ENGINEERING RESEARCH INSTITUTE
UNIVERSITY OF MICHIGAN
ANN ARBOR

Quarterly Progress Report No. 5

DEVELOPMENT OF GENERALIZED MATHEMATICAL PROCEDURES FOR OPTIMUM ASSEMBLY OF POTENTIALLY EFFECTIVE COMBAT CREWS

March 1, 1955 to May 31, 1955

Paul S. Dwyer

Project 2226

U.S. AIR FORCE
AIR RESEARCH AND DEVELOPMENT COMMAND
CONTRACT NO. AF 18(600)-1050

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Contract No.:	AF 18(600)-1050
Budget Project No.:	670-193
Contract Title:	Development of Generalized Mathematical Procedures for Optimum Assembly of Potentially Effective Combat Crews
Issuing Office:	The Air Research and Development Command
Contractor:	The Regents of the University of Michigan
Monitoring Agency:	Director, Detachment 4 (Crew Research Laboratory), Air Force Personnel and Training Research Center, Randolph Field, Texas
Principal Investigator:	Dr. Paul S. Dwyer
Period:	March 1, 1955 to May 31, 1955

PERSONNEL

Name	Title	Portion of Time Devoted to Contract Work
Dwyer, Paul S.	Professor of Mathematics, Consultant in Statistical Laboratory	Up to 40 hours[1] per month
Taylor, Patricia	Assistant in Research	Half time
Graves, Glenn	Assistant in Research	Varied[2]
Bassett, Karen	Typist	Half time[3]

[1]During this quarter, Dr. Dwyer worked full time on his University duties, and his work on the project was limited to 40 hours per month.

[2]Mr. Graves, who is assisting in translating some of the methods to routines which can be performed on MIDAC, has worked a total of 62 hours during this quarter.

[3]Mrs. Bassett has been working full time on the contract since April 11. She terminated her work on the contract on May 27, 1955.

ABSTRACT

This report indicates the reports written and the additional results obtained during the fifth quarter of the term of the contract. The official report "Generalized Mathematical Procedures for the Optimal Assembly of Potentially Effective Combat Crews," has been prepared and sent to the monitoring agency. Several additional chapters of the extended report have been written. Results include a general mathematical formula for group scores, methods for grouping the individual scores into classes, the relation of this problem to other linear programming problems, and a machine method (using MIDAC) for the determination of the reduced grouped matrix.

OBJECTIVE

The general objective of this contract is the development of generalized mathematical procedures for optimum assembly of potentially effective combat crews. More detailed objectives call for (a) a study of the general mathematical theory underlying the group assembly problem, (b) the determination of suitable methods of predicting crew scores from individual scores, (c) the development of a method and technique for finding the maximum assembly sum, (d) the practical adaptation of this technique to a high-speed digital computer, (e) the development of methods for obtaining approximate solutions, and (f) the determination of suitable measures of the adequacy of an approximation.

RESEARCH PROGRESS

1. GENERAL INFORMATION

Additional results, dealing for the most part with the materials of Chapters V, VI, VII, and XIII of the extended report, have been obtained during this quarter. Research has led to a general mathematical model for the group scores, to a development of the theory of the method of reduced matrices which is independent of such classical theory of linear programming as the simplex method, and to satisfactory results in adapting the method of reduced matrices to a digital computer.

Much of the working time on the contract during this quarter has been spent in preparing the necessary reports which describe the results of the contract work. All chapters of the extended report, with the exception of Chapters VII, XIII, and XIV, have been placed in the hands of Dr. Roby. The official report, which contains 70 pages, has also been prepared. The three copies specified in the contract have been sent.

2. THE OUTLINE OF THE EXTENDED REPORT

The outline of the fourteen chapters of the extended report is presented. A copy of all chapters except VII, XIII, and XIV has been sent to Dr. Roby; a copy of these chapters will be sent him prior to June 30.

Chapter	Contents
I	The general group assembly problem 1. Introduction 2. Group scores 3. Assembly scores 4. Mathematical statement of the problem 5. Groupings 6. Relation to personnel classification problem and similar problems 7. Use of permutation sets 8. Restatement of the problem using permutation sets

Chapter	Contents

Chapter	Contents

Chapter	Contents

3. WORK ON SPECIFIC TOPICS

A considerable portion of the working time of this quarter was spent writing additional chapters of the extended report and in preparing the official report. In addition there was further research dealing with the specific topics:

A. the determination of a suitable general mathematical model for the group scores,
B. the condensation of the N^k group scores into a much smaller number of scores representing the groups,
C. a study of the assembly problem as a problem in linear programming, and
D. the determination of suitable techniques for using an electronic digital computer (MIDAC) in obtaining the reduced grouped matrix when $k = 3$.

4. RESULTS

The main results of this quarter deal with the topics listed above.

A. A mathematical model of considerable generality was developed. This model is based on the statistical properties established in Chapters III and IV and it is broad enough to cover the various cases described by Dr. Roby.

B. Methods were established for grouping the individual scores into classes so that the matrix of group scores would be considerably condensed.

C. The study of the assembly problem as a problem in linear programming showed that this problem is, mathematically, a generalization of the transportation problem and the personnel classification problem. The usual solutions to these problems are related to dual problems dealing with the maximization or minimization of the sum of the bounding set. The theorem relating the solutions of these dual problems, when $k = 2$, does not hold generally when $k > 2$. The modification necessary to make it applicable to the $k > 2$ case has been worked out.

D. Techniques have been completed during this quarter for using MIDAC in reducing $k = 3$ problems with N any number and $n < 6$ to reduced grouped matrices. In no case thus far did the actual reduction take more than 65 seconds, though the print-out time was somewhat larger.

The main results of the research are now complete and the official report is written. During the remaining month of the term of the contract, time will be spent in writing the remaining chapters of the extended report.