ENGINEERING RESEARCH INSTITUTE
THE UNIVERSITY OF MICHIGAN
ANN ARBOR

Scientific Report No. 1

# CONFERENCE

# ON

# HIGH POWER ELECTRICAL IMPULSE TECHNIQUES

January 28, 29, 1957

W. G. Dow
H. C. Early
D. B. Miller

Project 2522

GEOPHYSICS RESEARCH DIRECTORATE
AIR FORCE CAMBRIDGE RESEARCH CENTER
AIR RESEARCH AND DEVELOPMENT COMMAND
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## ABSTRACT

The discussions of the conference which this document records brought out aspects of both acceleration by and storage of electromagnetic energy. The electromagnetic gun for acceleration of masses to high velocities and its inverse as a source of high power pulses were both treated at length. The various means for storing energy and their applications to fast rise time circuits as well as the rapid switching of energy between circuits were also discussed.

## OBJECTIVE

Project 2522 is an investigation on storage of electromagnetic energy and acceleration by electromagnetic forces.

UNIVERSITY OF MICHIGAN

ELECTRICAL ENGINEERING DEPARTMENT

Ann Arbor, Michigan
January 8, 1957

Informal Invitational Conference on High-Power Electrical Impulse Techniques

Dates: Monday and Tuesday, 28 and 29 January, 1957.

Place and Time: Gather initially at 9:30 a.m., Monday, January 28, at Room 3517 East Engineering Building, University of Michigan, Ann Arbor, Michigan.

Subject for Discussion: Methods for producing single electrical impulses at peak powers of the order of one billion watts, duration 1 to 100 microseconds.

Expected Attendance: 15 to 25 participants.

Agenda: As it develops.

Organized by: Gaseous Electronics Laboratory, Electrical Engineering Department, University of Michigan, at the request of Dr. Marcus O'Day, Air Force Cambridge Research Center, Boston, Massachusetts.

Reservations: Those accepting the invitations to attend should immediately advise the undersigned and request room reservations, giving approximate arrival and departure times; rooms will be reserved at the Michigan Union on the University of Michigan campus.

Transportation: The primary Detroit Air Lines Terminal, the Willow Run Airport, is 14 miles from Ann Arbor. Transportation between Ann Arbor and the Airport is by direct taxicab at standard rates.

Ann Arbor is on the main line of the New York Central (Michigan Central Route) between New York and Chicago. There is overnight Pullman service from New York and Boston.

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I. Introduction

A. This document is a report on an informal invitational conference on high-power electrical impulse techniques, held in the Electrical Engineering Department of The University of Michigan on January 28 and 29, 1957. This conference was organized by Professor W. G. Dow and Mr. H. C. Early of The University of Michigan at the request of Dr. Marcus O'Day of the Air Force Cambridge Research Center, Boston, Massachusetts.

The report consists of a reproduction of notes taken at the conference in addition to material submitted by participants after the conference further to document details of their contributions. The report has been prepared by Mr. D. B. Miller of the research staff of The University of Michigan.

B. The announced subject for the conference, as contained in the letter of invitation dated January 8, 1957, and included herewith, concerned methods for producing single electrical impulses at peak powers of the order of one billion watts, duration perhaps 1 to 100 microseconds, with extremely fast rise times.

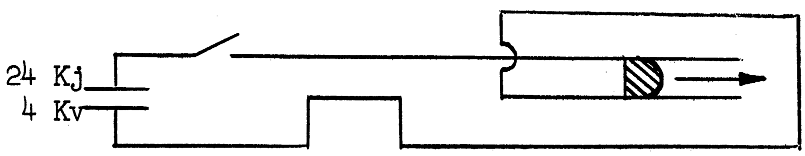
C. Much of the discussion at the conference centered on proposals to use an inverse electromagnetic gun to provide the required impulse power, but other feasible means were also considered.

D. An outline form is employed in the text of the report, to permit quick review and reference. The contents are presented approximately as told to the conferees at the meeting, but with some supplementation in the interest of clarity. When an entry has been submitted subsequent to the conference, an asterisk (*) will follow the name of the participant to whom the statement is attributed.

II. Air Force Cambridge Research Center Report (M. A. Levine reporting)

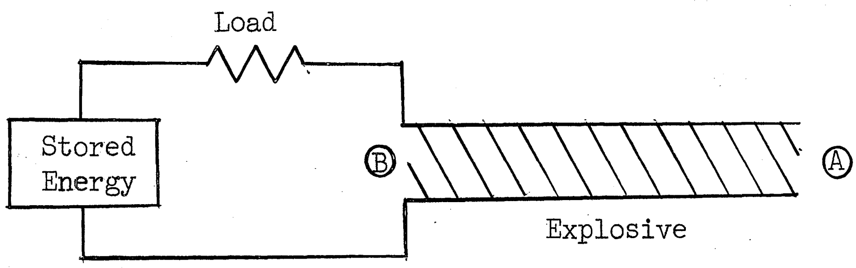
A. Initial work dealt with studies of gases in an electromagnetic field for methods of controlling a thermonuclear reaction. This led to research on the interaction of magnetic fields with various materials which in time led to the electric gun.

B. Rail type of electromagnetic gun:



Interest is in high velocity rather than momentum. 4000 feet per second was the maximum attainable velocity in an experimental gun, perhaps limited by contact difficulties.

C. Suggestion for a hypothetical inverse gun:




1. Begin explosion at A, causing ionization and allowing stored energy source to send current $I_o$ around circuit. As explosion proceeds toward B, the circuit L decreases. Since flux linkage LI remains constant,

$$LI = L_o I_o ,$$

and the decrease in L generates an increase in I. The energy ($1/2\ L I^2$) stored in the field increases. The [electrical] energy amplification factor may be measured as

$$u = \frac{\text{Energy out}}{\text{Energy in}} = \frac{\frac{1}{L}}{\frac{1}{L_o}} = \frac{L_o}{L} .$$

This derivation is for an idealized circuit with all resistance neglected and the exciting energy applied as a step function. (Submitted subsequent to conference.)

2. The limiting velocity of propagation of the explosion wave is 6-8 millimeters per microsecond and limits the rate at which this system reacts.

3. Estimate $H/H_o = 10^6/10^5$ (gauss/gauss)

4. Resistance of an explosion gas between 3 centimeters spaced rail is on the order of 2-10 ohms.

5. The possibility exists that the magnetic field may limit the burning rate (or explosion wave velocity) of the explosion.

6. Experiment confirms that the explosion gas is diamagnetic. Although conductivity in the explosion gas behind the shock front will not decrease appreciably in times involved, because of the dimagnetism

of the gas, the magnetic field will be pushed along in front of the shock wave.

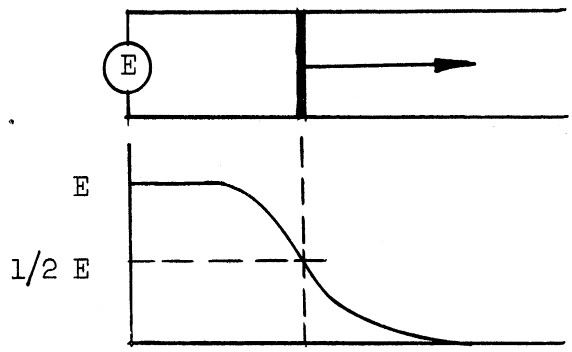
7. There appears to be a difference between this situation and that of a copper slug traveling A→B in that the conductor is moving in the copper case.

8. Discussion

a. Conductivity at 10,000 atmospheres is .1 to .01 that of copper. (E. A. Martin)

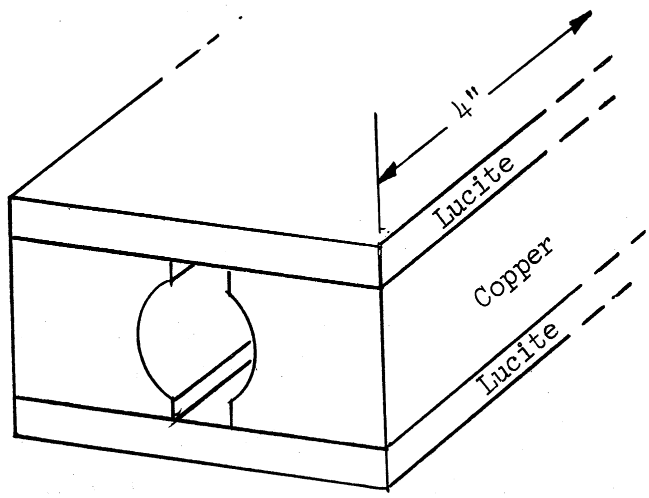
b. Phenomenon of the magnetic field pushed along by the shock wave might be analyzed by the skin depth concept. (W. G. Dow)

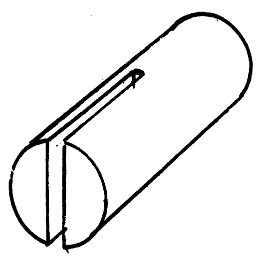
c. Considering a copper crossbar traveling on rails, there is appreciable voltage in front of the bar which might appear as loss in the explosive shock-wave case. (K. W. Miller)




III. Zenith Research Corporation Report (W. W. Salisbury reporting)

A. Early work on a capacitor-rail gun system was discouraging due to collapse of projectile at fairly low velocities.



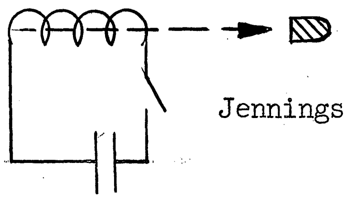
Projectile

1/2 gm (Al or Cu)

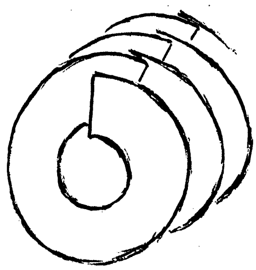
When this device was pulsed at $10^5$ amperes and 4 kilovolts, the projectile collapsed (due to pressure, not melting) when it attained about 200-feet-per-second velocity.

B. Subsequent work was on coil or inductive type of gun.

1. Description




Coil: series of split copper discs, 5 to 6 centimeters long epoxy impregnated glass cloth barrel liner.



Projectile: 3 to 4 centimeters long, 1/2-inch diameter, 2 to 15 grams.

2. Attained about 500 feet per second (measured with ballistic pendulum).

3. Multiple identical coils along same axis.

a. Attained up to 1000 feet per second with three coils.

b. Projectile spends approximately 2 periods in first coil and about 1-1/2 periods in the second.

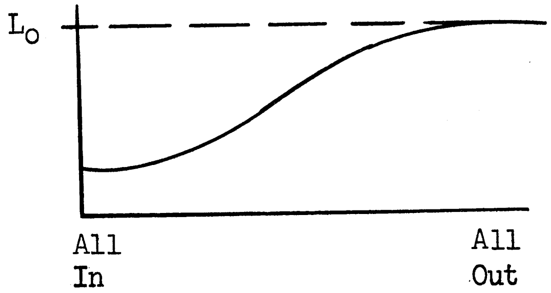
c. Temperature of emerging pellet depends greatly on synchronization, but the ultimate velocity is not as rapid a function of timing.

4. Efficiency = pellet energy/capacitor stored energy.

a. Efficiency increases with pellet diameter and velocity.

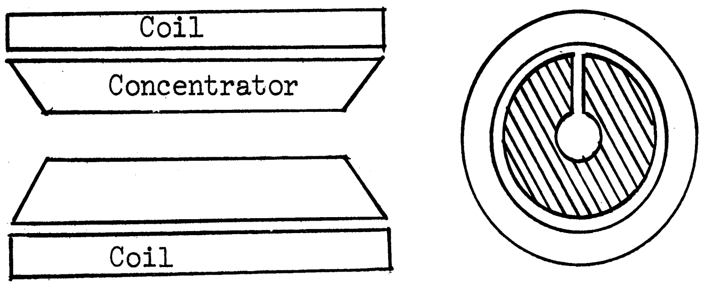
b. Efficiency is low (1 to 2 percent) since the damping of the oscillatory coil current due to the pellet is much less than damping due to circuit resistance.

5. As the pellet moves from completely within the coil to completely out, the coil inductance changes. (Here the pellet is shorter than the coil length.)




Discussion: Cambridge people got a sharp break in this curve. Their pellet length was equal to or greater than coil length. (J. L. Sampson)

6. A concentrator was used to reduce the effect of the end turns.




Field inside concentrator greater than outside due to reduced inside length. Same total current in a long width outside as in a shorter width inside.

7. This gun is essentially a linear induction motor and can therefore operate as a generator. (Induction generator need not be connected to line and load to determine frequency.)

IV. University of Utah Report

A. L. D. Harris' comments*

1. It should be remembered that Mr. Salisbury's objectives and results compared with those of the University of Utah group were very similar. However, our theoretical approach to the problem of accelerating a projectile with a coil or a series of coils differed

*An asterisk following the name of a contributor indicates that the text has been prepared and submitted by him after the conference.

markedly in some aspects. The University of Utah group has attempted to analyze this problem from three different independent points of view.

a. The projectile and the accelerating coil represents a coupled circuit. The mutual inductance between the two circuits decreases as the projectile moves from the coil. Where the projectile is assumed to have self-inductance and not resistance, a very simple expression can be written giving the kinetic energy in the pellet in terms of the energy delivered to the accelerating coil. It is easily demonstrated that the kinetic energy in the projectile is proportional to the mutual inductance between the projectile and the accelerating coil when the projectile is in the starting position. Because of the assumption of no resistance in the projectile, the results of this type of calculation have an unknown accuracy and give no insight into the temperature rise of the pellet due to the electrical energy converted to heat within the pellet. However, this type of analysis yields a good guide to the optimum geometry of the problem and to the timing of the frequency of the sinusoidal current in the accelerating coil compared to the motion of the projectile.

b. We have made an analysis of this problem by writing the differential equation of motion of the projectile where the force on the projectile is written in terms of the radial component of the field produced by the accelerating coil and the current in the pellet. This approach leads to a nonlinear second order differential equation and has been solved for a limited number of numerical situations. It seems that perhaps this equation should be solved for a wide range of variation of parameters by a digital computer.

c. One might think of both the accelerating coil as well as the projectile as consisting of a finite number of circular current paths. A particular circular current path in the accelerating coil is concentric but not in the same plane as a particular circular current path in a projectile. The force of reaction on this pair of circular current paths is rather easily written. The total force of reaction between the two circuits is equal to the sum of all of the separate forces, each due to a pair of current paths. This type of analysis has been initiated but not yet completed.

2. Some discussion took place at the Conference that seemed to imply that the situation of a projectile moving through a series of coils in acceleration was analogous to a linear induction motor. Early in our studies of this problem the University of Utah group adopted this attitude, but later we abandoned this analogy, thinking that it was totally inadequate. In the induction motor, because of the multiplic-

ity of the phases, the magnetic field (constant in magnitude) moves past the rotor conductor at a constant velocity. In the single coil and projectile problem, the magnetic field changes in respect to time but has no apparent motion. The idea of slip frequency comes from induction motor theory. Applying this notion to the coil accelerator problem, Dr. Salisbury concluded that the frequency of the current in the pellet would decrease at later stages in the electromagnetic gun as the velocity of the projectile increases. This would be true if the situation is analogous to a linear induction motor. But at the higher velocity stages in the gun, either or both the axial length of the coil or the frequency in the coil must be increased. We believe that probably but not necessarily the frequency of the current in the pellet will be increased in the high velocity stages of the gun compared to the low velocity stages.

3. The University of Utah group is currently making a preliminary study of the problem of the "inverse electromagnetic gun." As the projectile enters a current carrying coil, its kinetic energy may be extracted. We are considering this problem from two different points of view: (1) as a means of decelerating a projectile, and (2) as a means of generating an electrical pulse at a high level of power.

4. Discussion arising from Dr. Harris' report was concerned with the problem of efficiency of conversion from electrical to kinetic energy in electromagnetic gun, or efficiency from kinetic to electric energy in the inverse gun.

a. Cooling the pellet increases efficiency and perhaps a large advantage would result from making it superconducting when entering the first coil of the gun. (A. Ellett, W. W. Salisbury)

b. A necessary problem which ought to be solved is a theoretical estimate of the maximum efficiency attainable. Best conjecture is that 100 percent is theoretically attainable. (W. G. Dow)

c. One can define another type of efficiency which becomes important in considering maximum velocities attainable:

$$e = \frac{\text{projectile kinetic energy}}{\text{projectile heat energy}} \quad \text{(W. W. Salisbury)}$$

B. W. S. Partridge's comments.* Explosives in contradistinction to propellants used in guns can be used to fire projectiles at very high velocities. Unfortunately, the very high energies encountered in the detonation wave associated with the detonating explosive have a tendency to break the projectile being fired into small fragments. Under some conditions this tendency does not interfere with the experiment, such as when shaped charges are used to penetrate metal targets. In this case the collapsing cone forms an elongated jet which is composed of small fragments, each of which contributes to the penetration of the target. The disadvantage here is that it is impossible to separate the effect of one particular fragment in the jet.

Pugh was able to project a single fragment by means of explosives using a modified-shaped charge technique in which the pellet was shaped like a very shallow but thick-walled cone with the cone angle adjusted in such a way that the detonation wave flattened it into one single plate-like fragment. Rinehart, Allen, and White[1] at the U. S. Naval Ordnance Test Station in California extended this technique and attained velocities of the order of 5 kilometers/second for some pellets. Van Valkenburg[2] at the University of Utah modified this method by placing an oil layer between the explosive and the pellet. This gave better control of velocity and also decreased the tendency of the pellet to break up. Recently, Cook at the University of Utah has attained velocities of 7 to 7.2 kilometers/second for single 5-gram diamond-shaped pellets by shaping the detonation wave before it strikes the pellet. Using this technique, he has been able to obtain pellet velocities which were nearly equal to the detonation-wave velocity in the explosive. Without shaping the detonation wave before it strikes the pellet, one usually expects to obtain the pellet velocity of approximately twice the value of the particle velocity of the material behind the detonation wave. The particle velocity is related to the detonation velocity in an explosive in the following manner.

$$W = \frac{D}{4}$$

where $W$ = Particle velocity
$D$ = Detonation velocity.

Listed below is a table giving the detonation velocities of several common explosive materials.

| Explosive | Detonation Velocity |
|---|---|
| Composition C-4 | 8200 meters/sec |
| Composition B | 7800 meters/sec |
| 50/50 Pentolite | 7400 meters/sec |
| TNT | 6900 meters/sec |
| 50/50 Amatol | 6500 meters/sec |
| 65/35 Baratol | 5600 meters/sec |
| Ammonium Nitrate | 1700 meters/sec |

The use of explosives for propelling pellets has several disadvantages, one of which is the fact that it is very hard to aim a pellet being fired in this

1. Rinehart, J. S., Allen, W. A., and White, W. C., J. Appl. Phys. 23, (1952), 132, 198, 297.

2. Van Valkenburg, M. E., and Hendricks, C. D., J. Appl. Phys. 26 (1955), 776.

manner accurately due to nonhomogeneity of the explosive and variations in density. It is also necessary to keep all test equipment and other instrumentation at considerable distance from the point of detonation; otherwise they will be destroyed. By the nature of the energy release from an explosive, it must needs be an inefficient process when trying to transfer this energy into a fast-moving pellet so that the efficiency of energy conversion is extremely low. In considering the possibility of propelling a mass at a high velocity by explosive means and transferring that energy with an inverse gun system to another mass, I think one would finally conclude that the use of propellants in a conventional gun would be much more attractive. Under these conditions a large mass traveling down a barrel with coils spaced around it would be essentially a linear generator. It might prove to be easier to use a flywheel with its attendant weaknesses rather than try to overcome the problem of tolerance and spacing in such a system. Both the flywheel and the inverse gun have this in common, that the energy would be derived from a mass moving at a high speed and would then have to be transformed into electrical energy.

V. Armour Research Foundation Report (K. W. Miller* reporting)

A. Based on reports and material captured from a German research laboratory, original research on the electric gun, about ten years ago, was to confirm and extend work done by the Germans. Briefly, the Germans had early discarded solenoid types of electric guns in favor of the rail type. They had also discarded iron in the gun itself and had settled upon batteries or homopolar generators as most likely power sources. Captured equipment included a device which might have been intended as an inverse rail-type gun wrapped around a cylindrical flywheel. However, this device was incomplete and the method of operation could not be reconstructed in the absence of any explanatory papers, if indeed this device was actually intended to be an inverse gun type of power source.

The Germans had obtained muzzle velocities up to over 1000 meters/second on bullet-shaped projectiles of a few grams.

Work at Armour Research Foundation consisted, first, in analyzing and confirming German work and, secondly, in attempting to design an electric gun to accelerate anti-aircraft shells of approximately 14 pounds mass to velocities of the order of 2000 meters/second. Calculations indicated that the impulse energy required for even one shot was enormous, and for sustained bursts of fire from one or more anti-aircraft batteries electric power requirements become fantastic. On the basis of this study, which indicated impracticality of the electric gun as a large caliber weapon, Ordnance terminated the contract and all the German equipment was destroyed under bond - the project being classified at the time. Experimental work was limited to a brief program on storage-battery power sources.

The most important portions of the work were discussed in Technical Report

No. 3, dated 5-1-47, entitled "Electric Gun and Power Source," Project 15-391E, to the Ordnance Department, U. S. Army Service Forces, under contract W11-0220RD-11164.

Of greatest present interest in this report are the sections on electrical-gun equations, design of full-sized (14.3 pound projectile) and one-pound guns, and storage batteries and homopolar generators as pulse power sources.

Recent work by the Foundation under Contract AF 18(603)-56 with Air Research and Development Command, by contract stipulation, has been limited primarily to a theoretical study of rail-type guns and power sources. Based on this work, it appears that maximum attainable velocity is limited by melting of the projectile or crossbar whether or not the gun is of solenoid type or of rail type. In the latter case, velocity is not limited to some still lesser value by sliding electrical contact difficulties. In either case, a long "slenderness ratio" (length/diameter) of the pellet is desirable, permitting maximum current through the pellet before melting. Since energy of the pellet is proportional to $I^2t$, the maximum velocity of the projectile for given mass can be made greater as the slenderness ratio is increased. With ultimate velocity limited by melting of the pellet or crossbar "pusher," analysis has indicated that ultimately a muzzle velocity can be substantially increased by artificial cooling, specifically by ice. If the conducting material of the pellet or pusher is given a honeycomb structure intimately mixed with ice so that use is made of the high specific heat capacity and large latent heats of fusion and vaporization of ice, then the effective thermal capacity of the conductor can be increased several fold with only a 100 percent increase in mass due to the ice. If properly designed, the ice mass will be exhausted by the time the muzzle is reached whence, approximately, the effective mass is only increased 50 percent for the entire path length of acceleration. Only a slight further gain may be realized by directing the generated steam rearward as a jet to accelerate the pellet additionally.

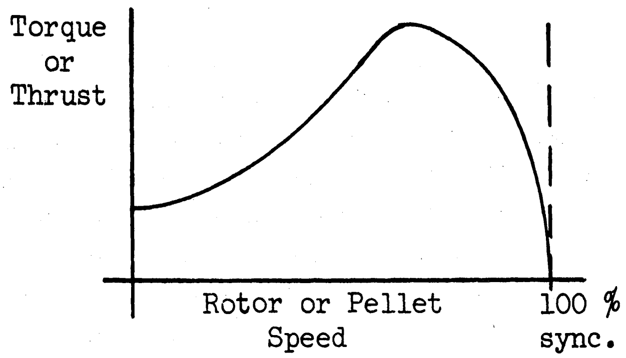
The theoretically attainable peak velocity, using a copper pellet with $H_2O$ (ice) cooling and reasonably high slenderness ratio, is about $10^5$ feet per second. Skin effect will tend to concentrate the current along the inner rail surfaces, thereby raising the rail temperature.

$$\begin{aligned} h &= \text{"effective" skin depth} \\ &= h(t) = 11.6\sqrt{t\,(\text{sec})}\ \text{cm} \\ &= .116\ \text{mm in 1}\ \mu\text{sec.} \end{aligned}$$

Magnetic flux due to the main current will have components perpendicular to the direction of the motion of the projectile, and so "parasitic currents" will be induced, circulating between rail and pellet at each rail contact, causing some heating. This effect will be decreased by increasing the slenderness ratio of the pellets.

B. Discussion

1. Melting of the projectile is also the eventual limiting factor in an induction-type gun. For an induction motor (or linear induction gun), the following curve is valid:




If the pellet speed stays on the thrust-speed curve peak, the heat and kinetic energies of the pellet will be equal. Beyond the peak toward 100-percent synchronous, kinetic energy is greater than pellet heating. (W. W. Salisbury)

2. With regard to Dr. Salisbury's analogy of the pellet and induction-motor characteristics, it might be pointed out that motor characteristics are derived for a particular set of conditions which are not at all similar to those which exist in the solenoid gun. For one thing, the solenoid gun is a variable frequency device. Another important factor is that induction-motor torque is steady, even though currents are continually reversing, which is not at all true of the gun. As a major objection, it might be pointed out that the gun is essentially a repulsion device and it will operate satisfactorily if coils along the path of the pellet are energized as the pellet passes through. This means that it can operate as a zero slip device. It is well known and easily demonstrable that an induction motor will have no torque at zero slip. (H. L. Garbarino*)

3. During the discussion two points arose which, the conference participants agreed, needed further analysis:

a. Formulation of force on a pellet being accelerated in an induction-type gun. Possible expressions are: that the force equals either the difference in $(H^2/8\pi)$ between front and back projectile, or the derivative $d(H^2/8\pi) \sim H\Delta H$.

b. Determination of whether or not the inductive-type gun could attain a higher theoretical maximum pellet velocity than could the conductive or rail-type gun.

VI. Harvard University Report (R. W. Waniek* reporting). The project originated at Harvard out of the need of establishing fields in excess of 100,000 gauss to impart a curvature to atomic particles traversing nuclear track emulsions. The magnets developed for this purpose would consist of stacked disks of beryllium copper with a central hole and insulation between turns. Alternatively spirals of the same material have also been used. With geometries of this type, fields up to 750 kilogauss could be reached. The coils were operated at room temperature or at liquid nitrogen temperature to improve their electrical and mechanical properties. With the addition of pulse transformers (iron and air core), the pulse length could be varied between 50 and 10,000 microseconds according to the experimental requirements. The source was a capacitor bank of 8000 joules operated at 3 kilovolts. Detailed considerations about the features of such magnets and their mode of operation are given in an article published in the *Review of Scientific Instruments*, *27*, (1956), 195.

Studies at even higher flux densities were carried out at the A.F.C.R.C. in cooperation with M. A. Levine. With a 24,000-joule capacitor bank, fields up to 1.6 megagauss were obtained in single turns of hardened beryllium copper. These fields were achieved in discharge times of about 20 microseconds. The energy density of the magnetic field exceeded by far the intrinsic strength of the material. Magnetic saw effects and cold flow of the material would set in during these extremely short periods of time. To avoid the disruptive effects caused by these very high currents, the idea of force-free configurations was put to practice and is currently being experimented upon. The results obtained during this second phase of experimentation will be published shortly in *Rev. Sci. Inst.*

VII. Tufts University Report (L. S. Combes reporting)

A. Summary of the work on the pinch effect at Tufts University[3] (submitted subsequent to conference)

1. The pinch effect may be defined as the constriction of a compressible conductor, such as an ionized gas, by the self-magnetic field of a current in that conductor. Assuming an energy balance between the ionized gas (nkT) and the magnetic field ($H^2/8\pi$), and neglecting the electric field in the gas, elementary theory leads to the following relationship:

$$a = (25/I)^{3/2} \, r^{5/2} \, (n_0 k T_0)^{3/4} \qquad (1)$$

where a is the radius of the constricted plasma (cm),

3. This work was sponsored by the Geophysics Research Directorate of the Air Force Cambridge Research Center, Air Research and Development Command, under Contract No. AF 19(122)-89.

$I$ = the current (amperes),
$r$ = the radius of the discharge tube (cm),
$n_o$ = the initial ion density (ions/$cm^3$), and
$T_o$ = the initial temperature (°K).

Equation (1) is confirmed by experimental evidence when the current is not over 5,000 amperes but may be in error by as much as 70 percent for currents up to 20,000 amperes.

Pinch phenomena were studied in various toroidal-shaped glass discharge tubes. The ionized gas or plasma in the glass toroidal discharge tube is the secondary of a transformer-like circuit with primary windings on transformer cores. Later the primary windings were formed by plating several turns of copper coaxially on the outside of the glass toroid itself.

Spectrographic evidence for the pinch effect was found. A quartz lens was used to cast a real image of the toroidal current across the slit of a grating spectrograph in such a way that spectral lines on the film will be short if the current is pinched but long if unpinched. With nitrogen in the toroid, the lines due to doubly ionized nitrogen atoms ($N^{++}$) showed the greatest pinching, while those due to unionized nitrogen molecules ($N_2$) were unpinched. Two possible explanations are: (a) high-energy electrons concentrated in the center of the tube produce high levels of ionization and excitation; (b) positive ions are actually concentrated at the center by the action of the self-magnetic field of the current.

A photomultiplier tube was used to measure the relative intensities of various spectral lines in the nitrogen spectrum and also the variation in intensity along each line. These data showed that the ionized atoms were in the pinched portions of the plasma and also that the ratio of the intensity of the $N^+$ to the intensity of the $N^{++}$ lines was fairly constant across the pinch, indicating a fairly constant temperature in that region.

In various toroids used, instabilities in the pinch have been evident. Two types of instability are possible, kink and sausage-type instability. It was found that copper plating on the outside of the glass toroid helped to stabilize the pinch and prevent the discharge from striking the walls of the toroid.

An extremely promising method of preventing or decreasing both types of instability in the pinch appears to be the inclusion of a longitudinal magnetic field inside the conducting plasma, thus forming a hollow or H-centered pinch.

Recently, a pulsed image-converter tube has been used to photograph the pinched current at any instant during the discharge cycle. A series of these pictures definitely shows that the discharge starts at the walls of the toroid and then constricts, remaining in the form of a hollow cylinder, until it finally merges into a solid axial filament. Instabilities then develop.

2. Items pertinent to pinch effect brought out in discussion:

a. Discharge-tube pressure is about 100 μ Hg (L. S. Combes)

b. Approximately a 10:1 ratio exists between pinched and unpinched current densities. (L. S. Combes)

c. Langmuir-probe technique can be used to measure electron temperature of the discharge. This, coupled with spectroscopically measured ion temperature, will indicate whether or not thermal equilibrium exists. (W. G. Dow)

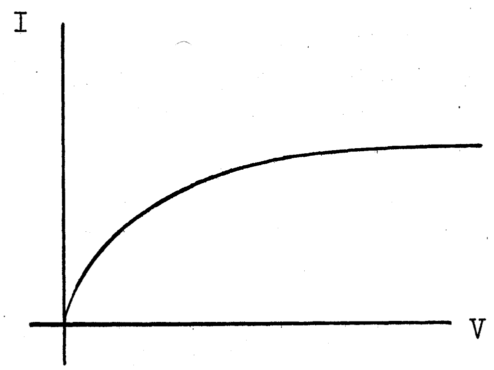
d. A problem exists in further increasing the temperature of the pinched discharge since it is very difficult to match the low impedance of the low-pressure discharge to an external energy source. (M. A. Levine and W. G. Dow)

e. Use of the stabilizing longitudinal field increases the pinch diameter and lowers the internal temperature. (M. A. Levine)

f. The Tufts group is, in the near future, bringing out a detailed report on its pinch effect work. (M. D. O'Day)

g. A 4- to 5-kilovolt discharge across a one-inch gap in an unpublished Tullahoma experiment appeared to be critically damped. This would only be possible if the discharge had a high impedance, perhaps from some instability. (H. C. Early)

h. Current-voltage curve from the condensor bank into the Tufts toroid. (M. A. Levine)



B. Energy storage in a rotating mechanical device has an advantage in being more controllable than a gun.

Discussion:

1. Means of getting energy out of a flywheel: (M. D. O'Day, W. G. Dow)

a. By slowing wheel down. Retarding force must be applied uniformly around entire wheel.

b. By firing a projectile from the rim.

c. By running the periphery of the wheel in curved tracks as an inverse rail gun.

2. Disadvantages of flywheel type storage (M. D. O'Day, H. C. Early)

a. Ultimate material strength of flywheel limits its energy storage to less than can be stored in a projectile.

b. Large weight and gyroscopic effect are poor for aircraft applications.

c. Development of flywheel would mean loss of time already spent on the electric gun.

C. Another possibility for creating high-energy electrical pulses might be to separate mechanically the plates of a charged capacitor, thereby increasing the stored energy.

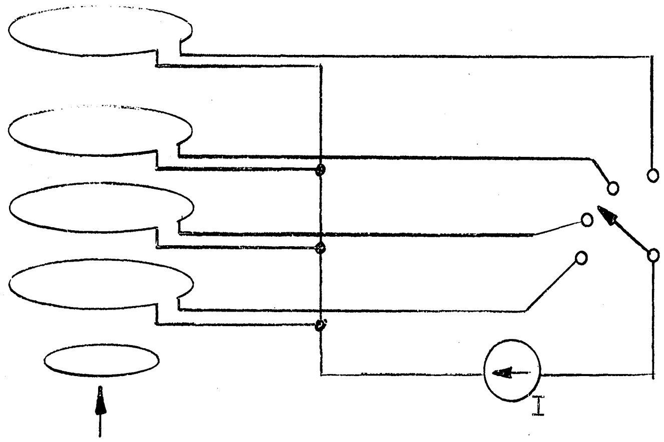
Discussion:

1. Explosive might be used as the capacitor dielectric to provide very fast separation of charged plates and consequent fast reduction of capacitance. (H. L. Garbarino)

2. Energy storage per unit volume is much more easily accomplished in a magnetic than in an electric field. For instance, considering 20,000 gauss as a high but handleable field, it is equivalent energy storage-wise to an electric field of $6 \times 10^6$ volts/centimeter. (M. D. O'Day, W. G. Dow, M. A. Levine)

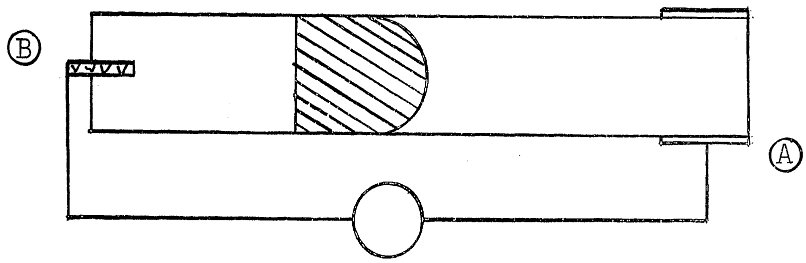
VIII. The University of Michigan Report (W. G. Dow, H. C. Early, and R. C. Walker reporting)

A. Suggestion for a solenoid-type induction gun:




This gun would consist of several loops connected in parallel with a projectile made of a single conductive ring. The ring is fired into the solonoid, and current is sent through the first coil after the ring has passed through the plane of this coil. As the ring approaches the second coil, current is switched from the first to second coil accelerating the ring. Switching the current in step with the ring can therefore give the ring an overall upward acceleration. The difficulty in the above circuit due to having to open a current carrying circuit may be overcome by making rather than breaking current in successive coils and accelerating the ring by repulsion from previous coil rather than attraction as in the first case. This, however, is magnetically unstable. The eventual limit to both systems is the burnout point of the ring, since its current is continually increasing. This might be overcome by alternately pushing and pulling the ring by a combination of the above two circuits.

B. Suggestion for an arc-driven gun.




This hypothetical gun uses a dielectric gun barrel with metal electrodes A and B and a metal projectile. The rear of the projectile and walls of the barrel are lined with material which evolves gas in the presence of an arc (as oil will do). Evacuate barrel forward of projectile and establish an arc from A through pellet to B. Evolving gas would produce approximately 30,000 psi pressure behind projectile. For 3-gram aluminum pellet, this method could attain 8000-meter-per-second pellet velocity in 4-meter barrel.

C. Inductive energy storage - see Appendix, "The Economics of Multimillion Watt-Second Inductive Energy Storage," H. C. Early and R. C. Walker, Conference Paper No. 56-333, presented at the AIEE Winter Meeting, 1956.

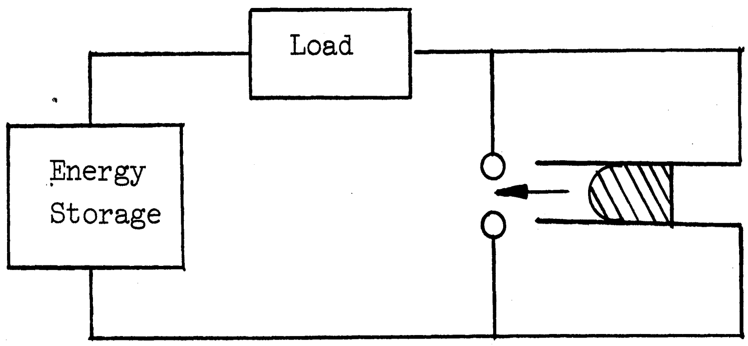
Discussion:

In designing the Harvard-M. I. T. accelerator, the energy stored in the deflection coils must be shifted to a second storage area each cycle. The original thought was to use capacitors for the alternate storage requirement, but an inductance proved to be more economical in the $10^5$ to $10^6$-joule region, in keeping with Mr. Early's and Mr. Walker's disclosure. (R. W. Waniek)

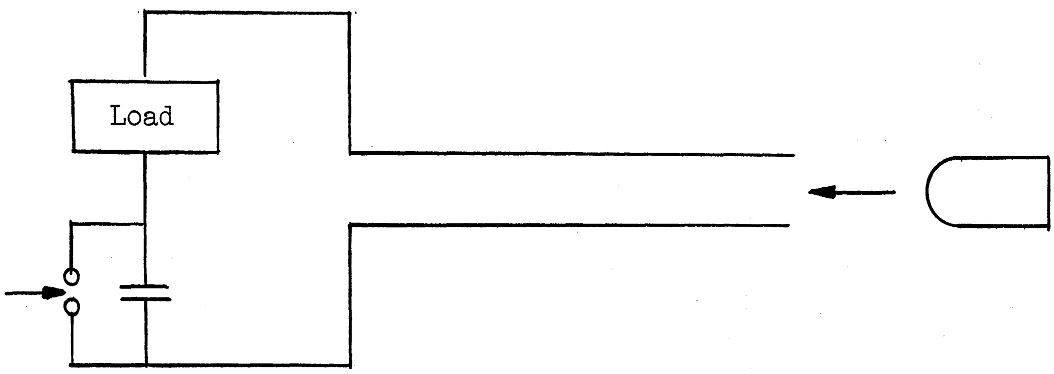
IX. General Discussion on Fast Discharge Circuits

A. Use of the electric gun in connection with fast discharge circuits.

1. The electric gun originally became of interest as a means of accomplishing rapid switching of energy from storage to load by firing a conducting medium across an insulating gap. Switching of 60,000 amperes in less than one microsecond was accomplished by firing a plasma into the gap; a solid bullet is not yet practical due to the velocity limit mentioned by Mr. Levine. (M. D. O'Day)




2. In the case of a projectile closing a circuit, as in the accompanying sketch,




the differential equation of the circuit may be written as:

$$\frac{d}{dt}(Li) + Ri + \frac{g}{C} = 0$$

$$L\frac{di}{dt} + i\left(R + \frac{dL}{dt}\right) + \frac{g}{C} = 0$$

Since $dL/dt < 0$, the second term $i(R + dL/dt)$ might presumably be made negative. (M. D. O'Day)

3. To get high power in a very short pulse duration from a projectile, the projectile must have sufficiently high energy (i.e., high velocity), and the deceleration must therefore take place in a very short length. For instance, to remove energy in one microsecond from a projectile traveling at 2000 meter/second, the energy removal must take place in a 2-millimeter distance. (W. G. Dow)

A further example: A 5-gram projectile traveling at 20,000 feet per second has 80,000 joule of kinetic energy. If 20,000 joule are wanted to be removed in a 10-centimeter distance, power is $10^9$ watt, force on projectile is 50,000 pounds, and time is about 20 microseconds. (E. A. Martin)

B. Means of energy storage.

1. Coal. Amplifying previous remarks by Dr. O'Day, the following figures apply to chemical storage of energy in coal:

$$\text{One lb coal} = \frac{14{,}000 \text{ Btu}}{3420 \text{ Btu/kwhr}} = 4 \text{ kwhr}$$

$$= 1.5 \times 10^7 \text{ joule}$$

$$12 \text{ lb coal} + 32 \text{ lb } O_2 = 44 \text{ lb } CO_2$$

$$44 \text{ lb } CO_2 + 160 \text{ lb air} = 172 \text{ lb flue gas}$$

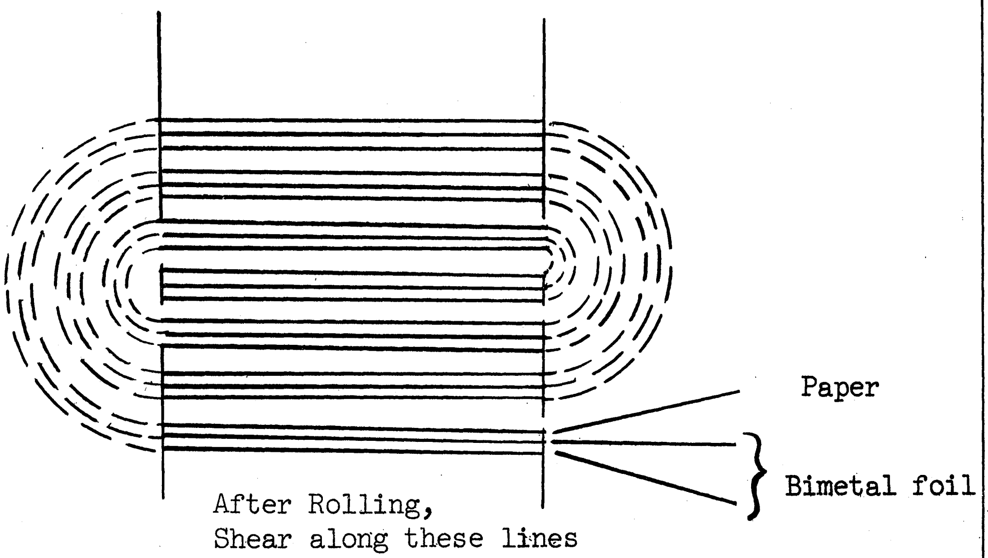
$$\text{flue gas/coal} = 15/1$$

$$CO_2/\text{coal} = 4/1$$

$$\frac{1.5 \times 10^7}{4} \sim 4 \times 10^6 \text{ joule per pound } (C + O_2)$$

The point to note here is that the chemical energy is not stored in coal alone but requires oxygen or air in the approximate ratios of 3 or 14 to 1 for combustion. (K. W. Miller*)

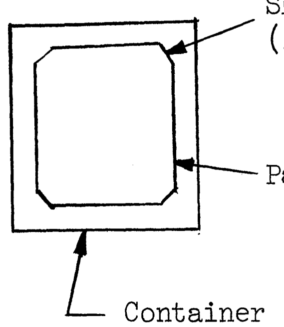
2. Battery. These figures for coal are to be compared with a foil or film type of primary battery advocated for certain types of high pulse power for expendable use. For example, if a composite slab of two metals are welded together and then rolled to a foil one mil or so thick and then if this foil is rolled together with a thin, chemically treated paper (much as foil condensers are wound) then the flattened roll may be sheared at its two sides as shown in the sketch, thus forming two Voltaic piles. When subsequently moistened with water or suitable electrolyte, approximately 5 amperes per square inch of area should be obtainable at 2 volts per cell or approximately 1000 volts per inch of stack. This corresponds to 5 kilowatts per cubic inch of battery. Such a battery should have a discharge life of approximately 10 seconds. An average mass density of approximately 4 appears reasonable for the battery, i.e., 7 cubic inches per pound giving a total stored electrical energy of 350,000 joules per pound. This compares not unfavorably with the energy storage content of coal, particularly considering that, with static apparatus, the conversion has already been made from chemical into usable electrical form.




Note: separation between layers has been increased in this drawing to show detail.

Alternative to bimetal foil, metal foil could be prepared with opposite faces rendered anode and cathode surfaces by electroplating, evaporating, and/or differential chemical treatment. Indeed, batteries could be constructed from only paper and appropriate minerals without use of metal foil at all.

Presumably, foil batteries so constructed would have all four sheared edges sealed with some insulating varnish and then the one or more corners sheared off as shown in the sketch, the whole to be enclosed in an insulating evacuated container, desirable for indefinite shelf life. When desired for use, the seal is to be broken, under water or electrolyte immediately before use. Presumably, such a battery would have very low internal resistance and inductance, and the time for current rise should be very short, perhaps only a few microseconds. (K. W. Miller*)




3. Kapitza battery. Under the title "A Method of Producing Strong Magnetic Fields" in the Proceedings of the Royal Society of London 105A, 1924, pp. 691-710, P. L. Kapitza described a special storage battery of his design for extra-large power pulses of a few milliseconds duration. (This reference is substituted for a brief description given at the meeting.) (K. W. Miller*)

4. Inductance

a. For comparative purposes, the energy/mass ratio was estimated for the large inductance coil built by The University of Michigan with results as follows:

15,000 lb aluminum + 5,000 lb reel plus power source stores $10^6$ joule.

$$\text{Therefore} \quad \frac{10^6 \text{ joule}}{2 \times 10^4 \text{ lb}} = 50 \frac{\text{joule}}{\text{lb}} .$$

Although far less favorable than the primary battery on an energy/mass basis, it should be noted that the primary battery is expendable wherein the inductance coil is a permanent reusable power source. (K. W. Miller*)

b. The size and cost of an inductance coil for storing energy is much less if the storage interval is short. If sufficient peak charging power is available to store the energy very quickly, a much smaller coil of lower Q is feasible. With The University of Michigan storage coil, the energy was stored over a relatively long 5-second interval to reduce the peak power requirement from the power line.

In some applications, there would be advantages in combining a mechanical storage system with an inductive storage system. Energy would be stored in a flywheel over an interval of several minutes; it would be delivered to the inductance coil during an interval of about one second, and then transferred from the inductance to the load in a few milliseconds or less. (H. C. Early*)

5. Homopolar generator

a. A homopolar generator which was designed by the Foundation under the Ordnance contract ten years ago, was briefly described. Full design particulars for this pulse power source are given in Technical Report No. 3.[4] This design consisted of a heavy flywheel approximately 6 feet in diameter carrying six generating disks, three nested on each side of the flywheel. The return current path for the rotating disks was also disks fixed to the pole faces, each subdivided into six pie-shaped segments. The brushes to the edges of the rotating disks were systematically connected through short, circumferentially disposed leads to the outer periphery of the segments of the static disk on the pole faces. The inner portions of the rotating disks were flanged into hub tubes to accommodate the other set of brushes which were connected systematically to the inside ends of the pie-shaped stationary segments (and to the outside circuit).

Because, in this design, the heavy currents are carried in closely spaced "current sheets," the computed internal inductance of the machine was surprisingly low, permitting current rise time of a millisecond or so. The design was calculated to give $10^6$ amperes at 1000 volts sustained for 10 to 15 milliseconds with a 10 to 15 percent reduction of flywheel energy. The total computed gross weight of 55 tons gives a power to mass ratio of approximately 15 kilowatts/pound for delivered pulse power. (K. W. Miller*)

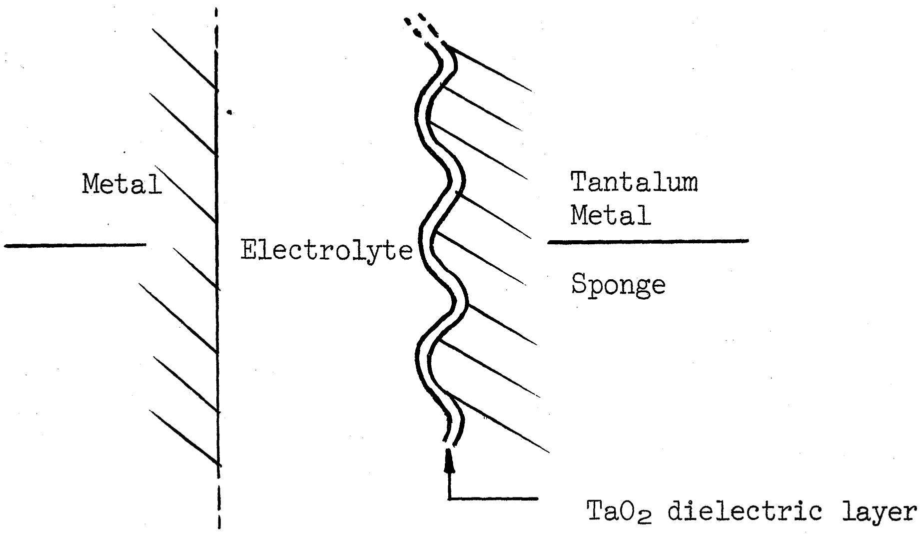
b. Oliphant's ideas about homopolar generators. For the purpose of energizing the magnet of a proton synchrotron, Oliphant is envisaging the use of a homopolar generator. By lucky coincidence an old cyclotron magnet of 148-inch pole diameter and 4-

4. "Electric Gun and Power Source," Project 15-391E, to the Ordnance Department, U. S. Army Service Forces under Contract W11-022ORD-11164, dated 5-1-47.

inch gap was available and it was put to use. Four mild steel discs, 139 inches in diameter and 10 inches thick, form the two rotors. The material was originally intended to be pole tips for the cyclotron magnet. Each has a mass of 19 tons. The two pairs of discs are supported from shafts inserted through the centers of the upper and the lower pole piece. The two sets are counterrotated to eliminate net moments. The discs rotating at 900 rpm in a magnetic field of 16,000 gauss have an energy of $5 \times 10^8$ joules and develop an emf of 800 volts when in series. The rotors are accelerated by passing through them 3,300 amperes from a grid-controlled rectifier. Brush contacts for passing the accelerating and the discharge current are made via a liquid jet of a sodium-potassium alloy. When the current passes through zero the rotors have lost 80 percent of their energy. It is proposed to extract a pulse every ten minutes. (R. W. Waniek*)

6. Capacitors

a. Tantalum Capacitors




This type of capacitor may prove to be superior to most other types in terms of energy storage capacity per unit volume. An inherent feature is that it has a high effective series resistance, which should mean a longer discharge time or time constant RC than most other types. Therefore, it might be considered for use in power sources operating in the millisecond range. It might be comparable to the inductance coil being developed at Michigan for their application. There are two major types, one having a sponge-like electrode as typified by the Fansteel product and one having a foil electrode as typified by the G. E. product. Of these two, the sponge type has the higher specific energy capacity. (H. L. Garbarino*)

The tantalum capacitor gives high energy storage at about 1000 volts. An aluminum type electrolytic capacitor will store 4 to 5 joule per cubic inch at 450 volts. (H. L. Garbarino)

Possible sources of high resistance in the tantalum capacitor are the metal sponge and the electrolyte. (W. W. Salisbury, M. A. Levine)

b. Commercially available fast discharge capacitors (R. W. Waniek*)

| Manufacturer and Serial Number | Rated Voltage kv | Energy joule | C μfd | L mμh | F Ringing Freq. mc | T=1/F μsec |
|---|---|---|---|---|---|---|
| Axel Bros. | | | | | | |
| 125 E 101 | 125 | 8000 | 1.0 | 65 | 0.625 | 1.60 |
| 125 E 102 | 125 | 4000 | 0.5 | 65 | 0.880 | 1.137 |
| 125 E 103 | 125 | 2000 | 0.25 | 65 | 1.25 | 0.80 |
| 60 E 101 | 60 | 4000 | 2.2 | 40 | 0.540 | 1.85 |
| 60 E 102 | 60 | 2000 | 1.1 | 40 | 0.760 | 1.316 |
| 60 E 103 | 60 | 1000 | 0.55 | 40 | 1.08 | 0.926 |
| 25 E 101 | 25 | 2000 | 6.4 | 25 | 0.400 | 2.50 |
| 25 E 102 | 25 | 1000 | 3.2 | 25 | 0.560 | 1.78 |
| 25 E 103 | 25 | 500 | 1.6 | 25 | 0.800 | 1.25 |
| Tobe Deutschmann | | | | | | |
| 1 "Cake" | 8 | 1300 | 14 | 20 | 0.177 | 5.65 |
| L A | 20 | 1500 | 7.5 | 80 | 0.205 | 4.87 |
| XN 249 | 100 | 4000 | 0.8 | 74 | 0.654 | 1.53 |
| Aerovox | 4 | 100 | 12.5 | 51 | 0.200 | 5.00 |

| Manufacturer and Serial Number | $I_{peak} = V\sqrt{C/L}$ megamp | $I_{pk}/T$ megamp/μsec | Cost $ | Cost/Energy $/joule | T x (Cost/Energy) $ μsec/joule | $\frac{\text{Cost/Energy}}{I_{pk}/T}$ $\frac{\$/\text{joule}}{\mu\text{sec/megamp}}$ |
|---|---|---|---|---|---|---|
| Axel Bros. | 0.490 | 0.306 | 4300 | 0.537 | 0.861 | 1.76 |
| | 0.346 | 0.302 | 2500 | 0.626 | 0.712 | 2.07 |
| | 0.245 | 0.306 | 1600 | 0.80 | 0.640 | 2.61 |
| | 0.444 | 0.240 | 2300 | 0.576 | 1.064 | 2.40 |
| | 0.314 | 0.238 | 1470 | 0.735 | .967 | 3.09 |
| | 0.245 | 0.275 | 950 | 0.950 | 1.025 | 3.45 |
| | 0.400 | 0.160 | 1350 | 0.676 | 1.690 | 4.22 |
| | 0.282 | 0.164 | 875 | 0.875 | 1.56 | 5.33 |
| | 0.200 | 0.160 | 570 | 1.140 | 1.42 | 7.11 |
| Tobe Deutschmann | 0.211 | 0.037 | 250 | 0.19 | 1.07 | 5.14 |
| | 0.194 | 0.040 | 250 | 0.17 | 0.828 | 4.25 |
| | 0.300 | 0.196 | 1500 | 0.374 | 0.573 | 1.91 |
| Aerovox | 0.0627 | 0.0125 | 900 | 0.11 | 0.45 | 7.20 |

c. Water dielectric capacitor. (R. W. Waniek)

1) Constants of water:

a) dielectric strength (at 500 kc) = 149 kv/cm
373 v/mil

decreases as T decreases.

b) dielectric constant (to $10^8$ cps):

$$k = 80-0.4(t-20°)$$

1/80 increase for 2-1/2°C cooling.

Maximum is 17 megohm-cm for purest water; may be as low as $10^4$ ohm-cm and still be suitable for a short pulse capacitor.

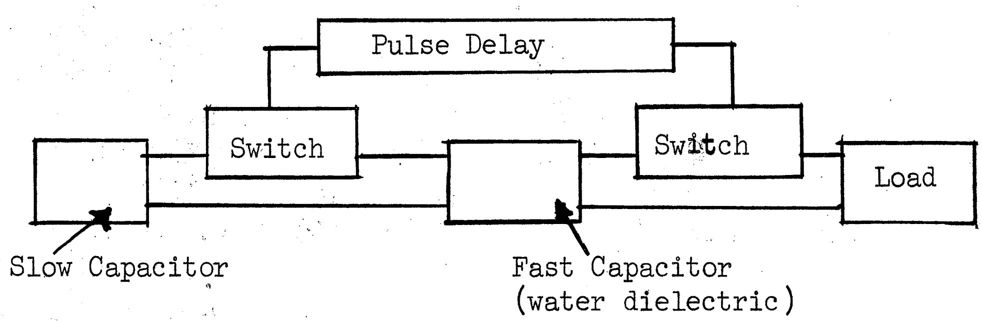
2) Water capacitors are generally designed as coaxial cylinders and some have been built to hold to 100 kilovolts.

3) Charging time must be short (less than 100 microseconds to a millisecond) so water does not boil to joulean heating.

4) In construction of a water capacitor, the electrodes must be degassed by r-f firing, distilled water should be employed, and an ion exchange cylinder is needed to keep water free of ions. Tank ought to be closed, although air probably would not greatly affect performance.

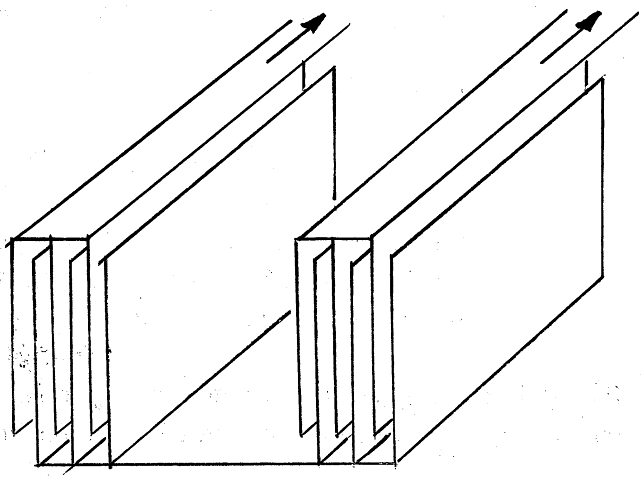
5) Example: Using one-meter diameter with 3-4 millimeter spacing between coaxial electrodes, the capacitance was 3 microfarads per meter of length.

Circuit in which this capacitor was used:




d. Barium titanate is also a good dielectric for very short durations. (A. Ellett)

e. Better capacitors (with respect to inductance) than commercial designs ought to be produced, and the speed with which energy can be removed from them ought to approach the relaxation time of the dielectric. Most of the inductance present in commercial capacitors is in the leads; a reduction in lead inductance could be accomplished by a capacitor design similar to the following:

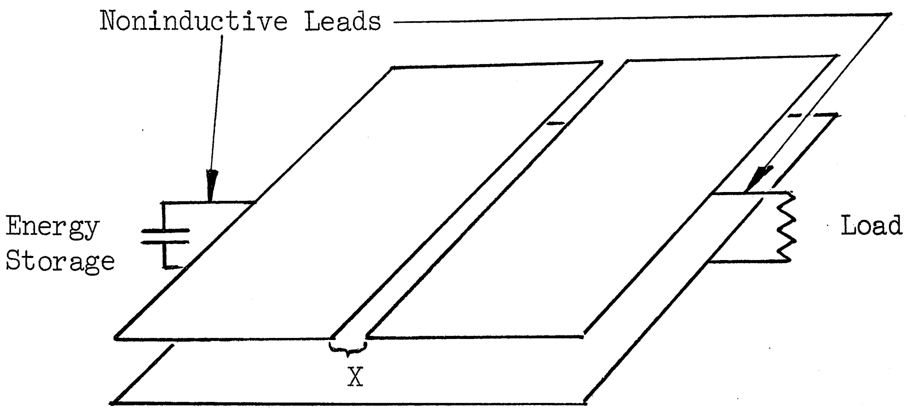


The cost of reducing leadout inductance ought not to be greater than 10 percent over present price. (W. W. Salisbury)

C. Switching of fast circuits.

1. Any switching method which is employed (e.g., to switch energy from a capacitor to a load or between storage places in a stepped energy transfer system) will add inductance.

Consider the following circuit in which inductance is minimum:




The problem in this circuit is to close the gap "X" with small additional inductance compared with the rest of the circuit. A bullet driven into the gap "X" has been proposed. (M. A. Levine)

Perhaps an exploding wire in the gap would initiate a broad low inductance discharge across "X." (H. C. Early)

2. The high pressure air switching gap has been quite successful. If air is loaded with a gas such as $CCL_4$, Freon, or $SF_6$, the switching speed might further be increased. (H. C. Early)

3. The use of a fuse in parallel with a mechanical circuit breaker for switching large currents at high voltages is discussed in the paper "The Economics of Multimillion Watt-Second Inductive Energy Storage," by H. C. Early and R. C. Walker, included as the appendix of this report. To minimize the amount of stored energy dissipated in the switching process, it is important that the switching take place in a very short interval of time. Presumably the switch-fuse arrangement described in this paper could be refined so that the circuit could be opened in about a millisecond. A very interesting German publication[5] describes methods of interrupting circuits with explosives

5. E. Marx, "Explosive-Action Circuit-Breakers," The Engineers' Digest, 16 (December, 1955), p. 563.

in which the total time lapse from a trigger signal to the interruption of a large current in 0.2 to 0.3 millisecond. (H. C. Early*)

X. Conclusion. This report has been prepared to present the essential information which came out in the conference discussions as well as to bring together later elaborations thereon by the various participants. Any further comments which the reader feels might be useful in furtherance of this program are invited.

Additional copies of this report are available at The University of Michigan and may be obtained through Professor Dow.

# APPENDIX

Conference Paper No. 56 - 333

# THE ECONOMICS OF MULTIMILLION WATT-SECOND INDUCTIVE ENERGY STORAGE

Presented as a Conference Paper
at the AIEE Winter Meeting
January 30 - February 3, 1956

H. C. Early and R. C. Walker
Department of Electrical Engineering
University of Michigan

## INTRODUCTION

In many electrical energy storage applications the objective is to store energy slowly over a relatively long interval and then withdraw the energy quickly to obtain a very high peak power. For certain applications of this type, the inductance coil has significant advantages over capacitors or rotating machinery. The inductive storage method is particularly advantageous in situations where millions of watt-seconds are to be stored, and the energy is to be expended in a time interval of a few milliseconds.

This paper is based on the experience gained in building an inductance coil for storing three megajoules of energy, which is used to produce a high-power arc at high gas pressure. The design and performance of the present installation is described in this paper; also some of the merits and limitations of other coil designs and alternative energy storage systems are considered.

The point of view of the discussion is based on the assumption of a low duty cycle operation where cumulative heating due to repetitive operations can be ignored and where only the instantaneous ratings of equipment are the limiting factors. This assumption results in a very large reduction in the size and cost of the equipment.

## COMPARISON OF STORAGE METHODS

The four most useful devices for storing large quantities of electrical energy appear to be: capacitors, inductances, rotating machinery and storage batteries. For certain applications the choice between competing methods can be made primarily on the basis of cost, but in most situations, factors other than the cost per watt-second are at least of equal importance.

Probably the most important differentiating characteristic between these systems is the length of the discharge time. For the most rapid release of energy, when the discharge interval must be short compared to a millisecond, capacitors have the advantage. For cases in which the discharge interval is of the order of milliseconds, the inductance coil is particularly attractive. For slower rates of discharge, corresponding to an interval of perhaps a second, rotating machinery is advantageous, while for still longer discharge periods (and long storage periods) storage batteries have an unusual advantage. In brief, as a rough rule of thumb, the four systems might be characterized by their possible minimum discharge periods: capacitor, microseconds; inductance, milliseconds; machinery, seconds; battery, minutes.

While capacitors have a unique advantage in their short discharge time, they suffer from a serious economic limitation in comparison to inductance coils, machinery, and batteries in that they cannot be temporarily overloaded no matter how low the duty cycle might be. The cost of a capacitor bank varies linearly with the amount of energy to be stored. At the present time, energy storage capacitors cost about six cents per watt-second and their volume is approximately one cubic inch per watt-second. The cost of the complete installation is much greater than these figures would indicate because of space requirements, shelf-type mounting arrangements, and wiring which requires an extensive protective system of relays and interlocks.

In comparing capacitive vs. inductive storage, the differences in circuit behaviour of the two systems may be a deciding factor. For instance, when a capacitor bank is discharged through a high-pressure electric arc, the arc will often extinguish when the capacitors are only partially discharged. On the other hand, if the arc energy is supplied from an inductance coil, the arc cannot extinguish until all the stored energy has been expended. In general, the capacitor tends to act as a voltage generator and the current is determined by the load characteristics, while the inductance tends to act as a current generator and the voltage is determined by the load characteristics.

The present project involves the storage of one to three million watt-seconds of energy to be dissipated in an arc discharge in a period of about ten milliseconds. The short discharge time ruled out the possibility of using machinery or batteries, while the linear increase in cost with respect to capacitors made their use extremely expensive. Therefore, the possible use of a coil was examined. At the million joule level, it was found that an inductive storage system would be cheaper than the capacitive system. However a more important conclusion was that the inductive storage installation becomes very much cheaper, in terms of watt-seconds per dollar, as the size increases. This is primarily because of a scaling factor whereby the ratio of stored energy to peak charging power of an inductance increases with the square of a linear dimension of the coil. Hence the charging period for a larger coil can be extended over a longer interval and the peak power requirement from the d-c supply is greatly reduced.

## SPECIFICATIONS OF UNIVERSITY OF MICHIGAN COIL

Fig. 1 is a photograph of the inductance which was designed for storing one million joules when operated as an air core inductance, and three million joules when the center is filled with transformer iron. The wood reel is ten feet in diameter, three feet wide and the inside core or drum is five feet in diameter. The winding consists of 5000 feet of 3,000,000 circular mil, 169 strand, aluminum cable; the total weight of the conductor is 15000 lbs. Mylar insulation was used for taping the cable and was also used between the layers of the winding. There are 12 1/2 layers on the winding and a total of 220 turns.

The coil has a d-c resistance of 0.03 ohms, an inductance of 75 millihenrys, a time constant of 2.5 seconds and self resonant frequency of 16 kilocycles. Without iron in the core a current of approximately 5200 amperes is required to store one megajoule.

The 5200 amperes of current requires 800 kw of d-c power which is furnished by a three phase ignitron rectifier. The current builds up at an exponential rate during the charging period which lasts approximately six seconds.

## THE ECONOMICS OF SCALING FACTORS

An economical design of energy storage apparatus requires a proper balance between the cost of the coil and the cost of the power supply. For a given storage capacity, the more money that is spent on the coil, the less is required for the power supply and vice versa. This section will consider the manner in which these relative costs and also the total costs vary with the capacity of the installation. The discussion is based on several simplifying assumptions which appear to be generally true. However, in certain special cases, they might require reviewing. These assumptions are:

1) The duty cycle is very low and the only limit to the instantaneous current density in the winding is the economic factor involving the cost of the power supply. (Perhaps the only situation where this assumption seriously breaks down is when the discharge time is unusually short and the peak power and voltage are so large that insulation breakdown becomes a serious factor.)

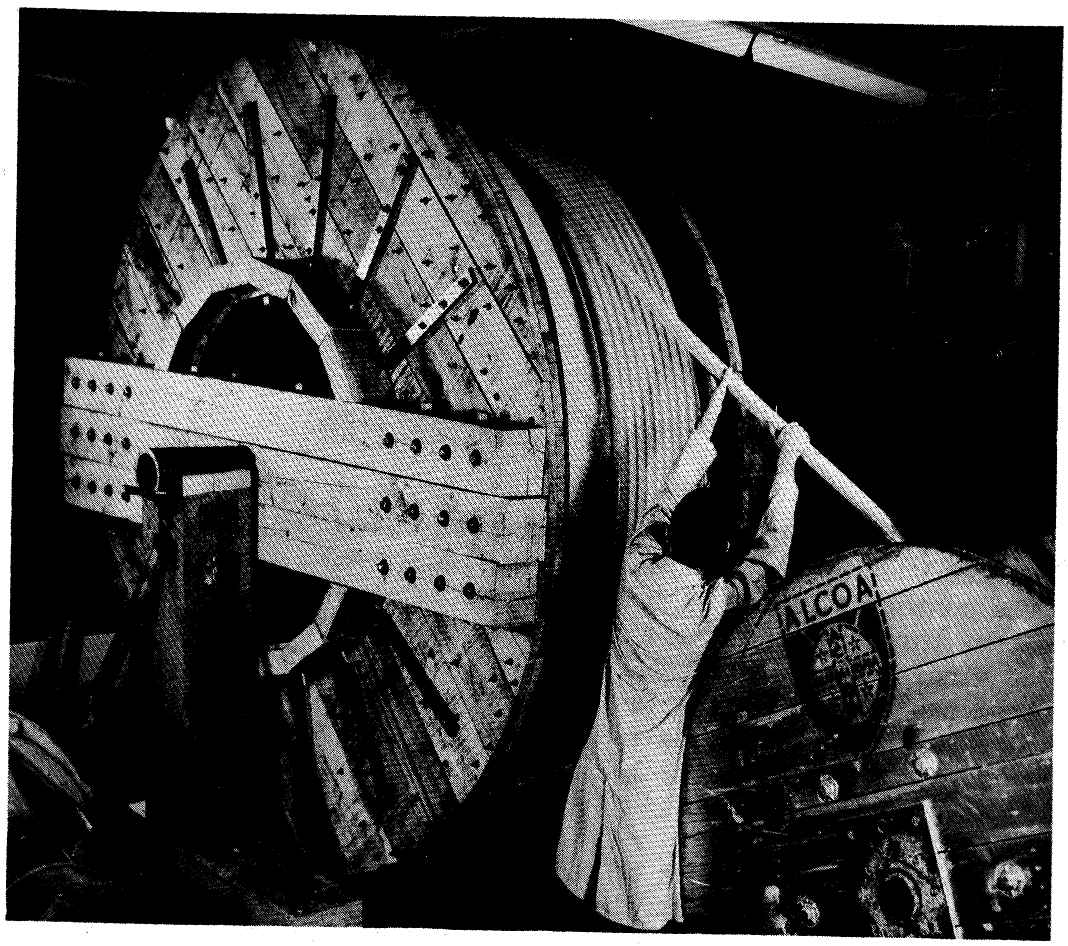



Fig. 1.
Energy
Storage
Cell

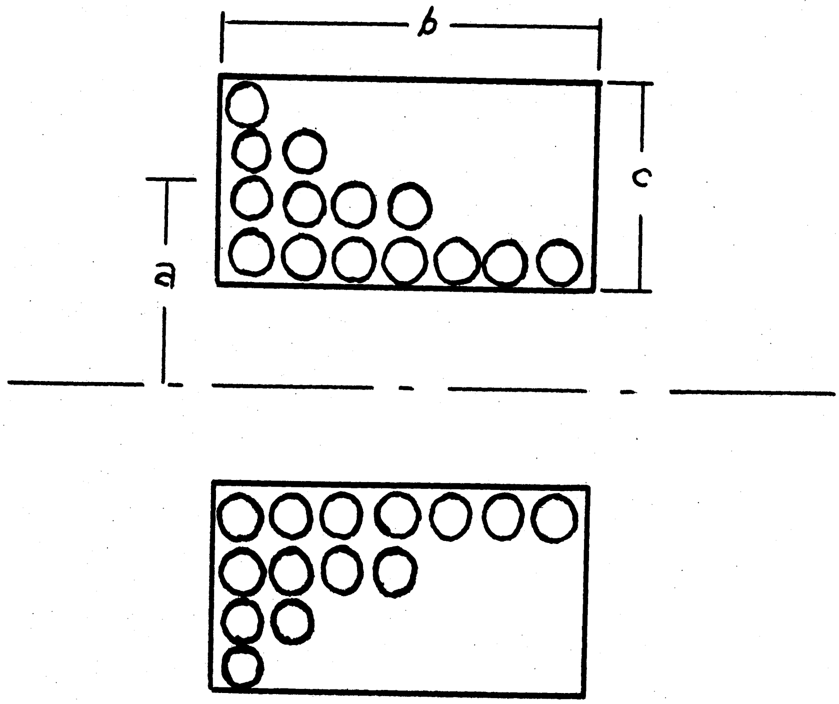



FIG. 2- COIL DIMENSIONS

2) The cost of the d-c power supply is proportional to the peak capacity in kilowatts. (This assumption roughly holds over a considerable range, but at sufficiently large power levels the cost per kilowatt tends to become less.)

3) The cost of the coil is proportional to its volume, i.e., to the cube of a linear dimension.

In evaluating various coil designs, a figure of merit may be defined which relates the total energy stored at peak current to the peak charging power:

$$\text{Figure of merit} = \frac{\text{stored energy}}{\text{peak charging power}} = \frac{1/2\, L\, I^2_m}{I^2_m\, R}$$

$$= \frac{L}{2R} = \text{one-half coil time constant}$$

where $L$ = inductance of the coil, henrys

$R$ = resistance of the coil, ohms

$I_m$ = peak coil current, amperes

A most important result that follows from this definition of a figure of merit follows from the dependence of the inductance and resistance of the coil upon its shape.

Consider a multiple-layer coil as shown in Fig. 2. The low-frequency inductance is given by [1]

$$L = J\, a\, n^2$$

where $a$ = mean radius

$n$ = number of turns

$b$ = axial length

$c$ = radial depth of the winding

$J$ = constant which depends on the ratios of $c/a$ and $b/c$

The resistance of the coil is

$$R = \rho \frac{l}{A}$$

where $\rho$ = resistivity

$l$ = total length of conductor

$A$ = cross-sectional area of the conductor

Since the length of conductor, l, equals $2\pi an$ and the cross-sectional area of the conductor equals the coil cross-section divided by the number of turns, or bc/n, the resistance of the coil is

$$R = \rho \frac{2\pi an}{bc/n} = (\text{constant}) \frac{a}{bc} n^2$$

For a given shape, b and c will be constant fractions of a, so that

$$R = (\text{constant}) \frac{n^2}{a}$$

Therefore, for any given shape,

$$\text{Figure of merit} = 1/2 \; L/R = 1/2 \frac{(\text{constant}) \; an^2}{(\text{constant}) \; \frac{n^2}{a}} = (\text{constant}) \; a^2$$

and the relation of stored energy to peak charging power varies as the square of the coil radius. Thus, for a given volume or weight of conductor, the figure of merit is immediately specified regardless of whether the coil is wound of many turns of fine wire or a few turns of large wire. (This statement neglects the effect on L and R of insulation spacing.) From the above relationship, it can be seen that increasing the linear dimensions of a coil by a factor k will increase the ratio of stored energy to peak power by a $k^2$ factor.

The inductance that can be obtained with a given length of conductor is a function of the coil geometry. Maximum inductance is obtained when b=c=0.66a. For this relation, $L = (4.2 \times 10^{-8}) \; an^2$, where a is measured in inches.[1]

For the maximum-inductance geometry:

$$\text{Figure of merit for copper} = (1.35 \times 10^{-3})a^2$$

$$\text{Figure of merit for aluminum} = (0.825 \times 10^{-3})a^2$$

From an economic standpoint, it is well to define the merit ratio in terms of conductor weight, rather than in terms of coil mean radius. The result is:

$$\text{Figure of merit for copper} = (1.7 \times 10^{-3})(\text{weight in pounds})^{2/3}$$

$$\text{Figure of merit for aluminum} = (2.24 \times 10^{-3})(\text{weight in pounds})^{2/3}$$

Thus, for a specified figure of merit, the total weight and cost of the conductor can be evaluated. As of December 19, 1955, the ingot price was $0.24 per pound for aluminum and $0.43 per pound for copper.

On the basis of the assumptions stated above, several important relations can be derived.

Let $S_p$ = cost of power supply

$S_c$ = cost of coil

$$\text{Then, figure of merit} = \frac{(\text{joules})}{(\text{watt})} = (\text{constant})\,(\text{coil volume})^{2/3}$$

$$= (\text{constant})\,(S_c)^{2/3}$$

$$\text{Storage Capacity} = \frac{(\text{joules})}{(\text{watt})}\,(\text{peak power}) = (\text{constant})\,(S_c)^{2/3}\,(S_p)$$

$$\frac{\text{Joules}}{\text{Dollar}} = \frac{\text{Storage Capacity}}{\text{Total Cost}} = (\text{constant})\,\frac{S_c^{2/3}\,S_p}{S_c + S_p}$$

For a fixed total cost ($S_c + S_p$), maximum stored energy is obtained for an optimum ratio $S_p/S_c$. The optimum ratio is 3/2. However, the ratio $S_p/S_c$ can vary from unity to 5/2 without decreasing the joules/dollar ratio more than four per cent.

Substituting $S_p/S_c = 3/2$ in the above relations, the following results are obtained:

$$\text{Total Cost} = S_c + S_p = 5/2\,S_c = 5/3\,S_p$$

$$\text{Storage Capacity} = \text{constant}\,(S_c)^{5/3} = \text{constant}\,(\text{total cost})^{5/3}$$

$$\text{Joules/dollar} = \text{constant}\,(\text{total cost})^{2/3}$$

$$= \text{constant}\,(\text{storage capacity})^{2/5}$$

Fig. 3 illustrates the trend of joules/dollar of capacitive and inductive storage as storage capacity of a system is increased. For the small installation, capacitors are less expensive, but the inductance becomes progressively more advantageous as the size of the system increases. The location of the crossover point depends on many factors which cannot be analytically expressed. However, based on present experience, the crossover point is probably at energy levels of the order of $10^5$ watt-seconds.

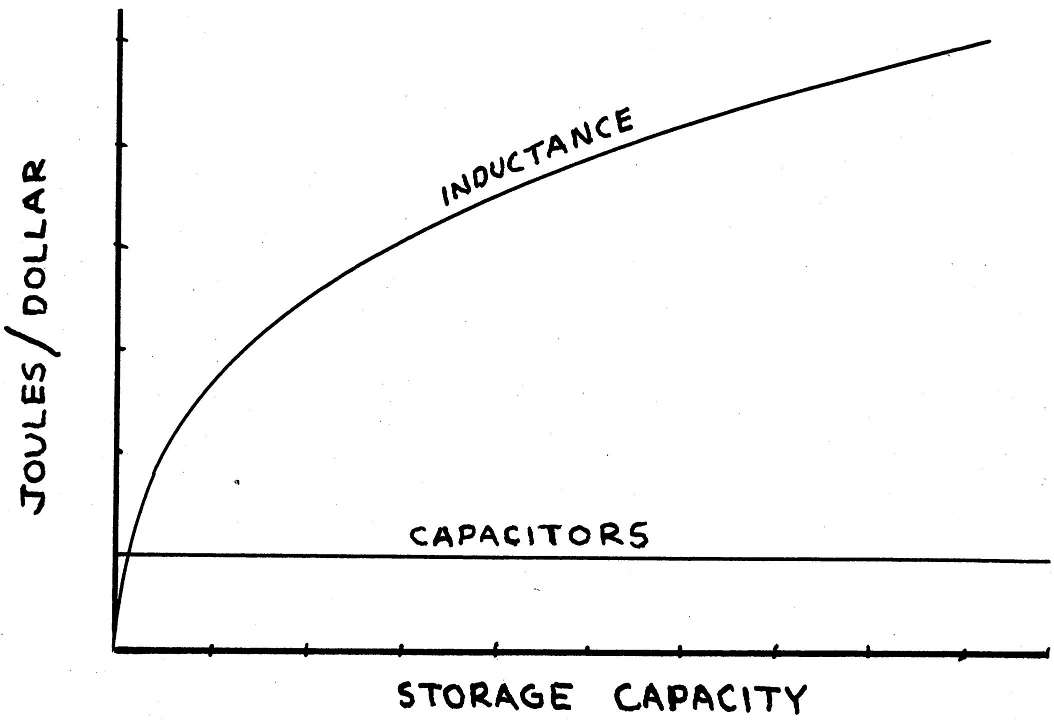



FIG. 3-TREND OF JOULES/DOLLAR VS. STORAGE CAPACITY

## CIRCUIT SWITCHING

The simplest possible circuit switching arrangement for charging and discharging the coil is shown in Fig. 4. During the charging interval while the coil current is building up, the switch across the load is closed. After the coil current has reached the desired value, the shorting switch across the load is opened, and the relatively high resistance of the load is inserted into the circuit. Two modifications of this basic circuit were found to be desirable. (1) Since the experimental tests produced fast high-voltage transients which might damage the rectifier or possibly reflect into the power line, the system was completely disconnected from the power source before the stored energy was delivered to the load. (2) In order to open the switch across the load without arcing, an auxiliary by-pass circuit, described below, was connected across the switch terminals.

The coil and discharge circuit are disconnected from the rectifier, before starting the discharge through the load, by means of an auxiliary ignitron as shown in Fig. 5. At the completion of the charging period, opening switch $S_1$ disconnects the rectifier. The voltage induced by the coil fires the auxiliary ignitron and the current is switched from the rectifier to the discharge ignitron. The switch $S_1$ is not required to handle any high voltages since the maximum voltage is limited to the d-c rectifier voltage which is about 200 volts.

The switching operation at $S_2$, (Fig. 5) whereby the coil current is switched into the load, is more difficult. Because the current is switched into a high impedance load, voltages of tens of thousands of volts are developed across the switch. The load is an electric arc inside a confined pressure chamber. The arc chamber is approximately 36 inches long and 3/4 inch in diameter. The arc is initiated by exploding a length of #30 Chromel wire connected between electrodes at opposite ends of the pressure chamber. A current of 3500 amperes will explode this wire and develop arc voltages of 20,000 volts in less than a millisecond. This switching problem was solved by the construction of an unusually fast opening switch, which was used in conjunction with a high-voltage fuse as shown in Fig. 5. The operation is as follows:

(1) When the switch $S_2$ opens, the current is first transferred to the high voltage fuse. Because of this short-circuiting fuse, there is no arcing as the switch opens. This allows the switch several milliseconds, before the fuse blows, to develop a high breakdown strength.

(2) The fuse opens after about five milliseconds and the current is transferred to the initiating wire in the arc chamber.

(3) The initiating wire in the arc chamber explodes and the stored energy is delivered to the arc.

The fast-acting switch $S_2$ is sketched in Fig. 6. One terminal of the

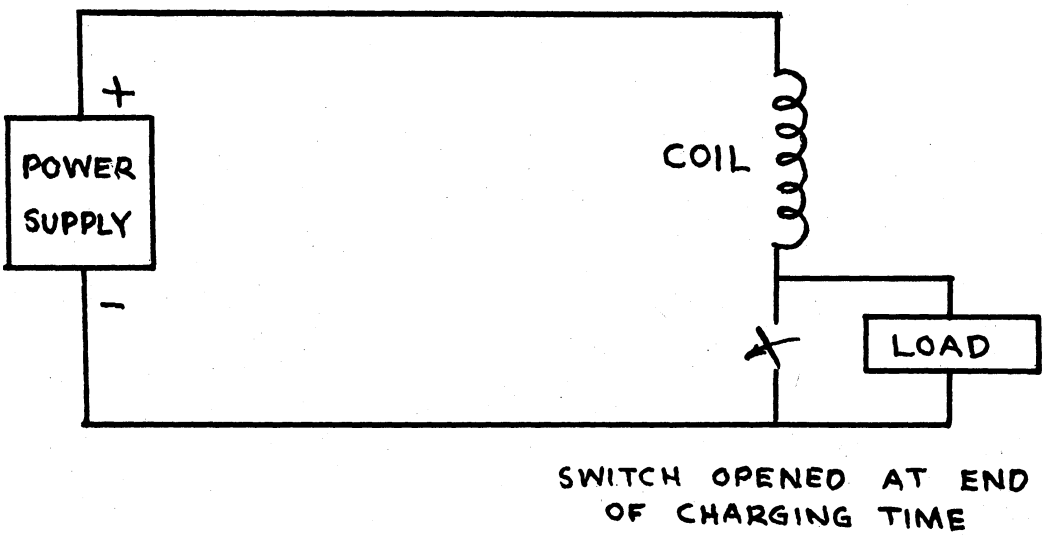



FIG. 4 -BASIC CIRCUIT FOR INDUCTIVE ENERGY STORAGE SYSTEM

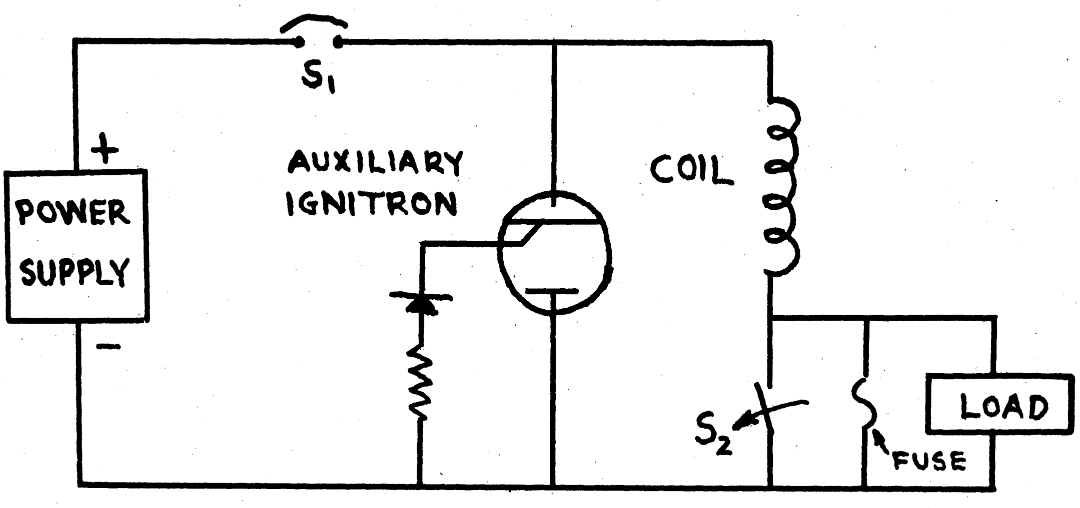
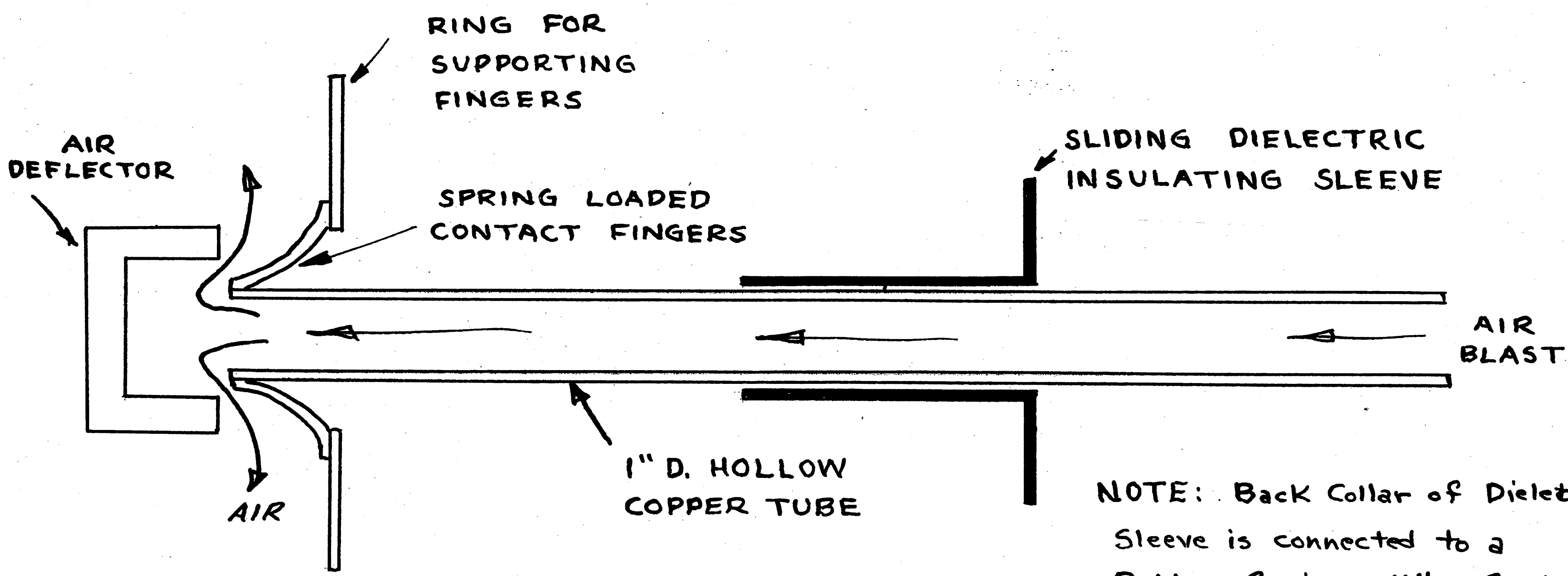



FIG.5 - MODIFIED CIRCUIT




NOTE: Back Collar of Dielectric Sleeve is connected to a Rubber Spring. When Spring is released, Sleeve is Pulled between Copper Tube and Contact Fingers, Opening the Circuit.

FIG. 6 - FAST ACTING SWITCH

switch is a one inch diameter copper tube and the other terminal is a circular ring of spring-loaded fingers which make contact with one end of the pipe when the switch is closed. A thin cylindrical sleeve covered with Mylar insulation slides freely along the pipe and opens the circuit when it passes between the spring-loaded fingers and the tube. Since this dielectic sleeve has a very small mass, it can easily be accelerated to a velocity of about 200 feet per second in two feet of travel along the tube by means of a stretched rubber spring. This velocity is more than adequate for present requirements. In order to minimize any small arc which might tend to form across the opening switch, a stream of compressed air is used to blast the dielectic surfaces along which an arc might form. This air blast arrangement is shown in the figure.

The expendable high-voltage fuse consists of an 18 inch length of #18 copper wire immersed in an oil filled fiber tube. Experience with this switching arrangement indicates that it could easily be adapted to handle much higher voltages and currents than have been encountered in the present installation.

## IMPEDANCE MATCHING

When the coil is discharged into a load of constant resistance the current decreases in an exponential manner during the discharge interval. The power during this interval varies as the square of the current and thus decreases at an exponential rate.

Other power pulse shapes are produced when the load is a non-linear resistance, such as an arc. A number of methods have been tested for varying the arc resistance during the discharge interval and a variety of power pulse shapes have been obtained. Tests with this coil have involved typical arc impedances of five to twenty ohms. For many applications of a stored energy system, operation with much lower load impedances and voltages is desirable. This can be accomplished by using a coil with fewer turns and larger currents. However, charging a very low impedance coil requires inconveniently large currents at such low voltages that the cost of the d-c power supply becomes more significant.

One method of reducing the charging current requirement of a very low impedance coil is to divide the winding into two or more sections which can be charged in series and discharged in parallel. A two section coil operated in this manner requires only one half of the peak charging current that would otherwise be required. This method is analogous to the impulse generator technique of charging capacitors in parallel and discharging them in series. In the case of the two section coil, the series to parallel switching can be accomplished with a minimum of complexity since only one additional discharge ignitron is required as is shown in Fig. 7. This arrangement has been tested at low power levels, and has been found to operate satisfactorily.

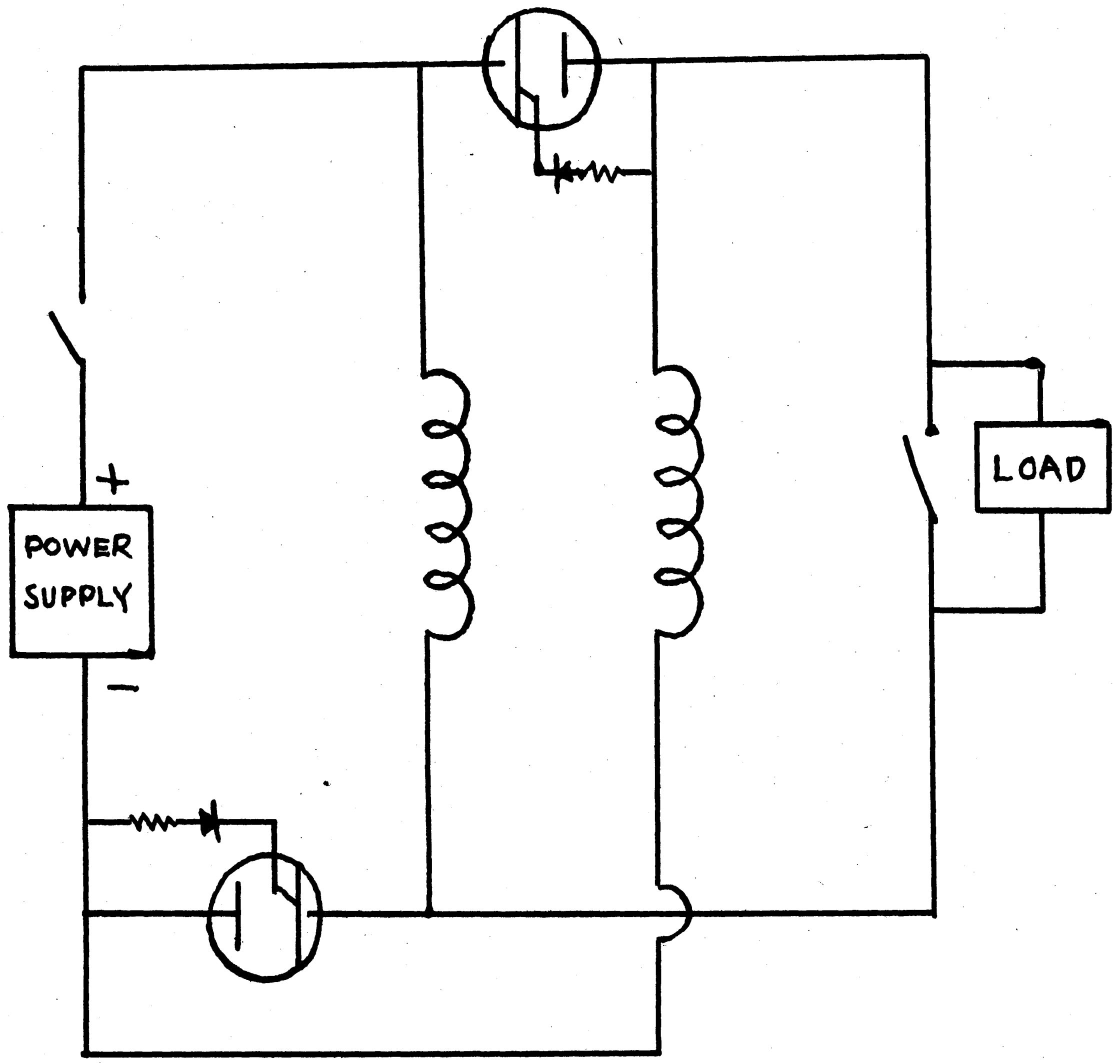



FIG. 7 - IGNITRON SWITCHING CIRCUIT FOR CHARGING COILS IN SERIES AND DISCHARGING IN PARALLEL.

An alternate method of very low impedance operation would be to use a homopolar generator as a source of d-c power. The Electric Products Company of Cleveland has built homopolar generators with a continuous rating of 21,000 amperes at 18.7 volts. This Company reports that with little change in size and design, this machine could deliver 100,000 or more amperes. Such a machine could presumably be used in connection with a flywheel to deliver five to ten times its continuous current rating for a few seconds during the final stages of current build up in an energy storage coil. It is reported that Allis Chalmers is currently engaged in development work on very high current homopolar generators.

## MULTIPLE-LAYER PROXIMITY EFFECTS

If most of the energy stored in a coil is to be delivered to an external load, the load resistance must be much greater than the coil resistance. The discharge power pulse has a frequency spectrum dependent upon its shape and duration. If the coil has a low Q (i.e., a high effective a-c resistance) for the important components of the frequency spectrum, the energy dissipated in the coil will be much greater than that predicted by a simple comparison of the coil d-c resistance to the load resistance. Therefore, an investigation of the increase in coil resistance due to skin and proximity effects was made. A more detailed account of this investigation is given in the Appendix. The results will be summarized here.

In the case of a multiple-layer coil, skin effect is of much less importance than proximity effect. Skin effect is a self-induced phenomenon in the conductor while proximity effect is a mutual effect between different turns. In the multiple-layer coil, the magnetic fields produced by all the surrounding turns reacting with the current in a particular turn produces a marked effect on the current distribution in that turn. For example, consider a coil with a mean radius of 45 inches and b=c=30 inches, and having 225 turns, wound with 1.6 inch diameter aluminum cable. At 100 cycles, skin effect will increase the $R_{ac}/R_{dc}$ ratio by a factor of 1.5 while proximity effect will increase it 165 times.

Butterworth[2] has published a comprehensive investigation of coil losses due to skin and proximity effects. His formulas are developed for two types of conductors: (1) solid round wire, and (2) litz wire, in which the individual strands are insulated from each other. A coil wound of litz wire often has a $R_{ac}/R_{dc}$ ratio which is smaller by at least an order of magnitude than the ratio for a similar coil wound of solid wire. The intermediate case of a stranded conductor in which there is no inter-strand insulation lies between these two extremes and is difficult to evaluate. Because of the contact resistance between the strands, the stranded conductor tends to have a partial "litzing" effect and in many cases has a lower $R_{ac}/R_{dc}$ ratio than the equivalent solid conductor. This effect appears to be more marked in aluminum stranded cables than it is in copper cables. The contact resistance is non-linear since it depends on the characteristics of oxide film between the strands and it is difficult to measure or estimate. Tests at the University of Michigan on a one-million circular mil, 61 strand, aluminum cable indicated that this inter-strand resistance was several times greater than that of similar copper cable. A recently published paper[3] also reports lower cable losses for aluminum as compared to copper, at 60 cycles in stranded cables of the 2,250,000 circular mil size.

As shown in Appendix the Butterworth formulas express the ratio of a-c to d-c resistance as a function of both frequency and conductor diameter (see Fig. 8). In the case of proximity effect, the resistance ratio for a given frequency increases with the square of conductor diameter until a peak is reached. After this peak, the resistance ratio decreases with the square root of diameter. Conversely, for a given diameter, the resistance ratio at first increases with the square of the frequency, then goes through a transition period, after which it increases with the square root of frequency.

Since inductance coils which would be built for megajoule (or greater) energy storage are designed for operation at thousands of amperes, the diameter of the individual turns will be of the order of an inch or more. At this conductor diameter, the $R_{ac}/R_{dc}$ ratio due to proximity effect can be very high, depending on frequency. Therefore it is necessary to consider the frequencies which will appear in the spectrum of the discharge current pulse.

For example, the coil that was built at Michigan is wound with 3,000,000 circular mil cable. Calculations for a solid cable of this size at the frequencies which would be important in a one millisecond discharge period indicated that a conductor with enameled strands would be mandatory. However, for a discharge time of 10 milliseconds it was possible to use ordinary stranded aluminum cable. For this discharge time, only the components of the frequency spectrum of the discharge pulse of low frequencies (perhaps up to 25 cycles) have sufficient magnitude to be important. The increase in effective coil resistance due to these low frequency components did not appear to be great enough to warrant the cost of individual strand enameling. Estimates of the additional cost of enameling the aluminum cable strands indicated that the resulting cost of the conductor would be more than doubled.

The ten millisecond discharge from the 75 millihenry coil is in the form of a power arc which has a resistance of the order of five to twenty ohms. Oscilloscopic measurements made during many discharge tests of this time duration have shown that the a-c resistance of the coil has not exceeded one ohm.

## COIL GEOMETRY AND WINDING

As previously noted, maximum inductance can be obtained with a given length of conductor by winding it as a multiple-layer coil with $b=c=0.66a$

(Fig. 2). However it is important to point out that if the coil is wound as a long solenoid, only a small decrease in inductance will result even when b becomes as much as four times c. At b=4c=1.2a the coil inductance is still 88 per cent of the maximum inductance.

The long solenoid has several advantages over the square cross-section coil. The number of layers in the coil is reduced so that in the coil construction, many winding problems are minimized. Also, during the charging and discharging of the coil, the magnetic forces acting on the turns tend to scuff and abraid the insulation. By reducing the number of layers, the load that is supported on the inner layers of the coil is reduced so that there is less wear on the insulation of these turns. From an electrical standpoint, the increase in a-c resistance of the coil during discharge due to proximity effect is greatly reduced, since proximity effect is proportional to the square of the number of layers (see Appendix).

The conductor insulation should be as thin as possible. The usual commercial practice is to insulate heavy cables with thick insulation such as neoprene or many layers of varnished cambric. The resulting increased spacing between turns substantially reduces the inductance that can be obtained with a given length of wire. The insulation of the present 2 inch diameter cable consists of two half-lapped wrappings of 5 mil Mylar tape covered with two wrappings of 8 mil adhesive paper tape. The total thickness is only about 1/32 inch. The inter-layer insulation consists of Mylar sheets for electrical insulation, and of canvas for mechanical stress distribution.

## USE OF IRON IN THE MAGNETIC CIRCUIT

The University of Michigan coil has so far been used without any iron in the magnetic circuit. For one million joules of energy storage, the flux density in the center of the air coil is 5000 gauss and the peak d-c charging power is 800 kilowatts. The planned addition of 30000 pounds of transformer iron into the hollow core of the wooden coil form will increase the inductance by a factor of three, and the stored energy, for the same peak charging power, will be correspondingly increased without saturating the iron. This calculation has been confirmed by scale model tests. With this coil, the cost of the iron is economically justified because if the same increase in storage capacity were to be obtained by increasing the charging current, the peak d-c power requirement would increase by a factor of three.

Because of magnetic saturation effects, only a fraction of the length of the magnetic circuit can be iron. When the added iron is located in the center of the coil the added inductance per pound of iron is a maximum and additional iron becomes less and less effective as the iron is extended outside the coil. This relation applies at flux densities low enough so that iron saturation is not a factor. In general, the amount of iron is limited by the law of diminishing returns caused by an inter-related combination of the two factors, 1) less added inductance per pound of iron, 2) increased saturation of the iron.

The advantage of using iron tends to diminish with the size and storage capacity of the coil because of saturation effects. Above some energy level, perhaps about ten megajoules, iron is no longer justified. This is because the flux density of any realistic coil design increases with the size of the coil until the use of iron can no longer economically be justified.

## CHARGING METHODS

As pointed out in the section on impedance matching, most coil designs require a power source capable of supplying d-c power at low voltages. This high current low voltage charging requirement stems from the fact that the induced voltage during discharge is hundreds of times larger than the charging voltage. Hence, if excessively high discharge voltages and load impedances are to be avoided, it is necessary to design a coil that requires large charging currents at low voltages.

Several sources of d-c power which may be advantageous for charging purposes are:

1) Ignitron rectifier
2) Germanium and silicon rectifier
3) Selenium rectifier
4) Low voltage conventional type d-c generator
5) Homopolar generator

High-current rectifiers are attractive for this application. Above 200 volts, ignitron rectifiers have long been efficient and satisfactory. In the range up to 50 volts, the selenium rectifier has usually been used for high currents in the past. However, in the intermediate range of 50 to 200 volts, there has long been a need for an efficient rectifier. The development of germanium and silicon rectifiers is filling this need and is of special interest in connection with inductive energy storage. Germanium rectifier installations ranging from 160,000 amperes at 15 volts (2.4 megawatts) to 42,000 amperes at 65 volts (2.9 megawatts) have been recently made.[4] The field of semi-conductor rectifiers is progressing rapidly and undoubtedly in the near future, units will be available at higher voltage and current ratings.

Conventional d-c machines are available at ratings that range from 9 volts to several hundred volts. The low voltage machines, used in electroplating applications, have an economic design limit of 10,000 to 20,000 amperes.

For extremely low voltages, homopolar machines are more advantageous.

If the peak power requirement of the energy storage coil is greater than that which can be supplied by the power line capacity which is available, an intermediate storage system, e.g., a flywheel, can be used. Energy is taken from the power lines over a relatively long time period, perhaps a minute, to be stored in the flywheel. This kinetic energy is transferred to the coil in seconds and then discharged to the load in milliseconds.

An example of what can be done with flywheel energy storage is the bevatron installation at the University of California. Two 67-ton flywheels store 675 million watt-seconds at synchronous speed. 80 million watt-seconds are taken from the flywheels over a period of 1.7 seconds. The speed change of the flywheel is about 6 per cent during this discharge period. Adapting a system similar to this for use in charging an energy-storage coil, would seem to be feasible.

## APPENDIX

### Skin and Proximity Effect Calculations in Multiple-Layer Coils

Formulas for determining the increase in a-c resistance of a multiple-layer coil due to skin and proximity effects have been developed by Butterworth[2] and are available in Radio Engineers' Handbook.

From equation 96, p. 81 of the Handbook, the resistance ratio for a coil wound with solid round wire can be expressed as:

$$\frac{R_{ac}}{R_{dc}} = H + CG$$

where C depends on the coil shape and the number of layers in the winding.

H and G are functions of a parameter x and are tabulated. H represents skin effect; the term containing G represents proximity effect.

x is proportional to $d\sqrt{f}$ for a conductor of given resistivity.

The skin effect term, H, of the above equation is usually much smaller than the proximity effect term for the frequencies and conductor diameters which are encountered in inductive energy storage applications. Therefore, it can be neglected.

However, the proximity effect term is very important. For the case of an unstranded-solid-round conductor, Fig. 8 shows the ratio $R_{ac}/R_{dc}$ (for

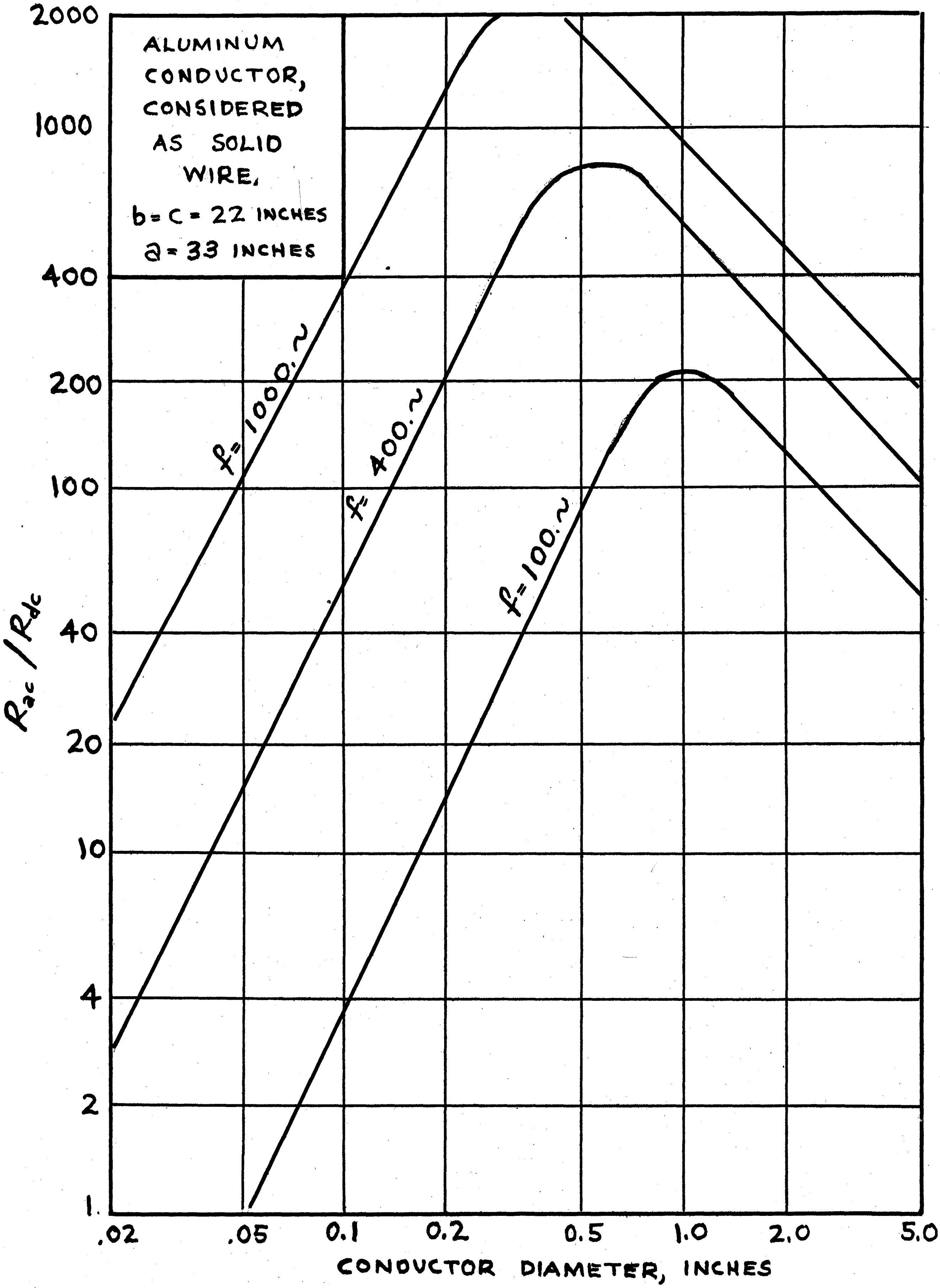



FIG. 8 - VARIATION OF $R_{ac}/R_{dc}$ DUE TO PROXIMITY EFFECT

proximity effect only) as a function of conductor diameter for frequencies of 100, 400, and 1000 cycles. To the left of the maximum points, the curves increase with $d^2$; to the right, they decrease as $1/d$. To the left of the maximum points, the spacing between curves depends on $f^2$; to the right, it depends on $\sqrt{f}$.

If a stranded cable is used the $R_{ac}/R_{dc}$ ratio can be reduced materially by enameling the individual strands so they are insulated from each other. An equation similar to the above is available for calculations of coils wound with insulated strands (equation 97, p. 82). A cable having insulated strands, instead of a solid conductor, can reduce the $R_{ac}/R_{dc}$ ratio of the coil by an order of magnitude.

For a given shape and number of turns, the parameter that determines the resistance ratio is $x = (\text{constant})\, d\sqrt{f}$. Therefore if a small model of a particular coil design is built in which all dimensions are reduced by a factor k, testing this inductance at a frequency equal to $k^2 f$ is electrically equivalent to testing the full-sized coil at a frequency f. ~~The "Q" of the model at $k^2 f$ will be the same as the full-size coil at f.~~

Many model tests were made in the preliminary design of the large coil described in this paper. Inductances were wound in various shapes, using solid wire conductors and conductors with enameled strands (litz wire). The tests all confirmed the predictions of the Butterworth formulas for the magnitude of the increase of a-c resistance due to skin and proximity effects.

The intermediate case between a solid wire and a cable having enameled strands is a stranded cable in which there is appreciable inter-strand resistance due to the presence of an oxide on the strands. Unfortunately, this case does not readily lend itself to design calculations based on direct measurement of the inter-strand resistance. The change in the amount of oxide with time, flexing of the strands by electromagnetic forces during charging and discharging, and the non-linear resistance of the oxide itself all combine to make it difficult to predict the a-c resistance of a coil. However, it is believed from experience with the present inductance wound with stranded aluminum cable, that the presence of the oxide on the strands does reduce the $R_{ac}/R_{dc}$ ratio below the value that would have resulted if solid unstranded conductor had been used.

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