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# Diffusion of Directed Polymers in a Strong Random Environment

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We consider a system of random walks or directed polymers interacting with an environment which is random in space and time. It was shown by Imbrie and Spencer that in spatial dimensions three or above the behavior is diffusive if the directed polymer interacts weakly with the environment and if the random environment follows the Bernoulli distribution. Under the same assumption on the random environment as that of Imbrie and Spencer, we establish that in spatial dimensions four or above the behavior is still diffusive even when the directed polymer interacts strongly with the environment. More generally, we can prove that, if the random environment is bounded and if the supremum of the support of the distribution has a positive mass, then there is an integer $d_0$ such that in dimensions higher than $d_0$ the behavior of the random polymer is always diffusive.

**KEY WORDS:** Random walks; directed polymers; random environment; martingales.

## INTRODUCTION

Let $\xi(t)$, $t \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$, be a symmetric nearest neighbor walk on $\mathbb{Z}^d$ starting at 0 and let $h(t, x)$, $t \in \mathbb{N}$, $x \in \mathbb{Z}^d$, be independent and identically distributed random variables which are also independent of $\xi$. We denote by $\langle \cdot \rangle$ the expectation with respect to $\xi$ and by $E(\cdot)$ the expectation with respect to the random environment $h$. For any $\beta > 0$ and for $t \in \mathbb{N}$, define

$$Z(t) = \exp\left[\beta \sum_{j=1}^{t} h(j, \xi(j))\right]$$

$(t, \xi(t))$ can be used to model a directed polymer. Physicists are interested

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in determining the growth speed of the directed polymer in the random environment, more precisely, the asymptotic behavior of the quotient

$$\frac{\langle |\xi(t)|^2 Z(t)\rangle}{\langle Z(t)\rangle}$$

as $t \to \infty$. The parameter $\beta$ measures the extent to which the directed polymer interacts with the random environment. When $\beta > 0$ is small, the interaction is weak, and when $\beta > 0$ is large, the interaction is strong.

The following result was first proved by Imbrie and Spencer,[3] and later Bolthausen[1] gave a simple proof of it using martingale theory.

**Theorem 1.** Suppose that $h$ follows the Bernoulli distribution and $d \geqslant 3$. If $\beta > 0$ is small enough, then

$$\lim_{t \to \infty} \frac{\langle |\xi(t)|^2 Z(t)\rangle}{t\langle Z(t)\rangle} = 1 \tag{1}$$

almost surely, where $|\cdot|$ is the Euclidean norm.

This theorem tells us that in dimension three or above, when the directed polymer interacts weakly with the environment, the behavior of the directed polymer is diffusive, which means that the speed of growth of the square displacement of the random directed polymer is of order $t$, the same as that of the free random walk. It is conjectured in the physics literature that when $\beta > 0$ is large, i.e., when the directed polymer interacts strongly with the environment, the behavior of the directed polymer is non-diffusive, i.e., the speed of growth of the square displacement of the random directed polymer is not of order $t$ (see, for instance, the references in ref. 3). There are numerical studies in the physics literature which support the conjecture above in the case of $d = 3$.

We give here a modification of the argument of ref. 1 to show that the conjecture above is incorrect when the environment $h$ follows the Bernoulli distribution. More precisely, we are going to show that if $h$ follows the Bernoulli distribution and if $d \geqslant 4$, then for any $\beta > 0$, (1) is true almost surely. More generally, we can prove that if $h$ is bounded and if the supremum of the support of $h$ has a positive mass, then there is an integer $d_0$ such that in dimensions higher than $d_0$, (1) is true for any $\beta > 0$.

## 1. SOME BASIC FACTS ON THE SIMPLE RANDOM WALK

In order to prove our main results, we need to recall some basic facts about the symmetric nearest neighbor random walk $\xi$ on $\mathbb{Z}^d$ starting from the origin first.

Let $p_d$ be the probability that a symmetric nearest neighbor random walk $\xi$ on $\mathbb{Z}^d$ starting from the origin ever returns to the origin, i.e.,

$$p_d = P(\xi(2s) = 0 \text{ for some } s > 0)$$

The following fact is intuitively clear and almost needs no proof. Since we could not find a proof of it in the literature, we present a proof here which was communicated to us (through e-mail) by Greg Lawler.

**Lemma 1.** If $d > 3$, then $p_d < p_{d-1}$.

*Proof.* By symmetry, it is easy to see that $p_d$ equals the probability that the $d$-dimensional random walk starting at $(1, 0, 0, ..., 0)$ ever reaches the origin $(0, 0, ..., 0)$. However, by considering the embedded $(d-1)$-dimensional random walk, we can see that $p_{d-1}$ is the probability that a $d$-dimensional random walk starting at $(1, 0, 0, ..., 0)$ ever reaches the set

$$\{(0, 0, 0, ..., 0, n) : n \in \mathbb{N}\}$$

Clearly, this probability is larger than $p_d$. QED

For some values of $d$, the values of $p_d$ have been calculated; see ref. 5 for the case of $d = 3$ and ref. 2 for the other cases. From refs. 5 and 2 we know that

$$p_3 = 0.340537329550999...$$

and

$$p_4 = 0.193201673224984...$$

In this note we only need the facts that $p_3 < 1/2$ and $p_4 < 1/5$.

We are also going to need the following obvious fact: for any $\eta > 0$, there exists a positive integer $d_0$ such that when $d > d_0$, $p_d < \eta$.

## 2. THE CASE OF A BERNOULLI ENVIRONMENT

Now we are going to assume that the environment variables follows a Bernoulli distribution. That is, for any $t \in \mathbb{N}$, $x \in \mathbb{Z}^d$, $h(t, x)$ takes on the values $+1$ and $-1$ with probability $1/2$ each.

For any fixed $0 < \varepsilon < 1$ and any $t \in \mathbb{N}$, define

$$\kappa(t) = \prod_{j=1}^{t} (1 + \varepsilon h(j, \xi(j)))$$

If we set $\varepsilon = \tanh \beta$, we can easily see that

$$Z(t) = \cosh^t(\beta)\, \kappa(t)$$

Thus

$$\frac{\langle |\xi(t)|^2 Z(t)\rangle}{t\langle Z(t)\rangle} = \frac{\langle |\xi(t)|^2 \kappa(t)\rangle}{\langle \kappa(t)\rangle}$$

$\varepsilon = \tanh \beta$ gives the relationship between the two parameters $\beta > 0$ and $0 < \varepsilon < 1$. The parameter $\beta > 0$ being small is equivalent to the fact that $0 < \varepsilon < 1$ is small, and $\beta > 0$ being large is equivalent to the fact that $0 < \varepsilon < 1$ is large. So in this section we are going to work with the parameter $0 < \varepsilon < 1$.

Let $\mathscr{F}_t$ be the $\sigma$-field generated by the variables $h(s, x)$, $s \leqslant t$, $x \in \mathbb{Z}^d$ and let $\mathscr{G}_t$ be the $\sigma$-field generated by the variables $\xi(s)$, $s \leqslant t$. The following lemma is due to ref. 1; we give the proof here for the sake of completeness.

**Lemma 2.** $\langle \kappa(t)\rangle$ is a nonnegative $\mathscr{F}_t$-martingale satisfying $E(\langle \kappa(t)\rangle) = 1$.

*Proof.* $E(\langle \kappa(t)\rangle) = 1$ is obvious and

$$\begin{aligned} E(\langle \kappa(t)\rangle \mid \mathscr{F}_{t-1}) &= \langle E(\kappa(t) \mid \mathscr{F}_{t-1})\rangle \\ &= \langle \kappa(t-1)\, E(1 + \varepsilon h(t, \xi(t)))\rangle \\ &= \langle \kappa(t-1)\rangle \end{aligned}$$

Thus the lemma is valid. QED

The following result is an improved version of Lemma 2 of ref. 1. Here we do not require that $\varepsilon > 0$ be small.

**Lemma 3.** If $d \geqslant 3$, then $\langle \kappa(t)\rangle$ converges almost surely to a random variable $\zeta$ satisfying

$$E(\zeta) = 1 \qquad \text{and} \qquad P(\zeta = 0) = 0$$

*Proof.* $\langle \kappa(t)\rangle$ converges almost surely by the martingale limit theorem (see, for instance, Theorem II-2-9 of ref. 4), say, to $\zeta$.

We consider two independent copies $\xi^{(1)}$ and $\xi^{(2)}$ of the random walk $\xi$ with corresponding quantities

$$\kappa^{(i)}(t) = \prod_{j=1}^{t} (1 + \varepsilon h(j, \xi^{(i)}(j)))$$

The random environment remains independent of $\xi^{(1)}$ and $\xi^{(2)}$. Then

$$\begin{aligned}
E(\langle \kappa(t)\rangle^2) &= E(\langle \kappa^{(1)}(t)\,\kappa^{(2)}(t)\rangle) \\
&= \left\langle E\left(\prod_{j=1}^{t}\,[1+\varepsilon h(j,\xi^{(1)}(j))][1+\varepsilon h(j,\xi^{(2)}(j))]\right)\right\rangle \\
&= \langle (1+\varepsilon^2)^{n_t(\xi^{(1)},\,\xi^{(2)})}\rangle \\
&\leqslant \langle (1-\varepsilon^2)^{n_\infty(\xi^{(1)},\,\xi^{(2)})}\rangle
\end{aligned}$$

where

$$n_t(\xi^{(1)},\xi^{(2)}) = \sum_{s=1}^{t} 1_{\xi^{(1)}(s)=\xi^{(2)}(s)} \leqslant n_\infty(\xi^{(1)},\xi^{(2)})$$

Since

$$\begin{aligned}
&P(\xi^{(1)}(s)=\xi^{(2)}(s) \text{ for some } s>0) \\
&\quad = P(\xi(2s)=0 \text{ for some } s>0)
\end{aligned}$$

we know that for any $k\geqslant 0$,

$$P(n_\infty = k) = p_d^k(1-p_d)$$

Therefore

$$\begin{aligned}
\langle (1+\varepsilon^2)^{n_\infty(\xi^{(1)},\xi^{(2)})}\rangle &\leqslant \langle 2^{n_\infty(\xi^{(1)},\xi^{(2)})}\rangle \\
&= \sum_{k=0}^{\infty} 2^k p_d^k(1-p^d) \\
&= (1-p_d)\sum_{k=0}^{\infty}(2p_d)^k < \infty
\end{aligned}$$

since $p_d < 1/2$ when $d\geqslant 3$. So it follows that

$$\sup E(\langle \kappa(t)\rangle^2) < \infty$$

Hence we can conclude that $\langle \kappa(t)\rangle$ converges to $\zeta$ in $L^1$ and $L^2$ (see, for instance, Proposition IV-2-7 of ref. 4). Therefore, $E(\zeta)=1$ and from this we see that $P(\zeta=0)$ is not equal to 1. It is easy to see that the event $\{\zeta=0\}$ belongs to the tail field

$$\bigcap_t \sigma(h(s,x)\colon s\geqslant t,\, x\in\mathbb{Z}^d)$$

Thus by Kolmogorov's zero–one law we know that $P(\zeta = 0) = 0$. QED

It is obvious that

$$M(t) = |\xi(t)|^2 - t$$

is a $\mathscr{G}_t$-martingale. If we define

$$Y(t) = \langle M(t)\, \kappa(t) \rangle$$

then $Y$ is an $\mathscr{F}_t$-martingale. In fact,

$$\begin{aligned} E(Y(t) \mid \mathscr{F}_{t-1}) &= \langle E(M(t)\, \kappa(t) \mid \mathscr{F}_{t-1}) \rangle \\ &= \langle M(t)\, E(\kappa(t) \mid \mathscr{F}_{t-1}) \rangle \\ &= \langle M(t)\, \kappa(t-1) \rangle \\ &= \langle\!\langle M(t)\, \kappa(t-1) \mid \mathscr{G}_{t-1} \rangle\!\rangle \\ &= \langle \kappa(t-1) \langle M(t) \mid \mathscr{G}_{t-1} \rangle\!\rangle \\ &= \langle M(t-1)\, \kappa(t-1) \rangle \end{aligned}$$

The following result is in the same spirit as Lemma 4 of ref. 1. Here we do not require that $\varepsilon > 0$ be small.

**Lemma 4.** If $d \geqslant 4$, then

$$\lim_{t \to \infty} \frac{Y(t)}{t} = 0$$

almost surely.

*Proof.* We are going to show that the martingale

$$\sum_{s=1}^{t} \frac{Y(s) - Y(s-1)}{s}$$

remains $L^2$-bounded. Once we prove that, we know that the martingale above converges almost surely, and then the result of this lemma will be a direct consequence of the Kronecker Lemma (see, for instance, Lemma VII-2-5 of ref. 4).

So we need only to show that the martingale above remains $L^2$-bounded. Noticing that

$$\begin{aligned}
&Y(t) - Y(t-1) \\
&\quad = \langle M(t)\, \kappa(t) - M(t-1)\, \kappa(t-1) \rangle \\
&\quad = \langle M(t)\, \varepsilon\kappa(t-1)\, h(t, \xi(t)) \rangle + \langle (M(t) - M(t-1))\, \kappa(t-1) \rangle \\
&\quad = \langle M(t)\, \varepsilon\kappa(t-1)\, h(t, \xi(t)) \rangle
\end{aligned}$$

we get that

$$\begin{aligned}
&E((Y(t) - Y(t-1))^2) \\
&\quad = E(\langle M(t)\, \kappa(t) - M(t-1)\, \kappa(t-1) \rangle^2) \\
&\quad = E(\langle M(t)\, \varepsilon\kappa(t-1)\, h(t, \xi(t)) \rangle^2) \\
&\quad = \varepsilon^2 E(\langle M^{(1)}(t)\, \kappa^{(1)}(t-1)\, h(t, \xi^{(1)}(t)) \\
&\qquad \times M^{(2)}(t)\, \kappa^{(2)}(t-1)\, h(t, \xi^{(2)}(t)) \rangle)
\end{aligned}$$

where $\xi^{(i)}$, $\kappa^{(i)}$ are as in proof of Lemma 3 and

$$M^{(i)}(t) = |\xi^{(i)}(t)|^2 - t$$

Using the assumptions about $h$, we get that

$$\begin{aligned}
&E(\langle M^{(1)}(t)\, \kappa^{(1)}(t-1)\, h(t, \xi^{(1)}(t)) \\
&\qquad \times M^{(2)}(t)\, \kappa^{(2)}(t-1)\, h(t, \xi^{(2)}(t)) \rangle) \\
&\quad = \langle M^{(1)}(t)\, M^{(2)}(t)\, E(\kappa^{(1)}(t-1)\, \kappa^{(2)}(t-1) \\
&\qquad \times h(t, \xi^{(1)}(t))\, h(t, \xi^{(2)}(t))) \rangle \\
&\quad = \langle M^{(1)}(t)\, M^{(2)}(t) (1+\varepsilon^2)^{n_{t-1}(\xi^{(1)}, \xi^{(2)})}\, 1_{\xi^{(1)}(t) = \xi^{(2)}(t)} \rangle \\
&\quad \leqslant \langle M^{(1)}(t)\, M^{(2)}(t) (1+\varepsilon^2)^{n_\infty(\xi^{(1)}, \xi^{(2)})}\, 1_{\xi^{(1)}(t) = \xi^{(2)}(t)} \rangle \\
&\quad \leqslant \langle (M^{(1)}(t)\, M^{(2)}(t))^{50} \rangle^{1/50} \\
&\qquad \times \langle (1+\varepsilon^2)^{(50/49) n_\infty}\, 1_{\xi^{(1)}(t) = \xi^{(2)}(t)} \rangle^{49/50} \\
&\quad \leqslant \langle (M(t))^{100} \rangle^{1/50} \langle (1+\varepsilon^2)^{(50/49)(11/5) n_\infty} \rangle^{(49/50)5/11} \\
&\qquad \times (P(\xi^{(1)}(t) = \xi^{(2)}(t)))^{(49/50)6/11}
\end{aligned}$$

where in the last and next-to-last relations we used Hölder's inequality.

We know that

$$(P(\xi^{(1)}(t) = \xi^{(2)}(t))^{(49/50)6/11}$$

is of order

$$(t^{-d/2})^{(49/50)6/11} \leqslant t^{-(12/11)49/50} = t^{-294/275}$$

and that

$$\langle (1+\varepsilon^2)^{(50/49)(11/5)n_\infty} \rangle \leqslant \langle (2^{110/49})^{n_\infty} \rangle$$
$$\leqslant \langle 5^{n_\infty} \rangle$$
$$= \sum_{k=0}^{\infty} (5p_d)^k (1-p_d) < \infty$$

It is clear that

$$\langle (M(t))^{100} \rangle = O(t^{100})$$

Therefore

$$\sup_t E\left( \sum_{s=1}^{t} \frac{Y(s)-Y(s-1)}{s} \right)^2$$
$$= \sup_t \sum_{s=1}^{t} s^{-2} E((Y(s)-Y(s-1))^2) < \infty$$

The proof is now complete. QED

From these two lemmas we immediately get the main result of this paper.

**Theorem 2.** If $d \geqslant 4$, then for any $\varepsilon \in (0, 1)$,

$$\lim_{t\to\infty} \frac{\langle |\xi(t)|^2 \kappa(t) \rangle}{t \langle \kappa(t) \rangle} = 1$$

almost surely.

If we introduce the $\mathscr{G}_t$-martingales $W_n(t, x)$ as Bolthausen[1] did and combine the argument in the proof of Lemma 4 above with the proofs of Lemma 4 and Theorem 2 of ref. 1, we can prove the following result, which is more general than Theorem 2 above.

**Theorem 3.** If $d \geqslant 4$, then for any $\varepsilon \in (0, 1)$ and all $n_1, \ldots, n_d \in \mathbb{N}_0$,

$$\lim_{t\to\infty} \frac{\langle \prod_{j=1}^{d} (\xi_j(t)/\sqrt{t})^{n_j} \kappa(t) \rangle}{\langle \kappa(t) \rangle} = \prod_{j=1}^{d} \gamma(n_j)\, d^{-n_j/2}$$

almost surely, where $\xi_j$, $j = 1, \ldots, d$, are the components of $\xi$, and $\gamma(n) = 0$ if $n$ is odd, and $\gamma(2k) = 1 \cdot 3 \cdots (2k-1)$.

For a given realization of the environment $h$, we define the probability measure $\mu_h^t$ on $\mathbb{R}^d$ by

$$\mu_h^t(A) = \frac{\langle 1_A(\xi(t)/\sqrt{t})\, \kappa(t)\rangle}{\langle \kappa(t)\rangle}$$

Then Theorem 3 implies the following central limit theorem.

**Theorem 4.** If $d \geqslant 4$, then for almost all $h$, $\mu_h^t$ converges to the centered normal law with covariance matrix $(1/d)$ times the identity matrix.

## 3. GENERALIZATIONS

Now we are going to generalize the results of the previous section to the case where the distribution of $h$ may be more general. Throughout this section we are going to assume that the environment variables $h$ are bounded and that the supremum of the support of $h$ has a positive mass.

For any $\beta > 0$, $t \in \mathbb{N}$, and $x \in \mathbb{Z}^d$, define

$$\Lambda_\beta = E \exp(\beta h(t, x))$$

$$H_\beta(t, x) = \frac{\exp(\beta h(t, x)) - \Lambda_\beta}{\Lambda_\beta}$$

Then we have

$$Z(t) = \Lambda_\beta^t \prod_{j=1}^{t} [1 + H_\beta(j, \xi(j))]$$

Therefore if we define

$$\tilde{\kappa}(t) = \prod_{j=1}^{t} [1 + H_\beta(j, \xi(j))]$$

then we have

$$\frac{\langle |\xi(t)|^2 Z(t)\rangle}{\langle Z(t)\rangle} = \frac{\langle |\xi(t)|^2 \tilde{\kappa}(t)\rangle}{\langle \tilde{\kappa}(t)\rangle}$$

The following result in an immediate consequence of our assumptions on the environment variables $h$. The proof is very straightfoward and so we omit it.

**Lemma 5.** If the supremum of the support of $h$ is $M$, then for any $\beta > 0$,

$$EH_\beta^2(t, x) \leqslant \frac{1 - [P(h = M)]^2}{[P(h = M)]^2}$$

The following result is similar to that of Lemma 2 above.

**Lemma 6.** $\langle \tilde{\kappa}(t) \rangle$ is a nonnegative $\mathscr{F}_t$-martingale satisfying $E(\langle \tilde{\kappa}(t) \rangle) = 1$.

**Lemma 7.** There exists a positive integer $d_1$ depending only on the distribution of $h$ such that if $d \geqslant d_1$, then $\langle \tilde{\kappa}(t) \rangle$ converges almost surely to a random variable $\zeta$ satisfying

$$E(\zeta) = 1 \qquad \text{and} \qquad P(\zeta = 0) = 0$$

*Proof.* The proof of this result is similar to that of Lemma 3 above; we only need to prove the fact that there exists a positive integer $d_1$ depending on the distribution of $h$ such that if $d \geqslant d_1$, then

$$\sup E(\langle \tilde{\kappa}(t) \rangle^2) < \infty$$

The rest of the proof is exactly the same as that of Lemma 3.

We consider two independent copies $\xi^{(1)}$ and $\xi^{(2)}$ of the random walk $\xi$ with corresponding quantities

$$\tilde{\kappa}^{(i)}(t) = \prod_{j=1}^{t} [1 + H_\beta(j, \xi^{(i)}(j))]$$

The random environment remains independent of $\xi^{(1)}$ and $\xi^{(2)}$. Then

$$\begin{aligned}
E(\langle \tilde{\kappa}(t) \rangle^2) &= E(\langle \tilde{\kappa}^{(1)}(t)\, \tilde{\kappa}^{(2)}(t) \rangle) \\
&= \left\langle E\left( \prod_{j=1}^{t} [1 + H_\beta(j, \xi^{(1)}(j))][1 + H_\beta(j, \xi^{(2)}(j))] \right) \right\rangle \\
&\leqslant \langle (1 + C)^{n_t(\xi^{(1)}, \xi^{(2)})} \rangle \\
&\leqslant \langle (1 + C)^{n_\infty(\xi^{(1)}, \xi^{(2)})} \rangle \\
&= \sum_{k=0}^{\infty} (1 + C)^k p_d^k (1 - p_d) \\
&= (1 - p_d) \sum_{k=0}^{\infty} ((1 + C)\, p_d)^k
\end{aligned}$$

where we have used Lemma 5 and

$$C = \frac{1 - [P(h = M)]^2}{[P(h = M)]^2}$$

From Section 1 we know that there exists a positive integer $d_1$ such that when $d \geqslant d_1$,

$$p_d < \frac{1}{1 + C}$$

Thus we have that if $d \geqslant d_1$, then

$$\sup E(\langle \tilde{\kappa}(t) \rangle^2) < \infty \quad \text{QED}$$

If we define

$$\tilde{Y}(t) = \langle M(t)\, \tilde{\kappa}(t) \rangle$$

then similar to Lemma 4 we have the following:

**Lemma 8.** There exists a positive integer $d_2$ depending only on the distribution of $h$ such that if $d \geqslant d_2$, then

$$\lim_{t \to \infty} \frac{\tilde{Y}(t)}{t} = 0$$

almost surely.

From this lemma we immediately get the following generalization of Theorem 2.

**Theorem 5.** If $d \geqslant d_0 = d_1 \vee d_2$, then for any $\beta > 0$,

$$\lim_{t \to \infty} \frac{\langle |\xi(t)|^2\, Z(t) \rangle}{t \langle Z(t) \rangle} = 1$$

almost surely.

Of course Theorems 3 and 4 can also be generalized to this setting.

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## REFERENCES

1. E. Bolthausen, A note on the diffusion of directed polymers in a random environment, *Commun. Math. Phys.* **123**:529–534 (1989).
2. P. Griffin, Accelerating beyond the third dimension: Returning to the origin in simple random walk, *Math. Scientist* **14**:24–35 (1990).
3. J. Z. Imbrie and T. Spencer, Diffusion of directed polymers in a random environment, *J. Stat. Phys.* **52**:609–626 (1988).
4. J. Neveu, *Discrete-Parameter Martingales* (North-Holland, Amsterdam, 1975).
5. F. Spitzer, *Principle of Random Walk*, 2nd ed. (Springer-Verlag, New York, 1976).

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