

ENGINEERING RESEARCH INSTITUTE
UNIVERSITY OF MICHIGAN
ANN ARBOR

Quarterly Progress Report No. 5

DEVELOPMENT OF GENERALIZED MATHEMATICAL
PROCEDURES FOR OPTIMUM ASSEMBLY OF
POTENTIALLY EFFECTIVE COMBAT CREWS

March 1, 1955 to May 31, 1955

Paul S. Dwyer

Project 2226

U.S. AIR FORCE
AIR RESEARCH AND DEVELOPMENT COMMAND
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Contract No.: AF 18(600)-1050

Budget Project No.: 670-193

Contract Title: Development of Generalized Mathematical Procedures for Optimum Assembly of Potentially Effective Combat Crews

Issuing Office: The Air Research and Development Command

Contractor: The Regents of the University of Michigan

Monitoring Agency: Director, Detachment 4 (Crew Research Laboratory), Air Force Personnel and Training Research Center, Randolph Field, Texas

Principal Investigator: Dr. Paul S. Dwyer

Period: March 1, 1955 to May 31, 1955

PERSONNEL

Name	Title	Portion of Time Devoted to Contract Work
Dwyer, Paul S.	Professor of Mathematics, Consultant in Statistical Laboratory	Up to 40 hours ¹ per month
Taylor, Patricia	Assistant in Research	Half time
Graves, Glenn	Assistant in Research	Varied ²
Bassett, Karen	Typist	Half time ³

¹During this quarter, Dr. Dwyer worked full time on his University duties, and his work on the project was limited to 40 hours per month.

²Mr. Graves, who is assisting in translating some of the methods to routines which can be performed on MIDAC, has worked a total of 62 hours during this quarter.

³Mrs. Bassett has been working full time on the contract since April 11. She terminated her work on the contract on May 27, 1955.

ABSTRACT

This report indicates the reports written and the additional results obtained during the fifth quarter of the term of the contract. The official report "Generalized Mathematical Procedures for the Optimal Assembly of Potentially Effective Combat Crews," has been prepared and sent to the monitoring agency. Several additional chapters of the extended report have been written. Results include a general mathematical formula for group scores, methods for grouping the individual scores into classes, the relation of this problem to other linear programming problems, and a machine method (using MIDAC) for the determination of the reduced grouped matrix.

OBJECTIVE

The general objective of this contract is the development of generalized mathematical procedures for optimum assembly of potentially effective combat crews. More detailed objectives call for (a) a study of the general mathematical theory underlying the group assembly problem, (b) the determination of suitable methods of predicting crew scores from individual scores, (c) the development of a method and technique for finding the maximum assembly sum, (d) the practical adaptation of this technique to a high-speed digital computer, (e) the development of methods for obtaining approximate solutions, and (f) the determination of suitable measures of the adequacy of an approximation.

RESEARCH PROGRESS

1. GENERAL INFORMATION

Additional results, dealing for the most part with the materials of Chapters V, VI, VII, and XIII of the extended report, have been obtained during this quarter. Research has led to a general mathematical model for the group scores, to a development of the theory of the method of reduced matrices which is independent of such classical theory of linear programming as the simplex method, and to satisfactory results in adapting the method of reduced matrices to a digital computer.

Much of the working time on the contract during this quarter has been spent in preparing the necessary reports which describe the results of the contract work. All chapters of the extended report, with the exception of Chapters VII, XIII, and XIV, have been placed in the hands of Dr. Roby. The official report, which contains 70 pages, has also been prepared. The three copies specified in the contract have been sent.

2. THE OUTLINE OF THE EXTENDED REPORT

The outline of the fourteen chapters of the extended report is presented. A copy of all chapters except VII, XIII, and XIV has been sent to Dr. Roby; a copy of these chapters will be sent him prior to June 30.

<u>Chapter</u>	<u>Contents</u>
I	The general group assembly problem <ol style="list-style-type: none"> 1. Introduction 2. Group scores 3. Assembly scores 4. Mathematical statement of the problem 5. Groupings 6. Relation to personnel classification problem and similar problems 7. Use of permutation sets 8. Restatement of the problem using permutation sets

<u>Chapter</u>	<u>Contents</u>
II	Transformations <ol style="list-style-type: none"> 1. Introduction 2. Subtraction of a constant 3. Deviate transformations 4. Approximate deviate transformations 5. Large deviate transformations 6. Extreme transformations
III	The distribution of all possible assembly sums <ol style="list-style-type: none"> 1. Introduction 2. The mean and variance of the distribution of all possible assembly sums for any k 3. The third central moment of the distribution of all possible assembly sums for $k = 2, 3,$ and 4 4. The fourth central moment when $k = 2$
IV	Application of analysis of variance and determination of a mathematical model appropriate to empirical data <ol style="list-style-type: none"> 1. Introduction 2. Analysis of variance when $k = 2$ 3. Analysis of variance when $k = 3$ 4. Analysis of variance when $k = 4$ 5. Analysis of variance for higher values of k 6. Determination of a mathematical model appropriate to empirical data
V	Mathematical models for group scores <ol style="list-style-type: none"> 1. Introduction 2. The observed score matrix for individuals 3. The rating matrix for individuals 4. The rating matrix for subgroups 5. The observed score matrices for subgroups 6. A mathematical model for group scores 7. Special cases of the general model 8. The simplification of the mathematical model by ignoring the main effects 9. Determination of a mathematical model from empirical group scores without

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	the necessity of ratings for individuals
	10. Conclusion
VI	Condensation of group scores
	1. Introduction
	2. Groupings of observed scores for individuals
	3. Groupings by ratings for individuals
	4. Disregard of ratings for individuals
	5. Groupings of ratings for subgroups obtained from scores or from ratings for individuals or classes
	6. Ratings based on scores for subgroups only
	7. Groupings of observed scores for subgroups
	8. Groupings of ratings for subgroups
	9. Effective reduction of k
	10. Precise functional models
	11. Conclusion
VII	The group assembly problem as a problem in linear programming
	1. Linear programming problems
	2. The two-dimensional problem
	3. The general assembly problem
	4. Methods of solution when $k = 2$
	5. The method of reduced matrices
VIII	The two-dimensional assembly problem
	1. Introduction
	2. Conditions of solution
	3. Use of extreme transformations
	4. Method of bounding sets
	5. Marginal zero transformations
	6. Determination of a completely reduced matrix
	7. Determination of an optimal solution from a completely reduced matrix
	8. Solution with the method of reduced matrices
	9. Solution of the quota problem with the detailed method of optimal regions

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IX	<p>Successive applications of two-dimensional techniques</p> <ol style="list-style-type: none"> 1. Introduction 2. A succession of two-dimensional problems 3. Approximate solution of the general problem, using totals of subclasses 4. Use of deviate scores in determining suitable subclasses 5. Use of approximate deviate scores in determining suitable subclasses 6. Use of results of analysis of variance in determining suitable subclasses 7. Conclusion
X	<p>The three-dimensional assembly problem</p> <ol style="list-style-type: none"> 1. Introduction 2. The use of extreme transformations 3. The use of marginal zero transformations 4. The reduction of the G_{ijh} matrix 5. The determination of the solution from the reduced matrix 6. The three-dimensional problem with frequencies 7. The reduced grouped matrix 8. Transformations leading to reduced grouped matrices 9. Reduced grouped matrix transformations 10. The determination of the solution from the reduced grouped matrix 11. Conclusion
XI	<p>The general assembly problem</p> <ol style="list-style-type: none"> 1. Introduction 2. Solution of the general problem with $k = 4$ and $N = 2$ 3. Solution of the general problem with $k = 4$ and $N = 3$ 4. Solution of the general problem with $k = 4$ and $N = 3$, using the method of reduced matrices 5. The solution of a frequency problem with $k = 4$ and $n = 3$

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6. The condensed solution of the frequency problem of section 11.5
7. The solution of a problem having unit frequencies with $k = 5$ and $N = 3$
8. The detailed solution of a frequency problem with $k = 5$ and $n = 3$
9. The generality of the method of reduced matrices

XII

Approximate solutions

1. Introduction
2. Approximate solutions using deviates
3. Approximate solutions using approximate deviates
4. Approximate solutions using large deviates
5. Approximate solutions using large row deviates
6. Approximate solutions of problems with $k > 2$, using a succession of two-dimensional techniques
7. Approximate solutions using reduced matrices
8. Approximate solutions using reduced matrices and successive interchanges
9. Measures of the adequacy of an approximation
10. Conclusion

XIII

Punched-card and machine methods

1. Use of marginal punched cards
2. Use of IBM punched cards and machines
3. Use of electronic digital computers

XIV

Concluding remarks

1. Summary
2. Recommendations for further research

3. WORK ON SPECIFIC TOPICS

A considerable portion of the working time of this quarter was spent writing additional chapters of the extended report and in preparing the official report. In addition there was further research dealing with the specific topics:

- A. the determination of a suitable general mathematical model for the group scores,
- B. the condensation of the N^k group scores into a much smaller number of scores representing the groups,
- C. a study of the assembly problem as a problem in linear programming, and
- D. the determination of suitable techniques for using an electronic digital computer (MIDAC) in obtaining the reduced grouped matrix when $k = 3$.

4. RESULTS

The main results of this quarter deal with the topics listed above.

A. A mathematical model of considerable generality was developed. This model is based on the statistical properties established in Chapters III and IV and it is broad enough to cover the various cases described by Dr. Roby.

B. Methods were established for grouping the individual scores into classes so that the matrix of group scores would be considerably condensed.

C. The study of the assembly problem as a problem in linear programming showed that this problem is, mathematically, a generalization of the transportation problem and the personnel classification problem. The usual solutions to these problems are related to dual problems dealing with the maximization or minimization of the sum of the bounding set. The theorem relating the solutions of these dual problems, when $k = 2$, does not hold generally when $k > 2$. The modification necessary to make it applicable to the $k > 2$ case has been worked out.

D. Techniques have been completed during this quarter for using MIDAC in reducing $k = 3$ problems with N any number and $n < 6$ to reduced grouped matrices. In no case thus far did the actual reduction take more than 65 seconds, though the print-out time was somewhat larger.

The main results of the research are now complete and the official report is written. During the remaining month of the term of the contract, time will be spent in writing the remaining chapters of the extended report.

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