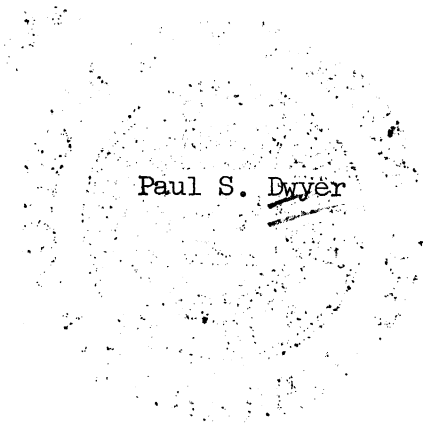
ENGINEERING RESEARCH INSTITUTE
THE UNIVERSITY OF MICHIGAN
ANN ARBOR

Quarterly Progress Report No. 4

DEVELOPMENT OF MATHEMATICAL PROCEDURES AND MULTIPLE CRITERIA FOR ASSEMBLY OF LARGE WORK GROUPS

April 16, 1956 — July 15, 1956



Project 2413

U. S. AIR FORCE
AIR RESEARCH AND DEVELOPMENT COMMAND
CONTRACT NO. AF 41(657)-9

July 1956

Contract No.:	AF 41(657)-9
Budget Project No.:	7-7713
Contract Title:	Development of Mathematical Procedures and Multiple Criteria for Assembly of Large Work Groups
Issuing Office:	The Air Research and Development Command
Contractor:	The Regents of The University of Michigan
Monitoring Agency:	Director, Crew Research Laboratory, Air Force Personnel and Training Research Center, Randolph Field, Texas
Principal Investigator:	Dr. Paul S. Dwyer
Period:	April 16, 1956 to July 15, 1956

PERSONNEL

Name	Title	Portion of Time Devoted to Contract Work
Dwyer, Paul S	Professor of Mathematics Consultant in Statistical Research Laboratory	Variable[1]
Galler, Bernard A.	Instructor in Mathematics Mathematics Consultant	Variable[2]
Graves, Patricia	Assistant in Research	Half Time

[1]During the first two months of this quarter, Dr. Dwyer worked full time on his university duties and his work on the project was limited to 40 hours per month. During the summer months he is working full time on the contract.

[2]During the first two months of this quarter, Dr. Galler worked full time on his university duties and his work on the project was limited to 40 hours per month. During the summer months he is working full time on the contract.

ABSTRACT

This report provides information about the progress of the work on the project during its fourth quarter. It presents:

1. A statement of the general objectives as indicated in the specifications of the contract and as amplified during the July, 1955 conference with Dr. Roby;

2. A statement of the objectives of the work of this quarter.

3. A discussion of the procedure used in carrying out the work of the quarter;

4. A statement of the general results obtained during the quarter;

5. A general discussion of the work on the contract to date with plans for the remaining term of the contract;

6. A summary statement.

I. OBJECTIVES

The general objectives of the work on this contract are the development of mathematical procedures for assembling individuals in large work units and employing multiple criteria for assembly. More specific objectives indicated in the contract include:

1. The extension of results obtained for 3- to 5-man group assemblies so as to secure the optimal assignment of individuals to groups of larger sizes;

2. The development of economical means of accomplishing data transformations and practical simplifications of the problem encountered in reducing grouped matrices;

3. The translation of these procedures into programs suitable for use with electronic digital computers;

4. The use of appropriate approximate solutions when the criteria for assembly are based on fallible scores and consideration should be given to errors in group scores;

5. The feasibility of simultaneously employing multiple criteria of classification such as may be desirable if both technical qualifications and social factors are to be considered in forming work groups.

Additional specific objectives were agreed on during the July, 1955 conference with Dr. Roby. These include:

6. A revision of substantial portions of the 14-chapter report of the previous contract, necessitated by the development of improved techniques and theory in the later periods of the contract which were not incorporated in the draft presented;

7. Additional work directed toward the further analysis and improvement of the reduced grouped matrix transformations;

8. The identification of the basic mathematical problem in group assembly with the basic mathematical problem of other applied problems such as the general transportation problem;

9. The determination of the extent of the relationship existing between the group-assembly problem and other related problems in linear programming;

10. An investigation of the effects of coarser groupings.

Objective 7 is closely related to objectives 1 and 2.

II. OBJECTIVES OF THE WORK OF THIS QUARTER

In general, the objectives of the work of this quarter were related to objectives 3, 4, 5, 7, and 8 stated above. More explicitly, the objectives of this quarter, in the order of time spent, were

1. The design of the theory and techniques appropriate to, and the preparation of programs for, electronic digital computers such as MIDAC, IBM Type 650, and IBM Type 704, along with certain modifications and adjustments in the method of reduced matrices to make it more conveniently applicable to machine techniques;

2. The development of a more adequate mathematical theory for obtaining an integral solution to the general problem once a fractional solution is available;

3. The determination of suitable approximate solutions where the group scores are fallible and subject to error;

4. The investigation of the feasibility of simultaneously employing multiple criteria of classification;

5. The preparation of certain results of the study for publication or an address.

III. PROCEDURE

The procedure during this quarter was similar to that of the previous ones. Dr. Galler has been working on the translation of the general results into programs suitable for electronic digital computers. Mrs. Graves and the project supervisor have been investigating theoretical questions raised in this process and have worked considerably on the problem of fractional solutions.

IV. RESULTS

The chief results of this quarter are summarized below.

1. A major result is the adaptation of the general method of reduced matrices to successful solution on an electronic digital computer. Originally we had planned to use MIDAC for this work, but with the availability of the IBM Type 650 in the Statistical Research Laboratory we shifted to the use of this machine. In some respects (storage and hence speed in the case of this problem) the IBM Type 650 is not as satisfactory as the MIDAC. However, the difference in costs to us ($28 per hour vs $68 per hour) favored the Type 650 and was more appropriate to the total of $4000 in the budget for machine time. A single program has now been prepared for the Type 650 so that we can run problems with k = 2, 3, 4, 5, 6, or 7 and obtain answers consisting of integral or fractional positive values. We have not yet worked on the elimination of the fractional answers with the machine. The solution of the k = 7 problem of Research Report No. 2 (see Table 6.1) took 45 minutes, while the solution of the k = 2 Easterfield problem (see Table 2.3.1 of the extended report of the previous project) took 15 minutes. This solution is not fully automatic but demands the attention of an operator for feeding cards and manipulating switches. The computation time should be greatly diminished with a machine such as the IBM Type 704, in which the technique is fully automatic. However, we feel that the more general accessibility of the Type 650 makes the program for this machine useful even though the machine time is somewhat longer.

2. Related to this is the fact that we have obtained the use of an IBM Type 704, without financial expense to the project, and have made considerable progress in programming the method for very general problems. Dr. Galler is working every day now on this at General Motors Technical Center and is making considerable progress with the program. He reports that his program will handle problems up to k = 20 with the total number of subdimensions, $n_1 + n_2 + \cdots + n_k$, equal to several hundred. This general program works also for the important special case (the personnel classification problem and the Hitchcock transportation problem) with k = 2. It now seems reasonable to expect that we will be able to obtain positive solutions (though not yet necessarily integral ones) on this machine for very general problems by the end of July.

3. The adaptation of the general mathematical method to machine computation has led to general questions which we have resolved. For example, we have found that the transformation based on the sum of the linear inconsistent forms is as effective as is the transformation based on the particular form having maximum inconsistency. This fact does not lead to an improvement with hand methods, but Dr. Galler finds it preferable for use with the IBM Type 704.

4. We have continued the improvement of the general mathematical theory. During this quarter this has centered, for the most part, on the determination of the integral solution from a fractional solution. This has turned out to be a combinatorial problem of some magnitude. We have now worked out $n_1 + n_2 + \ldots n_k$ conditions which must be satisfied in order for a nonzero element to be added to the permutation set of zeros and yield an integral solution.

5. Preliminary studies have been made on the problem of determining appropriate answers where the group scores are fallible and subject to error. Our results thus far indicate that, for many problems, errors of small size seem to have little effect on the solution.

6. Studies have also been made on the feasibility of simultaneously employing multiple criteria of classification. This theory is developing nicely. The basic tool which we use is the completely reduced matrix (or the almost completely reduced matrix) associated with each criterion. Then the effectiveness of one criterion can be measured against the effectiveness of the other criteria by using total errors, measures of efficiency, etc.

7. One paper, "The Method of Reduced Matrices for a General Transportation Problem," was prepared during this quarter by the project supervisor and Dr. Galler for presentation by Dr. Galler at the Los Angeles meeting of the Association for Computing Machinery. This paper contains a description of the method of reduced matrices as well as a description of an approximation method we have developed.

V. DISCUSSION

On the whole, we feel that we are accomplishing the objectives of the contract. The time taken to put the method on an immediately practicable basis using a large commercial machine (in addition to the demonstration that the method can be prepared for machines as provided in the contract) has slowed up the schedule some, since we wish to use machine solutions to problems of some magnitude in the study of general objectives 4 and 5.

The work of the remaining period of the contract will feature

1. The completion of the programming of the method for IBM Type 704;
2. The study of general objective 4;
3. The study of general objective 5;
4. Further study on the topics indicated by objectives 8, 9, and 10;

5. The preparation of research reports;
6. The preparation of a final report;
7. The writing of two or three papers for possible publication in scientific journals.

VI. SUMMARY STATEMENT

Though we have been slowed up a bit by taking time to put the method of reduced matrices on one of the very largest commercial electronic digital computers, we feel that the resulting practical availability of the method makes the results more valuable. We are more enthusiastic than ever about the method of reduced matrices and feel that if provides an effective solution not only for the group-assembly problem but also for several other problems in which the Government is interested.