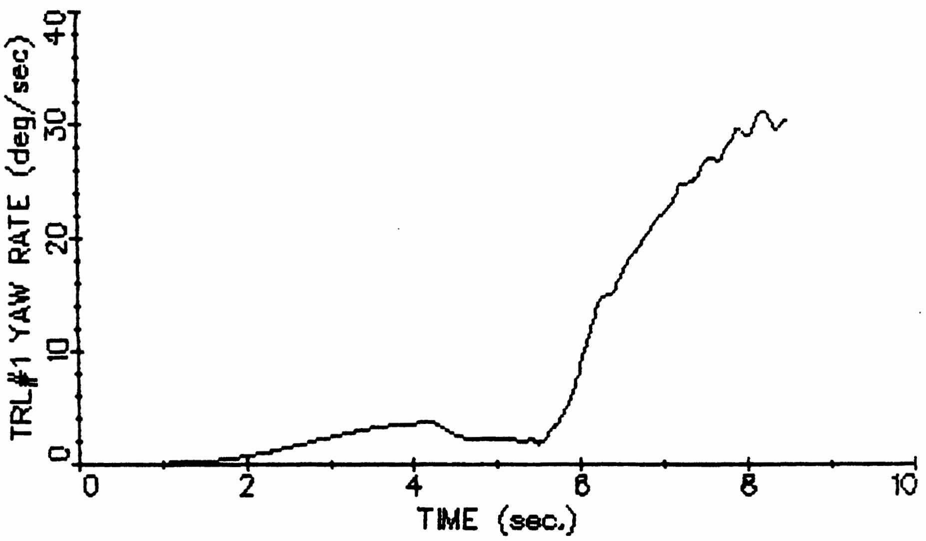


Figure 171d. Braking in a turn, loaded double



EMPTY DOUBLE (TIRE MODEL-WET SURFACE) BRAKING IN A 1500' TURN

Figure 172b. Braking in a turn, empty double



EMPTY DOUBLE (TIRE MODEL-WET SURFACE) BRAKING IN A 1500' TURN

Figure 172c. Braking in a turn, empty double

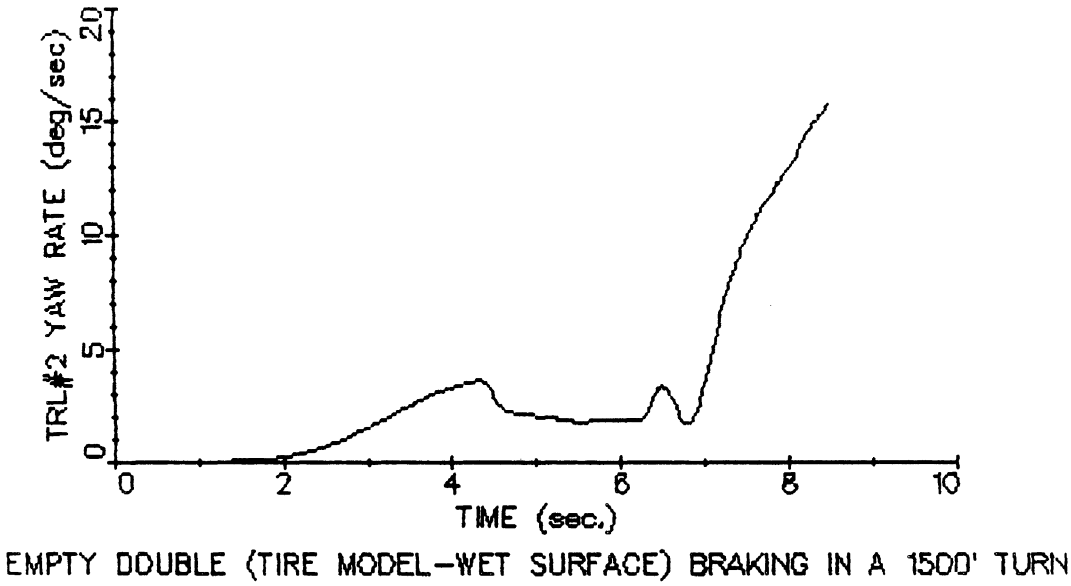


Figure 172d. Braking in a turn, empty double

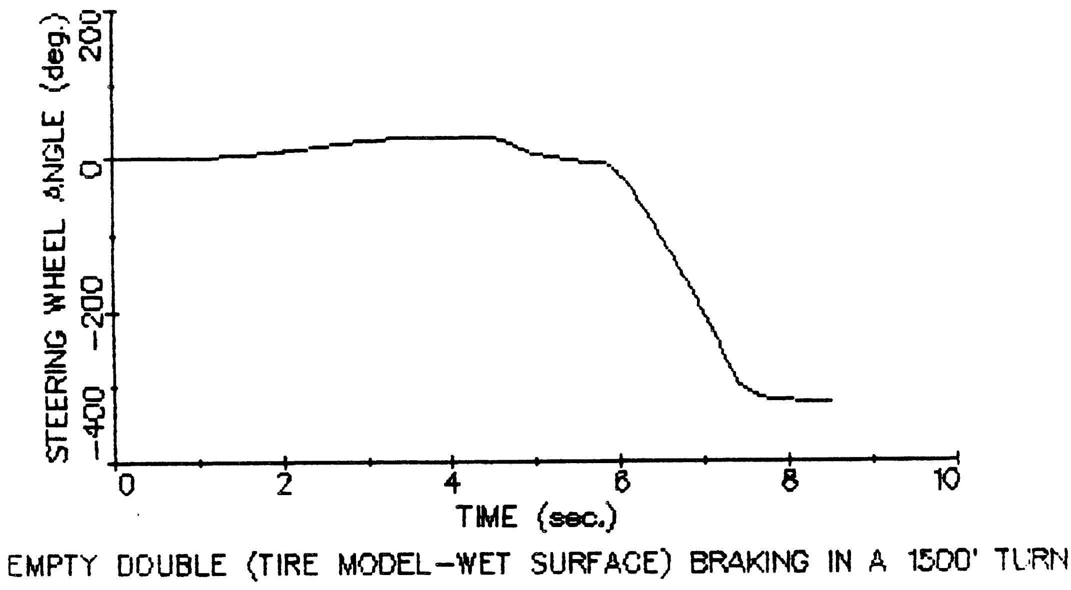


Figure 172e. Braking in a turn, empty double

rollover thresholds of both the full trailer and the 2S2 portion are computed separately. The lateral force at the pintle hitch is small, implying that (1) handling is not influenced significantly by properties of the full trailer and (2) the full trailer may have lateral motions that are amplified versions of the motion of the rear of the 2S2 portion.

During braking, the converter dolly transfers vertical load to the rear of the first semitrailer and, since the dolly is short, this load transfer is large enough to substantially load the rear axle of the first semitrailer. This leads to a different arrangement of friction utilizations than that which would occur if a fixed dolly were employed. In the case of a fixed dolly, the rear axle of the first semitrailer would carry less load than it would with the converter dolly. This difference will be important if brakes on the first semitrailer are more effective than the other brakes.

The additional articulation points on the double (compared to the tractor-semitrailer) contribute to improved low-speed offtracking, but also to increased high-speed, outboard offtracking thereby making the occurrence of curb tripping and subsequent rollover more likely unless the driver is careful to keep the tractor in the center of its lane on ramps and other curbed turns.

8.0 TRIPLES

8.1 Baseline Values of Pertinent Mechanical Properties

The benchmark triple is simply an extension of the benchmark double described in Section 7. This triple employs two identical full trailers that are identical to the full trailer specified in Table 37 and Figure 137. The tractor and first semitrailer of the triple are also identical to the tractor and semitrailer utilized in the benchmark double. Given these choices of units, the loaded triple has a gross combination weight of 115,000 lbs. (This is more weight than is allowed in most States. However, this arrangement is satisfactory for illustrating how triples would behave.) The distance from the front axle to the rearmost axle would be 91.33 feet.

8.2 Low-Speed Cornering - Tractrices

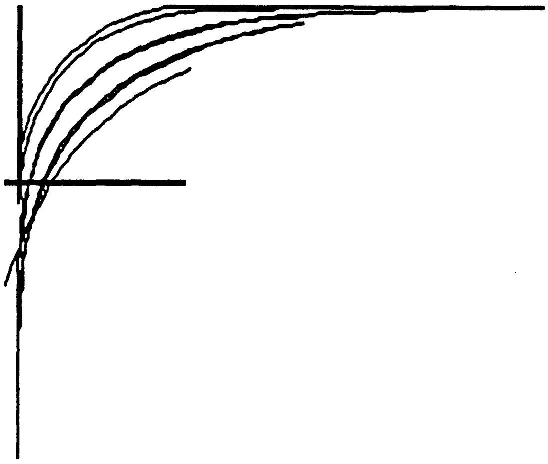
Benchmark performance. The performance signature consists of the paths of the axle centers as plotted in Figure 173. The maximum offtracking of the rearmost axle is 15.9 feet in this right angle turn with a radius of 41 feet to the center of the front axle.

Parametric variations. The effects of changes in the offtracking dimensions of the last dolly and semitrailer are studied here using the deviations listed in Table 46. The results presented in Figures 174 and 175 show that the changed trailer wheelbase is the only change in length that is large enough to have a significant influence on low-speed offtracking.

8.3 Constant-Deceleration Braking

Benchmark performance. The triple has high braking efficiencies in the loaded condition (0.92 at 0.2 g and 0.83 at 0.4 g) and lower efficiencies in the empty condition (0.7 at 0.2 g and 0.66 at 0.4 g). For the benchmark vehicle, the controlling axle (that is, the one requiring the highest friction) is the rearmost axle, the seventh axle of the combination.

Parametric sensitivities. The results for the variations listed in Table 47 are presented in Figures 176 through 179. Due to the similarities between this vehicle and the double treated in Section 7.3, the results presented here are much the same as those presented there. Interestingly, none of the deviations in mechanical properties have a significant effect on the poor braking efficiency of the empty triple. If triples are to be used to transport empty trailers, advanced or modified braking systems are needed to achieve high efficiencies.



MINIMUM TRACK

Susp. No.	Radius (ft)	Angle (deg)
1	41.00	0.00
2	39.67	75.32
3	33.43	63.93
4	33.94	61.92
5	29.44	56.73
6	29.02	56.22
7	25.10	54.08

Maximum Offtracking =15.90 ft.

Figure 173. Tractrices, triple, 41 foot turn

Table 46. Triple - Low-Speed Offtracking

Parameter Name	Benchmark Value	Minimum Value	Maximum Value	Notes
Second dolly's wheelbase	80 inches	72 inches	96 inches	
Third trailer's wheelbase	252 inches	240 inches	336 inches	

Table 47. Triple - Braking

Parameter Name	Benchmark Value	Minimum Value	Maximum Value	Notes
Front brake gain	2000 in.lb/psi	2000 in.lb/psi	3000 in.lb/psi	Both brakes combined
*First trailer's c.g. height	78.4 inches	60 inches	105 inches	
First dolly wheelbase	80 inches	72 inches	96 inches	
*Second trailer's c.g. height	78.5 inches	60 inches	105 inches	
Second dolly wheelbase	80 inches	72 inches	96 inches	
*Third trailer's c.g. height	78.5 inches	60 inches	105 inches	
Third trailer's wheelbase	252 inches	240 inches	336 inches	

* For the loaded vehicle

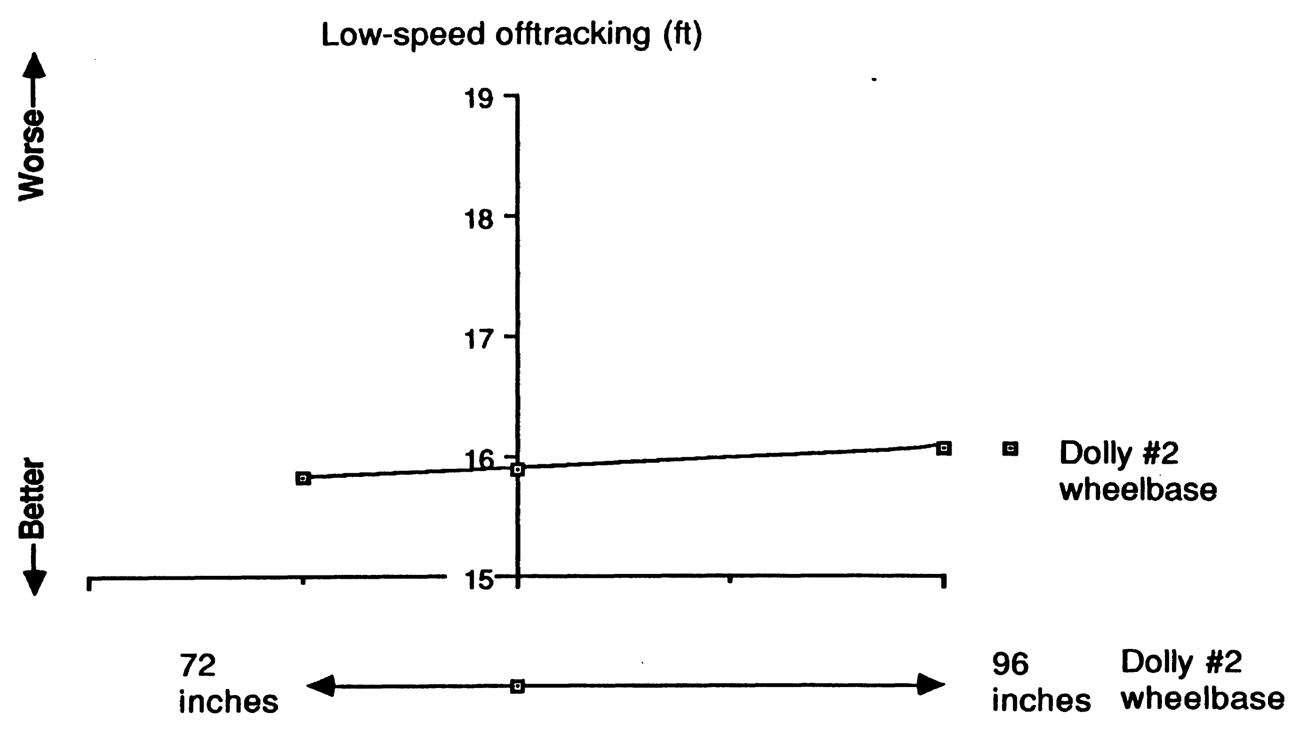


Figure 174. Parameter sensitivity diagram: low-speed offtracking, triple.

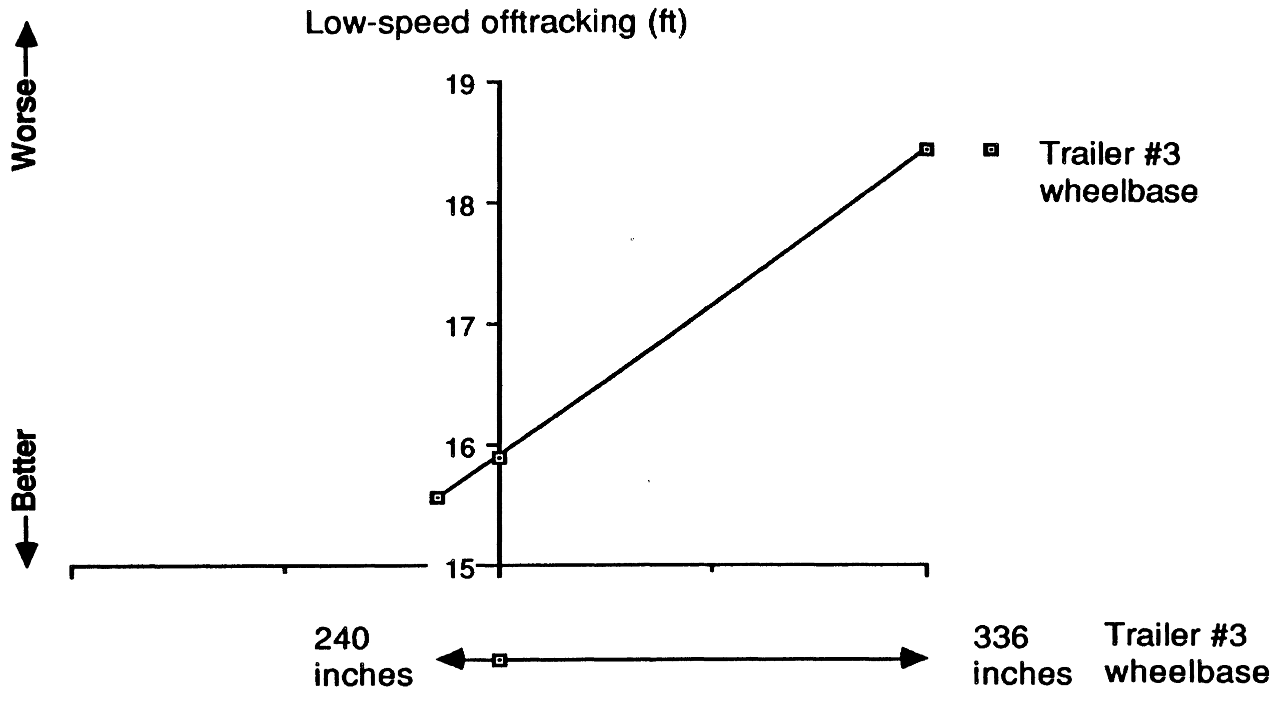


Figure 175. Parameter sensitivity diagram: low-speed offtracking, triple.

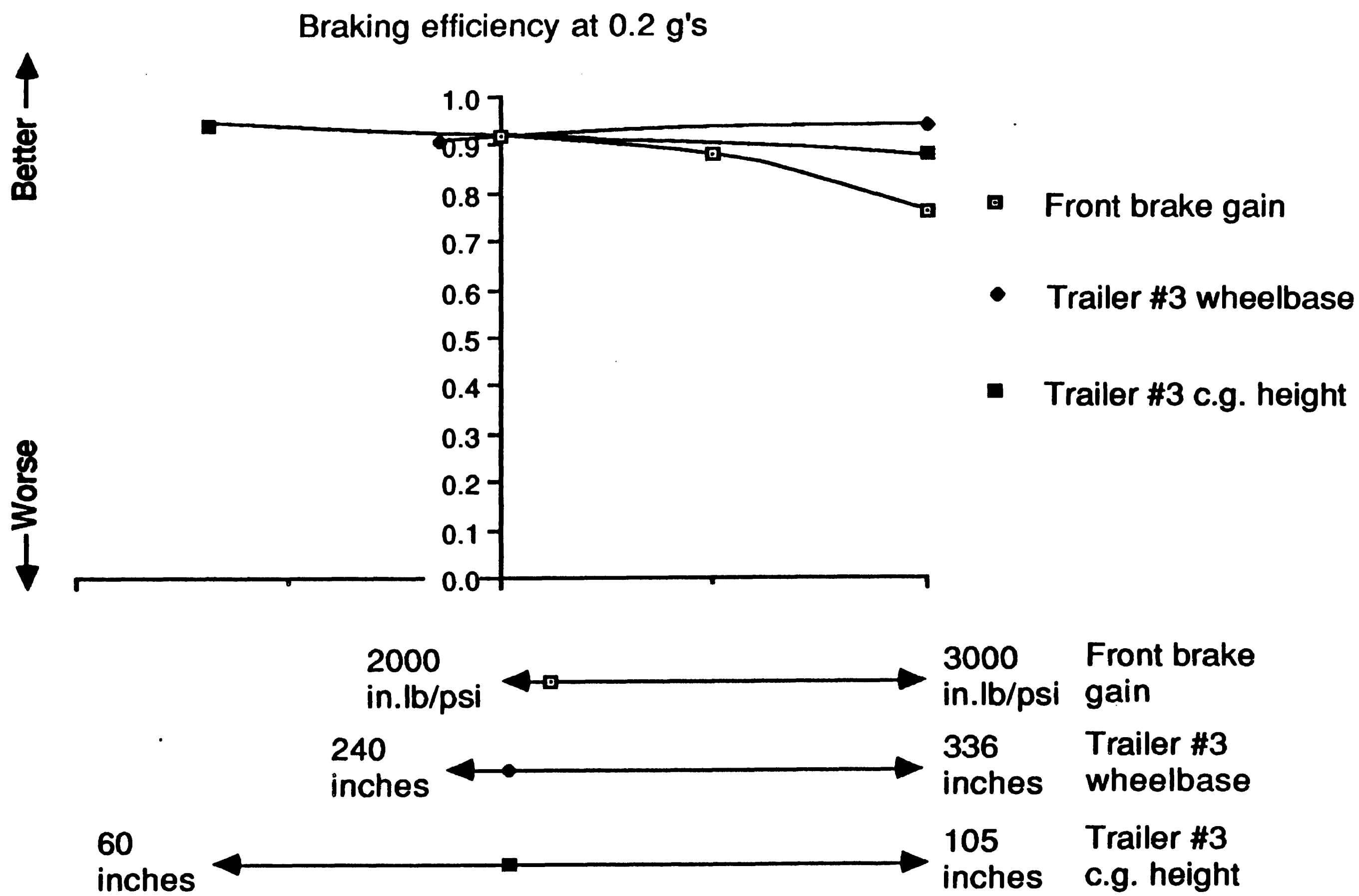


Figure 176. Parameter sensitivity diagram: braking, loaded triple.

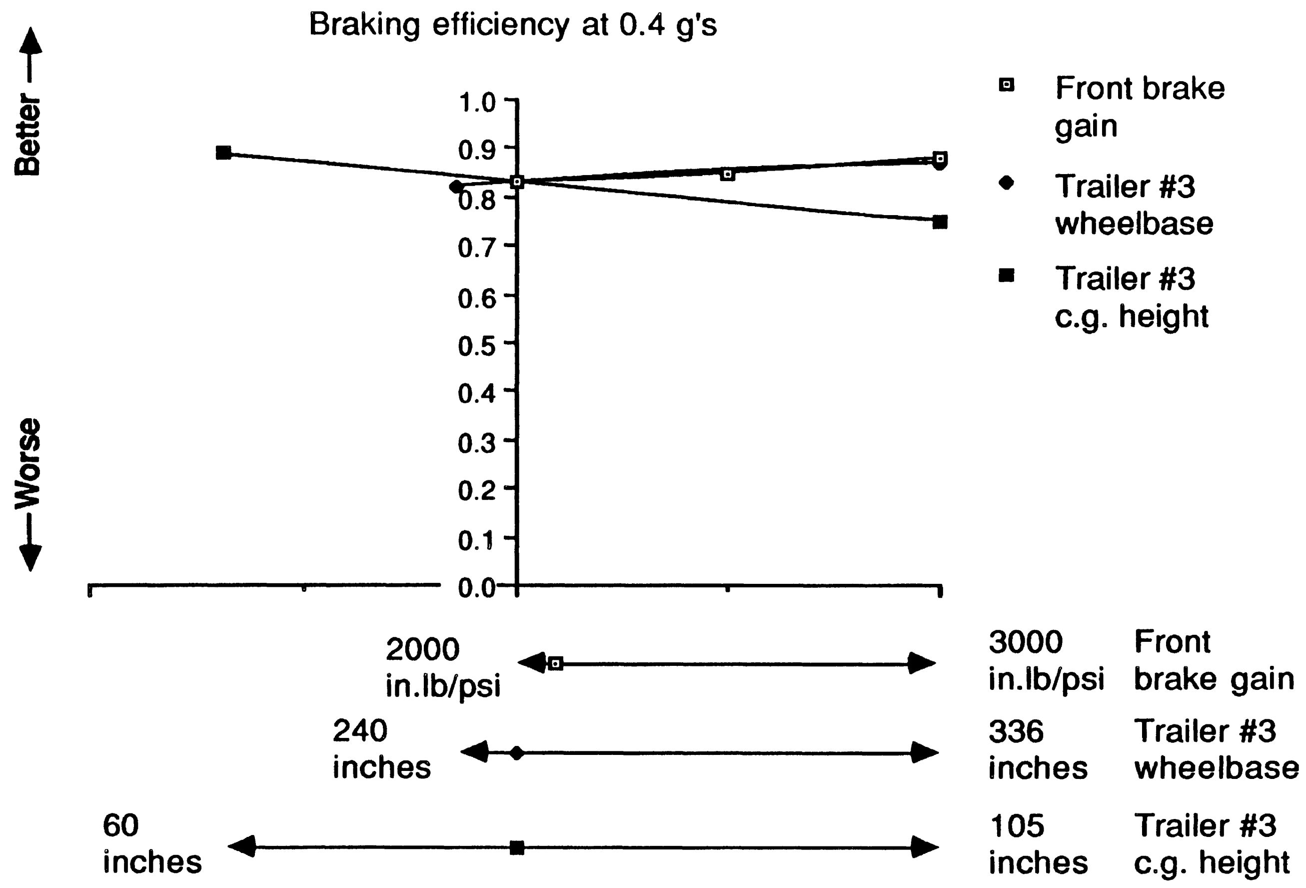


Figure 177. Parameter sensitivity diagram: braking, loaded triple.

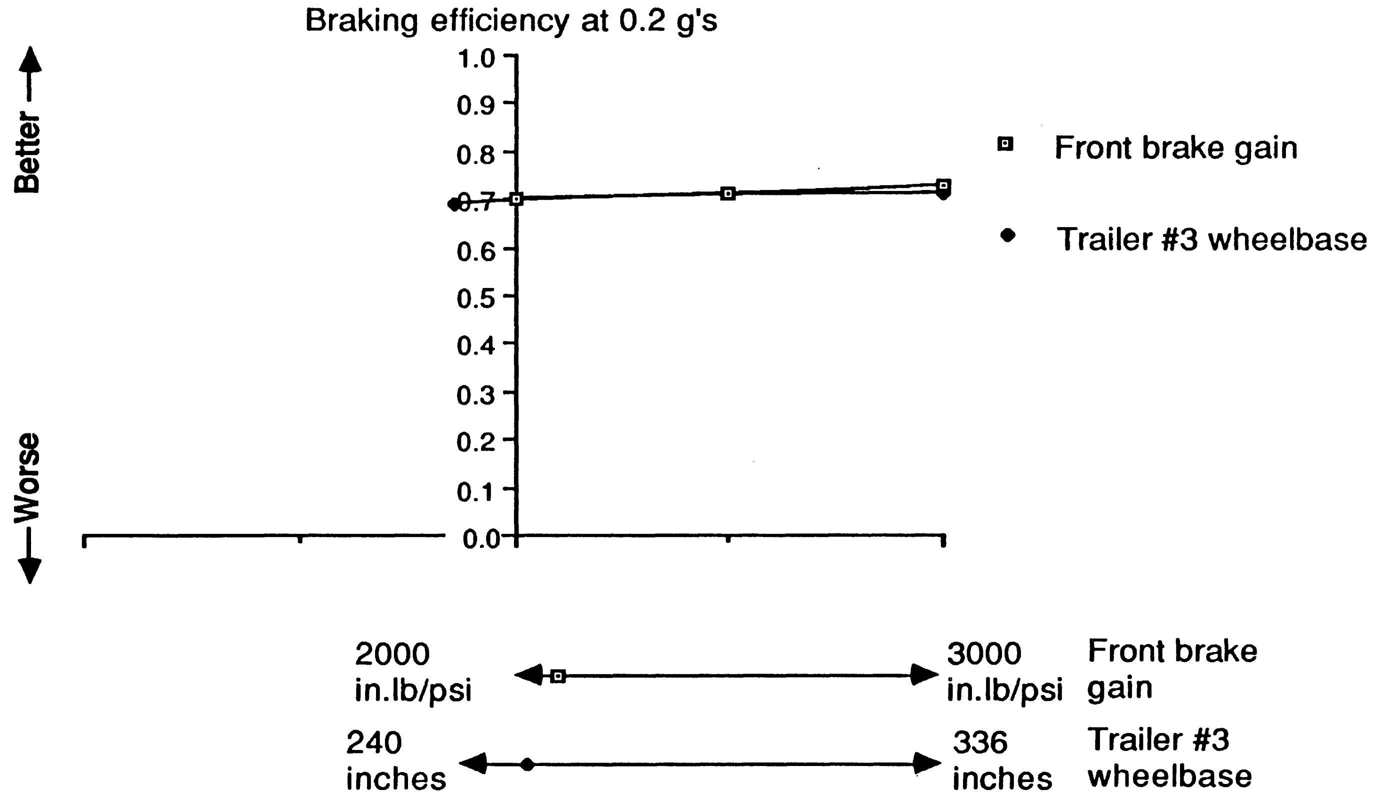


Figure 178. Parameter sensitivity diagram: braking, empty triple.

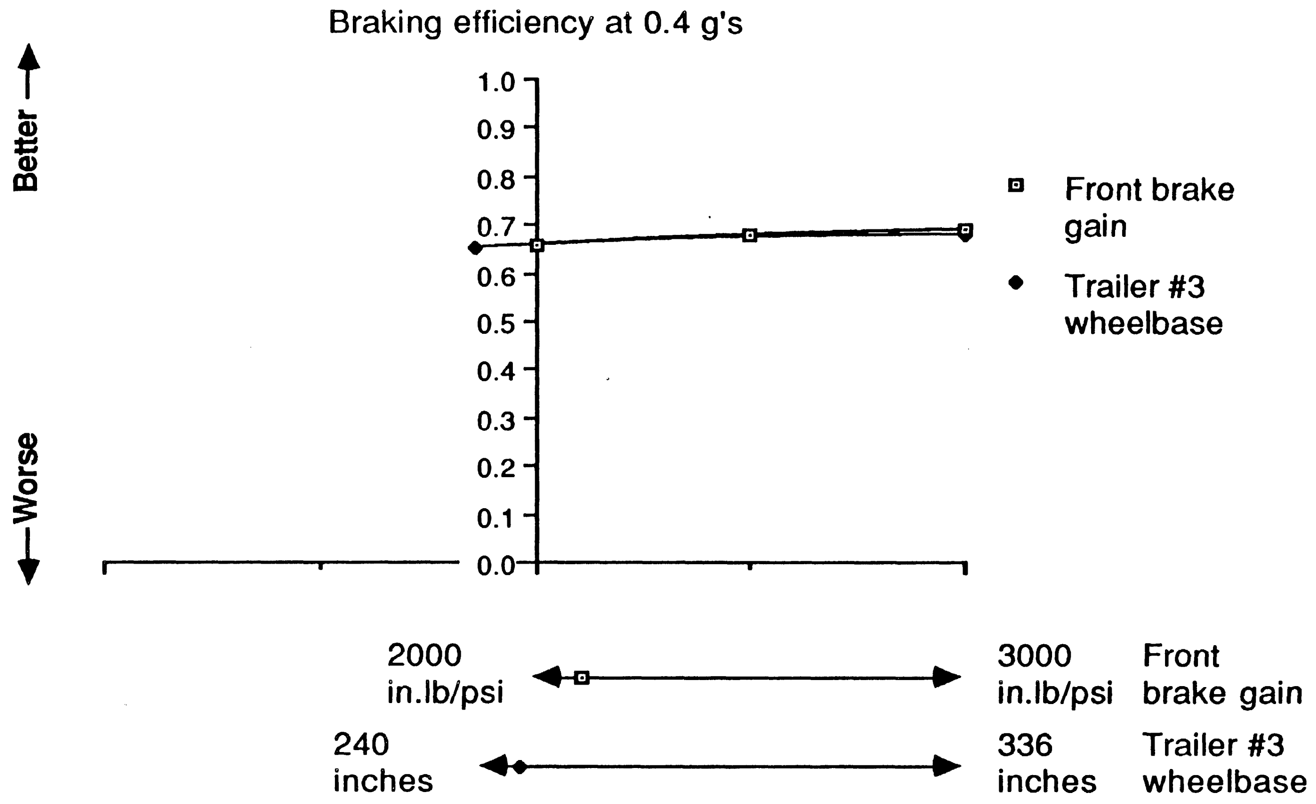


Figure 179. Parameter sensitivity diagram: braking, empty triple.

8.4 High-Speed Offtracking (Steady Turn - Tracking)

Benchmark performance. The offtracking of pertinent points along the centerline of the benchmark triple are given in Table 48. At 55 mph on a 1,200-foot-radius turn, the last axle offtracks by 1.83 feet. On a 600-foot turn with the vehicle traveling at 38 mph the last axle offtracks by 1.1 feet.

Parametric sensitivities. The mechanical properties of the dolly and semitrailer of the last full trailer are examined here (see Table 49). The results (see Figures 180 through 183) provide illustrations of the fundamental principles that (1) increasing cornering stiffnesses of the tires will decrease high-speed offtracking and (2) there is a worst-case wheelbase for each speed.

8.5 Steady Turn - Roll

The rolling performance of the triple is similar to that of the double because the last trailer of the triple is like the last trailer of the double and the vehicles are similar in other respects. Accordingly, the roll analyses for these two vehicles are practically equivalent. See the discussion in Section 7.5 to obtain information on the influences of changes in mechanical properties on the rollover thresholds of this type of vehicle combination whether it is a double or triple.

8.6 Steady Turn - Handling

The pintle hitch transmits no moment and little side force between the last two trailers when the triple is executing a steady turn. Since the front of the triple is the same as the double, the previous analysis for the double in Section 7.6 pertains to the triple, also. Changes in the mechanical properties of the last full trailer will have negligible influences on handling.

8.7 Rearward Amplification

The benchmark triple has a high level of rearward amplification. Its amplification is equal to the amplification of the double multiplied by the amplification between the motion of the c.g. of the first full trailer and the motion of the c.g. of the second full trailer in the triple. To first approximation, the amplification between the first and second full trailer is 2.2 at 3.0 radians/sec. Since the double has a maximum amplification of 2.1, the maximum amplification for the corresponding triple is roughly 4.5. This result is very sensitive to the amplification between the c.g.'s of adjacent trailers. For example, if a double has an amplification of 2.0, the amplification of the triple obtained by adding another identical trailer would be roughly 4.0, but if the amplification between adjacent trailers were 1.8, the overall amplification would be approximately 3.24.

Table 48

Simple Models:O.Triple

Velocity = 80.6667 ft/sec
Radius of the turn = 1200 ft

Suspension #	Rear Articulation Point on Unit #	Zero Speed Radius	High Speed Radius
1		1200.00	1200.00
2		1199.95	1200.25
	1	1199.95	1200.23
3		1199.77	1200.61
	2	1199.76	1200.70
4		1199.74	1200.86
	3	1199.74	1200.86
5		1199.56	1201.25
	4	1199.56	1201.33
6		1199.54	1201.50
	5	1199.54	1201.50
7		1199.35	1201.83
	6	1199.35	1201.92

Zero Speed Offtracking = 0.65 ft
High Speed Offtracking = -1.92 ft

Table 49. Triple - High-Speed Offtracking

Parameter Name	Benchmark Value	Minimum Value	Maximum Value	Notes
Second dolly's wheelbase	80 inches	72 inches	96 inches	
Second dolly's cornering stiffness	1909.1 lb/deg	954.5 lb/deg	2863.6 lb/deg	Cornering stiffness is determined from loaded vehicle data
Third trailer's wheelbase	252 inches	240 inches	336 inches	
Third trailer's cornering stiffness	1885.2 lb/deg	942.6 lb/deg	2827.8 lb/deg	Cornering stiffness is determined from loaded vehicle data

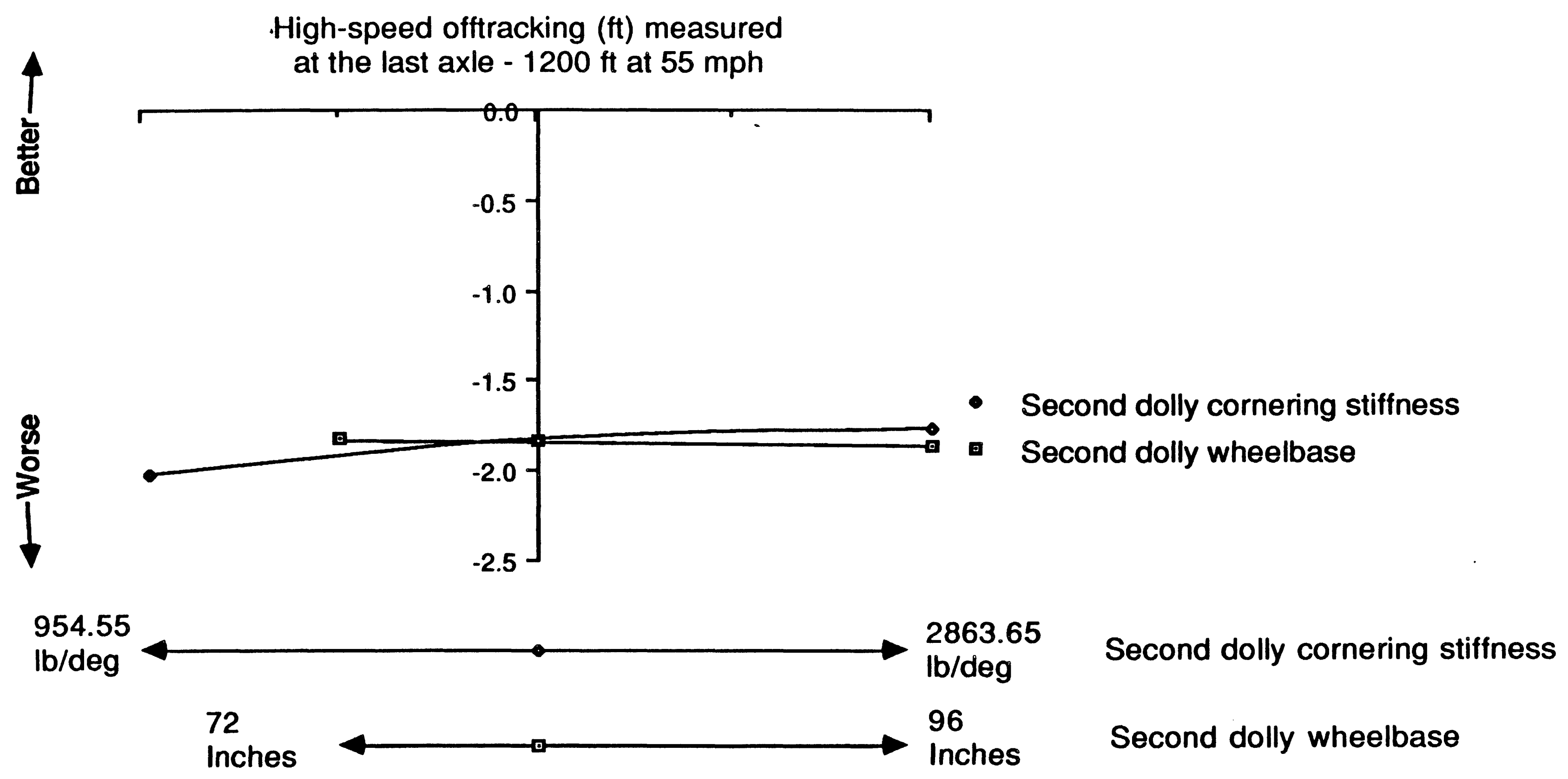


Figure 180. Parameter sensitivity diagram: high-speed offtracking, triple.

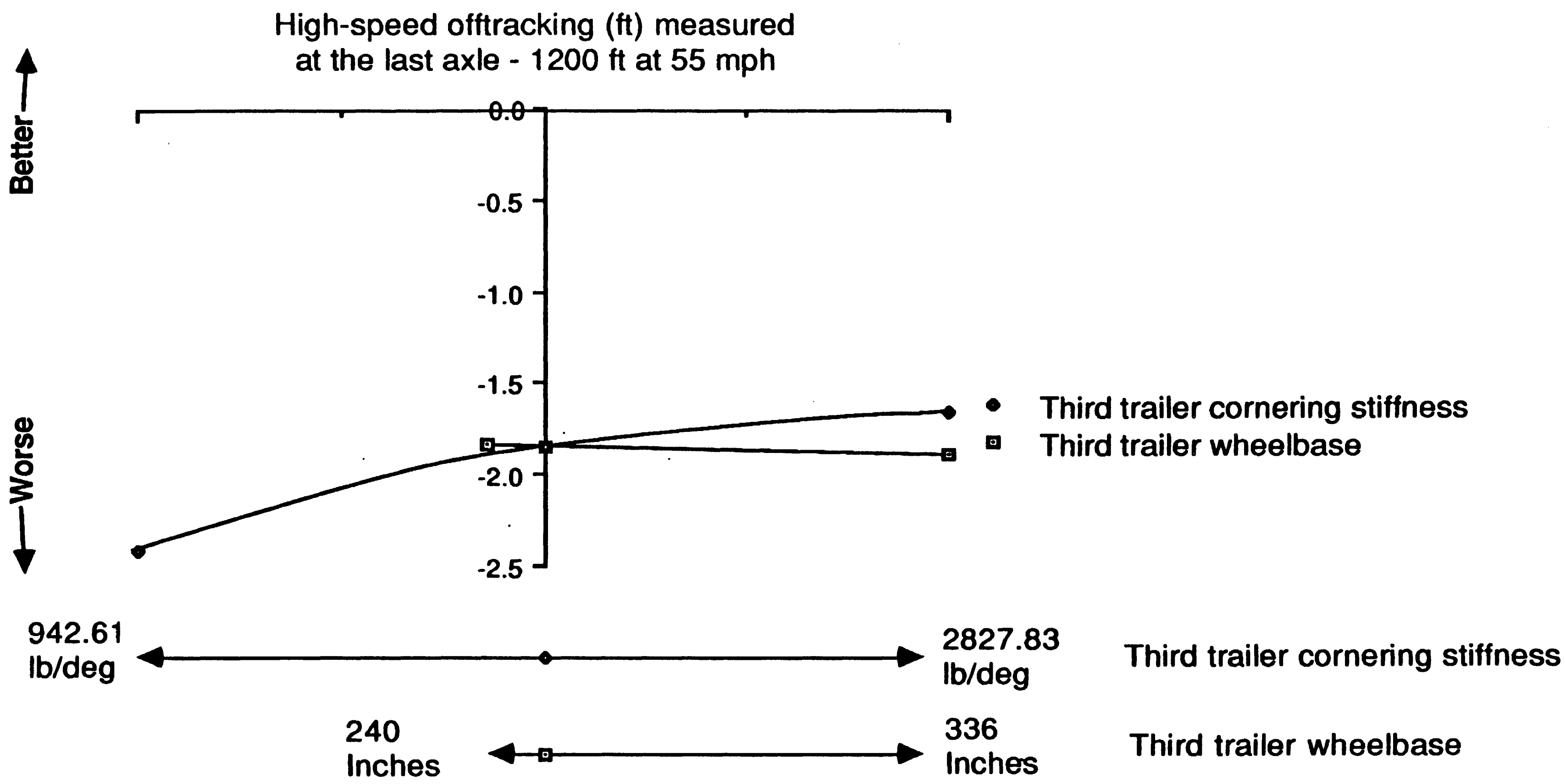


Figure 181. Parameter sensitivity diagram: high-speed offtracking, triple.

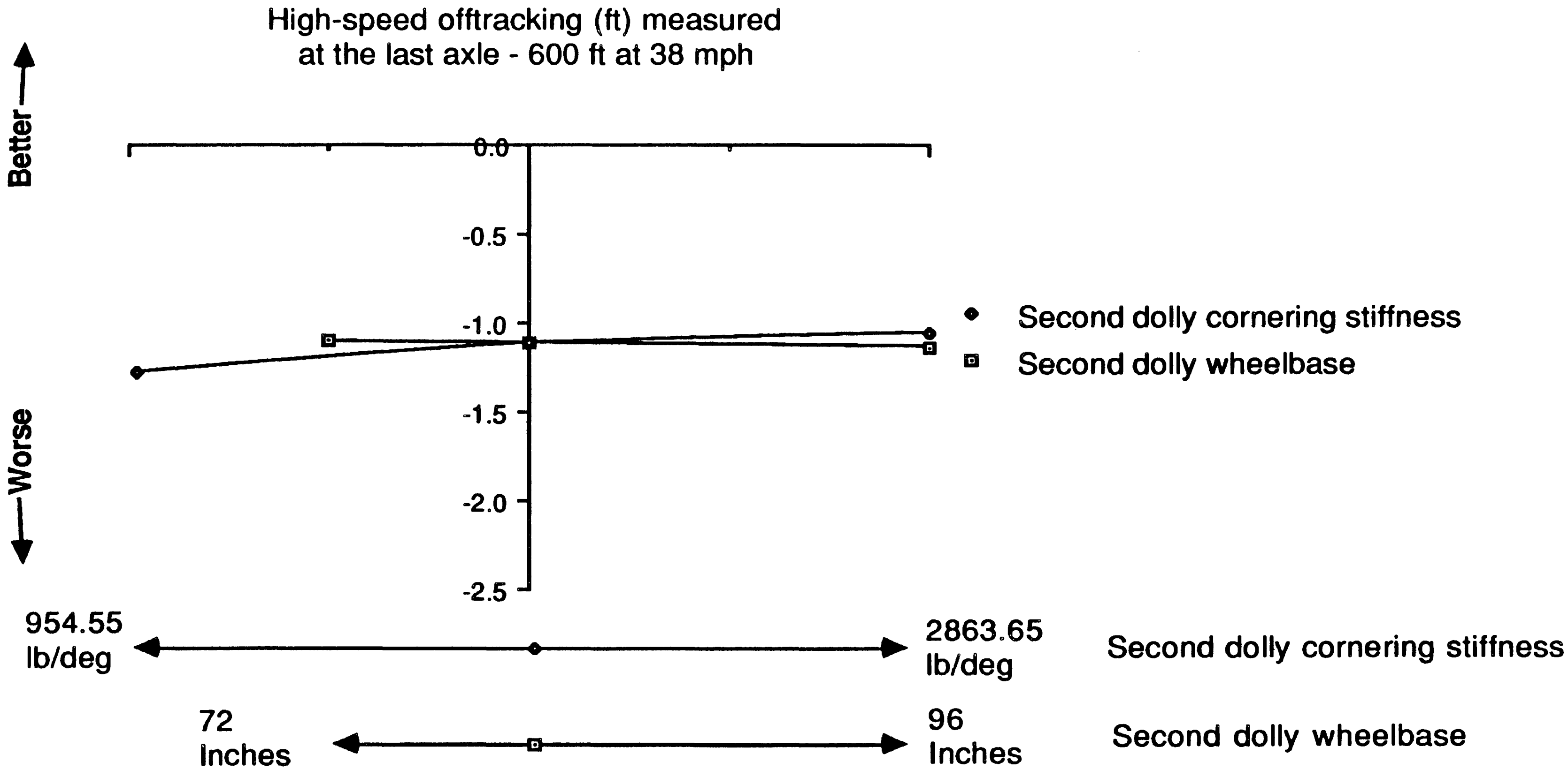


Figure 182. Parameter sensitivity diagram: high-speed offtracking, triple.

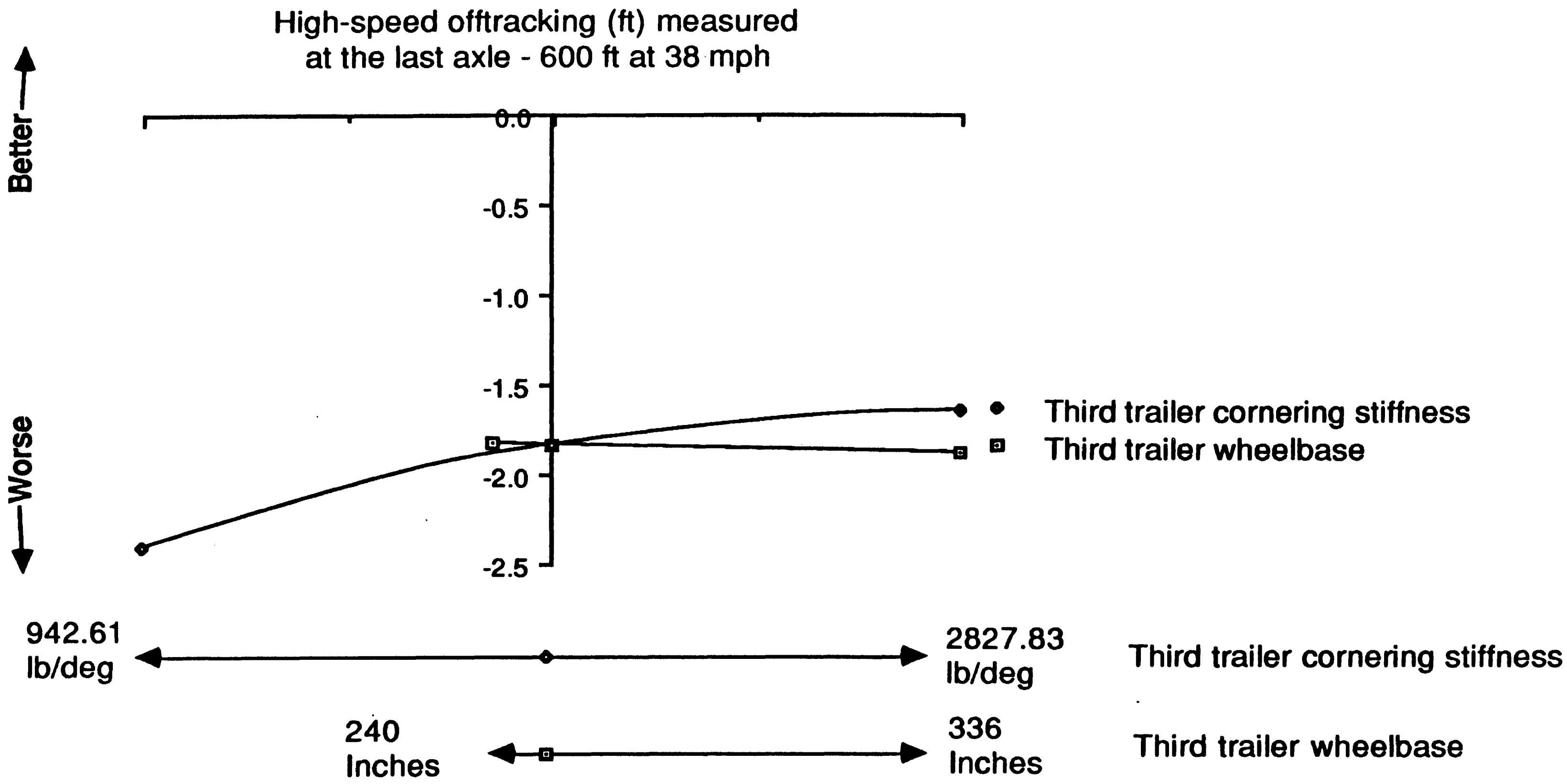


Figure 183. Parameter sensitivity diagram: high-speed offtracking, triple.

As discussed in Section 2.5, the primary mechanical factors influencing rearward amplification are wheelbases, hitch overhangs, and tire cornering stiffnesses. The performance sensitivities presented in Section 7.8 apply to the triple as well as to the double.

8.8 Concluding Remarks on the Triple

The triples configuration serves to amplify the performance limitations possessed by its corresponding doubles configuration in trailing fidelity, that is , in offtracking and rearward amplification. The drivers of currently assembled triples need to be very careful to avoid traffic conflicts that require sudden maneuvers to resolve. The danger of rolling over the last trailer is extraordinarily large if the unit is loaded so that the c.g. is high. In any case, the path of the rear axle of the vehicle will deviate considerably from that of the rearmost axle in low- and high-speed turning maneuvers and in obstacle-avoidance situations. A possible approach for alleviating these difficulties is to develop improved dolly concepts that will provide enhanced trailing fidelity.

9.0 A PROCEDURE FOR USING THE HANDBOOK TO EVALUATE A PROPOSED VEHICLE CONFIGURATION

Even though this handbook contains a large base of information on vehicle performance, the application of this information to a variety of performance goals is an art requiring skillful interpretations of principles applying to the dynamics of heavy trucks. To aid in employing and developing the knowledge-base presented in the handbook, relevant dynamical principles have been incorporated into simplified models that may be used for making first order predictions of vehicle performance in selected maneuvers. Given the understanding and knowledge-base provided by the Component Factbook and the Vehicle Dynamics Handbook, the purpose of this section is to outline how the knowledge-base presented in the handbook might be applied in a vehicle synthesis process aimed at improving braking and steering performances. (See SAE paper No. 870494 for an expanded version of this material [32])

9.1 Basic Features of a Vehicle Synthesis Process

"Vehicle synthesis" implies the use of performance objectives or targets in specifying or designing a vehicle [31]. The general intention is to emphasize solving for properties of a vehicle system that will satisfy performance targets rather than developing a vehicle design or a prototype vehicle first and then assessing its performance. In a systems analysis context, the idea is to change from an analysis problem to a synthesis problem. As illustrated in Figure 184, that is to change from (a) knowing the input and the system, and then solving for the output to (b) knowing the input and the desired output, and then "solving" for the system.

The synthesis process depends upon the ability to predict the influences of changes in mechanical properties on vehicle performance. The existence of conceptual models of the vehicles involved is implicit in this approach. Analysis-software and parametric information on the mechanical properties of components are assumed to be available. (In this regard, the component factbook and the vehicle dynamics handbook represent data and capabilities that are currently available.)

Vehicle synthesis, given an emphasis towards predictions based on simplified conceptual models, results in only a first order approximation to the exact performance desired. Nevertheless, the resulting designs are expected to lead to prototype vehicle systems that come close to the targets

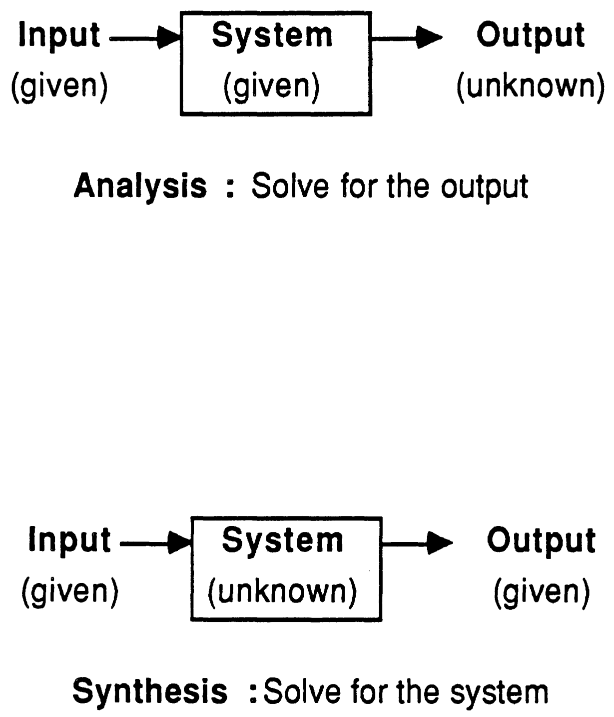


Figure 184. Synthesis compared to analysis.

used in the synthesis process. These prototypes should only require optimization rather than complete redesign during development.

9.2 Elements of a Proposed Synthesis Process

The notion of "chassis design synthesis" as described by Topping [31] has been adapted herein to fit the objective of aiding in improving braking and steering performances of heavy trucks. Although Topping addressed passenger cars and the three performance modes associated with ride and pitch control, roll, and steering-induced lateral dynamic motions of passenger cars, his generalized presentation of the design synthesis method provides a logical framework for the flow of the technical, practical, and pragmatic decisions that will be discussed here.

The proposed vehicle synthesis process is portrayed by the flow chart presented in Figure 185. As illustrated in Figure 185, certain initial decisions are required before beginning the solution of the first performance mode corresponding to the first maneuvering situation. The process begins with the selection of the maneuvering situations to be used in the synthesis. For each of these situations, one needs a basis for evaluating vehicle performance. Clearly, the intention here is to use the maneuvering situations, performance signatures, and measures employed in the handbook.

In addition to establishing performance measures (that is, performance variables), the mechanical properties whose values will be determined by the synthesis process need to be established. The pertinent mechanical properties (PMP's) are those that have strong influences on vehicle performance in one or more of the selected maneuvering situations. The determination of a set of values for the PMP's constitutes a solution to the synthesis problem.

Once the initial decisions are made, the next step is to set performance targets (see Figure 185). Implicit in this is an understanding of vehicle performance in the selected maneuvering situations. The knowledge base presented in the handbook provides one source of information on performance capabilities. Other sources of information could be the results of vehicle tests, the findings of additional analyses and simulations, or knowledge of the capabilities of competitive vehicles. However, the synthesis process has an iterative feature that provides for the possibility of changing performance targets if initial choices prove to be untenable.

Given performance targets, the process of finding solutions depends upon more choices and decisions. As indicated in Figure 185, the PMP's are divided into dependent and independent categories. The independent PMP's are those that have important influences in the maneuvering situations, but their values are set by considerations other than steering or braking performance.

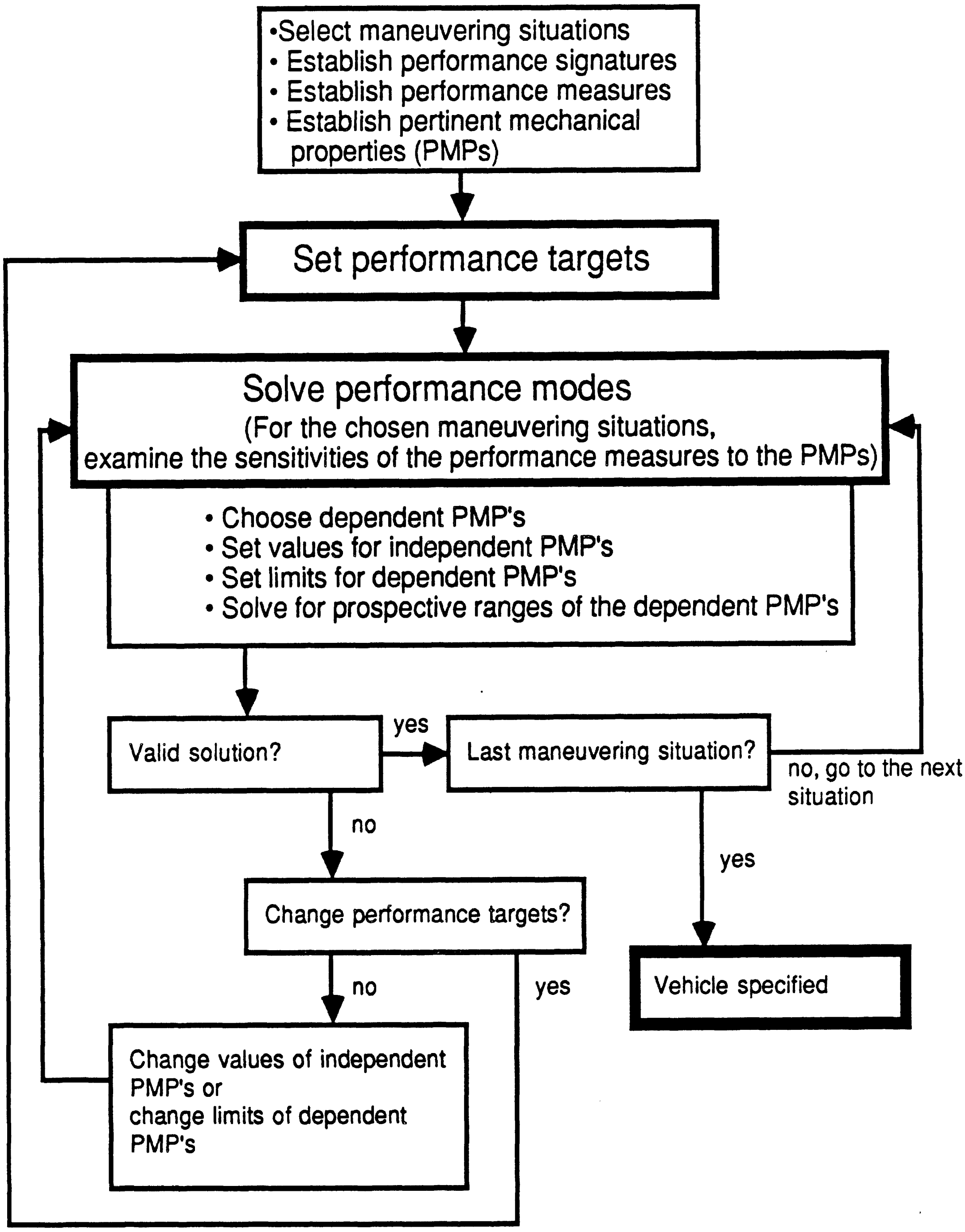


Figure 185. A vehicle synthesis procedure [32].

For example, wheelbases and c.g. heights may be determined by economic factors or the transportation mission of the vehicle. In that case, those pertinent mechanical properties will be independent variables in a synthesis problem in which the values of the other pertinent mechanical properties are the dependent variables. As already discussed, the sequence of maneuvering situations has been organized so that dependent mechanical properties from earlier maneuvering situations may become independent mechanical variables for later maneuvers (performance modes).

There are also other types of constraints on the dependent pertinent mechanical properties. The values of these properties are limited to acceptable and reasonable ranges determined by the characteristics of currently available components and the types of components the synthesizer chooses to employ. Information on the ranges of mechanical properties of basic components such as tires, suspensions, steering systems, brakes, and other items is presented in the Component Factbook. Nevertheless, the choices of ranges of properties may again depend upon matters in addition to braking and steering performance—for example, matters such as cost, endurance, licensing agreements, etc.

After finding a prospective solution for a maneuvering situation, the flow chart (Figure 185) directs the synthesizer to ask if this is a valid solution. If the solution does not violate any constraints, the process proceeds to the next maneuvering situation until all the performance targets are satisfied. If a suitable solution cannot be found, the synthesizer has three choices: (1) change the performance targets, (2) change the values of independent PMP's (which implies delving into matters other than steering and braking), or (3) change the limits of the dependent PMP's (which implies using new types of components or, at least, ones with different ranges of performance). If changes are made in any of these three items, the synthesis process may need to be repeated. However, an important function of the synthesis process is to identify and quantify conflicts between performance targets and other factors constraining the form of the vehicle system.

REFERENCES

1. Fancher, P.S. "An Evaluation of the Obstacle-Avoidance Capabilities of Articulated Commercial Vehicles." 10th Interna. Conf. on Experimental Safety Vehicles, Oxford, England, July 1985.

2. Fancher, P.S., et al. "A Factbook of the Mechanical Properties of the Components for Single-Unit and Articulated Heavy Trucks." Final Rept., Contract DTNH22-83-C-07187, Rept. No. DOT HS 807 125, December 1986.

3. Ervin, R.D., et al. "Influence of Size and Weight Variables on the Stability and Control Properties of Heavy Trucks." Final Rept., Contract No. FH-11-9577, Univ. of Mich. Transp. Res. Inst., Rept. No. UMTRI-83-10, May 1983.

4. Segel, L., et al. "Mechanics of Heavy-Duty Trucks and Truck Combinations." Univ. of Mich. Engrg. Summer Conference, June 13-17, 1983.

5. Bernard, J.E. and Vanderploeg, M. "Static and Dynamic Offtracking of Articulated Vehicles." SAE Paper No. 800151, 1980.

6. Mallikarjunarao, C., Ervin, R.D., and Segel, L. "Roll Response of Articulated Motor Trucks During Steady-Turning Maneuvers." ASME, Winter Annual Meeting, November 1982.

7. Winkler, C.B., Fancher, P.S., and MacAdam, C.C. "Parametric Analysis of Heavy Duty Truck Dynamic Stability." Final Rept., Contract DTNH-22-80-C-07344, Univ. of Mich. Transp. Res. Inst., Rept. No. UMTRI-83-13, March 1983.

8. Ervin, R.D., et al. "The Yaw Stability of Tractor-Semitrailers During Cornering." Final Rept., Contract No. DOT-HS-7-01602, Highway Safety Res. Inst., Univ. of Mich., Rept. No. UM-HSRI-79-21, June 1979.

9. Mallikarjunarao, C. and Fancher, P. "Analysis of the Directional Response Characteristics of Double Tankers." SAE Paper No. 781064, December 1978.

10. Mallikarjunarao, C. "Road Tank Design: Its Influence on the Risk and Economic Aspects of Transporting Gasoline in Michigan." Ph.D. Thesis, The University of Michigan, 1982.

11. MacAdam, C.C., et al. "A Computerized Model for Simulating the Braking and Steering Dynamics of Trucks, Tractor-Semitrailers, Doubles, and Triples Combinations - Users' Manual." Final Rept., MVMA Proj. 1197, Highway Safety Res. Inst., Univ. of Mich., Rept. No. UM-HSRI-80-58, September 1, 1980.

12. MacAdam, C.C. "Application of an Optimal Preview Control for Simulation of Closed-Loop Automobile Driving." IEEE Trans. on Systems, Man, and Cybernetics, Vol. SMC-11, No. 6, 1981, pp. 393-399.

13. Nordstrom, O. and Nordmark, S. "Test Procedures for the Evaluation of the Lateral Dynamics of Commercial Vehicle Combinations." Automobil-Industrie, Edition 2, June 1978.

14. Pacejka, H.B. "Simplified Analysis of Steady-State Turning Behaviour of Motor Vehicles." Vehicle System Dynamics, 2pp. 161-183, pp. 185-204, 1973.

15. Fancher, P.S. "The Static Stability of Articulated Commercial Vehicles." Vehicle System Dynamics, Vol. 14, 1985.

16. Ervin, R.D., et al. "Ad Hoc Study of Certain Safety-Related Aspects of Double-Bottom Tankers." Final Rept. to Michigan State Office of Highway Safety Planning, Rept. No. UM-HSRI-78-18, May 7, 1978.

17. McRuer, D.T., et al. "Measurement of Driver/Vehicle Multiloop Response Properties with a Single Disturbance Input." IEEE Trans. on Systems, Man, and Cybernetics, Vol. 5, No. 5, 1975.

18. MacAdam, C.C. "Frequency Domain Methods for Analyzing the Closed-Loop Directional Stability and Maneuverability of Driver/Vehicle Systems." Interna. Conf. on Modern Vehicle Design Analysis, London, 1983.

19. Fancher, P.S. "The Transient Directional Response of Full Trailers." SAE Paper No. 821259, 1982.

20. Fancher, P.S., et al. "Tracking and Stability of Multi-Unit Truck Combinations." Univ. of Mich. Transp. Res. Inst., Rept. No. UMTRI-84-25, July 1984.

21. Uffelmann, F. "Steering Behaviour of Motor Vehicles Under Braking." 2nd Course on Advanced Vehicle System Dynamics, Interna. Center for Transp. Studies, Amalfi, Italy, May 28-June 1, 1984.

22. Rompe; K. "Vehicle Handling Characteristics During Braking in a Turn." Presented at 8th Interna. Techn. Conf. on Experimental Safety Vehicles, Wolfsburg, October 21-24, 1980.

23. Fancher, P.S. "Integrating Antilock Braking Systems with the Directional Control Properties of Heavy Trucks." Interna. Conf. on Antilock Braking Systems for Road Vehicles, Inst. of Mech. Engrs., Oxford, England, September 1985.

24. Mischke, et al. "Contribution to the Development of a Concept of Driving Mechanics for Commercial Vehicles." Daimler Benz AG. SAE Paper No. 830643, 1983.

25. Rompe, K. and Heissing, B. "Methods of Objective Description of the Handling Characteristics of Heavy Commercial Vehicles." XXth FISITA Congress, May 1984.

26. Ervin, R.D., et al. "Effects of Tire Properties on Truck and Bus Handling." Final Rept., Contract No. DOT-HS-4-00943, Highway Safety Res. Inst., Univ. of Mich., Rept. No. UM-HSRI-76-11, December 1976.

27. Morrison, W.R.B. "A Swept Path Model Which Includes Tyre Mechanics." Proceedings, Sixth Conference, Australian Road Research Board, Vol. 6, Part 1, Canberra, 1972.

28. Sayers, M.W. "Vehicle Off-Tracking Models." Symposium on Geometric Design for Large Trucks." Transportation Research Board, Denver, Col., August 1985.

29. Gillespie, T.D., et al. "Truck and Tractor-Trailer Dynamic Response Simulation--T3DRS." Final Rept., Contract No. FH-11-9330, Highway Safety Res. Inst., Univ. of Mich., Rept. No. UM-HSRI-79-38, March 1979.

30. Bernard, J.E., Winkler, C.B., and Fancher, P.S. "A Computer-Based Method for Predicting the Directional Response of Trucks and Tractor-Trailers." Phase II Rept., Highway Safety Res. Inst., Univ. of Mich., Rept. No. UM-HSRI-PF-73-1, June 1, 1973.

31. Topping, R. W. "Chassis Design Synthesis with Application to Determination of Tire Force and Moment Coefficients." Symposium on Cornering and Handling Characteristics of Tires, Rubber Div., Amer. Chem. Soc., Los Angeles, April 23-26, 1985.

32. Fancher, P. S. and Mathew, A. "Using a Vehicle Dynamics Handbook as a Tool for Improving the Steering and Braking Performance of Heavy Trucks." SAE Paper No. 870494, Feb. 1987.

APPENDICES

SIMPLIFIED MODELS

TABLE OF CONTENTS

1.0 INTRODUCTION AND GENERAL INFORMATION

1.1 INTRODUCTION

The simplified models discussed in this report were developed by the University of Michigan Transportation Research Institute (UMTRI) under sponsorship of the National Highway Traffic Safety Administration (NHTSA). This document contains instructions for using the various computerized models and provides the specific information required for performing a simulation. It also contains a detailed description of the simplified models employed in this study.

The equilibrium analyses used here are simplified procedures that have been programmed in BASIC for use on Apple MacIntosh computers. The analyses address three types of vehicle maneuvers:

1. Steady turning at constant velocity and constant lateral acceleration,
2. Constant deceleration braking, and
3. Turning a corner at low speed.

During steady turning maneuvers, three different procedures are used to study tracking, rolling and handling.

The objectives in developing the simplified models were to,

- provide a tool for the first-order analysis of the performance of an articulated vehicle, and
- narrow the models' data requirements to only those vehicle parameters that affect the tracking/braking/rolling/handling performance of the vehicle.

With the first order estimates obtained from these models, the vehicle dynamicist is in a better position to use the more complex simulation models that include detailed representations of the components of the driver-vehicle system.

The outputs from these simulations are specific to the models used and could be numerical and/or graphical. Even at the output stage, reporting of the results is restricted to include only pertinent information.

1.2 ENGINEERING AND COMPUTER REQUIREMENTS

Throughout the models, the English system of units is used. With the exception of forward velocity which is entered in feet/second and turn radius which is entered in feet, all input data are given in the units of pounds, inches, degrees, and seconds. Masses and weights are in units of pounds, with a gravitational constant of 386 in/sec/sec assumed.

The simplified models are programmed in Microsoft Basic for use on an Apple Macintosh computer having a minimum of 512 Kilobytes of random access memory (RAM). All printer control instructions have been written for an Apple ImageWriter I.

2.0 USING THE MODELS

2.1 ON THE DESKTOP

The disk "Simple Models" consists of four folders and a BASIC program called "Models".

- The System Folder contains seven files which are required for the error-free execution of the various models. These files should remain on the disk at all times.
- The remaining folders named Brake (for straight–line braking), T/R (for handling and static roll), and Offtrack (for high and low speed offtracking) each contain their respective models and a selection of example data sets.
- The program "Models", which can be opened from the desktop, allows the user to access the different models (Figure 2.1).

2.2 ACCESSING THE VARIOUS MODELS

Selecting a particular model is as easy as pointing to the desired name (in Figure 2.1, the Braking model) and clicking the mouse button. The circle to the left of the model gets filled to confirm the user's choice. Clicking on another model would automatically deselect the previous choice. A simulation is launched as soon as the OK button is selected.

Two points are worth noting at this point,

- the OK button cannot be selected until a menu selection has been made, and
- selecting the CANCEL button returns the user to the desktop.

2.3 PRE-SIMULATION PROCESSING

To reduce confusion and any additional programming effort, the different models use similar pre and post processing procedures. The starting menu of each simulation looks very similar to the one shown in Figure 2.2.

2.3.1 New Data. If a data set has not been defined (as is the case soon after entering the pre–processor) the starting menu looks a little different from the menu shown in Figure 2.2 - the user is restricted to selecting,

- New Data,
- Other Models (see Section 2.3.6), or
- CANCEL (which returns the user to the desktop).

Selecting the first button in Figure 2.2, gives the user the option of entering data from the keyboard or accessing data from an existing data file. In the case of keyboard entry, the pre–processor guides the user through the different parameters required by the models (the models' data requirements are discussed in Chapter 3).

Due to the difference in the parameters required by the various models, each simulation has its own data set. As a result, each model can only read disk files that were created by itself (in other words, each model can only access data sets that it considers to be "valid"). Therefore, three data sets (for Braking, Offtracking, and Roll/Handling) are required to completely describe a given vehicle. To differentiate between themselves, the names of the example data sets have been prefixed with B, O, or T (for Braking, Offtracking, and Roll/Handling). For example, B.Tract./Semi is the name given to the Braking model's data set for a tractor and semi-trailer.

If the case of accessing data from a disk file, the pre–processor gives the user a list of all the text files on the disk (see Figure 2.3). The pre–processor continues to display these text file names until a "valid" data set is chosen or until the CANCEL button is selected.

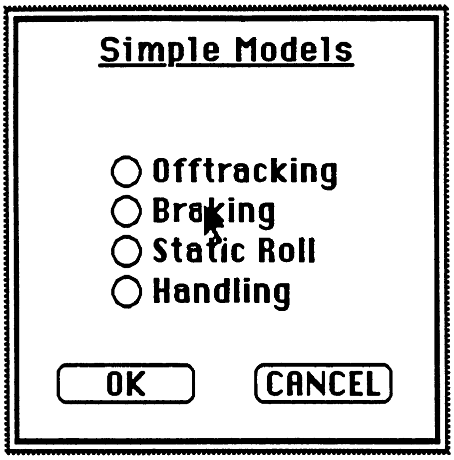


Figure 2.1 Simple models available

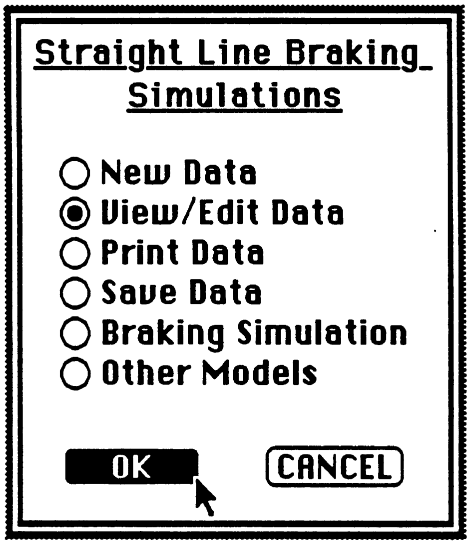


Figure 2.2 Menu displaying various choices

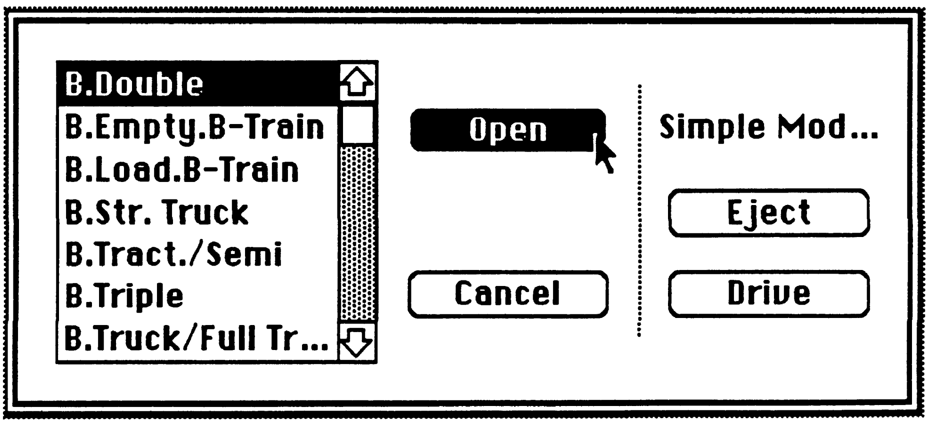


Figure 2.3 Data files being accessed from the disk

2.3.2 View/Edit Data. The second menu item in Figure 2.2 gives the user the opportunity to see and change values of parameters. Figures 2.4 and 2.5 are examples of one such exercise where the parameters pertain to the wheel and brake information of a particular axle. Though the menu shown in Figure 2.4 might differ among the models (depending upon the organization of the data), the menu of Figure 2.5 is standard.

Clicking the mouse button with the pointer on a particular parameter, selects the variable for change (Refer to the brake gain in Figure 2.5). Once a number has been selected, the keyboard can be used to enter the new value.

2.3.3 Print Data. This option uses a printer to generate a hard copy of the current data set. Note: All printer control instructions have been written for an Apple Imagewriter I.

2.3.4 Save Data. The fourth menu item of Figure 2.2 allows the user to save the current data set as a disk file. The pre–processor prompts the user for a file name and allows him/her to switch between floppy disks.

2.3.5 Simulation. This option transfers control from the pre–processor to the specific simulation program. Section 2.4 addresses the simulation process in more detail.

2.3.6 Other Models. Selecting the last menu item returns the user to the menu of Figure 2.1.

2.3.7 OK and CANCEL. As mentioned earlier, selecting CANCEL would return the user to the desktop. Also, the OK button remains inactive until a menu selection has been made.

2.4 THE SIMULATION PROCESS

Before a simulation can begin, the user must define the simulation parameters. Simulation parameters vary depending upon the model being used.

Model	Simulation Parameter
1. Low Speed Offtracking	- Radius of the turn - Angle of the turn - Increment of the calculation (distance along the track in feet)
2. Straight–Line Braking	- Printed copy of the results - Increment of the calculation (treadle pressure in psi)
3. High Speed Offtracking	- Printed copy of the results - Radius of the turn - Forward velocity
4. Static Roll	- Printed copy of the results - Increment of the calculation (total roll angle in radians)
5. Handling	- Printed copy of the results - Method of computation (partial derivatives vs. finite differences) - Forward velocity - Increment of the calculation (lateral acceleration in g's)

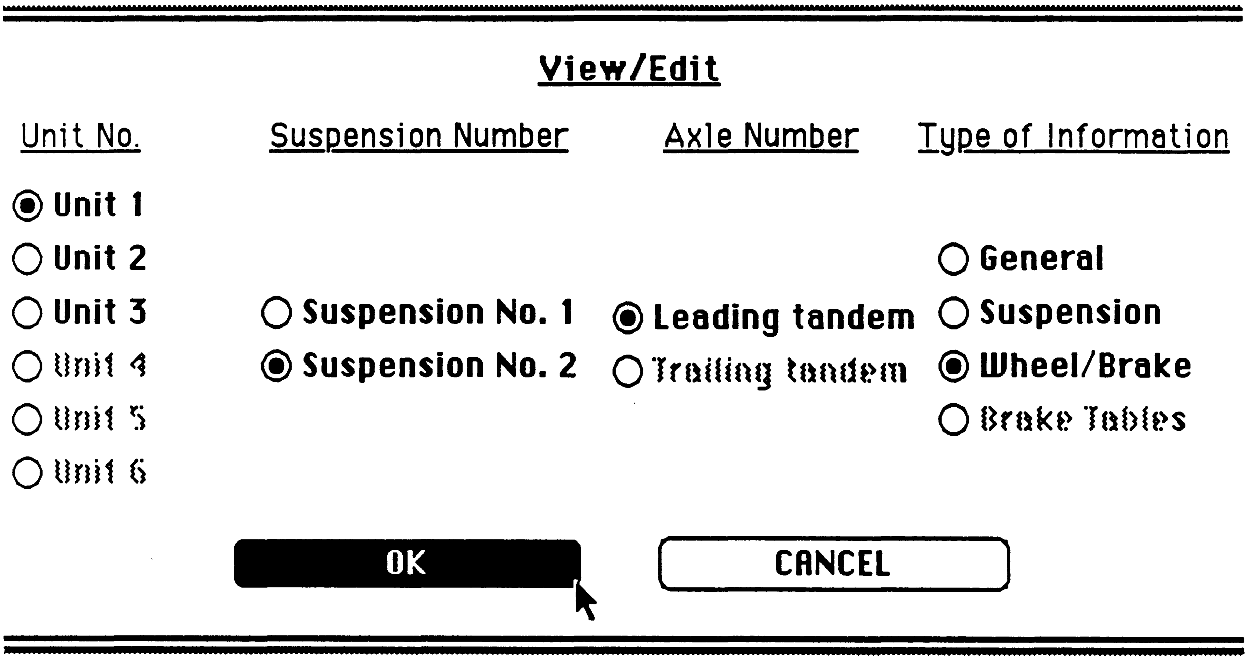


Figure 2.4 Selecting the type of information to be viewed/changed

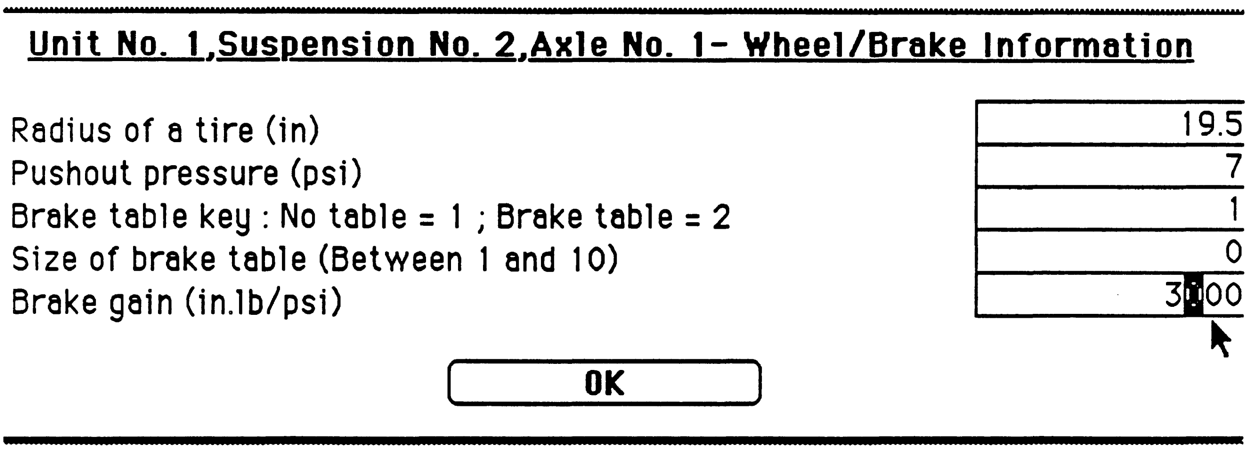


Figure 2.5 Special windows allow the use of the mouse and the keyboard while changing variables on the screen

The simulation parameters are assigned default values but can be set with the help of a menu, very similar to the one shown in Figure 2.6. It should be easy to determine that the menu in Figure 2.6 pertains to a braking simulation.

All the increments of calculation are additive and play a fairly important role in the simulation procedure. Decreasing the size of the increment increases the accuracy of the results but would also increase the time required for computation.

Once a simulation gets underway there is very little interaction between the user and the program. In addition to the results displayed on the screen (see Figure 2.7) and possibly on paper, the simulation program saves selected results on the disk for future processing. It is therefore good practice to have at least 10 Kilobytes of disk space before starting a simulation.

The simulation procedure can be terminated in one of two ways,

i. by the simulation program itself, or
ii. by the user (see Figure 2.7)

The following table displays the rules for an internal termination of a simulation.

Model	Rule
1. Low Speed Offtracking	- Completion of the turn.
2. Straight–Line Braking	- Treadle pressure equal to 100 psi - A friction utilization becomes greater than one or becomes negative.
3. High Speed Offtracking	- Completion of the turn.
4. Static Roll	- Lateral acceleration starts to decrease with increasing total roll angle.
5. Handling	- An axle lifts off.

At the end of a simulation, the program gives the user the option of returning to the pre–processor (Section 2.3/Figure 2.2) or of transferring control to the post–processor.

2.5 THE POST-PROCESSOR

The models use a general purpose plotting routine which produces graphs of variables generated during the simulation procedure. The post–processor allows the user to create both single and multiple plots.

Figure 2.8 displays the menu that the post–processor generates after a braking simulation. The round buttons determine the variables available for plotting. The square buttons, on the other hand, are plot control variables. Selecting "Scatter" forces the post–processor to locate points on the chart without connecting them with line segments. Selecting "Multiple" allows the user to select the variables for a multiple plot. The square buttons are of the "On/Off" variety, in that, they get selected and deselected by alternate mouse clicks.

The first step in creating a chart is to determine the variables to be plotted. For a single plot, the post–processor allows the user to select any two variables and assigns the most recently picked variable to the X axis (see Figure 2.8). For multiple plots, the program allows the user to select any number of variables to be plotted against the first variable in the list (in Figure 2.9, Pressure is the independent variable). The procedure to clear a selection of variables is to toggle between single and multiple plots (Note: CANCEL will return the user to the pre–processor - Section 2.3

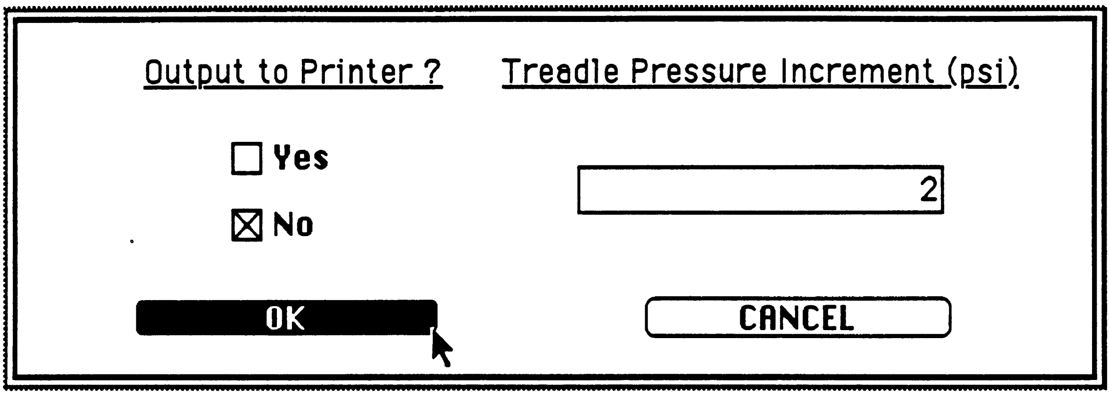


Figure 2.6 Setting simulation parameters

Simulation Control

Stop Simulation

1	1	[illegible]	0.00	10007.96	0.0000
1	2	1	0.00	17486.90	0.0000
2	1	1	0.00	17505.14	0.0000
3	1	1	0.00	17500.00	0.0000
3	2	1	0.00	17500.00	0.0000

Treadle Pressure = 10 psi

Deceleration = 2.692308E-02 g's

Braking Efficiency = .8989313

Unit No.	Suspension No.	Axle No.	Brake Force (lb)	Vertical Load (lb)	Friction Utilization
1	1	1	307.69	10273.50	0.0300
1	2	1	461.54	17412.11	0.0265
2	1	1	461.54	17314.39	0.0267
3	1	1	461.54	17782.24	0.0260

Figure 2.7 Window displaying results

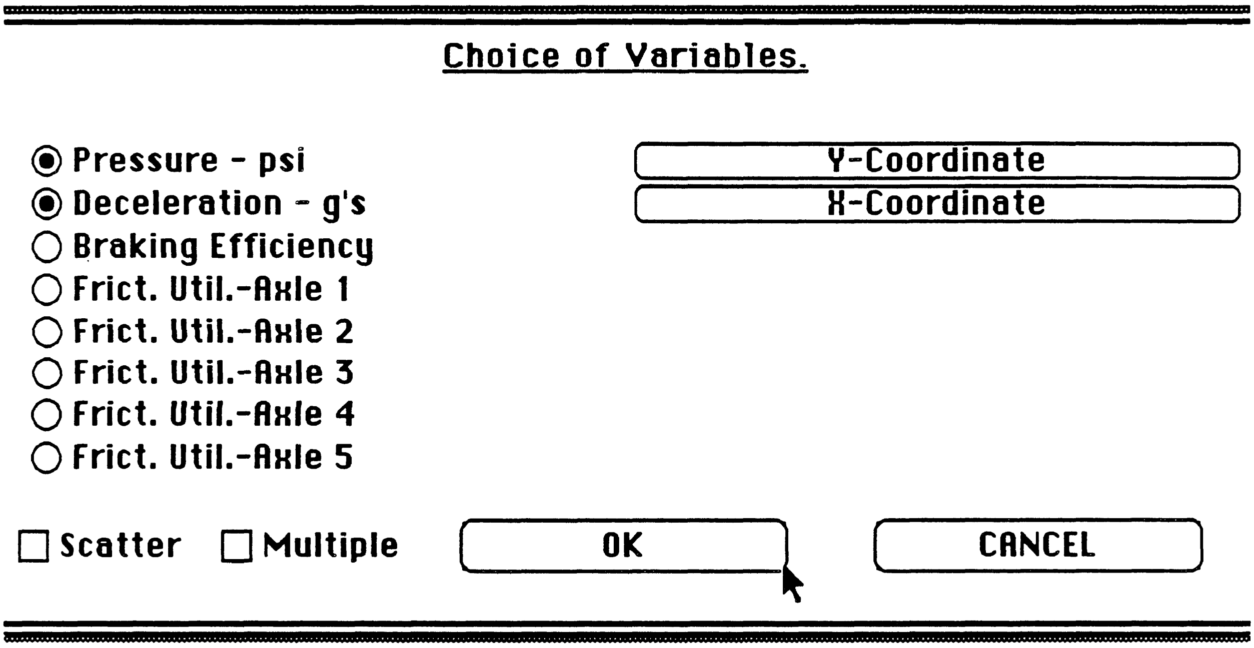


Figure 2.8 Choosing two variables to be plotted against each other

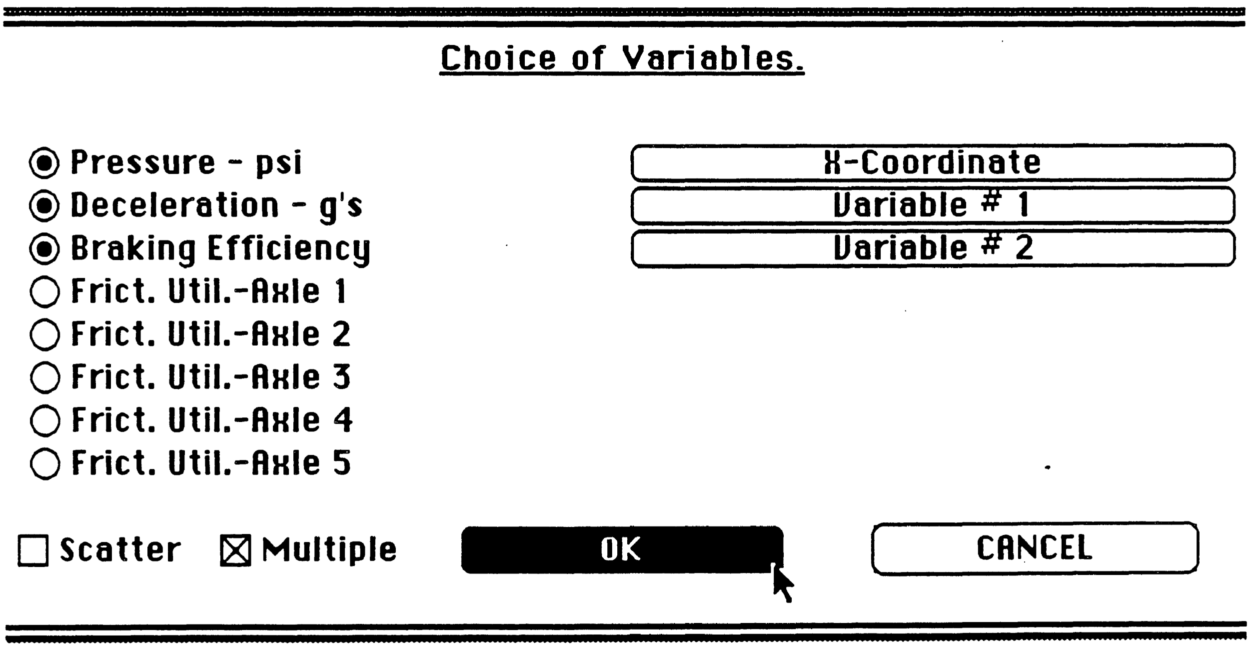


Fig 2.9 Choosing more than one variable to be plotted against Pressure

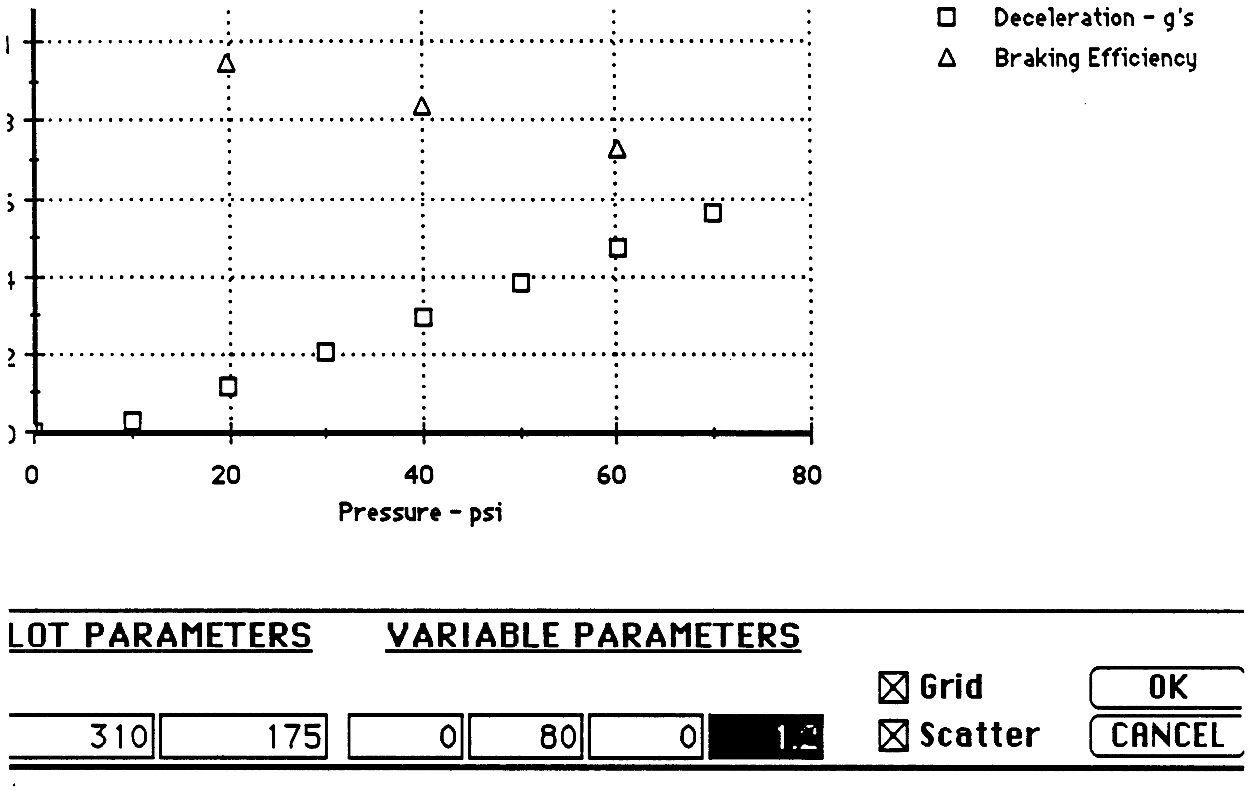


Figure 2.10 Controlling the appearance of the plot

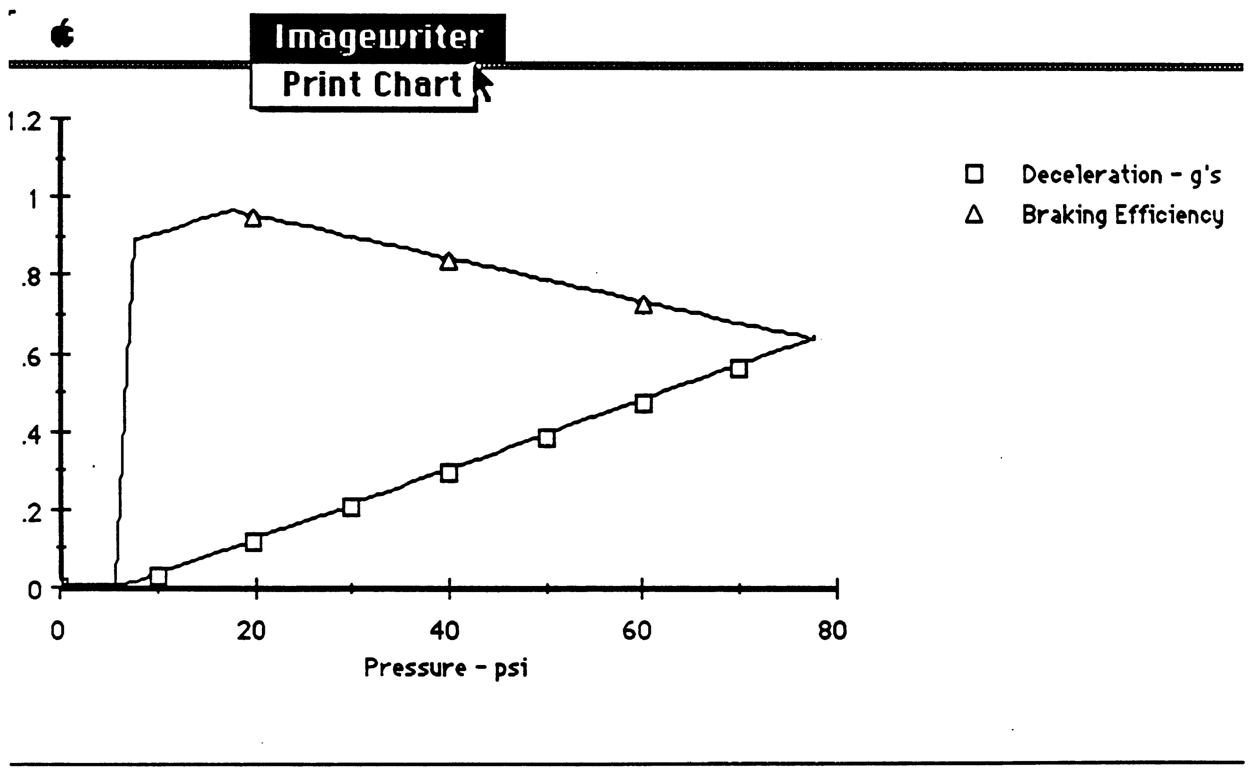


Figure 2.11 Getting a hard copy of the plot

and Figure 2.2). As usual, the OK button can be selected only after a "valid" menu selection has been made.

The post–processor automatically scales the plot and creates the displays shown in Figures 2.10 and 2.11. The lower section of Figure 2.10 shows the parameters that can be controlled by the user. The "Plot Parameters" define the size of the plot in pixels (X dimension, Y dimension of the box). The "Variable Parameters" define the variable ranges for the two axes (X minimum, X maximum, Y minimum, and Y maximum). The two square buttons are plot parameters which are used in the same fashion as the square buttons of Figure 2.8. Selecting OK returns a chart with the most recent choices of the plot settings. Selecting CANCEL, on the other hand, would return the user to the menu of Figure 2.9.

A printed copy of the chart can be generated by selecting the menu item "Imagewriter" as shown in Figure 2.11.

Note : Every effort has been made to ensure that the program statements are correct and result in a solution of the problem to a reasonable level of precision. Nevertheless, if programming errors are discovered, the user should contact the Engineering Research Division, The University of Michigan Transportation Research Institute, Ann Arbor, Michigan 48109.

If any of the simulations should "crash", the user should press the Control (the key to the immediate left of the Space Bar) and Period (.) keys together. To return to the Desktop, the user should type the words System in the "Command Window" that appears.

3.0 DATA REQUIREMENTS

3.1 LOW SPEED OFFTRACKING

3.1.1 Data Required. The Low Speed Offtracking Model incorporates a simple algorithm that computes the offtracking of the various units of any vehicle that can be modeled as a train of semitrailers. The analysis makes use of the most fundamental of vehicle parameters - the "offtracking dimensions".

The "offtracking dimensions" are defined as follows ,

1. Wheelbase: Distance [in inches] between the rear suspension and
   - the front suspension (for trucks and tractors),
   - the forward articulation point (for semitrailers and dollies).
2. Hitch location: Location of the rear articulation point [in inches] with respect to,
   - the front suspension (for trucks and tractors)
   - the forward articulation point (for semitrailers and dollies)

The "offtracking dimensions" described above are to be entered into the model in inches. Exhibit 3.1 contains a low speed offtracking data set for a tractor and semitrailer.

3.1.2 Overlap with the High Speed Offtracking Model. To conserve disk space, the High and Low Speed Offtracking models share the same data set. Suspension loads and cornering stiffnesses are vehicle parameters that are used solely by the High Speed Offtracking model. Setting these variables to zero, before running the Low Speed Offtracking model, is perfectly acceptable. Note: Zero values for these parameters would cause the High Speed Offtracking model to "crash".

3.2 BRAKING

3.2.1 Data Required. This constant deceleration braking procedure examines the proportioning of the braking system by calculating the friction level required at each axle to prevent a wheel lock at an axle. The ratio of deceleration to the highest friction level, required at any axle, is the braking efficiency of the vehicle at that level of deceleration.

This procedure distinguishes between single suspension (semitrailers and dollies) and dual suspension (tractors, trucks and full trailers) units and uses the following information to predict friction utilization at each axle:

For each unit:

1. Total (Sprung + Unsprung) weight [in pounds].
2. Height of the total (Sprung + Unsprung) center of gravity [in inches].
3. Longitudinal location of the front and rear articulation points with respect to the total center of gravity (discussed above) [in inches].
4. Height of the front and rear articulation points with respect to the ground [in inches].

For each suspension:

5. Longitudinal location of the suspension with respect to the total center of gravity [in inches].
6. Tandem axle separation [in inches].
7. Dynamic load transfer coefficient [-1 < Coefficient < 1].

For each axle/wheel/brake:

8. Radius of a tire on the wheel [in inches].

Exhibit 3.1

OFFTRACKING SIMULATIONS

Date: 12-20-1985 Simple Models:O.Tract./Semi Time: 12:12:30

The following information refers to Unit # 1

General Information

* Wheelbase (in) = 144
* Distance of rear articulation point from front suspension (in) = 129.6

* Load on the front suspension (lb) = 0
* Total cornering stiffness of tires on the front suspension (lb/deg) = 0
* Load on the rear suspension (lb) = 0
* Total cornering stiffness of tires on the rear suspension (lb/deg) = 0

The following information refers to Unit # 2

General Information

* Wheelbase (in) = 432
* Distance of rear hitch location from forward articulation point (in) = 468

* Load on the rear suspension (lb) = 0
* Total cornering stiffness of tires on the rear suspension (lb/deg) = 0

---

9. Pushout pressure [in psi].
10.a. Brake gain for the axle [in inch.pounds/psi per axle], or
10.b. Brake table for the axle [[psi vs. inch.pounds per axle]

Note: The brake table is limited to ten pressure/brake torque entries. Exhibit 3.2 contains a braking data set for a tractor and semitrailer.

3.3 HIGH SPEED OFFTRACKING

3.3.1 Data Required. This analysis applies to operation on highway curves at highway speeds. Besides the "offtracking dimensions" (see Section 3.1.1), this procedure uses suspension loads and suspension cornering stiffnesses.

Therefore, in addition the the "offtracking dimensions", the High Speed Offtracking model uses:

3. Suspension loads: The total suspension load (the sum of the loads on all the axles of the suspension) is entered [in pounds].
4. Suspension cornering stiffnesses: The sum of the cornering stiffnesses of all the tires on the suspension is entered [in pounds/degree].

As the analysis makes several small angle assumptions, this model is limited to high speed, large radius maneuvers. Exhibit 3.3 contains a high speed offtracking data set for a tractor and semitrailer.

3.4 STATIC ROLL

3.4.1 Data Required. The Static Roll model makes calculations that represent the rolling performance obtained during steady turning at various levels of lateral acceleration. They represent analytical equivalents of tilt-table experiments. The Static Roll model uses the following information to estimate the "rollover threshold" of any vehicle that can be modeled as a train of semitrailers (though the pintle hook of a dolly decouples various units in roll, it is still modeled as a semitrailer with a short wheel base and a low center of gravity).

For each unit:
1. Total (Sprung + Unsprung) weight [in pounds].
2. Height of the total (Sprung + Unsprung) center of gravity [in inches].

For each axle:
3. Track width of the axle [in inches].
4. Load borne by the axle [in pounds].
5. Weight of the axle/Unsprung weight [in pounds].
6. Height of the roll center [in inches].
7. Spring stiffness [in pounds/inch/side].
8. Spacing between springs [in inches].
9. Auxiliary roll stiffness for the axle [in inch.pounds/degree].

For each wheel:
10. Radius of a tire on the wheel [in inches].
11. Vertical stiffness of a tire on the wheel [in pounds/inch].

3.4.2 Overlap with the Handling Model. Due to the similarities in their respective data requirements, the Static Roll and Handling models share the same data set. The steering system and cornering stiffness information and the "offtracking dimensions" are used solely by the Handling model. These variables can be set to zero, before running the Static Roll model. Note:

Exhibit 3.2

Simple Models:B.Tract./Semi

STRAIGHT LINE BRAKING SIMULATIONS

Date: 02-04-1986 Time: 09:49:00

The following requests refer to a dual suspension unit (Unit # 1)

General Information
* Total weight (lb) = 15500
* Total c.g. height (in) = 34.83
* Location (wrt C.G.) of rear articulation point - X axis (in) = 68.8
* Location (wrt ground) of rear articulation point - Z axis (in) = 48

Suspension # 1
* Location (wrt C.G.) of supension - X axis (in) = 60.8

Wheel/Brake Information : Unit 1 Suspension 1 Axle 1
* Radius of a tire (in) = 19.5
* Pushout pressure (psi) = 7
* Brake table Key : No brake table = 1 ; Brake table = 2 1
* Brake gain (in.lb/psi) = 2000

Suspension # 2
* Location (wrt C.G.) of supension - X axis (in) = 83.2
* Tandem axle separation (in) = 48
* Dynamic load transfer coefficient (Between -1 and 1) = 0

Wheel/Brake Information : Unit 1 Suspension 2 Axle 1
* Radius of a tire (in) = 19.5
* Pushout pressure (psi) = 7
* Brake table Key : No brake table = 1 ; Brake table = 2 2
* Size of brake table (Between 1 and 10) = 4

Pressure vs. Brake torque Table

Point 1
* Treadle pressure (psi) = 20 * 2. Brake torque (in.lb) = 39000

Point 2
* Treadle pressure (psi) = 40 * 2. Brake torque (in.lb) = 99000

Point 3
* Treadle pressure (psi) = 60 * 2. Brake torque (in.lb) = 159000

Point 4
* Treadle pressure (psi) = 80 * 2. Brake torque (in.lb) = 219000

Wheel/Brake Information : Unit 1 Suspension 2 Axle 2
* Radius of a tire (in) = 19.5
* Pushout pressure (psi) = 7
* Brake table Key : No brake table = 1 ; Brake table = 2 1
* Brake gain (in.lb/psi) = 3000

The following requests refer to a single suspension unit (Unit # 2)

General Information
* Total weight (lb) = 64500
* Total c.g. height (in) = 81.44
* Location (wrt C.G.) of front articulation point - X axis (in) = 227.7
* Location (wrt ground) of front articulation point - Z axis (in) = 48

Suspension # 1
* Location (wrt C.G.) of supension - X axis (in) = 204.3

* Tandem axle separation (in) = 48
* Dynamic load transfer coefficient (Between -1 and 1) = .2

Wheel/Brake Information : Unit 2 Suspension 1 Axle 1

* Radius of a tire (in) = 19.5
* Pushout pressure (psi) = 7
* Brake table Key : No brake table = 1 ; Brake table = 2 1
* Brake gain (in.lb/psi) = 3000

Wheel/Brake Information : Unit 2 Suspension 1 Axle 2

* Radius of a tire (in) = 19.5
* Pushout pressure (psi) = 7
* Brake table Key : No brake table = 1 ; Brake table = 2 1
* Brake gain (in.lb/psi) = 3000

Exhibit 3.3

OFFTRACKING SIMULATIONS

Date: 12-20-1985 Simple Models:O.Tract./Semi Time: 12:13:14

The following information refers to Unit # 1

General Information
* Wheelbase (in) = 144
* Distance of rear articulation point from front suspension (in) = 129.6

* Load on the front suspension (lb) = 12000
* Total cornering stiffness of tires on the front suspension (lb/deg) = 705.56
* Load on the rear suspension (lb) = 17000
* Total cornering stiffness of tires on the rear suspension (lb/deg) = 1872

The following information refers to Unit # 2

General Information
* Wheelbase (in) = 432
* Distance of rear hitch location from forward articulation point (in) = 468

* Load on the rear suspension (lb) = 17000
* Total cornering stiffness of tires on the rear suspension (lb/deg) = 1872

Zero values for these parameters would cause the Handling model to "crash". Exhibit 3.4 contains a static roll data set for a tractor and semitrailer.

3.5 HANDLING

3.5.1 Data Required. "Handling" calculations are concerned with the steering angles required for a given type of steady turn. In addition to the level of steering, these calculations predict the level of lateral acceleration at which a vehicle might become statically unstable.

In addition to the data mentioned in section 3.4.1, the Handling model requires the following information (see section 3.4.2).

For each unit:

12. Longitudinal location of the front and rear articulation points with respect to the total center of gravity [in inches].

For each axle:

13. Longitudinal location of the axle with respect to the total center of gravity [in inches]. Axles located behind the center of gravity are given negative values.

For each tire:

14. Cornering stiffness vs. Vertical load [pounds/degree vs. pounds]. Three pairs of values are required to completely describe the cornering stiffness variation (with respect to vertical load) of a tire.
15. Static or Nominal load [in pounds].

For the steering system:

16. Steering gear ratio.
17. Steering system stiffness [inch.pound/degree].
18. Tie rod stiffness [inch.pound/degree].
19. Mechanical trail [inches].
20. Aligning moment for a tire on the front axle [inch.pound/degree].

Exhibit 3.5 contains the Handling model's data set for a tractor and semitrailer.

Exhibit 3.4

STEADY TURN/STATIC ROLL SIMULATIONS

Date: 12-20-1985 Simple Models:T.Tract./Semi Time: 12:24:01

The following information refers to the towing unit (Unit # 1)

General Information
* Total Weight (lb) = 15500
* C.G Height (in) = 34.83
* C.G - Rear articulation point distance (in) = 0
* Total number of axles on the unit (1-8)= 3

Steering System Information
* Steering gear ratio = 0
* Steering stiffness (in.lb/deg) = 0
* Tie rod stiffness (in.lb/deg) = 0
* Mechanical trail (in) = 0
* Aligning moment per tire (in.lb/deg) = 0

Axle # 1

Axle Information
* C.G-Axle distance (in) (-ve if axle is rear of C.G.) = 0
* Track width of the axle (in) = 80
* Axle load (lb) = 12000
* Unsprung mass of the axle (lb) = 1200
* Height of the roll center (in) = 23
* Suspension stiffness - per spring (lb/in) = 1200
* Total spacing between suspension springs (in) = 32
* Auxiliary roll stiffness (in.lb/deg) = 1072.33

Tire Information
* Number of tires on the axle = 2
* Vertical stiffness of a tire (lb/in) = 4500
* Radius of a tire (in) = 19.5
* Static or Nominal load for the tire (lb) = 0

Cornering Stiffness Table

Point 1
* 1. Cornering stiffness (lb/deg) = 0 * 2. Vertical Load (lb) = 0

Point 2
* 1. Cornering stiffness (lb/deg) = 0 * 2. Vertical Load (lb) = 0

Point 3
* 1. Cornering stiffness (lb/deg) = 0 * 2. Vertical Load (lb) = 0

Axle # 2

Axle Information
* C.G-Axle distance (in) (-ve if axle is rear of C.G.) = 0
* Track width of the axle (in) = 72
* Axle load (lb) = 17000
* Unsprung mass of the axle (lb) = 2300
* Height of the roll center (in) = 29
* Suspension stiffness - per spring (lb/in) = 600
* Total spacing between suspension springs (in) = 38
* Auxiliary roll stiffness (in.lb/deg) = 7560.76

Tire Information
* Number of tires on the axle = 4
* Vertical stiffness of a tire (lb/in) = 4500
* Radius of a tire (in) = 19.5

* Static or Nominal load for the tire (lb) = 0

Cornering Stiffness Table

Point 1

* 1. Cornering stiffness (lb/deg) = 0 * 2. Vertical Load (lb) = 0

Point 2

* 1. Cornering stiffness (lb/deg) = 0 * 2. Vertical Load (lb) = 0

Point 3

* 1. Cornering stiffness (lb/deg) = 0 * 2. Vertical Load (lb) = 0

Axle # 3

<u>Axle Information</u>

* C.G-Axle distance (in) (-ve if axle is rear of C.G.) = 0
* Track width of the axle (in) = 72
* Axle load (lb) = 17000
* Unsprung mass of the axle (lb) = 2300
* Height of the ro[illegible] = [illegible]
* Suspension stiffness - per spr[illegible]lb/in) = 6000
* Total spacing between suspension springs (in) = 38
* Auxiliary roll stiffness (in.lb/deg) = 7560.76

<u>Tire Information</u>

* Number of tires on the axle = 4
* Vertical stiffness of a tire (lb/in) = 4500
* Radius of a tire (in) = 19.5
* Static or Nominal load for the tire (lb) = 0

Cornering Stiffness Table

Point 1

* 1. Cornering stiffness (lb/deg) = 0 * 2. Vertical Load (lb) = 0

Point 2

* 1. Cornering stiffness (lb/deg) = 0 * 2. Vertical Load (lb) = 0

Point 3

* 1. Cornering stiffness (lb/deg) = 0 * 2. Vertical Load (lb) = 0

The following information refers to the semitrailer (Unit # 2)

General Information

* Total Weight (lb) = 64500
* C.G Height (in) = 78.47
* C.G - Front articulation point distance (in) = 0
* Total number of axles on the unit (1-8)= 2

Axle # 1

<u>Axle Information</u>

* C.G-Axle distance (in) (-ve if axle is rear of C.G.) = 0
* Track width of the axle (in) = 72
* Axle load (lb) = 17000
* Unsprung mass of the axle (lb) = 1500
* Height of the roll center (in) = 29
* Suspension stiffness - per spring (lb/in) = 9000
* Total spacing between suspension springs (in) = 38
* Auxiliary roll stiffness (in.lb/deg) = 11341.15

<u>Tire Information</u>

* Number of tires on the axle = 4
* Vertical stiffness of a tire (lb/in) = 4500
* Radius of a tire (in) = 19.5

* Static or Nominal load for the tire (lb) = 0

Cornering Stiffness Table

Point 1

* 1. Cornering stiffness (lb/deg) = 0   * 2. Vertical Load (lb) = 0

Point 2

* 1. Cornering stiffness (lb/deg) = 0   * 2. Vertical Load (lb) = 0

Point 3

* 1. Cornering stiffness (lb/deg) = 0   * 2. Vertical Load (lb) = 0

Axle # 2

Axle Information

* C.G-Axle distance (in) (-ve if axle is rear of C.G.) = 0
* Track width of the axle (in) = 72
* Axle load (lb) = 17000
* Unsprung mass of the axle (lb) = 1500
* Height of the roll center (in) = 29
* Suspension stiffness - per spring (lb/in) = 9000
* Total spacing between suspension springs (in) = 38
* Auxiliary roll stiffness (in.lb/deg) = 11341.15

Tire Information

* Number of tires on the axle = 4
* Vertical stiffness of a tire (lb/in) = 4500
* Radius of a tire (in) = 19.5
* Static or Nominal load for the tire (lb) = 0

Cornering Stiffness Table

Point 1

* 1. Cornering stiffness (lb/deg) = 0   * 2. Vertical Load (lb) = 0

Point 2

* 1. Cornering stiffness (lb/deg) = 0   * 2. Vertical Load (lb) = 0

Point 3

* 1. Cornering stiffness (lb/deg) = 0   * 2. Vertical Load (lb) = 0

Exhibit 3.5

STEADY TURN/STATIC ROLL SIMULATIONS

Date: 12-20-1985 Simple Models:T.Tract./Semi Time: 12:29:31

The following information refers to the towing unit (Unit # 1)

General Information
* Total Weight (lb) = 15500
* C.G Height (in) = 34.83
* C.G - Rear articulation point distance (in) = 68.8
* Total number of axles on the unit (1-8)= 3

Steering System Information
* Steering gear ratio = 28
* Steering stiffness (in.lb/deg) = 11000
* Tie rod stiffness (in.lb/deg) = 11000
* Mechanical trail (in) = 1
* Aligning moment per tire (in.lb/deg) = 1600

Axle # 1
Axle Information
* C.G-Axle distance (in) (-ve if axle is rear of C.G.) = 60.8
* Track width of the axle (in) = 80
* Axle load (lb) = 12000
* Unsprung mass of the axle (lb) = 1200
* Height of the roll center (in) = 23
* Suspension stiffness - per spring (lb/in) = 1200
* Total spacing between suspension springs (in) = 32
* Auxiliary roll stiffness (in.lb/deg) = 1072.33

Tire Information
* Number of tires on the axle = 2
* Vertical stiffness of a tire (lb/in) = 4500
* Radius of a tire (in) = 19.5
* Static or Nominal load for the tire (lb) = 4000

Cornering Stiffness Table

Point 1
* 1. Cornering stiffness (lb/deg) = 500 * 2. Vertical Load (lb) = 5000

Point 2
* 1. Cornering stiffness (lb/deg) = 523.334 * 2. Vertical Load (lb) = 6000

Point 3
* 1. Cornering stiffness (lb/deg) = 525.002 * 2. Vertical Load (lb) = 7000

Axle # 2
Axle Information
* C.G-Axle distance (in) (-ve if axle is rear of C.G.) =-59.2
* Track width of the axle (in) = 72
* Axle load (lb) = 17000
* Unsprung mass of the axle (lb) = 2300
* Height of the roll center (in) = 29
* Suspension stiffness - per spring (lb/in) = 6000
* Total spacing between suspension springs (in) = 38
* Auxiliary roll stiffness (in.lb/deg) = 7560.76

Tire Information
* Number of tires on the axle = 4
* Vertical stiffness of a tire (lb/in) = 4500
* Radius of a tire (in) = 19.5

* Static or Nominal load for the tire (lb) = 4000

Cornering Stiffness Table

Point 1

* 1. Cornering stiffness (lb/deg) = 500 * 2. Vertical Load (lb) = 5000

Point 2

* 1. Cornering stiffness (lb/deg) = 523.334 * 2. Vertical Load (lb) = 6000

Point 3

* 1. Cornering stiffness (lb/deg) = 525.002 * 2. Vertical Load (lb) = 7000

Axle # 3

Axle Information

* C.G-Axle distance (in) (-ve if axle is rear of C.G.) =-107.2
* Track width of the axle (in) = 72
* Axle load (lb) = 17000
* Unsprung mass of the axle (lb) = 2300
* Height of the roll center (in) = 29
* Suspension stiffness - per spring (lb/in) = 6000
* Total spacing between suspension springs (in) = 38
* Auxiliary roll stiffness (in.lb/deg) = 7560.76

Tire Information

* Number of tires on the axle = 4
* Vertical stiffness of a tire (lb/in) = 4500
* Radius of a tire (in) = 19.5
* Static or Nominal load for the tire (lb) = 4000

Cornering Stiffness Table

Point 1

* 1. Cornering stiffness (lb/deg) = 500 * 2. Vertical Load (lb) = 5000

Point 2

* 1. Cornering stiffness (lb/deg) = 523.334 * 2. Vertical Load (lb) = 6000

Point 3

* 1. Cornering stiffness (lb/deg) = 525.002 * 2. Vertical Load (lb) = 7000

The following information refers to the semitrailer (Unit # 2)

General Information

* Total Weight (lb) = 64500
* C.G Height (in) = 78.47
* C.G - Front articulation point distance (in) = 227.7
* Total number of axles on the unit (1-8)= 2

Axle # 1

Axle Information

* C.G-Axle distance (in) (-ve if axle is rear of C.G.) =-180.3
* Track width of the axle (in) = 72
* Axle load (lb) = 17000
* Unsprung mass of the axle (lb) = 1500
* Height of the roll center (in) = 29
* Suspension stiffness - per spring (lb/in) = 9000
* Total spacing between suspension springs (in) = 38
* Auxiliary roll stiffness (in.lb/deg) = 11341.15

Tire Information

* Number of tires on the axle = 4
* Vertical stiffness of a tire (lb/in) = 4500
* Radius of a tire (in) = 19.5

* Static or Nominal load for the tire (lb) = 4000

Cornering Stiffness Table

Point 1

* 1. Cornering stiffness (lb/deg) = 500 * 2. Vertical Load (lb) = 5000

Point 2

* 1. Cornering stiffness (lb/deg) = 523.334 * 2. Vertical Load (lb) = 6000

Point 3

* 1. Cornering stiffness (lb/deg) = 525.002 * 2. Vertical Load (lb) = 7000

Axle # 2

Axle Information

* C.G-Axle distance (in) (-ve if axle is rear of C.G.) =-228.3
* Track width of the axle (in) = 72
* Axle load (lb) = 17000
* Unsprung mass of the axle (lb) = 1500
* Height of the roll center (in) = 29
* Suspension stiffness - per spring (lb/in) = 9000
* Total spacing between suspension springs (in) = 38
* Auxiliary roll stiffness (in.lb/deg) = 11341.15

Tire Information

* Number of tires on the axle = 4
* Vertical stiffness of a tire (lb/in) = 4500
* Radius of a tire (in) = 19.5
* Static or Nominal load for the tire (lb) = 4000

Cornering Stiffness Table

Point 1

* 1. Cornering stiffness (lb/deg) = 500 * 2. Vertical Load (lb) = 5000

Point 2

* 1. Cornering stiffness (lb/deg) = 523.334 * 2. Vertical Load (lb) = 6000

Point 3

* 1. Cornering stiffness (lb/deg) = 525.002 * 2. Vertical Load (lb) = 7000

4.0 THE MODELS

4.1 LOW SPEED OFFTRACKING

4.1.1 Nomenclature and list of symbols used. In the following model the subscript "i" corresponds to a unit number. Also, all dollies (converter and fixed) are assumed to be semitrailers with wheelbase dimensions equal to the length of their drawbars.

R	Radius of the turn (ft)
WB_i	Wheelbase (in)
HI_i	Location of the rear articulation point, relative to the unit's rear suspension (in)

Note: In the computer model, hitch locations are determined relative to the front suspension (for trucks and tractors) and relative to the forward articulation point (for trailing units). This convention eliminates the ambiguity of entering negative/positive values for articulation locations that are forward/aft of the rear suspension.

4.1.2 The Low Speed Offtracking Model. This model covers the kinematic problem posed by the steady-turning of vehicles having single axles. As the model does not include the effects of tandem axles, wheelbase parameters should be determined from the center of any suspension having tandemized axles. Besides the vehicle parameters listed above, the model requires the radius of the path subtended by the front axle of the towing unit.

When a tractor-semitrailer tracks a steady-state circular trajectory, the tractor rear axle and trailer rear axle each subtend circular paths of different radii. The various radii associated with this steady-turning condition are shown by the terms labeled in Figure 4.1, namely, R_1 for the tractor steering axle, R_{H1} for the fifth wheel kingpin (rear hitch point), R_2 for the tractor rear tandem, and R_3 for the trailer tandem.

The tractor subtends a steady-state circular path with its front axle center tracking at a turn radius, R_1, and with the center of its rear suspension tracking about the same center at a radius, R_2, where

$$R_2^2 = R_1^2 - (WB_1/12)^2 \qquad (1)$$

The determination of the path radius R_2, is based simply upon the Pythagorean formula. The square root of the difference between the square of the hypotenuse, R_1, and the square of the side, WB_1, defines the length of the turn radius R_2.

The length of the turn radius R_3 depends upon the radius of the path subtended by the fifth wheel kingpin, R_{H1}. The radius R_{H1}, is determined from

$$R_{H1}^2 = R_1^2 + (HI_1/12)^2 - (WB_1/12)^2 \qquad (2)$$

$$R_3^2 = R_{H1}^2 - (WB_2/12)^2 \qquad (3)$$

The offtracking of a vehicle is defined as the difference between the "front" and "rear" radii. Or

$$OT = R_1 - R_3 \qquad (4)$$

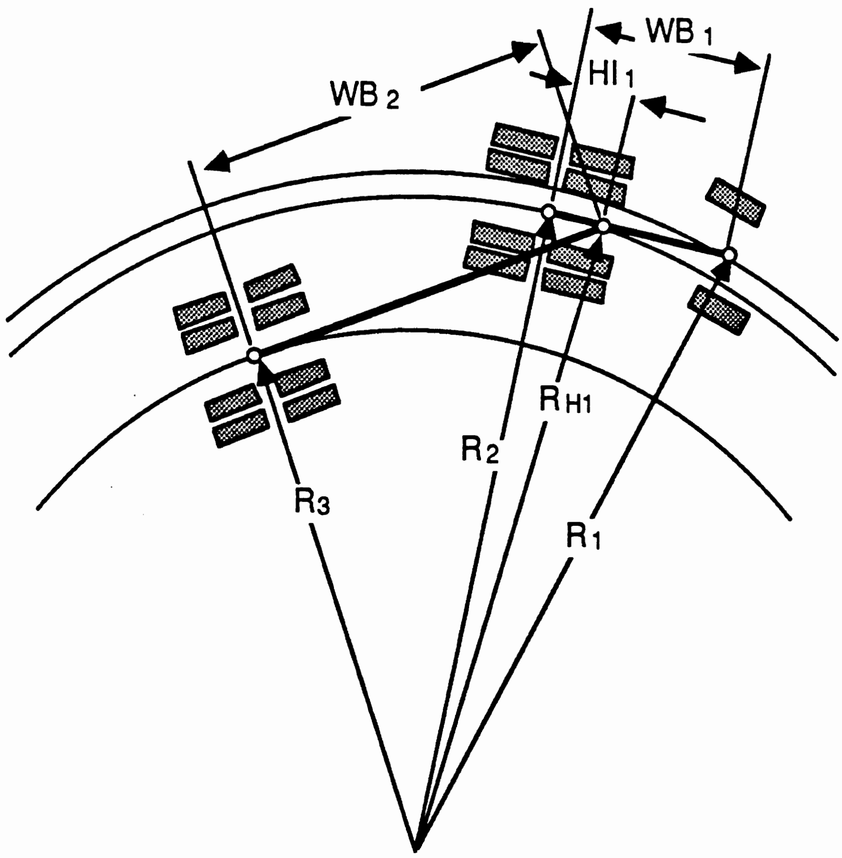


Figure 4.1. Use of the Pythagorean Theorem to analyze maximum (steady-state) off-tracking of a tractor-semi-trailer in a constant-radius turn

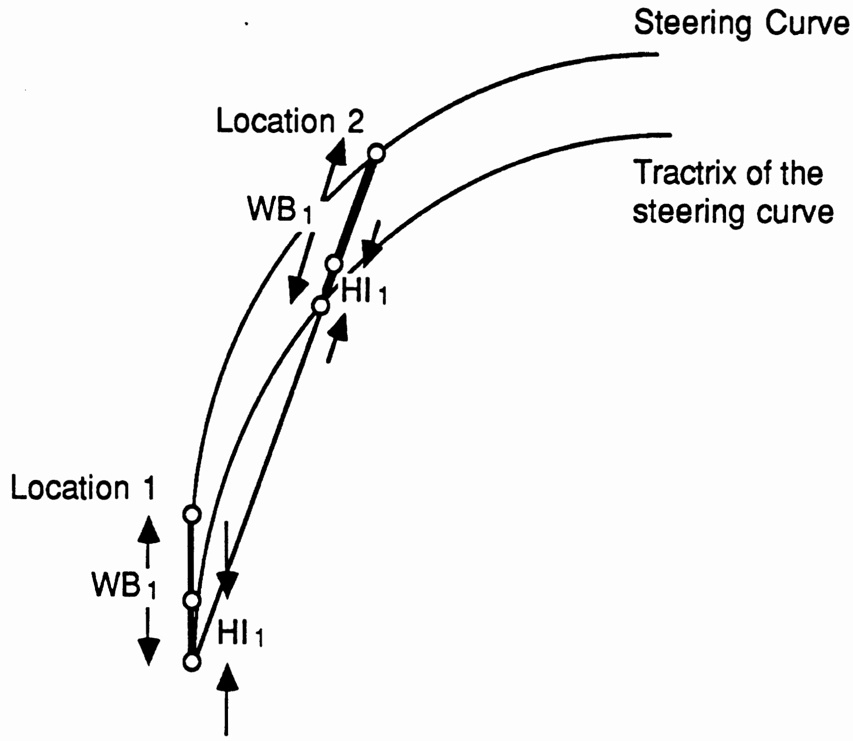


Figure 4.2. One step in determining the tractrix of the original steering curve

In the case of a doubles configuration, three additional lengths affect the offtracking expression.

HI_2 the rearward overhang of the first semitrailer (in)
WB_3 the length of the dolly's drawbar (in), and
WB_4 the wheelbase of the second semitrailer (in)

Note: There is virtually no kingpin offset in the design of a conventional dolly.

Following the same naming convention, the various radii are given by, R_{H2} for the path subtended by the semitrailer's (unit 2) pintle hitch, R_4 for the dolly's axle, R_{H3} for the path of the dolly's fifth wheel/turntable, and R_5 for the second (unit 4) semitrailer's axle.

$$R_{H2}^2 = R_{H1}^2 + (HI_2/12)^2 - (WB_2/12)^2 \quad (5)$$

$$R_4^2 = R_{H2}^2 - (WB_3/12)^2 \quad (6)$$

$$R_{H3} = R_4 \quad (7)$$

$$R_5^2 = R_{H3}^2 - (WB_4/12)^2 \quad (8)$$

And the total offtracking for a doubles configuration is given by

$$OT = R_1 - R_5 \quad (9)$$

4.1.3 The Transient Offtracking Model. The transient path followed by the trailing axle of a truck or tractor, as the result of a steering input at the front axle, is called a general tractrix of the original steering curve. In a tractor semitrailer configuration, a second general tractrix is produced for the path of the rear axle of the semitrailer (the "front axle" of trailing units, such as semitrailers, is the forward articulation point). Therefore, the problem of transient offtracking can be reduced to finding a consecutive series of general tractrix curves.

The general tractrix is characterized by the property that the distance along its tangent, taken from the point of tangence to the point of intersection with the leading curve, is constant and is equal to the pertinent wheelbase dimension. Shown in Figure 4.2 is a step in the calculation of the tractrix of the original steering curve.

4.2 BRAKING

4.2.1 Nomenclature and list of symbols used. In the following model the subscript "i" corresponds to a unit number, while "j" refers to the j'th axle on the i'th unit. In some cases the subscript "k" is used to distinguish between different suspensions on a unit. For example, in the case of a full trailer with a fixed dolly, the rear axles would be a part of suspension 2.

Due to the difference in the manner that fixed and converter dollies transfer vertical and horizontal loads, full trailers with fixed dollies are analyzed as composite units.

W_i Total weight (lb)
a Longitudinal deceleration of the vehicle combination (g's)
h_i Total (sprung+unsprung) mass c.g. height (in)
x_{Fi} Longitudinal distance between total c.g. position and forward articulation point (in)

h_{Fi}	Height (measured from ground) of the forward articulation point (in)
F_{xFi}	Longitudinal force at the forward articulation point (lb)
F_{zFi}	Vertical force at the forward articulation point (lb)
x_{Ri}	Longitudinal distance between total c.g. position and rear articulation point (in)
h_{Ri}	Height (measured from ground) of the rear articulation point (in)
F_{xRi}	Longitudinal force at the rear articulation point (lb)
F_{zRi}	Vertical force at the rear articulation point (lb)
x_{ki}	Longitudinal distance between total c.g. position and suspension "k" (in)
P_{ki}	Dynamic load shift parameter - for tandem axle suspensions only
M_{ki}	Moment due to dynamic load transfer - for tandem axle suspensions only (in.lb)
xts_{ki}	Tandem spread on the k'th suspension (for single axle suspensions $xts_{ki} = 0$)
F_{zji}	Axle load (lb)
F_{Bji}	Braking level at an axle (lb)

4.2.2 The Braking Model. This model determines braking performance assuming that the vehicle is making a constant deceleration stop. In addition to the vehicle parameters, the level of braking, F_{Bji}, is required as an input.

The response to the applied braking forces is described in terms of the longitudinal deceleration, a, and the vertical loads, F_{zji}, carried by each axle. For each level of braking input, the "minimum" value of friction needed to avoid wheel lockup is determined. Under the assumptions of the analysis, the wheels on the axle with the largest ratio of F_{Bji} to F_{zji} will lock up first. That is, the maximum ratio of F_{Bji}/F_{zji} represents the friction coefficient, μ_{ji}, required to perform a wheels–unlocked stop at the calculated level of deceleration, a.

The method used to represent inter-axle load transfer depends upon a special parameter, P_{ki}, that is used to describe the load transfer between the two axles in a tandem pair. This parameter not only describes the amount of load transfer, but also the pitch moment reacted by the sprung mass.

The first step in the calculation is to determine the longitudinal deceleration of the total vehicle. The deceleration of the vehicle combination is given by equation (10)

$$a = \{\Sigma_i \Sigma_j F_{Bji}\}/\{\Sigma_i W_i\} \quad \text{(10)}$$

Then, starting with the last unit in the train, longitudinal, pitch, and vertical equations are solved for each unit.

Most vehicle combinations can be broken down into distinct units, which can be further sub–divided into three categories.

1. Towing units
2. Semitrailers and Converter dollies, and
3. Full trailers with fixed dollies.

Note: A full trailer with a converter dolly can be further subdivided into two units, which can then be described by the equations in category 2.

4.2.2.1 Towing Units. By default, the towing unit is the first unit (i = 1) in the train. Refering to the geometric layouts and free body diagrams of the two towing units shown in Figure 4.3, the equations of motion are determined as follows.

The horizontal force balance equation is given by,

$F_{xR1} + [W_1 * a] - F_{B31} - F_{B21} - F_{B11} = 0$ ....(11)

If a suspension load is defined as F_{Ski}, then,

$F_{S11} = F_{z11}$ ....(12)

$F_{S21} = F_{z21} + F_{z31}$ ....(13)

Summing the moments about a point in the ground, vertically below the front axle, the moment balance equation can be written as,

$\{F_{S21} * [x_{11} + x_{21}]\} - M_{21} + [F_{xR1} * h_{R1}] - [F_{zR1} * (x_{R1} + x_{11})] + [W_1 * a * h_1]$

$- [W_1 * x_{11}] = 0$ ....(14)

Where,

$M_{21} = P_{21} * [F_{B31} + F_{B21}] * xts_{21}$ ....(15)

The axle loads F_{z21} and F_{z31} are given by,

$F_{z21} = [F_{S21}/2] + [M_{21}/xts_{21}]$ ....(16)

$F_{z31} = [F_{S21}/2] - [M_{21}/xts_{21}]$ ....(17)

Note: Equations (15) - (17) apply to suspensions with tandem axles.

The vertical force balance equation is given by,

$F_{zR1} + W_1 - F_{S11} - F_{S21} = 0$ ....(18)

Note: For the last unit in the train, $F_{zRi} = F_{xRi} = 0$

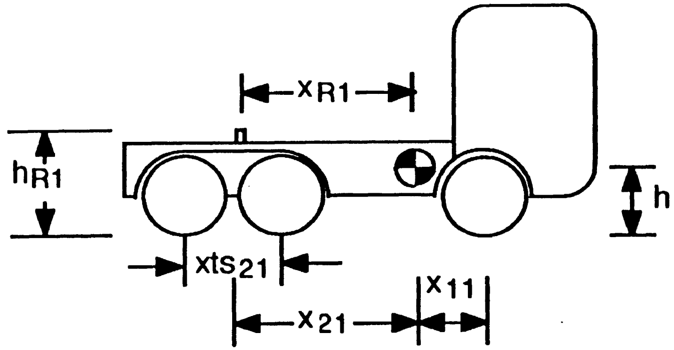
4.2.2.2 Semitrailers and Converter dollies. In the braking model the semitrailer and the converter dolly are modeled as identical units. The equations of motion can be developed based on the geometric layout and free body diagrams of Figure 4.4.

The horizontal force balance equation is given by,

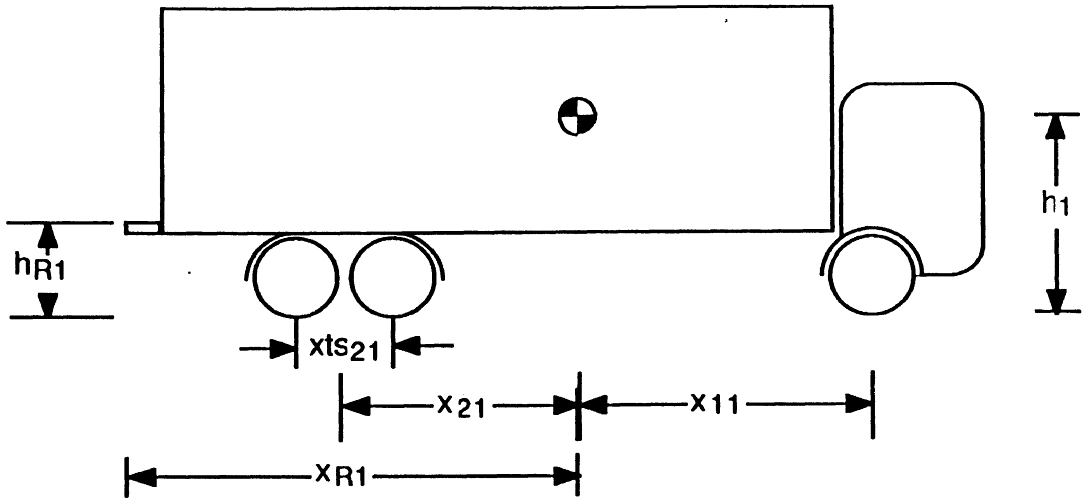
$F_{xRi} + [W_i * a] - F_{xFi} - F_{B2i} - F_{B1i} = 0$ ....(19)

If a suspension load is defined as F_{Ski}, then,

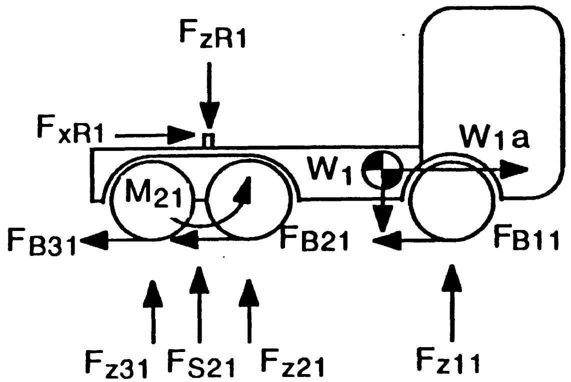
$F_{S1i} = F_{z1i} + F_{z2i}$ ....(20)



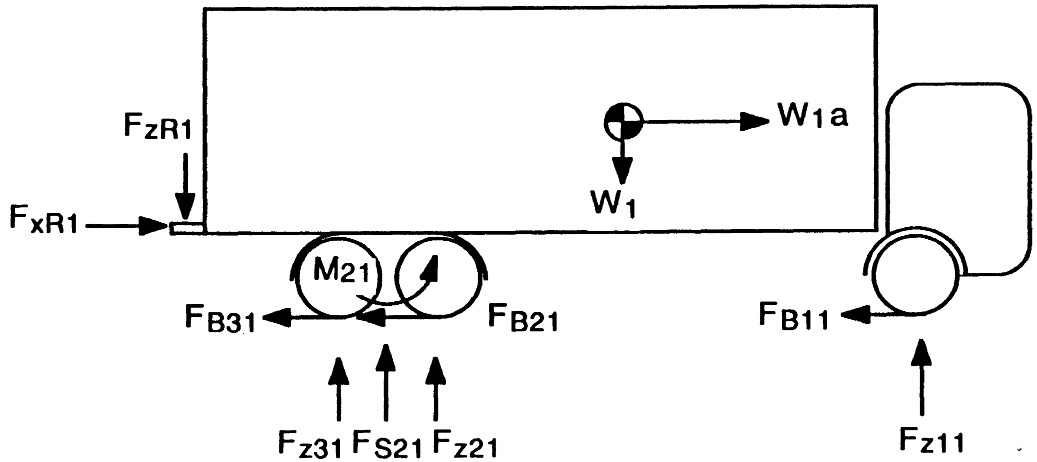
Geometric Layout of a Tractor



Geometric Layout of a Truck

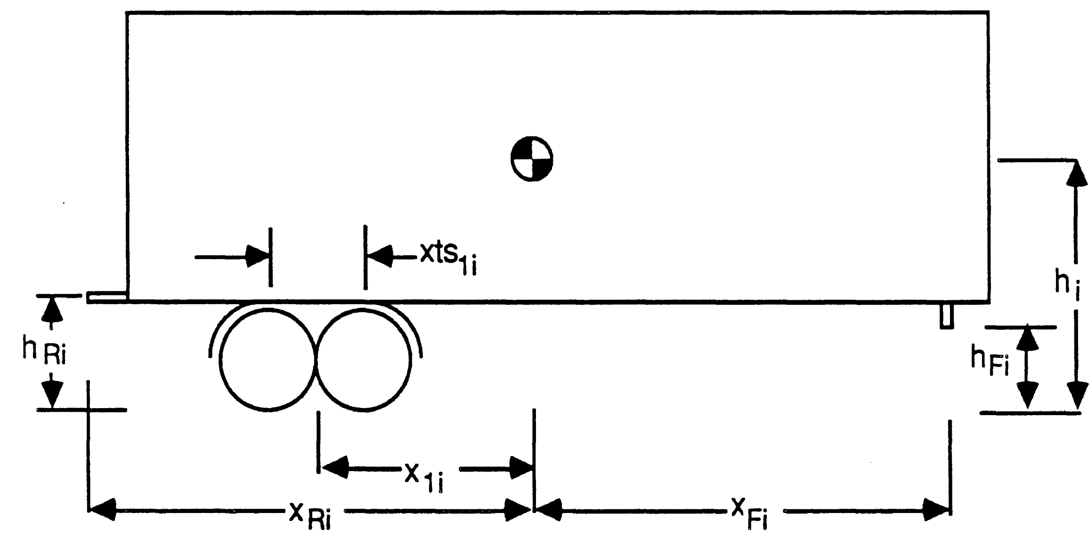


Forces and Moments (Tractor)

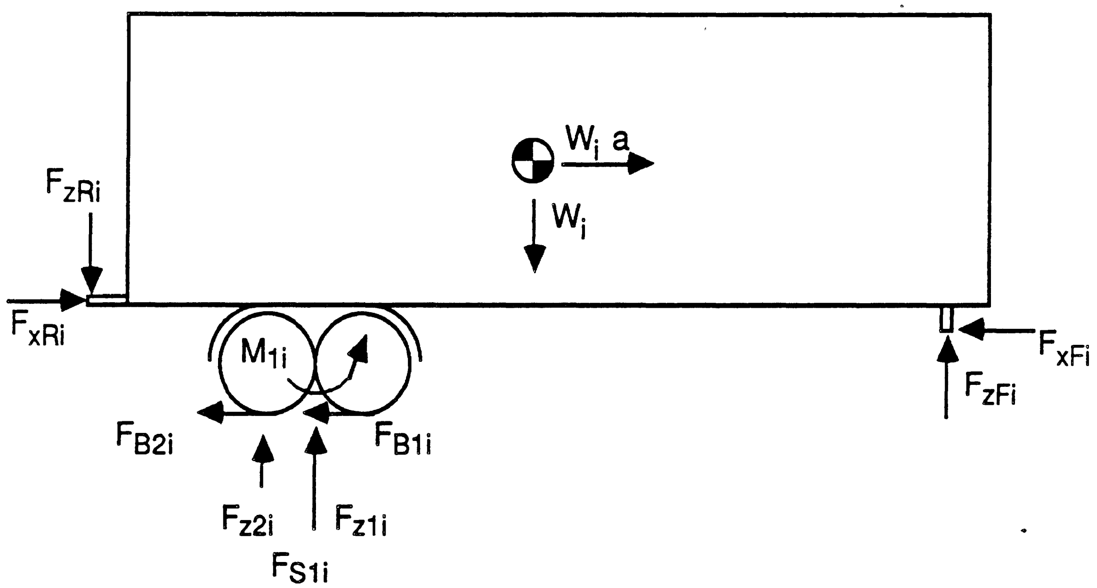


Forces and Moments (Truck)

Figure 4.3. Geometric Layouts and Free-body diagrams of towing units



Geometric Layout of a Semitrailer



Forces and Moments (Semitrailer)

Figure 4.4. Geometric Layouts and Free-body diagrams of a semitrailer

Summing the moments about the forward articulation point, the moment balance equation can be written as,

$$\{F_{S1i} * [x_{1i} + x_{Fi}]\} - M_{1i} + [F_{xR1} * (h_{Ri} - h_{Fi})] - [F_{zRi} * (x_{Ri} + x_{Fi})] + [W_i * a * (h_i - h_{Fi})]$$

$$- [W_i * x_{Fi}] + [(F_{B2i} + F_{B1i}) * h_{Fi}] = 0 \quad (21)$$

Where,

$$M_{1i} = P_{1i} * [F_{B2i} + F_{B1i}] * xts_{1i} \quad (22)$$

The axle loads F_{z1i} and F_{z2i} are given by,

$$F_{z1i} = [F_{S1i}/2] + [M_{1i}/xts_{1i}] \quad (23)$$

$$F_{z2i} = [F_{S1i}/2] - [M_{1i}/xts_{1i}] \quad (24)$$

Note: Equations (22) - (24) apply to suspensions with tandem axles.

The vertical force balance equation is given by,

$$F_{zRi} + W_i - F_{S1i} - F_{zFi} = 0 \quad (25)$$

Note: For the last unit in the train, $F_{zRi} = F_{xRi} = 0$

4.2.2.3 Full trailers with Fixed dollies. The fixed dolly differs in its basic design from a converter dolly.

1. The drawbar of a fixed dolly is hinged, and cannot transfer any of its pitching motion to the preceding unit in the train - the moment is therefore reacted out at the axles.
2. Fixed dollies normally use turntables instead of fifth wheels, which in turn, introduce an extra pitch moment into the equations of motion.

Due to the reasons listed above, the two unit trailer/fixed dolly combination is more easily modeled as a single unit. The free body diagram of such a full trailer is shown in Figure 4.5.

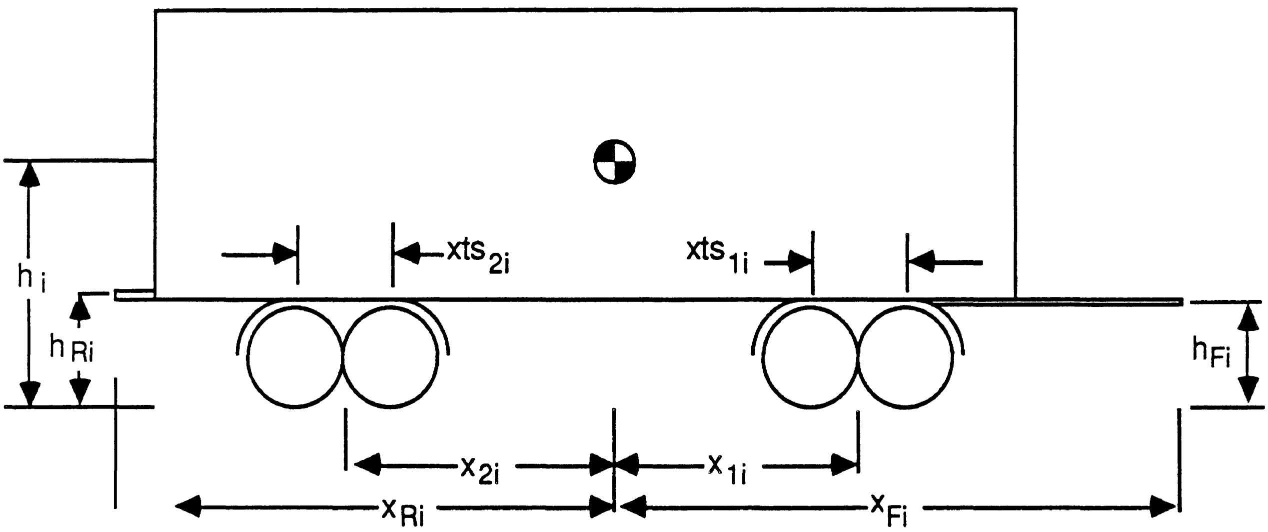
The horizontal force balance equation is given by,

$$F_{xRi} + [W_i * a] - F_{xFi} - F_{B4i} - F_{B3i} - F_{B2i} - F_{B1i} = 0 \quad (26)$$

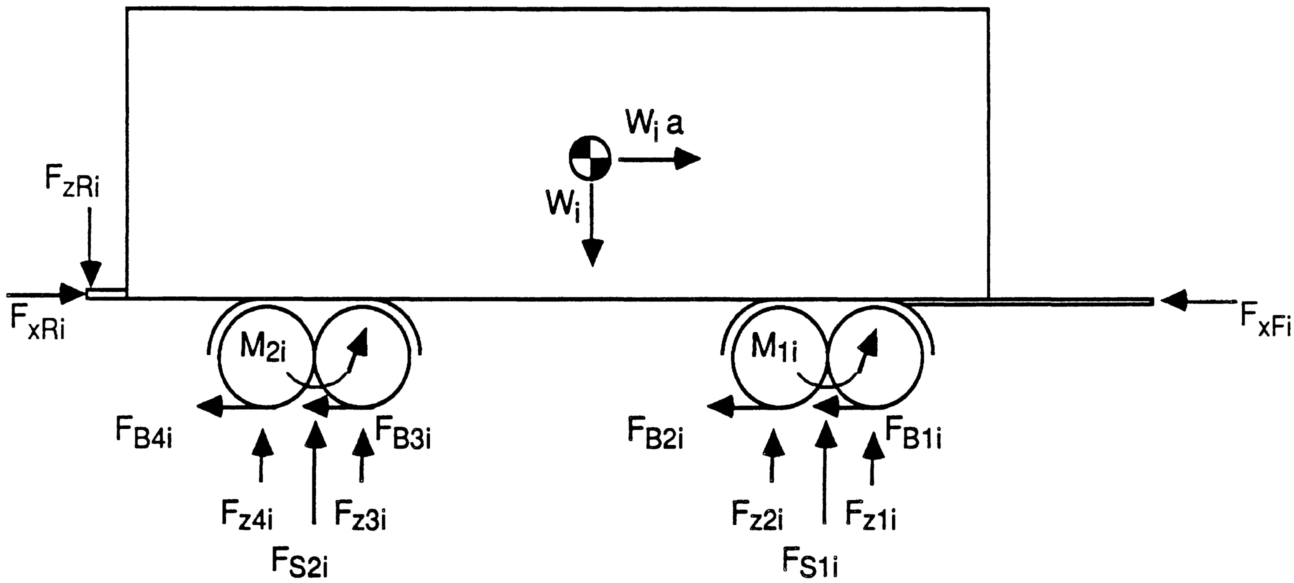
If a suspension load is defined as F_{Ski}, then,

$$F_{S1i} = F_{z1i} + F_{z2i} \quad (27)$$

$$F_{S2i} = F_{z3i} + F_{z4i} \quad (28)$$



Geometric Layout of a Full trailer and Fixed dolly



Forces and Moments (Full trailer and Fixed dolly)

Figure 4.5. Geometric Layouts and Free-body diagrams of a full trailer and fixed dolly

Summing the moments about a point in the ground, vertically below the front suspension, the moment balance equation can be written as,

$$\{F_{S2i} * [x_{1i} + x_{2i}]\} - M_{2i} - M_{1i} + [F_{xR1} * h_{Ri}] + [W_i * a * h_i] - [W_i * x_{1i}]$$

$$- [F_{zRi} * (x_{Ri} + x_{1i})] = 0 \quad (29)$$

Where,

$$M_{1i} = P_{1i} * [F_{B2i} + F_{B1i}] * xts_{1i} \quad (30)$$

$$M_{2i} = P_{2i} * [F_{B3i} + F_{B4i}] * xts_{2i} \quad (31)$$

The axle loads are given by,

$$F_{z1i} = [F_{S1i}/2] + [M_{1i}/xts_{1i}] \quad (32)$$

$$F_{z2i} = [F_{S1i}/2] - [M_{1i}/xts_{1i}] \quad (33)$$

$$F_{z3i} = [F_{S2i}/2] + [M_{2i}/xts_{2i}] \quad (34)$$

$$F_{z4i} = [F_{S2i}/2] - [M_{2i}/xts_{2i}] \quad (35)$$

Note: Equations (30) - (35) apply to suspensions with tandem axles.

The vertical force balance equation is given by,

$$F_{zRi} + W_i - F_{S1i} - F_{S2i} = 0 \quad (36)$$

Note: For the last unit in the train, $F_{zRi} = F_{xRi} = 0$

4.2.2.4 Friction utilizations and Braking efficiency. At each level of braking, the friction utilization at each axle is given by,

$$\mu_{ji} = F_{Bji}/F_{zji} \quad (37)$$

and, the "Braking Efficiency" is given by,

$$\text{Braking Efficiency} = a/\text{Max}(\mu_{ji}) \quad (38)$$

where, $\text{Max}(\mu_{ji})$ is the maximum friction utilization at an axle, at a given level of braking.

4.3 HIGH SPEED OFFTRACKING

4.3.1 Nomenclature and list of symbols used. In the High Speed Offtracking model the subscript "i" corresponds to a unit number. Also, all dollies (converter and fixed) are assumed to be semitrailers with wheelbase dimensions equal to the length of their drawbars.

R	Radius of the turn (ft)
WB_i	Wheelbase (in)
HI_i	Location of the rear articulation point, relative to the unit's rear suspension (in)

$C_{\alpha i}$ Sum of the cornering stiffnesses of the tires installed on the rear suspension (lb/rad)
F_{zi} Total load borne by the rear suspension (lb)
U Forward velocity (ft/sec)

Note: In the computer model, hitch locations are determined relative to the front suspension (for trucks and tractors) and relative to the forward articulation point (for trailing units). This convention eliminates the ambiguity of entering negative/positive values for articulation locations that are forward/aft of the rear suspension. Also, wheelbase parameters should be determined from the center of any suspension having tandemized axles.

4.3.2 The High Speed Offtracking Model. Generally it can be expected that articulated commercial vehicles will exhibit an outboard, rather than inboard, offtracking at highway speeds. For multiply articulated vehicles, this offtracking may become quite large.

This analysis assumes a linear relationship between tire lateral force and slip angle and further applies to curved paths in which the radius of curvature greatly exceeds the wheelbase of the vehicle unit. It also assumes that tire aligning moment effects are negligible and that zero roll steer is present.

The unit vehicle develops a certain level of tire slip in achieving the centripetal acceleration which is associated with the defined values of turn radius and velocity. The magnitude of the lateral slip is, of course, dependent upon the type of tires and the tire loads. Given the vehicles' wheelbase, this slip condition determines the outboard offtracking of the rear suspension's center point, as seen in Figure 4.6. The figure displays the geometric layout of the high speed offtracking of a semitrailer (unit #2), where the "offtracking dimension" is given by the difference between R_{H1} and R_3.

This analysis makes use of the Law of Cosines and makes the "Small Angle Assumption" with respect to tire slip angles. Figure 4.7 contains a complete layout (which will form the basis for future reference) for a tractor and semitrailer tracking a circular path of radius R_1.

Using the Law of Cosines and the triangle formed by the three sides R_1, R_2, and WB_1,

$$(R_1)^2 = (WB_1/12)^2 + (R_2)^2 - \{2 * (WB_1/12) * R_2 * \text{Cos}\,(90 - \alpha_1)\} \ldots\ldots (39)$$

Noting that $\text{Cos}(90 - \alpha_1) = \text{Sin}(\alpha_1)$, assuming that the slip angle α_1 is small, and using a temporary variable L_i which is the wheelbase of the unit in feet, equation (39) reduces to,

$$(R_1)^2 = (L_1)^2 + (R_2)^2 - \{2 * (L_1) * R_2 * \alpha_1\} \ldots\ldots (40)$$

As the relation between lateral force and slip angle is given by,

$$C_{\alpha i} * \alpha_i = F_{yi} = F_{zi} * a_{yi} \ldots\ldots (41)$$

where, F_{yi} is the lateral force, and a_{yi} is the lateral acceleration at the suspension center, and

$$a_{yi} = U^2/[R_i * g] \ldots\ldots (42)$$

equation (40) can be written as,

$$(R_1)^2 = (L_1)^2 + (R_2)^2 - \{2 * L_1 * [F_{z2}/C_{\alpha 2}] * [U^2/g]\} \ldots\ldots (43)$$

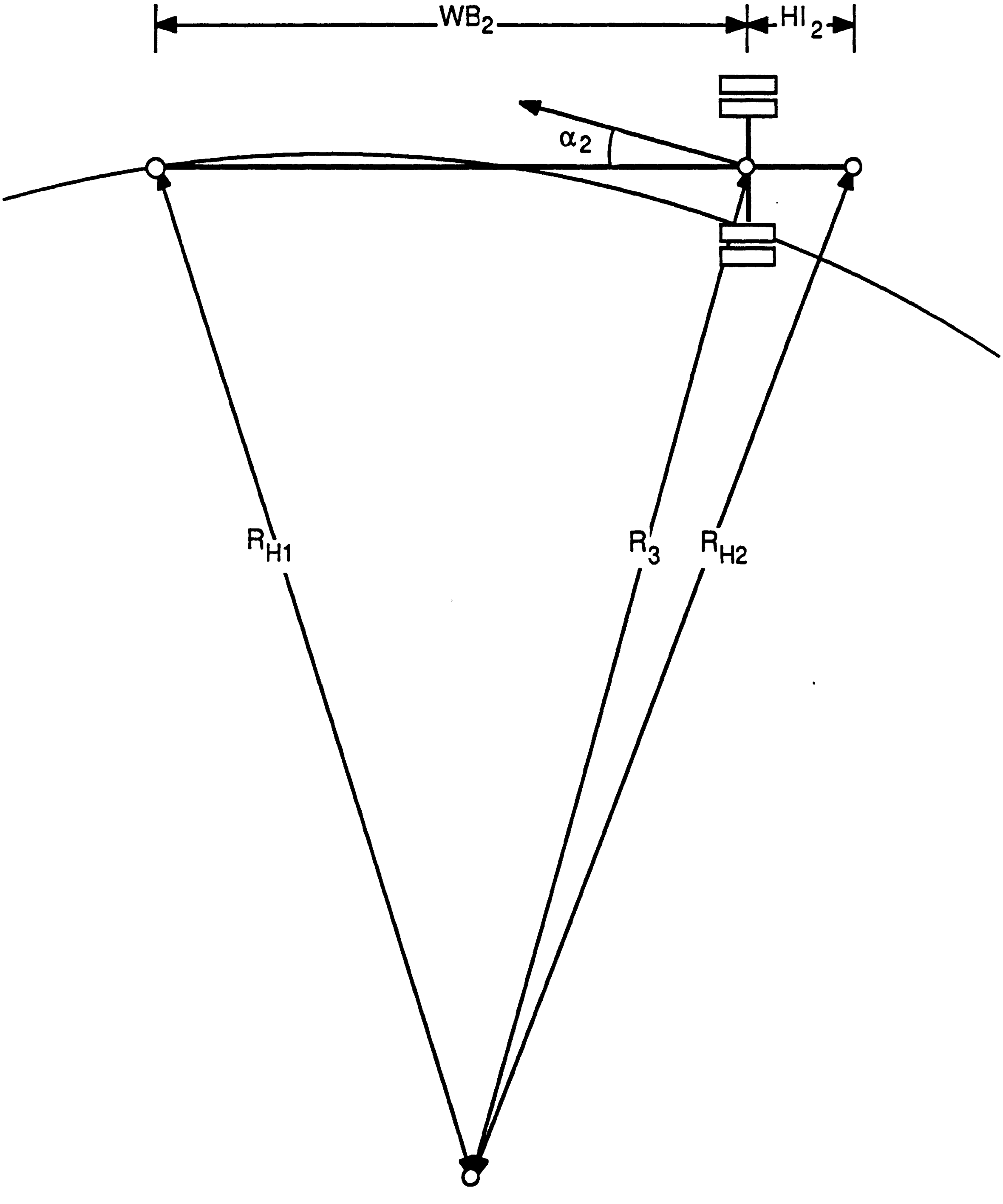


Figure 4.6. Layout of the High Speed Offtracking of a Semitrailer

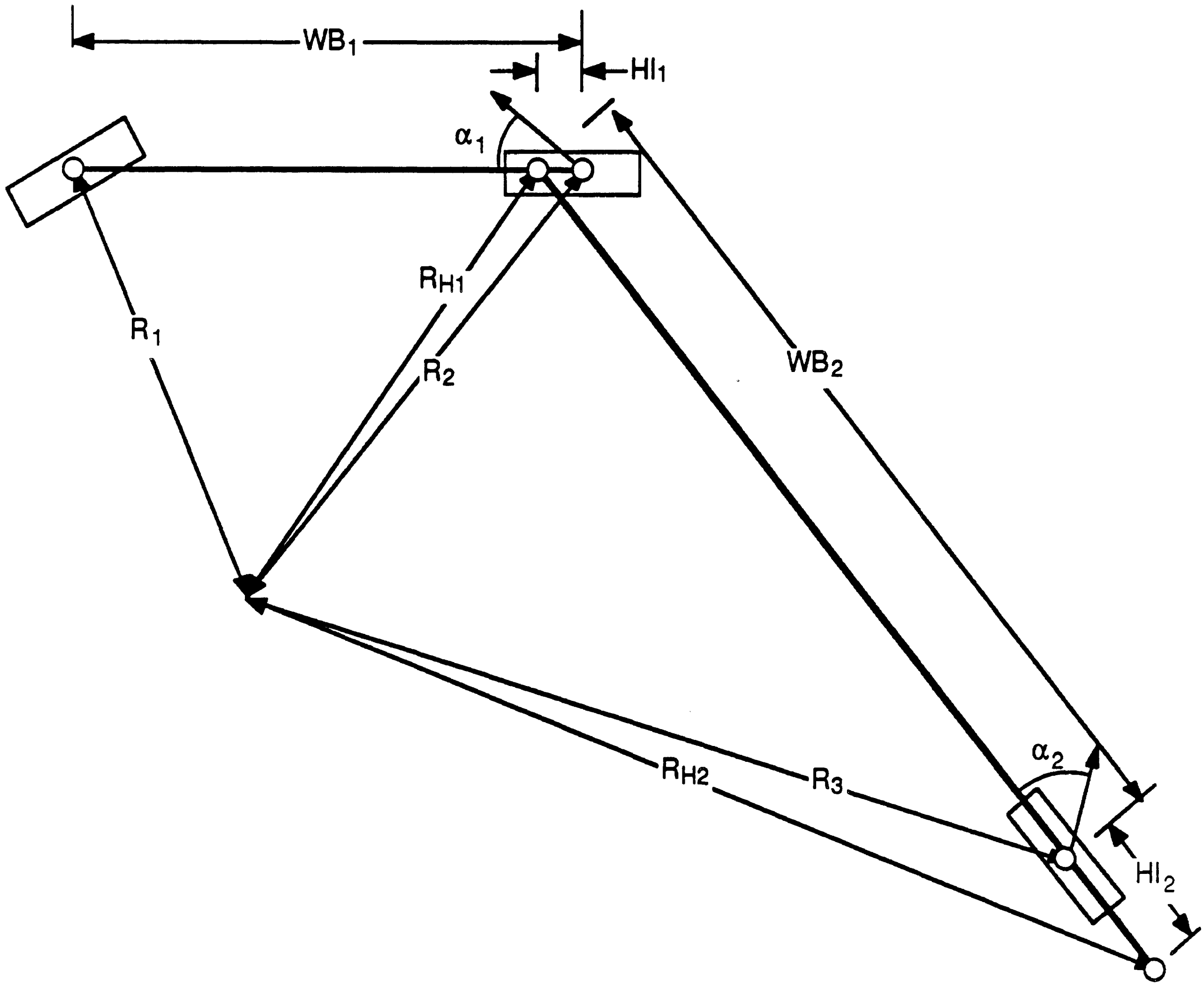


Figure 4.7. High Speed Offtracking Geometry for a Tractor and Semitrailer

Or,

$$(R_2)^2 = (R_1)^2 - (L_1)^2 + \{2 * L_1 * [F_{z2}/C_{\alpha 2}] * [U^2/g]\} \quad (44)$$

To determine the total offtracking of a multi-unit vehicle combination, the offtracking at each of the coupling points must be determined. The triangle formed by R_{H1}, R_2, and HI_1, can be used to determine R_{H1}.

$$(R_2)^2 = (R_{H1})^2 - (HI_1/12)^2 + \{2 * (HI_1/12) * (F_{z2}/C_{\alpha 2}) * (U^2/g)\} \quad (45)$$

Equation (45) reduces to,

$$(R_{H1})^2 = (R_2)^2 + (HI_1/12)^2 - \{2 * (HI_1/12) * (F_{z2}/C_{\alpha 2}) * (U^2/g)\} \quad (46)$$

The triangle formed by R_{H1}, R_3, and WB_2, helps determine R_3.

$$(R_3)^2 = (R_{H1})^2 - (L_2)^2 + \{2 * L_2 * (F_{z3}/C_{\alpha 3}) * (U^2/g)\} \quad (47)$$

As the trailer's hitch point is aft of the rear suspension, the Law of Cosines results in the following expression,

$$(R_{H2})^2 = (HI_2/12)^2 + (R_3)^2 - \{2 * (HI_2/12) * R_3 * \mathrm{Cos}\,(90 + \alpha_2)\} \quad (48)$$

As $\mathrm{Cos}(90 + \alpha_2) = -\mathrm{Sin}(\alpha_2)$, and $-\mathrm{Sin}(\alpha_2) = -\alpha_2$,

$$(R_{H2})^2 = (R_3)^2 + (HI_2/12)^2 + \{2 * (HI_2/12) * (F_{z3}/C_{\alpha 3}) * (U^2/g)\} \quad (49)$$

The "offtracking dimension" at the rearmost axle would be given by the difference between R_1 and R_3.

4.4 STATIC ROLL

4.4.1 Nomenclature and list of symbols used. In the following model the subscript "i" corresponds to a unit number, while "j" corresponds to the j'th axle.

W_i	Total weight (lb)
W_{si}	Sprung weight (lb)
W_{uji}	Unsprung weight of an axle (lb)
h_i	Total (sprung + unsprung) mass c.g. height (in)
h_{si}	Sprung mass c.g. height (in)
R_{Tji}	Unsprung mass c.g. height = Radius of a tire (in)
T'_{ji}	Total track width of the axle (in)
F_{zji}	Axle load (lb)
h_{rji}	Height of the roll center (in)
K_{Sji}	Spring stiffness - per side (lb/in)
S_{ji}	Total spacing between springs (in)
K_{SAji}	Auxiliary roll stiffness (in.lb/rad/axle)
N_{ji}	Number of tires on the axle

K_{Tji} Vertical stiffness of a tire on the axle (lb/in/tire)
$K_{\phi sji}$ Roll stiffness at an axle (in.lb/rad)
ϕ Total roll angle (rad)
ϕ_{sji} Sprung mass roll angle (rad)
ϕ_{uji} Unsprung mass roll angle (rad)
a_y Lateral acceleration (g's)

As most weight and c.g. locations are entered on a composite basis, the parameters must be reduced to their corresponding sprung and unsprung values.

$$W_{si} = W_i - \Sigma_j W_{uji} \quad \text{.......(50)}$$

$$h_{si} = \{[W_i * h_i] - [\Sigma_i W_{uji} * R_{Tji}]\}/W_{si} \quad \text{.......(51)}$$

Also, defining the following intermediate variables,

$$T_{ji} = T'_{ji}/2 \quad \text{.......(52)}$$

$$K_{\phi sji} = (2 * S_{ji}^2 * K_{Sji}) + K_{SAji} \quad \text{.......(53)}$$

$$F_{uji} = F_{zji} - W_{uji} \quad \text{.......(54)}$$

$$h_{uji} = h_{si}/[h_{si} - h_{rji}] \quad \text{.......(55)}$$

4.4.2 <u>The Static Roll Model.</u> The static roll model helps determine the rollover threshold of articulated vehicles during steady turning maneuvers. The roll response in a steady turn is computed by repeatedly solving, for small increments of roll angle, a set of equations which describe the static equilibrium of the vehicle in the roll plane.

4.4.2.1 <u>Unsprung and sprung mass moment equations.</u> Refering to Figure 4.9, and summing moments about a point in the ground,

$$[K_{\phi sji} * \phi_{sji}] + [(F_{zji} - W_{uji}) * h_{rji} * a_y] + [(F_{zji} - W_{uji}) * \phi_{uji} * h_{rji}]$$

$$+ [W_{uji} * R_{Tji} * (a_y + \phi_{uji})] = \{[(F_{zji}/2) + (K_{Tji} * \phi_{uji} * T_{ji})] * T_{ji}\}$$

$$- \{[(F_{zji}/2) - (K_{Tji} * \phi_{uji} * T_{ji})] * T_{ji}\} \quad \text{.......(56)}$$

From equations (54) and (56),

$$[K_{\phi sji} * \phi_{sji}] + [F_{uji} * h_{rji} * (a_y + \phi_{uji})] + [W_{uji} * R_{Tji} * (a_y + \phi_{uji})]$$

$$= 2 * K_{Tji} * \phi_{uji} * T_{ji}^2 \quad \text{.......(57)}$$

Performing the same calculations with the sprung mass (with moments being summed about a point in the ground - refer to Figure 4.10),

$$[W_{si} * h_{si} * (a_y + \phi)] - [\Sigma_j F_{uji} * h_{rji} * (a_y + \phi_{uji})] = \Sigma_j K_{\phi sji} * \phi_{sji} \quad \text{.......(58)}$$

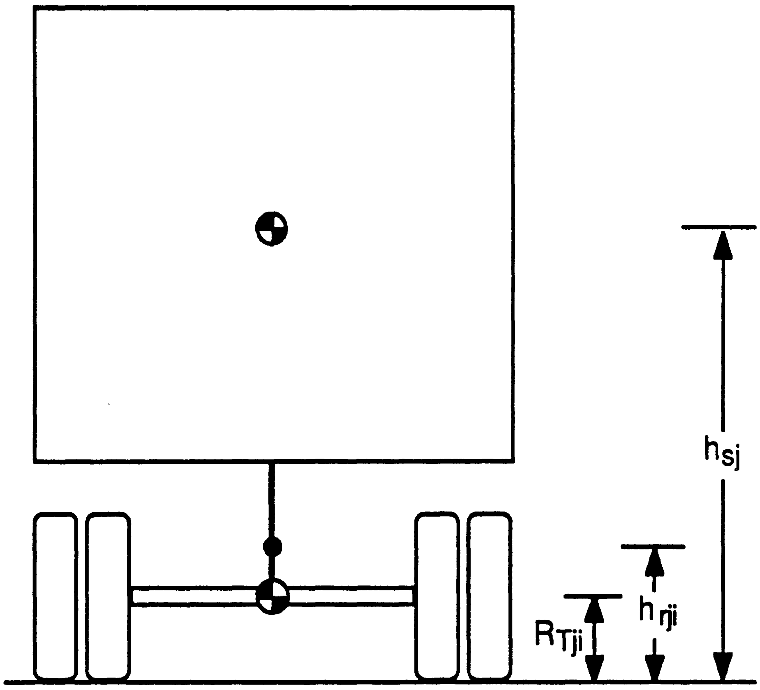


Figure 4.8. Some of the dimensions used in the static roll model

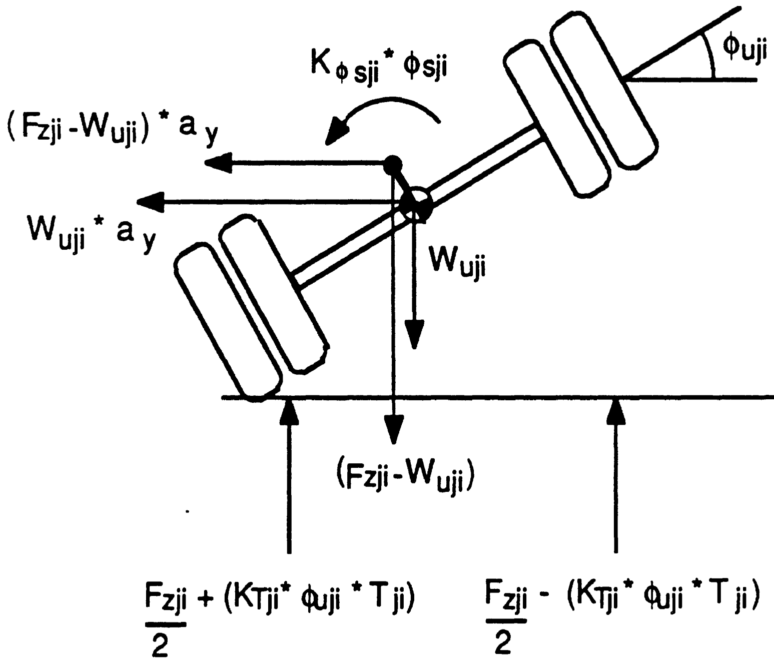


Figure 4.9. Representation of the forces and moments acting on the unsprung mass

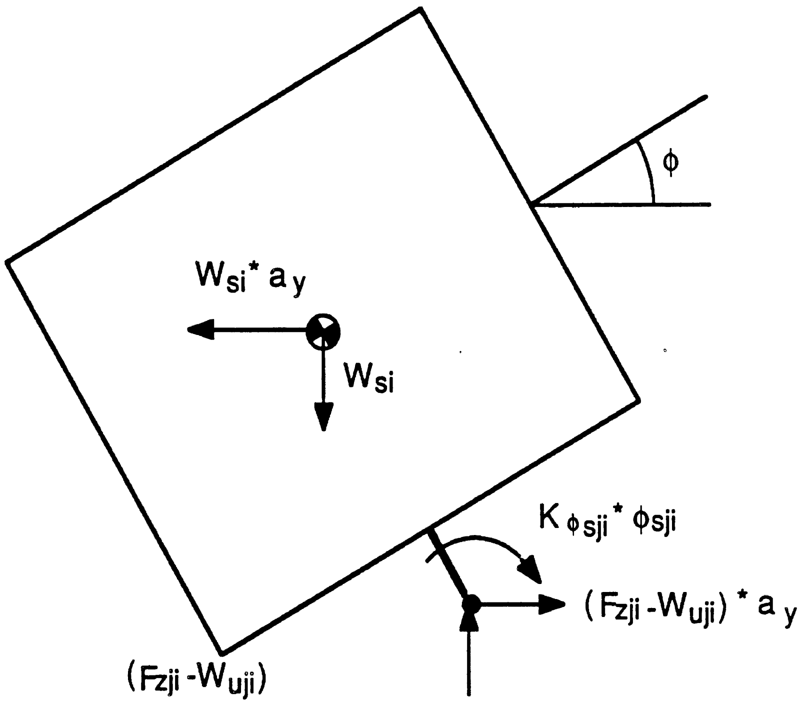


Figure 10. Representation of the forces and moments acting on the sprung mass

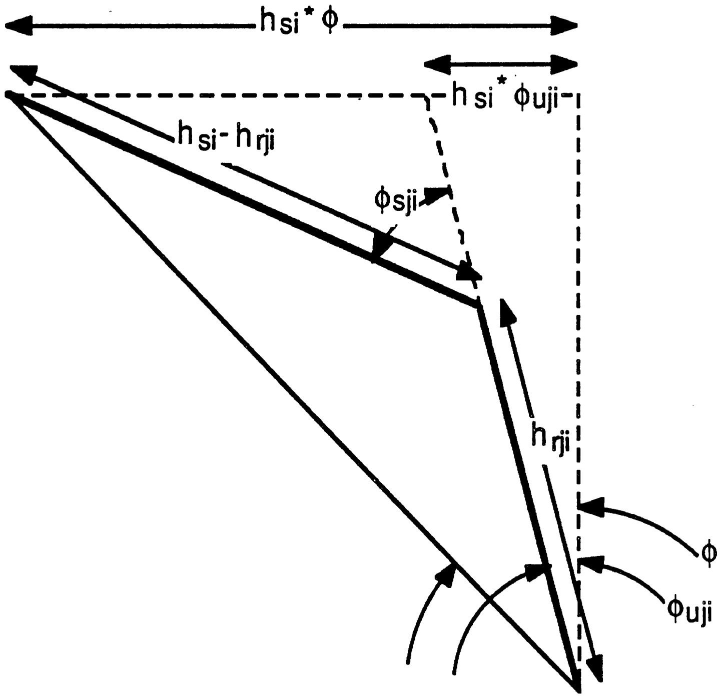


Figure 11. Geometry between sprung and unsprung mass roll angles

4.4.2.2 <u>Roll angle geometry.</u> If the roll angles in figure 4.11 are assumed to be small, the relation between ϕ_{sji}, ϕ_{uji} and ϕ can be determined.

$$\phi = \phi_{uji} + \{[(h_{si} - h_{rji}) * \phi_{sji}]\}/h_{si} \quad \text{......(59)}$$

Therefore from equations (55) and (59),

$$\phi_{sji} = (\phi - \phi_{uji}) * h_{uji} \quad \text{......(60)}$$

Substituting equation (60) in equation (57),

$$\{K_{\phi sji} * h_{uji} * \phi\} - \{K_{\phi sji} * h_{uji} * \phi_{uji}\} + \{F_{uji} * h_{rji} * (a_y + \phi_{uji})\}$$

$$+ \{W_{uji} * R_{Tji} * (a_y + \phi_{uji})\} = 2 * K_{Tji} * \phi_{uji} * T_{ji}^2 \quad \text{......(61)}$$

Rearranging the previous equation,

$$\{K_{\phi sji} * h_{uji}\} * \phi + \{[F_{uji} * h_{rji}] + [W_{uji} * R_{Tji}]\} * a_y$$

$$= \{[K_{\phi sji} * h_{uji}] - [F_{uji} * h_{rji}] - [W_{uji} * R_{Tji}] + [2 * K_{Tji} * T_{ji}^2]\} * \phi_{uji} \quad \text{.... (62)}$$

The unsprung mass roll angle, ϕ_{uji}, can be written as,

$$\phi_{uji} = \frac{\{K_{\phi sji} * h_{uji}\} * \phi + \{[F_{uji} * h_{rji}] + [W_{uji} * R_{Tji}]\} * a_y}{\{[K_{\phi sji} * h_{uji}] - [F_{uji} * h_{rji}] - [W_{uji} * R_{Tji}] + [2 * K_{Tji} * T_{ji}^2]\}} \quad \text{......(63)}$$

Defining the following intermediate variables,

$$A_{ji} = \frac{\{[F_{uji} * h_{rji}] + [W_{uji} * R_{Tji}]\}}{\{[K_{\phi sji} * h_{uji}] - [F_{uji} * h_{rji}] - [W_{uji} * R_{Tji}] + [2 * K_{Tji} * T_{ji}^2]\}} \quad \text{......(64)}$$

$$B_{ji} = \frac{\{K_{\phi sji} * h_{uji}\}}{\{[K_{\phi sji} * h_{uji}] - [F_{uji} * h_{rji}] - [W_{uji} * R_{Tji}] + [2 * K_{Tji} * T_{ji}^2]\}} \quad \text{......(65)}$$

Equation (63) can then be written as,

$$\phi_{uji} = \{A_{ji} * a_y\} + \{B_{ji} * \phi\} \quad \text{......(66)}$$

Substituting equation (60) in equation (58),

$$[W_{si} * h_{si} * (a_y + \phi)] - [\Sigma_j F_{uji} * h_{rji} * a_y] - [\Sigma_j F_{uji} * h_{rji} * \phi_{uji}]$$

$$= [\Sigma_j K_{\phi sji} * \phi * h_{uji}] - [\Sigma_j K_{\phi sji} * \phi_{uji} * h_{uji}] \quad \text{......(67)}$$

Rearranging the previous equation,

$$\{[W_{si} * h_{si}] - [\Sigma_j K_{\phi sji} * h_{uji}]\} * \phi + \{[W_{si} * h_{si}] - [\Sigma_j F_{uji} * h_{rji}]\} * a_y$$

$$= \Sigma_j\{[F_{uji} * h_{rji}] - [K_{\phi sji} * h_{uji}]\} * \phi_{uji} \quad \text{.......}(68)$$

and, defining another intermediate variable, C_{ji},

$$C_{ji} = [F_{uji} * h_{rji}] - [K_{\phi sji} * h_{uji}] \quad \text{.......}(69)$$

Using the equations (66), (68), and (69), the lateral acceleration, a_y, is defined in terms of total roll angle, ϕ.

$$a_y = \frac{-\{[W_{si} * h_{si}] - [\Sigma_j K_{\phi sji} * h_{uji}] - [\Sigma_j B_{ji} * C_{ji}]\} * \phi}{\{[W_{si} * h_{si}] - [\Sigma_j F_{uji} * h_{rji}] - [\Sigma_j A_{ji} * C_{ji}]\}} \quad \text{.......}(70)$$

After a_y has been determined, ϕ_{uji} and ϕ_{sji} can be calculated from equations (60) and (63). The load transfer on axle "j" is given by,

$$\Delta F_{zji} = K_{Tji} * T_{ji} * \phi_{uji} \quad \text{.......}(71)$$

4.4.2.3 Axle Liftoff. If axle "k" lifts off (that is, the load transfer exceeds half of the axle load), then another set of equations is used. From Figure 4.9, and equation (57),

$$[F_{uki} * h_{rki} * (a_y + \phi_{uki})] + [W_{uki} * R_{Tki} * (a_y + \phi_{uki})] + [K_{\phi ki} * \phi_{ski}]$$

$$= F_{zki} * T_{ki} \quad \text{.......}(72)$$

Rearranging,

$$\{[F_{uki} * h_{rki}] + [W_{uki} * R_{Tki}]\} * a_y + [K_{\phi ki} * \phi * h_{uki}] = F_{zki} * T_{ki}$$

$$- \{[F_{uki} * h_{rki}] + [W_{uki} * R_{Tki}] - [K_{\phi ki} * h_{uki}]\} * \phi_{uki} \quad \text{.......}(73)$$

Or,

$$\phi_{uki} = \frac{-\{[F_{uki} * h_{rki}] + [W_{uki} * R_{Tki}]\} * a_y + [K_{\phi ki} * \phi * h_{uki}] - F_{zki} * T_{ki}}{\{[F_{uki} * h_{rki}] + [W_{uki} * R_{Tki}] - [K_{\phi ki} * h_{uki}]\}} \quad \text{.......}(74)$$

Using equation (69) and defining the intermediate variables D_{ki}, and E_{ki},

$$D_{ki} = [F_{uki} * h_{rki}] + [W_{uki} * R_{Tki}] - [K_{\phi ki} * h_{uki}] \quad \text{.......}(75)$$

$$E_{ki} = C_{ki}/D_{ki} \quad \text{.......}(76)$$

the reactions on the sprung mass from the lifted axle are given by,

$$\{[W_{si} * h_{si}] - [K_{\phi ski} * h_{uki}]\} * \phi + \{[W_{si} * h_{si}] - [F_{uki} * h_{rki}]\} * a_y$$

$$= \{[F_{uki} * h_{rki}] - [K_{\phi ski} * h_{uki}]\} * \phi_{uki} \quad \text{.......}(77)$$

Rearranging equation (77) - Refer to equations (69), (73), (74), (75) and (76),

$$a_y = -\frac{\{[W_{si} * h_{si}] - [K_{\phi ski} * h_{uki}] + E_{ki} * [K_{\phi ski} * h_{uki}]\} * \phi + E_{ki} * F_{zki} * T_{ki}}{\{W_{si} * h_{si} - [F_{uki} * h_{rki}] + E_{ki} * ([F_{uki} * h_{rki}] + [W_{uki} * R_{Tki}])\}}$$

4.5 HANDLING

4.5.1 Nomenclature and list of symbols used. In the following model the subscript "i" corresponds to a unit number, while "j" corresponds to the j'th axle.

W_i	Total weight (lb)
W_{si}	Sprung weight (lb)
W_{uji}	Unsprung weight of an axle (lb)
h_i	Total (sprung + unsprung) mass c.g. height (in)
h_{si}	Sprung mass c.g. height (in)
R_{Tji}	Unsprung mass c.g. height = Radius of a tire (in)
x_{Fi}	Longitudinal distance between total c.g. position and forward articulation point (in)
x_{Ri}	Longitudinal distance between total c.g. position and rear articulation point (in)
x_{ji}	Longitudinal distance between total c.g. position and axle "j" (in)
T'_{ji}	Total track width of the axle (in)
F_{zji}	Axle load (lb)
F0zji	Static load on the tire (lb)
h_{rji}	Height of the roll center (in)
K_{Sji}	Spring stiffness - per side (lb/in)
S_{ji}	Total spacing between springs (in)
K_{SAji}	Auxiliary roll stiffness (in.lb/rad/axle)
N_{ji}	Number of tires on the axle
K_{Tji}	Vertical stiffness of a tire on the axle (lb/in/tire)
$K_{\phi sji}$	Roll stiffness at an axle (in.lb/rad)
ϕ	Total roll angle (rad)
ϕ_{sji}	Sprung mass roll angle (rad)
ϕ_{uji}	Unsprung mass roll angle (rad)
a_y	Lateral acceleration (g's)
$C_{\alpha 0ji}$	Cornering stiffness at the static load (lb/deg)
$C_{\alpha 1ji}$	Linear variation of cornering stiffness with vertical load, about the static load (1/deg)
$C_{\alpha 2ji}$	Quadratic variation of cornering stiffness with load, about the static load $(1/deg)^2$
N_G	Steering gear ratio
K_{SC}	Steering stiffness (in.lb/deg)
K_{TR}	Tie rod stiffness (in.lb/deg)
x_m	Mechanical trail (in)
A_T	Aligning moment per tire (in.lb/deg)

4.5.2 The Handling (Steady Turn) Model. The handling model helps determine the static yaw-stability of articulated vehicles during turning maneuvers. The yaw response in a steady turn is computed by repeatedly solving, for small increments of lateral acceleration, a set of equations which describe the static equilibrium of the vehicle in both roll and yaw planes.

4.5.2.1 Characteristics of truck tires. The truck tire shows a cornering stiffness that increases monotonically with load. Nevertheless, the curvature in the cornering stiffness characteristic (see Figure 4.12) causes a significant change in the yaw-stability of the vehicle during a steady turn involving considerable side-to-side load transfer.

If the cornering stiffness, $C'_{\alpha ji}$, of a single tire is treated as the following function of vertical load, F_{zji},

$$C'_{\alpha ji}(F_{zji}) = C_{\alpha 0ji} + [C_{\alpha 1ji} * (F_{zji} - F0_{zji})] + [C_{\alpha 2ji} * (F_{zji} - F0_{zji})^2] \text{ ...... (78)}$$

then, with three reference points (see Figure 4.12), the tire parameters $C_{\alpha 0ji}$, $C_{\alpha 1ji}$, and $C_{\alpha 2ji}$ can be determined.

The total cornering stiffness of an axle is given by,

$$C_{\alpha ji} = [N_{ji} * C_{\alpha 0ji}] + \{(N_{ji}/2) * C_{\alpha 1ji} * [F_{left} + F_{right} - (2 * F0_{zji})\}$$

$$+ \{(N_{ji}/2) * C_{\alpha 2ji} * [(F_{left} - F0_{zji})^2 + (F_{right} - F0_{zji})^2\} \text{ .... (79)}$$

Where,

$$F_{right} = F'_{zji} + \Delta F_{zji}$$

$$F_{left} = F'_{zji} - \Delta F_{zji}$$

$$F'_{zji} = F_{zji}/2$$

and ΔF_{zji} is the side-to-side load transfer on axle "j" - see equation (71) in the static roll model.

4.5.2.2 Steering system compliance. The compliance in the steering system reduces the cornering stiffness of the first axle of the towing unit.

The lateral force at the steering axle is given by,

$$F_{y11} = - C'_{\alpha 11} * (\alpha_{left} + \alpha_{right}) \text{ .... (80)}$$

where,

$C'_{\alpha 11}$	Cornering stiffness of one tire on the steering axle (lb/deg)
$\alpha_{left,\ right}$	Slip angles at left and right tires (deg)

If the following variables are defined as follows,

δ_{SW}	Steering wheel angle (deg)
$\delta_{left,\ right}$	Steer angles at left and right tires (deg)
x_p	Pneumatic trail (in)

$$x_p = A_T/C'_{\alpha 11} \text{ .... (81)}$$

$$\delta_{FW} = \delta_{SW}/N_G \text{ .... (82)}$$

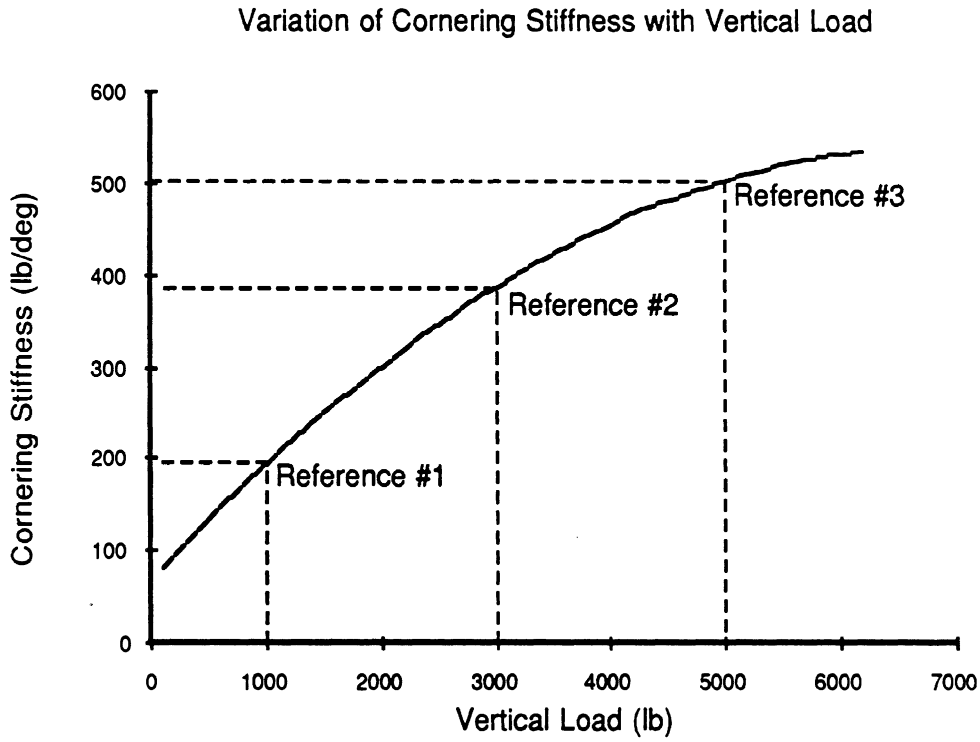


Figure 4.12. Influence of vertical load on the cornering stiffness of a truck tire

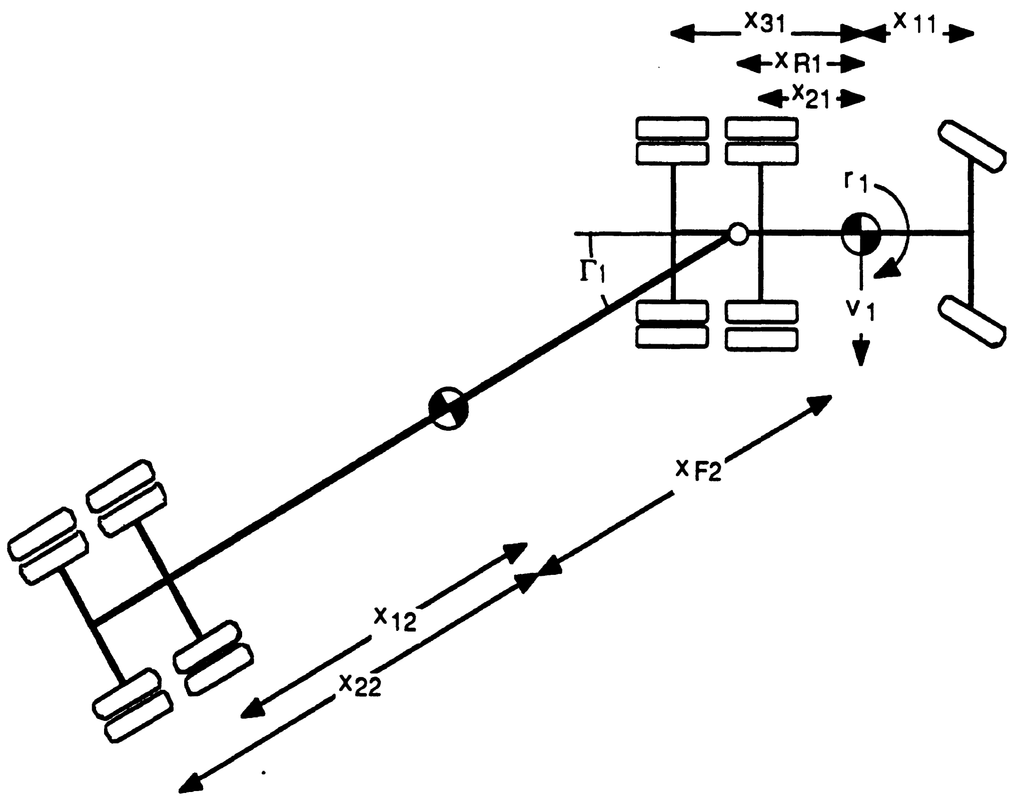


Figure 4.13. Geometric layout of a tractor and semitrailer

$$\delta_{left} = \delta_{FW} + [C'_{\alpha 11} * \frac{(x_p + x_m)}{K_{SC}} * (\alpha_{left} + \alpha_{right})] \quad \text{.......................... (83)}$$

$$\delta_{right} = \delta_{left} + [C'_{\alpha 11} * \frac{(x_p + x_m)}{K_{TR}} * \alpha_{right}] \quad \text{.................................. (84)}$$

From equations (80), (81), (83), and (84),

$$C_{\alpha 11} = \frac{C'_{\alpha 11} * \{2 + \frac{[C'_{\alpha 11} * (x_p + x_m)]}{K_{TR}}\}}{1 + \{\frac{[C'_{\alpha 11} * (x_p + x_m)]}{K_{SC}} * [\frac{2}{(K_{TR} * K_{SC})} + \frac{C'_{\alpha 11} * (x_p + x_m)}{K_{TR}} + 1]\}} \quad \text{.................. (85)}$$

4.5.2.3 Equilibrium in the yaw plane. For convenience, a special sign convention is used to indicate whether axles are forward or aft of the center of gravity of their unit; specifically in the equations ultimately used, x_{ji} is positive if axle "j" is forward of unit i's center of gravity, otherwise, x_{ji} is negative. Articulation points, which may be either at the front or the rear of the unit, are always positive.

Using the slip angles at each axle of the towing unit to develop expressions for the lateral forces (see Figure 4.14),

$$m_1 * ([\delta v_1/\delta t] + u * r_1) = [C_{\alpha 11} * \delta] - [(v_1/u) * (\Sigma_j C_{\alpha j1})]$$

$$- \{(r_1/u) * [\Sigma_j (x_{j1} * C_{\alpha j1})]\} - F_{F2} \quad \text{... (86)}$$

$$I_1 * [\delta r_1/\delta t] = (x_{11} * C_{\alpha 11} * \delta) + (x_{R1} * F_{F2}) - \{(v_1/u) * [\Sigma_j (x_{j1} * C_{\alpha j1})]\}$$

$$- \{(r_1/u) * [\Sigma_j (x_{j1}^2 * C_{\alpha j1})]\} \quad \text{... (87)}$$

For steady turning conditions,

$[\delta v_1/\delta t] = [\delta r_1/\delta t] = 0$, and, $r_i = r$, where v_1 is the lateral velocity, r_1 is the yaw rate and the subscript "i" refers to the unit number.

If the following intermediate variables are defined as follows (g is the acceleration due to gravity),

$a_y = u * r/g$ .... (u is the forward velocity)

$1/R = r/u$ .... (R is the path radius)

$F_{v1} = (1/u) * [\Sigma_j C_{\alpha j1}]$ .... (88)

$F_{r1} = (1/u) * [\Sigma_j (x_{j1} * C_{\alpha j1})]$ .... (89)

$T_{r1} = (1/u) * [\Sigma_j (x_{j1}^2 * C_{\alpha j1})]$ .... (90)

$T_{v1} = F_{r1}$ .... (91)

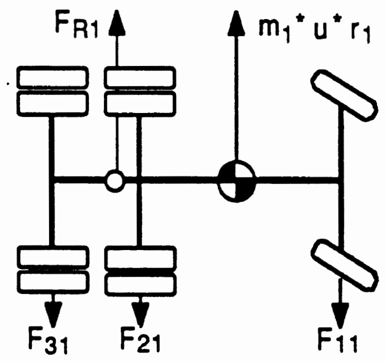


Figure 4.14. Free body diagram of the tractor

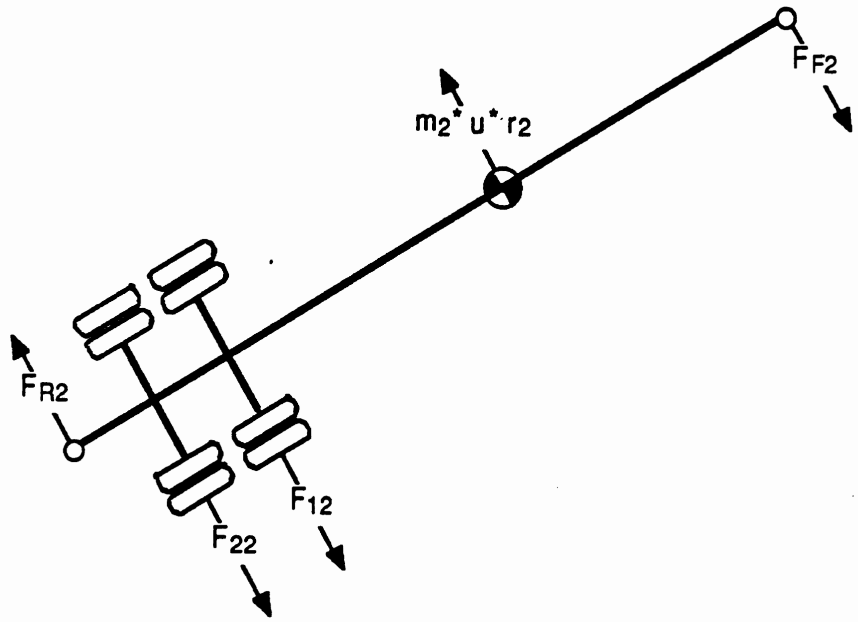


Figure 4.15. Free body diagram of the semitrailer

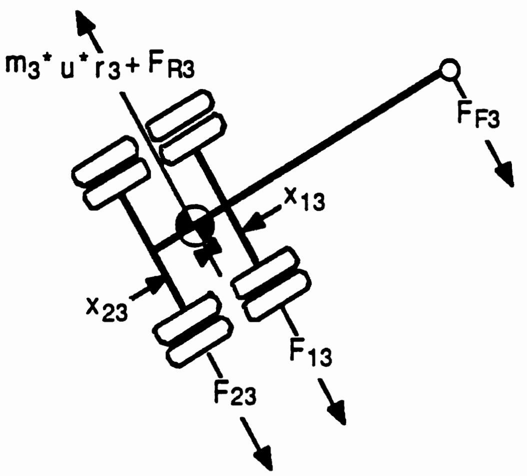


Figure 4.16. Free body diagram of the dolly

$F_\delta = C_{\alpha 11}$ .... (92)

$T_\delta = x_{11} * C_{\alpha 11}$ .... (93)

Equations (86) and (87) can be reduced to,

$\{[(m_1 * u) + F_{r1}] * r\} + [F_{v1} * v_1] = [F_\delta * \delta] - F_{F2}$ .... (94)

$[T_{r1} * r] + [T_{v1} * v_1] = [T_\delta * \delta] + [x_{R1} * F_{F2}]$ .... (95)

In a similar fashion, the steady turn equations can be developed for trailing units. From Figure 4.15, the equations for a semitrailer can be described as follows.

$\{[(m_2 * u) + F_{r2}] * r\} + [F_{v2} * v_2] = F_{F2} - F_{F3}$ .... (96)

$[T_{r2} * r] + [T_{v2} * v_2] = [x_{F2} * F_{F2}] + [x_{R2} * F_{F3}]$ .... (97)

The full trailer of a double includes a dolly which is a special kind of semitrailer. The dolly has a center of gravity and rear articulation point that are located close together. For multiple axle arrangements, the articulation point and center of gravity are located near the center of the axle set (see Figure 4.16). Because the yaw moment of inertia of the dolly is smaller than those of either of its adjacent units, it cannot accelerate rapidly in yaw.

Refering to Figure 4.16, and making the following assumptions,

$x_{23} = x_{13}$

$\alpha_{13} = [v_3 + (x_{13} * r)]/u$

$\alpha_{23} = [v_3 - (x_{13} * r)]/u$

$C_{\alpha 13} = C_{\alpha 23}$

then,

$F_{F3} = [2 * C_{\alpha 13} * x_{13}^2 * r]/[x_{F3} * u]$ .... (98)

As illustrated in equation (98), F_{F3} for the dolly depends on path curvature ($1/R = r/u$) and not F_{R3} or lateral acceleration a_y. Furthermore, F_{F3} tends to be small for typical highway curves with radii of 300 km or more. The dolly practically removes the influence of the full trailer on the semitrailer.

4.5.2.3 <u>Stability and control of the towing unit.</u> Matrix methods provide a relatively easy way to develop the handling equations for multiply articulated vehicles. The equations to be solved are of the forms,

$[A_1] * (r, v_1) = \{\mathbf{b_1} * F_{F2}\} + \{\mathbf{a_1} * \delta\}$ .... (for the tractor)

$[A_2] * (r, v_2) = \{\mathbf{c_2} * F_{F2}\} + \{\mathbf{b_2} * F_{F3}\}$ ....(towed units except dollies)

Where, $[A_1]$ and $[A_2]$ are 2 x 2 matrices reduced from equation (94) - (97). The column vectors $\mathbf{a_1}$, $\mathbf{b_1}$, $\mathbf{c_2}$, $\mathbf{b_2}$, (r, v_1) and (r, v_2) are reduced from the same set of equations.

Starting with a zero hitch force at the rearmost semitrailer the forces of constraint can be determined in sequence until the fifth wheel force F_{F2} is specified for use in the handling equation of the tractor.

As an example, F_{F2} represented in terms of r and F_{F3},

$r = |\underline{1,0}|[A_2]^{-1}[\mathbf{c_2} F_{F2} + \mathbf{b_2} F_{F3}]$, where, $|\underline{1,0}|$ is a row vector.

Similarly, F_{F3} can be expressed in terms of r and F_{F4} (and so on until F_{F2} is represented in terms of r and the lateral force at the last articulation point).

The steady turn equation for the tractor, given by,

$$[A_1] * (r, v_1) = \{\mathbf{b_1} * F_{F2}\} + \{\mathbf{a_1} * \delta\} \quad (99)$$

can be expressed in the form of a "handling" equation,

$$\delta = [L_e/R] + [U_e * a_y] \quad (100)$$

where δ is the steer angle, L_e is the "effective" wheelbase and U_e is the "understeer" gradient.

Representing the force at the fifth wheel,

$$F_{F2} = [A_2 * a_y] + [B_2/R] \quad (101)$$

where, A_2 and B_2 are the constants resulting from accumulating hitch forces from the last trailer in the train. For a tractor and semitrailer, it can be shown that,

$$A_2 = \frac{W_2 * \Sigma_j x_{j2} * C_{\alpha j2}}{[\Sigma_j x_{j2} * C_{\alpha j2}] - [x_{F2} * \Sigma_j C_{\alpha j2}]} \quad (102)$$

$$B_2 = \frac{[\Sigma_j (x_{j2} * C_{\alpha j2})]^2 - \{[\Sigma_j C_{\alpha j2}] * [\Sigma_j (x_{j2}^2 * C_{\alpha j2})]\}}{[\Sigma_j x_{j2} * C_{\alpha j2}] - [x_{F2} * \Sigma_j C_{\alpha j2}]}, \quad (103)$$

The variables in equation (100) are given by,

$$L_e = \frac{[\Sigma_j (x_{j2} * C_{\alpha j2})]^2 - \{[\Sigma_j C_{\alpha j2}] * [\Sigma_j (x_{j2}^2 * C_{\alpha j2})]\} + [A_1 * B_2]}{C_{\alpha 11} * \{[\Sigma_j x_{j1} * C_{\alpha j1}] - [x_{11} * \Sigma_j C_{\alpha j1}]\}}, \quad (104)$$

$$U_e = \frac{\{W_1 * [\Sigma_j (x_{j1} * C_{\alpha j1})]\} + [A_1 * A_2]}{C_{\alpha 11} * \{[\Sigma_j x_{j1} * C_{\alpha j1}] - [x_{11} * \Sigma_j C_{\alpha j1}]\}} \quad (105)$$

where,

$$A_1 = [\Sigma_j (x_{j1} * C_{\alpha j1})] - [x_{R1} * \Sigma_j C_{\alpha j1}] \quad (106)$$

Equation (100) can be rewritten as,

$$\delta = \frac{[g * L_e * a_y]}{u^2} + [U_e * a_y] \quad \text{.....(107)}$$

The perturbation equation derived from equation (107),

$$\Delta\delta = (g/u^2) * \{[(\delta L_e/\delta a_y) * a_y + L_e] + [(\delta U_e/\delta a_y) * a_y + U_e]\} * \Delta a_y \quad \text{.....(108)}$$

The condition for static instability is that $(\Delta a_y/\Delta\delta)$ approach infinity, that is the vehicle will be statically unstable if $u > u_c$ (u_c is the critical velocity), where,

$$u_c^2 = -g * \frac{[(\delta L_e/\delta a_y) * a_y + L_e]}{[(\delta U_e/\delta a_y) * a_y + U_e]} \quad \text{.....(109)}$$

5.0 REFERENCES

[1] Engineering Summer Conferences., "Mechanics of Heavy-Duty Trucks and Truck Combinations," Course Notes, University of Michigan Transportation Research Institute, Ann Arbor, June 1985

[2] Fancher, P.S., M., "Directional Stability and Control of Articulated Heavy Trucks," Lecture Notes, Amalfi Conferences, Italy, June 1984

[3] MacAdam, C.C., "A Computer Based Study of the Yaw/Roll Stability of Heavy Trucks Characterized by High Centers of Gravity," SAE Paper #821260, 1982

[4] MacAdam, C.C., et al, "A Computerized Model for Simulating the Braking and Steering Dynamics of Trucks, Tractor-Semitrailers, Doubles, and Triples Combinations," Users' Manual, University of Michigan Highway Safety Research Institute, Ann Arbor, September 1980

[5] Gillespie, T.D.,MacAdam, C.C., "Constant Velocity Yaw/Roll Program," Users' Manual, University of Michigan Transportation Research Institute, Ann Arbor, October 29, 1980