Review of Some Methods for Dispersion Calculation and Impact Prediction for Sounding Rockets

by

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1. Introduction

Range safety regulations require mostly a rather accurate prediction of the impact area of the debris of unguided rockets (as are most sounding rockets). The usual way to meet this requirement is to establish some nominal impact point by calculation of the rocket trajectory for ideal circumstances and defining some area around this point to account for the generally unknown differences between the actual and the ideal trajectory. The magnitude of this area is determined by the maximum expected deviation of the actual impact point and is called the dispersion area.

This report originated from a search in the available literature for a simple method for dispersion calculation and impact prediction. This objective could not be met, because many different methods are currently in use for these calculations, all of which have in common extensive calculation schemes which make the use of an electronic computer a necessity.

Due to the limited time available for this project this study was stopped with no result except for a better understanding of dispersion and of the characteristic properties of the methods reviewed. In view of a possible later continuation of this study it was felt worthwhile to lay down these experiences in writing, which is done in this report. All mathematics and equations used for the different methods were left out on purpose, as these can be readily looked up in the references themselves. Instead the emphasis was laid on the considerations and assumptions, which underlie the methods, as well as on their physical meaning.

2. Dispersion

The main causes for the deviation of the actual point of impact from its nominal location, which will be considered in this report, are

a. winds
b. thrust misalignments
c. structural misalignments

Other causes which give deviations of the point of impact but which are not explicitly included in the following treatment are:

d. errors in the burnout speed

e. errors in the launch angle

f. errors in the angle of attack at ignition of later stages

g. stage separation disturbances

h. disturbances due to the launcher (tipoff effects)

These last causes usually do not change the overall dispersion in an appreciable way, partly because they are implicitly taken into account (d, e, f) partly because the disturbances are in general very small (g, h).

For a well constructed rocket, it turns out that winds on the trajectory cause the largest deviation of the impact point as compared with the impact point calculated for no wind. This led to the present practice of taking the winds into account in the determination of the nominal impact point. This is done either by calculating different impact points for different wind models or by calculating for different wind models the launch angle compensation required to make all nominal impact points coincide. In both cases there will remain a small dispersion due to winds as a result of differences between the wind model used in the calculations and the actual winds at launch. This last dispersion is determined in the same way that the dispersion is calculated in general, namely by calculating the deviation of the impact point due to an estimated maximum error (in the wind in this case), the magnitude of which is dependent on the accuracy with which the model used for the calculation of the nominal impact point corresponds to the actual situation at launch.

By far the largest part of the deviation of the impact point due to the winds and misalignments is originated during the burning time of the rocket. This is readily understood if one takes a better look at how dispersion is caused. In a quiet atmosphere the only forces which determine the trajectory of a stable rocket without misalignment are the gravity force, the thrust and the drag. Due to the stability of the rocket and usual slow pitch rate during the first part of the trajectory of a sounding rocket, the longitudinal axis will be tangent to the trajectory as will be the thrust and the drag. This changes in the case of a wind as a stable rocket will tend to weathercock into the direction of the relative air velocity. This weathercocking for a rocket with finite inertia will be accompanied by aerodynamic moments and forces perpendicular to the trajectory, the main effect during burning however will be that the longitudinal axis and hence the thrust will be no longer tangent to the trajectory and will get a component perpendicular to the flight path. This thrust component affects the local slope of the trajectory and hence the final location of the impact point. The displacement of the impact point due to this effect will be such that the range of the rocket is extended in the direction from which the wind comes. In the case of structural misalignments (also called fin misalignments) a similar phenomenon occurs as a stable rocket will tend to make an angle with the trajectory equal to the equivalent trim angle.

After burnout and during coasting the axis of the rocket will still make an angle with the trajectory in the case of winds or structural misalignments. The perpendicular thrust component will not be present anymore and the only forces affecting the trajectory besides the gravity force will be the much smaller forces of aerodynamic origin. The effect of these in case of a wind will be in general a displacement of the impact point in the direction in which the wind is blowing and hence a diminution of the extension of the range caused by the wind from the same direction during burning. The effect however will be relatively small as is easily understood once one considers the trajectory of the rocket in a frame moving with the wind velocity. In this frame there will be either none or only oscillating aerodynamic forces perpendicular to the trajectory as a stable rocket will tend to align itself with the incoming air velocity. The range of this trajectory will be enlarged in the direction opposite to the motion of the moving frame as the horizontal velocity of the rocket seems to be larger in this direction, by an amount approximately equal to the wind velocity. The displacement of the impact point in the fixed frame will be the difference between this extension of the range of the trajectory in the moving frame and the displacement of the moving frame in the same time, which difference is obviously very small.

Most of the dispersion due to the winds during burning is caused in the lowest few thousand feet as the velocity of the rocket is still small. (The smaller the velocity the larger the dispersion.) The influence of the winds decreases thereafter rapidly with increasing height and in general some height (50,000 - 100,000 ft) can be established above which the winds have practically no influence.

The dispersion due to misalignments can be greatly reduced by giving the rocket a small roll rate. Caution in determining this roll rate is needed, however, as the coincidence of the roll frequency and the natural yaw or pitch frequency can lead to resonance effects and unacceptably large yaw and pitch angles. By a careful choice this difficulty can be overcome as discussed in reference 1.

As a final introductory remark it should be mentioned that the picture of dispersion causes sketched so far becomes much more complicated if various parts (booster, first stage) of the sounding rockets are considered. This is due to the fact that these parts after stage separation will have completely different and often unknown aerodynamic and inertial characteristics. Each case has therefore to be considered separately. Of interest in this context is reference 12 in which the attempts are discussed to predict the impact point of the booster of the Aerobee 150 sounding rocket. The calculated trajectories showed a summit altitude of 10, 000 ft after launcher from 4000 ft. The predicted impact distances for five flights turned out to have an error, which varied from 405 to 719 ft, while the azimuth predictions yielded errors in between 24.0° and 85.0°. Dispersion calculations in contrast herewith gave only a dispersion radius of about 200 ft!

3. Review of some dispersion calculation methods from the literature

Most important for the accuracy of dispersion calculations is the method used for the trajectory calculation. Different trajectory calculations are discussed in references 2, 3, 4, 5 and 6. (Reference 1 discusses the method described in reference 5 and 8, but does not give any details.) From these the first three treat also the determination of the dispersion. Reference 8 deals exclusively with the launch angle compensation to correct for the winds at launch, while reference 7 discusses in detail the complete setup of another approach to launch angle compensation. A short review of the basic features of the methods described in these references will be given here below. For details the references themselves should be consulted.

Reference 2 is one of the first and also one of the most extensive treatments of dispersion. It gives a good survey of the sources of dispersion and their relative merit. The rockets considered are artillery- and aircraft-rockets for which reasons the examples treated in detail are not significant for sounding rockets. The underlying theory is thorough and can be easily applied for sounding rocket calculations. The discussion of the dispersion due to winds is unfortunately rather brief.

The assumptions made for the determination of the trajectory are the following:

- the rocket is fairly symmetrical about its longitudinal axis, as well in aerodynamic as in inertial aspect.
- the rocket is not spinning or only slow spinning, so that gyroscopic and aerodynamic effects dependent on the spin can be neglected.
- only small misalignments, which are dependent on the orientation of the rocket, and only small yaw angles are considered.
- the trajectories are such that the gravitational acceleration can be considered constant.

The reference frame considered consists of a vertical plane and a cylindrical plane perpendicular to this, and the planes intersect each other along the "average" trajectory. The actual trajectory is oscillating around this average trajectory. The reason for choosing this reference frame is that in this way the yawing motion in the two perpendicular planes can be considered to be independent.

The resulting equations of motion are two sets of three simultaneous equations in the perpendicular planes (two translations and one rotation). From these sets the first equations regarding the motion along the average trajectory are the same. (There is no equation for the rolling motion, which can be chosen arbitrarily.) By introducing complex variables both sets of equations can be combined to reduce the total number of equations to three.

A large part of the reference is devoted to solving these three equations analytically after several appropriate assumptions. This turns out to be a very complicated matter, which complexity is only slightly reduced by the introduction of so-called rocket functions, which are given in tabulated form.

The final conclusion on this reference is that the described method, although it has its value as analytical solution, is very complicated for hand calculation, whereas later methods which are more accurate by having no analytical solutions are perhaps easier to work with when a modern computer is available. Another objection to this method for use with sounding rockets is that too little attention is paid to the dispersion caused by winds encountered on the trajectory.

Reference 3 gives one of the earliest and simplest methods to calculate the dispersion of sounding rockets due to winds. A clear description is given of the usual approach towards the calculation of the dispersion due to winds. This includes the following steps:

a. the atmosphere is divided into horizontal strata where the wind is constant in magnitude and direction.

b. the constant winds in the strata are weighted with windweighting factors to yield a so-called ballistic wind, which has per definition the same effect on the trajectory as the actual wind.

c. a unit wind effect or ballistic factor is calculated which is the dispersion caused by a wind of unit strength blowing out of one direction during the whole trajectory.

d. the total dispersion of the actual wind at launch is assumed to be equal to the product of the ballistic wind times the unit wind effect.

This approach is used, in principle, in nearly all methods for calculation of the dispersion due to winds. The main differences between the methods lie in the way the windweighting factors are determined as functions of altitude.

Reference 3 defines the windweighting factors for some altitudes as the ratio of the displacement caused by a unit wind up to that altitude and the displacement caused by a unit wind over the whole trajectory (unit wind effect). According to this scheme the winds in the mentioned horizontal strata are to be multiplied by the differences between the windweighting factors at the upper and lower limits of the strata.

The windweighting factor curves are determined by trajectory calculations. The trajectory calculation method discussed in reference 3 is well known and is referred to in the literature as the "Lewis method" after its author. The assumptions on which this method is based are the following:

- the rocket is fired vertically or nearly vertically so that the vertical velocities can be considered unchanged by the horizontal winds.
- the only forces acting on the rocket are the thrust, the drag and the gravity forces.
- the rocket weathercocks instantaneously in the direction of the relative air velocity (infinite stability!).

The reference system in which the motion of the rocket is considered consists of two vertical planes on which the trajectory is projected and which are moving with the wind velocity. The mentioned assumptions yield the possibility of using the vertical velocity-time functions as the solution of the equation of motion in the vertical direction. The horizontal directions the solutions of the equations of motion are found from the assumption that the only force perpendicular to the flight path in the moving frame is the gravity force. This results in a simple analytical solution for the horizontal velocity for each layer, which can be integrated over the known time to yield the horizontal displacement.

The final conclusion about this method may be formulated as follows. A means is given to calculate in a very simple way (the vertical velocity vs time relation is the only information required) the approximate displacement of the impact point caused by the winds on the trajectory for a large class of sounding rockets. Not for all sounding rockets, as the assumption of instantaneous weather-cocking prohibits the use of this method without additional assumptions for rockets launched from a zero length launcher.

It should be noted that reference 3 besides the above discussed method also treats the effect of a launch angle change and the effect of the rotation of the earth. This last effect can by good approximation be calculated separately as is done in nearly all the trajectory calculations which will be reviewed.

Reference 4, which is one of a whole series of reports (ref. 9-12) by the same author on this subject, treats the calculation of the dispersion due to winds as well as the dispersion due to structural and thrust misalignments. Moreover results are given of calculations of dispersion due to those other causes mentioned in the introduction.

The approach to the calculation of dispersion due to winds is exactly the same as given in ref. 3, which means that a windweighting factor curve and a unit wind effect are calculated, which are used respectively to determine the equivalent ballistic wind from the actual winds at launch and the total displacement of the impact point due to these winds, which is assumed to be equal to the product of the ballistic wind and the unit wind effect. The trajectory calculation used for the determination of the windweighting factors and the unit wind effect differs from the one discussed in ref. 3 in that the finite stability of the rocket is taken into account.

The main assumptions made for the general equations of motion, which in their general form yield the possibility of taking the misalignments into account, are:

- the pitch and yaw motions are independent, so that the trajectory can be treated as the sum of two two-dimensional trajectories (projections) in two perpendicular planes of an earth fixed frame.
- winds can be introduced in these equations as changes in the effective angle of attack, leaving the velocity of the rocket with respect to the air unchanged.

- misalignments can be introduced in the equations as extra normal forces proportional to the resulting trim angle.

The equations of motion obtained under these assumptions are in principle the same as those derived in ref. 2 and consist of three rather simple simultaneous equations (two translations and one rotation) in each of the reference planes, which can be solved by a numerical step-by-step integration. The range is found by integrating the horizontal velocity over the total flight time.

The conclusion about this method can be that the dispersion calculation is rather simple and fits very well to the likewise simple numerical methods presently in use to calculate the standard trajectories of sounding rockets. The method is made reasonably accurate by taking inertia into account for which reason it can also be applied to launchings from zero length launchers (slight modifications have to be introduced for the first part of the trajectory as the effective angles of attack due to the wind for low vehicle velocities may be too large to assume the lift proportional to the angle of attack). The method is slightly in error by its assumption of independency of the yaw and pitch motions as will be discussed in the review of ref. 8.

Besides the above discussed method itself the reference gives some calculated results for dispersion, which are of interest as they show the relative merit of the different sources of dispersion. The rocket considered was the Aerobee-300, a boosted two-stage sounding rocket which is launched from a 120 ft. high launch tower. The following main trajectory data apply:

launch angle	= 89^{o}
burnout height (2nd stage)	= 138, 000 ft (24. 9 mi)
burnout velocity (2nd stage)	= 9400 ft/sec
burnout time (2nd stage)	= 53. 5 sec
time coasting	= 0 sec
summit altitude	= 305 mi

The dispersion calculations in this case yield

Source	Non-rolling Vehicle (mi)	0.2 rev/sec roll rate at burnout 1st stage (mi)	2.0 rev/sec roll rate at burnout 1st stage (mi)
Error in ballistic wind (2mph)	27.4	27.4	27.4
Thrust misalignment of 1st stage and booster (0.125°)	53.5	16.7	13.8
Thrust misalignment of 2nd stage (0.125°)	26.7	19.8	1.2
Structural misalignments (trim angle 0.05°)	26.2	8.2	6.2
Error in angle of attack at ignition 2nd stage (4°)	1.6	1.6	1.6
Separation disturbance	3.0	3.0	3.0
Total dispersion (from sum of root mean squares)	71	39	32

Reference 5 deals only with trajectory determination taking into account thrust- and fin-misalignments and arbitrary winds. The setup of the equations of motion follows closely the lines of the aircraft flight mechanics of the motion of a rigid body with six degrees of freedom.

The assumptions made for the derivation of the equations are the following:

- The rocket is a rigid body with six degrees of freedom.
- The vehicle is symmetric in inertial sense with respect to some plane through the longitudinal axis.
- The vehicle has aerodynamic symmetry in roll except for small asymmetric misalignments.
- The aerodynamic forces and moments are assumed to be functions of the Mach number and nonlinear with the angle of attack.
- The acceleration of gravity is constant. (This assumption is not essential for the method and can easily be removed.)

The reference system consists of two Cartesian axes systems, one fixed to the flat non-rotating earth at the launching point, the other fixed in the center of gravity of the rocket. The orientation of this last reference system with respect to the first is given by the coordinates of the origin and by the Euler angles commonly used to fix aircraft altitudes.

The resulting equations of motion are the six well known Euler type equations commonly used to describe the motion of aircraft. The structural and thrust misalignments are introduced in these equations as forces and moments which magnitudes are determined by the geometry of the misalignments. The wind influence is brought in by changes in the aerodynamic forces due to changes in the angle of attack and changes in the relative velocity with respect to the air.

It may be concluded that this method for trajectory determination is basic as well as straight-forward. The accuracy will depend only on the inputs. The only disadvantage of the method is that a rather extensive computer program is needed to solve the complete equations as is a well known fact from aircraft flight mechanics.

It may be noted that the above method was setup to calculate launch angle compensations to be discussed in ref. 8. For this purpose a non-rotating earth and a constant acceleration of gravity could be assumed as only the first part of the trajectory had to be considered. For an actual trajectory calculation these assumptions should be removed.

Reference 6 also deals with determination of sounding rocket trajectories in which only the winds are taken into account. The setup of the equations of motion resembles very much that of ref. 2, however, no analytical solution is being sought. In comparison with the trajectory determination in ref. 2, this method is much more accurate taking into account the rolling motion of the rocket, the influence of the winds and the variation of the gravity force and the rotation of the earth. Moreover the aerodynamic forces and moments are introduced as arbitrary functions of the Mach number and angle of attack. However, no effort is made to calculate for the influence of misalignments.

The assumptions made are:

- The rocket is a rigid body with six degrees of freedom.
- The vehicle is axially symmetric in aerodynamic and in inertial respect.
- The thrust of the rocket is perfectly aligned.

The reference system used resembles that of ref. 5, one orthogonal system fixed to the earth, another fixed to the center of gravity of the rocket, which moves with the rocket, however, does not roll with it. The orientation of this reference system with respect to the earth-fixed system is given by the coordinates of the origin and by the direction cosines of the axes.

The resulting equations of motion are not simple as they contain many trigonometric functions as a result of the vector approach, which is used. There are six differential equations of the second order in the coordinates of the origin and the direction cosine angles.

Quite an extensive computer program is needed (as stated by the author himself) to calculate one trajectory exactly. However, several simplifications can be made to cope with this in most cases and furthermore perturbation equations are given, which can be used for similar trajectories once one is calculated in full.

These remarks may be concluded by stating that a very accurate means is given for the calculation of trajectories. The influence of the wind is taken well into account and for misalignments slight changes will suffice. The objection to this method is primarily the same as for the method of ref. 5, namely, that an extensive computer program is needed. Moreover, unlike the method of ref. 5 this method is less simple and straightforward and, although this is not a real objection, does not appeal so well to people used to the classical mechanical approach in flight mechanics.

Reference 7 differs from the foregoing references, as it does not deal with a specific method for calculation of the impact point and the launch angle compensation. The main subject instead is the computer setup of a procedure to perform such calculations in the field before an actual launching. The method of impact prediction, which underlies the procedure is in principle a refinement of the one discussed in ref. 2 making use of the assumption, that the deviation of the impact point due to winds can be found as the product of the ballistic wind and the unit wind effect.

The procedure supposes, that the no wind range, the windweighting factors and the unit wind effect for the rocket in question were calculated as functions of the launch angle well before the launch date. As the windweighting factors are not too sensitive for the launch angle, the ballistic wind can be calculated from the wind measurements before launch for a guessed value of the launch angle. This is done in steps in such a way that,after the bulk of the calculations are done, use can still be made of the latest data on the wind velocities in the most important lower altitude layers. With the ballistic wind, the precalculated tables for the no wind range and the unit wind effect,the impact point for each launch-and azimuth-angle can be calculated rapidly and plotted on a map. From these plots the required launch and azimuth angle are easily determined as those angles for which the predicted impact point will lie in the designated impact area. In practice an iteration procedure will be necessary to find the accurate values for these angles.

The description of the procedure in the reference is of much interest as it shows a glimpse of the practical aspects of a rather simple impact prediction method. The procedure is general in the sense that it can be used with different methods for impact prediction, provided the wind weighting procedure is similar to the one discussed in ref. 2. The accuracy of the eventual results will of course be dependent on these methods. A disadvantage of the launch angle compensation scheme is that an iteration procedure has to be performed shortly before launch.

Reference 8 describes a method for launch angle compensation, which differs substantially from the methods reviewed so far. For a standard trajectory two graphs are developed from which for each ballistic wind the launch- and azimuth-angles, which yield the same impact point with wind as the standard trajectory, can be read off directly. In the method several ideas are incorporated, which are helpful for the understanding of the wind problem. These will be discussed in more detail.

The main assumption in this method is that the bulk of the dispersion due to the winds is caused during the first part of the trajectory when the rocket is burning and that for this reason a height can be established above which the influence of the winds can be neglected completely. To obtain the same trajectory as in the no wind case it will then suffice, when such launch and azimuth angle corrections are determined and applied, that the direction of the trajectory at this height is the same as in the no wind case.

To apply the method a three-dimensional trajectory calculation method is needed, in which the non-linear behavior of the aerodynamic coefficients can be taken into account. (In ref. 8 use is made of the method described in ref. 5, other three-dimensional methods can be used as well.) By this choice of trajectory calculation two commonly used assumptions for launch angle compensation calculations, which may cause large errors are removed, i.e.:

1. the pitch and yaw motions of the vehicle are independent (which is commonly assumed in order to use a simple two-dimensional analysis)

2. the aerodynamic forces and moments are linearly proportional to the angle of attack (which is certainly not true for the large angles of attack which may occur directly after a launch from a zero length launcher).

The unique way in which the launch angle compensation graphs are developed, removes two other common assumptions, which can cause large errors, to wit:

3. the launch angle correction is equal to the launch angle change which yields in the no wind case the same dispersion in opposite direction as the one caused by the winds.

4. the azimuth angle correction can be calculated independently of the elevation angle.

In some methods the third assumption is removed by an iteration procedure. The fourth assumption is of special interest as this is related with the interdependency of the pitch and the yaw motion. The error made by this assumption becomes clear by a consideration of the effect of a pure side wind. With the assumption of independence of the pitch and the yaw motion, the pitch motion will not be affected by this side wind. What actually happens however is that, when the rocket yaws into the wind, this wind gets a component in the new pitch plane, which certainly will affect the pitch motion and eventually will produce a dispersion in the pitch plane. By removing this assumption the launch angle compensation method will be more accurate than previous methods.

The method to compose the launch and azimuth angle compensation graphs is relatively simple as it consists in principle of merely a cross-plotting of the results of trajectory calculations (up to the height above which the wind influence is assumed to be negligible) for several wind strength models out of different directions. Use is made of linearly increasing winds, which are fractional multiples of some standard wind profile, in order to get a better approximation of practical situations. This profile choice affects the wind-weighting technique, which is of interest for its procedure. It should be noted that this windweighting technique is not essential for the launch angle compensation method, although the use thereof will yield more accurate results.

As a consequence of the choice of a linearly increasing wind instead of a constant wind for the trajectory calculations the ballistic wind has to be defined as that linearly increasing wind, which is a fractional multiple of the standard profile which has the same effect on the trajectory as the actual winds at launch. To compute the ballistic wind from actual wind measurement data use is made of a wind sensitivity curve, which is determined in an analogous way as the wind weighting factor curve. In this case the sensitivity at some height is defined as the relative effect on the dispersion of a constant side wind, which is entered at this height and is assumed to be present up to the altitude above which the wind influence is neglected. From these sensitivity curves the limits are defined of 20 layers, of which each contributes 5% to the total wind effect. These limits then are drawn in a graph which shows the standard wind profile and some fractional multiples thereof as a function of the logarithm of the altitude. Due to the use of a logarithmic scale the layers so obtained will not differ too much in thickness in the graph as the thin layers at lower altitude (large wind influence) appear thicker, while the thick layers at higher altitudes appear thinner. The measured wind velocities and direction can be plotted in the so prepared graph. By suitable averaging of the wind velocities in each layer and expressing the result in terms as a fractional multiple of the standard wind in the same layer, the ballistic wind strength can be found by weighting these results further on in the ordinary way. The wind direction can be weighted directly in the ordinary way.

Except for the above reviewed launch angle compensation method and wind weighting technique ref. 8 also discusses the ways in which the accuracy of the method could be established. Results with the method are discussed in ref. 1.

As conclusion it can be stated that the method described in ref. 8 has several big advantages for which reason this method should be preferred above the earlier discussed methods, the main advantages being:

a. greater accuracy
b. simplicity of the graph preparation
c. simplicity of use before an actual launching

A disadvantage of the method is the use of the rather complicated trajectory analysis of ref. 5. It should be noted however that this last method is easilty adapted to yield valuable information for structural and stability analysis. If such information is also wanted, there is of course no objection against the use of this trajectory calculation method. In other cases, when only the knowledge of the dispersion is needed, it might be worthwhile to use a simpler method for the calculation of the trajectory, which takes the interdependence of the pitch and the yaw motion under the influence of the wind into account. A possible tentative setup of equations for such a method is given in the appendix. These equations may also serve as an illustration of the kind of equations in use for dispersion calculations.

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Appendix

As discussed in the conclusion of the foregoing review of the launch angle compensation method of ref. 8, it might be of value to use a simple method for trajectory calculation together with this compensation method. A requirement for the trajectory calculation method is in principle only that the interrelation of the pitch- and yaw-motion of the rocket as affected by winds be taken into account.

In this appendix equations for such a method are given tentatively. (No effort was made yet to solve and check them.) They are an extension of the equations used in ref. 4, which are in turn based on equations derived in ref. 2.

The assumptions for the equations are the following:

- the rocket is a rigid body, which is axial symmetric in inertial respect.
- the rocket has aerodynamic symmetry in roll except for asymmetric misalignments.
- the rocket is not spinning or only slowly spinning; so that gyroscopic and aerodynamic effects dependent on spin can be neglected.
- the angles between the rocket axis and the trajectory remain small during the powered part of the trajectory.
- the angles of attack will be small so that normal aerodynamic force coefficient can be considered to vary linearly with the angle of attack and the drag coefficient can be considered constant.
- the pitching motion in the vertical plane through the instantaneous velocity is independent of the yawing motion perpendicular to this plane.
- the range of the nearly vertical fired rocket is such that the earth can be considered flat and non-rotating. The gravity force is assumed to have a constant direction and to vary in magnitude as an inverse square force, such as in the case of a perfect vertical trajectory. It is further assumed that the effect of the earth's rotation on the trajectory can be calculated separately.

The reference system is an orthogonal frame fixed to the earth at the launching point. The direction of the trajectory is given by the angle γ between the instantaneous velocity and its projection on the horizontal plane and the azimuth angle ψ between this projection and some fixed direction (North direction) in the horizontal plane. (Fig. 1) The orientation of the rocket axis with respect to the trajectory is given by the angle α in the vertical plane through the instantaneous velocity and the yaw angle β in the plane through the instantaneous velocity perpendicular to the vertical plane.

With above assumptions the equations of motion of a sounding rocket in this reference system can be written as follows (see also notation list and fig. 1 and 2 at the end of this appendix):

$$\dot{V} = \frac{g_o}{W}\left[F - W\left(\frac{R_e}{R_e + h}\right)^2 \sin\gamma - C_D \cdot \tfrac{1}{2}\rho V^2 \cdot A\right] \quad (1)$$

$$\dot{\gamma} = \frac{g_o}{WV}\left[F\left\{\alpha + \delta_t \cos(\phi + \phi_t)\right\} - W\left(\frac{R_e}{R_e + h}\right)^2 \cos\gamma\right] +$$

$$+ \frac{g_o}{WV}\left[C_{N_\alpha}\left\{\alpha - \delta_f \cos(\phi + \phi_f) + \frac{u}{V}\cos(\Lambda - \psi)\sin\gamma\right\}\right] \quad (2)$$

$$\dot{\psi}\cos\gamma = \frac{g_o}{WV}\left[F\left\{\beta + \delta_t \sin(\phi + \phi_t)\right\}\right] +$$

$$+ \frac{g_o}{WV}\left[C_{N_\alpha}\left\{\beta - \delta_f \sin(\phi + \phi_f) - \frac{u}{V}\sin(\Lambda - \psi)\right\}\right] \quad (3)$$

$$\ddot{\alpha} + \ddot{\gamma} = -\frac{1}{I}\left[C_{N_\alpha} \cdot \tfrac{1}{2}\rho V^2 \cdot A \cdot L_v\left\{\alpha - \delta_f \cos(\phi + \phi_f) + \frac{u}{V}\cos(\Lambda - \psi)\sin\gamma\right\}\right] -$$

$$-\frac{1}{I}\left[C_{N_\alpha} \cdot \tfrac{1}{2}\rho V^2 \cdot A \cdot L_d \cdot \frac{L_d}{V} - \dot{W}\left(\frac{r_e r_t}{g_o}\right) + \dot{I}\right](\dot{\alpha} + \dot{\gamma}) - \quad (4)$$

$$-\frac{1}{I}\left[F r_t \delta_t \cos(\phi + \phi_t) - C_{M_f}\tfrac{1}{2}\rho V^2 \cdot A \cdot r \cdot \cos(\phi + \phi_m)\right]$$

$$\ddot{\beta} + \ddot{\psi}\cos\gamma = \dot{\psi}\dot{\gamma}\sin\gamma +$$

$$-\frac{1}{I}\left[C_{N_\alpha} \cdot \tfrac{1}{2}\rho V^2 \cdot A\, L_v\left\{\beta - \delta_f \sin(\phi + \phi_f) - \frac{u}{V}\sin(\Lambda - \psi)\right\}\right] -$$

$$-\frac{1}{I}\left[C_{N_\alpha} \cdot \tfrac{1}{2}\rho V^2 \cdot A \cdot L_d \frac{L_d}{V} - \dot{W}\frac{r_e r_t}{g_o} + \dot{I}\right](\dot{\beta} + \dot{\psi}\cos\gamma) - \quad (5)$$

$$-\frac{1}{I}\left[F r_t \delta_t \sin(\phi + \phi_t) - C_{M_f} \cdot \tfrac{1}{2}\rho V^2 \cdot A \cdot r \sin(\phi + \phi_m)\right]$$

The equations for this trajectory calculation, which was announced as simple, do not look very simple at first sight and by a further look they are even more complicated as several symbols stand for functions of time. It should be mentioned, however, that these equations are given in full and that for special cases simplifications are possible. In general, the aid of an electronic computer will be a necessity, especially when more trajectories are to be calculated. If this is the case, it would be worthwhile to consider the removal of the assumption of linear behavior of the normal aerodynamic force with the angle of attack, which removal probably can be done quite easily.

As conclusion of the discussion of this tentative method for trajectory calculation it may be of interest to list the needed information on the rocket, which includes

a. thrust vs. time
b. mass vs. time
c. moment of inertia vs. time
d. center of gravity vs. time
e. center of pressure (for the different misalignments) vs. Mach number
f. drag coefficient vs. Mach number
g. lift coefficient slope vs. Mach number
h. moment coefficient for the misalignments vs. Mach number
i. center of pressure of aerodynamic damping vs. Mach number

It turns out that this list is rather typical for this kind of trajectory calculation.

Notations

A -cross sectional area of rocket

$C_D = \frac{D}{\frac{1}{2}\rho V^2 A}$ -drag coefficient

$C_{M_f} = \frac{M_f}{\frac{1}{2}\rho V^2 . A . r}$ -maximum moment coefficient due to fin-misalignments for zero angle of attack

$C_N = \frac{N}{\frac{1}{2}\rho V^2 . A}$ - normal aerodynamic force coefficient

$C_{N_\alpha} = \frac{dC_N}{d\alpha}$ - slope of normal aerodynamic force coefficient

c. g - center of gravity

c. p - center of pressure

F - thrust

g_o - acceleration of gravity at earth surface

h - altitude

I - inertia moment of rocket

L_d - distance between c. g and c. p of the aerodynamic forces (L_d 0 if c. p behind c. g)

L_v - distance between c. g and c. p of the aerodynamic damping forces (L_v 0 if c. p behind c. g)

R_e - earth radius

r - radius of rocket

r_e - distance of rocket motor exit from c. g

r_t - distance of rocket motor throat from c. g

μ - wind velocity

V - velocity of rocket

W - weight of rocket

α - angle between the rocket axis and the trajectory in the vertical plane

β - angle between the rocket axis and the trajectory in a plane perpendicular to the pitch plane

γ - angle between the velocity and the horizontal plane

δ_f - maximum zero lift angle due to fin misalignments

δ_t - maximum thrust misalignment angle

Λ - direction of wind

ρ - air density

$\phi = \int_0^r \dot{\phi} \, dt$ - roll angle

ϕ_f - initial roll angle of plane belonging to δ_f

ϕ_m - initial roll angle of plane belonging to C_{M_f}

ϕ_t - initial roll angle of plane belonging to δ_t

ψ - azimuth angle of projection of V on horizontal plane

$\cdot$ - a dot above a variable indicates $\frac{d}{dt}$

$\cdot\cdot$ - two dots above a variable indicates $\frac{d^2}{dt^2}$

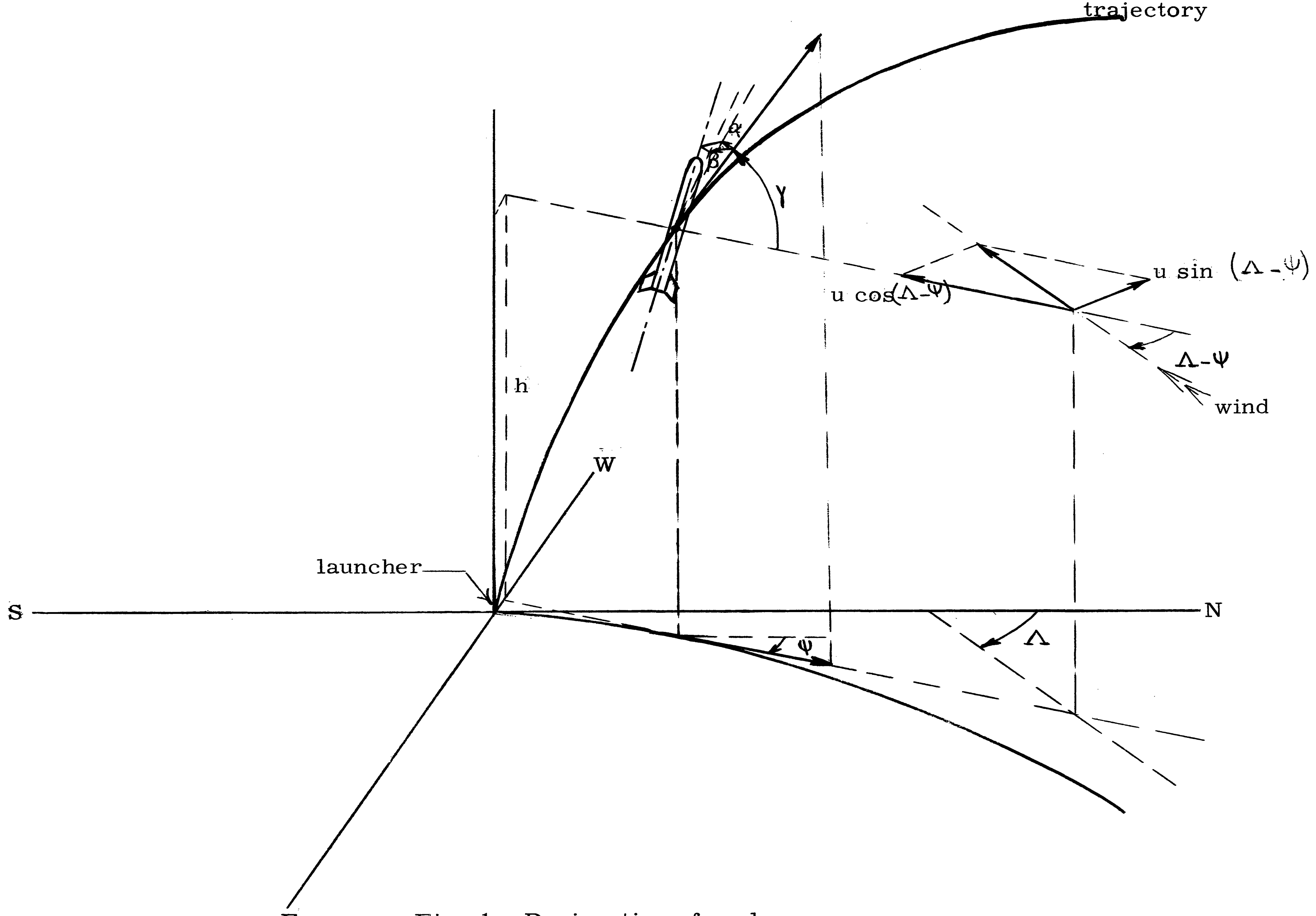

Fig. 1. Designation of angles.

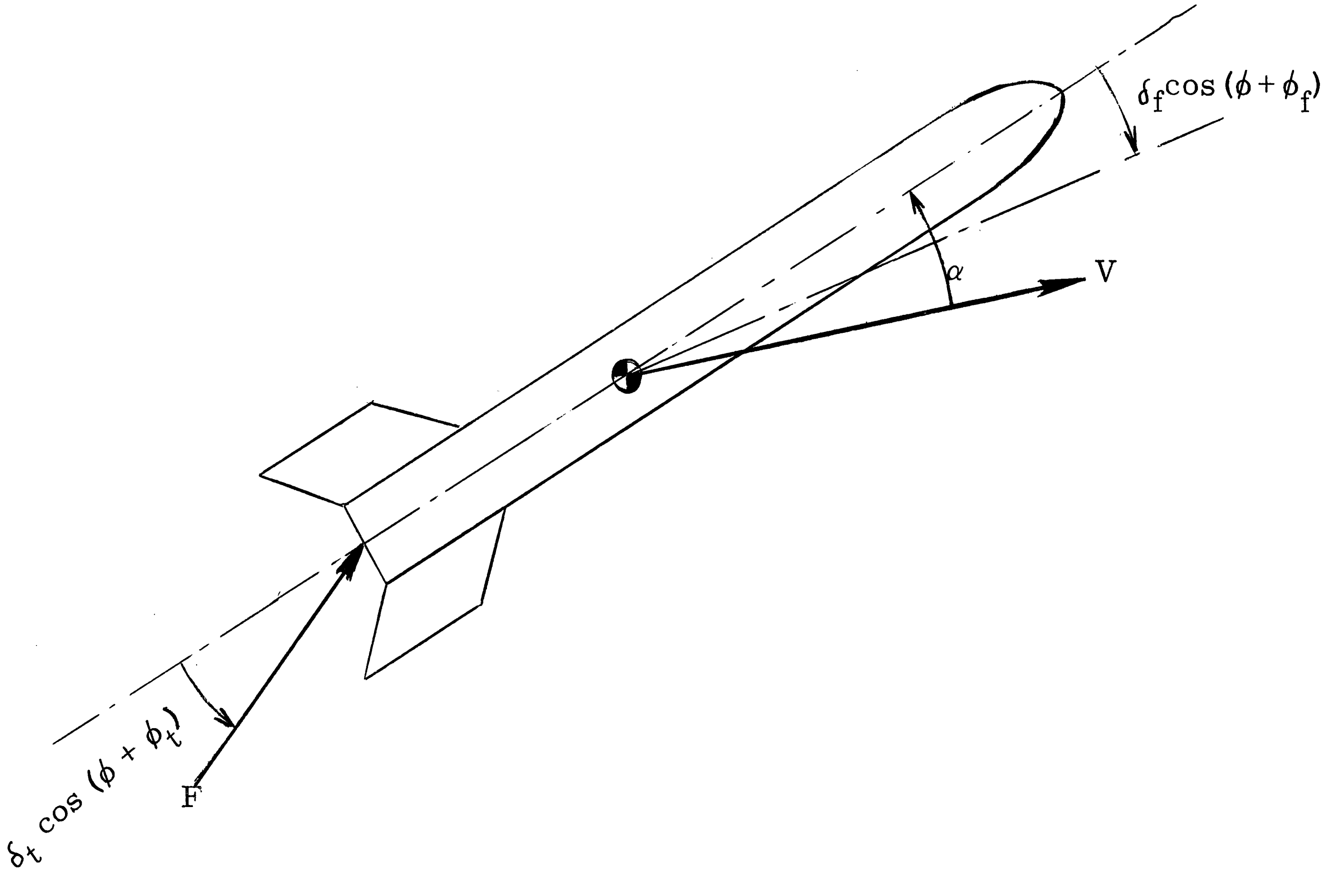

Fig. 2a. Angles in the pitch plane.

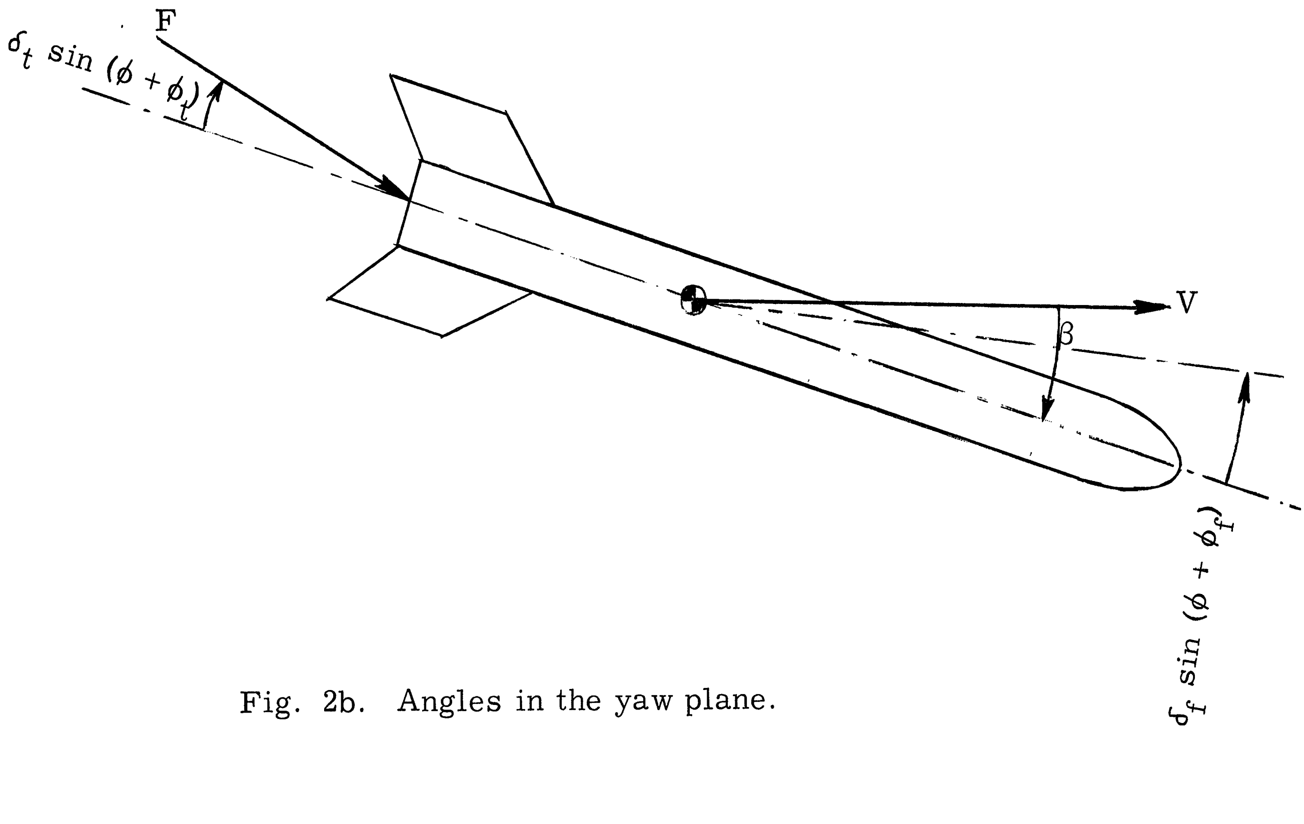

Fig. 2b. Angles in the yaw plane.