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Technical Report No. 7

INTERPLY STRESS AND LOAD DISTRIBUTION IN CORD-RUBBER LAMINATES

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NOMENCLATURE

English Letters:

a_{ij}	Constants associated with generalized Hooke's law, using properties based on cord tension.
a'_{ij}	Constants associated with generalized Hooke's law, using properties based on cord compression.
E, F, G	Elastic constants for orthotropic laminates with cords in tension.
E',F',G'	Elastic constants for orthotropic laminates with cords in compression.

Greek Letters:

α	One half the included angle between cords in adjoining plies.
ϵ	Strain.
ξ, ζ, η	Orthogonal co-ordinates aligned along and normal to the orthotropic axes in an orthotropic laminate.
σ	Stress
σ'	Interply stress
σ^*	External stress acting on a ply whose cords are in tension.
σ^{**}	External stress acting on a ply whose cords are in compression.
σ^+, σ^{++}	Sum of external and interply stresses on adjoining plies.
τ	Shear stress

I. FOREWORD

Among a series of technical reports issued by the Tire and Suspension Systems Research Group at The University of Michigan in October, 1960, was one which dealt with interply stresses set up in cord-rubber laminates due to external loads.[1] It was shown that the interply stresses set up in orthotropic laminates could be computed and presented graphically in a fairly general way. No effort was made to point out the role of the thickness of the plies in computing interply stresses, nor was a physical model set up which demonstrated clearly the origin of such stresses. Later work has shown that it is also possible to obtain such interply stresses for those cases where the laminate is not orthotropic, that is to say, in those cases where the cords in one ply are in a state of tension while the cords in another ply are in a state of compression. This latter case has been termed "Type 2," and that term will be used throughout this report to describe an anisotropic laminate. Orthotropic laminates, such as discussed in Ref. 1, will be termed "Type 1" laminates.

In view of the omissions made in the original report on this subject, and of the considerable new information now available, it seems desirable to review the entire subject again in more detail. For that reason, this report is intended to cover the general subject of interply stresses in laminates of both types, and in so doing will repeat small portions of Ref. 1. In those cases, this material will be clearly marked, since it is convenient to include the results previously published on "Type 1" laminates in this report too, as a means of comparing with the results from "Type 2" laminates and also to use as examples of the distribution of interply stresses as predicted by the various physical models.

II. SUMMARY

It may be shown from a rather simple physical model that interply stresses are necessary in those cord-rubber laminates which are made up by bonding two or more plies together. This physical model explains the origin of the interply stresses but does not explain quantitative information concerning them.

For orthotropic structures, that is, structures in which normal and shearing effects are not coupled, it was previously shown in Ref. 1 that interply stresses could be defined rather easily if the nature of the elastic constants of a single sheet was carefully considered. Numerical values of these interply stresses were presented in dimensionless form by means of graphs. By considering the thickness of a given set of plies, the actual interply stresses may be calculated.

Using digital computer solutions to the nine simultaneous equations governing the general anisotropic laminate, it is also possible to obtain interply stresses in a form similar to that used for orthotropic laminates. These are again presented in terms of different dimensionless ratios, spanning a range of variables which is believed large enough to encompass all those of technical interest. These interply stresses are shown to be generally similar to those previously calculated for orthotropic materials, with some variations.

It may also be shown, by similar solutions for the general anisotropic case, that dimensionless load-distribution curves may be presented for the two plies in a two-ply structure. These generally illustrate the phenomenon to be expected, namely, that stiffer plies carry greater portions of the load. When

external normal stresses are applied, this effect is most apparent at very small cord angles. For the application of shearing stresses, the differences in load distribution are greatest at cord angles of 45°.

III. PHYSICAL MODEL

The whole subject of interply stresses can be clarified considerably by reference to the simple process of extending two sheets whose cord directions are mirror images of one another. These are illustrated in Fig. 1 in which the ξ-direction is taken vertically while the η-direction is taken horizontally.

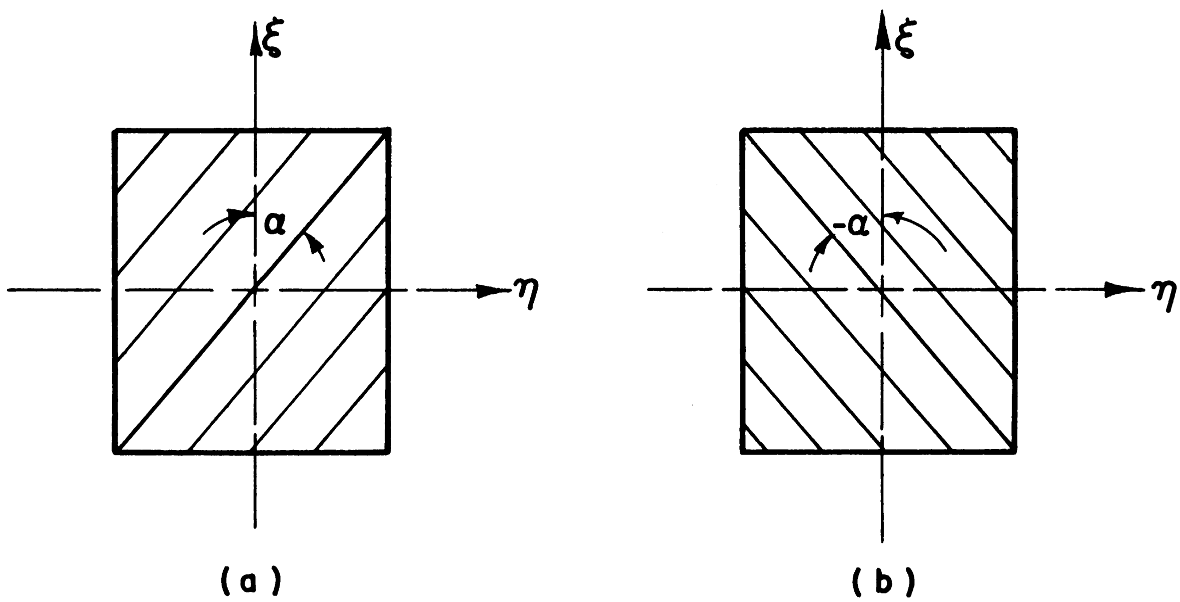

Fig. 1. Two separate plies with equal but opposite cord angles.

This is the same notation as used in all previous reports. Ref. 2 gave equations relating stresses and strains in general anisotropic materials, such as exhibited by either of the two plies in Fig. 1. The form of these equations is repeated here as Eqs. (1), for easy reference. These are the same as

Eqs. (9) of Ref. 2.

$$\begin{aligned} \epsilon_\xi &= a_{11}\sigma_\xi + a_{12}\sigma_\eta + a_{13}\sigma_{\xi\eta} \\ \epsilon_\eta &= a_{21}\sigma_\xi + a_{22}\sigma_\eta + a_{23}\sigma_{\xi\eta} \\ \epsilon_{\xi\eta} &= a_{31}\sigma_\xi + a_{32}\sigma_\eta + a_{33}\sigma_{\xi\eta} \end{aligned} \tag{1}$$

Recalling that in Eqs. (1) the positive angle α is that shown in Fig. 1a, extension of the two sheets shown in Fig. 1 may be accomplished in the unbonded state by gripping the upper and lower edges in some sort of grips and applying a load. In this case, no interply stresses can exist in the structure. Also, the only possible direction of stress is in the ξ-direction, or in the direction in which load is applied. Under those conditions, Eqs. (1) become, for this case, the following:

$$\begin{aligned} \epsilon_\xi &= [a_{11}(+\alpha)]\sigma_\xi & \qquad \epsilon_\xi &= [a_{11}(-\alpha)]\sigma_\xi \\ \epsilon_\eta &= [a_{21}(+\alpha)]\sigma_\xi & \qquad \epsilon_\eta &= [a_{21}(-\alpha)]\sigma_\xi \\ \epsilon_{\xi\eta} &= [a_{31}(+\alpha)]\sigma_\xi & \qquad \epsilon_{\xi\eta} &= [a_{31}(-\alpha)]\sigma_\xi \end{aligned} \tag{2}$$

Ply of Fig. 1a — Ply of Fig. 1b

Note that in Eqs. (2) the angular arguments of the a_{ij} terms are written in parentheses, and that it has been shown previously[2] that $a_{11}(+\alpha) = a_{11}(-\alpha)$, $a_{21}(+\alpha) = a_{21}(-\alpha)$, while $a_{31}(+\alpha) = -a_{31}(-\alpha)$. From these equations it may be seen at once that the extensions ϵ_ξ in the ξ-direction are equal, that the associated lateral contractions ϵ_η are also equal, but that the shear distortions $\epsilon_{\xi\eta}$ are equal but opposite in the two plies. This means that in extending two unbonded plies, with cords directed such as shown in Fig. 1 and with all properties identical except for the directions of the cord angles being mirror images of one another, equal distortions will not occur due to the imposition of a simple tensile stress. In particular, the shearing distortions will be different here. This case is, of course, that of a "Type 1" structure such as is exemplified by all cords in both plies being either in tension simultaneously or in compression simultaneously. The relations between the applied stress σ_ξ and the resulting strain in that direction ϵ_ξ represent a Young's modulus whose value is $1/a_{11}$ for this unbonded pair of plies. Similarly, a measure of lateral contraction of these two plies acting commonly but in an unbonded way is a_{21}. Finally, since $a_{11}(+\alpha) = a_{11}(-\alpha)$, any loads imposed on the over-all structure must, some distance away from local loading discontinuities, be distributed equally in the ξ-direction between the two plies. This is because of their equal stiffnesses.

The conditions which exist when the two plies of Fig. 1 are bonded together are obtained by setting each of the strains of Eqs. (1) equal to its corresponding strain for the other ply. In addition, it is necessary to assume that only external stresses in the ξ-direction exist and that any other stresses which are found in the η or $\xi\eta$ directions are stresses which must enter into the system through the interply bond. Under these conditions, Eqs. (3) are obtained.

Ply 1

$$\begin{aligned}
\epsilon_\xi &= [a_{11}(+\alpha)]\sigma_\xi + [a_{12}(+\alpha)]\sigma'_\eta + [a_{13}(+\alpha)]\sigma'_{\xi\eta} \\
\epsilon_\eta &= [a_{21}(+\alpha)]\sigma_\xi + [a_{22}(+\alpha)]\sigma'_\eta + [a_{23}(+\alpha)]\sigma'_{\xi\eta} \qquad (3) \\
\epsilon_{\xi\eta} &= [a_{31}(+\alpha)]\sigma_\xi + [a_{32}(+\alpha)]\sigma'_\eta + [a_{33}(+\alpha)]\sigma'_{\xi\eta} = 0
\end{aligned}$$

Ply 2

$$
\begin{aligned}
\epsilon_\xi &= [a_{11}(-\alpha)]\sigma_\xi - [a_{12}(-\alpha)]\sigma'_\eta - [a_{13}(-\alpha)]\sigma'_{\xi\eta} \\
\epsilon_\eta &= [a_{21}(-\alpha)]\sigma_\xi - [a_{22}(-\alpha)]\sigma'_\eta - [a_{23}(-\alpha)]\sigma'_{\xi\eta} \\
\epsilon_{\xi\eta} &= [a_{31}(-\alpha)]\sigma_\xi - [a_{32}(-\alpha)]\sigma'_\eta - [a_{33}(-\alpha)]\sigma'_{\xi\eta} = 0
\end{aligned}
\tag{3}
$$

Notice that in Eqs. (3) a prime is used to denote a stress arising due to the presence of the interply bond. Again, it is presumed here that a "Type 1" structure is being considered so that the various a_{ij} are symmetric except for a_{13} and a_{23} as previously discussed in Ref. 2. The resulting interply stresses and stiffnesses may be obtained by solution of these equations, this solution taking the form

$$
\begin{aligned}
\sigma'_{\xi\eta} &= -\frac{a_{31}(+\alpha)}{a_{33}(+\alpha)}\,\sigma_\xi \\
\epsilon_\xi &= \left\{a_{11}(+\alpha) - \frac{[a_{13}(+\alpha)][a_{31}(+\alpha)]}{a_{33}(+\alpha)}\right\}\sigma_\xi \\
\epsilon_\eta &= \left\{a_{21}(+\alpha) - \frac{[a_{23}(+\alpha)][a_{31}(+\alpha)]}{a_{33}(+\alpha)}\right\}\sigma_\xi
\end{aligned}
\tag{4}
$$

These equations are a special case of Eqs. (14) of Ref. 2, and as such have been discussed previously. They are shown here to illustrate that now the interply stress $\sigma'_{\xi\eta}$ may be determined in terms of the independent variable σ_ξ, and in addition the quantity equivalent to the elastic modulus is obtainable by forming the ratio σ_ξ/ϵ_ξ, which is seen from Eq. (4) to be somewhat different from a_{11}. This is of interest since it was previously shown that the stiffness of a pair of unbonded plies identical to those being considered here was $1/a_{11}$. It indicates that the elastic stiffnesses of bonded and unbonded structures are not the same. The correction terms are generally not large, however. In addition, Eqs. (4) also indicate that the effective Poisson's ratic, the quantity σ_ξ/ϵ_η, is not equal to a_{21}, the value previously obtained for the identical plies in the unbonded state. This indicates a similar stiffening in the lateral direction due to the presence of the bond. For this reason it may generally be concluded that bonding two plies results in a somewhat stiffer structure than if they remain unbonded, assuming identical materials.

It may also be seen from Fig. 1 and Eqs. (2) — (4) that physically one would not expect an interply stress σ'_η since the tendency for lateral contraction of each of the two plies shown in Fig. 1 is exactly the same. This is represented in the equations by means of the fact that $a_{21}(+\alpha) = a_{21}(-\alpha)$ as previously indicated. The strains in the ξ-direction were, of course, assumed to be equal as part of the conditions by which one would compare these two plies. Finally, the shear distortions of each of the two plies of Fig. 1 are not the same in the unbonded state. This is represented by the fact that $a_{31}(+\alpha)$ does not equal $a_{31}(-\alpha)$, as also previously indicated. Physically, this means that if one simultaneously extends the two plies shown in Fig. 1 an equal amount ϵ_ξ, the lateral contraction of each ply will be the same but the shearing distortions will be exactly opposite from one another.

The role of the interply bond is to set up those stresses which are necessary to force each ply to take on exactly the same strain state as its neighbor when it is loaded. From the discussion in the previous paragraph, it can be seen that the only interply stress necessary here is one whose role is to force the two shearing distortions to become equal to one another. This would normally be accomplished by an interply shear stress, and it is seen that Eqs. (4) do give an interply stress which has this result.

From this discussion, it may be seen that the formation of a two-ply structure of the first type, that is, one in which orthotropy prevails, can be explained on the basis of a very simple physical model. This model illustrates the need for the interply stress as well as the fact that the loads carried by each ply of the two-ply laminate are equal. A similar set of conclusions could be obtained by subjecting the pair of plies illustrated in Fig. 1 to an external shear stress rather than to an external normal stress. Under those conditions, the arguments are completely similar and result in the conclusions which are presented in some detail in Refs. 1 and 2. These should be consulted for a more thorough discussion of the equations used in describing bonded laminates.

Attention is next directed to the problem of creating a simple physical model for a "Type 2", or anisotropic, laminate. Again Fig. 1 may be used to represent two individual plies with cord angles which are mirror images of one another. It may now be imagined that for one reason or another the stiffnesses of one ply are different from those of the other, due say, to the use of different materials or, alternately, because one of the plies may have its cords in a state of compression while the other's cords are in a state of tension. This could be caused by the imposition of certain types of stress states. In any event, the elastic constants of the two plies will be differentiated by denoting the elastic constants of the ply illustrated in Fig. 1b by a'_{ij}, and those of the ply illustrated in Fig. 1a, by the usual a_{ij}.

Imagine that the two plies are again to be loaded simultaneously but without a bond between them. Let the loads be only in the ξ-direction, and of such a nature that the two strains ϵ_ξ in each ply are identical. It will require different stresses σ_ξ to achieve this purpose. It may be imagined that these different stresses are independent variables and may be gotten by some mechanism or device. Under those conditions, the equations which govern the elastic action of the two dissimilar plies are now

<u>Ply of Fig. 1a</u> | <u>Ply of Fig. 1b</u>

$$\begin{aligned} \epsilon_\xi &= [a_{11}(+\alpha)]\sigma_\xi^* & \qquad \epsilon'_\xi &= [a'_{11}(-\alpha)]\sigma_\xi^{**} \\ \epsilon_\eta &= [a_{21}(+\alpha)]\sigma_\xi^* & \qquad \epsilon'_\eta &= [a'_{12}(-\alpha)]\sigma_\xi^{**} \\ \epsilon_{\xi\eta} &= [a_{31}(+\alpha)]\sigma_\xi^* & \qquad \epsilon_{\xi\eta} &= [a'_{31}(-\alpha)]\sigma_\xi^{**} \end{aligned} \tag{5}$$

$$\epsilon_\xi = \epsilon'_\xi$$

$$2\sigma_\xi = \sigma_\xi^* + \sigma_\xi^{**}$$

where the asterisk and double asterisk continue a notation started Ref. 2, in which the single asterisk refers to the external stresses carried by one ply while the double asterisk refers to external stresses carried by the other ply. The a_{ij} describing each of the two plies not only has an angular argument different in sign but also has different values of the a'_{ij} from those of the first ply. Eqs. (5) indicate that in general the lateral contractions as well as the shearing distortions of such a pair of plies will be different upon loading to an equal strain in the ξ-direction. The stiffness of such a structure now depends on the elastic characteristics of both plies since, if one solves for the effective modulus in the ξ-direction of this unbonded structure, one obtains the result given in Eq. (6)

$$\sigma_\xi/\epsilon_\xi = \left[\frac{a_{11}(+\alpha) + \alpha'_{11}(-\alpha)}{2a_{11}(+\alpha)\cdot\alpha'_{11}(-\alpha)}\right] \tag{6}$$

The lateral contractions and the shearing distortions of each of the two plies discussed here will, of course, be different and not related to one another. Since these are not in general the same, it would be expected that if one wished to bond together two dissimilar plies such as these, it would be necessary to provide through the bond interply stresses in both the η-directions and in the distortional direction, or $\xi\eta$-direction. Thus, one would expect that the imposition of a normal stress in a "Type 2" laminate would result in interply stresses at right angles to this imposed stress as well as an interply shear stress. These physical expectations are analytically fulfilled as follows.

Consider now the problem of attempting to bond together the two dissimilar plies under discussion. To represent this condition in the equations, it is necessary to postulate that the strains ϵ_ξ, ϵ_η, and $\epsilon_{\xi\eta}$ are all equal. In addition, it is most convenient to imagine loading in one direction, say the ξ-direction by means of a normal stress σ_ξ. Under those conditions, Eqs. (7) represent a statement of Hooke's law for this particular case.

<u>Ply of Fig. 1a:</u>

$$\begin{aligned}
\epsilon_\xi &= [a_{11}(+\alpha)]\sigma_\xi^{+} + [a_{12}(+\alpha)]\sigma'_\eta + [a_{13}(+\alpha)]\sigma'_{\xi\eta} \\
\epsilon_\eta &= [a_{21}(+\alpha)]\sigma_\xi^{+} + [a_{22}(+\alpha)]\sigma'_\eta + [a_{23}(+\alpha)]\sigma'_{\xi\eta} \\
\epsilon_{\xi\eta} &= [a_{31}(+\alpha)]\sigma_\xi^{+} + [a_{32}(+\alpha)]\sigma'_\eta + [a_{33}(+\alpha)]\sigma'_{\xi\eta}
\end{aligned} \tag{7}$$

Ply of Fig. 1b:

$$\epsilon_\xi = [a'_{11}(-\alpha)]\sigma_\xi^{++} - [a'_{12}(-\alpha)]\sigma'_\eta - [a'_{13}(-\alpha)]\sigma'_{\xi\eta}$$

$$\epsilon_\eta = [a'_{21}(-\alpha)]\sigma_\xi^{++} - [a'_{22}(-\alpha)]\sigma'_\eta - [a'_{23}(-\alpha)]\sigma'_{\xi\eta}$$

$$\epsilon_{\xi\eta} = [a'_{31}(-\alpha)]\sigma_\xi^{++} - [a'_{32}(-\alpha)]\sigma'_\eta - [a'_{33}(-\alpha)]\sigma'_{\xi\eta}$$

Equation linking both plies: (7)

$$2\sigma_\xi = \sigma_\xi^{+} + \sigma_\xi^{++}$$

These equations are special cases of Eqs. (22) of Ref. 2, with the exception that the terminology σ^+ and σ^{++} has been adopted here to represent the total stress carried in the ξ-direction by the first and second plies, respectively. Some discussion of this notation is warranted. In general, it is not possible to bond two plies together as is done here, to require that their strains in the ξ-direction are equal, and simultaneously to require that the stresses imposed at their edges take on certain specified values consistent with the stiffnesses of the plies, in such a way that the strains in each ply would independently be the same. The reason for this is that in general it is not possible to control exactly the loads that are put into one ply as compared to those put into the other at places where load is applied, such as during an actual test of in a real pneumatic tire. If the loads applied at these edges or places of local loading are not consistent with those necessary to obtain equality of strain in the ξ-direction, then interply stresses will be generated to such an extent that the total net stress carried by each of the two plies, denoted here by σ^+ and σ^{++}, is equal to its proper value. By invoking Saint Venant's principle, it can be argued that some distance away from points of local loading these interply stresses in the ξ-direction actually disappear and the σ^+ or σ^{++} become equal to σ^* and σ^{**}, respectively. In other words, away from boundaries the interply stresses in the ξ-direction due to external loads in the ξ-direction diffuse out to zero.

Under these conditions, Eqs. (7) may be solved and the appropriate ratios of stress to resulting strain may be obtained in either the ξ-direction, the η-direction, or the $\xi\eta$-direction. These quantities represent the anisotropic counterparts of Young's modulus or elastic stiffness, of cross modulus or Poisson's ratio, and of the interply shear stress generated due to a normal external stress. In addition, the interply stress in the η-direction may also be obtained. Due to the number of equations involved here, no solution will be presented at this time for the particular case given, although the results of more complete solutions will be presented in the succeeding sections of this report. This example was designed to illustrate the physical basis of interply stresses, particularly those associated with these "Type 2," or anisotropic, structures.

INTERPLY STRESSES AND LOAD DISTRIBUTION IN "TYPE 1" LAMINATES

Both Refs. 1 and 2 gave expressions for values of the interply stresses generated in a "Type 1," or orthotropic, laminate due to the imposition of external loads. These expressions were obtained by properly equating the strains appearing in the generalized Hooke's law as written for each ply of a two-ply structure. By making some rather simple assumptions concerning the ratios of various elastic constants entering into these equations, based on a physical model of a single sheet or ply of cords imbedded in rubber, it is possible to reduce these expressions for interply stresses in a "Type 1" laminate to dimensionless form and to present these results graphically as a function of cord angle. This was done in Ref. 1 and will be repeated here in Figs. 2—5, where the meanings of the various terms may be seen by referring to Eqs. (8) which are also reproduced from Ref. 1. Note carefully that Figs. 2—5 apply only to "Type 1" structures since the a_{ij} used to evaluate Eqs. (8) were based on the same elastic properties in both plies of the two-ply structure.

$$\frac{\sigma_\eta'}{\sigma_{\xi\eta}} = \left\{\frac{[a_{12}(+\alpha)][a_{13}(+\alpha)] - [a_{11}(+\alpha)][a_{23}(+\alpha)]}{[a_{11}(+\alpha)][a_{22}(+\alpha)] - [a_{12}(+\alpha)]^2}\right\}$$

$$\frac{\sigma_\xi'}{\sigma_{\xi\eta}} = -\left[\left[\left\{\frac{a_{12}(+\alpha)}{a_{11}(+\alpha)}\right\}\left\{\frac{[a_{12}(+\alpha)][a_{13}(+\alpha)] - [a_{11}(+\alpha)][a_{23}(+\alpha)]}{[a_{11}(+\alpha)][a_{22}(+\alpha)] - [a_{12}(+\alpha)]^2}\right\} + \frac{a_{13}(+\alpha)}{a_{11}(+\alpha)}\right]\right]$$

$$\sigma_{\xi\eta}' = -\frac{a_{13}(+\alpha)}{a_{33}(+\alpha)}\sigma_\xi - \frac{a_{23}(+\alpha)}{a_{33}(+\alpha)}\sigma_\eta \qquad (8)$$

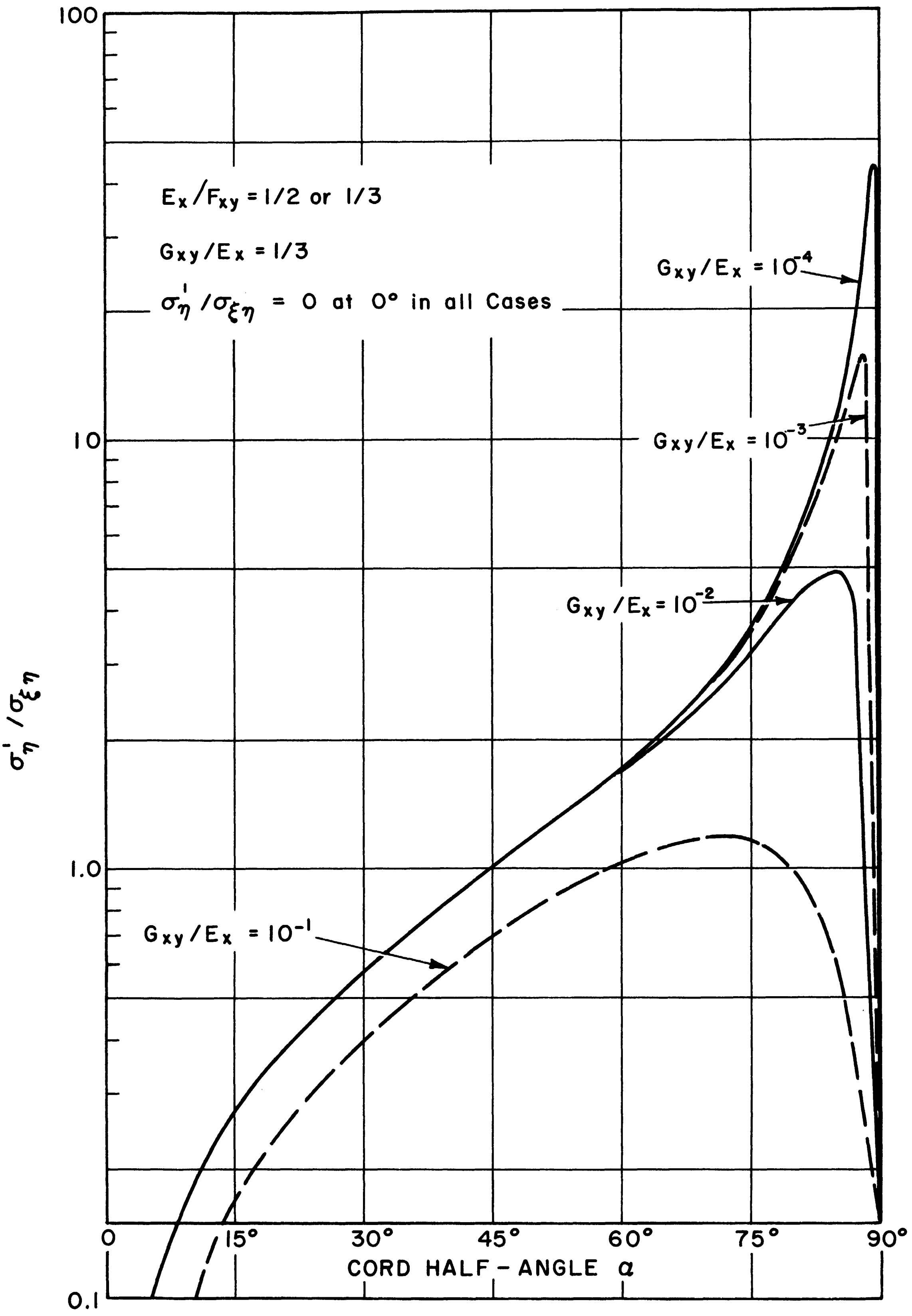

Fig. 2. $\sigma'_\eta/\sigma_{\xi\eta}$ vs. cord half-angle α for various values of G_{xy}/E_x.

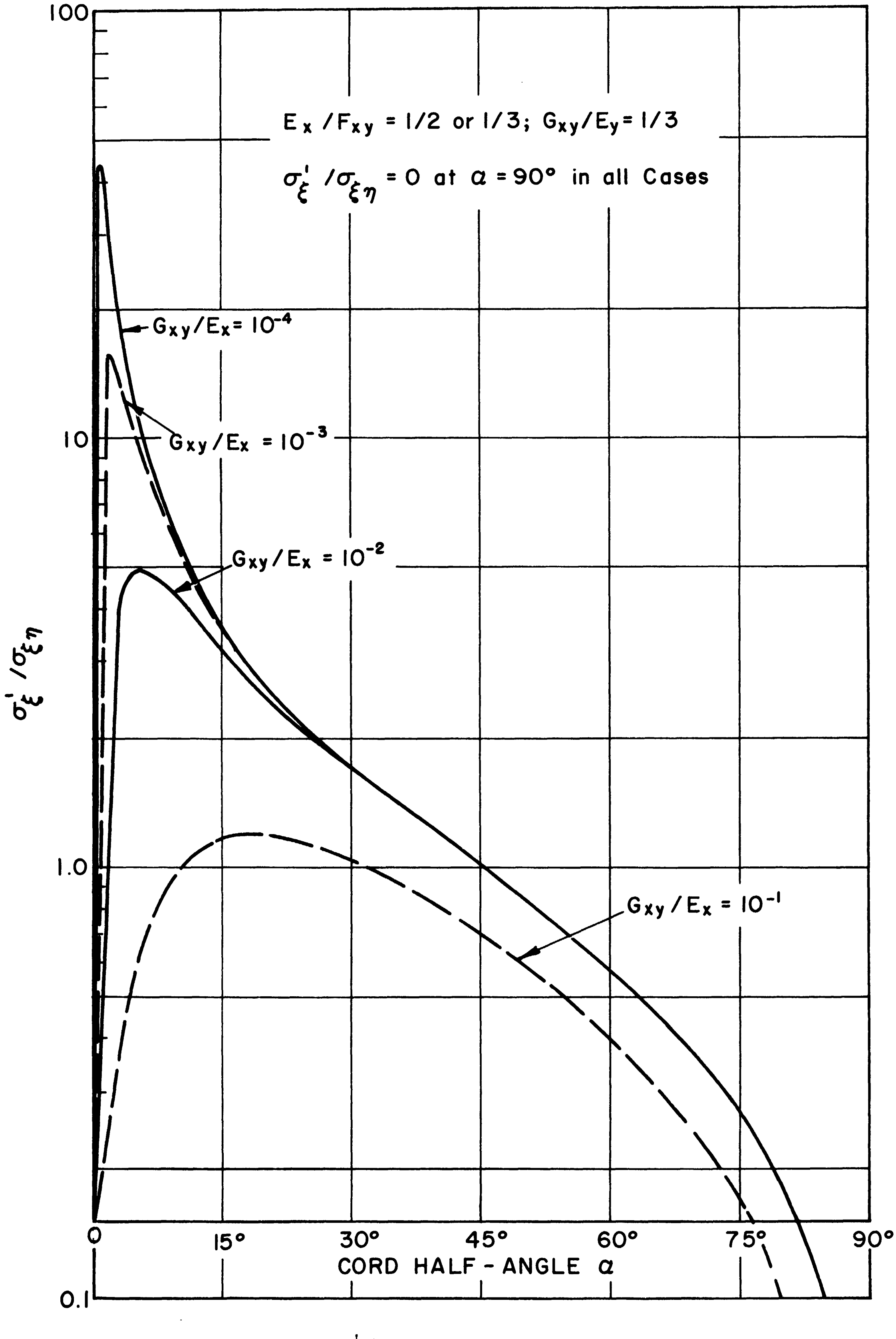

Fig. 3. $\sigma'_\xi/\sigma_{\xi\eta}$ vs. cord half-angle α for various values of G_{xy}/E_x.

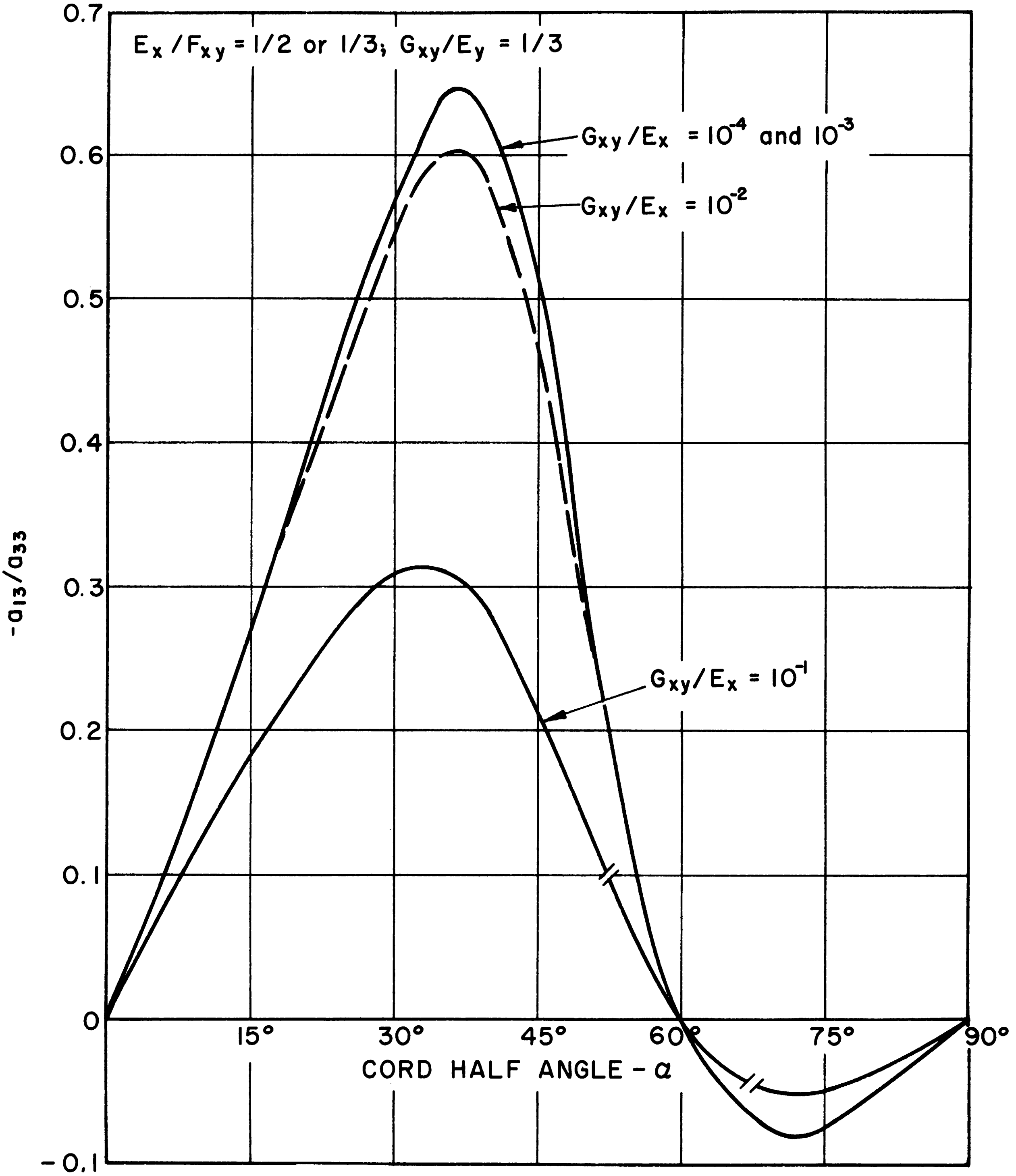

Fig. 4. $-(a_{13}/a_{33})$ vs. cord half-angle α for various values of G_{xy}/E_x.

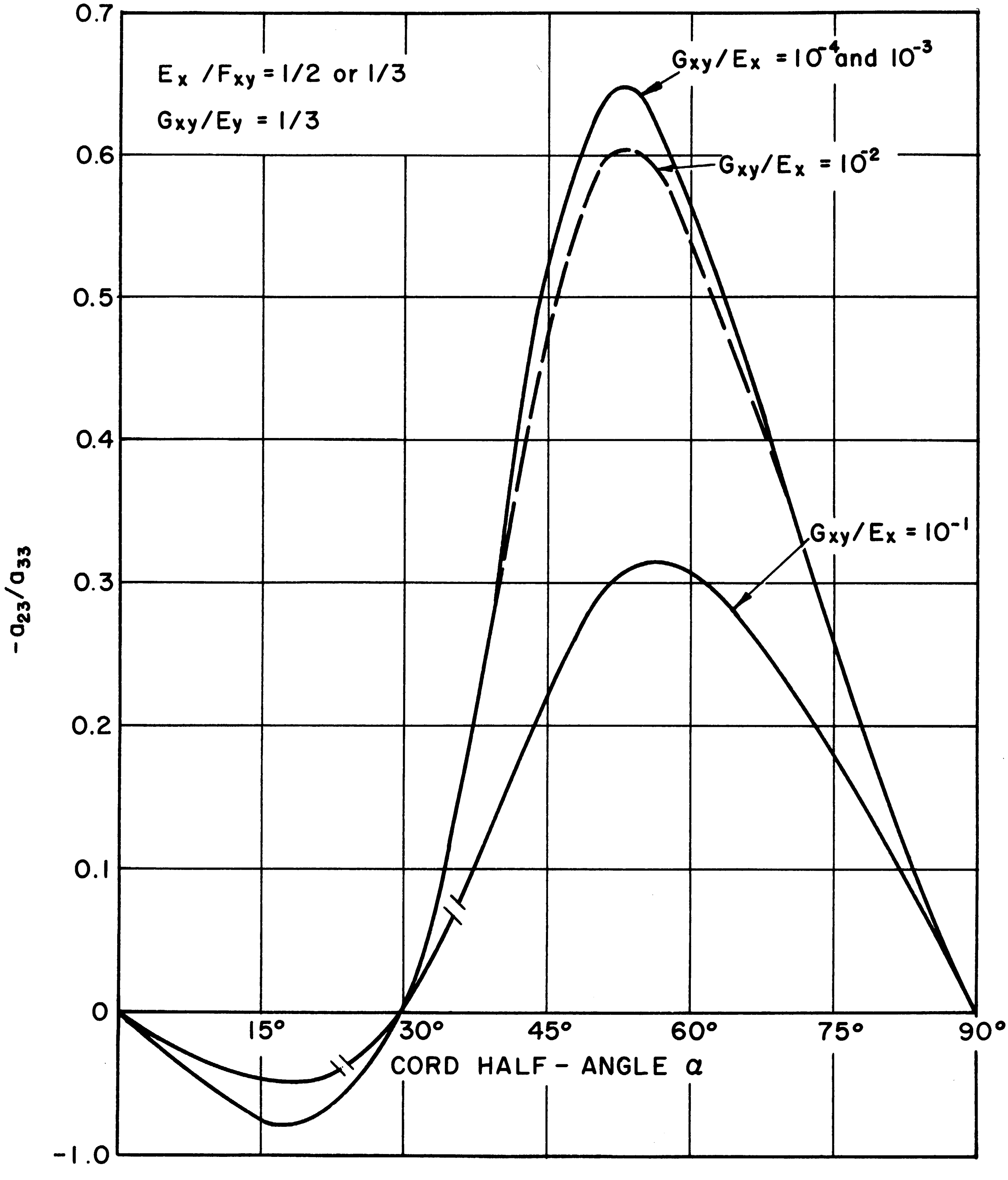

Fig. 5. $-(a_{23}/a_{33})$ vs. cord half-angle α for various values of G_{xy}/E_x.

These results are in complete accord with the discussion concerning the simple physical model in the preceding section. This is because external normal stresses cause only an interply shear stress, while an external shearing stress gives rise to normal components of the interply stress only.

Figures 2—5 represent plots of influence coefficients vs. cord angle from which interply stresses may be obtained directly. The method of using these curves may be seen most easily by referring to Fig. 6. In fig. 6a is shown a unit cube upon whose faces shear stresses equivalent to the interply stresses

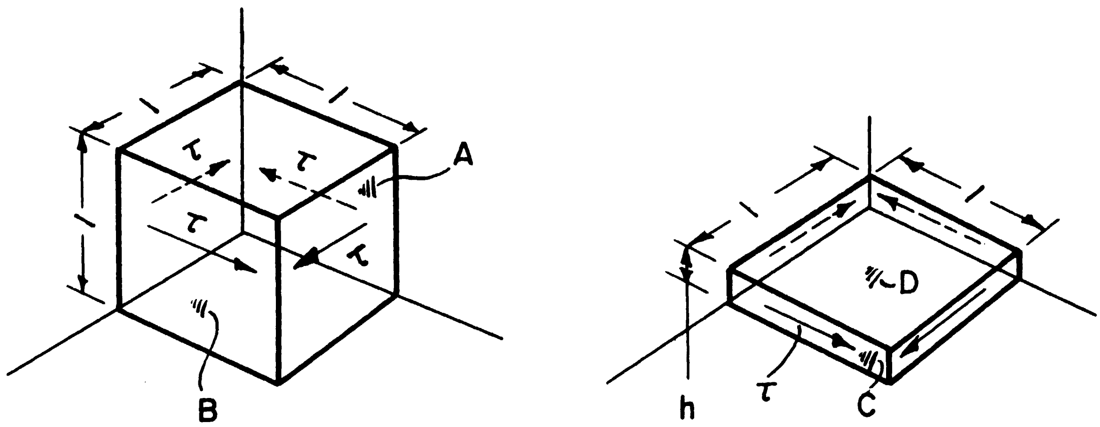

Fig. 6. Laminate elements of unit thickness
and of thickness h.

are presumed to act. It may be seen that these interply stresses will take on the same value in magnitude whether they act on faces A or B of Fig. 6a, since the areas of these two faces are equal. If one wishes to apply the results given in Figs. 2—5 to some element other than a unit cube, such as a unit area of thickness h shown in Fig. 6b, then it is necessary to note that a shear stress equivalent in force or moment to τ acting on face C of Fig. 6b is obtained by applying a shear stress of magnitude $\tau \times h$ on face D of this same figure, assuming it to be distributed uniformly over the area D and that this distribution results in the same net distortional effect as that of τ. Thus, in using the curves given in Figs. 2—5, it will be necessary to determine interply stresses by using the influence coefficients which are given and then by multiplying each of these interply stresses by the thickness h of the single ply being used to form the two-ply laminate. This will give the proper value of the interply stress between the two plies.

A direct consequence of this conclusion is that the distribution of interply stresses through a ply is linear with distance from the outer face. This is illustrated in Fig. 7. This conclusion is reached by considering the equilibrium of various strata of the two-ply structure of Fig. 7.

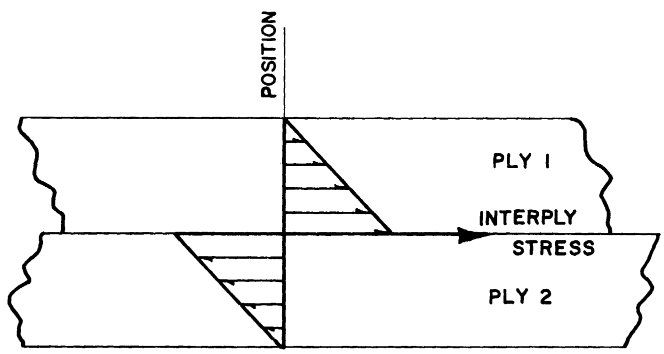

Fig. 7. Interply stress distribution through a pair of plies.

It was previously pointed out that these conclusions concerning interply stresses apply with equal validity to structures made up of any even number of plies. In considering such a situation it is apparent that pairs of plies will act together in just the same way that the two-plied structure discussed here acts. In regard to these multi-ply structures, it was previously stated in Ref. 1 that these interply stresses could occur only between alternate plies since the compatibility of strains required that the full value of the interply stress occur in the area lying between the first and second plies, and that, since equal but opposite moments were generated in this interply bond, the second ply was automatically held in a state of strain compatibility by these interply moments. All this is correct. It should be emphasized, however, that local interply stresses will still result, due to deformation of insulation rubber, between the second and third plies and in fact between any odd plies where such stresses are not required for compatibility purposes. The nature of these latter interply stresses is such that, when averaged over some area several times that of the square of the cord spacing, the net effective moment or force of such interply stresses <u>will vanish</u>. In the case where interply stresses <u>are</u> required for purposes of strain compatibility, such as between the first and second plies, the net effective moment or force over an area such as just mentioned does not vanish.

This means that while two fundamentally different things happen in the interply areas between alternate plies, hysteresis losses and local failure can occur in either type of area, and furthermore, it is not easy to pick out what the physical differences in action really are except to reiterate that the net effect of one is to transmit a moment or force while the effect of the other is to transmit no moment or force.

This discussion concerning interply stresses holds for loads applied in the <u>plane</u> of the carcass, i.e., membrane loads. It is clearly true that for bending loads, all the interply connections between laminates must carry shear forces and must possess structural capabilities. Thus it might be said in closing this discussion of interply stresses due to loads in the plane of the sheet that they represent only a portion of the total interply stress. The remainder is generated from bending effects and must be calculated on a separate basis.

V. INTERPLY STRESSES AND LOAD DISTRIBUTION IN "TYPE 2" LAMINATES

Equations (22) of Ref. 2 describe the elastic characteristics of two plies made from materials with different elastic constants and bonded together in such a way that their strains are the same. As was discussed previously in this report, it is necessary to consider Saint Venant's principle to argue that interply stresses in the ξ-direction set up due to external loads in that same direction vanish some little distance from boundaries or points of localized load application. The problem may be clarified either by assuming that this particular kind of interply stress, due to an external load in the same direction, vanishes, or alternately, by lumping together the external and interply stress on a given ply into a single stress value acting on that ply denoted by σ^{+} or σ^{++}. The results which one obtains from solution of these nine simultaneous equations in nine unknowns is the same in either case.

Two types of loads are of interest in connection with such a structure. The first is the influence of a normal stress on the resulting interply stresses and the way in which this stress is distributed between two plies. The other question of interest is the nature of the interply stresses and of the load distribution due to the imposition of an external shearing stress. Taking first the case of applying an external normal stress σ_ξ to such a structure, it is seen from our previous discussion of the physical model for such a case that in general interply stresses σ_η', a normal component of interply stress, and $\sigma_{\xi\eta}'$, a shearing component of interply stress, will be present. However, before such calculations can be made, it is necessary to specify numerical values for the constants a_{ij} and a'_{ij}. This is a considerably broader and more difficult problem than specifying a single numerical range over which constants a_{ij} may vary. In part, this is because it is more difficult to obtain good numerical data on cords in compression, which are used throughout this report as the basic method of generating a'_{ij} constants, but it is also true that one must specify here two numerical ranges for certain coefficients as well as the degree of their overlap. To cover this problem in a reasonably broad way without restricting solutions to a narrow numerical range, Table I has been used to generate the eight cases of interest which have been calculated.

TABLE I

$\frac{G_{xy}}{E_x} = 10^{-4}$	10^{-3}	10^{-2}
$\frac{G'_{xy}}{E'_x} = 10^{-4}$	10^{-3}	10^{-2}
10^{-3}	10^{-2}	10^{-1}
10^{-2}	10^{-1}	*

*This condition is omitted since it can be shown that G'_{xy}/E'_x is equal to 1/3 for isotropic materials, and so values greater are probably impossible.

Digital computer solutions of Eqs. (7), or of Eqs. (22) of Ref. 2, are presented in Figs. 8—13. These show that a normal stress σ_ξ generally results in shear components $\sigma'_{\xi\eta}$ of the interply stresses not widely different from those previously calculated for orthotropic, or "Type 1," materials. Some quantita-

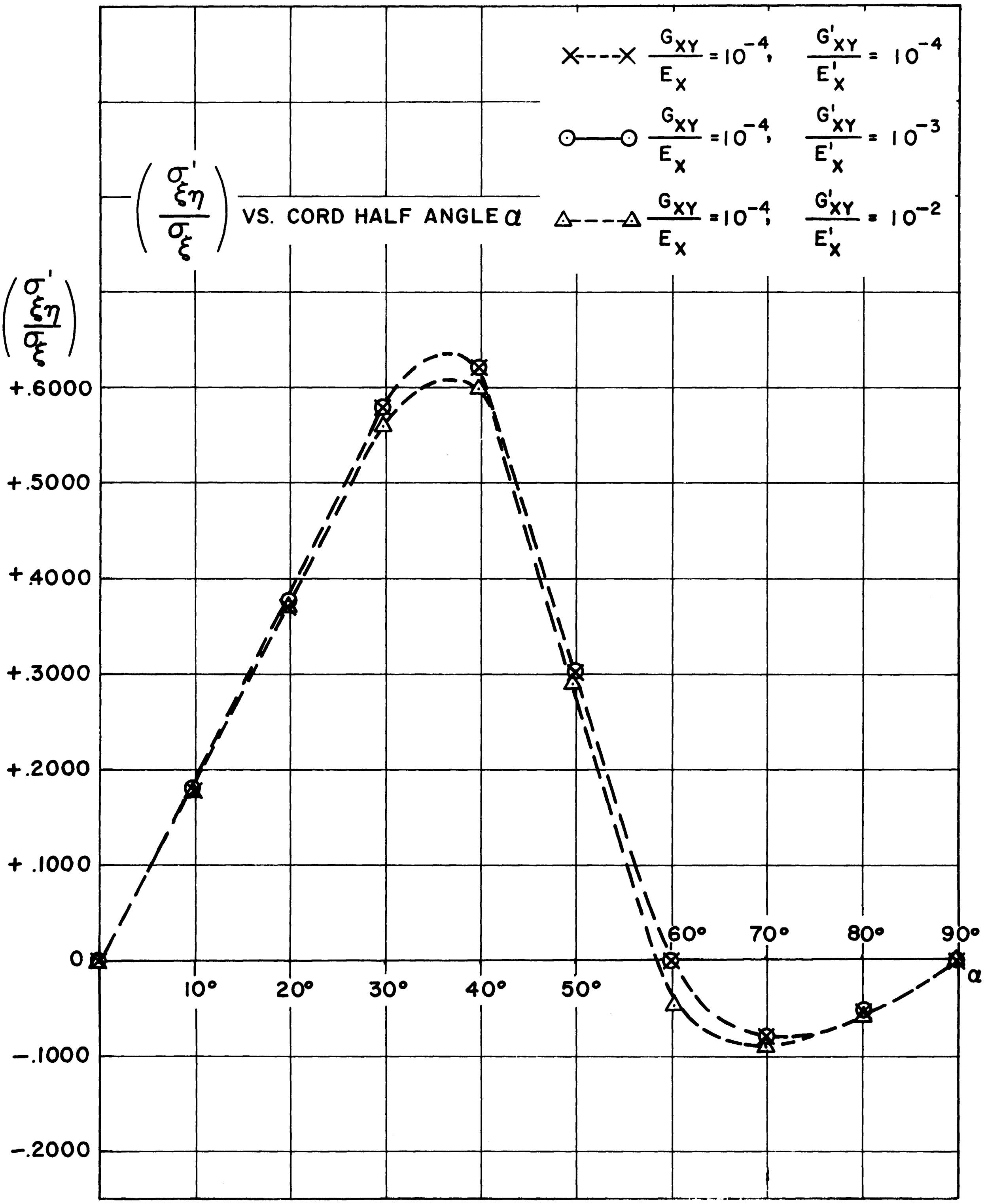

Fig. 8. $\sigma'_{\xi\eta}/\sigma_\xi$ vs. cord half angle α.

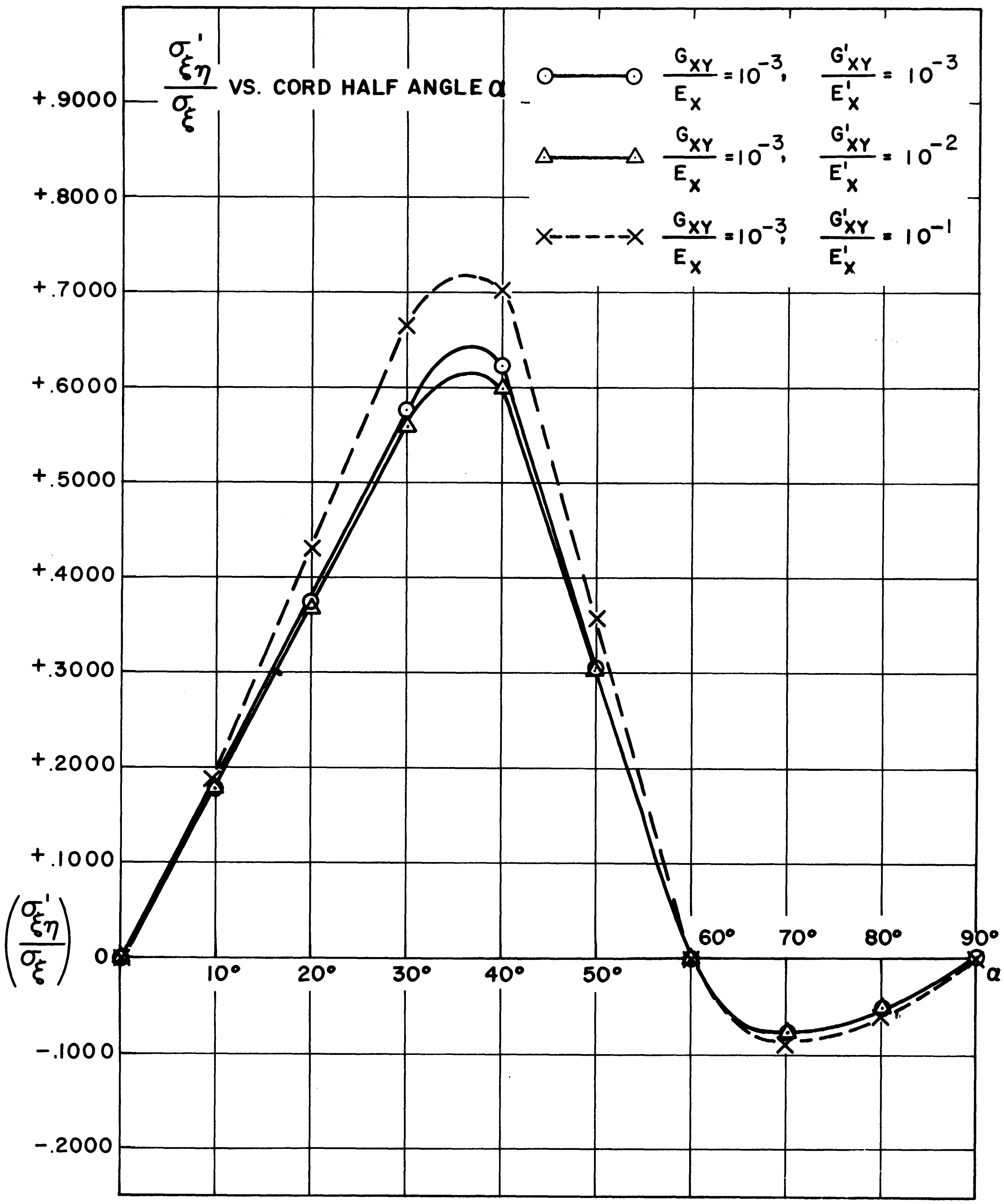

Fig. 9. $\sigma'_{\xi\eta}/\sigma_\xi$ vs. cord half angle α.

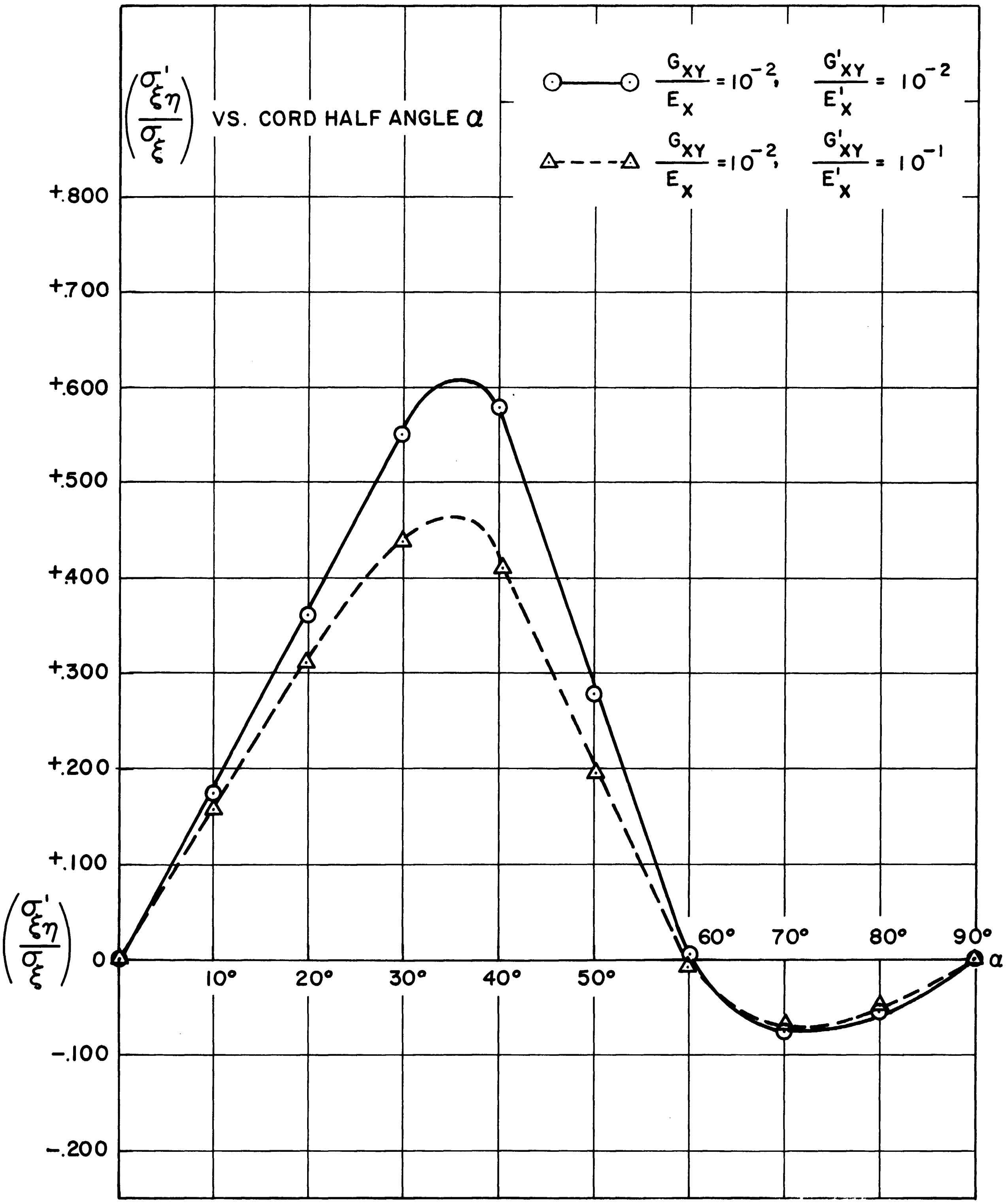

Fig. 10. $\sigma'_{\xi\eta}/\sigma_\xi$ vs. cord half angle α.

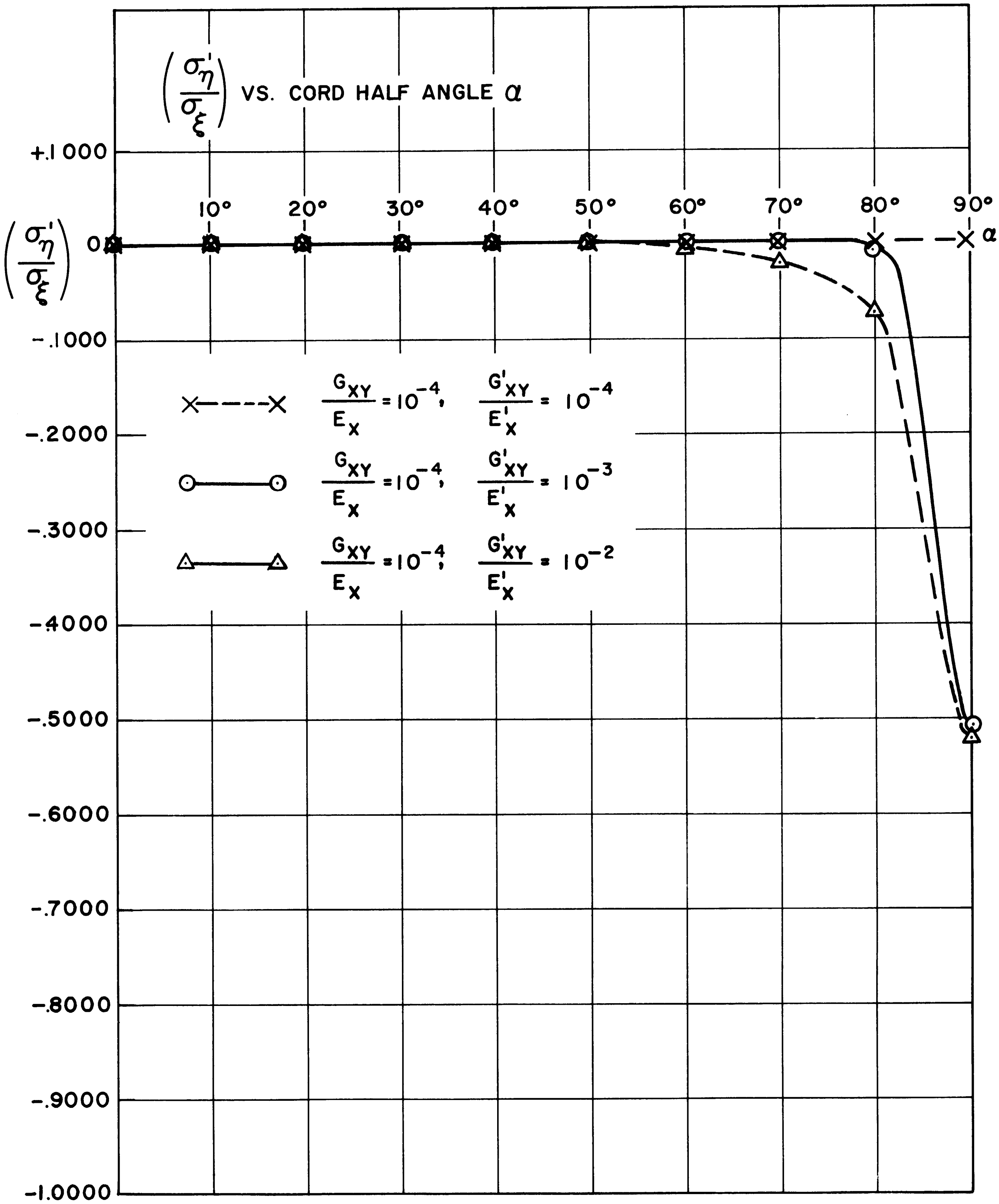

Fig. 11. σ'_η/σ_ξ vs. cord half angle α.

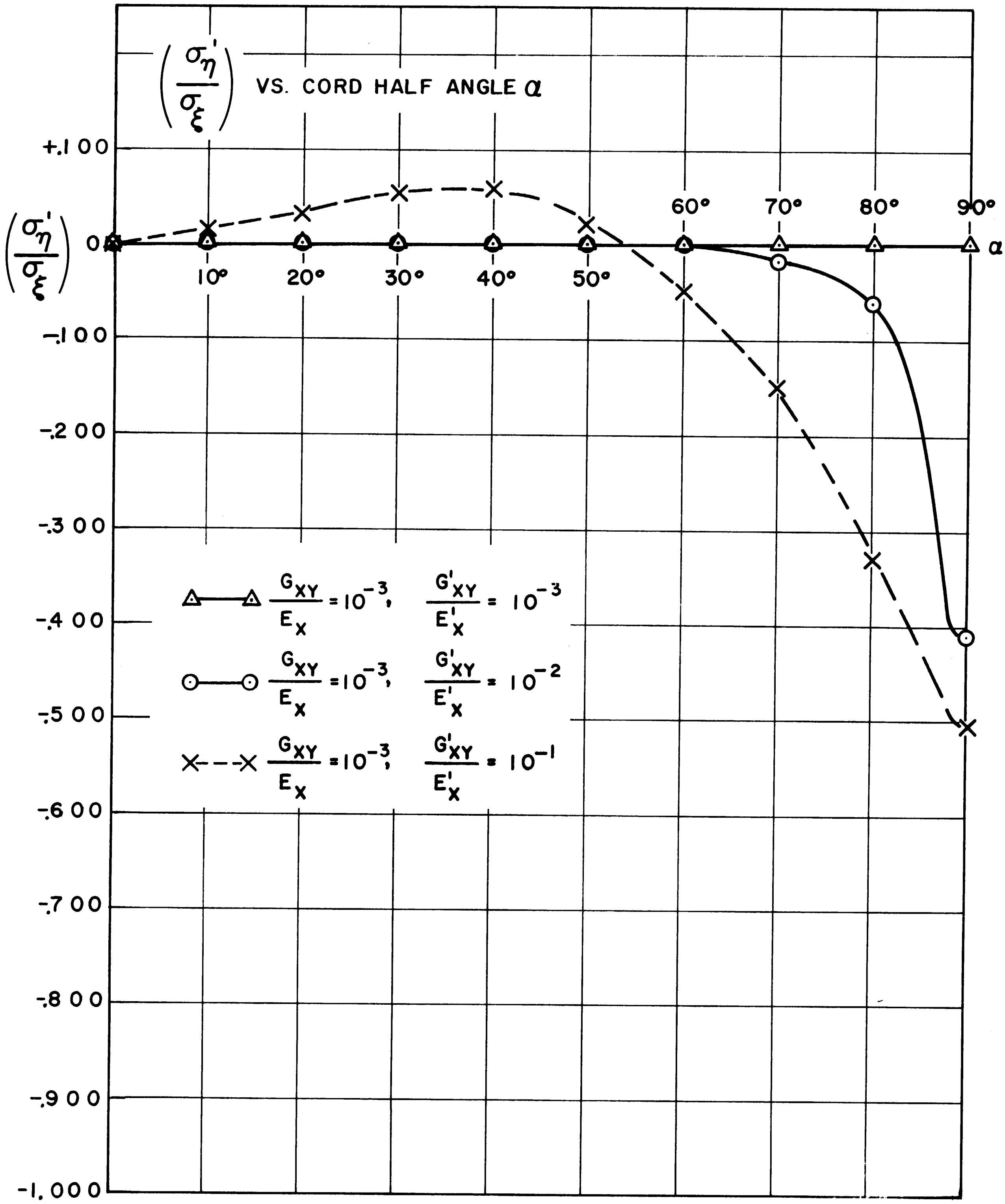

Fig. 12. σ_η'/σ_ξ vs. cord half angle α.

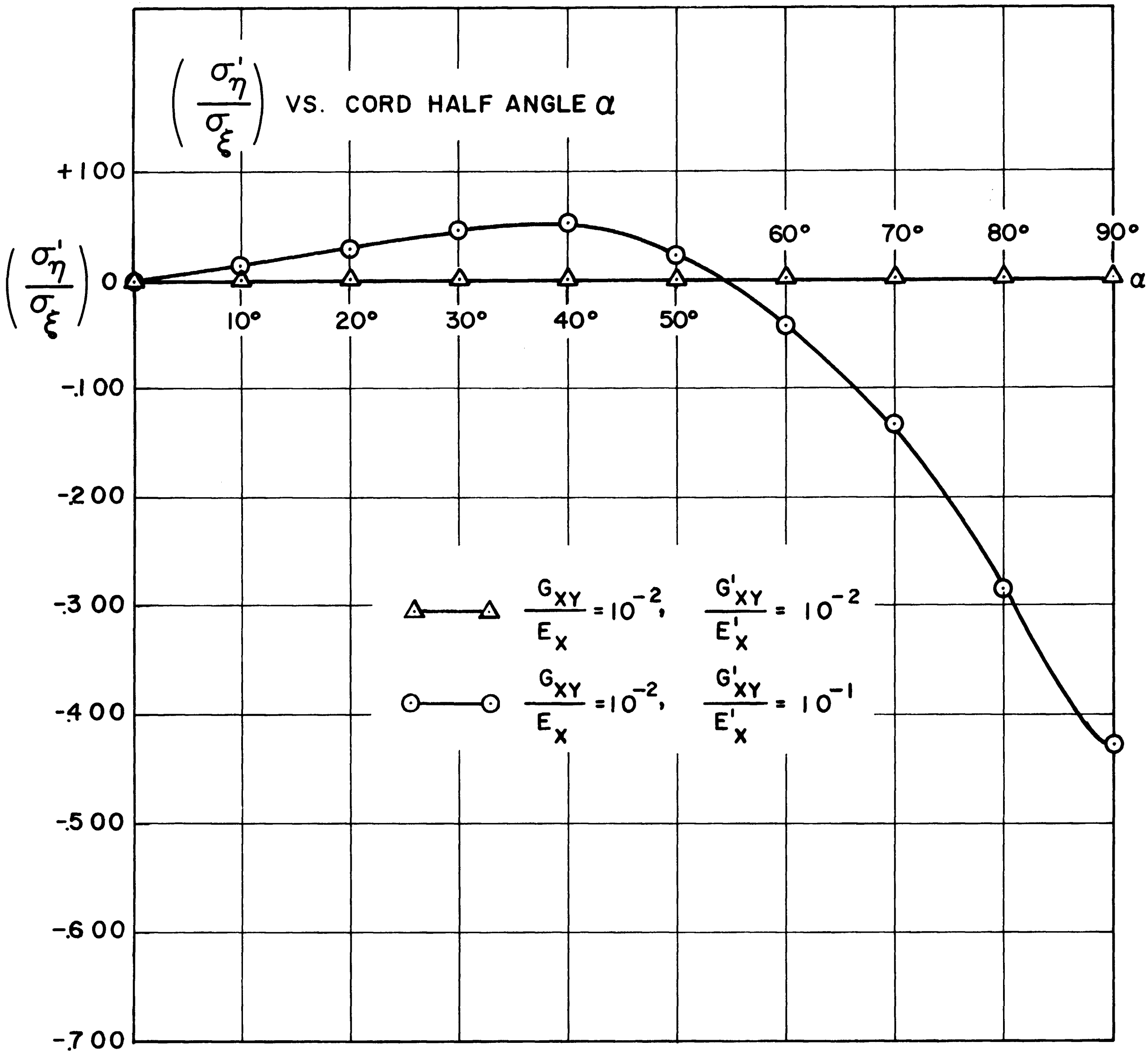

Fig. 13. σ'_η/σ_ξ vs. cord half angle α.

tive differences do exist, but the shapes of the curves generally are the same. From this it may be concluded that the shearing component of interply stress is not affected markedly by the bonding of dissimilar materials, as opposed to similar materials.

Orthotropic materials such as considered in Ref. 1 do not exhibit a normal component of interply stress when subjected to normal external stresses. Anisotropic materials, such as the "Type 2" structure being considered here, do exhibit this and the form of these normal interply components per unit external stress is shown in the figure to be important at very large cord angles but relatively unimportant at small cord angles. It is of considerable interest to note from these plots that it is quite easy to obtain a normal component of interply stress equal to one-half of the applied stress times the ply thickness, at angles between 80° and 90°.

Similar solutions may be obtained when only an external shearing stress $\sigma_{\xi\eta}$ is applied to the bonded dissimilar plies. The results, presented in Figs. 14–16, were again obtained from a computer solution of the equations and are given again for the range of values given in Table I.

From the data on the ratio $\sigma'_{\xi}/\sigma_{\xi\eta}$, it may be seen that this quantity is not very different from its value for an orthotropic, or "Type 1" material as presented in Ref. 1, Figs. 4 and 5. Here a linear scale is used for plotting $\sigma'_{\xi}/\sigma_{\xi\eta}$, and due to its restricted nature it is not possible to show the extremely high values of this interply stress component which can exist at angles close to zero. For a detailed view of them, Ref. 1 should be consulted. The use of the linear scale does give a somewhat more accurate view of this quantity at

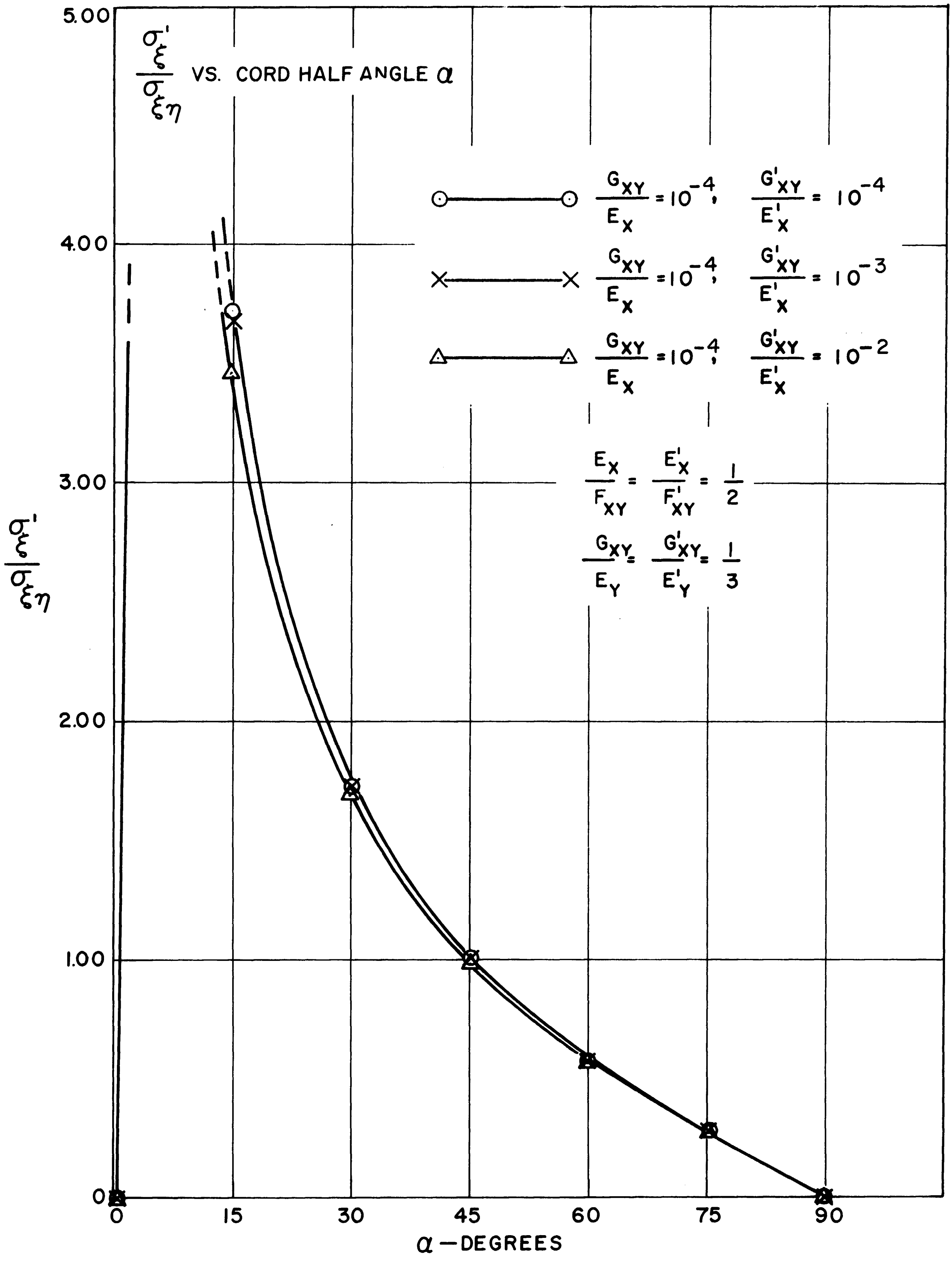

Fig. 14. $\sigma'_\xi/\sigma_{\xi\eta}$ vs. cord half angle α.

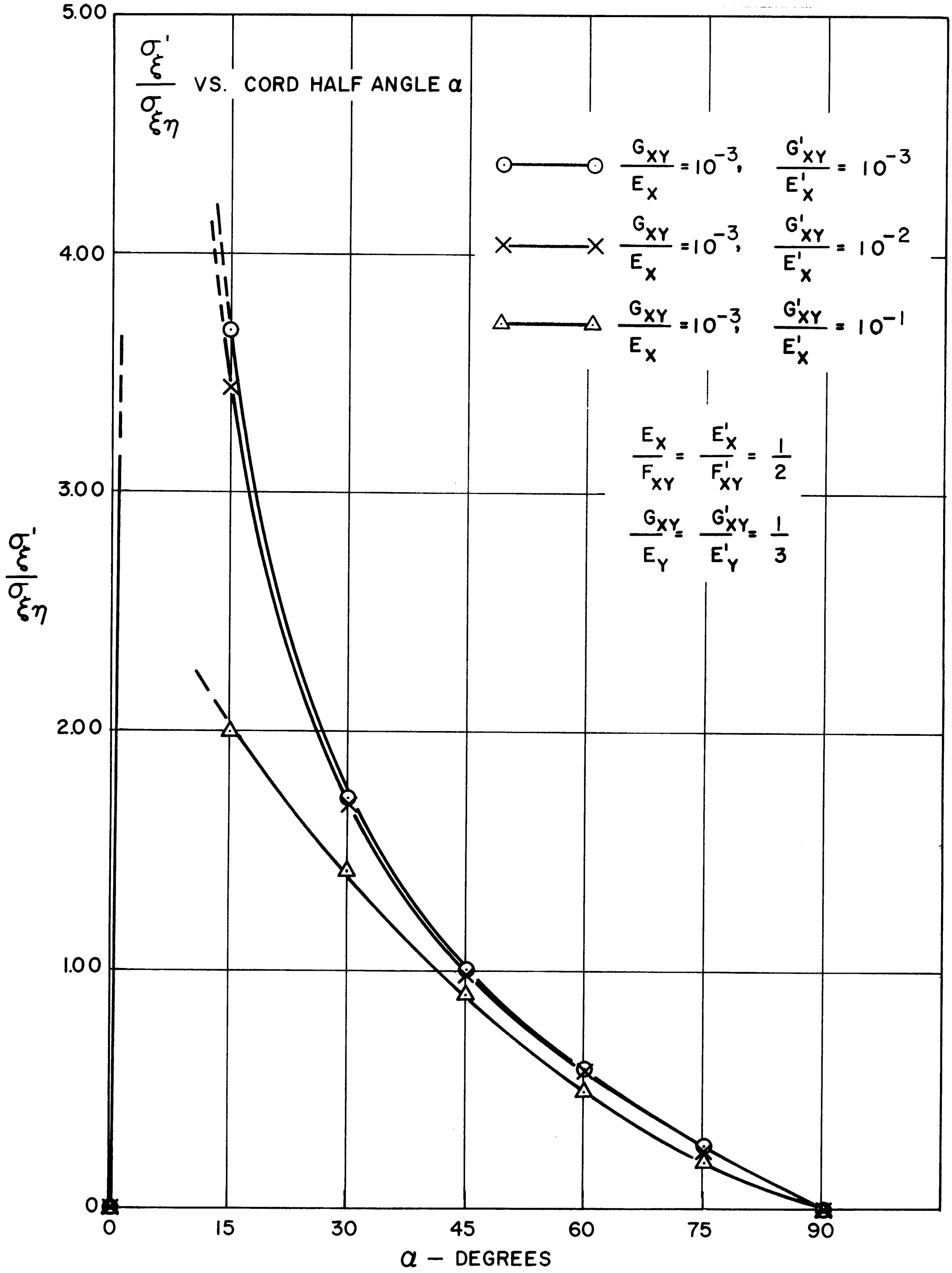

Fig. 15. $\sigma'_{\xi}/\sigma_{\xi\eta}$ vs. cord half angle α.

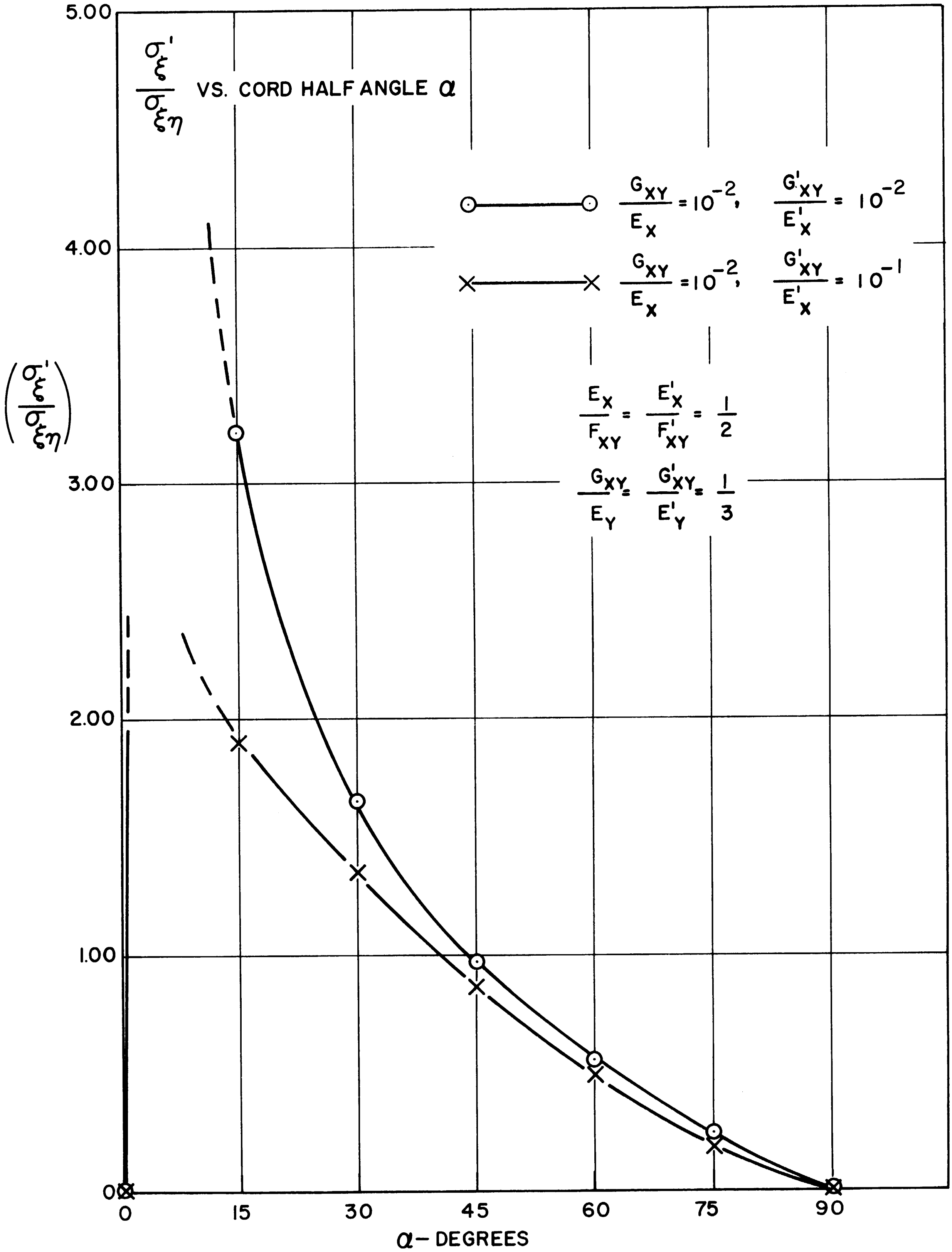

Fig. 16. $\sigma_\xi'/\sigma_{\xi\eta}$ vs. cord half angle α.

moderate cord angles, where most technical interest is centered. In summary, it might be said that, due to the similarity of the results with the orthotropic case, no particular penalty must be paid in the shearing component of interply stresses for bonding dissimilar plies. It is true, however, both here and in the orthotropic case, that higher ply stiffnesses and smaller cord angles result in larger values of this component.

The ratio $\sigma'_\xi/\sigma_{\xi\eta}$ can also be found by a solution of the simultaneous equations just described. If this ratio is plotted, it is seen to be an exact mirror image of Figs. 14—16. This is because $\sigma'_\eta(\alpha) = \sigma'_\xi(\frac{\Pi}{2} - \alpha)$, and so these data will not be presented separately here. They may be obtained directly from those plots.

Further experimental work on elastic properties of single sheets has indicated that the F_{xy} of a single ply may be somewhat smaller than the value originally given it in Ref. 1, due to the high lateral contraction of textile cord. To investigate the influence of this on the dimensionless curves of Figs. 14—16, data for a similar set of curves were constructed using the ratio $E_x/F_{xy} = 0.7$ instead of the value 0.5 presently being used. In all cases the differences were smaller than could be shown by the width of a line on the graph. Hence, it is believed that Figs. 14—16 are valid for a very wide range of cord-rubber materials, certainly for all those now being commercially used.

VI. LOAD DISTRIBUTION BETWEEN PLIES

The load distribution between plies in a two-ply structure subjected to membrane loads, that is, loads in the plane of the sheet should be discussed for completeness. As previously mentioned, "Type 1," or orthotropic, laminates in which both plies have equal stiffnesses distribute loads equally among the plies. Thus, all stresses which are imposed due to external sources are carried equally by the two plies of the structure. This conclusion also can be carried over to four-ply, six-ply, and all multi-ply structures which are orthotropic, provided that each ply is made of the same material and is laid up at the same cord angle. As a matter of practical interest, this may mean that, due to the different rises associated with the different plies in a pneumatic tire, some plies may tend to carry more load than others due to the slightly different cord angles involved. In general, however, it may be said that given the same cord angles in an orthotropic structure made up of identical plies, the loads will distribute such that all plies are equally stressed.

For "Type 2," or anisotropic, laminates, the problem of load distribution is considerably more complicated. The digital computer solutions of the equations governing a two-ply structure made up of plies of unequal stiffnesses have shown what would be generally expected, namely, that the ratio of normal stresses in one ply to the average external stress can be extracted from this set of equations. This quantity is plotted as a function of cord half angle in Figs. 17—19 for those cases where the externally applied stress is normal in nature, covering the range of variables given in Table I. It is seen that

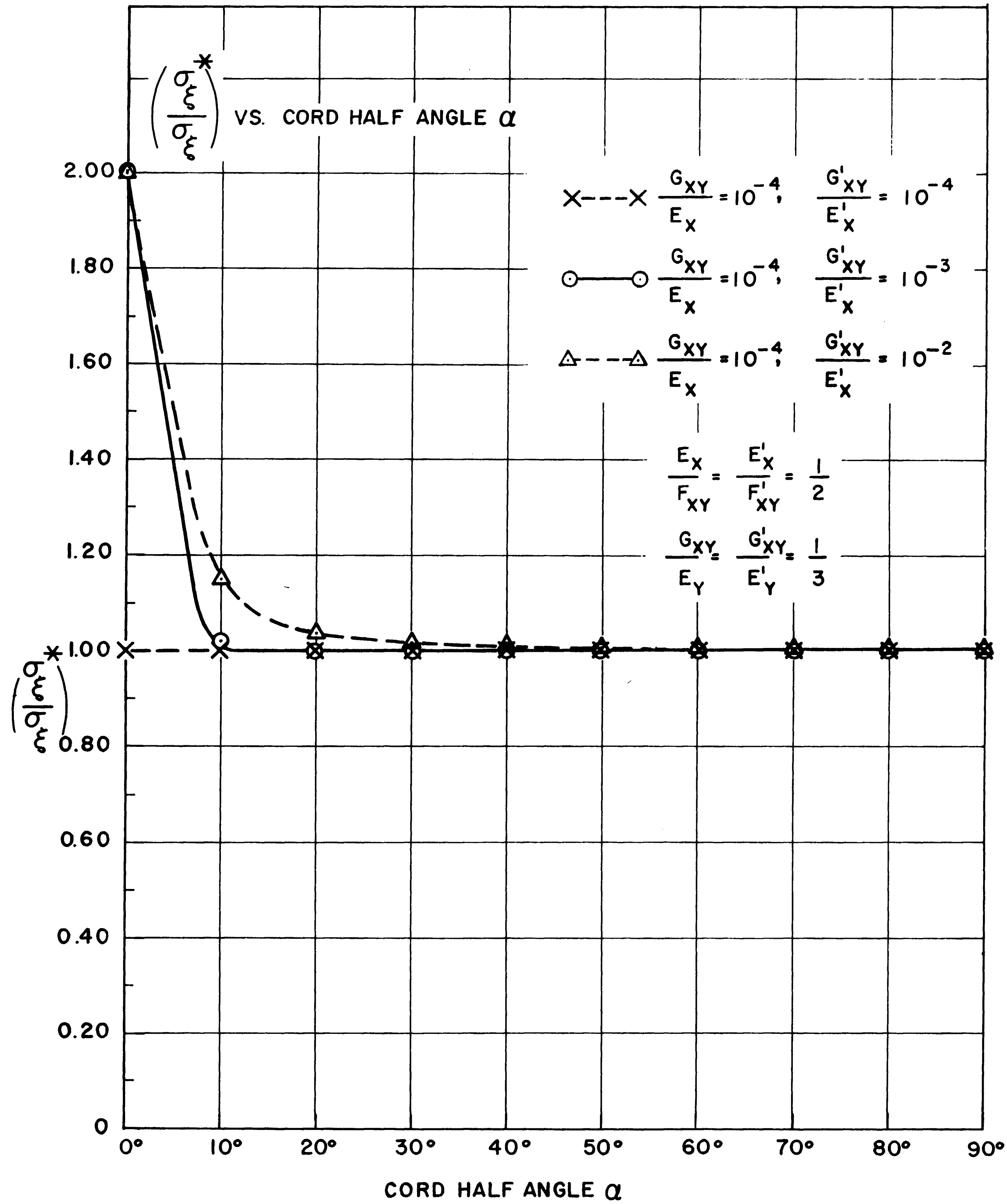

Fig. 17. σ_ξ^*/σ_ξ vs. cord half angle α.

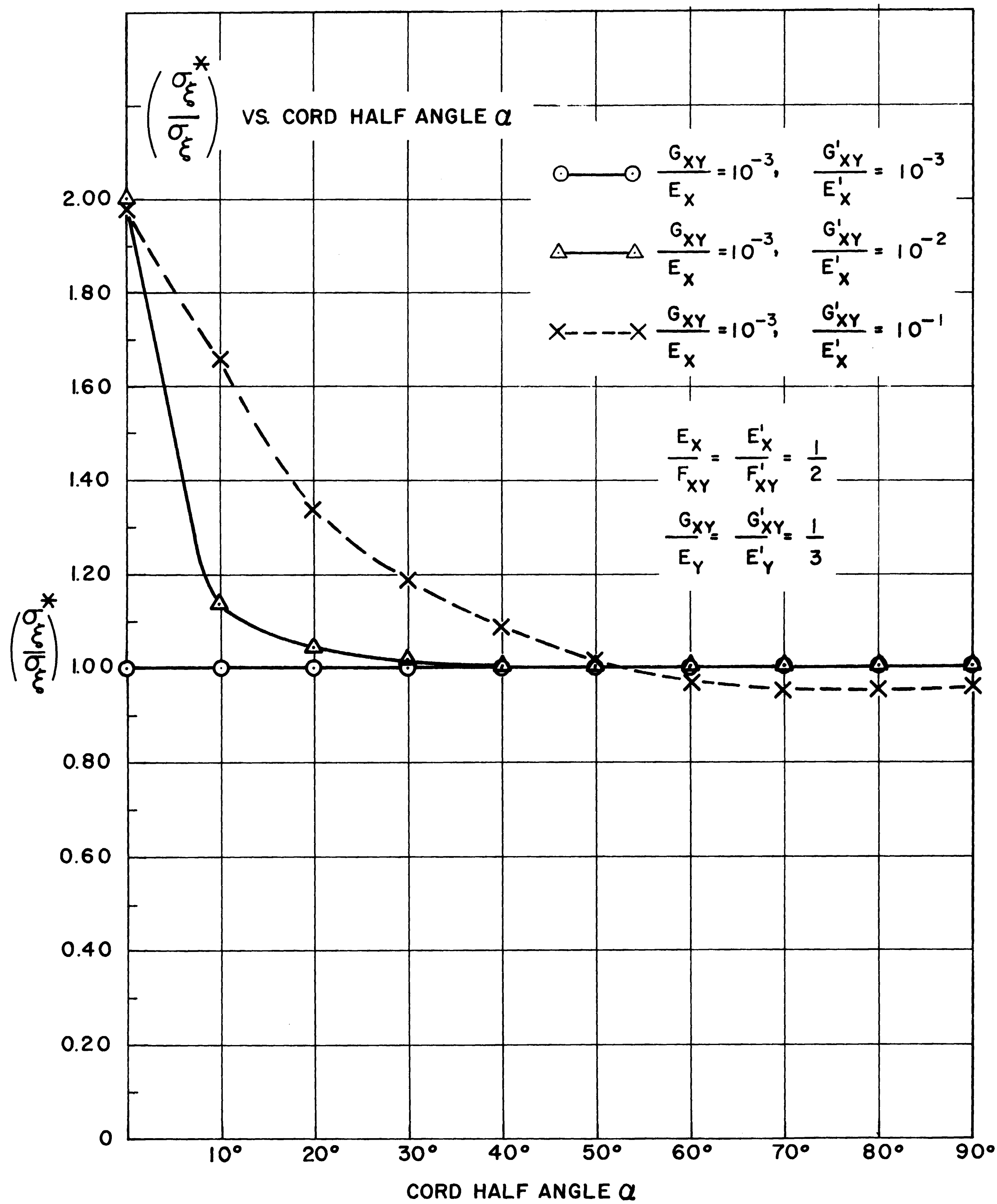

Fig. 18. σ_ξ^*/σ_ξ vs. cord half angle α.

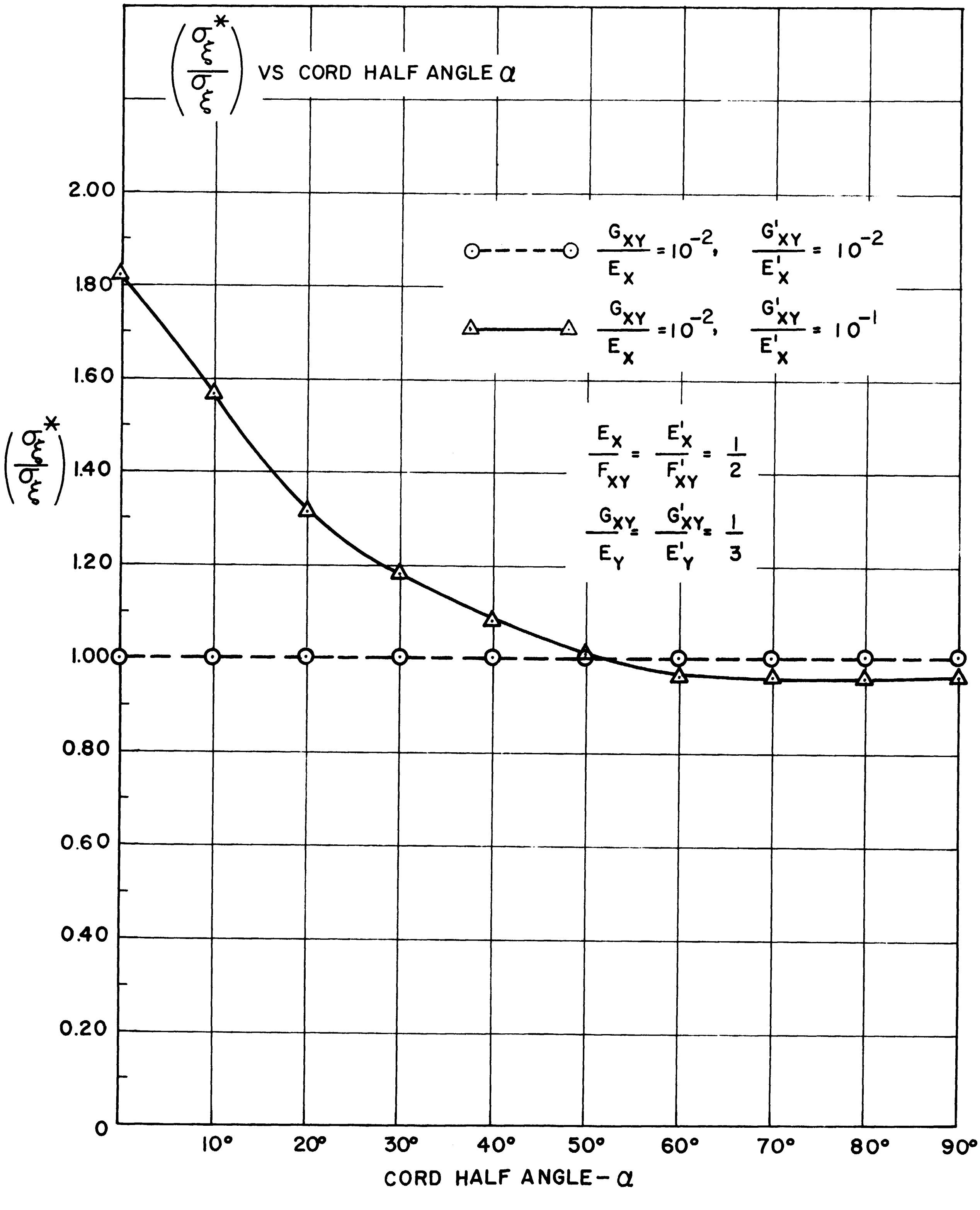

Fig. 19. σ_ξ^*/σ_ξ vs. cord half angle α.

for equal stiffnesses in the two plies equal stressing results, as would be expected. This is merely the "Type 1" structure previously discussed. As the stiffnesses become more and more unequal, an increasing tendency may be observed for a large portion of the load to be carried by one ply and a very small portion by the other ply. This effect is generally much more pronounced at low cord angles where stiffness differences are much greater.

A similar set of curves can be presented for solutions involving the application of external shear stresses only. These are given in Figs. 20—22. Here it may be seen that the bonding of dissimilar plies does not induce extreme variations in load distribution due to shear. The maximum recorded unbalance is about 35%. Maximum unbalances in load distribution between plies all occur at 45° cord half angle, as would be expected from physical considerations.

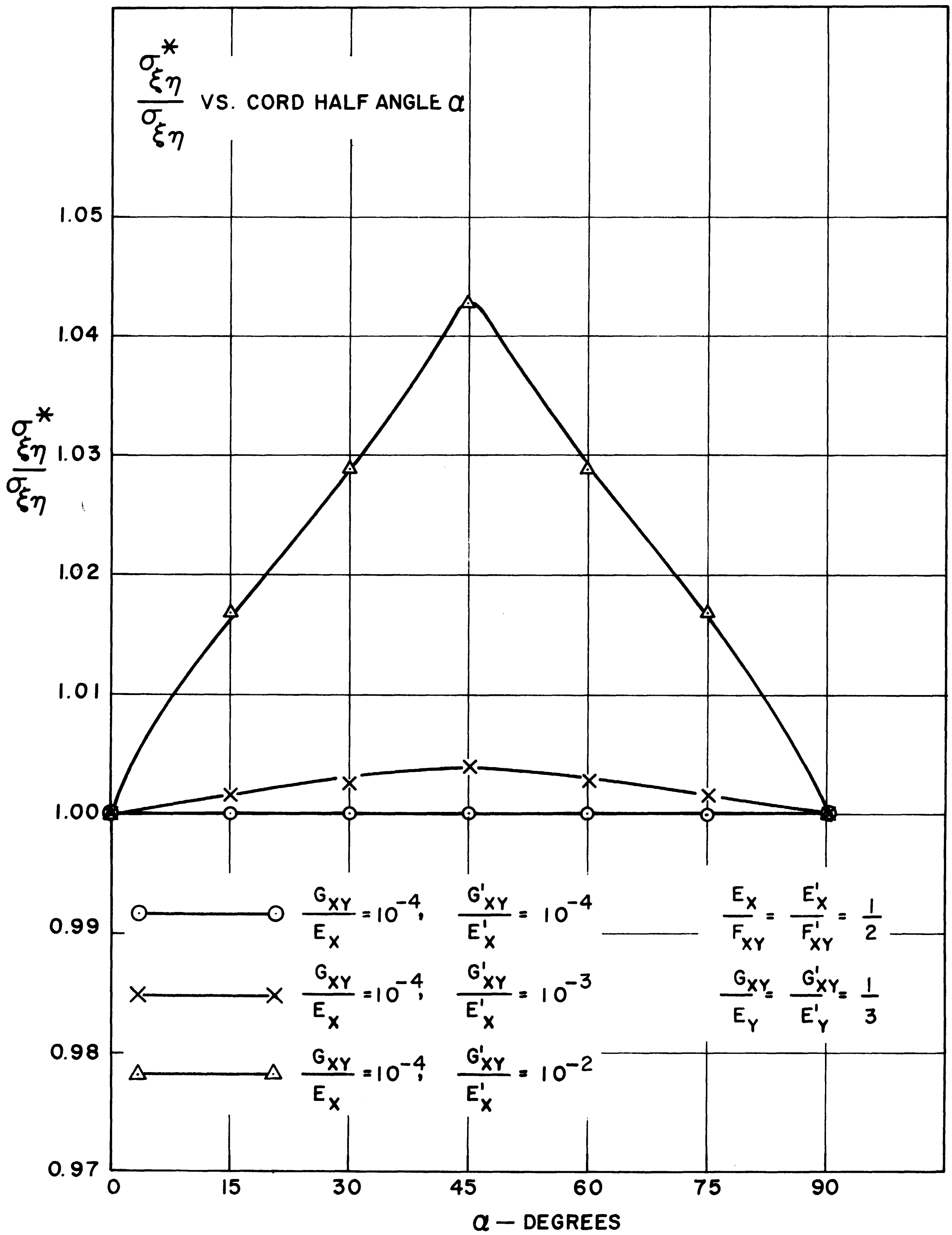

Fig. 20. $\sigma^*_{\xi\eta}/\sigma_{\xi\eta}$ vs. cord half angle α.

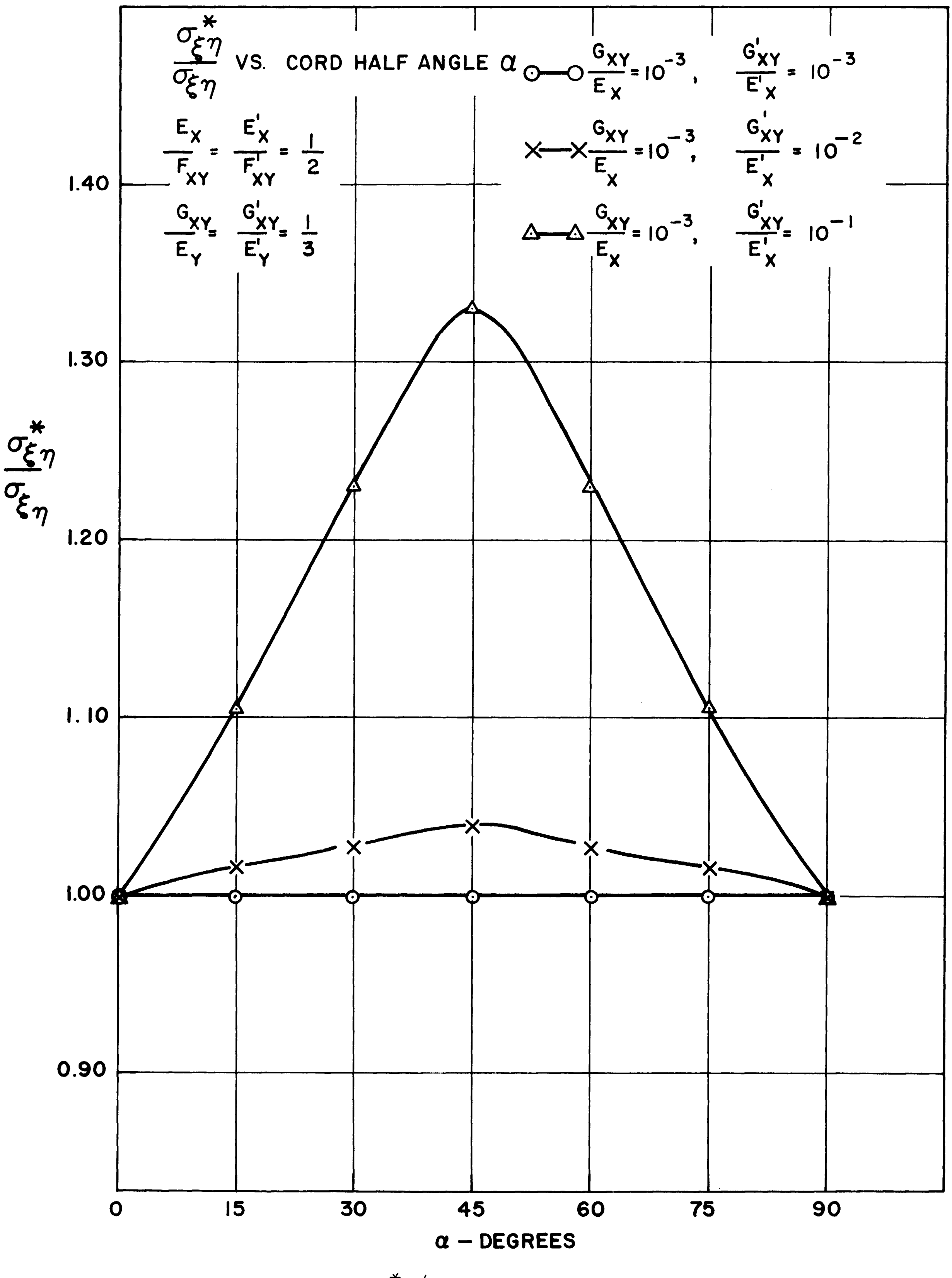

Fig. 21. $\sigma^*_{\xi\eta}/\sigma_{\xi\eta}$ vs. cord half angle α.

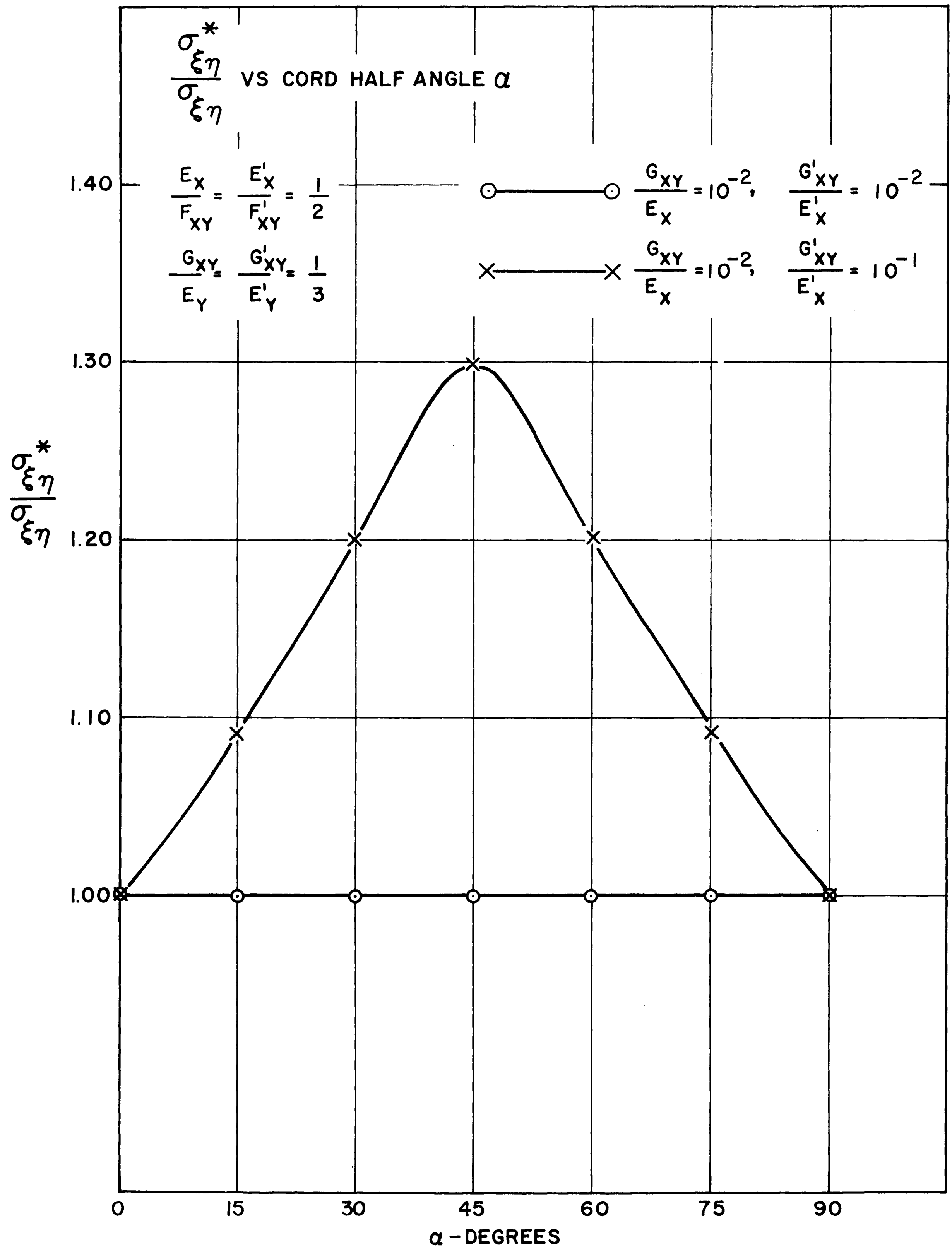

Fig. 22. $\sigma^*_{\xi\eta}/\sigma_{\xi\eta}$ vs. cord half angle α.

VII. EXAMPLE PROBLEM

The example problem given in Ref. 1 is presented again here with the correct solution for the interply stresses, taking into account the thickness of the plies. This solution originally given in Ref. 1 was incorrect. A second example illustrates use of material developed in connection with "Type 2" structures.

EXAMPLE 1

Given: a two-ply, laminated pressure vessel such as shown in Fig. 23, with end plugs so that an internal pressure of 5.0 psi can be carried. In addition, an external torque of 25.0 in. lb is to be carried.

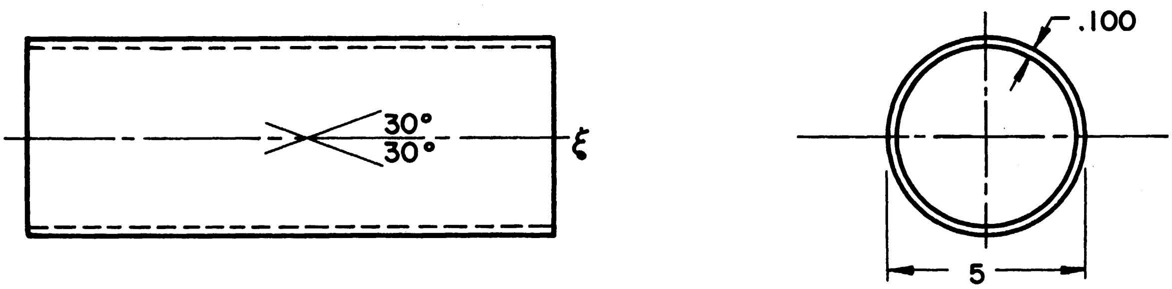

Fig. 23. Two-ply cylinder.

To find: the state of interply stress.

Solution: it will be assumed that Ref. 3 has been consulted and that it has been established that the cords in both plies are in a state of tension.

Following this, the external stresses may be calculated. The ξ-axis is taken arbitrarily to be in the longitudinal direction while the η-axis is in the circumferential direction. Thus, from simple pressure vessel expressions,

$$\sigma_\xi = \frac{5.0x2.5}{2x0.1} = 62.5 \text{ psi}$$

$$\sigma_\eta = \frac{5.0x2.5}{0.1} = 125.0 \text{ psi}$$

$$\sigma_{\xi\eta} = \frac{25.0}{\pi x0.1x5x2.5} = 6.38 \text{ psi}$$

It will also be assumed that the properties of each sheet used to laminate the tube are known and that G_{xy}/E_x is equal to 10^{-3}.

From Figs. 2—5 and Eq. (1),

$$\sigma'_{\xi\eta} = [(0.57)(62.5) + (0)(125.0)](0.05) = 1.78 \text{ psi}$$

$$\sigma'_\xi = [(1.7)(6.38)](0.5) = 0.542 \text{ psi}$$

$$\sigma'_\eta = [(0.58)(6.38)](0.05) = 0.185 \text{ psi}$$

Finally, from Eq. (2), the resultant value of the total interply stress is given by its maximum or minimum values as

$$\sigma'_{\substack{max\\min}} = \left(\frac{.542 + .185}{2}\right) \pm \left[\left(\frac{.185 - .542}{2}\right)^2 + (1.78)^2\right]^{\frac{1}{2}} = .364 \pm 1.79 \qquad (9)$$

$\sigma_{max} = 2.15$ psi $\qquad \sigma_{min} = -1.43$ psi

EXAMPLE 2

Given: a two-ply, laminated tube such as shown in Fig. 24, carrying a torque of 250 in. lb. Each ply is 0.050-in. thick, the cord angle is 45°, and the mean diameter is 5.0 in.

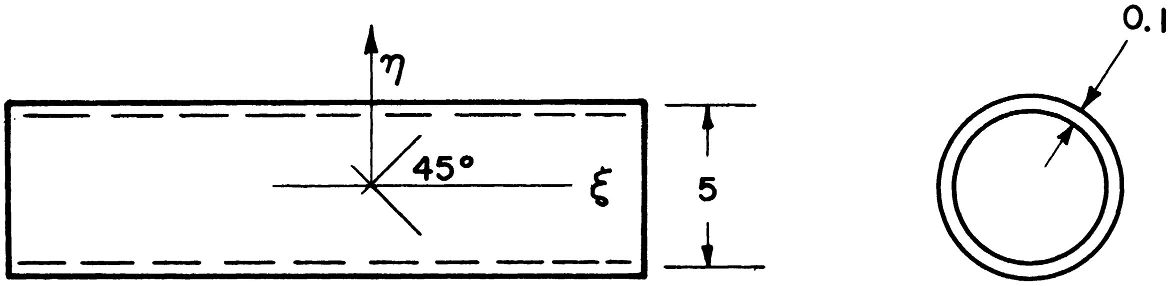

Fig. 24. Two-ply cylinder.

To find: the state of interply stress and the fraction of total load carried by each ply.

Solution: an applied torque such as occurs here tends to put the cords in one ply in a state of tension while the cords in another ply are forced into compression. It will be assumed that the load carried here is sufficient to do this, and that for the ply whose cords are in tension the ratio G_{xy}/E_x is equal to 10^{-3}, while for the ply whose cords are in compression this ratio equals 10^{-2}. Referring to Figs. 15 and 21, one obtains the following results:

$$\text{Basic shear stress} \quad \sigma_{\xi\eta} = \frac{250}{\pi x.1x5x2.5} = 63.8 \text{ psi}$$

$$\text{Interply stress} \quad \sigma'_{\xi} = [(63.8)(0.975)](0.05) = 3.11 \text{ psi from Fig. 15}$$

$$\text{Interply stress} \quad \sigma'_{\eta} = 3.11 \text{ psi from Fig. 15}$$

Maximum interply stress = 3.11 psi from Eq. (2), Ref. 1

Load distribution $\frac{\sigma^{*}_{\xi\eta}}{\sigma_{\xi\eta}}$ = 1.04 from Fig. 21

Hence, $\frac{\sigma^{**}_{\xi\eta}}{\sigma_{\xi\eta}} = (2.0 - 1.04) = 0.96$

VIII. ACKNOWLEDGMENT

The authors would like to acknowledge the assistance of Mr. David Dodge, who performed much of the numerical calculation necessary to obtain the curves presented in this report.

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