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"Theoretical Analysis of Coplanar Waveguide Open Circuit Discontinuity"

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THEORETICAL ANALYSIS OF COPLANAR WAVEGUIDE OPEN CIRCUIT DISCONTINUITY

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Abstract

The theoretical analysis of a coplanar waveguide open circuit discontinuity inside a rectangular cavity is presented. First, the dyadic Green's function of a y- and z- directed dirac delta magnetic currents inside a cavity will be derived. Then the method of moments will be used to solve the integral equation for the unknown magnetic current distribution in the slots. The scattering parameters of such discontinuity could be determined from the knowledge of the magnetic current distribution.

1 Introduction

The widespread use of microwave integrated circuits (MIC's) in recent years has caused rapid progress in its theory and technology. The first transmission line used in MIC's was, indeed, microstrip laid on dielectric substrate, and then other transmission lines such as slot lines, coplanar lines, finlines, and so on, were introduced and improved. Initially the analysis for this class of transmission lines was invariably a quasi-TEM approximation which can yield satisfactory results at low frequencies. However, at high frequencies its weakness becomes apparent. To feature the frequency dependence of these lines, a full wave analysis must be employed.

Recently, new uniplanar circuit configurations for monolithic MIC's were proposed [1]. The fundamental components in these uniplanar MIC's are the coplanar waveguides (CPW), slot lines and air bridges (Fig. 1).

Coplanar waveguides (CPW) offer several advantages over conventional microstrip line: there is no need for via holes which simplifies mounting of active and passive devices and they have low radiation loss. These as well as other advantages make CPW ideally suited for MIC's [2].

This report presents a full wave analysis of one type of coplanar discontinuity, namely the CPW open circuit. The ultimate goal of this study is to characterize various coplanar discontinuities up to the terahertz region. This is intended to be a step towards characterizing the coplanar air bridge discontinuity and other discontinuities.

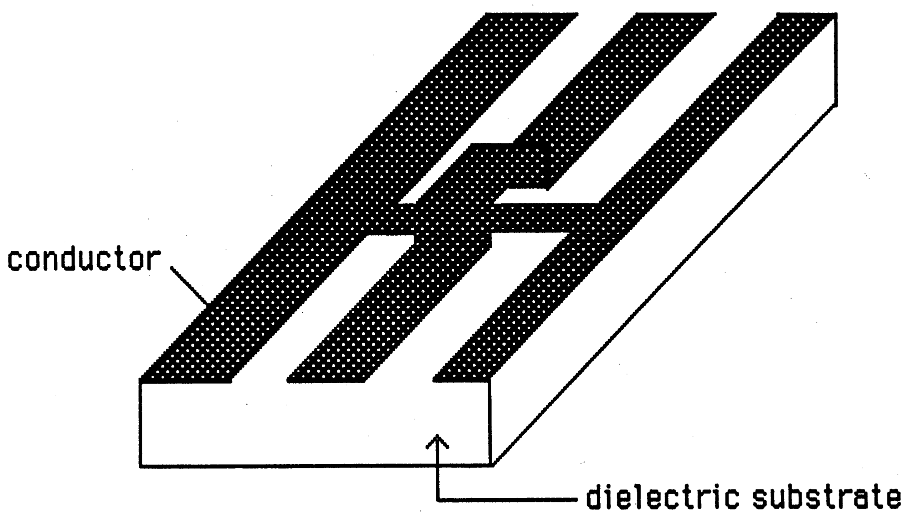

Fig.1 A bridged coplanar waveguide

2 Analysis

2.1 Introduction

A CPW open circuit is shown in Fig. 2. The CPW lies inside a rectangular cavity with a multidielectric structure. The main steps in the formulation of the problem are as follows:

1. Derive the fields in the two regions directly above and below the conductor strip.

2. Formulate the integral equation.

3. Solve this equation using the method of moments.

In the formulation, a few simplifying assumptions are made to reduce the complexity of the problem:

1. The width of the slots is small compared to the coplanar line wavelength λg. This will facilitate the assumption of undirectional magnetic currents in the slots with negligible loss in accuracy.

2. The dielectric layers are lossless and the conductors are perfect. However, the analysis can be easily extended to take losses into consideration.

3. The time dependence is of the form $e^{j\omega t}$ which will be suppressed throughout the analysis.

4. The input is a travelling wave with variation $e^{j\beta z}$ where β is the propagation constant of an infinite coplanar line [3].

2.2 Derivation of Green's Functions

In this section, the tensor Green's function [G] will be derived for the fields in regions (1) and (2) (see Fig. 2). The transmission line theory will be used to transform the surrounding layers into an impedance boundary. Throughout the analysis, $LSE(TE-x)$ and $LSM(TM-x)$ modes are used to derive the Green's function.

The dyadic Green's function denotes the fields of a point source. Hence, the electric field can be computed from

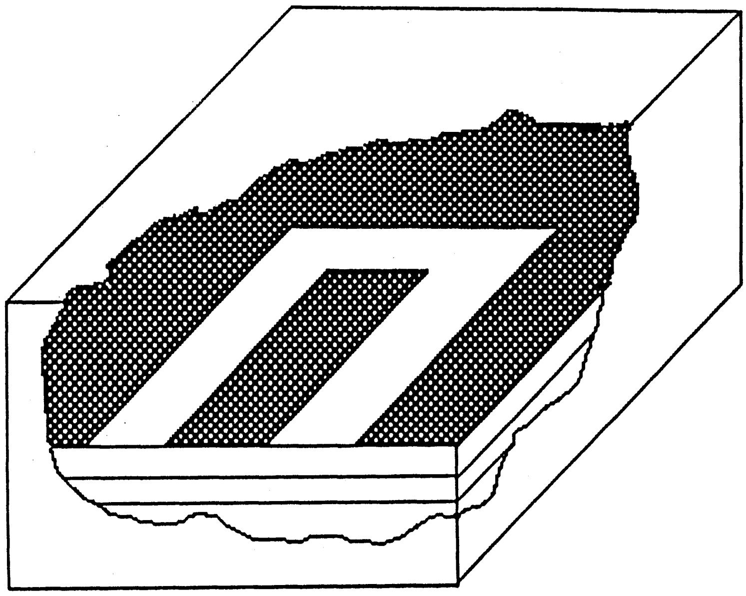

Fig.2 A cutview of a coplanar waveguide open circuit discontinuity inside a cavity .

$$\bar{E} = \int_{s'} \bar{J} \cdot \bar{\bar{G}}_E^e dS' + \int_{s'} \bar{M} \cdot \bar{\bar{G}}_E^m dS' \tag{1}$$

where the integration is done over the surface of the source. In rectangular coordinates $[G]_E^e$, for example, becomes

$$\begin{aligned} \bar{\bar{G}}_E^e &= G_{xx}\hat{x}\hat{x} + G_{xy}\hat{x}\hat{y} + G_{xz}\hat{x}\hat{z} \\ &+ G_{yx}\hat{y}\hat{x} + G_{yy}\hat{y}\hat{y} + G_{yz}\hat{y}\hat{z} \\ &+ G_{zx}\hat{z}\hat{x} + G_{zy}\hat{z}\hat{y} + G_{zz}\hat{z}\hat{z} \end{aligned} \tag{2}$$

where G_{ij} is the jth component of the electric field due to a unit i-directed electric current element. In the same manner, the magnetic field can be derived as

$$\bar{H} = \int_{S'} \bar{J} \cdot \bar{\bar{G}}_H^e dS' + \int_{S'} \bar{M} \cdot \bar{\bar{G}}_H^m dS' \tag{3}$$

In our problem, the two slots are assumed to have magnetic currents. In order to obtain the scattering parameters, the distribution of this magnetic current must be determined. Using the equivalence principle, our original problem is divided into four subproblems, Fig. 3. We have to solve for the Green's functions in both regions due to magnetic currents in the y and z directions. After that has been accomplished the continuity of the tangential fields at the interface will be used to arrive at the integral equation.

2.2.1 Green's function in region (1) for a z-directed magnetic current

The fields due to an infinitesimal z-directed magnetic current inside a cavity will be derived. Fig. 4 shows the structure with the magnetic current alleviated from the ground of the cavity. The magnetic current is assumed to be

$$\bar{M} = \hat{a}_z \delta(x - x')\delta(y - y')\delta(z - z') \tag{4}$$

Notice that at the end of the analysis x' will be substituted by zero. As mentioned before, a hybrid mode analysis (LSE and LSM) will be considered [4].

The following vector magnetic potential $\bar{A}$ and electric vector potential $\bar{F}$ for the LSM and LSE modes respectively are assumed

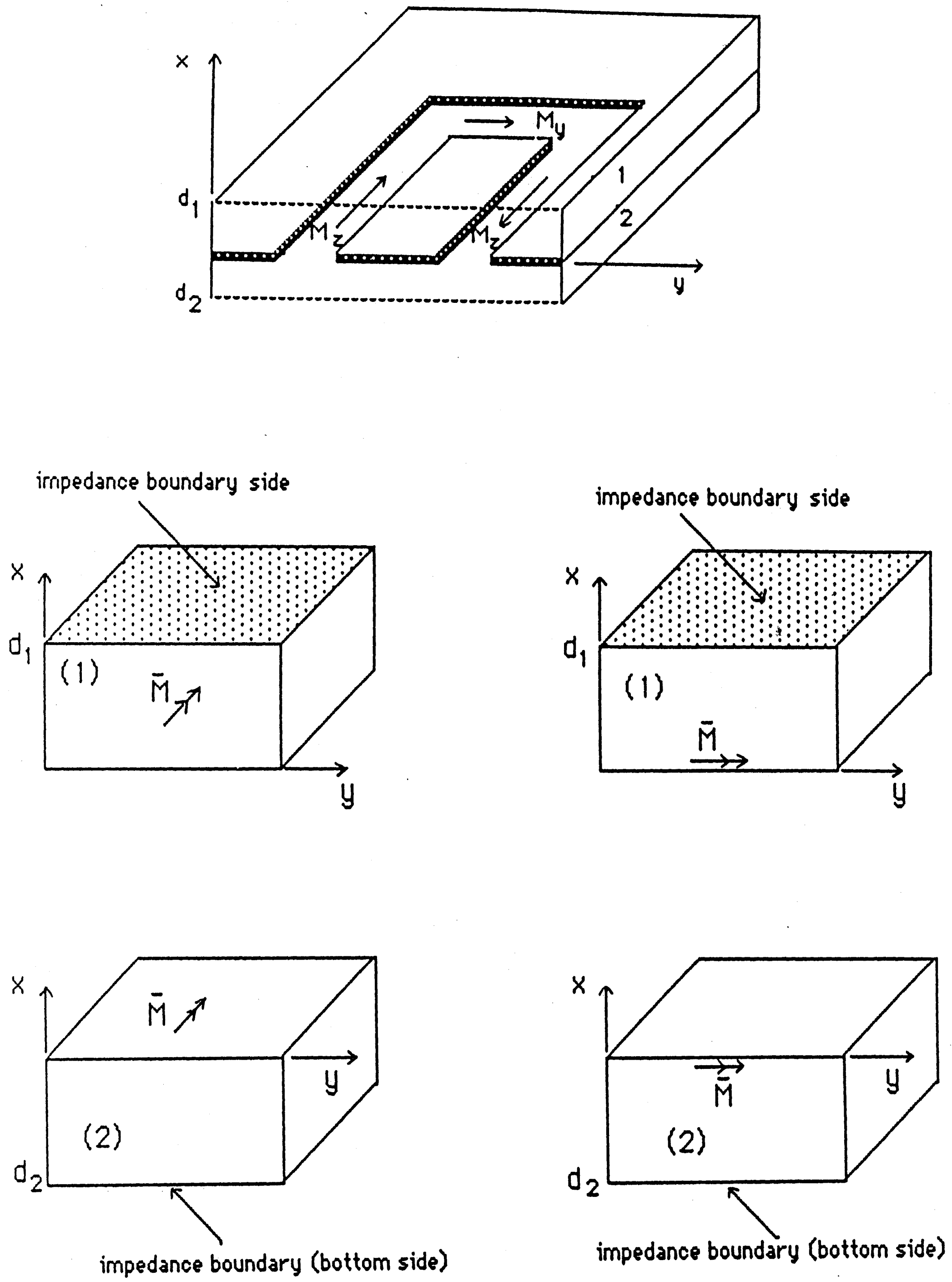

Fig.3 Four subproblems to be solved. Other than the impedance boundary side , the sides are assumed to be perfect conductors (cavity walls) .

$$\bar{A} = \hat{a}_x \psi \qquad , \qquad \bar{F} = \hat{a}_x \phi \tag{5}$$

Through the manipulation of Maxwell's equations

$$\bar{\nabla} \times \bar{E} = -jw\mu \bar{H} \tag{6}$$

$$\bar{\nabla} \times \bar{H} = jw\epsilon \bar{E} \tag{7}$$

along with

$$\bar{H} = \frac{1}{\mu} \bar{\nabla} \times \bar{A} \tag{8}$$

$$\bar{E} = -\frac{1}{\epsilon} \bar{\nabla} \times \bar{F} \tag{9}$$

one can obtain the field components in terms of (5) as

$$E_x = \frac{1}{jw\mu\epsilon}[-\frac{\partial^2 \psi}{\partial y^2} - \frac{\partial^2 \psi}{\partial z^2}] \tag{10}$$

$$E_y = \frac{1}{jw\mu\epsilon} \frac{\partial^2 \psi}{\partial x \partial y} - \frac{1}{\epsilon} \frac{\partial \phi}{\partial z} \tag{11}$$

$$E_z = \frac{1}{jw\mu\epsilon} \frac{\partial^2 \psi}{\partial x \partial z} + \frac{1}{\epsilon} \frac{\partial \phi}{\partial y} \tag{12}$$

$$H_x = \frac{1}{jw\mu\epsilon}[-\frac{\partial^2 \phi}{\partial y^2} - \frac{\partial^2 \psi}{\partial z^2}] \tag{13}$$

$$H_y = \frac{1}{\mu} \frac{\partial \psi}{\partial z} + \frac{1}{jw\mu\epsilon} \frac{\partial^2 \phi}{\partial x \partial y} \tag{14}$$

$$H_z = -\frac{1}{\mu} \frac{\partial \psi}{\partial y} + \frac{1}{jw\mu\epsilon} \frac{\partial^2 \phi}{\partial x \partial z} \tag{15}$$

Both vector potentials should satisfy the homogeneous wave equation (away from the source)

$$\nabla^2 \phi + k_1^2 \phi = 0 \tag{16}$$

$$\nabla^2 \psi + k_1^2 \psi = 0 \tag{17}$$

where $k_1^2 = w^2\mu\epsilon_1$.

As shown in Fig. 4, it is assumed that the current source divides the cavity into two regions I and II. Applying the method of separation of variables to solve (16) and (17) with the following boundary conditions

$$E^i_{x,y} = 0 \quad at \quad z = 0, l \tag{18}$$

$$E^i_{x,z} = 0 \quad at \quad z = 0, a \tag{19}$$

$$E^{II}_{y,z} = 0 \quad at \quad x = 0 \tag{20}$$

one can obtain

$$\begin{aligned} \phi^I &= \Sigma_{n=o}^{\infty}\Sigma_{m=o}^{\infty}[A_{mn}sin(k_x(x-d_1)) \\ &+ B_{mn}cos(k_x(x-d_1))]cos(\frac{m\tau}{a}y)cos(\frac{n\tau}{l}z) \end{aligned} \tag{21}$$

$$\phi^{II} = \Sigma_{n=o}^{\infty}\Sigma_{m=o}^{\infty}C_{mn}sin(k_x x)cos(\frac{m\tau}{a}y)cos(\frac{n\tau}{l}z) \tag{22}$$

$$\begin{aligned} \Psi^I &= \Sigma_{n=o}^{\infty}\Sigma_{m=o}^{\infty}[K_{mn}sin(k_x(x-d_1)) \\ &+ N_{mn}cos(k_x(x-d_1))]sin(\frac{m\tau}{a}y)sin(\frac{n\tau}{l}z) \end{aligned} \tag{23}$$

$$\Psi^{II} = \Sigma_{n=o}^{\infty}\Sigma_{m=o}^{\infty}D_{mn}cos(k_x x)sin(\frac{m\tau}{a}y)sin(\frac{n\tau}{l}z) \tag{24}$$

In the above equation, the following equation is satisfied

$$k_x^2 + k_y^2 + k_z^2 = k_1^2$$

where

$$k_y = \frac{m\pi}{a}, k_z = \frac{n\pi}{l} \quad and \quad k_1^2 = w^2\mu\epsilon_1$$

To simplify the notation, one can consider

$$\phi(x,y,z) = \Sigma_n\Sigma_m\tilde{\phi}(x)cos(k_y y)cos(k_z z) \tag{25}$$

$$\psi(x,y,z) = \Sigma_n\Sigma_m\tilde{\psi}(x)sin(k_y y)sin(k_z z) \tag{26}$$

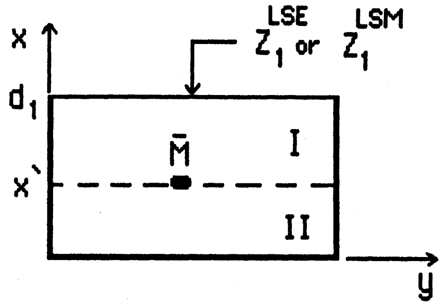

Fig.4 The magnetic source raised to apply the boundary conditions .

where

$$\tilde{\phi}(x) = \frac{4}{la}\int_o^l\int_o^a \phi(x,y,z)cos(k_y y)cos(k_z z)dydz \tag{27}$$

$$\tilde{\psi}(x) = \frac{4}{la}\int_o^l\int_o^a \phi(x,y,z)sin(k_y y)sin(k_z z)dydz \tag{28}$$

$\tilde{\phi}(x)$ and $\tilde{\psi}(x)$ can be considered as the coefficients of the double Fourier series of ϕ and ψ. In other words, one can consider them as ϕ and ψ in the Fourier domain.

Substituting (25) and (26) in (10) - (15) one can obtain the fields in the Fourier domain as

$$\tilde{E}_x = \frac{1}{jw\mu\epsilon_1}[k^2 - k_x^2]\tilde{\psi} \tag{29}$$

$$\tilde{E}_y = \frac{1}{\epsilon_1}k_z\tilde{\phi} + \frac{1}{jw\mu\epsilon_1}k_y\frac{\partial\tilde{\psi}}{\partial x} \tag{30}$$

$$\tilde{E}_z = -\frac{1}{\epsilon_1}k_y\tilde{\phi} + \frac{1}{jw\mu\epsilon_1}k_z\frac{\partial\tilde{\psi}}{\partial x} \tag{31}$$

$$\tilde{H}_x = \frac{1}{jw\mu\epsilon_1}[k^2 - k_x^2]\tilde{\phi} \tag{32}$$

$$\tilde{H}_y = -\frac{1}{jw\mu\epsilon_1}k_y\frac{\partial\tilde{\phi}}{\partial x} + \frac{1}{\mu}k_z\tilde{\psi} \tag{33}$$

$$\tilde{H}_z = -\frac{1}{jw\mu\epsilon_1}k_z\frac{\partial\tilde{\phi}}{\partial x} - \frac{1}{\mu}k_y\tilde{\psi} \tag{34}$$

where

$$E_x = \Sigma_m\Sigma_n\tilde{E}_x sin(k_y y)sin(k_z z) \tag{35}$$

$$E_y = \Sigma_m\Sigma_n\tilde{E}_y cos(k_y y)sin(k_z z) \tag{36}$$

$$E_z = \Sigma_m\Sigma_n\tilde{E}_z sin(k_y y)cos(k_z z) \tag{37}$$

$$H_x = \Sigma_m\Sigma_n\tilde{H}_x cos(k_y y)cos(k_z z) \tag{38}$$

$$H_y = \Sigma_m\Sigma_n\tilde{H}_y sin(k_y y)cos(k_z z) \tag{39}$$

$$H_z = \Sigma_m\Sigma_n\tilde{H}_z cos(k_y y)sin(k_z z) \tag{40}$$

Expressions for $\tilde{\phi}$ and $\tilde{\psi}$ are obtained from (21) - (24) as

$$\tilde{\phi}^{I} = A_{mn} sin(k_x(x - d_1)) + B_{mn} cos(k_x(x - d_1)) \quad (41)$$
$$\tilde{\phi}^{II} = C_{mn} sin(k_x x) \quad (42)$$
$$\tilde{\psi}^{I} = K_{mn} sin(k_x(x - d_1)) + N_{mn} cos(k_x(x - d_1)) \quad (43)$$
$$\tilde{\psi}^{II} = D_{mn} cos(k_x x) \quad (44)$$

Up to this point, one has to solve for the constants A, B, C, D, K and N, where the subscript mn will be suppressed for simplicity.

The following six boundary conditions will be employed to solve for the six unknowns,

$$E_z^I = E_z^{II} \quad at \quad x = x' \quad (45)$$
$$H_y^I = H_y^{II} \quad at \quad x = x' \quad (46)$$
$$H_z^I = H_z^{II} \quad at \quad x = x' \quad (47)$$
$$\left(\frac{E_y^I}{H_z^I}\right)^{LSE} = Z_1^{LSE} \quad at \quad x = d_1 \quad (48)$$
$$\left(\frac{E_y^I}{H_z^I}\right)^{LSM} = Z_1^{LSM} \quad at \quad x = d_1 \quad (49)$$

$$E_y^{II} - E_y^I = \delta(x - x')\delta(y - y')\delta(z - z') \quad (50)$$

In equation (48) and (49), Z_1^{LSE} and Z_1^{LSM} are the LSE and LSM impedances looking up at $x = d_1$. These can be computed using transmission line theory. That is, each layer is simply considered a transmission line with a characteristic impedance (Z_o^{LSE} or Z_o^{LSM}) and an eigenvalue k_x^i where

$$k_x^{i2} + k_y^2 + k_z^2 = w^2 \mu \epsilon_i$$

$$,(Z_o^{LSE})^i = \frac{w\mu}{k_x^i} \quad and \quad (Z_o^{LSM})^i = \frac{k_x^i}{w\epsilon_i}.$$

Equations (48) and (49) should be satisfied for each LSE and LSM mode, respectively.

Equation (50) signifies the discontinuity in the y-component of the electric field due to the magnetic source. In the transform domain, the boundary conditions (45) - (50) become

$$\tilde{E}_z^I = \tilde{E}_z^{II} \quad at \quad x = x' \tag{51}$$

$$\tilde{H}_y^I = \tilde{H}_y^{II} \quad at \quad x = x' \tag{52}$$

$$\tilde{H}_z^I = \tilde{H}_z^{II} \quad at \quad x = x' \tag{53}$$

$$(\frac{\tilde{E}_y^I}{\tilde{H}_z^I})^{LSE} = Z_1^{LSE} \quad at \quad x = d_1 \tag{54}$$

$$(\frac{\tilde{E}_y^I}{\tilde{H}_z^I})^{LSM} = Z_1^{LSM} \quad at \quad x = d_1 \tag{55}$$

$$(\tilde{E}_y^{II} - \tilde{E}_y^I) = \frac{2}{al}\epsilon_m sin(k_z z')cos(k_y y')\delta(x - x') \tag{56}$$

where

$$\epsilon_m = 1 \quad m = 0$$
$$= 2 \quad m \neq 0$$

Equation (56) can be obtained by substituting (36) in (50) and using the orthogonality properties of the sin's and cos's.

If (30) and (34) are substituted in (54) and (55), the following can be obtained

$$B = j\frac{k_x}{w\mu} Z_1^{LSE} A \tag{57}$$

and

$$K = -j\frac{w\epsilon_1}{k_x} Z_1^{LSM} N \tag{58}$$

This will reduce the problem to solving 4 equations for 4 unknowns. Taking into consideration the other 4 boundary conditions (51) - (53) and (56) and after simplification, one can obtain

$$C = \hat{c}A \tag{59}$$

$$D = \frac{jw\mu k_y}{k_x k_z}\frac{\hat{a}\hat{d}}{\hat{b}}A \tag{60}$$

$$K = w^2\mu\epsilon_1 Z_1^{LSM}\frac{k_y}{k_x^2 k_z}\frac{\hat{a}}{\hat{b}}A \tag{61}$$

$$N = jw\mu\frac{k_y}{k_x k_z}\frac{\hat{a}}{\hat{b}}A \tag{62}$$

$where$

$$A = \frac{2}{al}\epsilon_m \cdot \frac{\epsilon_1}{\hat{a}}\frac{k_z}{k_x^2 - k_1^2} sin(k_z z')cos(k_y y') \tag{63}$$

where a zero was substituted for x' and

$$\hat{a} = j\frac{k_x}{w\mu} Z_1^{LSE} cos(k_x d_1) - sin(k_x d_1) \tag{64}$$

$$\hat{b} = sin(k_x d_1) - j\frac{w\epsilon}{k_x} Z_1^{LSM} cos(k_x d_1) \tag{65}$$

In this manner, expressions for the six unknowns have been derived. Substituting (57) - (65) in (41) and (43), one can obtain complete expressions for $\tilde{\phi}^I$ and $\tilde{\psi}^I$ from which $\phi(x,y,z)$ and $\psi(x,y,z)$ can be derived. Using (29) - (40), the Green's functions (the fields due to a unit magnetic current element in the z-direction) for this subproblem are as follows

$$\begin{aligned}(G_{zx}^{E,M})^1 &= \sum_n \sum_m -\frac{2\epsilon_m}{jw\mu} \cdot \frac{1}{al} \cdot \frac{1}{\hat{b}} \cdot \frac{k_y}{k_x} \\ &* \quad [sin(k_x(x - d_1))]\frac{w^2\mu\epsilon_1}{k_x} Z_1^{LSM}\end{aligned}$$

$$+ \quad jw\mu cos(k_x(x-d_1))]$$
$$* \quad sin(k_z z')cos(k_y y')sin(k_y y)sin(k_z z) \qquad (66)$$

where $(G^{E,M}_{zx})^1$ is E_x in region 1 due to M_z.

$$
\begin{aligned}
(G^{E,M}_{zy})^1 &= \sum_n \sum_m \frac{2\epsilon_m}{al} \cdot \frac{\epsilon_1}{\hat{a}} \cdot \frac{k_z}{k_x^2 - k^2} \\
&\cdot \quad [(k_z - \frac{k_y^2 \hat{a}}{k_z \hat{b}}) \cdot \frac{1}{\epsilon_1} \cdot sin(k_x(x-d_1)) \\
&+ \quad (\frac{jk_x k_z}{w\mu\epsilon_1} Z_1^{LSE} - \frac{jwk_y^2}{k_x k_z} Z_1^{LSM} \frac{\hat{a}}{\hat{b}}) \cdot cos(k_x(x-d_1))] \\
&* \quad sin(k_z z')cos(k_y y')sin(k_z z)cos(k_y y) \qquad (67)
\end{aligned}
$$

$$
\begin{aligned}
(G^{E,M}_{zz})^1 &= \sum_n \sum_m \frac{2\epsilon_m}{al} \cdot \frac{\epsilon_1}{\hat{a}} \cdot \frac{k_z}{k_x^2 - k^2} \\
&\cdot \quad [(\frac{k_y Z_1^{LSM} \hat{a} w^2 \mu \epsilon_1 + k_x^2 k_y Z_1^{LSE} \hat{b}}{jw\mu\epsilon_1 k_x \hat{b}}) cos(k_x(x-d_1)) \\
&- \quad (\frac{k_y}{\epsilon_1} \cdot \frac{\hat{a}+\hat{b}}{\hat{b}} sin(k_x(x-d_1))] \\
&\cdot \quad sin(k_z z')cos(k_y y')sin(k_y y)cos(k_z z) \qquad (68)
\end{aligned}
$$

$$
\begin{aligned}
(G^{H,M}_{zx})^1 &= \sum_n \sum_m \frac{-2\epsilon_m}{al} \cdot \frac{k_z}{\hat{a}} \cdot \frac{1}{jw\mu} \\
&\cdot \quad [sin(k_x(x-d_1)) + \frac{jk_x}{w\mu} Z_1^{LSE} cos(k_x(x-d_1))] \\
&\cdot \quad sin(k_z z')cos(k_y y')sin(k_y y)cos(k_z z) \qquad (69)
\end{aligned}
$$

$$
\begin{aligned}
(G^{H,M}_{zy})^1 &= \sum_n \sum_m \frac{2\epsilon_m}{al} \cdot \frac{\epsilon_1}{\hat{a}} \cdot \frac{k_z}{k_x^2 - k_1^2} \\
&\cdot \quad [cos(k_x(x-d_1)) \cdot (\frac{jwk_y}{k_x} \frac{\hat{a}}{\hat{b}} - \frac{k_x k_y}{jw\mu\epsilon_1})
\end{aligned}
$$

$$+ \quad sin(k_x(x-d_1))\cdot(\frac{k_x^2 k_y}{w^2\mu^2\epsilon_1}Z_1^{LSE}+\frac{w^2\epsilon_1 k_y\hat{a}}{k_x^2\hat{b}}Z_1^{LSM})]$$

$$\cdot \quad sin(k_z z')cos(k_y y')sin(k_y y)cos(k_z z) \qquad (70)$$

$$(G_{zz}^{H,M})^1 = \sum_n\sum_m \frac{2\epsilon_m}{al}\cdot\frac{\epsilon_1}{\hat{a}}\cdot\frac{k_z}{k_x^2-k_1^2}$$

$$\cdot \quad [cos(k_x(x-d_1))\cdot(\frac{-k_x k_z}{jw\mu\epsilon_1}-\frac{jwk_y^2}{k_x k_z}\frac{\hat{a}}{\hat{b}})$$

$$+ \quad sin(k_x(x-d_1))\cdot(\frac{k_x^2 k_z}{w^2\mu^2\epsilon_1}Z_1^{LSE}-\frac{w^2\epsilon_1 k_y^2}{k_x^2 k_z}\frac{\hat{a}}{\hat{b}}Z_1^{LSM})]$$

$$\cdot \quad sin(k_z z')cos(k_y y')sin(k_z z)cos(k_y y) \qquad (71)$$

2.2.2 Green's function in region (1) for a y-directed magnetic current

Fig. 5 shows the structure under consideration with the magnetic current alleviated from the ground of the cavity. The magnetic current is assumed to be

$$\bar{M} = \hat{a}_y\delta(x-x')\delta(y-y')\delta(z-z') \qquad (72)$$

The same method used in 2.2.1 will be applied here. Equations (18) - (44) are applicable also to the structure of Fig. 3. Moreover, the boundary conditions (51) - (55) still hold in addition to

$$(E_z^I - E_z^{II}) = \delta(x-x')\delta(y-y')\delta(z-z') \qquad (73)$$

which can be written as

$$\tilde{E}_z^I - \tilde{E}_z^{II} = \frac{2}{al}\epsilon_n cos(k_z z')sin(k_y y')\delta(x-x') \qquad (74)$$

where

$$\begin{aligned}\epsilon_n &= 1 \quad n=0\\ &= 2 \quad n\neq 0\end{aligned}$$

Simplifying the boundary conditions equations, the following expressions are derived

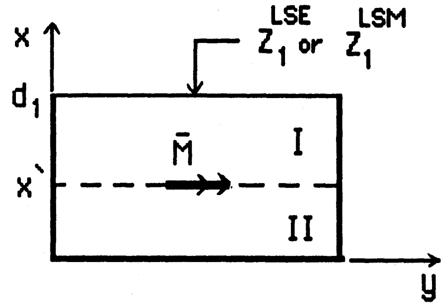

Fig.5 The magnetic current source raised to apply boundary conditions .

$$
\begin{aligned}
A &= \frac{2}{al}\epsilon_n \cdot \frac{\epsilon_1}{\hat{a}} \frac{k_y}{k_x^2 - k_1^2} sin(k_y y')cos(k_z z') && (75)\\
B &= j\frac{k_x}{w\mu} Z_1^{LSE} A && (76)\\
C &= \hat{c}A && (77)\\
D &= -jw\mu\frac{\hat{a}\hat{d}}{\hat{b}} \cdot \frac{k_z}{k_x k_y} A && (78)\\
K &= -w^2\mu\epsilon\frac{k_z}{k_x^2 k_y} \cdot \frac{\hat{a}}{\hat{b}} Z_1^{LSM} A && (79)\\
N &= -jw\mu\frac{k_z}{k_x k_y}\frac{\hat{a}}{\hat{b}} A && (80)
\end{aligned}
$$

where $\hat{a}$ and $\hat{b}$ are given by (64) and (65) and

$$
\begin{aligned}
\hat{c} &= cos(k_x d_1) + j\frac{k_x}{w\mu} Z_1^{LSE} sin(k_x d_1) && (81)\\
\hat{d} &= cos(k_x d_1) + j\frac{w\epsilon}{k_x} Z_1^{LSM} sin(k_x d_1) && (82)
\end{aligned}
$$

Finally, the dyadic Green's function is

$$
\begin{aligned}
(G_{yx}^{E,M})^1 &= \sum_n \sum_m \frac{2}{al}\epsilon_n \frac{1}{\hat{b}} \cdot \frac{k_z}{k_x}\\
&\cdot \left[-sin(k_x(x-d_1)) \cdot \frac{jw\epsilon_1}{k_x} Z_1^{LSM}\right.\\
&+ \left. cos(k_x(x-d_1))\right]\\
&\cdot sin(k_y y')cos(k_z z')sin(k_y y)sin(k_z z) && (83)
\end{aligned}
$$

$$
\begin{aligned}
(G_{yy}^{E,M})^1 &= \sum_n \sum_m \frac{2}{al}\epsilon_n \frac{\epsilon_1}{\hat{a}} \cdot \frac{k_y}{k_x^2 - k_1^2} \\
&\cdot \ [sin(k_x(x-d_1)) \cdot (\frac{k_z}{\epsilon_1} + \frac{k_z}{\epsilon_1}\frac{\hat{a}}{\hat{b}}) \\
&+ \ cos(k_x(x-d_1)) \cdot (j\frac{wk_z}{k_x} \cdot \frac{\hat{a}}{\hat{b}} \cdot Z_1^{LSM} + j\frac{k_z k_z}{\epsilon_1 w\mu} z_1^{LSE})] \\
&\cdot \ sin(k_y y')cos(k_z z')sin(k_z z)ws(k_y y) \qquad (84)
\end{aligned}
$$

$$
\begin{aligned}
(G_{yz}^{E,M})^1 &= \sum_n \sum_m \frac{2}{al}\epsilon_n \frac{\epsilon_1}{\hat{a}} \cdot \frac{k_y}{k_x^2 - k_1^2} \\
&\cdot \ [sin(k_x(x-d_1)) \cdot (\frac{-k_y}{\epsilon_1} + \frac{k_z^2}{k_y\epsilon_1}\frac{\hat{a}}{\hat{b}}) \\
&+ \ cos(k_x(x-d_1)) \cdot (-j\frac{k_x k_y}{w\mu\epsilon_1} Z_1^{LSE} + jw\frac{k_z^2}{k_x k_y} \cdot \frac{\hat{a}}{\hat{b}} Z_1^{LSM})] \\
&\cdot \ sin(k_y y')cos(k_z z')sin(k_y y)cos(k_z z) \qquad (85)
\end{aligned}
$$

$$
\begin{aligned}
(G_{yx}^{H,M})^1 &= \sum_n \sum_m \frac{-2}{al}\epsilon_n \frac{k_y}{\hat{a}} \cdot \frac{1}{jw\mu} \\
&\cdot \ [sin(k_x(x-d_1)) + j\frac{k_x}{w\mu} Z_1^{LSE} cos(k_x(x-d_1))] \\
&\cdot \ sin(k_y y')cos(k_z z')cos(k_y y)cos(k_z z) \qquad (86)
\end{aligned}
$$

$$
\begin{aligned}
(G_{yy}^{H,M})^1 &= \sum_n \sum_m \frac{2\epsilon_n}{al} \cdot \frac{\epsilon_1}{\hat{a}} \cdot \frac{k_y}{k_x^2 - k_1^2} \\
&\cdot \ [sin(k_x(x-d_1)) \cdot (\frac{k_x^2 k_y}{w^2\mu^2\epsilon_1} Z_1^{LSE} - \frac{w^2\epsilon_1 k_z^2}{k_x^2 k_y}\frac{\hat{a}}{\hat{b}} Z_1^{LSM}) \\
&+ \ cos(k_x(x-d_1)) \cdot (j\frac{k_x k_y}{w\mu\epsilon_1} - \frac{jwk_z^2}{k_x k_y}\frac{\hat{a}}{\hat{b}})] \\
&\cdot \ sin(k_y y')cos(k_z z')sin(k_y y)cos(k_z z) \qquad (87)
\end{aligned}
$$

$$
\begin{aligned}
(G_{yz}^{H,M})^1 &= \sum_n \sum_m \frac{2\epsilon_n}{al} \cdot \frac{\epsilon_1}{\hat{a}} \cdot \frac{k_y}{k_x^2 - k_1^2} \\
&\cdot \quad [sin(k_x(x-d_1)) \cdot (\frac{k_x^2 k_z}{w^2 \mu^2 \epsilon_1} Z_1^{LSE} + \frac{w^2 \epsilon_1 k_z}{k_x^2} \frac{\hat{a}}{\hat{b}} Z_1^{LSM}) \\
&+ \quad cos(k_x(x-d_1)) \cdot (j\frac{k_x k_z}{w\mu\epsilon_1} + j\frac{w k_z}{k_x} \frac{\hat{a}}{\hat{b}})] \\
&\cdot \quad sin(k_y y') cos(k_z z') sin(k_z z) cos(k_y y) \qquad (88)
\end{aligned}
$$

Up to this point, the dyadic Green's function

$$
\begin{aligned}
\bar{\bar{G}}_E^1 &= (G_{yx}^{E,M})^1 \hat{y}\hat{x} + (G_{yy}^{E,M})^1 \hat{y}\hat{y} + (G_{yz}^{E,M})^1 \hat{y}\hat{z} \\
&+ \quad (G_{zx}^{E,M})^1 \hat{z}\hat{x} + (G_{zy}^{E,M})^1 \hat{z}\hat{y}) + (G_{zz}^{E,M})^1 \hat{z}\hat{z} \qquad (89)
\end{aligned}
$$

$$
\begin{aligned}
\bar{\bar{G}}_H^1 &\doteq (G_{yx}^{H,M})^1 \hat{y}\hat{x} + (G_{yy}^{H,M})^1 \hat{y}\hat{y} + (G_{yz}^{H,M})^1 \hat{y}\hat{z} \\
&+ \quad (G_{zx}^{H,M})^1 \hat{z}\hat{x} + (G_{zy}^{H,M})^1 \hat{z}\hat{y} + (G_{zz}^{H,M})^1 \hat{z}\hat{z} \qquad (90)
\end{aligned}
$$

have been obtained.

2.2.3 Green's function in region (2)

The fields in region (2) (see Fig. 3) due to unit magnetic currents in the $\hat{y}$ and $\hat{z}$ direction are to be derived. Fig. 6 shows both structures to be solved. The scalar potentials in the Fourier domain can be written as

$$
\begin{aligned}
\tilde{\phi}^I &= Asin(k_x(x-d_2)) + Bcos(k_x(x-d_2)) \qquad (91) \\
\tilde{\phi}^{II} &= Csin(k_x x) \qquad (92) \\
\tilde{\psi}^I &= Ksin(k_x(x-d_2)) + Ncos(k_x(x-d_2))] \qquad (93) \\
\tilde{\psi}^{II} &= Dcos(k_x x) \qquad (94)
\end{aligned}
$$

The boundary conditions that apply for both structures in Fig. 6 are

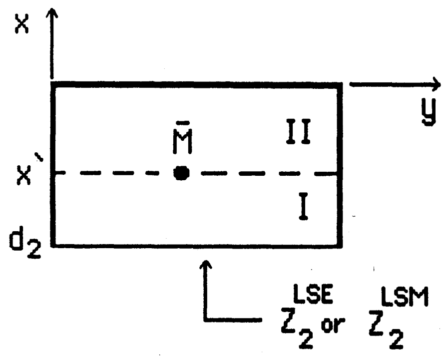

Fig.6a

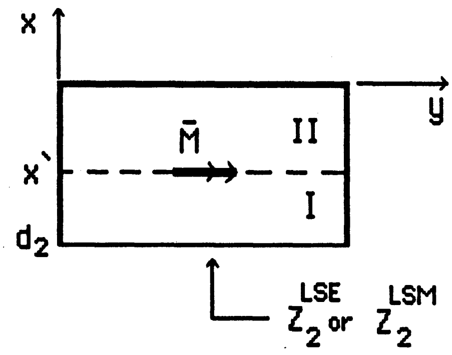

Fig.6b

Fig.6 Structures to be solved to obtain Green's function in region (2).

$$\tilde{E}_z^I = \tilde{E}_z^{II} \quad at \quad x = x' \tag{95}$$

$$\tilde{H}_y^I = \tilde{H}_y^{II} \quad at \quad x = x' \tag{96}$$

$$\tilde{H}_z^I = \tilde{H}_z^{II} \quad at \quad x = x' \tag{97}$$

$$(\frac{\tilde{E}_y^I}{H_z^I})^{LSE} = -Z_2^{LSE} \quad at \quad x = d_2 \tag{98}$$

$$(\frac{\tilde{E}_y^{II}}{H_z^{II}}) = -Z_2^{LSM} \quad at \quad x = d_2 \tag{99}$$

where Z_2^{LSE} and Z_2^{LSE} are the impedances at $x = d_2$ looking in the negative x-direction.

For Fig. 6a, a discontinuity in Ey exists such that

$$\tilde{E}_y^I - \tilde{E}_y^{II} = \frac{2}{al}\epsilon_m sin(k_z z')cos(k_y y')\delta(x - x') \tag{100}$$

While a discontinuity in E_z exists for Fig. 6b

$$\tilde{E}_z^{II} - \tilde{E}_z^I = \frac{2}{al}\epsilon_n cos(k_z z')sin(k_y y')\delta(x - x') \tag{101}$$

One can notice the similarity between (91) - (94) and (41) - (44). In addition, the boundary conditions (95) - (101) are the same as the ones applied in 2.2.1 and 2.2.2 except that $Z_2^{LSE(LSM)}$ replaces $Z_1^{LSE(LSM)}$, d_2 replaces d_1, $(-\epsilon_m)$ replaces (ϵ_m) and $(-\epsilon_n)$ replaces ϵ_n. So, in general, the Green's function in region (2) are similar to those in region (1) with the following changes

$$Z_1^{LSE} \longrightarrow Z_2^{LSE}$$

$$Z_1^{LSM} \longrightarrow Z_2^{LSM}$$

$$d_1 \longrightarrow d_2$$

$$\epsilon_n \longrightarrow -\epsilon_n$$

$$\epsilon_m \longrightarrow -\epsilon_m$$

$$\epsilon_1 \longrightarrow \epsilon_2$$

$$k_1^2 \longrightarrow k_2^2$$

In summary, the Green's function for the open circuit coplanar line discontinuity (inside a cavity) has been determined in this section. This was accomplished by working with Maxwell's equations and by representation of our source as dirac delta functions. Then, boundary conditions were applied to solve for the fields

2.3 Application of Method of Moments

In order to obtain the fields inside the cavity, one should integrate over the source coordinates (i.e. the slots).

$$\bar{E}^1 = \int\int_{s'} \bar{M} \cdot \bar{\bar{G}}_E^{(1)} ds' \quad (102)$$

$$\bar{E}^2 = -\int\int_{s'} \bar{M} \cdot \bar{\bar{G}}_E^{(2)} ds' \quad (103)$$

$$\bar{H}^1 = \int\int_{s'} \bar{M} \cdot \bar{\bar{G}}_H^{(1)} ds' \quad (104)$$

$$\bar{H}^2 = -\int\int_{s'} \bar{M} \cdot \bar{\bar{G}}_H^{(2)} ds' \quad (105)$$

The choice of the same magnetic current $\bar{M}$ to compute the fields in both regions reflects the continuity of the tangential electric field in the slot region. The negative sign that appears in (103) and (105) is due to the fact that the assumption

$$\bar{M}^{(1)} = \bar{E}^{(1)} \times \hat{a}_x = \bar{M} \quad (106)$$

leads to

$$\bar{M}^{(2)} = \bar{E}^{(2)} \times (-\hat{a}_x) = -\bar{M} \quad (107)$$

The remaining boundary condition to be used in order to arrive at the integral equation is the continuity of the tangential magnetic field

$$H^{(1)}_{tang.} = H^{(2)}_{tang.} \tag{108}$$

in the slots regions. Equation (108) may be written as

$$H^{(1)}_y = H^{(2)}_y \tag{109}$$
$$H^{(1)}_z = H^{(2)}_z \tag{110}$$

If the magnetic current is assumed to be

$$\bar{M} = \hat{a}_y M_y + \hat{a}_z M_z \tag{111}$$

the following equations for the magnetic field in both regions can be obtained

$$H^{(1)}_y = \int\int_{s'} [M_y (G^{H,M}_{yy})^1 + M_z (G^{H,M}_{zy})^1] ds' \tag{112}$$
$$H^{(1)}_z = \int\int_{s'} [M_y (G^{H,M}_{yz})^1 + M_z (G^{H,M}_{zz})^1] ds' \tag{113}$$
$$H^{(2)}_y = -\int\int_{s'} [M_y (G^{H,M}_{yy})^2 + M_z (G^{H,M}_{zy})^2] ds' \tag{114}$$
$$H^{(2)}_z = -\int\int_{s'} [M_y (G^{H,M}_{yz})^2 + M_z (G^{H,M}_{zz})^2] ds' \tag{115}$$

where $(G^{H,M}_{mn})^i$ is the H_n component due to M_m component in the ith region. Substituting in (109) and (110), one can obtain the following integral equations

$$\int\int_{s'} M_y (G^{(1)}_{yy} + G^{(2)}_{yy}) + M_z (G^{(1)}_{zy} + G^{(2)}_{zy}) ds' = 0 \tag{116}$$
$$\int\int_{s'} M_y (G^{(1)}_{yz} + G^{(2)}_{yz}) + M_z (G^{(1)}_{zz} + G^{(2)}_{zz}) ds' = 0 \tag{117}$$

where the superscript H, M is suppressed for simplicity. The integral equations (116) and (117) are to be solved for the unknown magnetic current distribution using the method of moments.

The method of moments is a numerical technique used for solving functional equation for which closed form solutions cannot be obtained [5]. By reducing the functional relation to a matrix equation, known methods can be used to solve for the unknown current distribution. The general steps involved in the moment method for the computation of surface currents can be summarized as follows:

1. The integral equation for the electric or magnetic field in terms of the unknown surface electric and/or magnetic currents is formulated. The resulting integral equation can be put in the form

$$L_{op}\binom{J_s}{M_s} = \bar{g}\binom{\bar{E}}{\bar{H}} \qquad (118)$$

where L_{op} is an integral operator on $\bar{J}_s$ and/or $\bar{M}_s$, and $\bar{g}$ is a vector function of either $\bar{E}$ and/or $\bar{H}$.

2. The unknown currents are expanded in terms of known, basis functions as

$$\bar{J}_s = \sum_{i=1}^{N_1} a_i \bar{\phi}_i \qquad (119)$$

$$\bar{M}_s = \sum_{j=1}^{N_2} b_j \bar{\psi}_j \qquad (120)$$

where the $a_i's$ and $b_i's$ are complex coefficients and N_1 and N_2 are the number of basis functions for $\bar{J}_s$ and $\bar{M}_s$ respectively.

3. A suitable inner product is defined and a set of test (or weighting) functions W is chosen. If (119) and (120) are substituted in (118) and the inner products with the weighting functions are performed, the results may be expressed as

$$\sum_{i=1}^{N_1} a_i < \bar{W}_q, L_{op}(\bar{\phi}_i) > + \sum_{j=1}^{N_2} b_j < \bar{W}_q, L_{op}(\bar{\psi}_j) > = < \bar{W}_q, \bar{g} > \qquad (121)$$

where the inner product is defined as

$$< \bar{a}, \bar{b} > = \int\int_s \bar{a} \cdot \bar{b} \; ds \qquad (122)$$

In Galerkin's procedure, which will be adopted here, the test functions are chosen to be the same with the basis functions.

4. A matrix equation is formed after the integrals (122) are computed. The unknowns in the matrix equation are the current amplitudes a_i and b_j which can be solved for by matrix inversion. One can notice that the method is computationally intensive, but with the advent of faster computers the moment method has become feasible.

In our problem, equations (116) and (117) represent the general integral equation (118). Now, applying step (2), the y-component of the magnetic current will be expanded as

$$M_y = \sum_{p=1}^{M} b_p \phi_p(y, z) \tag{123}$$

The $z-$ component of the magnetic current will be assumed to be composed of 5 components, incident and reflected travelling waves in each slot $(y_1 < y < y_1 + W_1, y_1 + W_1 + s < y < y_1 + W_1 + W_2 + s)$ up to some point $z = z_1$ and the sum of basis functions for $z > z_1$ (see Fig. 7). That is .

$$\begin{aligned} M_z &= [(A_1 \bar{e}^{j\beta z'} + B_1 e^{j\beta z'})(u(y - y_1) - u(y - y_1 - W_1)) \\ &+ (A_2 \bar{e}^{j\beta z'} + B_2 e^{j\beta z'}(u(y - y_1 - W_1 - s) - u(y - y_1 - W_1 - s - W_2))] \\ &* (u(z) - u(z - z_1)) \\ &+ [\sum_{n=1}^{N} a_n f_n(y, z)] u(z - z_1) \end{aligned} \tag{124}$$

where β is the propagation constant in the coplanar waveguide and $u(\cdot)$ is the unit step function. Substituting (123) and (124) in the integral equation (116) and (117). The following expressions can be obtained

$$\begin{aligned} &- \int_{s_1}\int A_1 \bar{e}^{j\beta z'} (G_{zy}^{(1)} + G_{zy}^{(2)}) ds' \\ &- \int_{s_2}\int A_2 \bar{e}^{j\beta z'} (G_{zy}^{(1)} + G_{zy}^{(2)}) ds' \\ &= \int_{s_1}\int B_1 e^{j\beta z'} (G_{zy}^{(1)} + G_{zy}^{(2)}) ds' + \int_{S_2}\int B_2 e^{j\beta z'} (G_{zy}^{(1)} + G_{zy}^{(2)}) ds' \end{aligned}$$

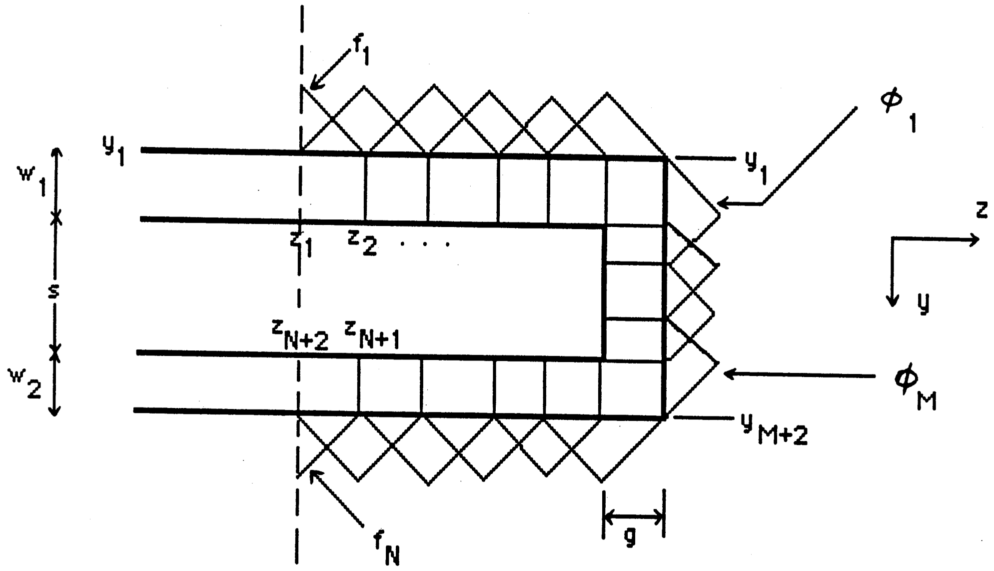

Fig. 7 Geometry for use in basis function expansion of magnetic current . .

$$
\begin{aligned}
&+ \sum_{p=1}^{M} b_p \int \int_{S_p} \phi_p(y',z')(G^{(1)}_{yy} + G^{(2)}_{yy})ds' \\
&+ \sum_{n=1}^{N} a_n \int_{S_n} \int f_n(y',z')(G^{(1)}_{zy} + G^{(2)}_{zy})ds'
\end{aligned}
\tag{125}
$$

and

$$
\begin{aligned}
&- \int_{s_1} \int A_1 \bar{e}^{j\beta z'}(G^{(1)}_{zz} + G^{(2)}_{zz})ds' \\
&- \int_{s_2} \int A_2 \bar{e}^{j\beta z'}(G^{(1)}_{zz} + G^{(2)}_{zz})ds' \\
&= \int_{s_1} \int B_1 e^{j\beta z'}(G^{(1)}_{zz} + G^{(2)}_{zz})ds' + \int_{S_2} \int B_2 e^{j\beta z'}(G^{(1)}_{zz} + G^{(2)}_{zz})ds' \\
&+ \sum_{p=1}^{M} b_p \int_{S_p} \int \phi_p(y',z')(G^{(1)}_{yz} + G^{(2)}_{yz})ds' \\
&+ \sum_{n=1}^{N} a_n \int_{S_n} \int f_n(y',z')(G^{(1)}_{zz} + G^{(2)}_{zz})ds'
\end{aligned}
\tag{126}
$$

where s_1 denotes the area for which $y_1 < y < y_1 + W_1$ and $o < z < z_1$, and s_2 denotes the area for which $y_1 + W_1 + s < y < y_1 + W_1 + s + W_2$ and $o < z < z_1$. S_p and S_n are the area over which ϕ_p and f_n are defined respectively. Galerkin's procědure will be applied where the test function are the same as the basis functions.

The inner product of (125) with $\phi_k(y,z), k = 1, \cdots m$, is performed which will result in M equations, each one of the following form

$$
\begin{aligned}
&- \int_{S_k} \int_{S_1} A_1 e^{-j\beta z'}(G^{(1)}_{zy} + G^{(2)}_{zy})\phi_k(y,z)ds'ds \\
&- \int_{S_k} \int_{S_2} A_2 \bar{e}^{j\beta z'}(G^{(1)}_{zy} + G^{(2)}_{zy})\phi_k(y,z)ds'ds \\
&= \int_{S_k} \int_{S_1} B_1 e^{j\beta z'}(G^{(1)}_{zy} + G^{(2)}_{zy})\phi_k(y,z)ds'ds \\
&+ \int_{S_k} \int_{S_2} B_2 e^{j\beta z'}(G^{(1)}_{zy} + G^{(2)}_{zy})\phi_k(y,z)ds'ds
\end{aligned}
$$

$$
\begin{aligned}
&+ \sum_{p=1}^{M} b_p \int_{S_k}\int_{S_p} \phi_p(y',z')(G_{yy}^{(1)} + G_{yy}^{(2)})\phi_k(y,z)ds'ds \\
&+ \sum_{n=1}^{N} a_n \int_{S_k}\int_{S_n} f_n(y',z')(G_{zy}^{(1)} + G_{zy}^{(2)})\phi_k(y,z)ds'ds \qquad (127)
\end{aligned}
$$

In the same manner, performing the inner product of (126) with $f_k(y,z)$, $k = 1, \cdots N, N$ equations are obtained as

$$
\begin{aligned}
&- \int_{S_k}\int_{S_1} A_1 e^{-j\beta z'}(G_{zz}^{(1)} + G_{zz}^{(2)})f_k(y,z)ds'ds \\
&- \int_{S_k}\int_{S_2} A_2 e^{-j\beta z'}(G_{zz}^{(1)} + G_{zz}^{(2)})f_k(y,z)ds'ds \\
&= \int_{S_k}\int_{S_1} B_1 e^{j\beta z'}(G_{zz}^{(1)} + G_{zz}^{(2)})f_k(y,z)ds'ds \\
&+ \int_{S_k}\int_{S_2} B_2 e^{j\beta z'}(G_{zz}^{(1)} + G_{zz}^{(2)})f_k(y,z)ds'ds \\
&+ \sum_{p=1}^{M} b_p \int_{S_k}\int_{S_p} \phi_p(y',z')(G_{yz}^{(1)} + G_{yz}^{(2)})f_k(y,z)ds'ds \\
&+ \sum_{p=1}^{N} b_p \int_{S_k}\int_{S_n} f_n(y',z')(G_{zz}^{(1)} + G_{zz}^{(2)})f_k(y,z)ds'ds \qquad (128)
\end{aligned}
$$

where S_k in (127) is the area over which $\phi_k(y,z)$ is defined, while in (128) denotes the area over which $f_k(y,z)$ is defined. Notice that the Green's functions are in terms of the source coordinates (y' and z') and the observation point coordinates (y and z). The Green's functions are obtained from (69) - (71) and (86) - (88).

Finally, the inner product equations (127) and (128) are solved to form the matrix equation. The matrix equation obtained will be of the following form

$$[Y]\begin{bmatrix} B_1 \\ B_2 \\ b_1 \\ \cdots \\ b_M \\ a_1 \\ \cdots \\ a_N \end{bmatrix} = [\underline{I}] \qquad (129)$$

where $[Y]$ is an $(M+N+2)\times(M+N+2)$ matrix and $[\underline{I}]$ is a vector of $(M+N+2)$ elements where

$$\begin{aligned} I_j &= -\int_{S_j}\int_{S_1} A_1\bar{e}^{j\beta z'}(G^{(1)}_{zy} + G^{(2)}_{zy})\phi_j(y,z)ds'ds \\ &- \int_{S_j}\int_{S_2} A_2\bar{e}^{j\beta z'}(G^{(1)}_{zy} + G_{zy})^{(2)}\phi_j(y,z)ds'ds \end{aligned} \qquad (130)$$

for $1 \leq j \leq M$ and

$$\begin{aligned} I_j &= -\int_{S}\int_{S_1} A_1\bar{e}^{j\beta z'}(G^{(1)}_{zz}) + G^{(2)}_{zz})f_{j-M}(y,z)ds'ds \\ &- \int_{S}\int_{S_2} A_2\bar{e}^{j\beta z'}(G^{(1)}_{zz} + G^{(2)}_{zz})f_{j-M}(y,z)ds'ds \end{aligned} \qquad (131)$$

for $M+1 \leq j \leq M+N$. The elements of $[Y]$ are obtained from (127) and (128) giving the following expressions. For $1 \leq i \leq M$

$$\begin{aligned} Y(i,1) &= \int_{S}\int_{S_1} e^{j\beta z'}(G^{(1)}_{zy} + G^{(2)}_{zy})\phi_i(y,z)ds'ds \\ Y(i,2) &= \int_{S}\int_{S_2} e^{j\beta z'}(G^{(1)}_{zy} + G^{(2)}_{zy})\phi_i(y,z)ds'ds \end{aligned} \qquad (132)$$

For $3 \leq j \leq M+2$

$$Y(i,j) = \int_{S}\int_{S'} \phi_{j-2}(y',z')(G^{(1)}_{yy} + G^{(2)}_{yy})\phi_i(y,z)ds'ds \qquad (133)$$

For $M+3 \leq j \leq M+N+2$

$$Y(i,j) = \int_S \int_{S'} f_{j-M-2}(y',z')(G_{zy}^{(1)} + G_{zy}^{(2)})\phi_i(y,z)ds'ds \quad (134)$$

For $M + 1 \leq i \leq M + N$

$$Y(i,1) = \int_S \int_{S_1} e^{j\beta z'}(G_{zz}^{(1)} + G_{zz}^{(2)})f_{i-M}(y,z)ds'ds \quad (135)$$

$$Y(i,2) = \int_S \int_{S_2} e^{j\beta z'}(G_{zz}^{(1)} + G_{zz}^{(2)})f_{i-M}(y,z)ds'ds \quad (136)$$

For $3 \leq j \leq M + 2$

$$Y(i,j) = \int_S \int_{S'} \phi_{j-2}(y',z')(G_{yz}^{(1)} + G_{yz}^{2})f_{i-M}(y,z)ds'ds \quad (137)$$

For $M + 3 \leq j \leq M + N + 2$

$$Y(i,j) = \int_S \int_{S'} f_{j-M-2}(y',z')(G_{zz}^{(1)} + G_{zz}^{(2)})f_{i-M}(y,z)ds'ds \quad (138)$$

It can be observed that two more equations are needed to solve for the $(M + N + 2)$ unknowns. The basis functions are chosen to be piecewise sinusoidal functions as shown in Fig. 7 such that

$$\begin{aligned} \phi_p(y) &= \frac{sin(k^*(y - y_p))}{sin(k^*(y_{p+1} - y_p))} \quad y_p \leq y \leq y_{p+1} \\ &= \frac{sin(k^*(y_{p+2} - y))}{sin(k^*(y_{p+2} - y_{p+1}))} \quad y_{p+1} \leq y \leq y_{p+2} \\ &= 0 \qquad elsewhere \end{aligned} \quad (139)$$

where $k* = w\sqrt{\mu\epsilon_{eff}}$ and ϵ_{eff} is the effective permitivity of the coplanar waveguide defined as

$$\epsilon_{eff} = (\frac{\beta}{\beta_o})_2 \quad (140)$$

where β_o is the free space propagation constant. The $z-$ variation of ϕ_p is assumed to be unity over the slot.

The basis functions for M_z are

$$\begin{aligned} f_1(z) &= \frac{sin(k^*(z_2 - z))}{sin(k^*(z_2 - z_1))} \quad z_1 \le z \le z_2 \\ f_p(z) &= \frac{sin(k^*(z_{p+1} - z))}{sin(k^*(z_{p+1} - z_p))} \quad z_p \le z \le z_{p+1} \\ &= \frac{sin(k^*(z - z_{p+1}))}{sin(k^*(z_p - z_{p-1}))} \quad z_{p+1} \le z \le z_p \end{aligned} \tag{141}$$

for $2 \le p \le \frac{N}{2}$ and $y_1 \le y \le y_1 + W_1$

$$\begin{aligned} f_p(z) &= \frac{sin(k^*(z_{p+3} - z))}{sin(k^*(z_{p+3} - z_{p+2}))} \quad z_{p+3} \le z \le z_{p+2} \\ &= \frac{sin(k^*(z - z_{p+1}))}{sin(k^*(z_{p+2} - z_{p+1}))} \quad z_{p+2} \le z \le z_{p+2} \end{aligned} \tag{142}$$

for $\frac{N}{2} + 1 \le p \le N - 1$ and $y_1 + W_1 + s \le y \le y_1 + W_1 + s + W_2$

$$f_N(z) = \frac{sin(k^*(z - z_{N+1}))}{sink^*(z_{N+2} - z_{N+1}))} \quad z_{N+2} \le z \le z_{N+1} \tag{143}$$

So, the other two equations needed can be obtained by imposing the continuity condition of the magnetic current M_z such that

$$A_1 e^{-j\beta z_1} + \beta_1 e^{j\beta z_1} = a_1 \tag{144}$$

and

$$A_2 e^{-j\beta z_1} + B_2 e^{j\beta z_1} = a_N \tag{145}$$

So, one can write

$$Y(M + N + 1, 1) = -e^{j2\beta z_1} \tag{146}$$

$$Y(M + N + 1, M + 3) = e^{j\beta z_1} \tag{147}$$

$$Y(M + N + 2, 2) = -e^{j2\beta z_1} \tag{148}$$

$$Y(M + N + 2, M + N + 2) = e^{j\beta z_1} \tag{149}$$

$$I(M + N + 1) = A_1 \tag{150}$$

$$I(M + N + 2) = A_2 \tag{151}$$

Finally, the integrals involved in the elements of $[Y]$ and $[I]$ can be performed analytically. In fact, one can find that seven integrals only have to be performed to get the elements of the matrices. Appendix A shows the derivation of these integrals. Once the element of $[Y]$ and $[I]$ are determined, (129) can be solved for the unknown current amplitudes by inverting $[Y]$.

Using the derived current distribution in the slots, one can determine the scattering parameters characterizing the open and coplanar waveguide discontinuity.

3 Summary

The open circuit CPW discontinuity has been analyzed theoretically in this report. The dyadic Green's functions for y and z directed dirac delta magnetic currents, placed in a rectangular cavity, were obtained. The fields were assumed to be a superposition of LSE and LSM modes. Then, the continuity of the tangential electric and magnetic fields in the slot region was used to arrive at the integral equation. Finally, the integral equation was solved using the method of moments to obtain the unknown magnetic current distribution from which the scattering parameters can be evaluated.

This study is intended to be a step towards characterizing various CPW discontinuities including the CPW air bridge. A computer program, that solves the CPW open circuit discontinuity, is in the process of writing.

Appendix A

$$
\begin{aligned}
I_{y_1} &= \int_{y_1}^{y_1+W_1} cos(k_y y) d_y \\
&= \frac{1}{k_y}[sin(ky(y_1+W_1) - sin(k_y y_1)] \qquad k_y \neq 0 \\
&= W_1 \qquad k_y = 0
\end{aligned}
$$

$$
\begin{aligned}
I_{y_2} &= \int_{y_1+W_1+S}^{y_1+W_1+S+W_2} cos(k_y y) dy \\
&= \frac{1}{k_y}[sin(k_y(y_1+w_1+s+w_2)) - sin(k_y(y_1+w_1+s))] \qquad k_y \neq 0 \\
&= W_2 \qquad k_y = 0
\end{aligned}
$$

$$
\begin{aligned}
I_{z1} &= \int_o^{z_1} e^{j\beta z} sin(k_z z) dz \\
&= \frac{1}{k_z^2 - \beta^2}[k_z - k_z e^{j\beta z_1} cos(k_z z_1) + j\beta e^{j\beta z_1} sin(k_z z_1)]
\end{aligned}
$$

$$
\begin{aligned}
I_{z2} &= \int_{z_1+l_d}^{z_1+l_d+g} cos(k_z z) dz \\
&= \frac{1}{k_z}[sin(k_z(z_1+l_d+g)) - sin(k_z(z_1+l_z))] \quad k_z \neq 0 \\
&= g \qquad k_z = 0
\end{aligned}
$$

$$
\begin{aligned}
I_{f_1} &= \frac{1}{sin(k^*(z_z - z_1))} \int_{z_1}^{z_2} sin(k_z z) sin(k^*(z_2 - z)) dz \\
&= \frac{1}{sin(k^*(z_2 - z_1))} \cdot \frac{1}{k_z^2 - k^{*2}} \cdot [-k^* sin(k_z z_2) \\
&+ k^* sin(k_z z_1) cos(k^*(z_2 - z_1)) + k_z cos(k_z z_1) sin(k^*(z_2 - z_1))]
\end{aligned}
$$

$$
\begin{aligned}
I_{f_N} &= \frac{1}{sin(k^*(z_{N+2}-z_{N+1}))}\int_{Z_{N+2}}^{Z_{N+1}} sin(k_z z)sin(k^*(z-z_{N+1}))dz \\
&= \frac{1}{sin(k^*(z_{N+2}-z_{N+1}))}\cdot\frac{1}{k_z^2-k^{*2}}\cdot[-k^* sin(k_z z_{N+1}) \\
&+ k^* sin(k_z z_{N+2})cos(k^*(z_{N+2}-z_{N+2})) \\
&+ k_z cos(k_z z_{N+2})sin(k^*(z_{N+1}-z_{N+2}))]
\end{aligned}
$$

$$
\begin{aligned}
I(y_i) &= \int_{y_i}^{y_{i+1}} sin(k_y y)\frac{sin(k^*(y-y_i))}{sin(k^*(y_{i+1}-y_i))}dy \\
&+ \int_{y_{i+1}}^{y_{i+2}} sin(k_y y)\frac{sin(k^*(y_{i+2}-y))}{sin(k^*(y_{i+2}-y_{i+1}))}dy \\
&= \frac{k^*}{k_y^2-k^{*2}}\cdot\frac{1}{sin(k^*(y_{i+1}-y_i))}sin(k^*(y_{i+2}-y_{i+1})) \\
&\cdot\ [-sin(k_y y_{i+z})sin(k^*(y_{i+2}-y_i)) \\
&-\ sin(k_y y_i)sin(k^*(y_{i+2}-y_{i+1})) \\
&+\ sin(k_y y_{i+1})sin(k^*(y_{i+2}-y_i))]
\end{aligned}
$$

4 References

1. T. Hirota, Y. Tarusawa and H. Ogawa, "Uniplanar MMIC Hybrids. A Proposed New MMIC Structure," IEEE Trns. Microwave Theory Tech., Vol. MTT-35, pp. 576-581, June 1987.

2. R. Simons and G. Ponchak, "Modeling of Some Coplanar Waveguide Discontinuities," IEEE Trans. Microwave Theory Tech., Vol. MTT-36, pp. 1796-1803, Dec. 1988.

3. G. Hasnain, A. Dienes and J. Whinnery, "Dispersion of Picosecond Pulses in Coplanar Transmission Lines," IEEE Trans. Microwave Theory Tech., Vol. MTT-34, pp. 738-741, June 1986.

4. R. Harrington, Time-Harmonic Electromagnetic Fields, New York: McGraw Hill, 1961, p. 152.

5. R. Mittra, Ed., Computer Techniques for Electromagnetics, Pergamon Press, 1973.

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