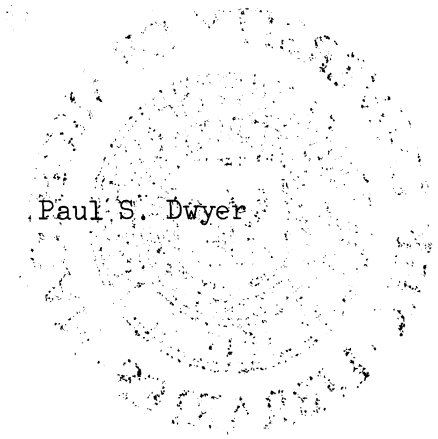
ENGINEERING RESEARCH INSTITUTE
UNIVERSITY OF MICHIGAN
ANN ARBOR

Summary Report

DEVELOPMENT OF GENERALIZED MATHEMATICAL PROCEDURES FOR OPTIMUM ASSEMBLY OF POTENTIALLY EFFECTIVE COMBAT CREWS

March 1, 1954, to June 30, 1955



Project 2226

U. S. AIR FORCE
AIR RESEARCH AND DEVELOPMENT COMMAND
CONTRACT NO. AF 18(600)-1050

June 1955

ensm

UMR0883

Contract No.:	AF 18(600)-1050
Budget Project No.:	670-193
Contract Title:	Development of Generalized Mathematical Procedures for Optimum Assembly of Potentially Effective Combat Crews
Issuing Office:	The Air Research and Development Command
Contractor:	The Regents of the University of Michigan
Monitoring Agency:	Director, Detachment 4 (Crew Research Laboratory), Air Force Personnel and Training Research Center, Randolph Field, Texas
Principal Investigator:	Dr. Paul S. Dwyer
Period:	March 1, 1954, to June 30, 1955

ABSTRACT

This report provides summary information about the work on the project. It presents:

(1) an outline of the contents of the extended research report, previously submitted, which gives the detailed results of the study,

(2) a statement of the basic conclusions and recommendations resulting from the study,

(3) the names, duties, extent of service, reimbursement, and work accomplished for the various personnel engaged on the project, and

(4) a summary accounting for the use of contract funds.

OBJECTIVE

The general objective of this contract is the development of generalized mathematical procedures for optimum assembly of potentially effective combat crews. More detailed objectives call for (a) a study of the general mathematical theory underlying the group assembly problem, (b) the determination of suitable methods of predicting crew scores from individual scores, (c) the development of a method and technique for finding the maximum assembly sum, (d) the practical adaptation of this technique to a high-speed digital computer, (e) the development of methods for obtaining approximate solutions, and (f) the determination of suitable measures of the adequacy of an approximation.

1. OUTLINE OF THE CONTENTS OF THE REPORTS PREVIOUSLY SUBMITTED

A copy of a research report, containing 14 chapters with references, appendix materials, and extensive illustrations, which may be said to give a detailed statement of the work on and results of the project, has been placed in the hands of Dr. Roby. Copies of an abbreviated report of about 80 pages, which may serve as the basis of a technical report in the AFPTRC series, have also been presented. The following outline of topics in the extended research report gives some indication of the various topics studied and the detail of the presentation. The outline of topics in the abbreviated report is similar.

2. OUTLINE OF THE EXTENDED REPORT

Chapter	Contents
I	The general group assembly problem 1. Introduction 2. Group scores 3. Assembly scores 4. Mathematical statement of the problem 5. Groupings 6. Relation to personnel classification problem and similar problems 7. Use of permutation sets 8. Restatement of the problem using permutation sets
II	Transformations 1. Introduction 2. Subtraction of a constant 3. Deviate transformations 4. Approximate deviate transformations 5. Large deviate transformations 6. Extreme transformations
III	The distribution of all possible assembly sums 1. Introduction 2. The mean and variance of the distribution of all possible assembly sums for any k

3. BASIC CONCLUSIONS AND RECOMMENDATIONS

A. BASIC CONCLUSIONS

The following conclusions parallel the objectives of the contract as stated above.

1. A study of the general mathematical theory of the group assembly problem has shown it to be more than a generalization of the transportation problem; many methods used in solving the transportation problem are not adequate for handling the group assembly problem. This study led to the method of reduced matrices, which is directly applicable to the transportation problem as well as to the general group assembly problem.

2. The study of group scores, and particularly the study of the variation of the group assembly sum by analysis of variance techniques, has led to a suitable mathematical model for determining the group scores. The analysis leads to the conclusion that the interaction terms are the important ones.

3. The study leads to the conclusion that the method of reduced matrices is a recommended method both for the transportation problems and for the general assembly sum problem.

4. The conclusion is drawn that the method of reduced matrices is adapted to high-speed digital computers. This was demonstrated by

making the reductions on M I D A C for many k = 3 problems.

5. Useful approximate solutions can be obtained by:
   (a) the earlier steps of the method of reduced matrices,
   (b) the results of the deviate transformation, and
   (c) the successive use of two-dimensional techniques.

6. The conclusion is drawn that suitable estimates of the adequacy of an approximation can be determined from the same partially reduced matrix which provides the value of the approximate sum.

In general, the objectives of the contract seem to be completed successfully, especially when k = 3, and the accomplishment of these objectives brings the need for additional studies into focus. These are indicated by the following recommendations for future study.

B. RECOMMENDATIONS

1. Additional study of the solution when the number of positions is large.

2. Extension of the techniques in which several alternative criteria of group effectiveness are to be applied simultaneously.

3. More study on the problem of grouping the group scores into classes.

4. Extended study of the use of electronic digital computers in obtaining exact and approximate solutions.

5. A study of the effects of the errors of the fallible group scores on the process of maximization.

6. An investigation leading to the identification of the group assembly problem with other problems which may be encountered by the Air Force, the Army, the Navy, or industrial organizations having contracts with the Armed Forces.

4. SUMMARY OF PERSONNEL ACTIVITY

The names of the persons participating in the work on the contract, together with the duties, period of service, reimbursement, and work accomplished, are indicated in the table and footnotes below.

Name	Title	Portion of Time Devoted to Contract Work
Dwyer, Paul S.	Principal Investigator (Professor of Mathematics, Consultant in Statistical Research Laboratory)	Up to 40 hours per month[1]
Graves, Glenn	Assistant in Research (Graduate Student)	Varied[2]
Hubbell, Charles	Assistant in Research (Graduate Student)	Full time[3]
Lott, Fred	Assistant in Research (Graduate Student)	Half time[4]
Rider, Leonard	Assistant in Research (Graduate Student)	Half time[5]
Taylor, Patricia	Assistant in Research (Graduate Student)	Half time[6]
Bassett, Karen	Typist	Half time[7]
Parker, Kathryn	Secretary (Student)	Varied[8]

[1] During the summer months of 1954, June 13 to September 13, Dr. Dwyer worked full time on the project. Throughout the other 13 months of the contract, be worked full time on his University duties, and his work on the project was limited to 40 hours per month. He had general charge of the work on the project and the preparation of the reports and worked a total of 982-1/2 hours at the rate of $8.40 per hour.

[2] Mr. Graves began working on the project January 27, 1955, and terminated June 30, 1955. His particular job was to assist in translating some of the methods to routines which could be performed on M I D A C (Michigan Digital Automatic Computer), which he did successfully. He worked a total of 100 hours at the rate of $3.00 per hour.

[3] Mr. Hubbell began work on the project June 7, 1954, and terminated September 13, 1955. He assisted in the determination of the general solution when $k = 3$, and applied the methods to IBM machines and

marginal punched cards. He worked a total of 417-1/2 hours at the rate of $2.00 per hour.

[4]Mr. Lott began work on the project June 7, 1954, and terminated September 13, 1955. He studied the problem of moments of the assembly sum and derived and illustrated the important formulas of Chapter III of the extended report. He worked a total of 310-1/2 hours at the rate of $2.00 per hour.

[5]Mr. Rider started work on the project on September 18, 1954, and terminated January 6, 1955. He assisted in perfecting the $k = 2$ and $k = 3$ techniques and in showing how the methods could be applied to $k = 4$ and $k = 5$ problems. He worked a total of 274-1/2 hours at $2.00 per hour.

[6]Miss Taylor started work on the project September 23, 1954, and terminated June 30, 1955. She contributed greatly to the development of the method of reduced matrices, particularly in formalizing the reduced grouped matrix transformations. She acted as a coauthor with the principal investigator in writing all reports prepared during 1955. She worked a total of 741-1/2 hours at $2.00 per hour.

[7]Mrs. Bassett began work on January 10, 1955, and terminated May 27, 1955. From April 11 until she terminated she worked full time. She typed the reports and numerical illustrations which describe the results of the various aspects of the contract work. She worked a total of 517-1/2 hours at $1.20 per hour.

[8]Miss Parker worked half time on the project from August 24, 1954, to September 20, 1954 typing the project reports. She worked a total of 60 hours at $1.40 per hour.

5. SUMMARY ACCOUNTING OF UTILIZATION OF CONTRACT FUNDS

This summary is made as of June 25, 1955. The exact final figures cannot at this date be determined since, for example, the cost of this summary report is not known though an estimate is available. It should be noted that the figures below apply to the 16-month contract as renegotiated rather that to the original 13-month contract.

USE OF CONTRACT FUNDS AS CALCULATED JUNE 25, 1955

Services of principal investigator	$8,253.00	
Services of assistants in research	3,788.00	
Clerical (for typing reports)	705.00	
Total personnel		$12,746.00
Service charge {35% on personnel during 1954 / 37% on personnel during 1955}		4,589.77
Reproducing Reports (includes $60.00 estimate for this summary report)		323.95
Travel (trips to San Antonio and New York)		354.13
Tabulating and M I D A C		237.90
Supplies		41.78
Remaining funds as of June 25, 1955		876.47
Total Funds		$19,170.00