

ENGINEERING RESEARCH INSTITUTE
UNIVERSITY OF MICHIGAN
ANN ARBOR

Quarterly Progress Report No. 4

December 1, 1954 to February 28, 1955

DEVELOPMENT OF GENERALIZED MATHEMATICAL
PROCEDURES FOR OPTIMUM ASSEMBLY OF
POTENTIALLY EFFECTIVE COMBAT CREWS

PAUL S. DWYER

Project 2226

U.S. AIR FORCE
AIR RESEARCH AND DEVELOPMENT COMMAND
CONTRACT NO. AF 18(600)-1050

March 1955

en 8n

UMR0882

no. 4

QUARTERLY PROGRESS REPORT NO. 4

December 1, 1954 to February 28, 1955

Contract No.: AF 18(600)-1050

Budget Project No.: 670-193

Contract Title: Development of Generalized Mathematical
Procedures for Optimum Assembly of Po-
tentially Effective Combat Crews

Issuing Office: The Air Research and Development Command

Contractor: The Regents of the University of Michigan

Monitoring Agency: Director, Detachment 4 (Crew Research
Laboratory), Air Force Personnel and Train-
ing Research Center, Randolph Field, Texas

Principal Investigator: Dr. Paul S. Dwyer

Period: December 1, 1954 to February 28, 1955

PERSONNEL

Name	Title	Portion of Time Devoted to Contract Work
Dwyer, Paul S.	Professor of Mathematics, Consultant in Statistical Laboratory	Up to 40 hours ¹ per month
Rider, Leonard	Assistant in Research	Half time ²
Taylor, Patricia	Assistant in Research	Half time
Graves, Glenn	Assistant in Research	Varied ³
Bassett, Karen	Typist	Half time ⁴

¹During this quarter, Dr. Dwyer worked full time on his university duties, and his work on the project was limited to 40 hours per month.

²Mr. Rider terminated his work on the project on January 6, 1955. He obtained his degree at the end of the first semester and left the University.

³Mr. Graves began working on the project on January 27, 1955. His particular job is to assist in translating some of the methods to routines which can be performed on MIDAC (Michigan Digital Automatic Computer). The arrangement with Mr. Graves specified that the total number of hours he would spend would not exceed 150.

⁴Mrs. Bassett began work on January 10, 1955. She is typing the reports and numerical illustrations which describe the results of the various aspects of the contract work.

RESEARCH PROGRESS1. General

Additional results, dealing for the most part with the materials of Chapters X, XI, XII, and XIII of the extended report (as designated in the third quarterly report) have been obtained during this quarter. These results have led to revisions of the extensive material of Chapter X (the first draft of which was indicated in the third quarterly report) and the initial drafts of Chapters I, XI, and XII. During this period the techniques for $k = 3$ advanced to the point where we felt they were ready to be adapted to a large-scale digital computer and we are able to report considerable progress in adapting them to MIDAC.

Early in January we secured the services of a half-time typist who is now typing the present drafts of the chapters of the extended report. Drafts for several of these chapters have been sent to Dr. Roby for his advice and suggestions and more will follow soon.

Some consideration has been given to the final official report, which will be in the nature of a shorter and less technical version of the material of the extended report, and the advice of Dr. Roby has been sought on this point. This report will be written when the preliminary drafts for all the chapters of the extended report are complete.

2. Outline of Topics

The outline of topics, drawn up at the conference in mid-June, reaffirmed at the conference in September, and restated in the third quarterly report, is not used as the basis of reference in this report. Rather, the extended form of this outline, which is the current outline of the contents of

the extended report, is used for this purpose. Though proposed outlines for the extended report appear in the report for the second quarter and in more detail in the report for the third quarter, comparison shows that the present outline of the extended report is more detailed than either of these, particularly in the material dealing with Chapters I, XI, and XII. Also, some changes resulting from the revisions of Chapter X which resulted from our improved knowledge of appropriate techniques, have been made in the outline for that chapter.

3. Present Outline of the Extended Report

The outline, as of February 28, 1955, of the fourteen chapters of the extended report is presented. The outline is given in more detail for those chapters which have been written.

The extent of the work on each chapter is indicated. A single asterisk indicates that some work has been done on the subject matter of that chapter. A double asterisk indicates that considerable work is completed, a triple asterisk, that at least a preliminary draft of the chapter is finished. A quadruple asterisk indicates that a typed copy of the chapter has been placed in the hands of Dr. Roby.

Chapter

Contents

I***

The general group assembly problem

1. Introduction
2. Group scores
3. Assembly scores
4. Mathematical statement of the problem
5. Groupings
6. Relation to personnel classification problem and similar problems
7. Use of permutation sets
8. Restatement of the problem using permutation sets

II****

Transformations

1. Introduction
2. Subtraction of a constant
3. Deviate transformations
4. Approximate deviate transformations
5. Large deviate transformations
6. Extreme transformations

<u>Chapter</u>	<u>Contents</u>
III****	The distribution of all possible assembly sums <ol style="list-style-type: none"> 1. Introduction 2. The mean and variance of the distribution of all possible assembly sums for any k 3. The third central moment of the distribution of all possible assembly sums for $k=2, 3,$ and 4 4. The fourth central moment when $k=2$
IV****	Application of analysis of variance and determination of a mathematical model appropriate to empirical data <ol style="list-style-type: none"> 1. Introduction 2. Analysis of variance when $k=2$ 3. Analysis of variance when $k=3$ 4. Analysis of variance when $k=4$ 5. Analysis of variance for higher values of k 6. Determination of a mathematical model appropriate to empirical data
V**	Mathematical models for group scores
VI**	Condensation by grouping <ol style="list-style-type: none"> 1. Reduction, in effect, of the number of classes by grouping
VII**	The group assembly problem as a problem in linear programming <ol style="list-style-type: none"> 1. Possibility of a dual and generalized conditions of solution 2. Extent of applicability of simplex method, method of bounding sets, method of interchange, and method of optimal regions to the general problem
VIII****	The two-dimensional assembly problem <ol style="list-style-type: none"> 1. Introduction 2. Conditions of solution 3. Use of extreme transformations 4. Method of bounding sets 5. Marginal zero transformations 6. Determination of a completely reduced matrix

<u>Chapter</u>	<u>Contents</u>
VIII**** (cont.)	7. Determination of an optimal solution from a completely reduced matrix 8. Solution with the method of marginal zeros 9. Solution of the quota problem with the detailed method of optimal regions
IX****	Successive applications of two-dimensional techniques <ol style="list-style-type: none"> 1. Introduction 2. A succession of two-dimensional techniques 3. Approximate solution of the general problem, using totals of subclasses 4. Use of deviate scores in determining suitable subclasses 5. Use of approximate deviate scores in determining suitable subclasses 6. Use of results of analysis of variance in determining suitable subclasses 7. Conclusion
X****	The three-dimensional assembly problem <ol style="list-style-type: none"> 1. Introduction 2. The use of extreme transformations 3. The use of marginal zero transformations 4. The reduction of the G_{ijh} matrix 5. The determination of the solution from the reduced matrix 6. The three-dimensional problem with frequencies 7. The reduced grouped matrix 8. Transformations leading to reduced grouped matrices 9. Reduced grouped matrix transformations 10. The determination of the solution from the reduced grouped matrix 11. Conclusion
XI****	The general assembly problem <ol style="list-style-type: none"> 1. Introduction 2. Solution of the general problem with $k=4$ and $N=2$ 3. Solution of the general problem with $k=4$ and $N=3$

Chapter

Contents

XI***
(cont.)

4. Solution of the general problem with $k=4$ and $N=3$, using the method of reduced matrices
5. The solution of a frequency problem with $k=4$ and $n=3$
6. The condensed solution of the frequency problem of section 11.5
7. The solution of a problem having unit frequencies with $k=5$ and $N=3$
8. The detailed solution of a frequency problem with $k=5$ and $n=3$
9. The generality of the method of reduced matrices

XII***

Approximate solutions

1. Introduction
2. Approximate solutions using deviates
3. Approximate solutions using approximate deviates
4. Approximate solutions using large deviates
5. Approximate solutions using large row deviates
6. Approximate solutions of problems with $k>2$, using a succession of two-dimensional techniques
7. Approximate solutions using reduced matrices
8. Measures of the adequacy of an approximation
9. Conclusion

XIII**

Punched-card and machine methods

1. Use of marginal punched cards
2. Use of IBM punched cards and machines
3. Use of electronic digital computers

XIV

Concluding remarks

**

References

4. Work on Specific Topics

A comparison of the outline of the extended report as it appears in this report with that appearing in the third quarterly report reveals the work done on specific topics during this fourth quarter. For the most part the work done during this quarter may be grouped under these headings:

- A. Work done in improving the theory and calculational techniques for solving $k=3$ problems as described in the current draft of Chapter X. (The original draft of Chapter X has been revised to incorporate these improvements.)
- B. Work done in developing the theory and calculational techniques, including the preparation of a preliminary draft of Chapter XI, for the general assembly problem.
- C. Work done on the problem of approximate solutions, including the methods of solution and the development of measures of the adequacy of the approximation, which has resulted in a preliminary draft of Chapter XII.
- D. Work in adapting the method solution when $k=3$, to a large scale electronic digital computer such as MIDAC. This work has not yet resulted in a completed method, but there are indications that a solution, or at least a very good approximation to a solution, will soon be obtained, for $k=3$ problems which the MIDAC can produce automatically.
- E. The collection of materials and the writing of the first draft of the introductory Chapter I.
- F. The typing of the current drafts of Chapters VIII, IX, and X. A copy of each of these chapters has been sent to Dr. Roby during this quarter.

5. Results

The main results of this quarter deal with improvements of the method of reduction using marginal zero transformations. The techniques have been perfected so that we can discover the successive transformations necessary for the reduction of the original matrix to a matrix from which the solution can be obtained.

- A. It was demonstrated in the first quarterly report that matrices exist which can not be completely reduced, i.e., that constants can not be

subtracted from the rows, columns, and layers of these matrices so as to result in a matrix with a permutation set of zeros. It is now known that marginal zero transformations exist, and can be explicitly stated, which reduce any matrix to a matrix having either a grouped or non-grouped permutation set of zeros or a grouped or non-grouped permutation set of zeros with fractions. The optimal solution may be obtained directly from the zero terms of the matrix having the grouped or non-grouped permutation set of zeros, if such a matrix exists, or, if it does not exist, from the matrix having the grouped or ungrouped permutation set of zeros with fractions.

- B. The successive steps in the method of reduction using marginal zero transformations, applicable to problems with $k=2$, $k=3$, and $k>3$, are now known to be
1. The determination and application of the (extreme) transformations which reduce the matrix to marginal zeros.
 2. Application of formal tests which enable us to determine if a reduced matrix is a reduced grouped matrix.
 3. The determination and application of the marginal zero transformations which reduce the reduced matrix to a reduced grouped matrix.
 4. The determination, using the equations of condition of the system, of the more complex transformations which further reduce the matrix.
 5. Formal test that a reduced grouped matrix is completely reduced, i.e., one having a grouped (ungrouped) permutation set of zeros.
 6. The reduction of the matrix using these more complex marginal zero transformations to a matrix which is either a completely reduced grouped (ungrouped) matrix, or, if such a matrix does not exist, to one which has a grouped (ungrouped) permutation set of zeros with fractions.
 7. The determination of the solution from the grouped (ungrouped) matrix which has a grouped (ungrouped) permutation set of zeros or from the grouped (ungrouped) matrix which has a grouped (ungrouped) permutation set of zeros with fractions.
- C. The basic distinction between the method of reduction using marginal zero transformations and the simplex method may now be stated concisely. The simplex method starts with feasible solutions which satisfy the equations of condition and, by changes of bases which

are equivalent to transformations, moves in the direction of maximizing the assembly sum. The method of reduction using marginal zero transformations, on the other hand, features maximal elements which do not initially satisfy all the equations of condition and successive transformations which are designed to satisfy more and more of the equations of condition until, finally, all the equations of condition are satisfied.

- D. This method, with slight modifications, is now known to be useful not only when $k=2$, but also when $k=3, 4$, etc.
- E. This method is very useful also when $k=2$ for minimization problems. A paper in process of preparation, designed for eventual publication in Econometrica and entitled "A Solution of the Hitchcock Transportation Problem with a Method of Transformations", features the method of reduction using marginal zero transformations as applied to $k=2$ minimization problems. At the completion of the first draft of this paper, a copy will be placed in the hands of Dr. Roby.
- F. A paper entitled "The Detailed Method of Optimal Regions" has been written during this quarter and will be submitted to Psychometrika. This paper describes a method worked out by Dr. Dwyer while employed as Consultant, Personnel Research Branch, Department of the Army. While no time from this project was used in writing this paper, the paper is essentially a description of material which might have been included in section 8.9 of the extended report. As such, this paper may be considered to be a supplement to the extended report, so a copy is being sent to Dr. Roby.
- G. Indices of the degree of an approximation derived during the quarter and described in Chapter XII of the extended report, seem useful in measuring the extent of an error in an approximation.

UNIVERSITY OF MICHIGAN



3 9015 02082 8011