An Optimal Repeat Inspection Plan Under Varying Inspection Errors

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Technical Report Number 94-25

July 1994

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Repeat Inspection Plans have been proposed for the quality control of critical multicharacteristic components. The optimal plan has been obtained via a cost minimization model. In all the models type I and type II errors are assumed to be fixed and known. In this paper this assumption is relaxed and a model with varying inspection error is developed for obtaining the optimal inspection plan. The model generalizes the model in the literature. The generalization is obtained through the development of a procedure that relates the level of the errors to incoming quality. The procedure utilizes, Receivers's Operating Characteristic (ROC) Curve, Signal Detection Theory (SDT) and constrained regression for obtaininhg a functional relationship that relates the errors to incoming quality. Results indicated that ignoring the variation in the errors grossly under estimates the expected total cost (ETC) and may provide more confident in the quality of accepted components than it is true quality level.

Key Words: Optimal Inspection Plan, Receiver's Operating Characteristic Curve, Constrained Regression, Signal Detection Theory

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1.0 Introduction

Inspection is carried out at many stages of the production process. Despite the increased effort in process control acceptance sampling will continue to be an important topic in quality control. In the past a large amount of theortical and applied research has been aimed at improving the inspection operation.(Drury et al. [1979], Harris and Chaney [1969], Rizzi et al. [1979].) Indeed, the quality control engineer faced with the design of an inspection plan has to address many factors related to the inspector, the process, and/or the manufacturing design. He has to understand all these factors and optimize their effects to reach to an optimal design of the plan.

Effect of inspection errors on single sampling plans have received considerable attention in the past. Ayoub et al. [1970] presented formula for the Average Outgoing Quality (AOQ) and Average Total Inspection (ATI) for a single sampling plan under inspection error. Latter Collins et al. [1972] relaxed the assumption of perfect replacement and allowed defective replacement in the formula of ATI. Case et al. [1973] developed similar results for Dodge's Sampling plan for continuous production and Collins et al. [1973] studied the effect of inspection errors on single sampling plan. Bennet et al . [1974, 1975] investigated the effect of inspection errors on a cost-based single sampling plans.

Repeat or multiple inspection has been proposed for improving the Average Outgoing Quality (AOQ) (Schlegel[1973]). Unfortunately multiple inspection tends to increase the overall cost of the operation, however this is not true for all operations, and cost considerations for each operation must be evaluated independently. Bennett, Case and Schmidt [1974] have shown through several examples how to conduct such an evaluation.

A succussfull use of repeat or multiple inspection for the quality control of critical multicharacteristic components have been demonstrated by Raouf et al. [1983]. A critical multicharacteristic component is a component having several characteristic and could cause disaster or high cost upon failure. Such components can be a part of an air craft, space shuttle or a complex gas ignition system. The justification for repeat inspection (more than 100% inspection) is that inspection is never perfect. Inspectors commit type I error (false rejection) and type II error (false acceptance). In case of critical components the cost of false acceptance is much higher in order of magnitude than the cost of false acceptance or the cost of inspection. Therefore it has been suggested and shown by Jain [1982] that repeat inspections is likely to reduce the expected cost of false acceptance and increase the expected cost of false rejection and inspection, but the expected total cost, which is the sum of the three costs is likely to reduce.

Another development which has significant impact on the quality control area is the increasing use of complete inspection plans. The development is mainly due to the growth in automatic manufacturing systems which makes complete inspection (100% inspection) inexpensive and reliable (Tang[1987]). Raouf et al. [1983], developed a minimum cost complete repeat inspection plan for multicharacteristic critical components. Lee [1988], simplified Raouf et al. model and obtained simple optimality conditions for the model. Jariedi et al. [1987] utilized multiple inspections to obtain a desired level of AOQ. Latter Duffuaa and Raouf [1989] developed three models for multicharacteristic critical components. The work of Duffuaa and Raouf [1989] extends the work of Raouf et al. [1983] by considering other objectives such as minimizing the probability of accepting a defective component. Maghsoodloo [1978], analyized the effect of error on multistage sampling plan and Tang and Schneider [1987], examined the effects of inspection errors on a complete inspection plan developed based on the models of Tang [1987]. Duffuaa [1994] studied the statistical and economic impact of inspection errors on performance measures of complete inspection plan. He concluded that the error has a significant impact and need to be incorporated in the design of repeat inspection plans.

All the modells that incorporate inspection error assume that type I and type II are fixed and known. This assumption is not true because these errors change with the incoming quality Harris [1968]. Repeat inspection means each characteristic is inspected more than once. After the first inspection the quality (probability that the characteristic is defective) of each characteristic changes. Therefore the characteristic arrives to the second inspection cycle with different quality level. This new level, of quality has different type I and type II errors. In other words inspectors commit different level of errors for different quality levels. This has not been incorporated in the models in the literature. To have a realistic inspection plan the change in the errors must be incorporated in the design of the inspection plan.

The purpose of this report is to extent the repeat inspection plan and its model developed by Raouf et al. [1983] by incorporating the dynamic changes in the error in the design of the plan. This is accomplished through the development of a procedure that estimates the two types of errors as a function of incoming quality. The procedure utilizes Receiver's Operating Characteristic (ROC) curve and constrained regression to obtain a functional relationship between incoming quality and type I and type II errors. Then the errors are updated using the functional relationship prior to each repeat inspection.

The rest of the report is organized as follows. Section 2 presents a summary of Raouf et al model. Section 3 provides the procedure used for errors estimation and in Section 4 the new model is outlined. Section 5 provides the results of the comparisons between Raouf et al model and the proposed model under varying inspection error. Section 6 concludes the report

2.0 Model and Plan Description

The complete repeat inspection plan in this paper is the one proposed by Raouf et al. [1983] for the inspection of critical components where incoming items are subjected to several cycles of inspections. The plan is applied as follows: An inspector inspects one particular characteristic for each component entering the inspection process and all the accepted components go to the second inspector, who inspects the second characteristic. This chain of inspection continues until all the characteristics are inspected once. This completes one cycle of inspection. All accepted components, if necessary, go to the next cycle of inspection, and the process is repeated a total of n times before the component is finally accepted. Here n is the optimal number of inspections necessary to minimize the total expected cost per accepted component. Finally, the accepted components will be those which are accepted in the nth cycle, and the totality of rejected components will be the sum of those rejected in the 1st, 2nd, ..., nth cycles.

The model is developed for components having N characteristics, with incoming quality P_i, i = 1, 2, ..., N. A component is classified as non defective only if all the characteristics met the quality specifications. Characteristics defective rates are assumed independent. The probabilities of type I error, E_{1i} and type II error E_{2i} , i = 1, 2, ..., N, are assumed to be known. Three different types of costs are considered: (1) cost due to false rejection of a non defective component, C_r, (ii) cost due to false acceptance of defective component, C_a, and, finally (iii) cost of inspection, C_i, i = 1, 2, ..., N. Prior to stating the model the following notations,which are consistent with previous notation on this subject are defined. In the notation i ranges from 1 to N, and j from 1 to n.

Notation:

M_j Number of components entering the jth cycle of inspection.

N Number of characteristics in each component to be inspected.

P_i Probability of ith characteristic in the sequence of inspection being defective entering the inspection.

PG Probability of a component being nondefective entering the inspection.

PGC	Probability of a component being defective entering the inspection, the complement of PG.
E_{1i}	Probability of classifying the ith nondefective characteristic in the sequence of inspection as defective (type I error).
E_{2i}	Probability of classifying the ith defective characteristic in the sequence of inspection as nondefective (type II error).
$P_i(j)$	Probability of the ith characteristic in the sequence of inspection being defective on entering the jth cycle.
PG(j)	Probability of a component being nondefective on entering the jth cycle.
PGC(j)	Probability of a component being defective on entering the jth cycle, complement of PG(j).
$M_{i,j}$	Extended number of components entering the ith stage of inspection in the jth cycle.
$PG_{i,j}$	Probability of a component being nondefective in the ith stage of the jth cycle.
$FR_{i,j}$	Expected number of falsely rejected components in the ith stage of the jth cycle.
$FA_{i,j}$	Expected number of falsely accepted components in the ith stage of the jth cycle.
$CA_{i,j}$	Extpected number of correctly accepted components in the ith stage of the jth cycle.
$R_{i,j}$	Rate of rejection of components due to ith characteristics in the sequence of inspection in the jth cycle.
A(j)	Extended number of accepted components in the jth cycle.
CFR(j)	Cost of false rejection in the jth cycle.
CFA(j)	Cost of false acceptance in the jth cycle.
CI(j)	Cost of inspection in the jth cycle.
TCFR	Total cost of false rejection.
TCFA	Total cost of false acceptance.
TCI	Total cost of inspection.

TA Total number of accepted components.

$E(tc)|_j$ Expected total cost per accepted component after j cycles of inspection

E ( ) Expected value of the argument inside the parentheses

Basic relationships in the model

The probability of the ith characteristics being defective will vary from cycle to cycle. The relationship between $P_i(j)$ and P_i is given below

$$P_i(1) = P_i \tag{1}$$

Using Baye's theorem

$$P_i(2) = P_iE_{2i}/[P_iE_{2i} + (1 - P_i)(1 - E_{1i})] \tag{2}$$

Similarly

$$P_i(3) = P_i(2)E_{2i}/[P_i(2)E_{2i} +(1 - P_i(2))(1 - E_{1i})] \tag{3}$$

and from the symmetry of expressions (2) and (3) we get

$$P_i(j) = P_i(j - 1) E_{2i}/[P_i(j - 1)E_{2i} +(1 - P_i (j - 1)(1 - E_{1i})] \tag{4}$$

The probability of a characteristic being defective changes in each cycle, hence the probability of a component being nondefective also changes. It is given below

$$PG = \prod_{i=1}^{N} (1 - P_i) \tag{5}$$

The probability of a component being defective is

$$PGC = 1 - PG \tag{6}$$

Clearly,

$$PG(1) = PG = \prod_{i=1}^{N} (1 - P_i(1)) \tag{7}$$

The probability of a component being nondefective entering the jth cycle is

$$PG(j) = \prod_{i=1}^{N} (1 - P_i(j)) \tag{8}$$

The probability of a component being defective entering the jth cycle is

$$PGC(j) = 1 - PG(j) \tag{9}$$

When there is no inspection, the expected total cost per accepted component will simply be the cost of false acceptance of all the defective components

$$E(tc)|_{j=0} = C_a (1 - PG) \tag{10}$$

The expected total cost per accepted component, after n cycles of inspection, can be written as

$$E(tc)|_{j=n} = [TCFR + TCFA + TCI]/TA \tag{11}$$

where TCFR, TCFA, TCI, and TA are as defined earlier.

Cost minimization model

The objective of this model is to determine the optimal inspection plan for multicharacteristic components. The model minimizes the expected total cost per accepted component resulting from type I errors, type II errors, and cost of inspection. Given the basic relationships in the previous section, a mathematical expression for expected total cost per accepted component will be obtained. Our objective is to minimize this cost subject to the relationships governing this situation.

In order to derive the expected total cost of inspection after n cycles of inspections, analysis of cycle 1 of inspection is necessary. All the components entering cycle 1 go to the first inspector, who inspects the first characteristic in each component in order to classify it as defective or nondefective. This is the first stage of inspection.

Stage 1 in cycle 1: Number of components entering this stage is

$$M_{1,1} = M_1 \tag{12}$$

The probability of a component being nondefective is

$$PG_{1,1} = PG \tag{13}$$

E (number of falsely rejected components) is

$$\begin{aligned} FR_{1,1} &= M_{1,1}PG_{1,1}E11 \\ &= M_1PGE_{11} \end{aligned} \tag{14}$$

E (number of falsely accepted components) is

$$FA_{1,1} = M_{1,1}[P_1E_{21} + (1 - PG_{1,1} - P_1)(1 - E_{11})]$$

$$= M[P_1E_{21} + (1 - PG - P_1)(1 - E_{11})] \quad (15)$$

E (number of correctly accepted components) is

$$CA_{1,1} = M_{1,1}PG_{1,1}(1 - E_{11})$$

$$= MP\ \ G\ (1\ E_{11}) \quad (16)$$

All accepted components in this stage go to the second inspector who inspects the second characteristic of each component in order to classify it as defective or nondefective.

Stage 2 of the first cycle

$$M_{2,1} = FA_{1,1} + CA_{1,1}$$

$$= M_1[P_1E_{21} + (1 - P_1)(1 - E_{11})] \quad (17)$$

Using equation (8), we get,

$$PG_{2,1} = \left(1 - P_1(2)\right) \prod_{i=2}^{N} (1 - P_i)$$

Substitute the value of $P_1(2)$, the following following formula is obtained,

$$= PG(1 - E_{11})/[P_1E_{21} + (1 - P_1)(1 - E_{11})] \quad (18)$$

$$FR_{2,1} = M_{2,1}PG_{2,1}E_{12}$$

$$= M_1PG(1 - E_{11})E_{12} \quad (19)$$

$$FA_{2,1} = M_{2,1}[P_2E_{22} + (1 - PG_{2,1} - P_2)(1 - E_{12})]$$

$$= M_1[P_2E_{22} + (1 - PG_{2,1} - P_2)(1 - E_{12})] \quad (20)$$

$$CA_{2,1} = M_{2,1}PG_{2,1}(1 - E_{12})$$

$$= M_1PG \prod_{i=1}^{2} (1 - E_{1i}) \quad (21)$$

By symmetry, we can obtain stage N of the first cycle

$$M_{N,1} = M_1 \prod_{i=1}^{N-1} [P_iE_{2i} + (1 - P_i)(1 - E_{1i})] \quad (22)$$

$$PG_{N,1} = PG \prod_{i=1}^{N-1} [(1 - E_{1i}) / (P_i E_{2i} + (1 - P_i)(1 - E_{1i}))] \tag{23}$$

$$FR_{N,1} = M_1 PG \prod_{i=1}^{N-1} (1 - E_{1i}) E_{1N}$$

(24)

$$FR_{N,1} = M_1 PG \prod_{i=1}^{N-1} (1 - E_{1i}) E_{1N}$$

$$x[P_N E_{2N} + (1 - PG_{N,1} - P_N)(1 - E_{iN})] \tag{25}$$

$$CA_{N,1} = M_1 PG \prod_{i=1}^{N} (1 - E_{1i}) \tag{26}$$

This completes one cycle of inspection, and the result of this cycle is described by the following equations.

Number of accepted components after completing the first cycle is,

$$A(1) = FA_{N,1} + CA_{N,1} \tag{27}$$

Cost of false rejection is

$$CFR(1) = C_r \sum_{i=1}^{N} (FR_{i,1}) \tag{28}$$

Cost of false acceptance is

$$CFA(1) = C_a(FA_{N,1}) \tag{29}$$

Cost of inspection is

$$CI(1) = \sum_{i=1}^{N} C_i M_{i,1} \tag{30}$$

E (total cost per accepted components after one cycle of inspection is)

$$E(tc)|_{j=1} = [CFR(1) + CFA(1) + CI(1)]/A(1) \tag{31}$$

where CFR(1), CFA(1), CI(1), and A(1) are given by equations (28), (29), (30) and (27), respectively.

Before proceeding to the second cycle, it was shown by Raouf et al. [1983] that the manner in which characteristics are ordered for inspection affects only the cost of inspection. Then Duffuaa and Raouf [1990] formulated an optimal rule f in [13] for minimizing the cost of inspection within each inspection cycle. To minimize the cost of inspection, this rule should be applied in each cycle. The rule states: at inspection cycle j, compute the ratio $C_i/R_{i,j}$ for all i. The formula for $R_{i,j}$ is given in equation (36). Then, for each component, first inspect the characteristic with the least ratio and, lastly, inspect the one with the highest ratio. This rule ensures that CI(j) is minimized within cycle j.

From the analysis of cycle 1, it can easily be seen that after this cycle we can compute the new values of $P_i(2)$, PG(2), M_2 and proceed in the same manner as in the first cycle to compute the cost of false rejection, cost of false acceptance and cost of inspection. Hence, by symmetry the results of the nth cycle can be obtained.

$$A(n) = FA_{N,n} + CA_{,n} \tag{32}$$

$$CFR(n) = C_r \sum_{i=1}^{N} (FR_{i,N}) \tag{33}$$

$$CFA(n) = C_a (FA_{N,n}) \tag{34}$$

$$CI(n) = \sum_{i=1}^{N} C_i M_{i,n} \tag{35}$$

The ratio used to determine the optimal ordering of characteristics in the nth inspection cycle is $C_i/R_{i,n}$, $i = 1, ..., N$ where

$$R_{i,n} = P_i(n)(1 - E_{2i}) + (1 - P_i(n))E_{1i} \tag{36}$$

After n cycles of inspection we must determine the total cost of inspection per accepted component. It is determined from the total cost of false rejection TCFR, total cost of false acceptance TCFA, and the total cost of inspection given below.

$$TCFR = \sum_{j=1}^{n} [CFR(j)] \qquad (37)$$

$$TCFA = CFA(n) \qquad (38)$$

$$TCI = \sum_{j=1}^{n} [CI(j)] \qquad (39)$$

Total accepted components

$$TA = A(n) \qquad (40)$$

The above equations (1) through (40) provide the basic relationship for the model; the purpose is to find the value of n which minimizes the expected total cost per accepted component. The above model can be stated as find n which minimizes equation (41).

$$\text{Min } E(tc)|_{j=n} \qquad (41)$$

The optimal n is found as follows: we start with no inspection and the expected total cost is computed. Then the number of inspections is incremented by one and the expected total cost is computed and compared with the previous one. As soon as the expected total cost starts to increase as a function of n we stop and the optimal n is the previous value of n.

3.0 Procedure for Error Probabilities Estimates

Signal detection theory (SDT) has been found to be useful in modeling the performance of industrial inspector (Wallack and Adams[1970]). In the context of repeat inspection the incoming quality is equivalent to the probability of occurrence of a signal. Jaraiedi [1983] has developed two procedures for error estimation using the concept of SDT and ROC curves. In this section the work of Jaraiedi is refined by using constraint regression and polynomial relationships to estimate the error probabilities as a function of incoming quality.

The ROC curve is graph of false alarm (type I error, E_1), versus the probability of a hit (which is 1-type II error ($1-E_2$) in percentages. The ROC analysis of the detection task Inspection), where the observation interval is constant, revealed that a change in *P* results in a significant change in the level of detection of defects (Fox and Haslegrave [1969]), and causes the observation to shift along the ROC curve due to criterion adjustments made by inspectors. In such a case, one can assume that the length of the ROC curve traveled due to change in *P* which is proportional to the amount of the change. An increase in *P* causes a shift towards the upper corner of the ROC space, while a decrease in *P* has the opposite effect. Based on this assumption, a method for estimation of two types of error probabilities is developed.

Given two known defect rates and their corresponding points on the ROC curve, we see that the problem is how to find the point on the curve which corresponds to a value of *P* between the two known points. To estimate the Type-I and Type-II error probabilities at a certain incoming fraction defective *P* which is between the two known points p_1 and p_2, can be used. Using the assumption that distance traveled on the curve is proportional to the change in signal rate.

$$\frac{S_{x_1, x_2}}{S_{x_1, x_p}} = \frac{p2 - p1}{p - p1} \tag{42}$$

$$\frac{h(x_2) - h(x_1)}{h(x_p - h(x_1)} = \frac{p2 - p1}{p - p1} \tag{43}$$

Solving for $h(x_p)$ the result is:

$$h(x_p) = \frac{p - p1}{p2 - p1} [h(x_2) - h(x_1)] + h(x_1) \tag{44}$$

Hence, the Type-I error probability, E_1, becomes:

$$E_1 = h^{-1}(x_p) \tag{45}$$

and given the equation for ROC curve, the Type-II error probability is given as:

$$E_2 = 100 - f\ (E_1) \tag{46}$$

Using stepwise regression and fractional power function Jaraiedi obtained functional relationships for E_{1i} and E_{2i} . Next a systematic procedure based on constrained regression and polynomial relationship is presented The steps of the procedure for developing functional relationships between incoming quality and type I and type II errors is as follows:

1. From the properties of ROC curve it must pass in the point (0, 100). The axis of the ROC are in percent. Second it is an increasing function and concone In other words:

$$f' = \frac{df}{dx} > 0 \quad \text{for } 0 < x \quad < 100$$

and

$$f'' = \frac{d^2 f}{dx^2} < 0 \quad \text{for } 0 < x \quad < 100$$

Therefore use a constrained linear regression to fit the ROC. The functional format used in the ROC curve is fractional power functin as suggested in by Jaraiedi [1983].

2. Use the assumption embeded in equations (42-46) to generate E_1 and E_2 and P from the ROC curve.

3. Use a polynomial relationship between the P and the errors and again employ constrained regression to obtain the estimates of the coefficient of the polynomial.

The above methodology is demonstrated using the data in the experiment reported by Harris in [1968]. The problem of estimating the ROC curves is formulated as a constrained regression and it is a non-Linear program as stated below:

$$\text{Min} \sum_{i=1}^{n} [(1-E_2) - f(E_1)]^2 \tag{47}$$

$$\text{s.t.}$$

$$f'(E_1) \geq 0$$

$$f''(E_1) \leq 0$$

$$f(0) = 0$$

$$f(100) = 100$$

The functional form used for the ROC curve is:

$$P(\text{Hit}) = (1\text{-}E_2) = A_1\,(E_1)^{1/N1} + A_2(E_1)^{1/N2} \tag{48}$$

The software GINO (General Interactive Optimizer) developed by Liebman, Lasdon, Sharge, and Warren [1986] is used to solve the program in (47) and the following estimates for A_1, A_2, N_1 and N_2 are obtained.

$A_1 = -54.6177$, $A_2 = 127.3531$, $N_1 = -16.6549$ and $N_2 = 74.7161$.

Hence the ROC curve is

$$(1\text{-}E_2) = -54{,}6177\ E_1^{-0.093862} + 127.3531\ E_1^{0.013384} \tag{49}$$

E_1 and E_2 in (49) is in percentages.

Once the ROC curve has been fitted, many points corresponding to Type-I and Type-II error can be generated, using equations (42-46). A Fortran program was developed that used the numerical integration method to generate the desired points for a given defective rate as given in Table 1 below:

P	$E_1(\%)$	$E_2(\%)$
0.01	2.001	26.668
0.02	2.047	26.418
0.025	2.188	25.683
0.03	2.375	24.765
0.04	2.750	23,102

Table 1. Corresponding E_1 and E_2 for a given P

After generating data for P and E_1 and E_2 the following models are developed for testing the functional relationships between P, E_1 and E_2. Experimentation with polynomials of different degrees, it has been found that a polynomial of degree 4 provides a smooth and minimum sum of sequence errors.

Model for E_1

$$\min \sum_{i=1}^{n} \left[E_{1i} - f_i(P_i)\right]^2 \tag{50}$$

s.t.

$$f_1'(P_i) \geq 0$$

$$f_1(P_1) \leq 0.5$$

$$f_1(P_i) \geq 0$$

$$n = 1, 2, \ldots 5.$$

Model for E_2

$$\min \sum_{i=1}^{n} \left[E_{2i} - f_2(P_i)\right]^2 \tag{51}$$

s.t.

$$f_2(P_i) \le 0$$

$$f_2(P_1) \le 0.5$$

$$f_2(P_1) \ge 0$$

f_1 and f_2 are restricted from the class of monotone polynomials functions. Using the date in Table 1 and GINO, the following two polynomials have been found the best to describe the data:

$$E_1 = 1.977 - 0.049p + 0.0449p^2 + 0.00419p^3 - 0.000041p^4 \qquad (52)$$

and

$$E_2 = 26.786 + 0.2789p - 0.2675p_2 - 0.0122p_3 + 0.00083p^4 \qquad (53)$$

The above procedure and functional relationship is the key to extending the model by Raouf et al. [1983] to varying inspection errors.

4.0 Inspection Plans Under Varying Inspection Errors

The models developed previously assume type I and type II errors are constant through the inspection process. However it has been verified empirically that the errors are function of incoming quality (Harris [1968]). Type I error increases as incoming quality P increases and type II error decreases with an increase in P.

The quality of characteristics P changes from cycle to cycle. Therefore in this paper it is proposed that after each cycle the functional relationships in equations (52 and 53) to be used to update the error probabilities before the next cycle. The equations in Section 2 are utilized with the new updated values of the error probabilities to estimate the optimal parameters of the inspection plan..

The updated algorithm for the extended model is as follows:

Step 1 Determine the PG and $E(tc)|_{j=0}$ from equations (5) and (10); respectively, set $j = 1$.

Step 2 Compute $P_i(j)$, $PG(j)$, $PG_{N,j}$, M_j and $C_i/R_{i,j}$ for i = 1, 2, ..., N using equations (4), (8), (23), (27), (36), respectively. Arrange the ratios $C_i/R_{i,j}$ (i = 1, 2, ..., N) in order of decreasing magnitude. This is the optimal sequence of inspection for the jth cycle.

Step 3 Rearrange the probabilities P_i, E_{1i}, E_{2i}, and the inspection cost C_i according to the optimal sequence obtained in step 2.

Step 4 Compute A(j), CFR(j), CFA(j), and CI(j) using equations (32) (33), (34), and (35) respectively.

Step 5 Compute TCFR, TCFA, TCI, and TA from equations (37), (38), (39), and (40) respectively.

Step 6 Compute $E(tc)|_j$, using equation (11).

Step 7 If $E(tc)|_j$ is less than $E(tc)|_{j-1}$, set j = j + 1, update E_{1i} and E_{2i} using equations (50) and (51). Go to step 2. Otherwise stop.

5. Comparison with Previous Model

In order to examine the effect of incorporating varying inspection error in the repeat inspection plan the following example is solved and compared with the model in the literature. The example consist of a batch of 100 ccomponents each has 3 characteristics. The rest of the data is in Table 2.

$N = 3$	$M = 100$	
$P_1 = 0.1$	$P_2 = 0.2$	$P_3 = 0.13$
$P_{11} = .05$	$P_{12} = 105$	$P_{13} = .05$
$P_{21} = 0.1$	$P_{22} = 0.1$	$P_{23} = 0.1$
$C_r = 100$	$C_r = 100$	$C_r = 100$
$C_r = 500$	$C_r = 500$	$C_r = 500$
	$C_a = 100000$	

Table 2. Data for the Example

The solution given by Raouf et al. model that assumes constant inspection errors is :

$n^* = 3$, ETC = 666.03   A(n) = 59

PG = .9975

The solution of the proposed model under varying inspection error is

$n^* = 3$, ETC = 2019.72   A(n) = 38

PG = 0.9916

From the solution of the above simple example it can be concluded that ignoring the concept of varying inspection error simplifies the inspection plan and model and grossly under estimates the expected total cost and provides higher values of PG. That may lead to believe that the quality of accepted components is higher than its true quality level.

6. Conclusion

In this paper the inspection plan and model proposed in [16] are generalized to handle varying inspection errors. This is accomplished by developing a systematic procedure for the errors estimation. The procedure utilizes signal detection theory, the ROC curve and constrained regression to come up with a functional relationship relating type I and type II errors to incoming quality.

It can be concluded that ignoring the variation in inspection error over simplifies the inspection plan and tend to provide more confidence in the quality of accepted components than it is true quality level.

Acknowledgment

The authors would like to acknowledge the support provided by King Fahd University of Petroleum and Minerals for conducting this research. Also the Department of Industrial and Operations Engineering, University of Michigan, Ann Arbor is Acknowledged for hosting the first author.

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