THE UNIVERSITY OF MICHIGAN

INDUSTRY PROGRAM OF THE COLLEGE OF ENGINEERING

THEORY AND PERFORMANCE OF TURBO-MACHINERY

Glenn V. Edmonson

March, 1963

IP-611

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THEORY AND PERFORMANCE OF TURBO-MACHINERY

Introduction

A pumping machine is a portion of a total fluid transport system where the required energy to move the fluid at a prescribed rate-of-flow is added. All types of pumping machinery can be generally classified either displacement or impulse pumps. This paper is concerned with the impulse machine.

The displacement machine is characterized by a variable volume cavity bounded on all sides by rigid walls and into which fluid is forced during expansion of the cavity volume by an energy gradient favorable to fluid flow toward the cavity. On contraction of the volume of the cavity fluid volume is forced to a higher energy level by an external energy source, usually mechanical in nature. The flow rate in the system, in all cases, is, to a degree, oscillatory in character, the amplitude of the variation being a function of the number of cavities in action and the frequency with which the cavities complete the expansion and contraction cycle.

The turbo-machine, on the other hand, is characterized by an in-line type of enclosure, unbounded by rigid surfaces at right angle to the flow path of fluid. Fluid is forced, by a favorable energy gradient, into the machine. Energy additions to the fluid, in all cases, occur by an impulse action of a rotating blade system. Energy is supplied to the rotating blade system usually by a mechanical source of power, is transferred to the fluid by blade system action, the transfer in all cases resulting in a "change-of-whirl." The passages for fluid flow are relatively unrestricted in the path of flow of the fluid. In the immediate vicinity of the rotating blade system, a periodic fluctation occurs that is damped out by resistance to flow such that, for all practical purposes, the machine may be regarded as a steady flow device under equilibrium system requirements.

Obviously, if an energy gradient exists without the inclusion of a pumping machine that will off-set the loss of energy in the transport system, a pump should not be included. This specific reference is intended to reinforce the following. "The characteristic behavior of an impulse pumping machine is completely regulated by the requirements of the transport system."

The term "external system" will be used throughout when discussing that portion of a system external to the flanges of the pump. The term "internal system" will be used in the discussion of that portion of the total system between the flanges of the pump.

The following sections constitute the body of this paper:

I) General Energy Relationships.

II) Behavior of Fluids.

III) Theory and Performance of Turbo-Machinery, and

IV) Capacity of a Pumping System.

I. GENERAL ENERGY RELATIONSHIP

The reason to use any form of pumping machinery is to utilize a source of energy and to convert that energy to a form that may be used to advantage. The pump depends on a source of mechanical energy which is converted into forms of stored energy required to transport fluid. It is a part of an engineering system.

The behavior in all engineering systems is governed by the natural and physical laws of the universe. Although these laws are intuitive, continued successful application has established their validity, and consequently form the basis for all engineering analytical procedures. The Laws of Thermodynamics, Newton's Laws of Motion and Gravitational Forces, and the Conservation of Mass form the basis for the engineering system analysis with which this paper is concerned. The laws of thermodynamics are accounting processes; the laws of motion provide an insight into the mechanism of energy transition.

The term, Work, is defined as follows:

$$\pm W = \int f ds \qquad (1)$$

where (W) with the minus sign (-) indicates work done upon a system; (f) is a force in pounds moving through an incremental distance, ds the product of which when integrated ($\int$) is the work done by the force (f) moving through a total distance.

The First Law of Thermodynamics states that the sum of heat added plus the work done upon (minus sign -) or the work done by (plus sign +) an engineering system is equal to the total change of energy in the system.

In equation form

$$q \pm W = \Delta e \qquad (2)$$

where (q) is the heat in BTU added to the system, $\pm$ W is the work done by or upon the system in BTU, and Δe is the total charge of energy of that system in BTU.

Figure 1 is a sketch of a machine where the items of stored energy, energy in transition, heat, and work are indicated. Since we are concerned with the pump, shaft work (W) is indicated as being gone upon the machine.

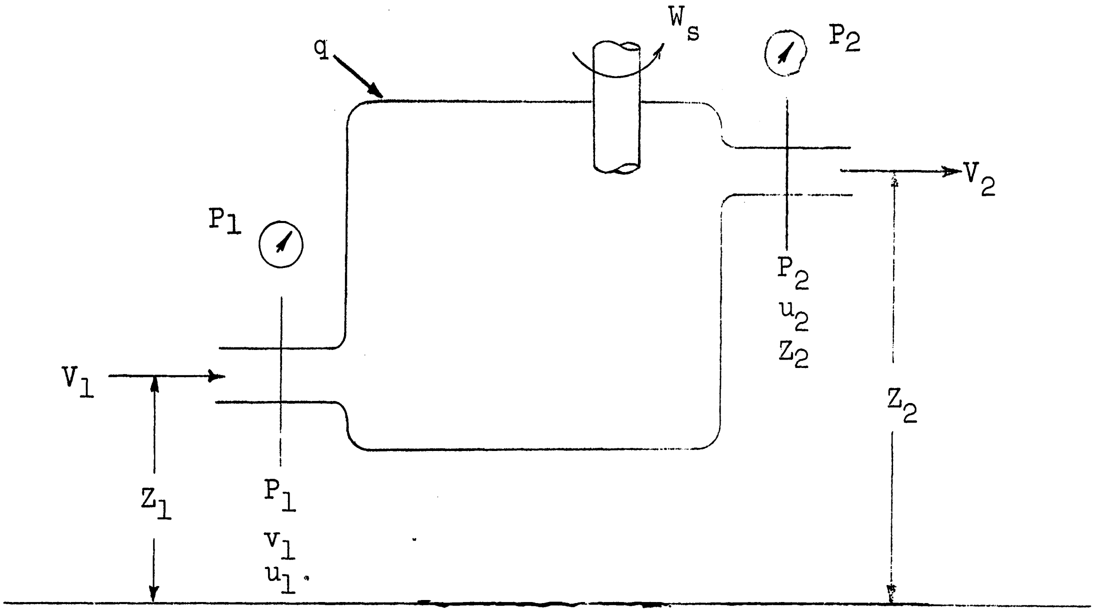


Figure 1.

Z = potential energy (ft.lbs/lb)
P = pressure - (lbs/sq.ft.)
v = specific volume - (cu.ft./lb.)
u = internal stored energy - (BTU/lb.)
V = mean absolute velocity - (ft./sec.)
J = 778 ft. lbs. per BTU.
W_s = Shaft work - (ft. lbs. per lb.)
q = heat added - (BTU/lb.)

Expansion of Equation (2) yields

$$q - \frac{W_s}{J} = \left(\frac{P_2v_2 - P_1v_1}{J}\right) + \left(\frac{V_2^2 - V_1^2}{2gJ}\right) + \left(\frac{Z_2 - Z_1}{J}\right) + (u_2 - u_1) \quad (3)$$

Item 1 on the right hand side of the equation being the total change in flow-work, item 2 being the total change in kinetic energy;

item 3 being the total change of potential energy; and item 4 the total change of internal energy as indicated by a temperature change.

Equation (3) has particular significance when written in the differential form.

$$dq - \frac{dW_s}{J} = \frac{Pdv - vdP}{J} + \frac{VdV}{gJ} + dz + du \qquad (4)$$

and when re-arranged

$$\frac{-dW_s}{J} = \left(\frac{Pdv}{J} + du - dq\right) + \frac{1}{J}\left(vdP + \frac{VdV}{g} + dz\right) \qquad (5)$$

It is important to understand the sum of the first three terms on the right hand side of Equation (5).

Supposing we construct a special closed cylinder (Figure 2) and piston arrangement which permits a volume expansion or contraction by a movement of the piston. Application of the First Law of Thermodynamics to the cylinder can be made where:

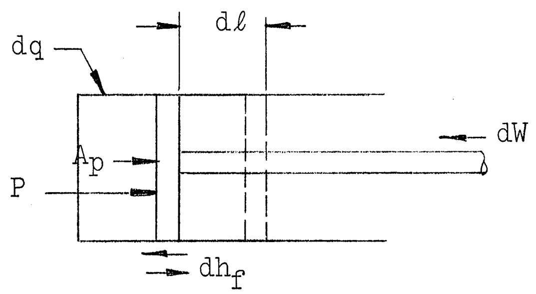


Figure 2

$d\ell$ = incremental length of piston movement.
A_p = area of piston
$A_p d\ell$ = dv (change of volume)

Then

$$PA_p d\ell = Pdv \qquad (6)$$

dh_f = resistance to piston motion (ft.lbs).

And

$$\frac{dW_s}{J} = -\frac{Pdv}{J} + \frac{dh_f}{J} \qquad (7)$$

In applying an energy balance to the closed cylinder on (2) for the closed cylinder

$$-\frac{dW_s}{J} + dq = du \qquad (8)$$

And substituting the value of dW_s from (7) the following results:

$$\frac{Pdv}{J} + du - dq = \frac{dh_f}{J} \qquad (9)$$

If the rigid boundary of the cylinder sketched in Figure 2 were replaced by a fluid particle with flexible boundaries and you, as an observer, were stationed at (A), and the particle was moving in a steady-flow process, the observer would see a contraction or expansion of the boundaries, would experience a rise or

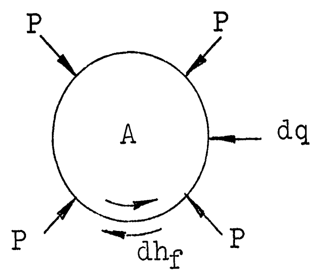


Figure 3.

fall of temperature, and would note a shear stress at the boundary of the particle and the adjacent fluid. The observer would recognize Equation (9) as the analytical description of his observation. And for the machine, Equation (5) becomes

$$-dW_s = (vdP + \frac{VdV}{g} + dz) + dh_f \qquad (10)$$

The above form of the General Energy Equation is of use where it is required to know the mechanism of fluid motion and will be found to be of particular benefit in solving the problems associated with relatively incompressible fluids.

In the case of the pump handling water the specific volume (v) can be treated as a constant and since $v = 1/\gamma$ (γ = specific weight) and $P = 144\ p$ (p = lbs. per sq.in.)

$$- W_s = \frac{(p_2 - p)144}{\gamma} + \frac{V_2 - V_1^2}{2g} + (Z_2 - Z_1) + h_{f_{2-1}} \qquad (11)$$

For the special case when $W_s = 0$ and the resistance to fluid flow = 0 ($h_f = 0$)

$$\frac{(p_2)144}{\gamma} + \frac{V_2^2}{2g} + Z_2 = \frac{(p_1)144}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \text{constant} \qquad (12)$$

which is recognized as Bernoulli's equation as derived from the concept of Conservation of Energy.

SUMMARY

1) Equation (10)

$$- dW_s = (vdP + \frac{VdV}{g} + dz) + dh_f$$

results from a general analysis of a mechanically driven machine. The pump that handles a liquid is a special case. Reasonable assumption that the specific weight (γ) and the inverse, specific volume (v) remains constant in the case of the pump leads immediately to Equation (11) in which all terms are measureable in terms of average values. Further, the term, $h_{f_{2-1}}$, becomes a portion of the "internal" system energy requirement which by necessity must be a part of total "work" supplied by the energy source.

2) The $h_{f_{2-1}}$ term requires attention. Referring to Equation (9)

$$\frac{Pdv}{J} + du - dq = dh_f$$

it is immediately apparent that Pdv = 0 following the assumption that (v) is constant. The term (dq) may or may not be zero. The ambient temperature surrounding the machine or more precisely, the differential temperature between the ambient and the temperature of the fluid passing through the system will determine the effect of the term. In any case

$$du = c(T_2 - T_1)$$

where c is a proportionality constant and $(T_2 - T_1)$ is the absolute temperature rise of the fluid.

Equation (9) may thus be rewritten

$$c(T_2 - T_1) = dh_{f_{2-1}} + dq$$

Thus indicating that effects of viscous force interaction between the boundaries of the machine passage (rotors and casings) and the fluid causes a temperature rise in the fluid. In many pumping applications, the temperature rise is so small as to be reasonably neglected, however, only after due consideration of the term.

II. BEHAVIOR OF FLUIDS

A. Properties of Fluids

Fluids can generally be classified as gases or liquids and for most engineering purposes can be judged to be homogeneous. Both fluids have properties as specific weight (r - lbs. per cu.ft.) specific volume (v = cu.ft. per lb.); density (ρ - mass/cubic volume = r/g); and vapor pressure (p_v = unit pressure/unit area at the temperature at which a liquid and its gaseous phase are in equilibrium).

A gas is defined as a fluid which will conform to the shape of its container and will occupy the entire enclosed volume. A liquid, on the other hand, will conform to the shape of its container and will occupy only that portion of its container proportional to the weight of fluid.

Both liquids and gas typically yield continuously under shear stresses; both fluids are capable of supporting compressive stress if confined within fixed rigid boundaries.

There are several properties of real fluids that need attention.

1. Viscosity

Viscosity is a measure of the resistance a fluid offers to a continous shear force. If two plates are separated a finite distance and one of the two plates is caused to move at a velocity, V_2, relative to the other, the fluid

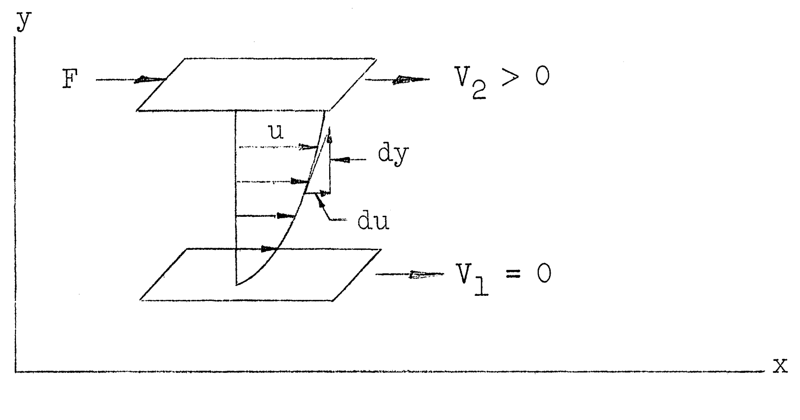


Figure 4.

between those plates will deform and will resist the force that is required to move the plates. The velocity of movement of a real fluid layer next to each plate is equal to the velocity of the respective plate. The velocity of fluid motion, u, intermediate between the plates will take on some distribution as indicated in Figure 4. The coefficient of viscosity, μ, of the fluid is defined as follows:

$$\frac{F}{A} = \mu \frac{du}{dy} \qquad (14)$$

$$\frac{F}{A} = \tau \text{ (Shear stress)}$$

$$\therefore \quad \tau = \mu \frac{du}{dy}$$

$$\text{or} \quad \mu = \frac{\tau}{\frac{du}{dy}}$$

2. Pressure at a Point in a Fluid

The pressure at a point in a fluid under both equilibrium and accelerated motion conditions is equal and opposite in all directions. This statement is subject to proof; however, it is stated here in order to develop a fundamentally important relationship between pressure and the height of a homogeneous fluid. Many fluid pressure measurements that are necessary to establish an energy level are made by measurement of height. For instance, the pressure of the atmosphere against the surface of the earth or the surface of a lake is proportional to the height of the atmosphere above a given point. The gauge pressure at the bottom of a lake is proportional to the height of water above the point of measurement. The absolute pressure at the bottom of the lake is the sum of the pressure exerted by the atmosphere against the surface of the lake plus the pressure exerted by the water above the same point.

Measurement of Pressure at any point in a fluid may or may not be a measure of a continuing energy supply.

3. Pressure-Height Relationship

The pressure-height relationship that measures pressure only is illustrated in Figure 5.

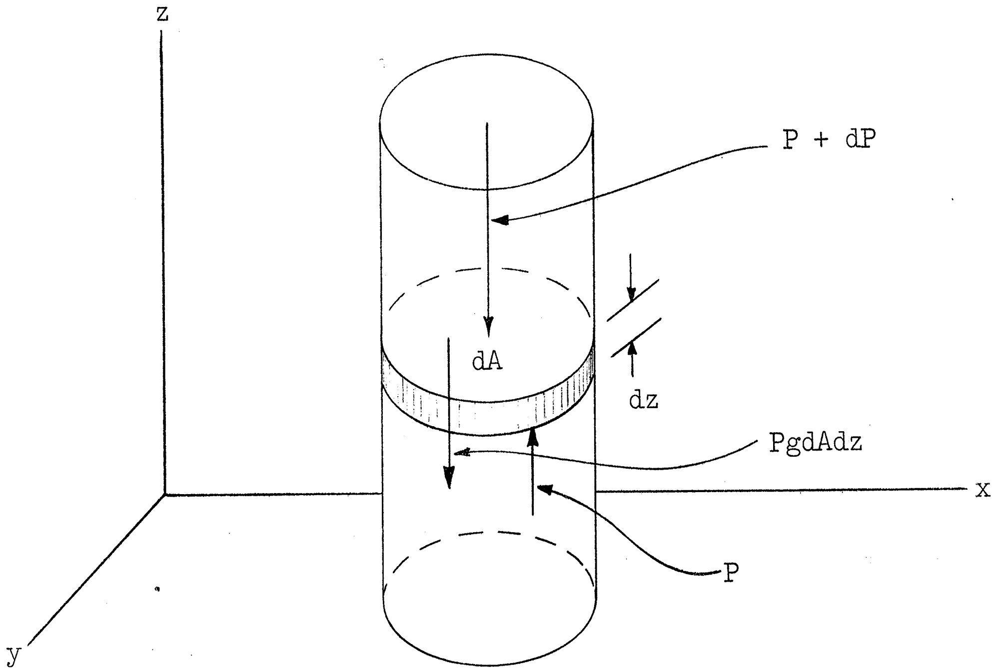


Figure 5.

$$\sum Fz = 0$$

$$PdA - (P + dP)\, dA - \rho g\, dAdz = 0$$

$$- dP - \rho gdz = 0$$

$$- \int dP = g \int_1^2 \rho dz$$

In the case of air the density, ρ, is a function of height, whereas, for water, ρ is considered constant over rather wide variations in height. The integral may be written for water as follows:

$$- \int_1^2 dP = \rho g \int_1^2 dz$$

$$\text{and } (P_1 - P_2) = \rho g(z_2 - z_1)$$

$$P = 144\ p$$

$$\rho g = \gamma$$

$$\therefore \quad \left(\frac{p_2 - p_1}{\gamma}\right) 144 = z_2 - z_1 \tag{16}$$

B. Fluids in Motion

At any instant in fluid motion and at a given point within the fluid body, it is theoretically possible to describe the motion in terms of an absolute velocity, V-ft./sec., and assign to an incremental mass a fluid vector which has both magnitude and direction. This vector may likewise describe the motion of an adjacent fluid increment in either magnitude or direction, or both. Depending upon such fluid parameters as density (ρ) and viscosity (μ) and upon the dimensions of the fluid channel, it is entirely possible, however, that the vector, V, that would describe the motion of an adjacent particle may be completely different.

Although the isolation of individual mass particles is practically impossible, it is necessary to understand incremental descriptions if one is to have an understanding of gross fluid flow behavior. The understanding of gross fluid flow behavior is an important part of all fluid machine design. Momentum and shear stress as applied to an incremental fluid volume lead to a regularly used term - Reynolds number , R_e.

The concepts of rotational and irrotational motion lead to an understanding of the term circulation and the fluid vortex. Further, the understanding of Reynolds number leads to the evaluation of laminar and turbulent motion and the accompanying fact of resistance to flow of all real fluids.

1. Reynolds Number

Momentum is defined as the product of the mass of a body times the velocity with which the mass is moving. If the mass is caused to accelerate an inertial force equal and opposite to an external force, causing the change in motion, exists. This is internal force as follows:

$$f = \frac{d}{dt}(mv) \tag{17}$$

Figure 6 is an isolated incremental mass of fluid in a rectangular coordinate system. A system of forces acts upon it, namely: 1) forces that will cause it to accelerate, 2) viscous forces from surrounding fluid at its boundaries tending to resist its motion, and 3) pressure forces normal to its boundaries which in a limiting case have been said to be equal and opposite in all directions. This latter statement is subject to proof not demonstrated in this paper. The following demonstration of effects in the y-direction can be extended to both the x and z directions of motion.

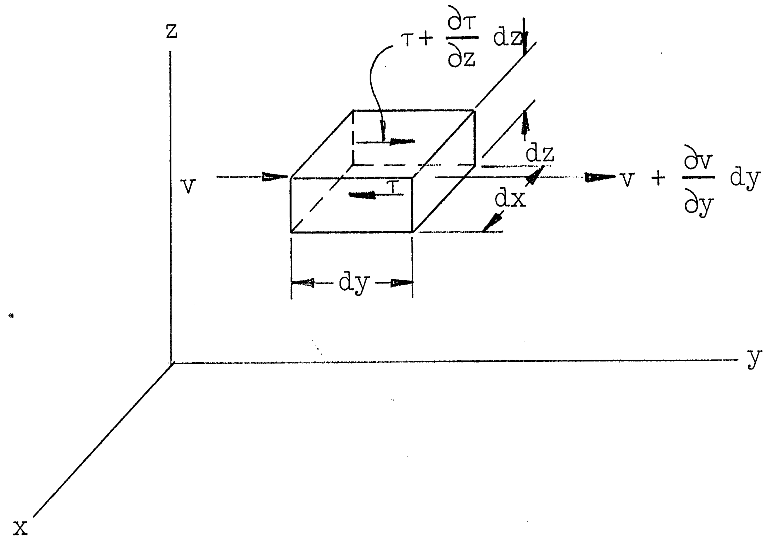


Figure 6.

If the general statement of (17) is applied to the fluid particle illustrated in Figure 6, the following results:

(Inertial forces) = f_i

$$f_i = \frac{d}{dt}(mv) = \rho dxdydz \frac{d}{dt}\left[\left(v + \frac{\partial v}{\partial y} dy\right) - v\right]$$

$$= \rho dxdydz \frac{dv}{dy}\frac{dy}{dt}$$

But $\frac{dy}{dt} = v$

$$\therefore \quad f_i = \rho v \, dxdydz \frac{\partial v}{\partial y} \qquad (18)$$

Further, the shear forces interacting on the surfaces of this incremental particle may be determined:

(Viscous Force) = f_v

$$f_v = \left[\left(\tau + \frac{\partial \tau}{\partial z} dz\right) - \tau\right] dxdy$$

$$= \frac{\partial \tau}{\partial z} \, dxdydz$$

Viscosity (μ) has been previously defined as the ratio of shear stress to (τ) to rate-of-shear strain. Therefore (referring to Figure 6):

$$\frac{\partial \tau}{\partial z} = \mu \frac{\partial}{\partial z}\left(\frac{\partial v}{\partial z}\right) = \mu \frac{\partial^2 v}{\partial z^2}$$

$$\therefore \; f_v = \mu \frac{\partial}{\partial z}\left(\frac{\partial v}{\partial z}\right) dxdydz \qquad (19)$$

The unit (pressure - p_i) force required to overcome the inertial effects of this fluid element is:

$$p_i = \frac{f_i}{A} = \frac{\rho v dxdydz}{dxdz} \frac{\partial v}{\partial y} = \rho v dy \frac{\partial v}{\partial y} \propto \rho V^2 \qquad (20)$$

The unit force (pressure - p_v) to overcome the resistance to fluid motion caused by shear stress is:

$$p_v = \frac{f_v}{A} = \frac{\mu \frac{\partial}{\partial z}\left(\frac{\partial v}{\partial z}\right) dxdydz}{dxdy} = \mu \frac{\partial}{\partial z}\left(\frac{\partial v}{\partial z}\right) dz \propto \frac{\mu V}{d} \qquad (21)$$

where V-ft./sec., is the absolute velocity, μ is the fluid viscosity and (d) is a linear dimension associated with the body of fluid such as boundary dimension location radially with respect to a center point of interest, etc.

Reynolds number is a ratio of inertia forces to viscous forces. It is a dimensionless number.

$$\text{Reynolds number } (R_e) = \frac{\rho V^2}{\frac{\mu V}{d}} = \frac{\rho V d}{\mu} \qquad (22)$$

As Reynolds number increases in value, inertia forces become predominant and, as a matter of fact, are the controlling forces associated with turbulent fluid motion.

2. Irrotational and Rotational Motion

In the preceding section some measures that help to predict the behavior of fluid have been developed. Since there are forces involved that cause the fluid to behave in a certain manner, it should be anticipated that a clear understanding of this fluid behavior is important to the performance of turbo-machinery.

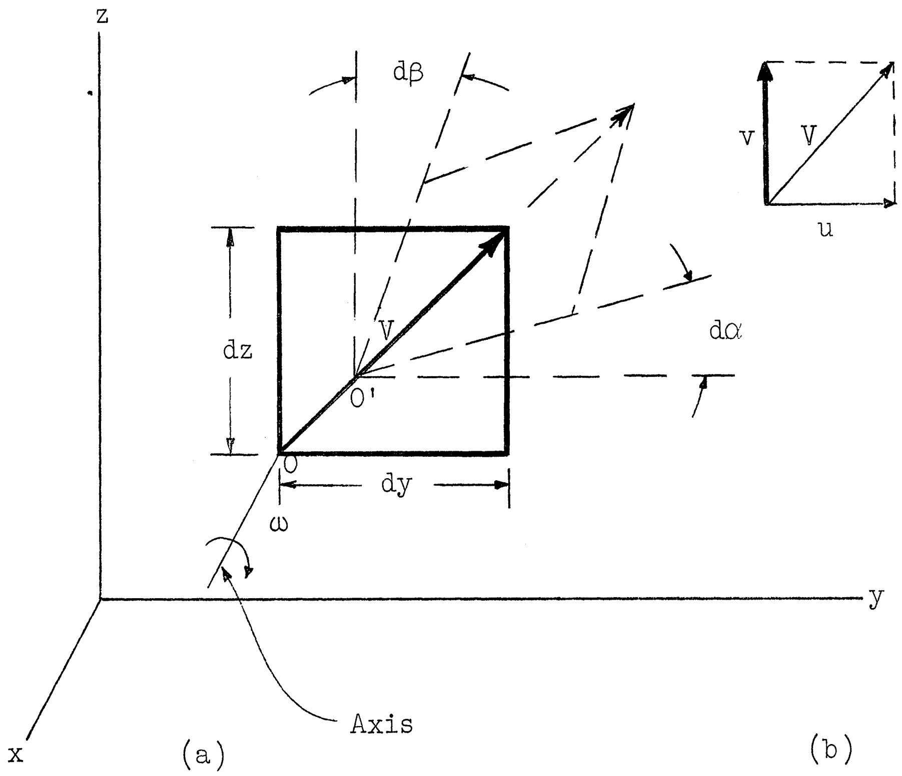


Figure 7.

Figure 7 (a) is developed from Figure 6 of the fluid element onto which certain forces were applied. Only the profile of the particle in the z-y plane is considered in Figure 7 (a);

however, a similar discussion of the particles deformation applied simultaneously to both the x-y and the x-y planes.

If the isolated particle is a part of a body of fluid in motion one of three deformations and consequently its orientation can occur. In each of the three cases the particle is assumed to move forward in a time, dt, from its initial position designated by (0) on the lower left hand corner to a new position (0') in the period of time under discussion. In each of the three cases the diagonal on the face of the particle extended from lower left to the upper right hand corner can be assumed to be the direction and velocity, V, that describes the fluid motion. The three cases are:

a) **The particle in moving from position (0) to (0') may do so with no deformation in which case (d) and (dβ) = 0,** the relationship of sides of the particle remains unchanged, and the magnitude and direction of the diagonal (V) remain unchanged. The indicated rotation, ω, around its axis parallel to the x-axis = 0.

b) The particle in moving from position (0) to (0') can deform in such a manner that dβ in the clockwise direction is equal to dα in the counterclockwise direction, in other words dα = -dβ. In this case the diagonal will remain in the same direction as when the particle was in position (0). **There is no rotation of the vector, V, about the axis parallel to the x-axis:**

The fluid motion for cases (1) and (2) is described as having an irrotational motion.

c) The particle in moving from position (0) to (0') can deform in a manner that value of dβ in both magnitude and direction can be different than dα. In this case the direction of the vector, V, cannot remain oriented to the same direction as it was in the initial (0) position.

In a descriptive manner, one can visualize that it rotates as does the hand of a clock. The name of this angular motion is vorticity and is assigned value of 2ω, twice the average angular motion of the vector, V, about the axis of the particle (0). The fluid motion resulting from this angular motion is said to be rotational.

A vector, V, may be resolved into components. In Figure 7 (b) the vector, V, can be resolved into two components, v, perpendicular to the x-axis and, w, perpendicular to the z-axis. Since the remainder of this discussion will be confined to a radial system of coordinates, the absolute velocity, V, will be resolved into a radial component, V_r, and a tangential component, V_u. Figure 8 is a fluid particle in a radial coordinate plane in which it is assumed that the particle has <u>unit depth</u> perpendicular to the plane of Figure 8.

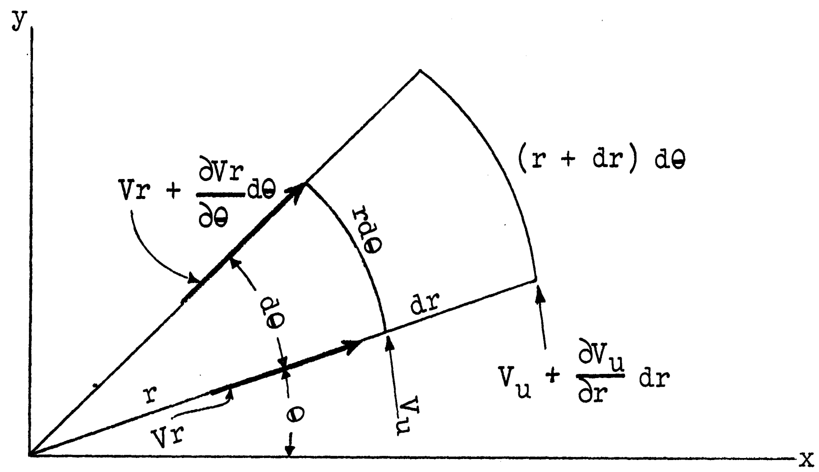


Figure 8.

3. Circulation

An initially abstract idea <u>Circulation</u> that will later have meaning is being introduced at this point. Circulation (K) is defined as the product of a velocity times the distance along the side of a fluid element in the direction of the velocity. In mathematical language:

$$K = \oint vds \quad \underline{\text{or}} \quad dK = vds \qquad (23)$$

where v is a velocity, ds is a distance in the direction of that velocity and the symbol $\oint$ is an indication that this product must be <u>summed</u> for <u>all</u> velocity vectors and distance products around the fluid element. This definition is applied to the radial element shown in Figure 8.

$$dK = V_r dr + \left[\left(V_u + \frac{\partial V_u}{\partial r} dr\right)\right](r + dr)\, d\theta - V_u r d\theta - \left(V_r + \frac{\partial V_r}{\partial \theta} d\theta\right)$$

After cancellation of terms

$$dK = \left[\left(V\, drd\theta + \frac{\partial V_u}{\partial r} rd\theta dr\right) - \frac{\partial V_r}{\partial \theta}\right]$$

$$\left\{\left[\frac{\partial}{\partial r}(V_u r)\right] - \frac{\partial V_r}{\partial \theta}\right\} drd\theta \qquad (24)$$

The area of the element shown in Figure 8 is $rdrd\theta$, therefore

$$dK = \frac{1}{r}\left\{\left[\frac{\partial}{\partial r}(V_u r)\right] - \frac{\partial V_r}{\partial \theta}\right\} rd\theta dr$$

$$\frac{dK}{rd\theta dr} = \frac{1}{r}\left\{\left[\frac{\partial}{\partial r}(V_u r)\right] - \frac{\partial V_r}{\partial \theta}\right\} \qquad (25)$$

It can be shown readily that the left hand side of Equation (24) is <u>vorticity</u> as discussed in Section 2-c and consequently

$$\frac{dK}{rd\theta dr} = 2\omega$$

where $|\omega| \geq 0$.

Therefore

$$\frac{1}{r}\left\{\left[\frac{\partial}{\partial r}(V\ r)\right] - \frac{\partial V_r}{\partial \theta}\right\} = |2\omega| \geq 0 \qquad (26)$$

If $\omega = 0$ (Zero Vorticity) then

$$\frac{\partial}{\partial r}(V_u r) - \frac{\partial V_r}{\partial \theta} = 0 \qquad (27)$$

4. The Fluid Vortex

a) Free Vortex

If either the first or second term in Equation (27) equals zero the remaining term is zero. The integration of either term alone is equal to a constant. These facts should be kept in mind for the following. Suppose that $V_r = 0$ and that $V_u \neq 0$ and, further, there are no additions or depletions of energy in a fluid flow, one element of which is shown in Figure 9.

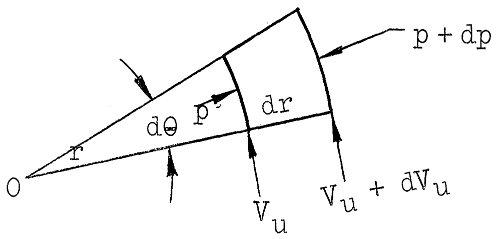


Figure 9.

In a flow of constant energy the results developed in Equation (12), Section I may be applied:

$$\left(\frac{p + dp}{\gamma}\right) 144 + \frac{(V_u + dV_u)^2}{2g} = \left(\frac{p}{\gamma}\right) 144 + \frac{V_u^2}{2g}$$

which is

$$+\left(\frac{dp}{\gamma}\right) 144 + \frac{V_u dV_u}{g} = 0 \qquad (28)$$

Further, the pressure rise across the element caused by centrifugal force of the mass on the surrounding fluid is:

$$\left(\frac{dp}{dr}\right) 144 = \frac{r}{g}\left(\frac{V_u^2}{r}\right)$$

or

$$\left(\frac{dp}{\gamma}\right) 144 = \frac{V_u^2}{g}\ \frac{dr}{r} \qquad (29)$$

Combined Equations (28) and (29) yield

$$\frac{dV_u}{V_u} + \frac{dr}{r} = \frac{d}{dr}\ (V_u r) = 0 \qquad (30)$$

or

$$V_u r = \text{Constant} \qquad (31)$$

From which it can be immediately stated that, in a free vortex motion of constant energy, the radial rate of change of angular momentum is zero and a condition of zero vorticity (see Equation 27) exists.

b) Forced Vortex

The solid line in Figure 10 is a plot of the equation $V_u r$ = constant. The exponent of (r) in Equation (31) is (1).

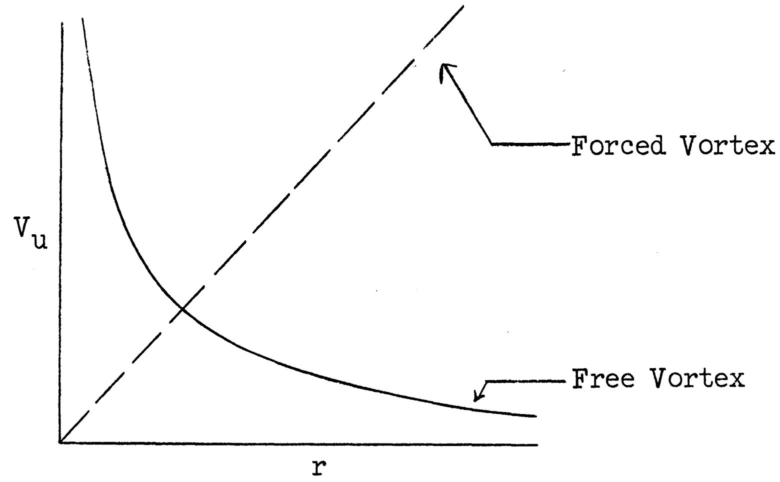


Figure 10.

There are many other exponents of (r) in a relationship between V_u and r. Each will describe a different pattern of flow.

$$V_u r^n = \text{constant}$$

A pattern of particular importance occurs when n = -1 yielding V_u/r = constant which is also plotted in dotted lines in Figure 10. This relationship is known as a <u>forced vortex</u> or <u>solid body rotation</u>.

SUMMARY

The above analysis concerning the behavior of fluids at first glance appears somewhat paradoxical. On one hand, the contention is held that a fluid is homogeneous, indicating at least, that there is molecular stability, if not, total uniformity. On the other hand, the mechanics of the analysis leads one to isolate a body of fluid surrounded by fluid of the same kind. The limiting process justifies the means, nevertheless, and, in turn, provides a gross understanding of the events in a pumping machine cycle which are of very great importance to the performance of the machine.

1. Viscosity (μ), density (ρ), and specific weight (γ), are all properties of a fluid which when introduced into certain combinations provide insight into the effects of interactions between the fluid and its surroundings. Reynolds number (Re) is a ratio of the only two active forces within a fluid body (pressure is considered inactive in that its effect is independent of the other forces), namely, inertia and visions forces. As will be noted later h_f is a function of Reynolds number, Re.

2. Although the significants of rotational and irrotational motion is not immediately evident, the relationship between Vorticity, Circulation, and Radial Rate - of - Change of Angular Momentum is important. Equation (26) provides the insight into the gross behavior of the fluid passing through a rotating blade system.

$$\frac{1}{r}\left\{\left[\frac{\partial}{\partial r}(V_u r)\right] - \frac{\partial V_r}{\partial \theta}\right. = |2\omega| \geq 0 \qquad (26)$$

3. The free and forced vortex are the theoretical extremes that might be expected in a pumping machine. The only ideal condition under which a free vortex may exist is; a) no addition of energy in a radial direction and, b) zero Vorticity ($|2\omega| = 0$). Since a radial pumping machine must add energy to the fluid (Equation 11) then the conditions for Circulation between the blades exist, Vorticity is to be expected and the angular distribution ($\partial V_r/\partial\theta$) of fluid flow with respect to the periphery of the rotor is almost certainly to be non-uniform

$$\left(\frac{\partial V_r}{\partial \theta} \neq 0\right)$$

III. THEORY AND PERFORMANCE OF TURBO-MACHINERY

A. Introduction

Although the theory of fluid flow behavior provides certain insight into the performance of pumping machinery a simple, straight forward approach through use of Newtons Laws of Motion applied to a fluid traversing a rotating blade system has formed the basis for initial design considerations. These considerations, the understandings of theoretical fluid behavior, and the associated Concepts of Similarity provide the bulk of the technology for impulse pumping machinery.

For the purpose of the following (one-dimensional) analysis of the stationary and rotating blade systems the following assumptions are made:

1. The fluid traversing the blade system is ideal, i.e., nonviscous.

2. The blades are infinitely thin in section and infinitely close together. Therefore the fluid is perfectly guided.

3. The fluid enters the vane system without shock, i.e., everywhere the fluid flows tangentially to the blade surface

B. Stationary Radial Blade System

Figure 11 is a radial blade assembly. Blade (a) above the horizontal center line is different than blade (b) below the horizontal center line, each illustrating a different set of blades for different assemblies will be accordingly analyzed.

Two items should be noted in Figure 11.

a) The absolute velocities (V) at entrance and exit from the blade section for both cases (a) and (b) are tangent to the blade surface.

b) All vanes throughout the periphery (assumption (2) above) are identical for cases (a) and (b).

Let

Q = fluid flow cu. ft./sec.
V = absolute velocity of fluid - ft./sec.
β = positive angle between tangent to circle and tangent to blade surface.
V_r = radial component of absolute velocity (V).
V_u = tangential component of absolute velocity (V).
1& 2 = subscripts to indicate entrance and exit, respectively, to rotor blade system.
F_t = tangential force acting perpendicular to radial line.
r = radius of point in fluid flow channel.
b = width of passage at any radius, r.

$$V_1 = \frac{Vr_1}{\text{Sin}\beta_1} \quad \text{and} \quad V_2 = \frac{Vr_2}{\text{Sin}\beta_2}$$

$$Vr_1 = \frac{Q}{2\pi r_1 b_1} \quad \text{and} \quad Vr_2 \quad \frac{Q}{2\pi r_2 b_2}$$

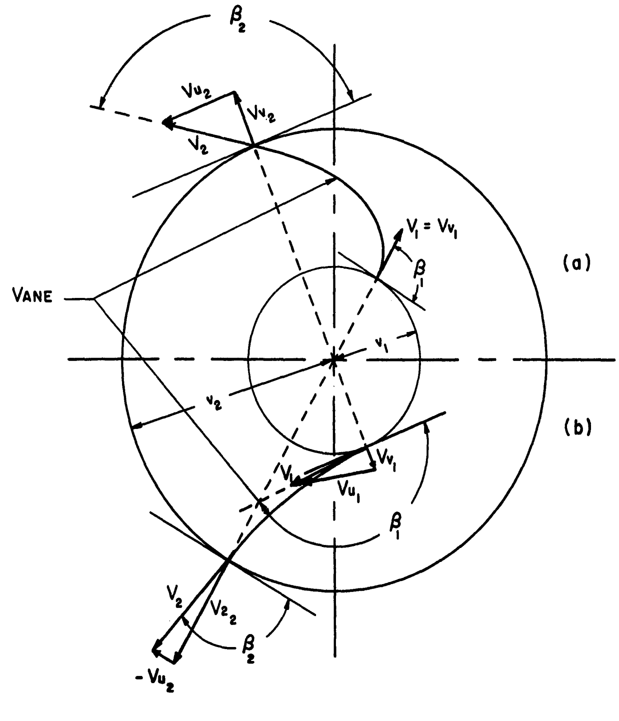


Figure 11.

For the ideal fluid (fractionless) the special form of the General Energy relationship (Equation 12) may be written as follows:

$$\left(\frac{p_2 - p_1}{\gamma}\right) 144 = \frac{V_1^2 - V_2^2}{2g}$$

$$\left(\frac{p_2 - p_1}{\gamma}\right) 144 = \frac{\left(\frac{Q}{2\pi}\right)^2}{2g} \left[\left(\frac{1}{r_1 b_1}\right)^2 \left(\frac{1}{\sin\beta_1}\right)^2 - \left(\frac{1}{r_2 b_2}\right)^2 \left(\frac{1}{\sin\beta_2}\right)^2\right] \quad (32)$$

Assuming Q, r_1, r_2, b_1, and b_2 are constant for cases (a) and (b), and the product rb = constant then

$$\left(\frac{p_2}{\gamma}\right) 144 = \left(\frac{p_1}{\gamma}\right) 144 + K \left[\left(\frac{C}{\sin\beta_1}\right)^2 - \left(\frac{C}{\sin\beta_2}\right)^2\right] \quad (33)$$

where K and C are constant.

If β_2 is an obtuse angle ($\beta_2 > 90^\circ$) and $\beta_1 = 90^\circ$ as shown in case (a) then $p_2 < p_1$ since $\sin \beta_2 < \sin \beta_1$. As a result, the absolute velocity (V_2) is greater than (V_1). The fluid has been accelerated with a resulting decrease in pressure. This is the action of a typical guide vane section to be found on a water turbine.

If $\beta_1 > \beta_2$ and β_1 is an obtuse angle as shown in case (b), Figure 11, then the $\sin \beta_2 > \sin \beta_1$ and $p_2 > p_1$. This is the action of the diffuser vanes on a diffuser type of centrifugal pump.

The acceleration of the fluid is caused by a force interaction between the fluid and the guiding vanes. From Figure 11

$$V_{u_1} = V_1 \sin\beta_1 \qquad \text{and} \qquad V_{r_1} = V_u \tan\beta_1$$

$$V_{u_2} = V_2 \sin\beta_2 \qquad \text{and} \qquad V_{r_1} = V_1 \tan\beta_1$$

and the magnitude of the force from Newton's Law is

F_t = Ma where the mass flow, M, in time, t,

is:

$$M = \left(\frac{\gamma Q}{g}\right) t$$

and

$$a = \frac{(V_{u_1} - V_{u_1})}{t}$$

Therefore

$$F_t = \frac{\gamma Q}{g}\, dt\, \frac{(V_{u_2} - V_{u_1})}{t}$$

$$= \frac{\gamma Q}{g} (V_{u_2} - V_{u_1}) \qquad (34)$$

Then from above:

$$F_t = \frac{\gamma Q}{g} \left[\left(\frac{Q}{2\pi r_2 b_2}\right)\left(\frac{1}{\tan\beta_2}\right) - \left(\frac{Q}{2\pi r_1 b_1}\right)\left(\frac{1}{\tan\beta_1}\right) \right]$$

$$= K\, (C_1 \cot\beta_2 - C_2 \cot\beta_1) \qquad (35)$$

The force interaction between the fluid and vane system produces a moment (T) on the vane system. This moment is frequently called shaft torque.

$$T = F_t r = \frac{\gamma Q}{g} (V_{u_2} r_2 - V_{u_1} r_1) \qquad (36)$$

C. Theoretical Performance of a Rotating Radial Blade System

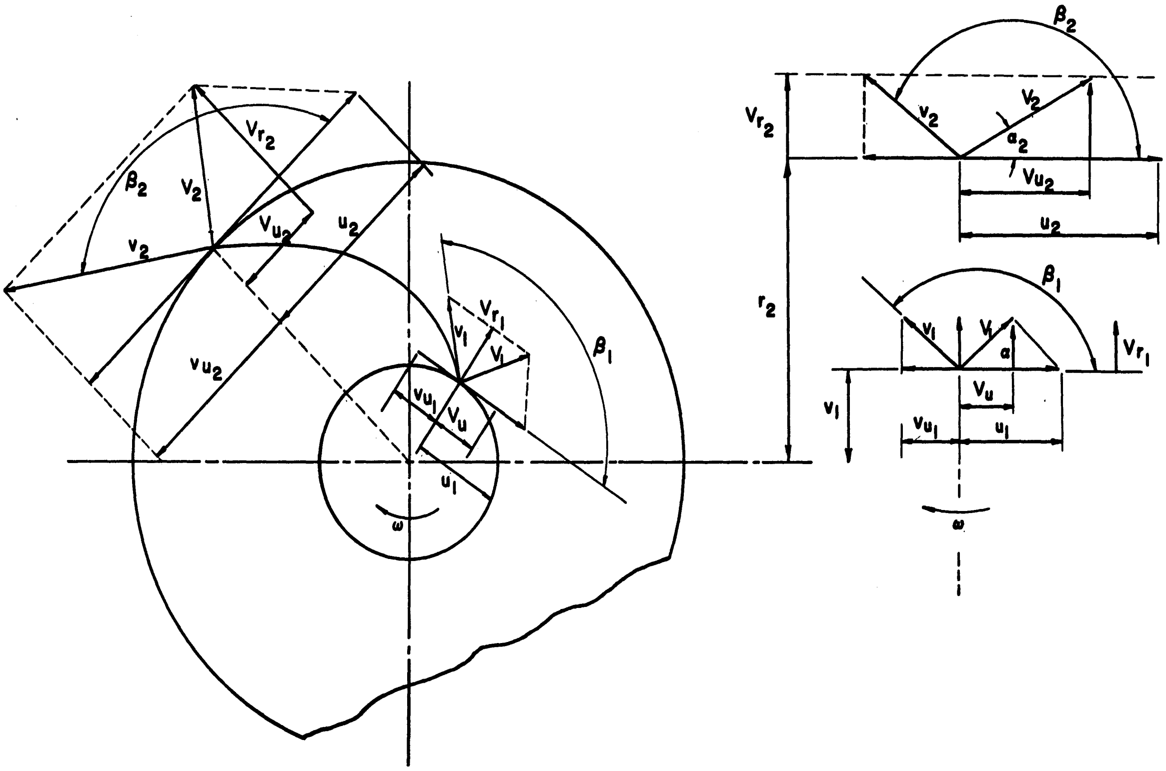
Figure 12 is a sketch showing one blade profile of a radial system, assuming the conditions for analysis as set-forth in the Introduction to Section III. The rotor is turning at a constant angular velocity, ω. The tangential velocity of the blade system at entrance is $u_1 = \omega r_1$ and at exit, $u_2 = \omega r_2$.

In a rotating vane system the relative velocity of fluid flow, v, is a component of the Absolute Velocity, V, its magnitude and direction being identically that of the absolute velocity of flow in the case of the stationary blade system. Then

$$v_{u_1} = v_1 \sin\beta_1 \quad \text{and} \quad V_{r_1} = v_{u_1} \tan\beta_1$$

$$v_{u_2} = v_2 \sin\beta_2 \quad \text{and} \quad V_{r_1} = v_{u_2} \tan\beta_2$$

where again $V_r = Q/2\pi rb$ at inlet and outlet respectively.



(a) (b)

Figure 12.

The absolute velocity, V, is the vector sum of the relative velocity, v, and the tangential velocity, u. In vector notation, $\vec{v} + \vec{u} = \vec{V}$. These vector sums are shown associated with the individual blade in Figure 12a and shown oriented to a common radius line in Figure 12b. The difference and the effect in magnitude of the vector, u, become more clearly displaced by the orientation.

The action of the rotating blade on the fluid causes an acceleration of the absolute velocity, V. The moment of that interacting force is

$$T = F_t r = \frac{\gamma Q}{g} (V_{u_2} r_2 - V_{u_1} r_1) \tag{36}$$

and the power (H.P.) required is:

$$H.P. = \frac{T\omega}{550} = \frac{\gamma Q}{550g} \left(V_{u_2} \omega r_2 - V_{u_1} \omega r_1 \right)$$

$$H.P. = \frac{\gamma Q}{550g} \left(V_{u_2} u_2 - V_{u_1} u_1 \right) \tag{37}$$

In pump applications power is frequently written:

$$H.P. = \frac{\gamma QH}{550}$$

where H is defined as head in ft.lbs/lb. of fluid being discharged by the machine and by Equation (37) is:

$$H = \frac{1}{g}\left(V_{u_2u_2} - V_{u_1u_1}\right)$$

or

$$H = \frac{\omega}{g}\left(V_{u_2r_2} - V_{u_1r_1}\right) \qquad (39)$$

The difference in absolute energy levels at entrance and exit must be equal to the energy (H) added to the fluid in its passage through the rotating blade system. Therefore,

$$\left(\frac{p_2 - p_1}{\gamma}\right)144 + \frac{V_2 - V_1^2}{2g} + Z_2 - Z_1 = \frac{1}{g}\left(V_{u_2u_2} - V_{u_1u_1}\right) \qquad (40)$$

Because of relative magnitudes of energy items and because of the center line reference, $Z_2 - Z_1 = 0$ is assumed. Equation (40) can be rearranged and developed to show some very important features in performance.

$$\left(\frac{p_2 - p_1}{\gamma}\right)144 + \left[\left(\frac{V_2^2 - 2u_2Vu_2}{2g}\right) - \left(\frac{V_1^2 - 2u_1Vu_1}{2g}\right)\right] = 0 \qquad (41)$$

Figure 12b and the Cosine Law yields

$$v_2^2 = V_2^2 + u_2^2 - 2u_2V_2\cos\alpha_2$$

$$v_1 = V_1 + u_1 - 2u_1V_1\cos\alpha_1$$

where α is the angle between the direction of the tangential velocity, u, and the absolute velocity vector, V. Certain substitutions may be made:

$$V_2 \cos\alpha_2 = V_{u_2}$$

$$V_1 \cos\alpha_1 = V_{u_1}$$

Then

$$v_2^{\,2} - u_2^{\,2} = V_2^{\,2} - 2_{u_2} V_{u_2}$$

$$v_1^2 - u_1^2 = V_1^2 - 2_{u_1} V_{u_1}$$

Therefore

$$\left(\frac{p_2 - p_1}{\gamma}\right) 144 = \frac{u_2^{\,2} - u_1^{\,2}}{2g} + \frac{v_1^{\,2} - v_2^{\,2}}{2g}$$

Further

$$v_2 \cos\beta_2 = v_{u_2} \qquad \text{or} \qquad v_2 = v_{u_2} \sec\beta_2$$

$$v_1 \cos\beta_1 = v_{u_1} \qquad \text{or} \qquad v_1 = v_{u_1} \sec\beta_1$$

and

$$\left(\frac{p_2 - p_1}{\gamma}\right) 144 = \frac{u_2^2 - u_1^2}{2g} + \frac{(v_{u_1} \sec\beta_1)^2 - (v_{u_2} \sec\beta_2)^2}{2g} \qquad (42)$$

Equation (42) indicates that the capability of a rotating blade system to develop a pressure increase is strongly dependent on, ω, and upon the difference between r_2 and r_1. ($u_2 = \omega r_2$) and ($u_1 = \omega r_1$). Further, the pressure rise is influenced by the rotor blade form since $0 \leqq \beta_1$ $\beta_2 \leqq 180^\circ$. For the special case where β_1 and $\beta_2 = 90^\circ$, the pressure rise is equivalent to that pressure corresponding to solid body rotation.

Equation (39) may be oriented into a second form of particular use in examining further the effect of the angles, β. From Figure 12b:

$$V_{u_2} = u_2 - v_{u_2}$$

$$v_{u_2} = \frac{V_{r_2}}{\tan\beta_2}$$

and

$$V_{u_1} = u_1 - v_{\omega 1}$$

$$v_{u_1} = \frac{V_{r_1}}{\tan\beta_1}$$

Substituting these values into Equation (39) yield

$$H = \frac{u_2^{\,2}}{g}\left(1 - \frac{V_{r_2}}{u_2 \tan\beta_2}\right) - \frac{u_1^{\,2}}{g}\left(1 - \frac{V_{r_1}}{u_1 \tan\beta_1}\right) \qquad (43)$$

Figure 13 is plotted to show the theoretical influences of β_2 upon the performance of a rotating blade system in solid line, assuming $\beta_1 = 90^{\circ}$.

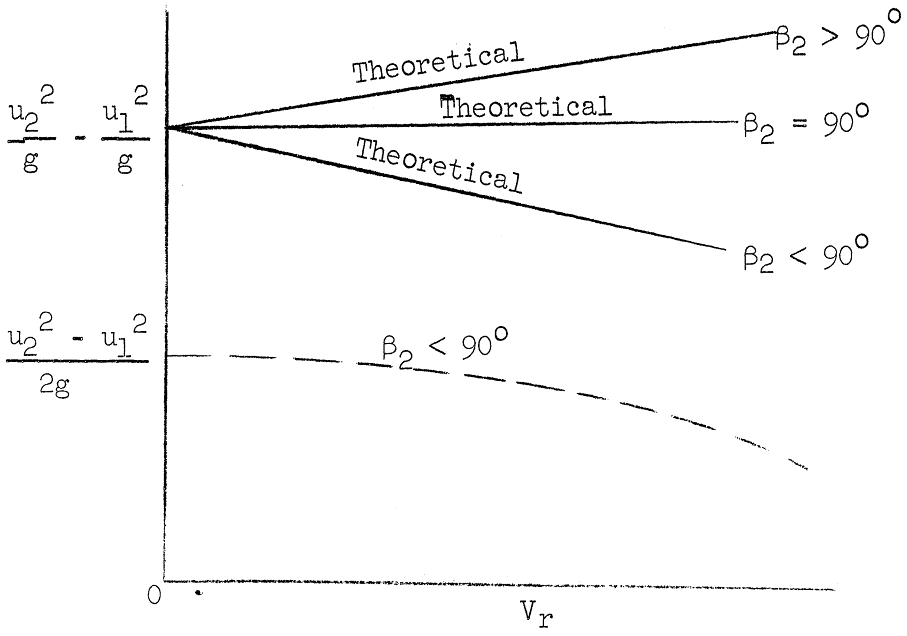


Figure 13.

D. Actual Performance of a Rotating Radial Blade System

The actual rotating radial blade system which is called an impeller in a centrifugal pump cannot be built to even closely approximate the assumption of an infinite number of blades, infinitely thin as initially assumed. It becomes necessary, therefore, to deal with several engineering considerations and to assay if possible, the effects of those considerations.

One of the initial limitations facing the engineer is the determination of the required area for fluid flow at the inner radius of the blade system. The minimum open area is that area of passage which will insure passage of that quantity of fluid for which the pump is designed.

Manufacturing methods have a substantial influence upon the number of blades that may be considered. The required practical thickness of blade as determined by the selected processing methods reduces the area for flow at the area of opening occupied by the number of blades of finite thickness. If the pump is designed to handle a fluid in which solids are suspended then the minimum area in any one passage must be large enough to allow the solid to pass through the rotor.

A third restriction on the number of blades in the rotor may be cost. The intricacies of manufacturing of a blade system is increased as the number of blades are increased. The problem of processing, cleaning, polishing and machining all contribute to increased costs of the more complex rotor.

The initial determination of the allowable number of blade in a rotor has direct influence on actual pump performance.

The function of a rotor of a pump is to add energy to the fluid. According to Equation (40) the energy forms are 1) (p_2-p_1/γ) 144 and 2) $(V_2^2-V_1^2/2g)$, namely flow-work (a force (pressure) moving a distance) and kinetic energy. This addition of energy from rotor action on the fluid results in a "change-of-whirl" $(V_{u2}u_2 - V_{u1}u_1)$. The pressure increase, item 1 above, is noted to depend to a major degree on the tangential velocity $(u = \omega r)$ of the rotor. At zero flow $(V_r = 0)$ the actual pressure rise across a pump is as shown by dashed lines on Figure 13 and is the exact rise one would experience in a forced vortex motion of a fluid body.

Item 2, above, is the kinetic component of the total energy added to the fluid which is likewise dependent for the magnitude on the tangential velocity, u. This may be quickly realized by examination of the summation of vectors as in Figure 12. In all cases the relatively high absolute velocity of flow at exit, V_2, requires special design considerations. The single volute and double volute casings have been designed and are found in use with moderate speed pumps producing modest pressure rises. The diffuser casing is in use where high pressure rises per stage are required and where high kinetic energy components would be expected. The efficient conversion of kinetic energy to equivalent flow-work is the objective in either case.

It is important to understand the fluid flow effects that the physical design requirement produce.

It will be recalled that considerable emphasis was placed upon the concept of "Vorticity," the analysis resulting in Equation (26) reproduced at this point.

$$\frac{1}{r}\left\{\left[\frac{\partial}{\partial r}(V_{ur})\right] - \frac{\partial V_r}{\partial \theta}\right\} = |2\omega| \geq 0 \qquad (26)$$

Further, it was shown that

$$\frac{d}{dr}(V_{ur}) = 0$$

could happen only if the fluid was in a region of constant energy. A pump rotor is in a region where energy is added according to Equation (39).

$$H = \frac{\omega}{g}(V_{u_2}r_2 - V_{u_1}r_1) \qquad (39)$$

or in differential form its value must be greater than zero. (Equation 44 below).

$$\frac{dH}{dr} = \frac{\omega}{g}\frac{d}{dr}(V_{ur}) > 0 \qquad (44)$$

Therefore, unless $\partial V_r/\partial\theta = \partial/\partial r\ (V_{ur})$(Equation 26) a condition exists for Vorticity in the spacing between rotor blades. A counter-current tendency as indicated in Figure 14 results and an accompanying deviation of flow from the theoretical at exit from the rotor occurs. The magnitude of the effect of Vorticity depends upon the spacing between the blades, the deviation from theoretical consideration increasing as the spacing accompanying a decreased number of vanes, become larger.

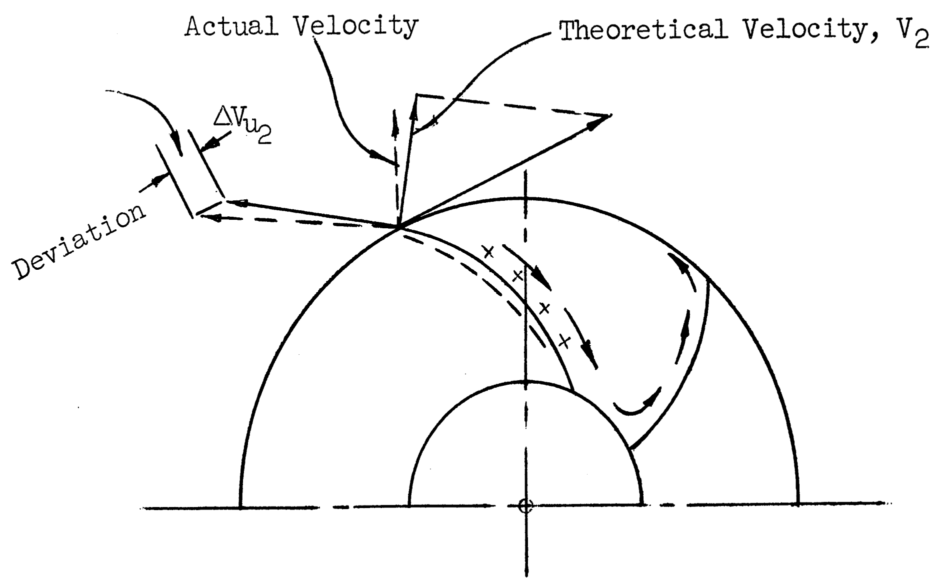


Figure 14.

E. Similarity Considerations for Pumping Machinery

As has been noted in the foregoing presentation several sets of variables have been important in establishing the performance of turbo-machinery. First, the geometric variables which establish the boundaries of fluid flow passages had a direct bearing on magnitude of the several energy items the sum of which determines the total energy stored in the fluid. Second, the kinematic variables such as V, v, and u, all directed quantities and all dependent on the passage configuration, were used in combination to establish machine performance. And, third, through the process of dynamic consideration both fluid characteristics and the characteristics performance of a machine have been determined, the former use leading to Reynolds number (Re) and the latter to Figure 13.

A very useful and reliable set of criteria known as the Affinity Laws of turbo-machinery are found by further extension of the use of the variables referred to above.

Geometric Similarity between two bladed rotors or casings is satisfied when a change in any one dimension of a design is accompanied by a proportional change of all other dimensions of the blade system and all angles remain constant.

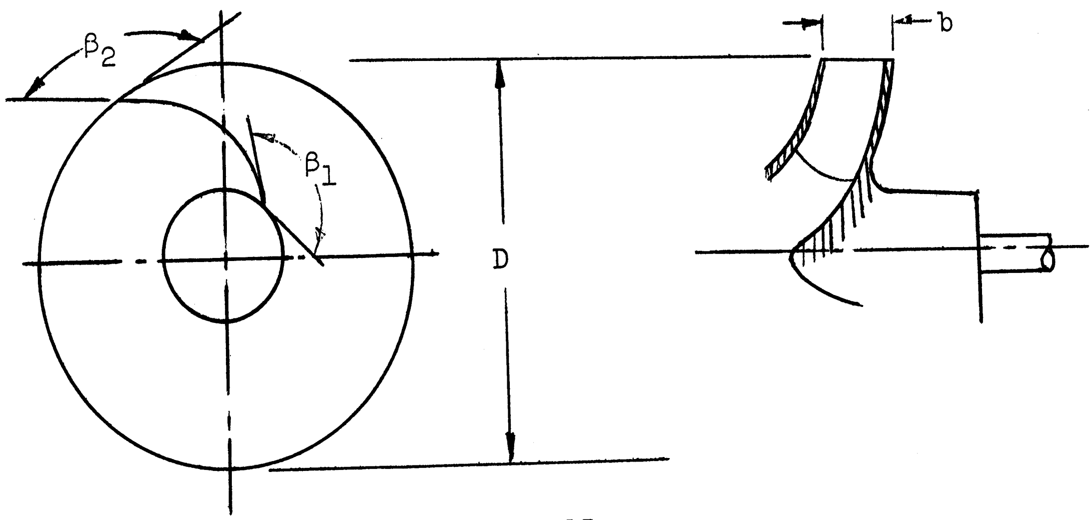


Figure 15.

From the condition for Geometric Similarity

$$b = k_1 D \quad \text{or} \quad k_1 = \frac{b}{D}$$

where k_1 is a proportional constant.

Kinematic Similarity is satisfied when any vector change in magnitude, not direction is accompanied by a like change in all other vectors. Figure 16 will illustrate this requirement.

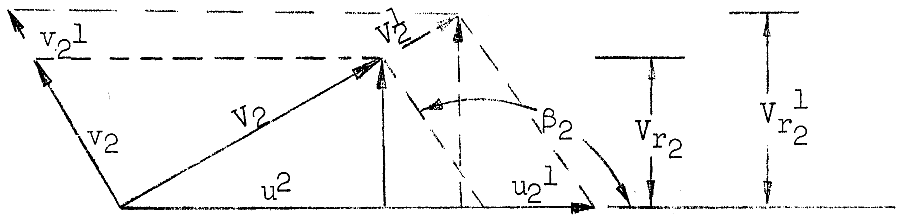


Figure 16.

For Kinematic Similarity

$$\frac{V_{r_2}}{u_2} = k_2 = \frac{V_{r_2}'}{u_2'}$$

where k_2 is a proportionally constant. Further,

$$b = k_1 D \quad \text{and} \quad V_{r_2} = \frac{Q}{k_1 \pi D^2} \quad \text{and} \quad u_2 = \frac{\pi D N}{60}$$

$$\frac{V_{r_2}}{u_2} = \frac{\frac{Q}{k_1 \pi D^2}}{\frac{\pi D N}{60}} = \left(\frac{60}{k_1 \pi^2}\right) \frac{Q}{ND^3} = k_2$$

From which the first of the Affinity Laws emerges.

$$\underline{\text{First Law}} \quad - \frac{Q}{ND^3} = \left(\frac{k\pi^2}{60}\right) k_2 = K_1 \qquad (45)$$

Dynamic Similarity is satisfied when the interacting forces between the rotor and the fluid varies in a manner proportional to the changes resulting from Kinematic Similarity.

The gross energy added to a fluid by a rotating blade system is:

$$H = \frac{u_2^2}{g}\left(1 - \frac{V_{r2}}{u_2 \tan\beta_2}\right) - \frac{u_1^2}{g}\left(1 - \frac{V_{r1}}{u_1 \tan\beta_1}\right) \qquad (43)$$

For Kinematic Similarity

$$\frac{V_{r1}}{u_1} = \frac{V_{r2}}{u_2} = k_2$$

For Geometric Similarity β_1 and β_2 remain constant. Therefore

$$H = k_4 u^2 = k_4 \left(\frac{\pi DN}{60}\right)^2$$

$$\underline{\text{Second}}\ \underline{\text{Law}} - \frac{H}{N^2 D^2} = K_2 \qquad (46)$$

Power Similarity is satisfied when both Kinematic and Dynamic Similarity is satisifed and when the value of the fraction factor, f, in the Fanning formula

$$h_f = f \frac{L}{D} \frac{V^2}{2g}$$

is independent of Reynolds number (Re).

The over-all efficiency of a pump (η_P) is defined as

$$\eta_P = \frac{\text{Energy added - Losses in Functional Resistance}}{\text{Energy Added}}$$

The power required to drive a pump is:

$$BHP = \frac{\gamma QH}{550\eta_P}$$

In the Fanning formula it will be noted that Geometric Similarity may be maintained by making

$$\frac{L}{D} = k_5 \text{ (constant)}$$

Further

$$V_r = k_2 u$$

$$V_r^2 = k_2^2 N^2 D^2$$

Then

$$\eta_P = \frac{K_2 N^2 D^2 - k_1^2 N^2 D^2}{K_2 N^2 D^2} = \frac{K_2 - k_1^2}{K_2} = k_4$$

Therefore,

$$BHP = \frac{\gamma}{550 k_4} N^3 D^5$$

$$\underline{\text{Third Law}} - \frac{BHP}{N^3 D^5} = K_4 \qquad (47)$$

The ratio of the two constants, K_1 and K_2 are of great importance in the technology of turbo-machinery. Both values may be raised to an exponential power without reducing their importance. Only the absolute value of the ratio is changed. Therefore,

$$\frac{(K_1)^{1/2}}{(K_2)^{3/4}} = \frac{\left(\frac{Q}{ND^3}\right)^{1/2}}{\left(\frac{H}{N^2D^2}\right)^{3/4}} = \frac{NQ^{1/2}}{H^{3/4}}$$

which is known as Specific Speed and is usually represented as:

$$N_S = \frac{NQ^{1/2}}{H^{3/4}} \qquad (48)$$

It will be noted that Specific Speed results from so maneuvering the Similarity Laws that the physical dimensions of a given machine is eliminated. The value of Specific Speed becomes an index type of number associated with the types of pumping equipment that will satisfy the capacity requirements of a fluid transport system. The index number concept is borne out by the frequently published chart reproduced in Figure 17.

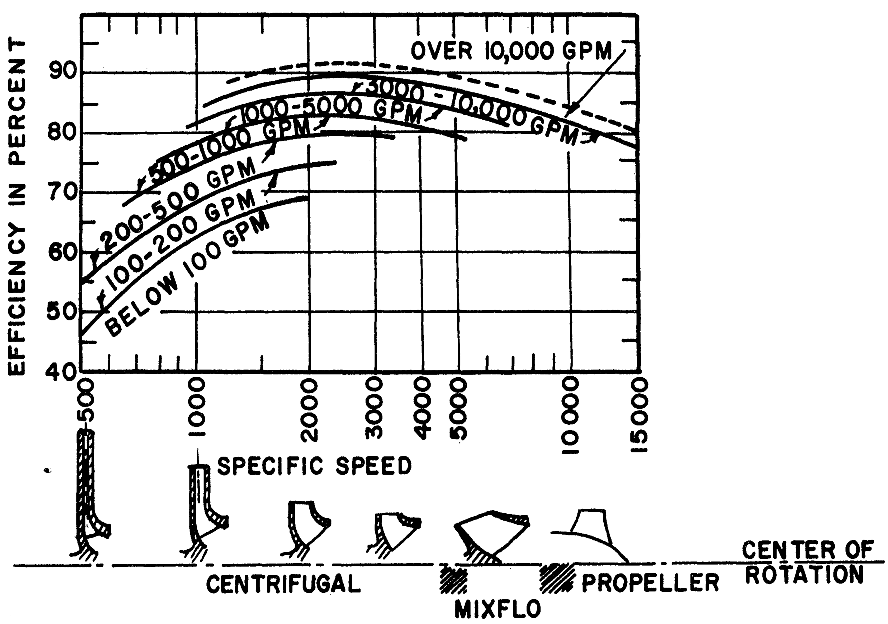


Figure 17.

Variation of Efficiency with Specific Speed for Various Sizes of Pump. (Courtesy of Worthington Corporation)

For complete similarity, it should be immediately apparent that similar fluid flow conditions must exist - in other words, Reynolds number

$$Re = \frac{\rho V D}{\mu}$$

and should remain constant or else there should be a logical explanation why the friction factor, f, as used above can be treated as being independent Re. It will be noted that from Reynolds number that

$$V \propto \frac{1}{D} \qquad (a)$$

while for Kinematic Similarity

$$\frac{V_r}{u} = \frac{V_r}{\pi DN} \qquad (b)$$

or

$$V_r \propto D \propto V$$

In case (a) V is inversely proportional to D and in case (b) V is directly proportional to D. The conditions as established for Similarity of Flow and Kinematic Similarity are incompatible. The use of the Affinity Laws are therefore confined to those regions in which resistance to flow of fluid through passages is independent of Reynolds number. Fortunately, but not without justification the Affinity Laws do provide a very powerful set of relationships in pumping machine technology.

F. Use of the Affinity Laws

The four relationships $Q/ND^3 = K_1$, $H/N^2D^2 = K_2$, $BHP/N^3D^5 = K_3$ and $N_S = NQ^{1/2}/H^{3/4}$ form a set of functions, the variable for which may be externally measured, of very great significants.

The first use of these has been previously indicated, namely, the cataloging of machine types based upon specific speed values (N_S). An extension of this concept to the inlet of a pumping machine is frequently found in the Net Positive Suction Head function

$$S = \frac{NQ^{1/2}}{H_{Sv}^{3/4}} \qquad (48)$$

where H_{Sv} = a value of energy level required at inlet to a pump of such total value that the liquid being pumped will not vaporize, thus causing a reduction in liquid capacity of the machine, possible damage to the rotor inlet, and in the extreme failure of the pump to perform. H_{Sv} is frequently given as follows

$$H_{Sv} = h_a \pm (h - h_v) \qquad (49)$$

where h_a = total potential energy of the atmosphere above the liquid source of supply, h_v is the equivalent head corresponding to the vapor pressure at the temperature of the liquid supply and h is the sum of

$$(\frac{p_1}{\gamma})\, 144 + \frac{V_1^2}{2g} + Z_1 \quad .$$

A second use of the Affinity Laws is the possibility that they provide to predict the capacity of a pump, handling a given liquid, at driving speeds N_1 different than the available capacity data. At constant diameter, D, the quantity of liquid delivered by the pump is proportional to the speed N as follows:

$$\frac{Q_2}{Q_1} = \frac{N_2}{N_1} \qquad \text{or} \qquad Q_2 = Q_1 \frac{N_1}{N_2}$$

and the energy delivered to the fluid by the rotor by that same pump of diameter (D) will vary as the speed ratio square:

$$\frac{H_2}{H_1} = \frac{{N_1}^2}{{N_2}^2} \qquad \text{or} \qquad H_2 = H_1 \frac{(N_1)^2}{(N_2)^2}$$

The result of the application of these functions is shown in Figure It should be borne in mind, further, that the efficiency of a unit is assumed constant for any condition of similarity.

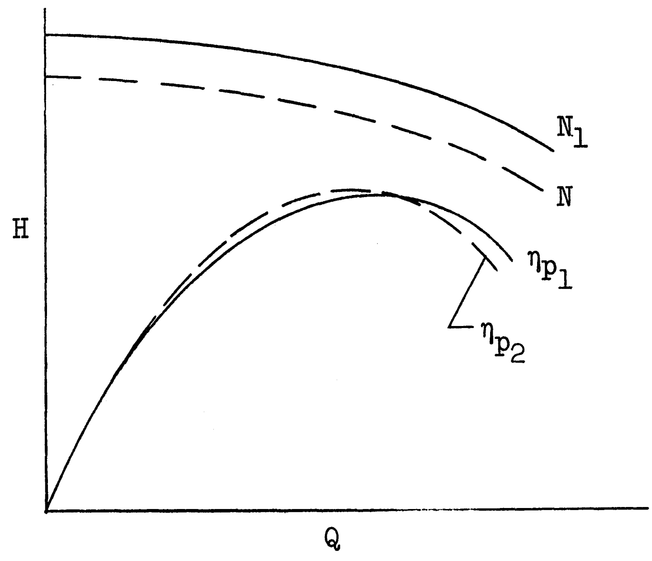


Figure 18.

A third use of the Affinity Laws is in the design considerations for a new size of pump. Based on the concepts of Geometric Similarity of design, predictions of capacity of the new machine from the known performance of an existing machine is possible. Conversely, when the capacity requirements are known it is equally possible to predict, with judgment, the required size of the machine.

IV. CAPACITY OF A PUMPING SYSTEM

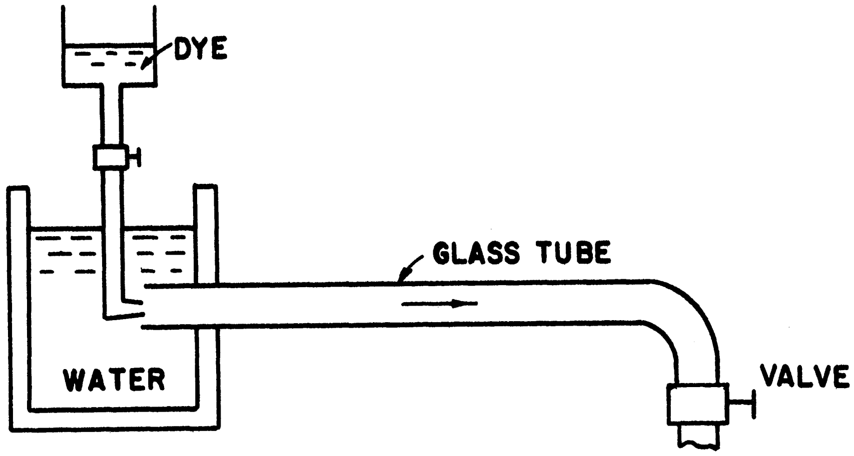
Introduction

The most important determination to be made in the consideration of any combination fluid system to transport fluid is the net work that must be done on the system to accomplish the entended result. Since any such system is subject to losses, it follows that a method of calculating those losses is necessary. In addition, methods of determining the level of the separate energy items such as flow work, potential and kinetic energy must be established. Obviously, the output energy of the internal system must be in those forms required by the external system if a pumping system is to function.

A. Hydraulic Losses

The determination of the mechanism of losses and the conduct of the theoretical and experimental studies of fluid flow have extended over many years. The efforts of many men, whose abilities are established world-wide, have gone into this task. To attempt a detailed study of the mechanism alone would require more space than can be allotted to this paper. It is the present purpose to establish a relatively simple analytical procedure such that a uniform calculation will result.

The flow of a fluid in a conduit is known to behave in two distinct manners depending upon a relation of physical quantities which in combination are dimensionless. This fact was established by Osborne Reynolds in a series of experiments reported in 1883. His experimental apparatus was essentially in accordance with Figure 18 in which fluid was caused to flow from a relatively large undisturbed reservoir. Within the tube, a small outlet for a die was installed. Essentially it was found that for various diameters of pipe and various fluids that below a certain numerical value of $R = \rho VD/\mu$. R being called Reynolds number, the die streak in the fluid downstream of the outlet remained in a constant relative location with respect to the conduit walls. In a range of increased values of R, it was found that the die streak dispersed through the channel in an entirely unpredictable manner. Later experimental work has shown that the exact value of R to predict the change from laminar to turbulent flow varies mainly depending upon conditions under which the tests were conducted. For a constant test condition, however, the original finding that a ratio as described above could predict the condition of flow was correct.



The concept of the boundary layer was introduced by L. Prandtl.

Suppose that a viscous fluid approaches a flat submerged plate at a uniform distributed velocity, V, as pictured in Figure 20a. At an incremental distance to the right of the leading edge of the plate only the layer of fluid next to the surface of the plate is affected. The adjacent layer to that layer of fluid on the plate experiences a reaction due to a change of velocity from V to 0. As the fluid front progresses across the plate, a velocity gradient such as is sketched in Figure 20b, is established which is proportional to the shear stress developed between adjacent layers.

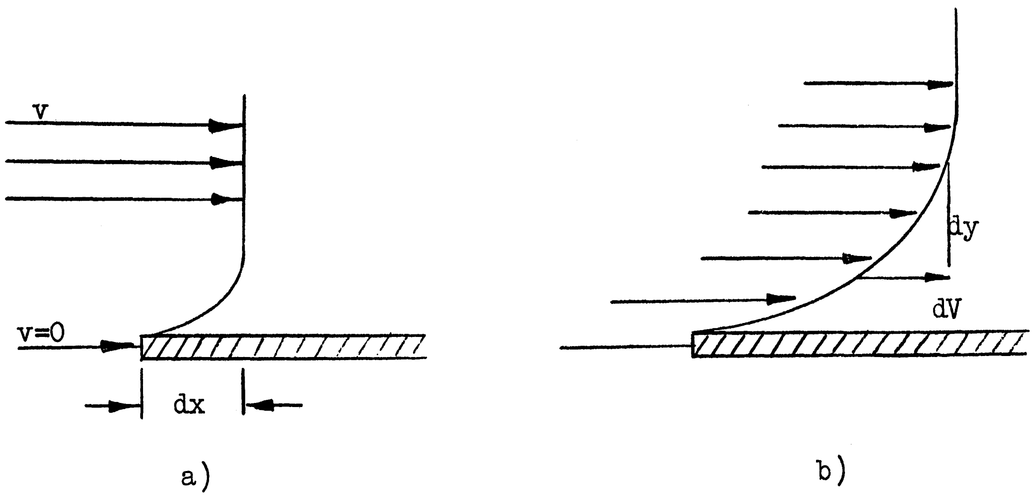


Figure 20.

It is apparent that this relation can hold only so long as the flow in the region of fluid space above the plate remains laminar in character.

Experimental work has established further information concerning the behavior of the boundary layer on the flat plate which can be reasonably used to establish flow conditions within a closed conduit. Assume a flat plate submerged in an extended fluid flow as is sketched in Figure 21a. At the leading edge of the plate, the boundary layer is essentially zero but increases in thickness, δ, as the fluid front progresses over the plate length. If the velocity is increased, a critical length, ℓ, from the leading edge is found beyond which the boundary layer changes from an ordered motion to a turbulent motion. Experimental work has established that ratio $PV\ell/\mu$ = constant predicts the point of change from laminar to turbulent flow (Figure 21b).

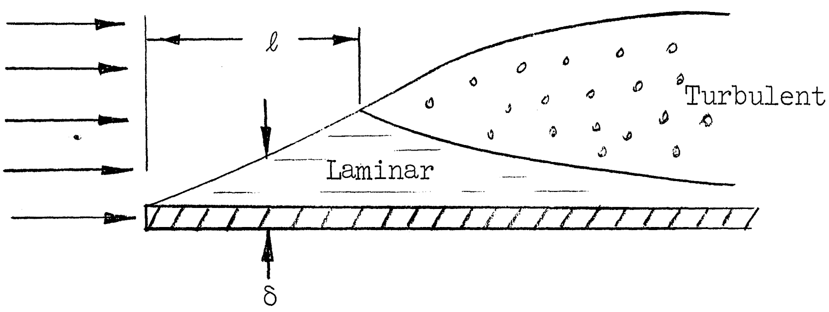


Figure 21a.

Subsequent work has established that there are additional zones in the process which can be identified. Figure 21b indicates the zones.

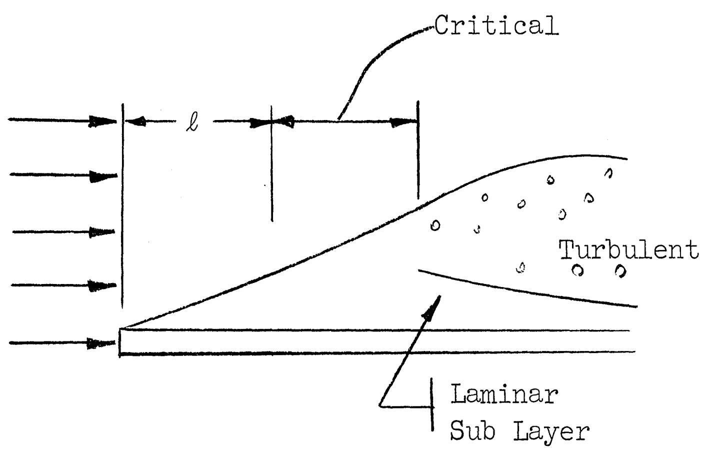


Figure 21b.

In the region between the laminar layer next to the plate at the leading edge and the turbulent layer beyond the critical length there is a transition zone in which turbulence is started by a type of oscillatory motion between the fluid layers. Within the turbulent region, a sub-layer has been identified which is laminar in character due to the approach to zero velocity at the surface of the plate.

If instead of a uniformly distributed fluid flowing over a flat plate it is directed into a tube, the entrance edge of which is well rounded, a velocity distribution and a boundary layer build-up similar to the flat plate is found. The velocity distribution as the fluid front progresses through the length of the tube progressively changes until a "developed flow" is reached where the velocity front is a stabilized pattern. The ratio of the length along the tube to where the flow is "developed" to the diameter of the tube, is a function of Reynolds number. If at the point where the velocity distribution is stabilized, the boundary layer has continued to build up in a laminar flow, the flow will be laminar and a parabolic velocity front will be developed. The pressure differential caused by such flow can be analytically predicted. If the boundary layer becomes turbulent as predicted by Reynolds number, before the point of "developed flow" is realized, the flow proceeding down the conduit will be turbulent. The result of such flow cannot be exactly predicted analytically. There are solutions for many special cases where simplifying assumptions can be made.

The physical phenomena resulting from turbulent flow were demonstrated in the dye-release experiment of Reynolds. Such a transfer of fluid across the fluid boundary must necessarily have resulted from a transfer of mass at right angles to the average flow direction. An exchange of momentum between successive layers of fluid was the result where a higher velocity mass particle from the inner stream filaments caused an acceleration of lower velocity particles in a layer closer to the boundary. In turn, the lower velocity mass particle due to continuity, must transfer to the higher velocity area, the net result being a complete random mixing of the fluid. In the transfer process, the velocity profile of the developed flow is more nearly uniform over a larger portion of the conduit passage than it was for the ordered laminar flow. This is illustrated by Figure 22.

A very high velocity gradient of the boundary layer accompanies the characteristic turbulent velocity profile since the layer of fluid next to the conduit wall has zero velocity. Within this boundary where a high velocity gradient exists a turbulent and a sublaminar flow may be found.

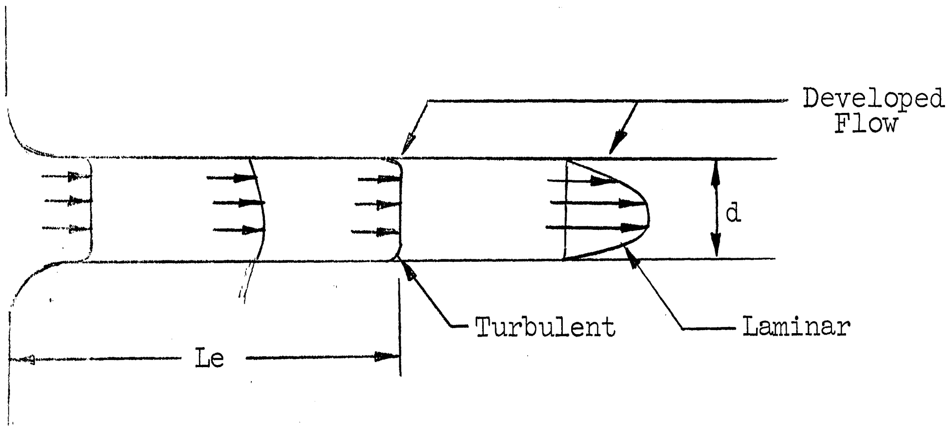


Figure 22.

It was proposed by Prandtl that it be considered that the total resistance for flow is developed within the boundary layer and that exterior to this layer the irreversibilities be treated as zero. Experimental results have substantiated this approximation.

With the exception of isolated cases, the engineer is concerned with turbulent flow, and because of this fact finds it necessary to depend to a considerable extent upon experimental results for his determination of system requirements. This certainly does not mean that the engineer is not benefitted by the large amount of effort that has been expended upon analytical procedures. Interpretations of experimental results must be made with full knowledge of the analytical techniques if correct correlations are made.

Based upon experimental evidence and analytical procedure, it becomes evident that the shear stress at the conduit walls, τ_o, in a fully developed flow in a closed conduit is a function of V, D, μ, ρ, and the roughness of the conduit wall which is given in terms of an absolute height, ϵ, of a projection above a line corresponding to a smooth wall. Without extending an analytical justification it can likely be seen and is being stated here that the friction coefficient, the values of which have been determined by the sum total of many investigators is

$$f = f\left(\frac{\epsilon}{D}, \frac{\rho V D}{\mu}\right) \qquad (47)$$

and the energy required to overcome the resistance to flow is given by the Fanning formula is

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \qquad (48)$$

where f = friction factor
V = average absolute velocity (ft/sec)
D = inside diameter of conduit (ft.)
L = length of conduit in ft.

and the energy equation for the length of pipe, L, folowing a real fluid is:

$$\frac{(P_2 - P_1)}{\gamma} 144 + \frac{V_2^2 - V_1^2}{2g} + Z_2 - Z_1 = f \frac{L}{D} \frac{V}{2g} \qquad (49)$$

B. Determination of the Friction Factor

Figure 23 is a chart prepared and presented by Mr. L. F. Moody, from numerous tests and data available to him to establish a relationship between the friction factor, f, Reynolds number, R, and the relative roughness of various new and clean pipes and conduits of circular cross section. The data specifically apply to steady, isothermal flow of a homogeneous fluid which is considered to be incompressible. This chart and others to be used will form a convenient presentation of the data for the engineer's use. Since Reynolds number includes fluid properties within its calculation, it can be readily seen that the chart can, within limits for which it was initially developed, be used to calculate the resistance offered to a wide variety of fluids in motion.

There are several reasons why a presentation such as Figure 23 cannot give to its user a high degree of accuracy in the determination of the friction factor, f. This is partially due to the more or less arbitrary procedure used in the determination of the relative roughness ratio, ϵ/D, where ϵ might be approximately defined as the height of a projection into the fluid stream from the nominal smooth inside diameter of the pipe. Other items such as the shape of the projection and the spacing of the projection have an effect on the relationship between the Reynolds number and the friction factor, f, such that an accuracy of $\pm$ 10 percent is usually predicted. For a smooth pipe where the ratio ϵ/D approaches zero, the probable accuracy of $\pm$ 5 percent might be considered.

For a Reynolds number less than 2000 it will be seen in Figure 23, the friction factor, f, has a value $f = 64/R$. Since f, for this zone is independent of roughness of conduit, it is reasonable to suppose that although projections from the pipe wall intefere with the flow in the boundary the changes in momentum of the fluid resulting from the interference cover the force of such small magnitude that the laminar flow is not disturbed.

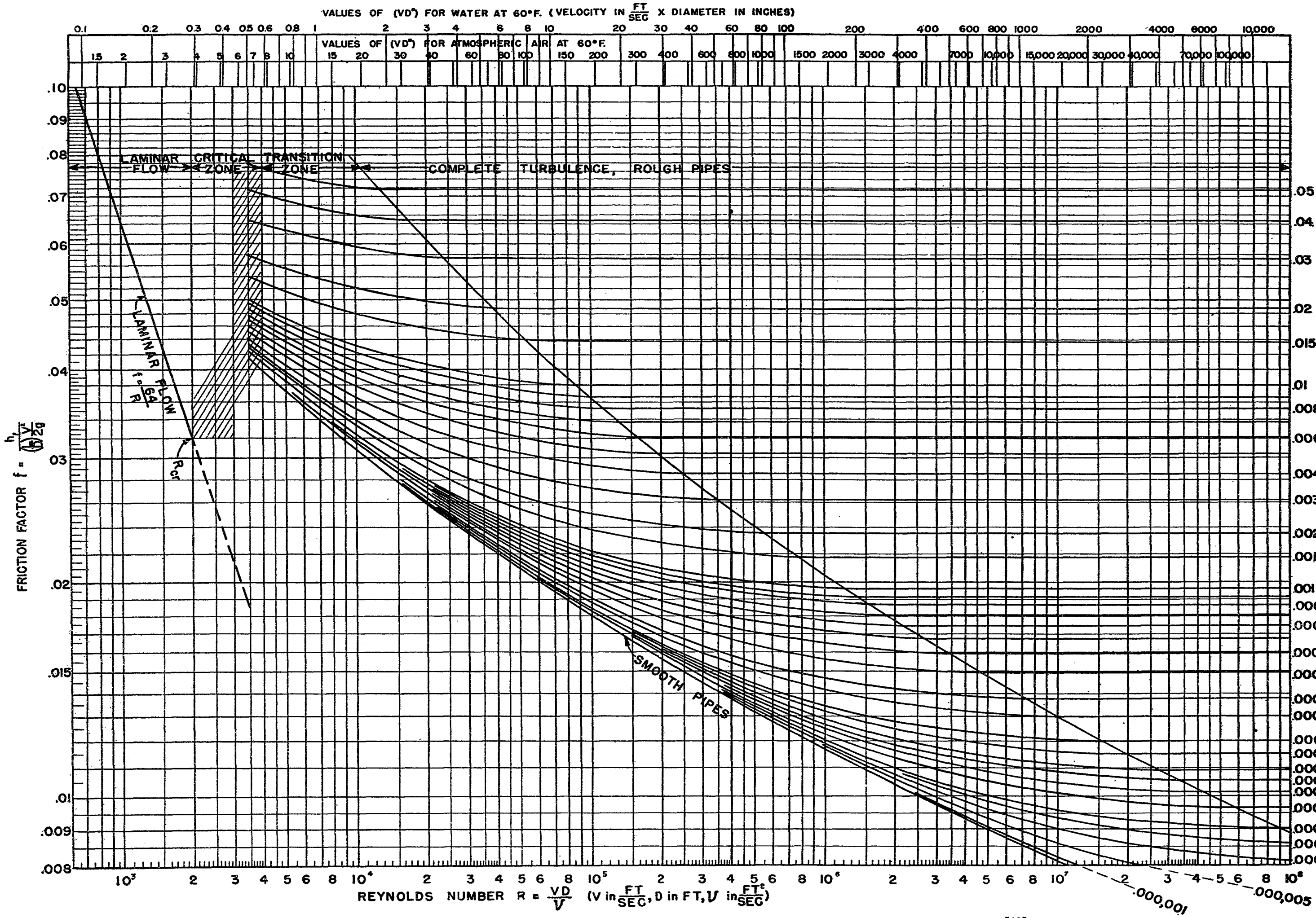


Figure 23. Relative Roughness Factors for New Clean Pipes. [18]

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There is a second region in values of Reynolds number between 2000 and 4000 in which either laminar or a degree of turbulence might be expected. The area of the chart is referred to as the critical zone. The uncertainty of the value of the fraction factor, f, in this region is indicated by the shaded area. Depending upon such influences as the entering of the fluid into the conduit, the exterior disturbances that are usually present to some degree, the flow may be laminar and independent of roughness, or turbulent.

Three additional regions at R greater than 4000 may be identified in Figure 23. Between the area of turbulent flow in the smooth pipe and the region where the value of f is practically independent of R, may be found a transition zone where the lines of constant relative roughness tend to converge to a value of f for a smooth pipe and also to a common value at the left hand side of the chart. The regions above the transition zone in the area of high relative roughness and below the line for a smooth pipe are of qualitative interest with respect to the momentum exchange of turbulent flow. In the region of high relative roughness, ϵ/D, it would be expected that the boundary layer would be turbulent because of the momentum changes due to roughness. The case of the smooth pipe would continue to exert influence. It is to be noted that for a smooth pipe the friction factor, f, is a function of R at least within the limits of the court.

Figures 24 and 25 are included to aid the solution of fluid-handling system problems. Figure 24 is a chart showing typical values for new clean commercial pipe. Since the relative roughness will vary because of method of manufacture and place of manufacture, the ratio ϵ/D should be considered carefully and accordingly. Figure 25 is a composite plot, the left hand side of which is a relation between kinematic viscosity, ρ, and the temperature of the various fluids. To the right of this chart and oriented to it is a plot of the VD product values, where V = the average velocity in feet per second, and D = the inside diameter of the pipe in inches. The value of Reynolds number required for Figure 23 can be read off the upper horizontal scale immediately above the intersection of the VD product line by the horizontal line connecting the fluid description with the constant VD product line. The combination of the three charts in Figures 23, 24, and 25 respectively, provides the engineer with information to establish the friction factor, f.

C. Energy Requirements of Systems

There are very few cases in the engineering application of fluid-flow systems where exact duplications can be found. Some piping systems

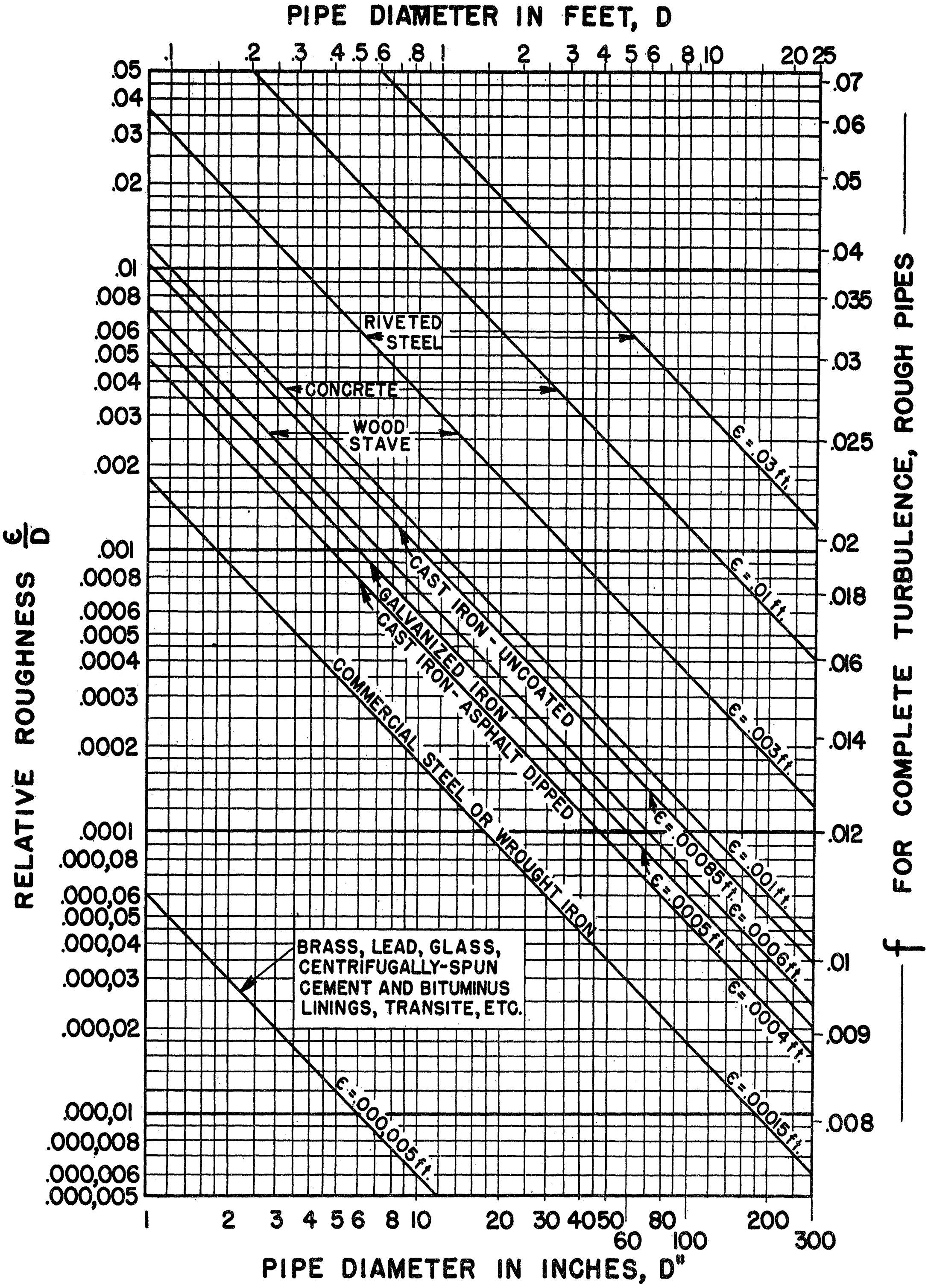

Figure 24. Relative Roughness Factors for New Clean Pipes.

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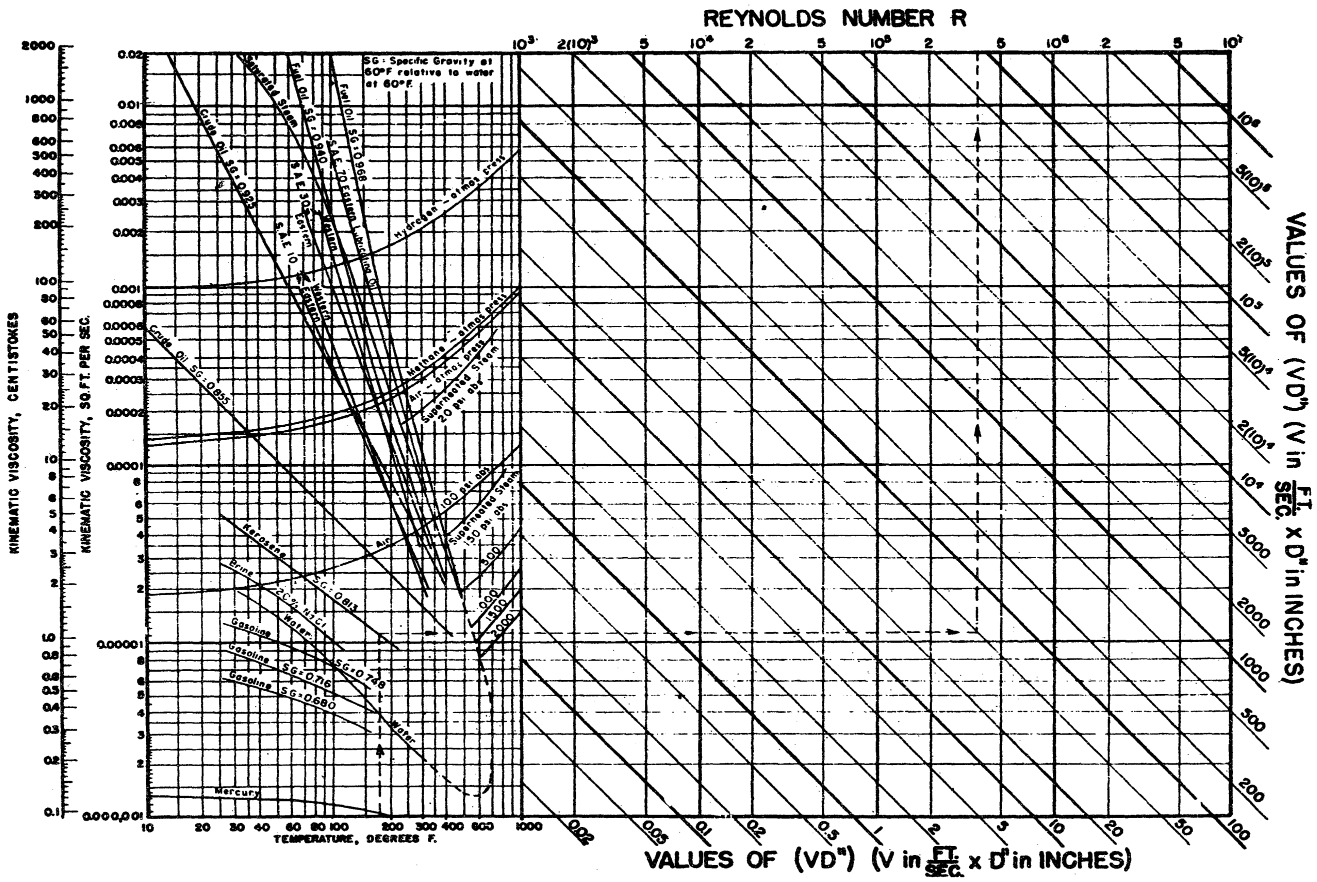

Figure 25. Kinematic Viscosity and Reynolds Number Chart.

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are very long, constant-diameter systems with few changes of section and gradual bends such as is found in cross-country pipe lines. The other extreme is the usual piping scheme found in industrial processes where fittings of all types in relatively close proximity are found. It is apparent that a friction factor, f, determined by an experimental determination in the region of developed flow could be applied with much more confidence in the first case than in the latter industrial system. Any fitting has a tendency to cause a disturbance downstream that will produce a distribution other than "developed flow" and will, to a limited extent, disturb the upstream flow to the fitting.

There are many equally successful empirical methods in use to establish the differential pressures which cause fluid to flow at a required rate. Certain methods seem to find favor in particular types of applications or in industries mainly due to the fact that engineers have found that that method or methods have given satisfactory results. Alternate methods are likely to be in favor in application of fluid systems in other industries.

There are many designs of valves and fittings, so many as to make impossible a detailed accounting of their individual characteristics. It is, however, possible to classify such items into groups. For instance, a valve can broadly be classified as a gate valve or a globe valve; fittings can be classified as branching, reducing, expanding, or deflecting. Any attempt to assign to these groups a value for the resistance to flow must necessarily be approximate. It has been found, however, that the error resulting from the assigning of values to the various groups, is nominally of sufficiently small magnitude to justify such procedures.

With the exception of the unusual designs, many laboratory experiments within the usual flow range have indicated that the pressure drop through valves and fittings caused by resistance offered by that fitting to fluid flow can be expressed

$$\Delta P = f(V^n) \qquad (45)$$

where n has a value very close to 2.0, which is usually used for engineering calculations. V is the average velocity of flow in feet per second leading to the fitting. Energy, unavailable for further use in the system because of the fitting or valve characteristics, is usually expressed in the following manner:

$$h_f = K \frac{V^2}{2g} \qquad (46)$$

The coefficient K is known as the valve or fitting coefficient. A similar value for orifices, changes of section of pipe, entrances and exits may also be developed. The determination of this coefficient is difficult due to the disturbance caused by the fitting in the connecting piping to either side of the fitting. Further, in the use of this coefficient, variations of the results sought through its use will vary depending upon the relative roughness of the pipe connected to the fitting and the spacing of one fitting relative to the second.

In any designed piping system the total length through which the fluid would pass would include both the length of pipe plus the sum of the lengths of all the required fittings in the system. Assume that a straight length of pipe as sketched in Figure 26 includes a fitting of length d. Assume further that there is more than the critical length of pipe to either side of the fitting, lengths a and b, such that the flow will be fully developed at sections corresponding to A, B, and C respectively. It would appear from intuition that p_1 is greater than p_2 even though c = a + b + d. This, of course, is the case and the difference in values between p_1 and p_2 would be directly assignable to the added resistance offered by the fitting of length d. In either case, the physical length of the fluid system from A to B is the same. It would be a comparatively simple matter to calculate the energy requirements for the system as sketched in Figure 26b using the system factor, f. Calculating a reliable value for the system as sketched in Figure 26a would be less accurate due to the undetermined portion of the length of the pipe, a and b, to which the fraction factor, f, could be applied.

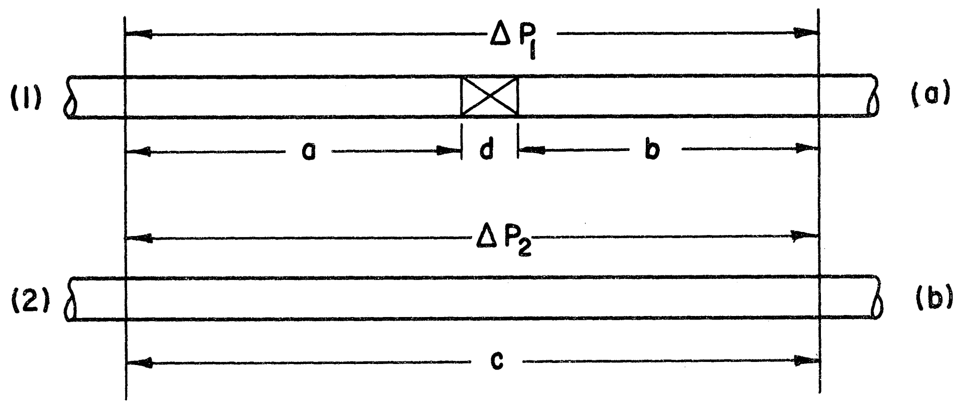

Figure 26.

Suppose that in an extension of the length of the pipe, c, as sketched in Figure 26b were made until the final length of the straight pipe of constant diameter, in which length the developed flow pattern remained constant, caused the same pressure drop at the same flow rate as occurred in Figure 26a. The resistance to flow caused by an added pipe length beyond the length of Figure 26b is obviously equal to the resistance to flow caused by the fitting in Figure 26a. Since the actual systems of Figure 26b and Figure 26a are the same physical length, it appears that the extended length of the piping proposed as an addition to the system of Figure 26b is the "equivalent" length of pipe added to the actual length of the system in which the length of fittings or valves is considered to be a length of pipe of a kind similar to that to which they were attached. This method of calculation is known as the "equivalent pipe length method."

Suppose that for a given value or fitting the energy required to overcome resistance to flow through that fitting could be given as follows:

$$h_{f_v} = K \frac{V^2}{2g} \tag{50}$$

The coefficient K which is known as the resistance coefficient, is a proportionality constant related to the particular type of fitting, valve, change of section, entrance, exit or pipe bend in a manner that resistance to flow of a fluid varies as the square of the velocity of flow through the particular section. The same energy is required to cause flow of fluid through a constant diameter of straight pipe and is given by

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \tag{51}$$

Equating Equation (50) with Equation (51) one obtains the following:

$$f \frac{L_e'}{D} \frac{V^2}{2g} = K \frac{V^2}{2g}$$

$$L_e' = D \frac{K}{f} \tag{52}$$

where L_e' is the equivalent length of pipe in feet which will require the same energy to overcome its resistance to flow as was required by fitting. If K is considered constant, then the equivalent length will vary inversely as the friction factor, f, which is a function of Reynolds number.

The value of the resistance coefficient K has been found to be essentially constant for a given type and size, for instance, of a valve. The value of the coefficient varies with size and to a degree for various designs of a given type and size. The variation of K in the latter case, that is design, an average technique produces results sufficiently accurate to justify the use of the pipe designation. Table I below is a representative list of average values of the coefficient K for various sections in common use.

TABLE I

VALUES OF THE COEFFICIENT K FOR VARIOUS FITTINGS

Fittings	K
Globe valve	10
Angle valve	5
Close return bend	2.2
Standard tee	1.8
Standard elbow	0.9
Medium sweep elbow	0.75
Long sweep elbow	0.60
45-degree elbow	0.42
Gate valve	0.19

In the actual physical system of length, L, which includes the length of all fittings and valves in the system, one must add an equivalent length L_e' of the same diameter as the pipe. The total equivalent length for the system of length L to which the fraction factor, f, may be applied is

$$L_e = L_e' + L \tag{53}$$

and the energy required for the piping system including the energy requirements of the piping, valves and fittings is given as follows:

$$h_f = f \frac{L_e}{D} \frac{V^2}{2g} \tag{54}$$

The resistance offered to the flow of fluid through the various types of flow section has been the subject of many experimental studies. The results are widely published.

D. Capacity in Parallel and Series Systems

Equation (49) may be re-written for a complete engineering system or any portion thereof as follows:

$$\left(\frac{p_2 - p_1}{\gamma}\right) 144 + \frac{V_2^{\,2} - V_1^{\,2}}{2g} + (Z_2 - Z_1) = h_{f_{2-1}} = f \frac{L_e}{D} \frac{V^2}{2g} \qquad (55)$$

Figure 27 is a sketch of a single length of pipe which is a part of a system and includes an medium sweep elbow of the same diameter as the pipe. The velocity of flow V in the entire length of pipe including the elbow for a given quantity, Q, is a function of the selection of the diameter, D

$$V = \frac{Q}{\frac{\pi D^2}{4}}$$

Since the pipe and fitting are the same diameter

$$\frac{V_2^{\,2} - V_1^{\,2}}{2g} = 0$$

Further, a length of pipe of diameter, D, that would cause the same loss as the medium sweep elbow as given by Equation (52). From Table I K = .75, f is given by Figure 23, and D is determined as related to the velocity, V. It is important to note that an engineering judgement or choice is needed at this point because of the dependence of two parameters. Equation (55) applied to Figure 27 may be written as follows:

$$\left\{ \left[\left(\left(\frac{p_2}{\gamma}\right) 144 + Z_2\right) \right] - \left[\left(\frac{p_1}{\gamma}\right) 144 + (-Z_1) \right] \right\} = f \frac{L_e}{D} \frac{V^2}{2g}$$

and if plotted as in Figure 28, the left hand side of the equation is an ordinate and the right hand side is an abscissa.

Figure 29 is a sketch of a complete pumping system. The system includes a closed source of supply and closed tanks at the points of discharge. It is important to understand the following:

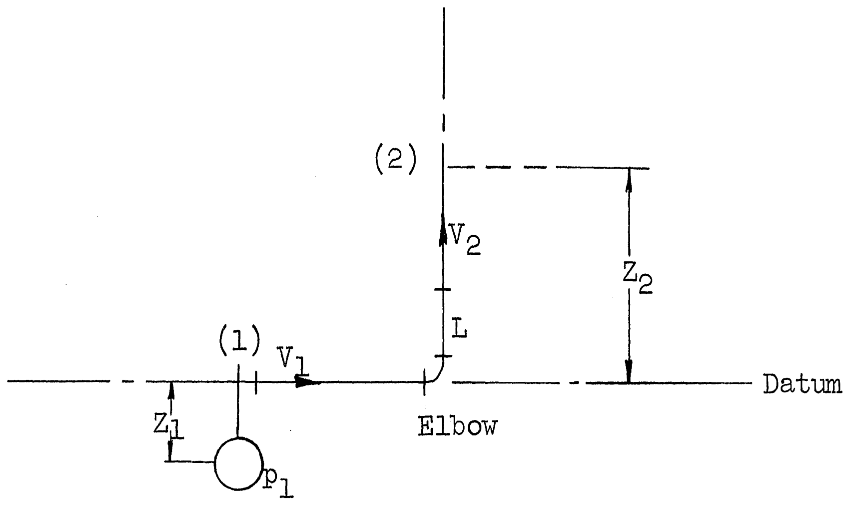

Figure 27.

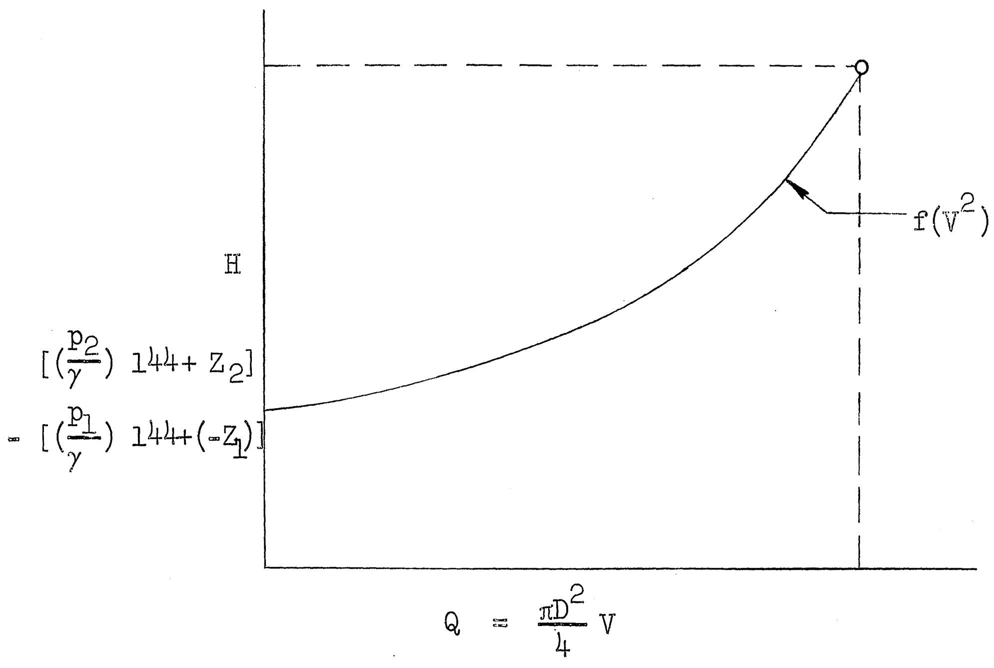

Figure 28.

1) The energy required to transport fluid to two or more portions of a series system is the sum of the energy requirement for the partial systems at constant quantities (Q) flowing through the series system.

For instance L_2 (the actual physical length) connecting points (2) and (7) is in series with two branching systems from point (7), L_3 and L_6.

2) At any point of entry to a parallel system the absolute energy level available to the parallel systems is constant for each system.

For instance point (7) is a junction of two parallel systems L_3 and L_6. Therefore the quantity of fluid flowing in each system will be a function of the energy requirements of each system at any given level of constant energy.

Finally the capacity of a pumping system may be found. Figure 30 is a plot which will determine the quantity distribution of fluid to the points of discharge (4), (5) and (6) based upon the characteristics of the pump (A).

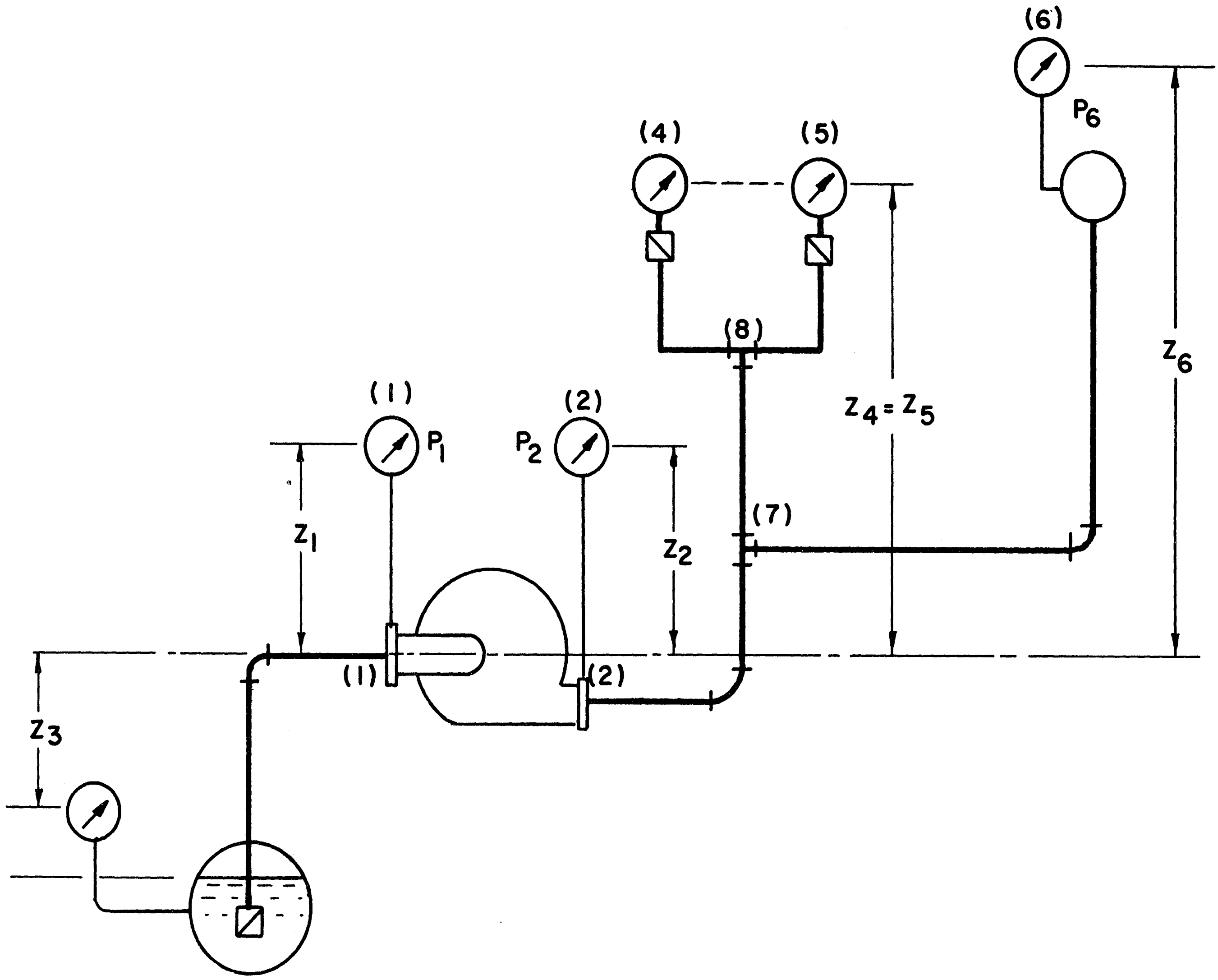

Figure 29.

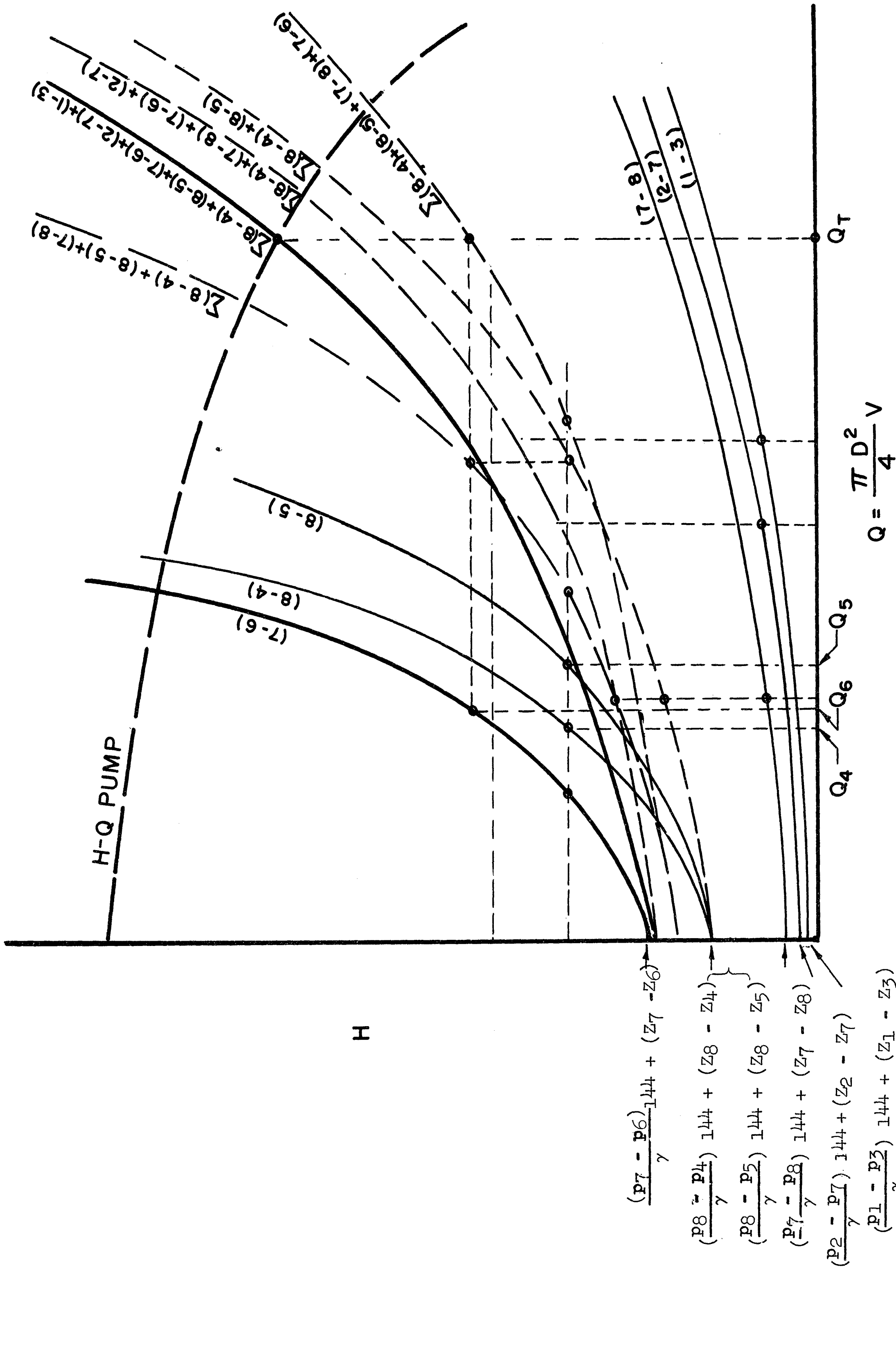
H-Q PUMP
H
Σ(8 - 4) + (8 - 5) + (7 - 8)
Σ(8 - 4)+(8-5)+(7-6)+(2-7)+(1-3)
Σ(8 - 4)+(7 - 8) + (7 - 6) + (2 - 7)
Σ(8 - 4) + (8 - 5)
Σ(8 - 4)+(8-5) + (7-8)+(7-6)
(8 - 5)
(8 - 4)
(7 - 6)
(7 - 8)
(2 - 7)
(1 - 3)
Q_T
Q_5
Q_6
Q_4
$Q = \frac{\pi D^2}{4} V$
$(\frac{p_7 - p_6}{\gamma})144 + (z_7 - z_6)$
$(\frac{p_8 - p_4}{\gamma})144 + (z_8 - z_4)$
$(\frac{p_8 - p_5}{\gamma})144 + (z_8 - z_5)$
$(\frac{p_7 - p_8}{\gamma})144 + (z_7 - z_8)$
$(\frac{p_2 - p_7}{\gamma})144 + (z_2 - z_7)$
$(\frac{p_1 - p_3}{\gamma})144 + (z_1 - z_3)$

Figure 30.