A General Inspection Plan For Critical Multicharacteristic Components

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Abstract

In this paper, a general inspection plan for critical multicharacteristic components is presented. In this plan the assumption in the literature that characteristics are inspected equal number of times is relaxed and allowed for different number of inspections for different characteristics depending on characteristic's defective rates and inspection cost. A mathematical model that depicts and represents the plan has been developed. A decent type algorithm is proposed to determine the optimal number of repeat inspections and sequence characteristics for inspection that minimizes the expected total cost. The expected total cost consists of the cost of false acceptence (cost if type II error), cost of false rejection (cost of type I error), and the cost of inspection. Emperical comparisons with the model in the literature on randomly generated problems has been conducted. The results have shown that the proposed plan performs better in terms of the expected total cost on 87 percent of the generated problems for the assumed specific parameters. The reduction in the expected total cost is up to 23.5 percent.

Key words Qualty Control, Optimal Inspection Plan, Optimal Sequence, Type I Error,Type II Error.

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1 Introduction

In production systems, inspection for acceptence purposes is carried out at many stages. This includes inspection of incoming material, in process inspection during the manufacturing operations and finished product inspection. Competitive forces and sometimes catastrophic failure have resulted in tight quality control of products. Componets whose failure result in catastrophy, serious hazard or very high cost are termed critical components. Such components usually have several characteristics and failure of one of them results in compenent failure. The quality requirements for such components are tight and field failure must be kept to the minimum level. Such components can be a part of an air-craft, a space shuttle or a complex gas ignition system. For critical components a common practice in industry is to institute multiple inspections. The reason for multiple inspection is that inspection is never perfect. There is always the possibility of false acceptence (type II error) and false rejection (type I error) . Both errors have costs i. e the cost of false acceptence and the cost of false rejection. In case of critical components the cost of false acceptence is much higher than the cost of false rejection, because falsely accepted components may result in system failure which may involve system loss and human lives losses. Therefore it is preceived and shown that repeat (multiple) inspections is likely to reduce the costs of the errors and increase the cost of inspection. However the expected total cost which is the the sum of the three costs is likely to reduce[11]. Hence a need exist to determine the optimal inspection plan and the optimal number of repeat inspection that minimizes the expected total cost.

Raouf and Elfeituri[12] conducted a study to investigate the factors which affect inspector accuracy and concluded that type II error is a more realistic criterion for measuring inspector accuracy. Ayoub et al [1] presented a formula for average outgoing quality(AOQ) and average total inspection(ATI) under inspection error. In a subsequent study, Collins et al [4] relaxed the assumption of perfect inspection of replacement and allowed defective replacement in the formula for AOQ and ATI. Case et al [3], presented similar results for Dodge's sampling plans for continous production. In [2], Bennett et al investigated the effect of inspection error on cost based single sampling plan design.

In the literature of multicharacteristic critical components inspection, Raouf et al proposed the following inspection plan [12]. The plan is described as follows: an inspector inspects one particular characteristic for each component entering the inspection process, and all the accepted components go to the second inspector who inspects the second characteristic. Then all accepted components go to the third inspector who inspects the third characteristic. The chain of inspection continues until all characteristics are inspected once. This completes one cycle of inspection and this process is repeated n before the component is finally accepted. Here n is the optimal number of repeat inspections needed to minimize total expected cost. Also in [13] they developed the initial model for determining the optimal number of repeat inspections which minimizes expected total cost. In [9] Duffuaa and Raouf established an optimal rule for sequencing characteristics for inspection in the plan proposed by Raouf. In [5], Duffuaa and Nadeem extended the model in [13] to situations where characteristic's defective rates are statistically dependent. Lee [11], simplified the model in [13] and obtained simple optimality conditions for the model. Using the plan in [13] Duffuaa and Raouf proposed three models for repeat multicharacteristic inspection [8]. In their model they considered another criterion which is minimizing the probability of accepting a defective component. Recently Duffuaa and Al-Najjar [6], proposed an alternative plan ,where the first inspector performs n inspections on the first characteristic, prior to passing the accepted components to the second inspector who inspects the second characteristic n times, prior passing the accepted compenents to the third inspector. This chain of inspection is continued untill all characteristics are inspected. In this plan inspection is done in stages and in a way decenteralized .

Both plans in the literature requires equal number of inspections for different characteristics. This requirement simplifies the model for the plan, but however it is unrealistic and not cost effective. The reason for that different characteristics do not have the same defective rate nor the same cost of inspection. Therefore it is more realistic to relax the assumption of equal number of inspections for different characterictics. The plan proposed by Duffuaa and Al-Najjar can be modified to allow different number of inspections for different characteristics. The general plan proposed in this paper is as follows: the first inspector inspects one particular characteristics n_1 times, then passes all accepted components to the second inspector who inspects the second characteristic n_2. This chain is continued untill all characteristics are inspected, n_1,$n_2, \ldots, n_N$, where N is the number of characteristics the component has.

The objective of this paper is to propose a general inspection plan for critical multicharacteristic components. Then develop a model that depicts the plan and outline an algorthm to determine the optimal inspection plan that minmizes the expected total cost per accepted component. Comparasions with the original model and the plans in the literature will be presented.The rest of the paper is organized as follows: Section 2 presents the problem and the general inspection plan. Section 3 contains model development. An algorthm for determing the optimal, $n_1, n_2, \ldots, n_N$, is outlined in section 4 and comparisons with the plans and models in the literature are given in section 5. Section 6 concludes the paper.

2 PROBLEM AND PLAN DEFINITION

Prior to problem definition and model formulation ,the following notation is adopted. In the notation i ranges from 1 to N and j ranges from 1 to n.

M	Number Of components to be inspected .
M_i	Number of components entering the i-th stage of inspection.
$M_{i,j}$	Number of components entering The j-th cycle of stage i.
N	Number of characteristics in each component to be inspected
P_i	Probability of the i-th characteristic being defective entering the inspection.
C_i	Cost of inspection of characteristi.
n_i	Optimal number of repeat inspections for characteristic .
C_a	Cost of false acceptance per component.
C_r	Cost of false rejection per component.
PG	Probability of a component being nondefective entering the inspection .

E_{1i}	Probability of classifying the i-th nondefective characteristic in the sequence of inspection as defective (type I error) .
E_{2i}	Probability of classifying the i-th defective characteristic in the sequence of inspection as nondefective (type I Ierror) .
$P_{i,j}$	Probability of the i-th characteristic in the sequence of inspection being defective entering the j-th cycle.
$PG_{i,j}$	Probability of a component being nondefective entering the j-th cycle of the ith stage .
PG_{i,n_i+1}	Probability of a component being nondefective after inspecting characteristic 1 througr i, each n_i times.
$FR_{i,j}$	Expected number of falsely rejected components in the j-th cycle of the i-th stage.
$FA_{i,j}$	Expected number of falsely accepted components in the j-th cycle of the i-th stage.
$CA_{i,j}$	Expected number of correctly accepted components in the j-th cycle of the i-th stag
$R_{i,j}$	Rate of rejection of components due to i-th characteristic in the sequence of inspection of the j-th cycle.
A_i	Expected number of accepted components in the i-th stage.
CFR_i	Cost of false rejection in the i-th stage.
CFA_i	Cost of false acceptance in the i-th stage.
CI_i	Cost of inspection in the i-th stage.
TCFR	Total cost of false rejection.
TCFA	Total cost of false acceptance.
TCI	Total cost of inspection.
TA	Total number of accepted components.
$E(tc)\|_j$	Expected total cost per accepted component after j-th stages of inspection.

The model is developed for critical component having N characteristics requiring inspection. The incoming quality of characteristic i is P_i. A component is classified as nondefective only if all the characteristics meet quality specifications. An inspector commits type I error E_{1i}, and type II error, E_{2i}, when he inspects characteristic i. Three different costs are considered : (*i*) Cost of false rejection of nondefective components, (cost of type I error), (*ii*) cost of false acceptance of defective components, (cost of type II error), and (*iii*) cost of inspection.

In order to control the quality of such critical components, the following general inspection plan is proposed. The plan is applied as follows: The first inspector inspects one particular characteristic n_1 times for each component entering the inspection process,(this is the first stage of inspection), all the accepted components go to the second inspector,who inspects the second characteristic n_2 times (this is the second stage of inspection). This chain of inspection continues untill all characteristics are inspected, $(n_1, n_2, \ldots, n_N)$. Here n_i is the optimal number of repeat inspections for characteristic i necessary to minimize the expected total cost per accepted component. Stage i has n_i cycles of inspection. The general inspection plan is shown in Figure 1. Finally, the accepted components will be those which are accepted at the N-th stage, and the rejected components is the sum of those rejected in the 1st, 2nd, ..., N-th stages.

The objective is to find the optimal number of repeat inspectons, n_i for characteristic i, $i = 1, \ldots, N$, in order to minimize the total expected cost. The expected total cost consists of the cost of false acceptance, cost of false rejection and cost of inspection. Next a mathematical model is developed to depict the proposed plan and aide in finding the optimal number of repeat inspections.

3 MODEL DEVELOPMENT

In the next three subsections the details of the model are presented.In the first subsection the basic relationships are derived,followed by the general expressions in the second subsection. Then the rule for finding the optimal sequence of inspection is given.

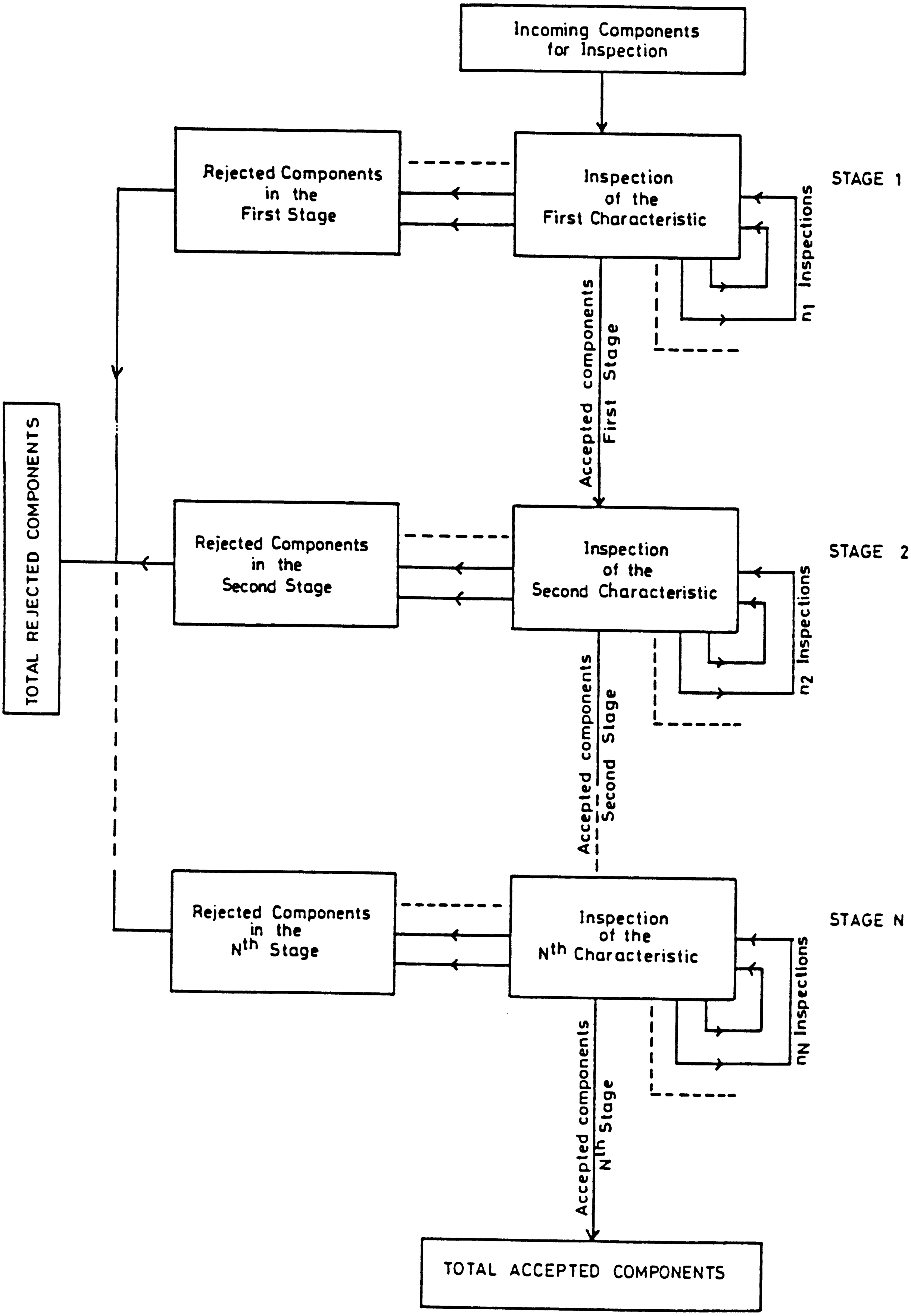

Figure 1. General Inspection Plan

3.1 Basic Relationships of the Model

This model and the one in the literature assume characteristic defective rates are statistically independent. This assumption is not highly restrictive and apply to many situations. For example when inspecting aircraft engines,characteristics crospond to different parts of the engine, which are made at different plants. It is highly likely that the defective rates of such parts are independent. Also independence can be used as a reasonable approximation for many other situations.

The probability of the i-th characteristic being defective will vary from cycle to cycle. First we shall establish the relationship between $P_{i,j}$ and P_i.
Expressing $P_{i,j}$ in terms of P_i.
Obviously

$$P_{i,1} = P_i \tag{1}$$

Using Bayes theorem,

$$P_{i,2} = \frac{P_i E_{2i}}{[P_i E_{2i} + (1 - P_i)(1 - E_{1i})} \tag{2}$$

Applying Bayes theorem again.

$$P_{i,3} = \frac{P_{I,2} E_{2i}}{[P_{i,2} E_{2i} + (1 - P_{I,2})(1 - E_{1i})]} \tag{3}$$

Substituting for $P_{i,2}$ from equation(2) into the above formula,gives after simplifiction,

$$P_{i,3} = \frac{P_i E_{2i}^2}{[P_i P_{2i}^2 + (1 - P_i)(1 - E_{1i})^2]} \tag{4}$$

Similarly,

$$P_{i,4} = \frac{P_i E_{2i}^3}{[P_i E_{2i}^3 + (1 - P_i)(1 - E_{1i})^3]} \tag{5}$$

In general from the symmetry of expressions (2), (3), and (4) the following is deduced:

$$P_{i,j} = \frac{PiE_{2i}^{j-1}}{[P_i E_{2i}^{j-1} + (1 - Pi)(1 - E_{1i})^{j-1}]} \tag{6}$$

The probability of a characteristic being defective changes in each cycle and so the probability of a component being nondefective. Bearing this in mind, we shall establish the relationship between $PG_{i,j}$, the probability of a componnt being nondefective entering the j-th cycle of the i-th stage, and the incoming quality, P_i.

Expressing PG in terms of P_i.

The probability of a component being nondefective is

$$PG = \prod_{i=1}^{N}(1 - P_i) \tag{7}$$

$$PG_{1,1} = PG \tag{8}$$

The probability of a component being nondefective after inspecting characteristic 1, n_1 times is:

$$PG_{1,n_1+1} = \prod_{i=2}^{N}(1 - P_i)[(1 - P_{1,n_1+1})] \tag{9}$$

The probability of a component being defective after inspecting all characteristics is :

$$PG_{N,n_N+1} = \prod_{i=1}^{N}(1 - P_{i,n_i+1}) \tag{10}$$

The probability of a component being defective after inspecting characteristic 1 through $i-1$, n_i times and characteristic i, m times and the other characteristics from $i+1$ through N are not inspected is given by:

$$PG_{i,k} = [\prod_{k=1}^{i-1}(1 - P_{k,n_k+1})][(1 - P_{i,m+1})][\prod_{k=i+1}^{N}(1 - P_k)] \tag{11}$$

When there is no inspection, the expected total cost per accepted component will simply be the cost due to false acceptance of defective components and is given by:

$$E(tc)|_{j=0} = C_a(1 - PG) \tag{12}$$

where C_a is the cost of false acceptance per component and PG is given by eqauation (7).

The expected total cost per accepted component, after inspecting all characteristics and characteristic i is inspected n_i times, is given as:

$$E(tc)|_{i=N,j=n_i} = [TCFR + TCFA + TCI]/TA \tag{13}$$

In order to determine TCFR, TCFA, TCI, and TA, an analysis of different stages of inspection is necessary.

3.2 Analysis of Stage (1)

All the components entering stage (1) go to the first inspector, who inspects the first characteristic in each component in order to classify it as defective or nondefective. The first stage has n_1 cycle of inspections. Following is the first cycle of inspection.

3.2.1 Cyle(1)

Number of components entering cycle 1 is

$$M_{1,1} = M_1 \tag{14}$$

The probability of a component being defective is

$$PG_{1,1} = PG \tag{15}$$

E(number of falsely rejected components) is

$$\begin{aligned} FR_{1,1} &= M_{1,1} PG_{1,1} E_{11} \\ &= M_1 PG E_{11} \end{aligned} \tag{16}$$

E(number of falsely accepted components) is

$$\begin{aligned} FA_{1,1} &= M_{1,1}[P_1 E_{21} + (1 - PG_{1,1} - P_1)(1 - E_{11})] \\ &= M_1[P_1 E_{21} + (1 - PG - P_1)(1 - P11)] \end{aligned} \tag{17}$$

E(number of correctly accepted componente) is

$$\begin{aligned} CA_{1,1} &= M_{1,1} PG_{1,1}(1 - E_{11}) \\ &= M_1 PG(1 - E_{11} \end{aligned} \tag{18}$$

All accepted components in this cycle go to the first inspector again to inspect the first characteristic for the second time. (perform cycle 2).

3.2.2 Cycle(2)

$$\begin{aligned}
M_{1,2} &= FA_{1,1} + CA_{1,1} \\
&= M_1[P_1E_{21} + (1 - PG - P_1)(1 - E_{11})] + M_1PG(1 - E_{11}) \qquad (19) \\
&= M_1[P_1E_{21} + (1 - P_1)(1 - E_{11}) \\
PG_{1,2} &= (1 - P_{1,2})[\prod_{i=2}^{N}(1 - P_i)] \\
&= PG(1 - E_{11})/[P_1E_{21} + (1 - P_1)(1 - E_{11})] \qquad (20) \\
FR_{1,2} &= M_{1,2}PG_{1,2}E_{11} \\
&= M_1PG(1 - E_{11}) \qquad (21) \\
FA_{1,2} &= M_{1,2}[P_{1,2}E_{21} + (1 - PG_{1,2} - P_{1,2})(1 - E_{11}) \\
&= M_1[P_1E_{21} + (1 - P_1)(1 - E_{11})][P_{1,2}E_{21} \\
&\quad +(1 - PG_{1,2} - P_{1,2})(1 - E_{11})] \qquad (22) \\
CA_{1,2} &= M_{1,2}PG_{1,2}(1 - E_{11}) \\
&= M_1PG(1 - E_{11}) \qquad (23) \\
& \qquad (24)
\end{aligned}$$

Similarly,

3.2.3 Cycle(3)

$$\begin{aligned}
M_{1,3} &= FA_{1,1} + CA_{1,1} \\
&= M_1[P_1E_{21} + (1 - P_1)(1 - E_{11})][^2P_1E_{21} + (1 - P_{1,2})(1 - E_{11}) \\
PG_{1,3} &= (1 - P_{1,3})[\prod_{i=2}^{N}(1 - P_i) \\
&= PG(1 - E_{11})^2/[P_1E_{21} + (1 - P_1)(1 - E_{11})] \times \\
&\quad [P_{1,2}E_{21} + (1 - P_{1,2})(1 - E_{11})] \qquad (25) \\
FR_{1,3} &= M_1PGE_{11}(1 - E_{11})^2 \qquad (26) \\
FA_{1,3} &= M_1\prod_{j=1}^{2}[P_{1,j}E_{21} + (1 - P_{1,j})(1 - E_{11})] \times
\end{aligned}$$

$$[P_{1,3}E_{21} + (1 - PG_{1,3} - P_{1,3})(1 - E_{11})] \quad (27)$$

$$CA_{1,2} = M_1 PG(1 - E_{11})^3 \quad (28)$$

$$(29)$$

3.2.4 Cycle(n_1)

From the symmetry of the expressions, the n_1 th cycle of the first stage results can be writtern as follows:

$$M_{1,n_1} = M_1 \prod_{j=1}^{n_1} [P_{1,j}E_{21} + (1 - P_{1,j})(1 - E_{11}] \quad (30)$$

$$PG_{1,n_1} = PG(1 - E_{11})^{n_1} / \prod_{j=1}^{n_1} [P_{1,j}E_{21} + (1 - P_{1,j})(1 - E_{11})] \quad (31)$$

$$FR_{1,n_1} = M_1 PGE_{11}(1 - P_{11})^{n_1} \quad (32)$$

$$FA_{1,n_1} = M_1 \left[\prod_{j=1}^{n_1-1} \{P_{1,j}E_{21} + (1 - P_{1,j})(1 - E_{11})\} \right] \times [P_{1,n_1}E_{21} + (1 - PG_{1,n_1} - P_{1,n_1})(1 - E_{11}) \quad (33)$$

$$CA_{1,n_1} = M_1 PG(1 - E_{11})^{n_1} \quad (34)$$

This completes stage one of the inspection.it has n_1 cycles.

3.3 Results of Stage (1)

E(number of accepted components) is

$$\begin{aligned} A(1) &= FA_{1,n_1} + CA_{1,n_1} \\ &= M_1 \left[[\prod_{j=1}^{n_1-1} \{P_{1,j}E_{21} + (1 - P_{1,j})(1 - E_{11})\}] \times [P_{1,n_1}E_{21} + (1 - PG_{1,n_1} - P_{1,n_1}) \times \right. \\ &\quad (1 - E_{11}) + PG(1 - E_{11})^{n_1}]] \end{aligned} \quad (35)$$

where PG_{1,n_1} is given in equation (9).

Cost of false rejection after one stag of inspection is completed is given by:

$$CFR_1 = C_r \sum_{j=1}^{n_1} FR_{1,j}$$

$$= C_r M_1 PGE_{11} \sum_{j=1}^{n_1} (1 - E_{11})^{j-1} \quad (36)$$

Cost of false acceptance after stage one is completed is:

$$\begin{aligned} CFA_1 &= C_a(FA_{1,n_1}) \\ &= C_a M_1 \left[\prod_{j=1}^{n_1-1} \{P_{1,j}E_{21} + (1 - P_{1,j})(1 - E_{11})\} \right] \times \\ &\quad [P_{1,n_1}E_{21} + (1 - PG_{1,n_1} - P_{1,n_1})(1 - E_{11})] \end{aligned} \quad (37)$$

Cost of inspection after stage one is compeleted is:

$$\begin{aligned} CI_1 &= C_1 \sum_{j=1}^{n_1} M_{1,j} \\ &= C_1 M_1 \left[\sum_{k=1}^{n_1} \prod_{j=1}^{k-1} \{P_{1,j}E_{21} + (1 - P_{1,j})(1 - E_{11})\} \right] \end{aligned} \quad (38)$$

E(total cost per accepted component after one stage of inspection) is:

$$E(tc)|_{i=1} = [CFR_1 + CFA_1 + CI_1]/A_1 \quad (39)$$

where A_1, CFR_1, CFA_1 and CI_1 are given by equations (35), (36), (37), and equation (38) respectively.

3.4 Analysis of Stage (2) of Inspection

All accepted components from stage one proceed to the second inspector who inspects the second characteristics.Therfore the expected number of components entering stage two is $M_{2,1} = A_1$ where A_1 is given by equation (37).

3.4.1 Cycle(1)

$$\begin{aligned} M_{2,1} &= FA_{1,n_1} + CA_{1,n_1} \\ &= M_1 \prod_{j=1}^{n_1} [P_{1,j}E_{21} + (1 - P_{1,j})(1 - E_{11})] \end{aligned}$$

$$
\begin{aligned}
PG_{2,1} &= PG(1-E_{11})^{n_1} / \left[\prod_{j=1}^{n_1}\{P_{1,j}E_{21} + (1-P_{1,j})(1-E_{11})\}\right] \\
FR_{2,1} &= M_1 PGE_{12}(1-E_{11})^{n_1} \\
FA_{2,1} &= M_1 \left[\prod_{j=1}^{n_1}\{P_{1,j}E_{21} + (1-P_{1,j})(1-E_{11})\}\right] \times [P_2E_{22} + (1-PG_{2,1}-P_2)(1-E_{12})] \qquad (40)\\
CA_{2,1} &= M_1 PG(1-E_{11})^{n_1}(1-E_{12})
\end{aligned}
$$

3.4.2 Cycle(2)

$$
\begin{aligned}
M_{2,2} &= FA_{2,1} + CA_{2,1} \\
&= M_1 \left[\prod_{j=1}^{n_1}\{P_{1,j}E_{21} + (1-P_{1,j})(1-E_{11})\}\right] \times [P_2E_{22} + (1-P_2)(1-E_{12}) \\
PG_{2,2} &= PG(1-E_{11})^{n_1}(1-E_{12}) / \left[\prod_{j=1}^{n_1} P_{1,j}E_{21} + (1-P_{1,j})(1-E_{11})\right] [P_2E_{22} + (1-P_2)(1-E_{12})] \\
FR_{2,2} &= M_1 PGE_{12}(1-E_{11})^{n_1}(1-E_{12}) \\
FA_{2,2} &= M_1 \left[\prod_{j=1}^{n_1}\{P_{1,j}E_{21} + (1-P_{1,j})(1-E_{11})\}\right] \times [P_2E_{22} + (1-P_2)(1-E_{12})] \times \\
&\quad [P_{2,2}E_{22} + (1-PG_{2,2}-P_{2,2})(1-E_{12})] \\
CA_{2,2} &= M_1 PG(1-E_{11})^{n_1}(1-E_{12})^2
\end{aligned}
$$

3.4.3 Cycle(n_2)

$$
\begin{aligned}
M_{2,n_2} &= M_1 \left[\prod_{j=1}^{n_1}\{P_{1,j}E_{21} + (1-P_{1,j})(1-E_{11})\}\right] \left[\prod_{j=1}^{n_2-1}\{P_{2,j}E_{22} + (1-P_{2,j})(1-E_{12})\}\right] \\
PG_{2,n_2} &= \frac{PG(1-E_{11})^{n_1}(1-E_{12})^{n_2-1}}{\left[\prod_{j=1}^{n_1}\{P_{1,j}E_{21} + (1-P_{1,j})(1-E_{11})\}\right]\left[\prod_{j=1}^{n_2-1}\{P_{2,j}E_{22} + (1-P_{2,j})(1-E_{12})\}\right]} \\
FR_{2,n_2} &= M_1 PGE_{12}(1-E_{11})^{n_1}(1-E_{12})^{n_2-1} \\
FA_{2,n_2} &= M_1 \left[\prod_{j=1}^{n_1}\{P_{1,j}E_{21} + (1-P_{1,j})(1-E_{11})\}\right] \times \left[\prod_{j=1}^{n_2-1}\{P_{2,j}E_{22} + (1-P_{2,j})(1-E_{12})\}\right] \times [P_{2,n} \\
&\quad +(1-PG_{2,n_2}-P_{2,n_2})(1-E_{12})] \\
CA_(2,n_2) &= M_1 PG(1-E_{11})^{n_1}(1-E_{12})^{n_2}
\end{aligned}
$$

3.5 Results of Stage (2)

$$\begin{aligned}
A_2 &= FA_{2,n_2} + CA_{2,n_2} \\
&= M_1 \left[\prod_{j=1}^{n_1}\{P_{1,j}E_{21} + (1-P_{1,j})(1-E_{11})\}\right]\left[\prod_{j=1}^{n_2-1}\{P_{2,j}E_{22} + (1-P_{2,j})(1-E_{12})\}\right] \\
&\quad \times [P_{2,n_2}E_{22} + (1-PG_{2,n_2} - P_{2,n_2})(1-E_{12})] + M_1 PG(1-E_{11})^{n_1}(1-E_{12})^{n_2} \\
CFR_2 &= C_r \sum_{j=1}^{n_2} FR_{2,j} \\
&= C_r M_1 PGE_{12}(1-E_{11})^{n_1} \sum_{j=1}^{n_2}(1-E_{12})^{j-1} \\
CFA_2 &= C_a(FA_{2,n_2}) \\
&= C_a M_1 \left[\prod_{j=1}^{n_1}\{P_{1,j}E_{21} + (1-P_{1,j})(1-E_{11})\}\right]\left[\prod_{j=1}^{n_2-1}\{P_{2,j}E_{22} + (1-P_{2,j})(1-E_{12})\}\right] \\
&\quad \times [P_{2,n_2}E_{22} + (1-PG_{2,n_2} - P_{2,n_2})(1-E_{12})] \\
CI_2 &= C_2 \sum_{j=1}^{n_2} M_{2,j} \\
&= C_2 M_1 \left[\prod_{j=1}^{n_1}\{P_{1,j}E_{21} + (1-P_{1,j})(1-E_{11})\}\right]\left[\sum_{k=1}^{n_2}\prod_{j=1}^{k-1}\{P_{2,j}E_{22} + (1-P_{2,j})(1-E_{12})\}\right]
\end{aligned}$$

3.6 Relationships needed to compute expected total cost

Using the symmetry in the results obtained from the analysis of stages 1 and 2, the following general expressions needed to compute the expected total cost can be derived.

Total number of accepted components after completing N stages of inspection, i.e, after inspecting the N-th characteristic is given as:

$$\begin{aligned}
A_N = M &\left[\prod_{k=1}^{N-1}\prod_{j=1}^{n_k}\{P_{k,j}E_{2k} + (1-P_{k,j})(1-E_{1k})\}\right] \times \\
&\left[\prod_{j=1}^{n_N-1}\{P_{N,j}E_{2N} + (1-P_{N,j})(1-E_{1N})\}\right] \times \\
&[P_{N,n_N}E_{2N} + (1-PG_{N,n_N} - P_{N,n_N})(1-E_{1N})] + \\
M &\left[PG\prod_{k=1}^{N}(1-E_{1k})\right] \qquad (41)
\end{aligned}$$

Cost of false acceptance at each stage i, $i = 1, ..., N$ is given as:

$$CFR_i = [C_r \times M \times PG \times E_{1i}] \left[\prod_{k=1}^{i-1} (1 - E_{1k})^{n_k}\right] \times \left[\sum_{k=1}^{n_i} (1 - E_{1i})^{k-1}\right] \tag{42}$$

Cost of false acceptance after completing the N-th stage of inspection is given as:

$$CFA_N = C_a M \left[\prod_{k=1}^{N-1} \prod_{j=1}^{n_k} \{P_{k,j} E_{2k} + (1 - P_{k,j})(1 - E_{1k})\}\right] \times \left[\prod_{j=1}^{n_N - 1} \{P_{N,j} E_{2N} + (1 - P_{N,j})(1 - E_{1N})\}\right] \times [P_{N,n_N} E_{2N} + (1 - PG_{N,n_N} - P_{N,n_N})(1 - E_{1N})] \tag{43}$$

Cost of inspection at each stage,for stage i,$i = 1, ..., N$, is given as:

$$CI_i = C_i M \left[\prod_{k=1}^{i-1} \prod_{j=1}^{n_k} \{P_{k,j} P_{2k} + (1 - P_{k,j})(1 - E_{1k})\}\right] \times \left[\sum_{k=1}^{n_i} \left\{\prod_{j=1}^{k-1} \{P_{i,j} E_{2i} + (1 - P_{i,j})(1 - E_{1i})\}\right]\right. \tag{44}$$

In order to determine the general expression for the expected total cost per accepted component, the following expressions must be determined. The expressions are, total cost of false rejection TCFR, total cost of false acceptance TCFA, total cost of inspection TCI and total number of accepted components, TA.

$$TCFR = \sum_{i=1}^{N} CFR_i \tag{45}$$

$$TCFA = CFA_N = C_a(FA_{N,n_N}) \tag{46}$$

$$TCI = \sum_{i=1}^{N} CI_i \tag{47}$$

$$TA = A_N = FA_{N,n_N} + CA_{N,n_N} \tag{48}$$

$$E(tc)|_{i=N} = \frac{TCFA + TCFR + TCI}{TA} \tag{49}$$

The objective is to find $n_1, n_2, ..., n_N$, that provide the minimum expected total cost.

3.7 Determing the Optimal Sequence of Inspection

The cost of inspection is influenced by the sequence in which characteristics are ordered for inspection, i.e, the order of the stages. The following rule provedes the optimal sequence of characteristic inspection.Let

$$r(i) = \frac{C_i f_1(R_{i,j})}{1 - f_2(R_{i,j})} \; j \overset{i=1,2,\ldots,N}{=} 1, 2, ..., n \tag{50}$$

where :

$$\begin{aligned} R_{i,j} &= P_{i,j}(1 - E_{2i}) + (1 - P_{i,j})E_{1i} \\ f_1(R_{i,j}) &= \sum_{j=1}^{n_i} \left[\prod_{k=1}^{j} (1 - R_{i,k-1}) \right] \\ f_2(R_{i,j}) &= \prod_{k=1}^{n_i} (1 - R_{i,k}) \end{aligned} \tag{51}$$

The characteristic with the lowest ratio is inspected first, next higher ratio second, and so on. The characteristic with the highest ratio is the N-th characteristic to be inspected. The optimality of this rule follows from the proof given by Duffuaa and Raouf[9]. Next a computational procedure is presented for finding the optimal $n_1, n_2, \ldots, n_N$ and hence the optimal inspection plan.

4 Computational Procedure

The computational procedure developed for finding the optimal parameters of the inspection plan depends on the concept of steepest decent. At iteration i characteristic i, has been inspected r_i times. The expected total cost at iteration i is:

$ETC(r_1, r_2, \ldots, r_N)$ The decent direction i,from $(r_1, r_2, \ldots, r_i, \ldots, r_N)$ to $(r_1, r_2, \ldots, r_i + 1, \ldots, r_N)$ is given by:

$$DIN(i) = ETC(r_1, r_2, \ldots, r_i + 1, \ldots, r_N) - ETC(r_1, r_2, \ldots, r_i, \ldots, r_N) \tag{52}$$

at each point the decent is computed in all directions. Then a move is made in the direction which has the largest decent. Suppose we are at the stage where each characteristic is inspected r_i. Then the steps of the algorithm are:

STEP(1):	Compute $ETC(r_1, r_2, \ldots, r_N)$
STEP(2):	Find $DIN(i), fori = 1, \ldots, N$.
	IF$DIN \geq 0$ for all i, go to step (6). Otherwise proceed.
STEP(3):	Find $\{\max_i \mid DIN(i) \mid \lvert DIN(i) < 0\} = DIN(k)$.
STEP(4):	Inspect characteristic k and compute
	$ETC(r_1, r_2, \ldots, r_k + 1, r_{k+1}, \ldots, r_N)$
STEP(5):	Go to step (2).
STEP(6):	Stop.The optimal number of inspections for each characteristics is
	$(r_1, r_2, \ldots, r_N)$ and the total expected cost is $ETC(r_1, r_2, \ldots, r_N)$.

The above algorithem is expected to provide a local minimum.

5 Results and Model Comparisons

In order to compare the developed model with the model in the literature the following example is given. A software is developed implementing the algorithm stated above and used to obtain the optimal number of repeat inspections. A batch consisting of 100 components is to be inspected .Each component has three characteristics. The data for the example is given in Table 1. The example is solved utilizing the model in the literature and the new model.

Table 1. Data for the example.

$M = 100$	$N = 8$		
$P_1 = 0.109$	$P_2 = 0.186$	$P_3 = 0.127$	$P_4 = 0.212$
$P_5 = 0.174$	$P_6 = 0.192$	$P_7 = 0.146$	$P_8 = 0.175$
$E_{11} = 0.126$	$E_{12} = 0.118$	$E_{13} = 0.075$	$E_{14} = 0.093$
$E_{15} = 0.051$	$E_{16} = 0.129$	$E_{17} = 0.102$	$E_{18} = 0.046$
$E_{21} = 0.088$	$E_{22} = 0.121$	$E_{23} = 0.112$	$E_{24} = 0.088$
$E_{25} = 0.130$	$E_{26} = 0.072$	$E_{27} = 0.077$	$E_{28} = 0.136$
$C_1 = 99$	$C_2 = 12$	$C_3 = 67$	$C_4 = 50$
$C_5 = 76$	$C_6 = 21$	$C_7 = 14$	$C_8 = 95$
	$C_a = 523248$		
	$C_r = 733$		

Solving the above example using Raouf et al model and the algorithm in [14], the following results has been obtained:

Optimal sequence 3, 7, 6, 4, 1, 3, 5, 8

Optimal number of repeat inspections = 3.

Minmun expected total cost = 16063.11

Probabilty of a component being nondefective is= 0.993716478

Solving the same example using the proposed plan and model, the following results has been obtained:

> Optimal sequence $2, 7, 6, 4, 1, 3, 5, 8$
>
> Optimal number of inspections in the order of the sequence above is $2, 3, 3, 3, 3, 2, 2, 3$
>
> Minmun expected total cost = 12280.25
>
> Probabilty of a component being nondefective is= 0.99365294

The proposed model showed improvement over the model in the literature in terms of the expected total cost. Actually the saving in expected total cost amount to 23.5 percent for the above example. This led to the following detailed comparisons on randomely generated inspection problems.

The parameters of the geneated problems are $N, C_i, C_a, C_r, P_i, E_{1i}$ and E_{2i},for $i = 1, 2, \ldots, N$. The parameters N, C_i, C_a, C_r are assumed to be uniformely distributed. The other parameters P_i, E_{1i}, E_{2i}. are assumed to be normally distributed with mean μ and variance σ^2 N is assumed to be a discrete uniform distribution that takes the values $1, 2, \ldots, 10$. C_i is uniformely distributed between 10 and 100. i.e $U(10, 100)$, $C_a \sim U(100000, 1000000)$ and $C_r \sim U(500, 1000)$. P_i is assumed normally distributed with mean $\mu = .05$ and variance $\sigma^2 = 0.014$ i.e $P_i \sim N(.05, .014)$. $E_{1i} \sim N(0.1, 0.0009)$ and $E_{2i} \sim N(0.1, 0.0009)$ for all i.

A program has been developed to generate 100 random inspection problems and solved to obtain the optimal inspection parameters using both models. From the experiment conducted using the generated problems with the above parameters assuming the given distributions, it was found that the proposed model gave better results in terms of the expected total cost in 87 percent of the generated problems. Raouf et al model gave better results on 3 percent of the generated problems. On 10 percent the two models gave the same results. In the problems where the two models agree all components have one characteristic and in that case the two plan become identical. The reduction in the total expected cost, when the proposed model performed better in comparison with Raouf et al model ranges from i to 23.5 percent,while the reduction ranges from 0.1 to 7 percent when Raouf et al performed better.The proposed model and plan tend to perfom better in terms of expected total cost when there is varability between the $P_i's$ i.e the characteristic defective rates vary

The above results demonstrate the superiority of the proposed approach and the resulting inspection plan over the one in the literature.

6 Conclusion

A general inspection plan has been proposed for the inspection of critical components with several characteristics. The plan allows for different number of inspections for different characteristics depending on characteristic's defective rate and cost of inspection. A moel has been formulated which depicts the plan and is employed in determing the optimal number of repeat inspections that minimizes the expected total cost per accepted component. A decent type algorithm has been developed for obtaining the optimal number of inspections for each characteristic. The model and the plan in this paper provide flexibility of variable number of inspections for different characteristics. A desirable property the models and plans in the literature lack. The model has few reaistic assumptions and as such may have a wide range of applicability. Results on randomly generated problems have shown that the new model and plan performed better in optimizing inspection plans for critical components than the ones in the literature.

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