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Topology optimization of compliant mechanisms with multiple outputs

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Abstract A procedure for the topology design of compliant mechanisms with multiple output requirements is presented. Two methods for handling the multiple output requirements are developed, a combined virtual load method and a weighted sum of objectives method. The problem formulations and numerical solution procedures are discussed and illustrated by design examples. The examples illustrate the capabilities of the design procedure, the effect of the direction of the output deflection requirements on the solution, as well as computational issues such as the effect of the starting point and effect of the material resource constraint.

1 Introduction

The optimization of structural systems for maximum stiffness and least weight has been studied extensively by many researchers (e.g. Prager and Rozvany 1977; Bendsøe and Kikuchi 1988; Bendsøe *et al.* 1993). Various computational techniques have been developed to predict the optimal topology, shape and size of such structural systems. In addition to these methods, new methods have been developed recently for the optimization and design of structural systems which consider flexibility (Frecker *et al.* 1997; Nishiwaki *et al.* 1998; Larsen *et al.* 1996; Sigmund 1996). These designs incorporate flexibility as a preferred effect, in contrast to stiffest least weight configurations. One example of such a structural system is a compliant mechanism.

A compliant mechanism can be defined as a single-piece flexible structure which uses elastic deformation to achieve force and motion transmission. Compliant mechanisms differ from conventional rigid-link mechanisms in that they contain no rigid links or joints and are intentionally flexible. Because of this fundamental difference from conventional mechanisms, the kinematic synthesis methods that exist for rigid-link mechanism design are inadequate for the design of compliant mechanisms. Similarly, because compliant mechanisms are intentionally flexible, the optimization methods that have been developed for stiffest structure design cannot be directly applied to the design of compliant mechanisms.

Early work in the related field of analysis of flexible-link mechanisms was conducted by researchers such as Burns and Crossley (1966, 1968) and Shoup and McLarnan (1971a,b). More recently, methods for synthesis of compliant mechanisms have been developed by Midha and others, which use kinematic techniques such as graph theory (Murphy *et al.* 1993) and Burmester theory (Mettlach and Midha 1996), as well as a pseudo rigid-body model (Howell and Midha 1994). These methods approach compliant mechanism design from a kinematic viewpoint, i.e. they begin with a known rigid-link mechanism and convert it to a compliant mechanism.

On the other hand, researchers such as Ananthasuresh and others have approached compliant mechanism design from a structural viewpoint, using topology optimization methods (Ananthasuresh *et al.* 1993, 1994a,b). A topology optimization approach is advantageous because it does not require a rigid-link mechanism configuration as a starting point, and can be used to design single-piece fully compliant mechanisms. Other efforts aimed at using optimization techniques to design mechanisms have been developed by Sigmund (1996) and Larsen *et al.* (1997). Furthermore, Frecker *et al.* (1997) and Nishiwaki *et al.* (1998) used multicriteria optimization to perform topological synthesis of compliant mechanisms. The focus of this paper is on a multicriteria optimization formulation for topology design of compliant mechanisms with multiple output requirements. The topology design problem is posed in terms of an applied load and specified output deflections. For the multiple output case, a single applied load and several output deflections are prescribed at various locations. A multicriteria optimization procedure for the single output case has been previously developed by Frecker *et al.* (1997), which serves as the basis for the formulation presented here. Multiple output requirements in topology design of compliant mechanisms have also been considered by Larsen *et al.* (1997) using a different formulation based on prescribed mechanical and geometric advantages.

This paper is organized as follows. Two different topology optimization formulations to handle multiple output requirements are presented along with a discussion of their numerical implementation. The basic computational procedure for the optimization algorithm is then discussed. Design examples are presented comparing the results of each formulation, illustrating the effect of the direction of the output deflection requirements on the optimal solution, and demonstrating the effect of the starting point and the material resource constraint on the optimal solution.

2 Topology optimization

2.1 General formulation

For many practical tasks, it is desirable to exploit the benefits of both stiffness and flexibility when designing compliant mechanisms. That is, a compliant mechanism should be flexible so that it can easily deform, but it should also be stiff enough to provide an adequate mechanical advantage. As a motivating example, consider the design of a general compliant gripper mechanism, as shown in Figs. 1a and b. We would like this device to be able to grasp and hold some object or workpiece when the load $\mathbf{F}_A$ is applied. In this load condition (Fig. 1a), the compliant gripper mechanism should be very flexible so that it can easily achieve the desired motion. Once the compliant gripper touches the workpiece, however, it should be very stiff so that it is able to resist the additional load that is exerted by the resistance of the object once it has been secured (Fig. 1b). This compliant gripper mechanism example can be generalized to apply for a broad class of compliant mechanism design problems, where the device must possess both a certain flexibility and stiffness for a particular task.

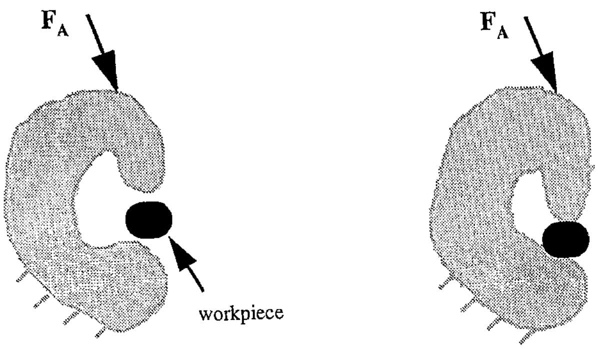

Fig. 1. (a) Load condition 1, (b) load condition 2

For the first loading condition, the flexibility of the structure should be maximized. Consider a general design domain as in Fig. 2. The applied load is represented as a traction $\mathbf{f}_A$ applied on boundary Γ_1. For the case of multiple output deflection requirements, the deflection at each specified point Δ_j should be maximized in the desired direction. These output deflection requirements are handled by applying a virtual force $\mathbf{f}_{Bj}$ at each point of interest in the desired directions, as shown in Fig. 2 for three output deflection requirements. The second loading condition is now considered in Fig. 3, where the stiffness of the structure is to be maximized as a way to control the mean compliance. The point of the applied load is considered fixed, and a virtual load $-\mathbf{f}_{Bj}$ is applied at each point of interest, representing the resistance of the workpiece(s). The flexibility and stiffness design portions of this problem can now be combined using multicriteria optimization in order to find a compromise solution between the two requirements. Two methods of formulating these requirements have been developed and are described below.

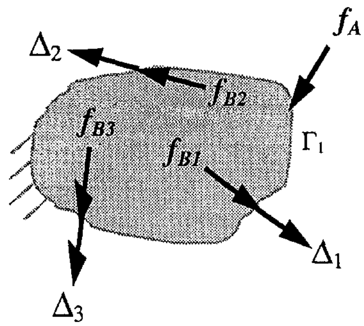

Fig. 2. Flexibility design

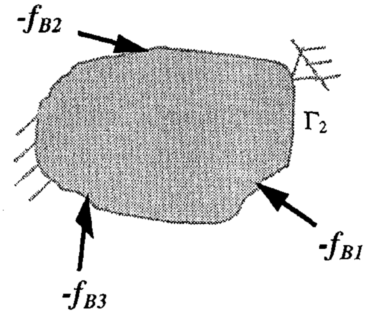

Fig. 3. Stiffness design

2.2 Combined virtual load

2.2.1 Formulation. As a method to combine the output deflection requirements, a single virtual load $\mathbf{f}_B$ can be formed by a vector combination of the individual virtual loads $\mathbf{f}_{Bj}$ as in (1), for a total of N_f loads. The mutual potential energy L_1 is formulated as a measure of the flexibility of the structure as in (2), where $\mathbf{u}_A$ are the nodal displacements due to the actual load (Shield and Prager 1970). Since the individual output deflection requirements have been combined into a single virtual load, a single mutual potential energy term is required. This formulation is the same as for the single-output case where the load $\mathbf{f}_B$ represents a single output deflection requirement

$$\mathbf{f}_B = \sum_{j=1}^{N_f} \mathbf{f}_{Bj} \,, \tag{1}$$

$$\max L_1(\mathbf{u}_a) = \int_{\Gamma_1} \mathbf{f}_B \bullet \mathbf{u}_A \, \mathrm{d}\Gamma \,. \tag{2}$$

For the second loading condition the stiffness of the structure is to be maximized. Here the strain energy L_2 is formulated as the design objective (3), where $\mathbf{u}_B$ are the nodal displacements due to the virtual load. This part of the formulation is equivalent to the minimization of mean compliance formulations used in many current structural optimization methods,

$$\min L_2(\mathbf{u}_B) = \int_{\Gamma_2} -\mathbf{f}_B \bullet \mathbf{u}_B \, \mathrm{d}\Gamma \,. \tag{3}$$

The flexibility design and stiffness design parts of the compliant mechanism design problem form a set of conflicting design requirements. These design objectives can be combined using multicriteria optimization in order to find a compromise solution. Generally there are two approaches to combining conflicting design objectives, a linear combination and a multiplication. Most multicriteria optimization methods use a linear combination of the two objectives as in (4), where α and β are positive scalar weighting factors. This approach was taken by Ananthasuresh *et al.* (1994b) for compliant mechanism design,

$$\max[\alpha L_1 - \beta L_2] \,. \tag{4}$$

There is a computational difficulty when using this approach, however. Often the values of L_1 and L_2 differ by several orders of magnitude depending on the problem specifications. When this difference occurs one term will dominate, which skews the optimal solution in favor of the larger term. This effect can be compensated for by choosing appropriate scalar weighting factors, but the values of these factors are strictly problem-dependent. It is not possible in general to predict the appropriate weighting factors so that both objectives are considered equally in the solution. Therefore, a new method of combining the two objectives is needed.

The second way to combine the two objectives is using multiplication. Minimization of strain energy can be expressed as maximization of its inverse, as in (5). The combined design objective can then be expressed as the product of this term and the mutual potential energy $L_1(\mathbf{u}_A)$. Since the mutual potential energy is to be maximized and the strain energy is to be minimized, the combined problem is posed as in (6). Using a ratio of the two design objectives rather than a linear combination avoids difficulties due to differences in orders of magnitude, and there is no need to select appropriate weighting factors for each problem. The constraints for this combined problem are the equilibrium equations for the actual displacements and the virtual displacements, an upper limit on the material resource, and upper and lower bounds on the design variables. This formulation represents a new method of incorporating both the flexibility and stiffness requirements into a single design objective,

$$\min L_2(\mathbf{u}_B) \Leftrightarrow \max \left[\frac{1}{L_2(\mathbf{u}_B)}\right] \,, \tag{5}$$

$$\max \left[\frac{L_1(\mathbf{u}_A)}{L_2(\mathbf{u}_B)}\right] = \left[\frac{\int_{\Gamma_1} \mathbf{f}_B \bullet \mathbf{u}_A \,\mathrm{d}\Gamma}{\int_{\Gamma_2} -\mathbf{f}_B \bullet \mathbf{u}_B \,\mathrm{d}\Gamma}\right] \,, \tag{6}$$

subject to: equilibrium equations, total material resource constraint, bounds on design variables.

The physical meaning of this type of objective function can be considered as follows. The mutual potential energy (MPE) in the numerator is intended to characterize the mechanism part of the design problem, where a compliant structure is to be designed which will undergo a displacement in a specified direction(s) when subject to a given applied load. This MPE term individually cannot be used as the objective function, however, because the resulting optimal designs would have maximum flexibility, i.e. each element would reach its lower bound constraint. In practical situations, not only is the motion of the compliant mechanism of concern, but also its ability to transfer force to the output location. That is, the compliant mechanism must possess sufficient stiffness after the motion is complete. As a way to meet this stiffness requirement, the strain energy (SE) is introduced. The strain energy is due to a resisting load(s) in the opposite direction to the desired output displacement, and the compliant mechanism is treated as a structure. Here the stiffness is maximized by minimizing the total strain energy or compliance. The two objectives, the MPE and the SE, are then combined into a single multicriteria objective function using the ratio formulation. A limitation to this type of formulation, however, is that there is no direct control over the value of the resulting mechanical and geometric advantage of the compliant mechanism.

2.2.2 Numerical implementation. To implement the multicriteria optimization problem formulation numerically, a ground structure of truss elements was chosen for the finite element analysis. As is commonly done in structural optimization problems, a dense ground structure of truss elements is used to approximate a continuous structural design domain. Since the individual elements and the resulting structures are permitted to undergo elastic deformation, the solutions obtained by the optimization procedure are not considered to be standard rigid-link mechanisms. Although the individual truss elements can support only tension and compression modes of loading, they were chosen as finite elements because of their simplicity in analysis. Clearly, incorporating bending modes of loading is important when modeling compliant mechanisms. However individual element bending is assumed to be small, and hence is not accounted for directly. The mechanics of bending can be modelled indirectly by using a sufficient number of truss elements. For instance, a pair of truss elements can simulate a beam in bending, where one element acts as the portion of the beam in tension, and the other element acts as the portion of the beam in compression. In fact, it has been shown that allowing individual element bending by using a ground structure of frame elements does not affect the topology of the optimal solution (Frecker *et al.* 1998).

The problem formulation for the case of a truss ground structure of N elements is shown in (7). The mutual potential energy is formulated as $\mathbf{v}_B{}^T\mathbf{K}_1\mathbf{u}_A$, where $\mathbf{v}_B$ are the nodal displacements due to the virtual load $\mathbf{f}_B$, and $\mathbf{K}_1$ is the symmetric global stiffness matrix. The strain energy is formulated as $\mathbf{u}_B{}^T\mathbf{K}_2\mathbf{u}_B$, where $\mathbf{K}_2$ is the symmetric global stiffness matrix. Note that $\mathbf{K}_1$ and $\mathbf{K}_2$ are different due to the different geometric constraints in the two loading conditions. The constraints are the equilibrium equations due to the applied load $\mathbf{f}_A$, the virtual load $\mathbf{f}_B$, and the resisting load $-\mathbf{f}_B$; the total material resource V^*; and bounds on the design variables A_{lower} and A_{upper}.

The design sensitivity of the objective function is shown in (8). Since the stiffnesses are linear functions of the design variables for truss structures, the sensitivities of the stiffness matrices are constants. The sensitivity analysis for the constraints is trivial; i.e. the sensitivities of equilibrium constraints are zero since the loads are independent of the design variables; and the other constraints are linear functions of the design variables, so their sensitivities are constants,

$$\max_{A_i} \left[\frac{\mathbf{v}_B{}^T \mathbf{K}_1 \mathbf{u}_A}{\mathbf{u}_B{}^T \mathbf{K}_2 \mathbf{u}_B} \right] ,$$

subject to

$$\mathbf{K}_1 \mathbf{u}_A = \mathbf{f}_A ,$$

$$\mathbf{K}_1 \mathbf{v}_B = \mathbf{f}_B ,$$

$$\mathbf{K}_2 \mathbf{u}_B = -\mathbf{f}_B ,$$

$$\sum_{i=1}^{N} A_i \ell_i \leq V^* ,$$

$$A_{\text{lower}} \leq A_i \leq A_{\text{upper}} , \tag{7}$$

$$\frac{\partial}{\partial A_i} \left[\frac{\mathbf{v}_B{}^T \mathbf{K}_1 \mathbf{u}_A}{\mathbf{u}_B{}^T \mathbf{K}_2 \mathbf{u}_B} \right] =$$

$$\frac{L_1 \left(\mathbf{u}_B{}^T \frac{\partial \mathbf{K}_2}{\partial A_i} \mathbf{u}_B \right) - L_2 \left(\mathbf{v}_B{}^T \frac{\partial \mathbf{K}_1}{\partial A_i} \mathbf{u}_A \right)}{L_2^2} . \tag{8}$$

2.3 Weighted sum of objectives

2.3.1 Formulation. A second method of handling multiple output requirements was developed by considering each output deflection requirement separately, then combining them into a weighted sum of design objectives. An individual ratio of mutual potential energy to strain energy is formulated due to each virtual load. For a total number of N_f output requirements, the problem formulation is given in (9), where w_j are scalar weighting factors. By selecting these weighting factors appropriately, the designer has the option to weight certain output deflection requirements more heavily than others if desired. The constraints are the equilibrium equations due to the applied load and due to each virtual load. In addition, there are constraints on the total material resource and bounds on the design variables,

$$\max \left[\sum_{j=1}^{N_f} w_j \frac{\int_{\Gamma_1} \mathbf{f}_{Bj} \bullet \mathbf{u}_{Aj} \, \mathrm{d}\Gamma}{\int_{\Gamma_2} -\mathbf{f}_{Bj} \bullet \mathbf{u}_{Bj} \, \mathrm{d}\Gamma} \right] , \tag{9}$$

subject to: $2N_f + 1$ equilibrium equations, total material resource constraint, bounds on design variables.

2.3.2 Implementation. This formulation is implemented in the same manner as the combined virtual load formulation using a ground structure of truss elements (10). The design sensitivity of this objective function is given in (11) for N_f output requirements. As in the formulation for a single output displacement, the sensitivities of the constraints are either constants or zero,

$$\max \left[\sum_{j=1}^{N_f} w_j \frac{L_{1j}}{L_{2j}} \right] = \max_{A_i} \left[\sum_{j=1}^{N_f} w_j \frac{\mathbf{v}_{Bj}{}^T \mathbf{K}_1 \mathbf{u}_A}{\mathbf{u}_{Bj}{}^T \mathbf{K}_2 \mathbf{u}_{Bj}} \right] , \tag{10}$$

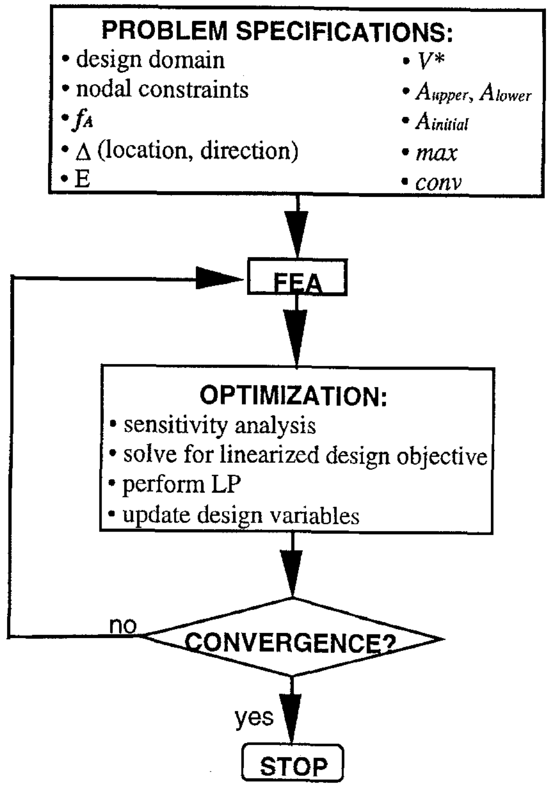

Fig. 4. Basic computational procedure

subject to

$$\mathbf{K}_1 \mathbf{u}_A = \mathbf{f}_A ,$$

$$\mathbf{K}_1 \mathbf{v}_{Bj} = \mathbf{f}_{Bj} ,$$

$$\mathbf{K}_2 \mathbf{u}_{Bj} = -\mathbf{f}_{Bj} ,$$

$$\sum_{i=1}^{N} A_i \ell_i \leq V^* ,$$

$$A_{\text{lower}} \leq A_i \leq A_{\text{upper}} ,$$

$$\frac{\partial}{\partial A_i} \left[\sum_{j=1}^{N_f} w_j \frac{\mathbf{v}_{Bj}{}^T \mathbf{K}_1 \mathbf{u}_A}{\mathbf{u}_{Bj}{}^T \mathbf{K}_2 \mathbf{u}_{Bj}} \right] =$$

$$\sum_{j=1}^{N_f} w_j \frac{L_{ij} \left(\mathbf{u}_{Bj}{}^T \frac{\partial \mathbf{K}_2}{\partial A_i} \mathbf{u}_{Bj} \right) - L_{2j} \left(\mathbf{v}_{Bj}{}^T \frac{\partial \mathbf{K}_1}{\partial A_i} \mathbf{u}_A \right)}{L_{2j}^2} . \tag{11}$$

Clearly this formulation for multiple outputs will require increased computation time compared to the single output case. This weighted sum of objectives formulation requires a separate finite element analysis for each virtual load, which can increase the required computation time significantly when using a large number of elements and/or output deflection requirements. Also, it may be more difficult for the optimization algorithm to converge when using a large number of terms in the multicriteria design objective.

2.4 *Solution technique*

The sequential linear programming (SLP) method for constrained minimization was chosen as the solution technique for both problem formulations. Although there are other more sophisticated solution methods such as sequential quadratic programming (SQP) which may provide faster convergence, these methods were not chosen for this problem because of the speciality of the design objective. The design objective is a ratio of two convex functions, which may not be adequately approximated by a quadratic function in a SQP approach. In general, the SLP method provides a good conservative approximation to the design objective, even though it may require numerous algorithm iterations.

An algorithm was written in FORTRAN to perform the SLP procedure. The basic computational procedure is outlined in Fig. 4. In the first step, the problem specifications are given by the user, including the geometry of the problem, the input force, the direction of desired output deflection, and other constraints. The move limit (*max*) is also provided, which was set to 0.15% of the previous value in most cases. In the second step, the finite element analysis is performed based on the starting point. The displacements were calculated using the pivoting solver SSPFA with SSPSL from the SLATEC library (Dongarra *et al.* 1979). Then the sensitivities and linearized design objective are calculated, and the update to the design variables is determined based on the simplex method using the SPLP solver from the SLATEC library (Hanson and Hiebert 1981). To avoid a solution with many intermediate values of design variables, a penalty function is used at this stage (Frecker *et al.* 1997). Algorithm convergence is based on the following two criteria: when the update to the design variables is sufficiently small ($\leq conv$), and when the design objective stops increasing, the algorithm is said to have converged.

2.5 *Solution existence and uniqueness*

Solution existence has not been proven mathematically, and experience using this algorithm demonstrates that a solution may not exist for every set of problem specifications. In addition, solution uniqueness has not been proven. Generally in topology design problems, the topology of the optimal solution is dependent on the starting point, indicating that there are several possible solutions or local minima. Further, the design objective, which is a ratio of two convex functions, is not itself a convex function. Generally neither the mutual potential energy nor the strain energy are convex functions. For a continuous problem where the number of design variables is infinite, infinitely many possible solutions may exist, and convexity is not guaranteed. However, for the case of a truss structure, where the size of the ground structure and hence the number of design variables is fixed and finite, we can say that the mutual potential energy and strain energy are individually convex functions since they are both linear in the design variables. Still, once the ratio of these two quantities is formed, convexity is not guaranteed. We can conclude, therefore, that there may be many solutions for this problem, and that the solution obtained by starting with an unbiased initial guess is not unique. This aspect is illustrated by an example in Section 3.4.

3 Design examples

3.1 *Two-output mechanism*

This example illustrates the results of both of the multiple outputs formulations for the case of two prescribed outputs. The design problem is shown in Fig. 5a, where the applied load is to result in the prescribed deflections Δ_1 and Δ_2 simultaneously. This example was motivated by the design of a disk-eject mechanism, where a single actuator (F_A) is used to move a floppy disk up and out of the disk drive (Δ_1 and Δ_2). The starting point for the topology design problem is a 7-node by 5-node full ground structure where each design variable is given an initial value between the upper and lower bound constraints (Fig. 5b). The optimal solution and corresponding finite element model from the sum of weighted objectives formulation is shown in Fig. 5c, where each of the deflection requirements has been weighted equally. The optimal solution and corresponding finite element model from the combined virtual load formulation is shown in Fig. 5d. Notice that the optimal topologies from each of the two problem formulations are similar, but not identical. These results indicate that either formulation generates a valid topology for this compliant mechanism design with two outputs. The topological results shown in Figs. 5c and d are threshold plots, where the elements which reached or were near the lower bound constraint are not pictured. These elements do provide a weak connection from the thicker elements to the support points, however. When interpreting the final topological solution to design a fully compliant mechanism, these minimum thickness elements are ignored, as they are significantly weaker ($\approx$ 100 times) than the elements with thicknesses at or near the upper bound constraint. The minimum thickness elements are needed in the mathematical sense, however, to prevent a singularity in the stiffness matrix. Grayscale plots of the topological solutions are also shown in Figs. 5e and f.

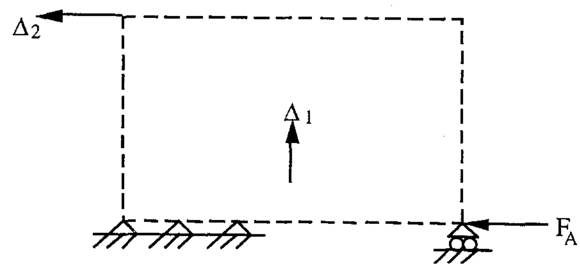

Fig. 5a. Two-output design problem

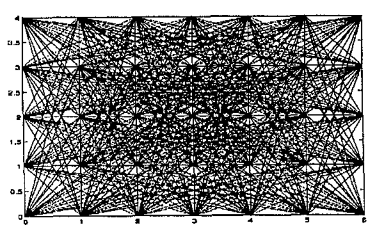

Fig. 5b. Starting point

The convergence history for both these solutions is given in Figs. 5g and h. In Fig. 5g the mutual potential energy

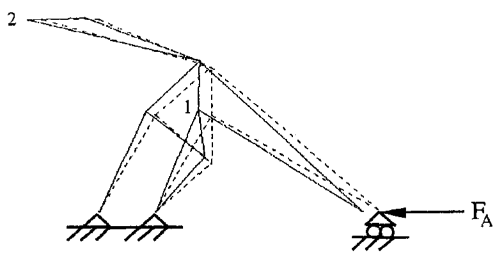

Fig. 5c. Optimal topology, sum of weighted objectives formulation

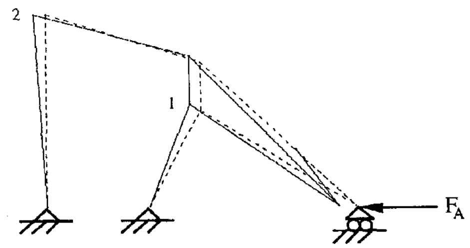

Fig. 5d. Optimal topology, combined virtual load formulation

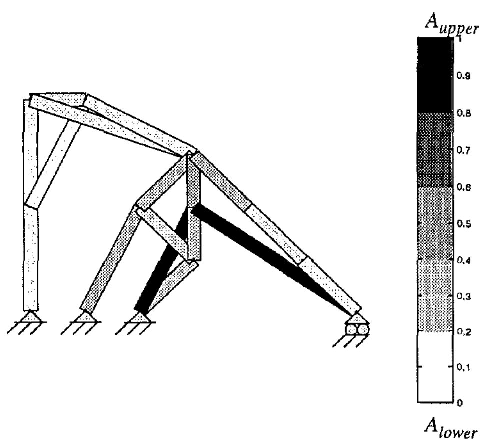

Fig. 5e. Grayscale plot, sum of weighted objectives formulation

for point 1 (MPE_1), the mutual potential energy for point 2 (MPE_2), and the objective function are maximized. The strain energy for point 1 (SE_1), and the strain energy for point 2 (SE_2) are minimized. In Fig. 5h, there is a single value of mutual potential energy and strain energy due to the combined virtual load.

Both formulations provided valid topological results in the above example. These solutions can be compared based on several criteria, as summarized in Table 1. The weighted sum of objectives formulation required more computation time and more algorithm iterations than the combined virtual load formulation. The increase in computation time can be attributed to the separate finite element analysis which is required for each virtual load in the weighted sum of objectives formulation. Additional algorithm iterations are required for the weighted sum of objectives formulation in order to find a compromise solution between the output deflection requirements. In this case, the values for the volume of the optimal solution, as well as the geometric advantage for each point of interest are comparable. Geometric advantage is defined to be the ratio of the magnitude of the output displacement

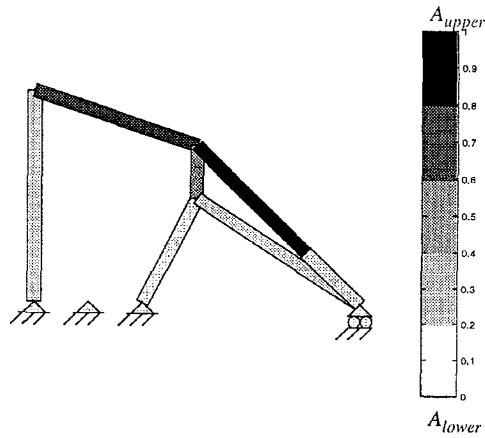

Fig. 5f. Grayscale plot, combined virtual load formulation

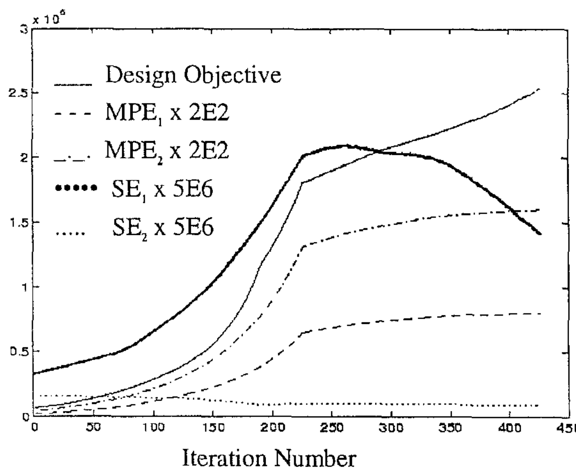

Fig. 5g. Convergence history, sum of weighted objectives formulation

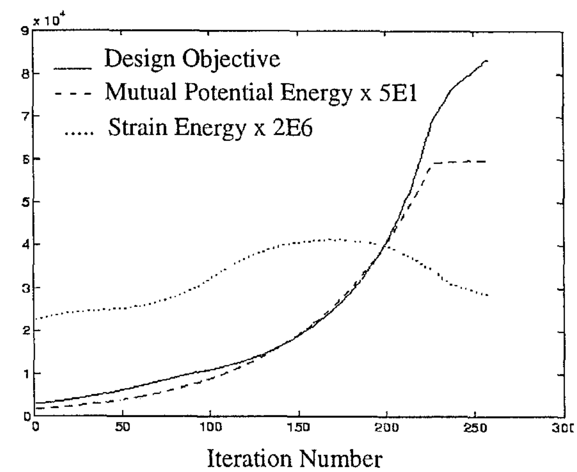

Fig. 5h. Convergence history, combined virtual load formulation

Table 1. Comparison of results

Formulation	Weighted sum of objectives	Combined virtual load
Number of iterations	426	258
Total computation time (min)	91	31
Volume of solution (m^3)	7.18	7.14
Geometric advantage 1	0.82	0.76
Geometric advantage 2	0.74	0.75

to the magnitude of the input displacement at the particular point of interest.

The remaining examples were generated using the combined virtual load method

3.2 *Effect of direction of output deflection requirements*

The purpose of this example is to illustrate the effect of the direction of the output deflection requirements on the topology of the solution. It is motivated by the design of a gripper-type mechanism with two prescribed outputs. The design domain is shown in Fig. 6a, where the applied force F_A is to cause two output deflections Δ_1 and Δ_2 at the points 1 and 2, respectively. The starting point for the algorithm is a 7-node by 5-node full ground structure where each design variable is given an initial value between the upper and lower bound constraints. By varying the direction of vertical output deflection requirements at the points 1 and 2, four cases are generated as shown in Figs. 6b-e. The topology results are shown as threshold plots in the undeformed shape (dashed lines) the deformed shape (solid lines), and are shown to vary depending on the direction of the output deflection requirement. Notice that in Case 4 the algorithm was not able to find a solution which satisfied both deflection requirements, i.e. although this topology is a mathematically feasible solution, the Δ_1 requirement is not satisfied. This result can be justified as follows. The purpose of using the virtual load method is to pose a local displacement-type constraint in a global form. By using the mutual potential energy in the objective function, the displacement of this particular point in the direction of the virtual load is maximized. However, this formulation does not guarantee that the actual output displacement will be exactly in the desired direction. As is evidenced by these examples, it is possible that the actual output displacement will also have a component in a direction perpendicular to the desired direction. A limitation of the current formulation is that there is no way to prohibit this other component. The optimizer attempts to find a solution that is as close as possible to the desired output displacement given the other requirements of the problem.

A comparison among the resulting values of mechanical and geometric advantage is made in Table 2. Note that the mechanical advantage quantities were calculated by considering each point separately, e.g. MA_1 is taken as the ratio of the reaction force when point 1 is fixed to the total input force. Also note that the geometric advantage quantities are calculated by considering the ratio of the magnitude of the total output displacement at the point of interest to the magnitude of the input displacement, rather than considering the component of the output deflection in the specified direction.

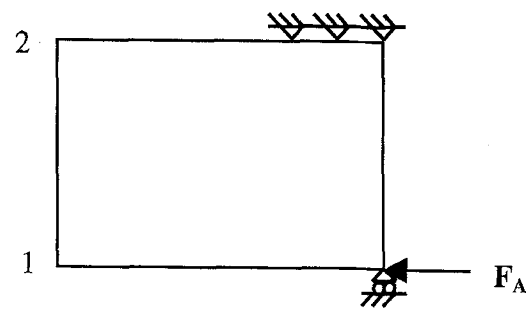

Fig. 6a. Design problem

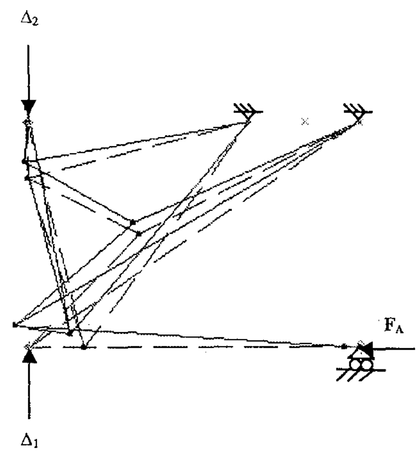

Fig. 6b. Case 1

3.3 *Effect of material resource constraint*

The purpose of this example is to illustrate the effect of the value of the material resource constraint on the topology of the optimal solution. The design problem is shown in Fig. 7a,

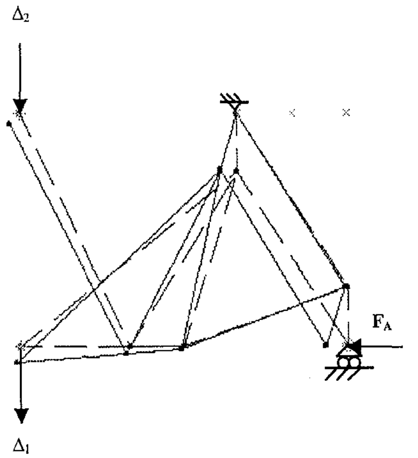

Fig. 6c. Case 2

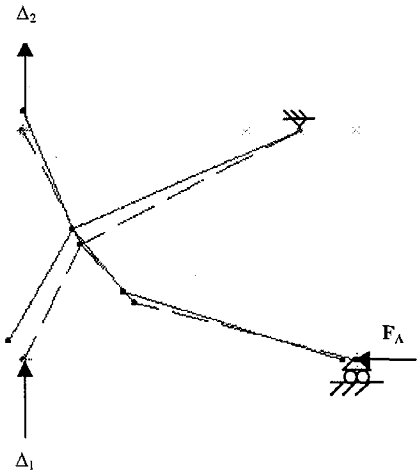

Fig. 6d. Case 3

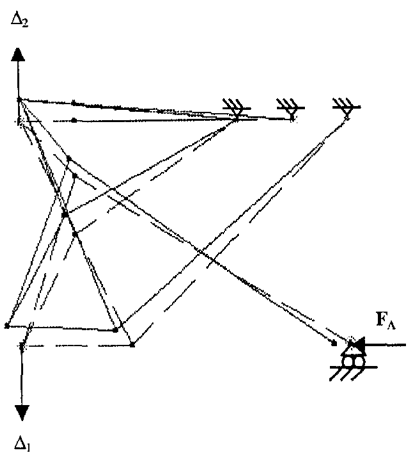

Fig. 6e. Case 4

where it is desired that a clamping mechanism be designed with output deflections Δ_1 and Δ_2. The starting point is a ground structure of elements shown in Fig. 7b. Note that the optimal topologies for two values of volume constraint

Table 2. Comparison of mechanical and geometric advantage values

	MA_1	GA_1	MA_2	GA_2
Case 1	0.98	1.71	0.94	1.02
Case 2	0.94	0.71	0.23	0.61
Case 3	0.59	1.64	0.83	1.32
Case 4	0.64	1.49	1.04	1.37

V^* (Figs. 7c and d) are slightly different. The fact that the value of the material resource constraint can affect the topology of the optimal solution should be expected because solution uniqueness is not guaranteed. Since a globally optimal solution is not guaranteed, the topology of the solution can depend on this constraint and also on the starting point, as will be discussed in the next section.

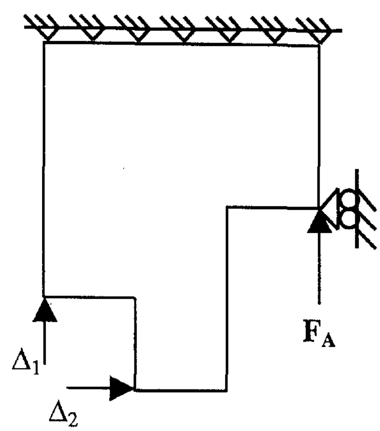

Fig. 7a. Design problem

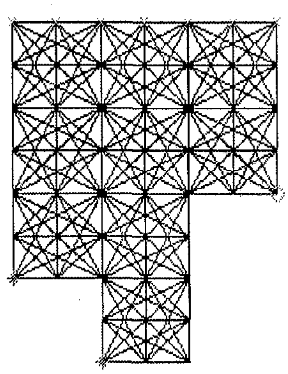

Fig. 7b. Starting point

3.4 *Effect of starting point*

Since a globally optimal solution is not guaranteed, the starting point can affect the topology of the optimal solution as

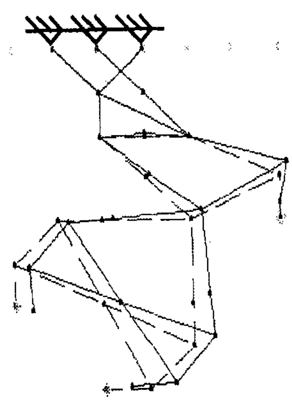

Fig. 7c. $V^* = 20$

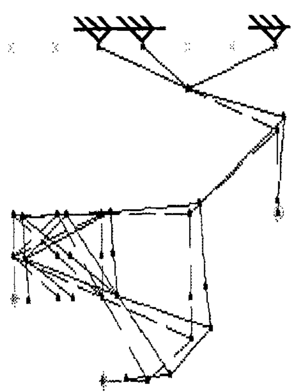

Fig. 7d. $V^* = 25$

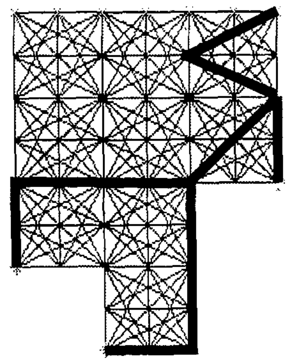

Fig. 8a. Nonuniform starting point

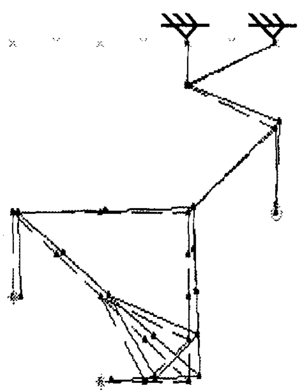

Fig. 8b. Solution

well. In the examples presented thus far, the starting point was a uniform ground structure of truss elements, each with an initial design variable between the upper and lower bound constraints. In this example, a nonuniform starting point is used for the same design problem of Fig. 7a, with certain design variables starting at the upper bound constraint (shown as bold lines in Fig. 8a). The remaining design variables are given an initial value between the upper and lower bounds. The solution based on this starting point is shown in Fig. 8b for a volume constraint $V^* = 20$. Note that this solution is a slightly less complex topology from the solution obtained using a uniform starting point in Fig. 7c, demonstrating that a local optimum may be present. These results confirm that there are multiple solutions or local optima that depend on the starting point and values of the constraints. As is expected when using mathematical programming methods, arrival at a globally optimal solution is not guaranteed and is usually unlikely.

4 Conclusions

A topology optimization procedure has been developed for design of compliant mechanisms with multiple output deflection requirements. This method incorporates both flexibility and stiffness requirements by forming a ratio of mutual potential energy to strain energy as the design objective. This type of objective has previously been developed by Frecker and Nishiwaki for topology optimization of compliant mechanisms with a single output deflection requirement, and in this paper the case of multiple outputs is considered. Two methods of formulating the optimal design problem for multiple outputs are presented, a combined virtual load method and a weighted sum of objectives method. It was found that both methods produce valid topological results, but that the combined virtual load method requires less computation time. This combined virtual load formulation requires essentially the same total computation time as the previously developed formulation for the single-output case. Design examples are presented for problems with two output requirements. The directions of the output deflection requirements are shown to have an effect on the topology of the solution as well as on the resulting mechanical and geometric advantages. Further, it was found that both the value of the material resource constraint and the starting point can have an effect on the topological solution, indicating the presence of local optima.

Clearly there are other issues which are important in the design of compliant mechanisms that are not addressed directly in this paper. For instance, the deflections experienced by these types of devices can easily exceed the linear range, so nonlinear finite element analysis can be incorporated to improve the analysis. Another practical concern is stress concentrations and fatigue. This issue can be partially addressed by the choice of an appropriate material. Polymers with high fatigue resistance such as Delrin should be used for applications requiring many loading cycles.

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