An Optimal Complete Inspection Plan For Critical Multicharacteristic Components

S.O.Duffuaa*
Department of Industrial and Operations Engineering
University of Michigan
Ann Arbor, MI 48109-2117, USA
and
H.J.Al-Najjar**
Mac-Doland International,Riyadh,Saudi Arabia

Technical Report 94-13

April 1994

An Optimal Complete Inspection Plan For Critical Multicharacteristic Components

S.O.Duffuaa*
Department of Industrial and Operations Engineering
University of Michigan
Ann Arbor, MI 48109-2117, USA
and
H.J.Al-Najjar**
Mac-Doland International,Riyadh,Saudi Arabia

Abstract

In this paper, a new inspection plan for critical multicharacteristic components is presented. A mathematical model that depicts and represents the plan has been developed. An algorithm is proposed to determine the optimal number of repeat inspections and sequence characteristics for inspection that minimizes the expected total cost. The expected total cost consists of the cost of false acceptence (cost of type II error), cost of false rejection (cost of type I error), and the cost of inspection. Emperical comparisons with Raouf et al model on randomly generated problems have been conducted. The results have shown that the proposed plan performs better in terms of the expected total cost on 82 percent of the generated problems for the assumed specific parameters. The reduction in the expected total cost ranges from 0.1- 10 percent.

Key words Expected Cost of Inspection, Quality Control, Optimal Inspection Plan, Type I Error, Type II Error.

*Author for correspondence, currently on sabbatical leave from King Fhad University of Petroleum and Minerals, Dhahran, 31261, Saudi Arabia.

1 Introduction

Quality control can be accomplished through process control or product control. Product control is achieved through inspection plans.The focuss of this paper is on modelling and determing optimal inspection plans for critical components. A component is critical if it causes disaster or very high cost upon failure.Such components can be a part of an air-craft, a space shuttle or a complex gas ignition system.It has been pointed out by Zunzaniyka and Druy[1] that there could be as many as 14 characteristic for which the component may fail. For critical components a common practice in industry is to institute multiple inspections. The reason for multiple inspection is that inspection is never perfect. There is always the possibility of false acceptence (type II error) and false rejection (type I error). Both errors have costs i.e the cost of false acceptence and the cost of false rejection. In case of critical components the cost of false acceptence is much higher than the cost of false rejection, because falsely accepted components may result in system failure which may involve system loss and human lives losses. Therefore it is preceived and shown that repeat (multiple) inspections is likely to reduce the costs of the errors and increase the cost of inspection. However the expected total cost which is the the sum of the three costs is likely to reduce,Jain[2]. Hence a need exists to determine the optimal inspection plan and the optimal number of repeat inspections that minimizes the expected total cost.

Raouf and Elfeituri[3] conducted a study to investigate the factors which affect inspector accuracy and concluded that type II error is a more realistic criterion for measuring inspector accuracy. Ayoub et al[4] presented a formula for average outgoing quality(AOQ) and average total inspection(ATI) under inspection error. In a subsequent study, Collins et al[5] relaxed the assumption of perfect inspection of replacement and allowed defective replacement in the formula for AOQ and ATI. Case et al[6] presented similar results for Dodge's sampling plans for continous production. Bennett et al[7] investigated the effect of inspection error on cost based single sampling plan design.

In the literature of multicharacteristic critical components inspection, Raouf et al[8], developed the initial model for determining the optimal number of repeat inspections that minimizes expected total cost. Duffuaa and Raouf[9] established an optimal rule for sequencing characteristics for inspection. Also Duffuaa and Raouf[10] extended the model in reference 9 to situations where characteristic defective rates are statistically dependent. Lee[11], simpilified the model in reference 9 and obtained simple optimality conditions for the model.Duffuaa and Raouf[12], presented three models for multicharacteristic inspection.In their models they considered minimizing the probability of accepting a defective compenent as another criterion. All the models in the literature are developed for a single plan given in Raouf et al[8] and Duffuaa and Raouf[12]. The plan is defined as follows: an inspector inspects one particular characteristic for each component entering the inspection process, and all the accepted components go to the second inspector, who inspects the second characteristic. All accepted components go to the third inspector who inspects the third characteristic. This chain of inspection is continued until all characteristics are inspected once. This completes one cycle of inspection.All accepted components. if necessary,go to the next cycle of inspection and this process is repeated n times before the component is finally accepted.Here n is the optimal number of repeat inspections needed to minimize the expected total cost per accepted component. No alternative plan for repeat multicharacteristic inspection has appeared in the literature.

In this paper, a new plan is proposed for the inspection of critical components. In the proposed plan an inspector performs n repeat inspections on the characteristic prior to passing the component to the next inspector. Here n is the optimal number of repeat inspections needed to minimize the expected total cost. In this plan the inspection process is decenteralized and the inspection is performed in stages. Each stage has n cycles. An optimization model that depicts the new plan has been developed to determine the optimal n which minimizes the total expected cost per accepted component. The primary difference between the model in this paper and the one in the literature is in the way characteristics are sequenced for inspection. In the new model the rule used for sequencing characteristics reflects multiple inspections for each characteristic, while in the previous model the sequencing is done at the beginning of each cycle, in the old plan, for a single inspection. This difference leads to a different sequence of inspection and therfore a different cost of inspection. The difference in models structure is due to the difference in the inspection plans.

The rest of the paper is organized as follows: first problem and plan definition are given. Then the model that depicts the plan is presented, followed by the algorithm for obtaining the optimal n. Next comparisons with the model in the literature are performed and results are presented. Finally the conclusions of the paper are outlined.

2 PROBLEM AND PLAN DEFINITION

Prior to problem definition and model formulation ,the following notation is adopted: In the notations i ranges from 1 to N and j ranges from 1 to n.

M	Number Of components to be inspected .
M_i	Number of components entering the i-th stage of inspection .
$M_{i,j}$	Number of components entering The j-th cycle of stage i .
N	Number of characteristics in each component to be inspected .
P_i	Probability of the i-th characteristic being defective entering the inspection .
C_i	Cost of inspection of characteristic i .
n	Optimal number of repeat inspections .
C_a	Cost of false acceptance per component .
C_r	Cost of false rejection per component .
PG	Probability of a component being nondefective on entering the inspection .
E_{1i}	Probability of classifying the i-th nondefective characteristic in the sequence of inspection as defective (type I error) .

E_{2i}	Probability of classifying the i-th defective characteristic in the sequence of inspection as nondefective (type I Ierror) .
$P_{i,j}$	Probability of the i-th characteristic in the sequence of inspection being defective on entering the j-th cycle .
$PG_{i,j}$	Probability of a component being nondefective on entering the j-th cycle of the i-th stage of inspection .
$PG_{i,n+1}$	Probability of a component being nondefective after inspecting characteristics 1 through i, each n times .
$FR_{i,j}$	Expected number of falsely rejected components in the j-th cycle of the i-th stage .
$FA_{i,j}$	Expected number of falsely accepted components in the j-th cycle of the i-th stage .
$CA_{i,j}$	Expected number of correctly accepted components in the j-th cycle of the i-th stage .
$R_{i,j}$	Rate of rejection of components due to the i-th characteristic in the sequence of inspection of the j-th cycle .
A_i	Expected number of accepted components in the i-th stage .
CFR_i	Cost of false rejection in the i-th stage .
CFA_i	Cost of false acceptance in the i-th stage .
CI_i	Cost of inspection in the i-th stage .
TCFR	Total cost of false rejection .
TCFA	Total cost of false acceptance .
TCI	Total cost of inspection .
TA	Total number of accepted components .
$E(tc)\|_j$	Expected total cost per accepted component after j-th stages of inspection .

The model is developed for critical component having N characteristics requiring inspection.The incoming quality of characteristic i is P_i. A component is classified as nondefective only if all the characteristics meet quality specifications. An inspector commits type I error E_{1i} and type II error,E_{2i}, when he inspects characteristic i. Three different types of costs are considered : (i) Cost of false rejection of non-defective components,(cost of type I error),(ii) cost of false acceptance of defective components,(cost of type II error),and (iii) cost of inspection.

In order to control the quality of such critical components, the following inspection plan is proposed. The new inspection plan is applied as follows: The first inspector inspects one particular characteristic n times for each component entering the inspection process,(this is the first stage of inspection), all the accepted components go to the second inspector, who inspects the second characteristic n times (this is the second stage of inspection). This chain of inspection continues untill all characteristics are inspected n times. Here n is the optimal number of repeat inspections necessary to minimize the expected total cost per accepted component. Each stage has n cycles of inspections. The new inspection plan is shown in Figure 1. Finally ,the accepted components will be those which are accepted at the N-th stage,and the rejected components is the sum of those rejected in the 1st,2nd, ..., N-th stages.When implementing this plan at each inspection station (inspector) a randamization process must be installed so that the inspector would not guess whether the component has been inspected or not. This can be implemented by designing a conveyor where components are mixed prior to arriving to the inspector. This may minimize the bias inspectors may have due to his previous inspection.

The objective is to find the optimal number of repeat inspectons,n, to minimize the expected total cost. The expected total cost consists of the cost of false acceptance,cost of false rejection and cost of inspection. Next a mathematical model is developed that depicts the proposed plan and used to find the optimal number of repeat inspections.

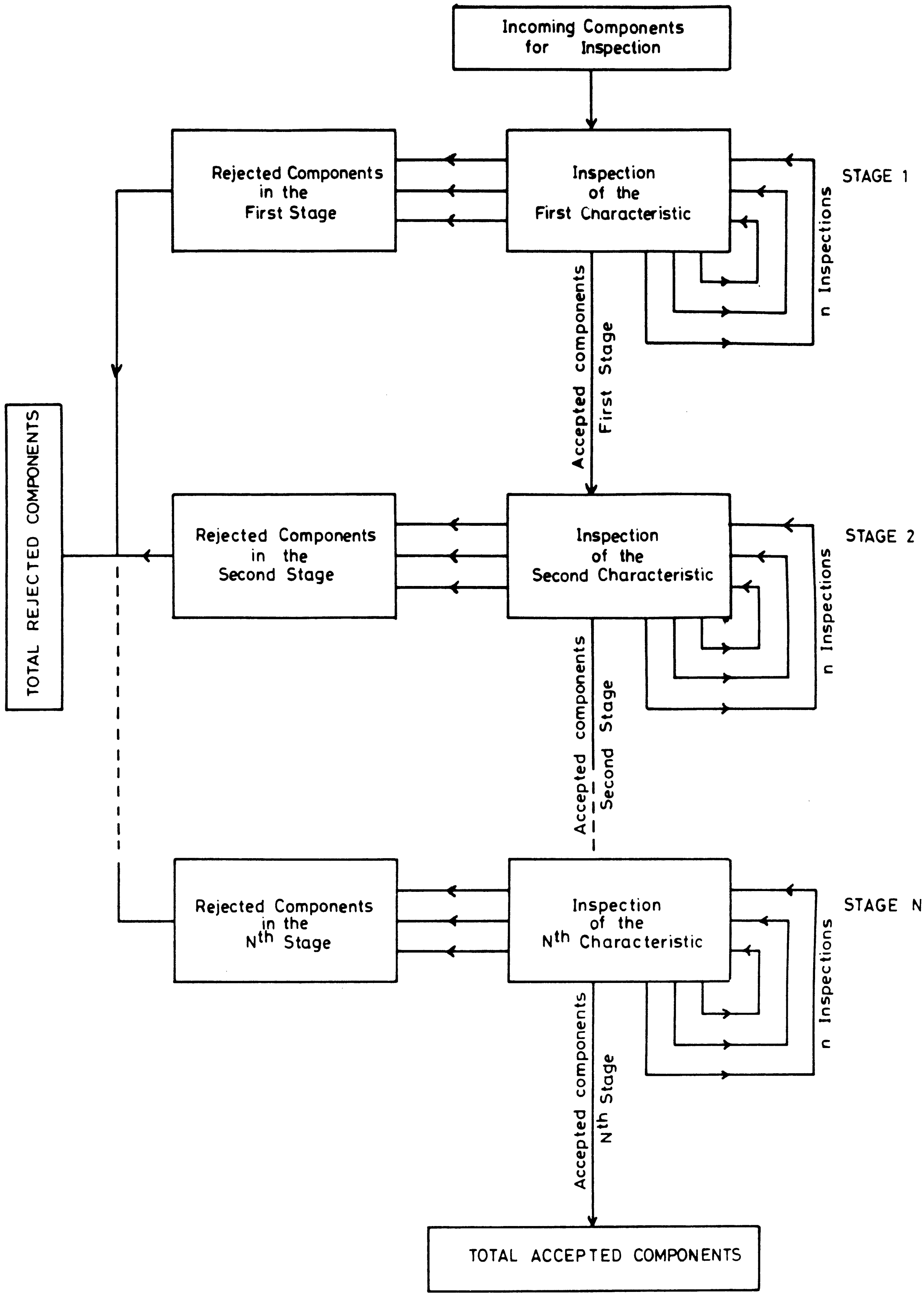

Figure 1, Proposed Inspection Plan.

3 MODEL DEVELOPMENT

In the next three subsections the details of the model are presented.In the first subsection the basic relationships are derived,followed by the general expressions in the second subsection. Then the rule for finding the optimal sequence of inspection is given.

3.1 Basic Relationships of the Model

This model and the one in the literature assume characteristic defective rates are statistically independent. This assumption is not highly restrictive and apply to many situations. For example when inspecting aircraft engines,characteristics crospond to different parts of the engine, which are made at different plants. It is highly likely that the defective rates of such parts are independent. Also independence can be used as a reasonable approximation for many other situations.

The probability of the i-th characteristic being defective will vary from cycle to cycle. First we shall establish the relationship between $P_{i,j}$ and P_i.

Expressing $P_{i,j}$ in terms of P_i

Obviously

$$P_{i,1} = P_i \tag{1}$$

Using Bayes theorem,

$$P_{i,2} = \frac{P_i E_{2i}}{[P_i E_{2i} + (1 - P_{i,1})(1 - E_{1i})]} \tag{2}$$

Applying Bayes theorem again,

$$P_{i,3} = \frac{P_{i,2} E_{2i}}{[P_{i,2} E_{2i} + (1 - P_{i,2})(1 - E_{1i})]} \tag{3}$$

Substituting for $P_{i,2}$ from equation(2) into the above formula,gives after simplifiction,

$$P_{i,3} = \frac{P_i E_{2i}^2}{[P_i E_{2i}^2 + (1 - P_i)(1 - E_{1i})^2]} \tag{4}$$

Similarly,

$$P_{i,4} = \frac{PiE_{2i}^3}{[PiE_{2i}^3 + (1-P_i)(1-E_{1i})]} \tag{5}$$

In general from the symmetry of expressions (2), (3), and (4) the following is deduced:

$$P_{i,j} = \frac{PiE_{2i}^{j-1}}{[P_iE_{2i}^{j-1} + (1-P_i)(1-E_{1i})]} \tag{6}$$

The probability of a characteristic being defective changes in each cycle and so the probability of a component being nondefective. Bearing this in mind, we shall establish the relationship between $PG_{i,j}$, the probability of a component being nondefective entering the j-th cycle of the i-th stage, and the incoming quality, P_i. Expressing PG in terms of P_i. The probability of a component being nondefective is

$$PG = \prod_{i=1}^{N}(1-P_i) \tag{7}$$

The probability of a component being nondefective after inspecting characteristic 1, n times is:

$$PG_{1,n+1} = \prod_{i=2}^{N}(1-P_i)[(1-P_{1,n+1})] \tag{8}$$

The probability of a component being defective after inspecting all characteristics n times is :

$$PG_{N,n+1} = \prod_{i=1}^{N}(1-P_{i,n+1}) \tag{9}$$

The probability of a component being defective after inspecting characteristic 1 through $i-1$, n times and characteristic i, k times and the other characteristics from $i+1$ through N are not inspected is given by:

$$PG_{i,k} = [\prod_{k=1}^{i-1}(1-P_{k,n+1})][(1-P_{i,k+1})][\prod_{k=i+1}^{N}(1-P_k)] \tag{10}$$

When there is no inspection ,the expected total cost per accepted component will simply be the cost due to false acceptance of defective components and is given by:

$$E(tc)|_{j=0} = C_a(1-PG) \tag{11}$$

where C_a is the cost of false acceptance per component and PG is given by equation (7).

The expected total cost per accepted component, after inspecting each characteristic n times, i.e performing n cycles of inspection in all stages, is given as:

$$E(tc)|_{j=n} = [TCFR + TCFA + TCI]/TA \tag{12}$$

In order to determine TCFR, TCFA, TCI, and TA, an analysis of different stages of inspection is necessary. The complete analysis of stages 1 and 2 are given in the appendix of the paper. From the analysis given in the appendix, the following general relationships are derived.

3.2 Relationships needed to compute expected total cost

Using the symmetry in the results obtained from the analysis of stages 1 and 2 ,the following general expressions needed to compute the expected total cost can be derived.

Total number of accepted components after completing N stages of inspection, i.e ,after inspecting the N-th characteristic is given as:

$$\begin{aligned} A_N &= M\left[\prod_{k=1}^{N-1}\prod_{j=1}^{n}[P_{k,j}E_{2k} + (1-P_{k,j})(1-E_{1k})]\right] \times \\ &\left[\prod_{j=1}^{n-1}[P_{N,j}E_{2N} + (1-P_{N,j})(1-E_{1N})]\right] \times \\ &[P_{N,n}E_{2N} + (1-PG_{N,n} - P_{N,n})(1-E_{1N}] + \\ &M\left[PG\prod_{k=1}^{N}(1-E_{1k})^n\right] \end{aligned} \tag{13}$$

Cost of false acceptance at each stage i, $i = 1, ..., N$, is given as:

$$\begin{aligned} CFR_i &= [C_r \times M \times PG \times E_{1i}]\left[\prod_{k=1}^{i-1}(1-E_{1k})^n\right] \times \\ &\left[\sum_{k=1}^{n}(1-E_{1i})^{k-1}\right] \end{aligned} \tag{14}$$

Cost of false acceptance after completing the N-th stage of inspection is given as:

$$\begin{aligned} CFA_N &= C_a M \left[\prod_{k=1}^{N-1} \prod_{j=1}^{n} [P_{k,j} E_{2k} + (1 - P_{k,j})(1 - E_{1k})] \right] \times \\ & \left[\prod_{j=1}^{n-1} [P_{N,j} E_{2N} + (1 - P_{N,j})(1 - E_{1N})] \right] \times \\ & [P_{N,n} E_{2N} + (1 - PG_{N,n} - P_{N,n})(1 - E_{1N}] \end{aligned} \quad (15)$$

Cost of inspection at each stage,for stage i, $i = 1, ..., N$, is given as:

$$\begin{aligned} CI_i &= C_i M \left[\prod_{k=1}^{i-1} \prod_{j=1}^{n} [P_{k,j} E_{2k} + (1 - P_{k,j})(1 - E_{1k})] \right] \times \\ & \left[\sum_{k=1}^{n} \left\{ \prod_{j=1}^{k-1} [P_{i,j} E_{2i} + (1 - P_{i,j})(1 - E_{1i})] \right\} \right] \end{aligned} \quad (16)$$

In order to determine the general expression for the expected total cost per accepted component, the following expressions must be determined.The expressions are, total cost of false rejection TCFR, total cost of false acceptance TCFA, total cost of inspection TCI, and total number of accepted components, TA.

$$TCFR = \sum_{i=1}^{N} CFR_i \quad (17)$$

$$TCFA = CFA_N = C_a(FA_{N,n}) \quad (18)$$

$$TCI = \sum_{i=1}^{N} CI_i \quad (19)$$

$$TA = A_N = FA_{N,n} + CA_{N,n} \quad (20)$$

$$E(tc)|_{i=n} = \frac{TCFA + TCFR + TCI}{TA} \quad (21)$$

The objective is to find n that provides the minimum of $E(tc)_{j=n}$.

3.3 Determing the Optimal Sequence of Inspection

The cost of inspection is influenced by the sequence in which characteristics are ordered for inspection, i.e, the order of the stages. The following rule provides the optimal sequence of characteristic inspection. Let

$$r(i) = \frac{C_i f_1(R_{i,j})}{1 - f_2(R_{i,j})} \qquad \begin{array}{l} i = 1, 2, \dots, N \\ j = 1, 2, \dots, n \end{array} \tag{22}$$

where :

$$\begin{aligned} R_{i,j} &= P_{i,j}(1 - E_{2i}) + (1 - P_{i,j})E_{1i} \\ f_1(R_{i,j}) &= \sum_{j=1}^{n} \left[\prod_{k=1}^{j} (1 - R_{i,k-1}) \right] \\ f_2(R_{i,j}) &= \prod_{k=1}^{n} (1 - R_{i,k}) \end{aligned} \tag{23}$$

The characteristic with the lowest ratio is inspected first, next higher ratio second,and so on.The characteristic with the highest ratio is the N-th characteristic to be inspected. The optimality of this rule follows from the proof given by Duffuaa and Raouf[9]. Next a computational procedure is presented for finding the optimal n and hence the optimal inspection plan.

4 Computational Procedure

STEP(1): Set $j = 0$,compute PG and $E(tc)|_j = 0$ using equations (7) ,(11) respectively .

STEP(2): Let $j = j + 1$, sequence characteristics according to equation (22)

STEP(3): Comput $P_{i,j}, PG_{N,n}, A_N, CFR_i, CFA_N, CI_i$ from equations (6), (9), (13), (14), (15) and (16),respectively .

STEP(4): Compute TCFR,TCFA,TCI,TA,and $E(tc)|_j$ from equations (17), (18), (19), (20),and (21) respectively .

STEP(5): If $E(tc)|_j < E(tc)|_{j-1}$, Go to 2,otherwise STOP ($n = j - 1$) .

5 Results and Model Comparisons

In order to compare the developed model with the model in the literature the following example is given. A software is developed implementing the algorithm stated above and used to obtain the optimal number of repeat inspections. A batch consisting of 100 components is to be inspected. Each component has eight characteristics. The data for the example is given in Table 1. The example is solved utilizing the model in the literature and the new model.

Table 1. Data for the example.

$M = 100$	$N = 8$		
$P_1 = 0.109$	$P_2 = 0.186$	$P_3 = 0.127$	$P_4 = 0.212$
$P_5 = 0.174$	$P_6 = 0.192$	$P_7 = 0.146$	$P_8 = 0.175$
$E_{11} = 0.126$	$E_{12} = 0.118$	$E_{13} = 0.075$	$E_{14} = 0.093$
$E_{15} = 0.051$	$E_{16} = 0.129$	$E_{17} = 0.102$	$E_{18} = 0.046$
$E_{21} = 0.088$	$E_{22} = 0.121$	$E_{23} = 0.112$	$E_{24} = 0.088$
$E_{25} = 0.130$	$E_{26} = 0.072$	$E_{27} = 0.077$	$E_{28} = 0.136$
$C_1 = 99$	$C_2 = 12$	$C_3 = 67$	$C_4 = 50$
$C_5 = 76$	$C_6 = 21$	$C_7 = 14$	$C_8 = 95$
	$C_a = 523248$		
	$C_r = 733$		

Solving the above example using Raouf et al model and algorithm[8] , the following results has been obtained:

Optimal number of repeat inspections = 3.

Minmun expected total cost = 16063.11

Optimal sequence 3, 7, 6, 4, 1, 3, 5, 8

Probabilty of a component being nondefective is= 0.9937165

Solving the same example using the proposed plan and model,the following results has been obtained:

Optimal number of repeat inspections = 3.

Minmun expected total cost = 14448.62

Optimal sequence 2, 7, 6, 4, 3, 1, 5, 8

Probabilty of a component being nondefective is= 0.9973872

The proposed model showed improvement over the model in the literature in terms of the expected total cost. Actually the saving in expected total cost amount to 10.1 percent for the above example. This led to the following detailed comparisons on randomely generated inspection problems.

The parameters of the geneated problems are $N, C_i, C_a, C_r, P_i, E_{1i}$ and E_{2i},for $i = 1, 2, \ldots, N$. The parameters N, C_i, C_a, C_r are assumed to be uniformely distributed. The other parameters P_i, E_{1i}, E_{2i} are assumed to be normally distributed with mean

μ and variance σ^2 .N is assumed to be a discrete uniform distribution that takes the values $1, 2, \ldots, 10$.C_i is uniformely distributed between 10 and 100,i.e $U(10, 100)$,C_a $\sim U(100000, 1000000)$ and $C_r \sim U(500, 1000)$.P_i is assumed normally distributed with mean $\mu = .05$ and variance $\sigma^2 = .014$ i.e $P_i \sim N(.05, .014)$.$E_{1i} \sim N(0.1, 0.0009)$ and $E_{2i} \sim N(0.1, 0.0009)$ for all i.

A program was written to generate 100 random inspection problems and solved to obtain the optimal inspection parameters using both models. From the experiment conducted using the generated problems with the above parameters assuming the given distributions, it was found that the proposed model gave better results in terms of the expected total cost in 82 percent of the generated problems. Raouf et al model gave better results on 4 percent of the generated problems.On 14 percent the two models gave the same results.

The reduction in the total expected cost, when the proposed model performed better in comparison with Raouf et al model ranges from 1 to 10 percent, while the reduction ranges from 0.1 to 7 percent when Raouf et al performed better. In general the model in ths paper performs better when the defect rates are in the ranges 0 to 10 percent and the probability type I error in the range 0 to 0.15 percent and type II error in the range 0 to .10. The above results demonstrate the superiority of the proposed approach and the resulting inspection plan over the one in the literature. Also it was noticed that the new plan has less material handling cost which may result in additional cost reduction.This aspect of material handling is under further research.

6 Conclusion

A new inspection plan has been proposed for the inspection of critical components with several characteristics. A model has been formulated which depicts the plan and is employed for determing the optimal number of repeat inspections that minmizes the expected total cost per accepted component. An algorithm has been developed for obtaining the optimal n The model has few reaistic assumptions and as such may have a wide range of applicability. Results on randomly generated problems have indicated that the new model and plan performed better in optimizing inspection plans for critical components than the ones in the literature. Further savings are expected if the material hadling cost is incorporated in both models. The incorporation of material handling cost in both models is under research.

7 Appendix

In this appendix complete analysis of stages 1 and 2 are given. From the symmetry noticed from the analysis of the two stages the general results are derived and given in the body of the paper.

7.1 Analysis of Stage (1)

All the components entering stage (1) go to the first inspector, who inspects the first characteristic in each component in order to classify it as defective or nondefective. The first stage has n cycles of inspections. Following is the first inspection cycle analysis .

7.1.1 Cyle(1)

Number of components entering cycle 1 is

$$M_{1,1} = M_1 \tag{24}$$

The probability of a component being defective is

$$PG_{1,1} = PG \tag{25}$$

E(number of falsely rejected components) is

$$\begin{aligned} FR_{1,1} &= M_{1,1}PG_{1,1}E_{11} \\ &= M_1 PGE_{11} \end{aligned} \quad (26)$$

E(number of falsely accepted components) is

$$\begin{aligned} FA_{1,1} &= M_{1,1}[P_1E_{21} + (1 - PG_{1,1} - P_1)(1 - E_{11})] \\ &= M_1[P_1E_{21} + (1 - PG - P_1)(1 - E_{11})] \end{aligned} \quad (27)$$

E(number of correctly accepted componente) is

$$\begin{aligned} CA_{1,1} &= M_{1,1}PG(1 - E_{11}) \\ &= M_1PG(1 - E_{11}) \end{aligned} \quad (28)$$

All accepted components in this cycle go to the first inspector again to inspect the first characteristic for the second time.,(perform cycle 2).

7.1.2 Cycle(2)

$$\begin{aligned} M_{1,2} &= FA_{1,1} + CA_{1,1} \\ &= M_1[P_1E_{21} + (1 - PG - P_1)(1 - E_{11}) + M_1PG(1 - E_{11}) \quad (29) \\ &= M_1[P_1E_{21} + (1 - P_1)(1 - E_{11})] \\ PG_{1,2} &= (1 - P_{1,2})[\prod_{i=2}^{N}(1 - P_i)] \\ &= PG(1 - E_{11})/[P_1E_{21} + (1 - P_1)(1 - E_{11})] \quad (30) \\ FR_{1,2} &= M_{1,2}PG_{1,2}E_{11} \\ &= M_1PGE_{11}(1 - E_{11}) \quad (31) \\ FA_{1,2} &= M_{1,2}[P_{1,2}E_{21} + (1 - PG_{1,2} - P_{1,2})(1 - E_{11})] \\ &= M_1[P_1E_{21} + (1 - P_1)(1 - E_{11})][P_{1,2}E_{21} \\ &\quad +(1 - PG_{1,2} - P_{1,2})(1 - E_{11})] \quad (32) \\ CA_{1,2} &= M_{1,2}PG_{1,2}(1 - E_{11}) \\ &= M_1PG(1 - E_{11})^2 \quad (33) \end{aligned}$$

Similarly,

7.1.3 Cycle(3)

$$\begin{aligned}
M_{1,3} &= FA_{1,2} + CA_{1,2} \\
&= M_1[P_1E_{21} + (1-P_1)(1-E_{11})][P_{1,2}E_{21} + (1-P_{1,2})(1-E_{11})] \quad (34) \\
PG_{1,3} &= (1-P_{1,3})[\prod_{i=2}^{N}(1-P_i)] \\
&= PG(1-E_{11})^2/[P_1E_{21} + (1-P_1)(1-E_{11})] \times \\
&\quad [P_{1,2}E_{21} + (1-P_{1,2})(1-E_{11})] \quad (35) \\
FR_{1,3} &= M_1PGE_{11}(1-E_{11})^2 \quad (36) \\
FA_{1,3} &= M_1\prod_{j=1}^{2}[P_{1,j}E_{21} + (1-P_{1,j})(1-E_{11})] \times \\
&\quad [P_{1,3}E_{21} + (1-PG_{1,3} - P1,3)(1-E_{11})] \quad (37) \\
CA_{1,2} &= M_1PG(1-E_{11})^3 \quad (38)
\end{aligned}$$

7.1.4 Cycle(n)

From the symmetry of the expressions, the n- th cycle of the first stage results can be writtern as follows:

$$\begin{aligned}
M_{1,n} &= M_1\prod_{j=1}^{n-1}[P_{1,j}E_{21} + (1-P_{1,j})(1-E_{11}] \quad (39) \\
PG_{1,n} &= PG(1-E_{11})^{n-1}/\prod_{j=1}^{n-1}[P_{1,j}E_{21} + (1-P_{1,j})(1-E_{11}] \quad (40) \\
FR_{1,n} &= M_1PGE_{11}(1-P11)^{n-1} \quad (41) \\
FA_{1,n} &= M_1\left[\prod_{j=1}^{n-1}\{P_{1,j}E_{21} + (1-P_{1,j})(1-E_{11})\}\right] \times \\
&\quad [P_{1,n}E_{21} + (1-PG_{1,n} - P_{1,n})(1-E_{11})] \quad (42) \\
CA_{1,n} &= M_1PG(1-E_{11})^n \quad (43)
\end{aligned}$$

This completes stage one of the inspection with n cycles.

7.2 Results of Stage (1)

E(number of accepted components) is

$$\begin{aligned} A_1 &= FA_{1,n} + CA_{1,n} \\ &= M_1 \left[\prod_{j=1}^{n-1} \{P_{1,j}E_{21} + (1 - P_{1,j})\,(1 - E_{11})\} \times [P_{1,n}E_{21} + (1 - PG_{1,n} - P_{1,n}) \times \right. \\ &\quad \left. (1 - E_{11})] + PG(1 - E_{11})^n \right] \end{aligned} \tag{44}$$

where $PG_{1,n}$ can be easily obtained from equation (9).

Cost of false rejection after one stag of inspection is completed is given by:

$$\begin{aligned} CFR_1 &= C_r \sum_{j=1}^{n} FR_{1,j} \\ &= C_r M_1 PGE_{11} \sum_{j=1}^{n} (1 - E_{11})^{j-1} \end{aligned} \tag{45}$$

Cost of false acceptance after stage one is completed is:

$$\begin{aligned} CFA_1 &= C_a(FA_{1,n}) \\ &= C_a M_1 \left[\prod_{j=1}^{n-1} [P_{1,j}E_{21} + (1 - P_{1,j})(1 - E_{11})] \right] \times \\ &\quad [P_{1,n}E_{21} + (1 - PG_{1,n} - P_{1,n})(1 - E_{11})] \end{aligned} \tag{46}$$

Cost of inspection after stage one is compeleted is:

$$\begin{aligned} CI_1 &= C_1 \sum_{j=1}^{n} M_{1,j} \\ &= C_1 M_1 \left[\sum_{k=1}^{n} \prod_{j=1}^{k-1} [P_{1,j}E_{21} + (1 - P_{1,j})(1 - E_{11})] \right] \end{aligned} \tag{47}$$

E(total cost per accepted component after one stage of inspection) is:

$$E(tc)|_{j=1} = [CFR_1 + CFA_1 + CI_1]/A_1 \tag{48}$$

where A_1, CFR_1, CFA_1 and CI_1 are given by equations (44),(45),(46), and equation (47) respectively

7.3 Analysis of Stage (2) of Inspection

All accepted components from stage one proceed to the second inspector who inspects the second characteristics. Therfore the expected number of components entering stage two is $M_{2,1} = A_1$ where A_1 is given by equation (44).

7.3.1 Cycle(1)

$$
\begin{aligned}
M_{2,1} &= FA_{1,n} + CA_{1,n} \\
&= M_1 \left[\prod_{j=1}^{n} \{P_{1,j}E_{21} + (1 - P_{1,j})\,(1 - E_{11})\} \right. \\
&\quad (1 - E_{11})] + PG(1 - E_{11})^n] \\
PG_{2,1} &= PG(1 - E_{11})^n / \left[\prod_{j=1}^{n} [P_{1,j}E_{21} + (1 - P_{1,j})(1 - E_{11})] \right] \\
FR_{2,1} &= M_1 PGE_{12}(1 - E_{11})^n \\
FA_{2,1} &= M_1 \left[\prod_{j=1}^{n} \{P_{1,j}E_{21} + (1 - P_{1,j})(1 - E_{11})\} \right] \times [P_2E_{22} + (1 - PG_{2,1} - P_2)(1 - E_{12})] \\
CA_{2,1} &= M_1 PG(1 - E_{11})^n (1 - E_{12})
\end{aligned}
$$

7.3.2 Cycle(2)

$$
\begin{aligned}
M_{2,2} &= FA_{2,1} + CA_{2,1} \\
&= M_1 \prod_{j=1}^{n} [P_{1,j}E_{21} + (1 - P_{1,j})(1 - E_{11})] \times [P_2E_{22} + (1 - P_2)(1 - E_{12})] \\
PG_{2,2} &= PG(1 - E_{11})^n (1 - E_{12}) / \left[\prod_{j=1}^{n} \{P_{1,j}E_{21} + (1 - P_{1,j})(1 - E_{11})\} \right] \times \\
&\quad [P_2E_{22} + (1 - P_2)(1 - E_{12})] \\
FR_{2,2} &= M_1 PGE_{12}(1 - E_{11})^n (1 - E_{12}) \\
FA_{2,2} &= M_1 \left[\prod_{j=1}^{n} \{P_{1,j}E_{21} + (1 - P_{1,j})(1 - E_{11})\} \right] \times [P_2E_{22} + (1 - P_2)(1 - E_{12})] \\
&\quad [^2 P_2E_{22} + (1 - PG(2,2) - 2P_2)(1 - E_{12})] \\
CA_{2,1} &= M_1 PG(1 - E_{11})^n (1 - E_{12})
\end{aligned}
$$

7.3.3 Cycle(n)

$$
\begin{aligned}
M_{2,n} &= M_1 \left[\prod_{j=1}^{n} \{^j P_{1,j} E_{21} + (1 - P_{1,j})(1 - E_{11})\}\right] \left[\prod_{j=1}^{n-1} \{P_{2,j} E_{22} + (1 - P_{2,j})(1 - E_{12})\}\right] \\
PG_{2,n} &= PG(1 - E_{11})^n (1 - E_{12})^{n-1} / \left[\prod_{j=1}^{n} \{P_{1,j} E_{21} + (1 - P_{1,j})(1 - E_{11})\}\right] \times \\
&\quad \left[\prod_{j=1}^{n-1} \{P_{2,j} E_{22} + (1 - P_{2,j})(1 - E_{12})\}\right] \\
FR_{2,n} &= M_1 PG E_{12} (1 - E_{11})^n (1 - E_{12})^{n-1} \\
FA_{2,n} &= M_1 \left[\prod_{j=1}^{n} \{P_{1,j} E_{21} + (1 - P_{1,j})(1 - E_{11})\}\right] \times \left[\prod_{j=1}^{n-1} \{P_{2,j} E_{22} + (1 - P_{2,j})(1 - E_{12})\}\right] \times \\
&\quad [P_{2,n} E_{22} + (1 - PG_{2,n} - P_{2,n})(1 - E_{12})] \\
CA_(2, n) &= M_1 PG (1 - E_{11})^n (1 - E_{12})^n
\end{aligned}
$$

7.4 Results of Stage (2)

$$
\begin{aligned}
A_2 &= FA_{2,n} + CA_{2,n} \\
&= M_1 \left[\prod_{j=1}^{n} \{P_{1,j} E_{21} + (1 - P_{1,j})(1 - E_{11})\}\right] \left[\prod_{j=1}^{n-1} \{P_{2,j} E_{22} + (1 - P_{2,j})(1 - E_{12})\}\right] \\
&\quad \times [P_{2,n} E_{22} + (1 - PG_{2,n} - P_{2,n})(1 - E_{12})] + M_1 PG (1 - E_{11})^n (1 - E_{12})^n \\
CFR_2 &= C_r \sum_{j=1}^{n} FR_{2,j} \\
&= C_r M_1 PG E_{12} (1 - E_{11})^n \sum_{j=1}^{n} (1 - E_{12})^{j-1} \\
CFA_2 &= C_a (FA_{2,n}) \\
&= C_a M_1 \left[\prod_{j=1}^{n} \{P_{1,j} E_{21} + (1 - P_{1,j})(1 - E_{11})\}\right] \left[\prod_{j=1}^{n-1} \{P_{2,j} E_{22} + (1 - P_{2,j})(1 - E_{12})\}\right] \\
&\quad \times [P_{2,n} E_{22} + (1 - PG_{2,n} - P_{2,n})(1 - E_{12})] \\
CI_2 &= C_2 \sum_{j=1}^{n} M_{2,j} \\
&= C_2 M_1 \left[\prod_{j=1}^{n} \{P_{1,j} E_{21} + (1 - P_{1,j})(1 - E_{11})\}\right] \left[\sum_{k=1}^{n} \prod_{j=1}^{k-1} \{P_{2,j} E_{22} + (1 - P_{2,j})(1 - E_{12})\}\right]
\end{aligned}
$$

8 Acknowledgment

The authors would like to acknowledge the support provided by King Fhad University of Petroleum and Minerals, Dhahran,31261, Saudi Arabia, for conducting this research. Especially the support for the sabbatical leave for the first author. Also the department of industrial and Operations Engineering,University of Michigan at Ann Arbor,is acknowledged for hosting the first author and extending it's facilities.

9 References

[1] X.K.Zunzaniyka, and C.G.Drury (1975)"Effet of information on industrial inspection performance", *Human Reliability in Quality Control* edited by Dury and J.G.Fox, Taylor and Francis Ltd.,London.

[2] J.K.Jain(1977) "A model for determing the optimal number of inspections minimizing inspection cost", Unpublished M.Sc.thesis, University of Windsor, Windsor , Ontario, Canada.

[3] A.Raouf and F.Elfeturi (1983) "Study of the effects of defect rates ,task complexity and inspection rates on inspection accuracy, *Procedings of the first Saudi Engineering Conference*, Jeddah, Saudi Arabia.

[4] M.M. Ayoub,B.K.Lambert and A.G.Walkevar(1975)"Effects of two types of inspector error on single sampling inspection plans", A paper presented at *Humam Factor Society Meeting*, Baltimore, U.S.A.

[5] R.D.Collins,K.E.Case and G.K.Bennett(1972)"The effects of inspection accuracy on statistical quality control, *Proceedings of the 23rd Annual Conference and Convention*,AIIE, Anaheim, California, U.S.A.

[6] K.E. Case,G.K.Bennett and J.W.Schmidt(1973)"The Dodge CSP-1 continous sampling plan under inspection error", AIIE Trans. 5, 193-202.

[7] G.K.Bennett,K.E.Case and J.W.Schmidt(1974) "The economic effects of inspector error on attribute sampling plans", *Naval Research Logistics Quarterly* 21, 431-443.

[8] A.Raouf,J.K.Jain and P.Y.Sathe(1983)"A cost minimization model for multicharacteristic component inspection", *IIE Trans.* 15, No. 3, 187-194.

[9] S.O.Duffuaa and A.Raouf (1990) "An optimal sequence in multicharacteristic inspection", *J.of Opt. Theory and App.(JOTA)* 67, No. 1 , 79-86.

[10] S.O.Duffuaa and A.Raouf(1987)"A cost minimization model for dependent multicharacteristic inspection", *Proceedings IXth International Conference on Prod. Res. and Exhibition*, Cincinati, Ohio, 738-746.

[11] H.L.Lee(1988) "On the optimality of a simplified multicharacteristic component inspection model" *IIE Trans.* 20, No.4, 392-398.

[12] S.O.Duffuaa and A.Raouf(1989)" Models for multicharacteristic repeat inspection", *App.Math.Modelling* 13, 408-412.